# 第三届粒子物理前沿研讨会

# Richness out of smallness: from Neutrino Collider to Muon Collider

22 July 2022

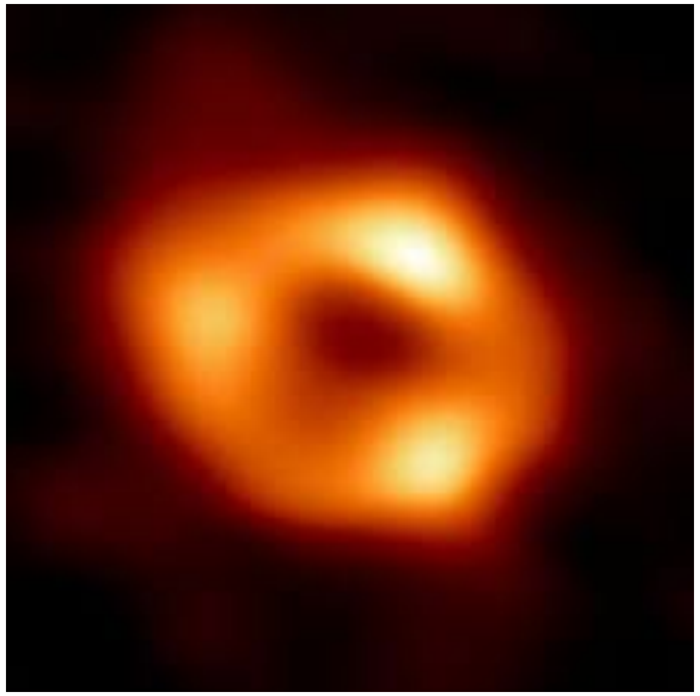
卢梦 中山大学

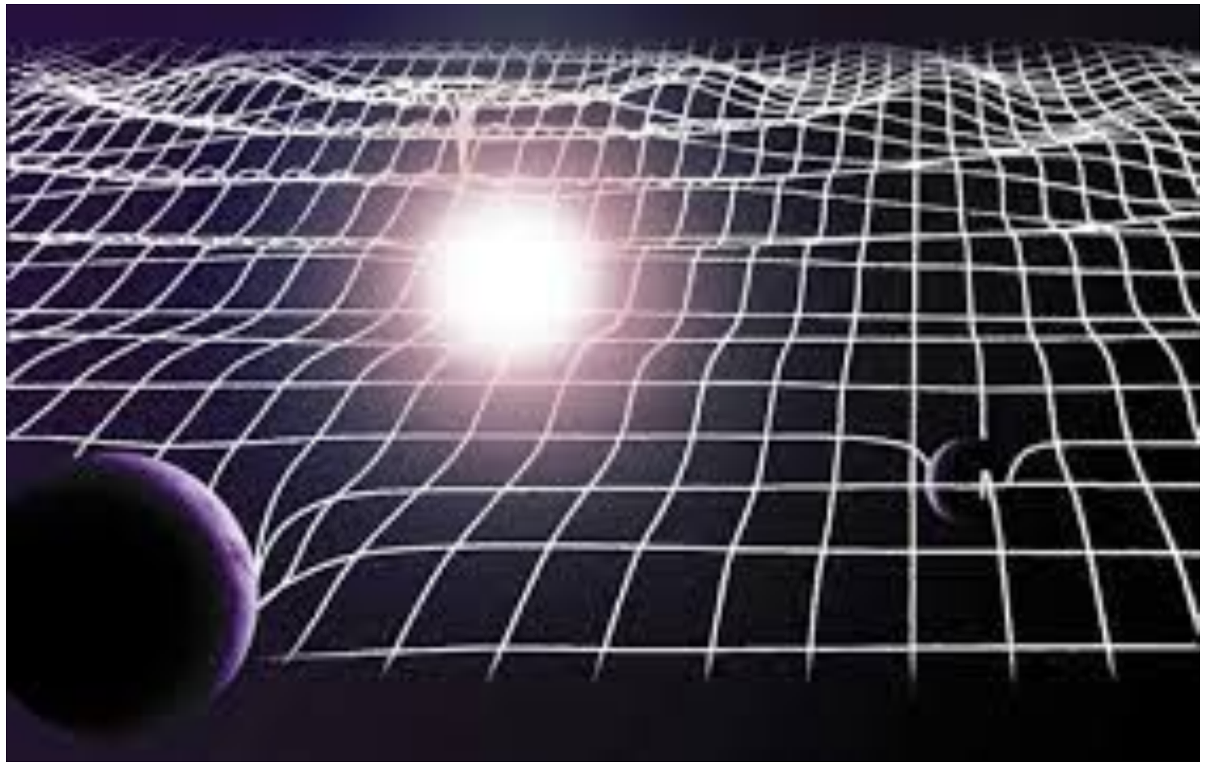
# Outline

- Prelude
- Introduction to HE-Colliders
- Muon Collider
- Neutrino beam
- Neutrino collision
- Electron-Muon Collider
- Summary

# Prelude

## Gifts from the sky:

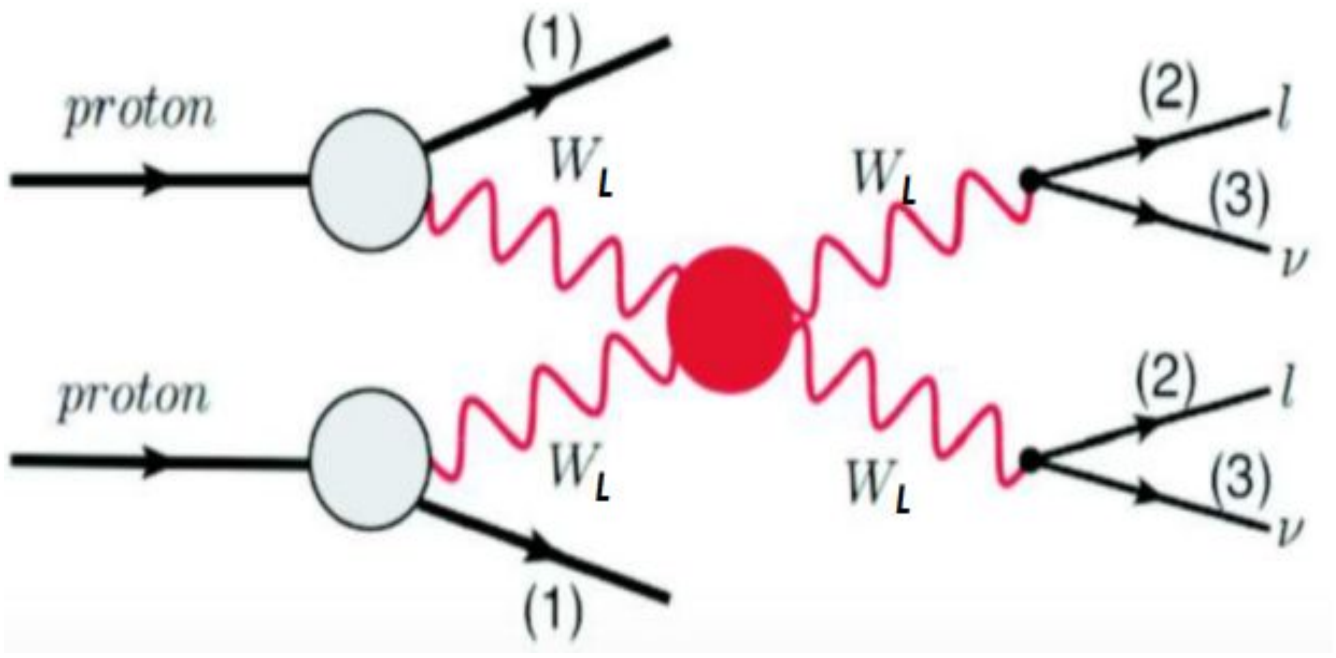


# Prelude

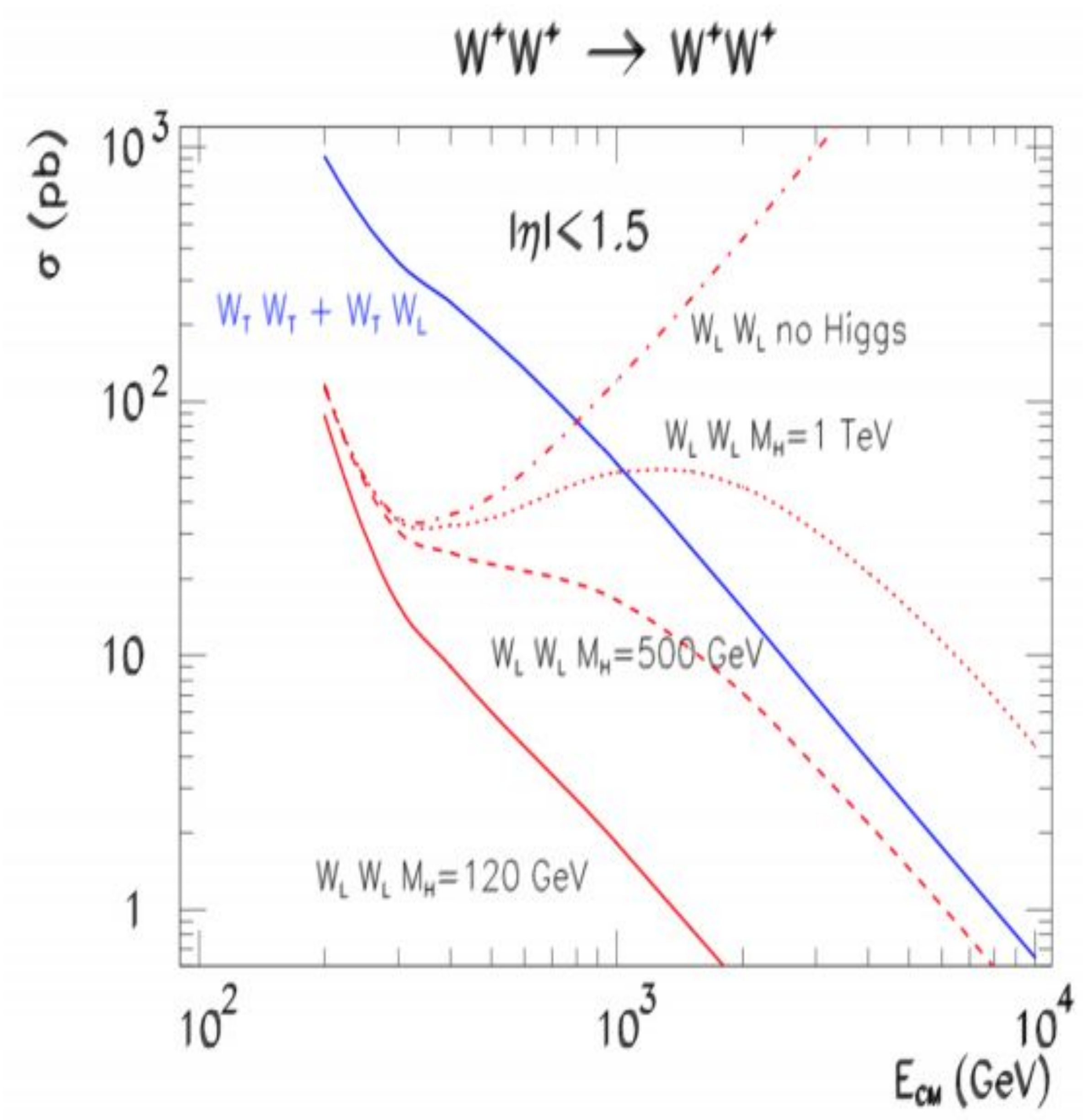
## Create rich physics from small: Collider

# Prelude: Vector Boson Scattering




- **杨米尔斯非阿贝尔相互作用**
  *反常耦合、有效理论EFT*
- **电弱对称性破缺**
  *希格斯幺正机制*
- **TeV能级的新物理**
  *Boosted喷注技术*

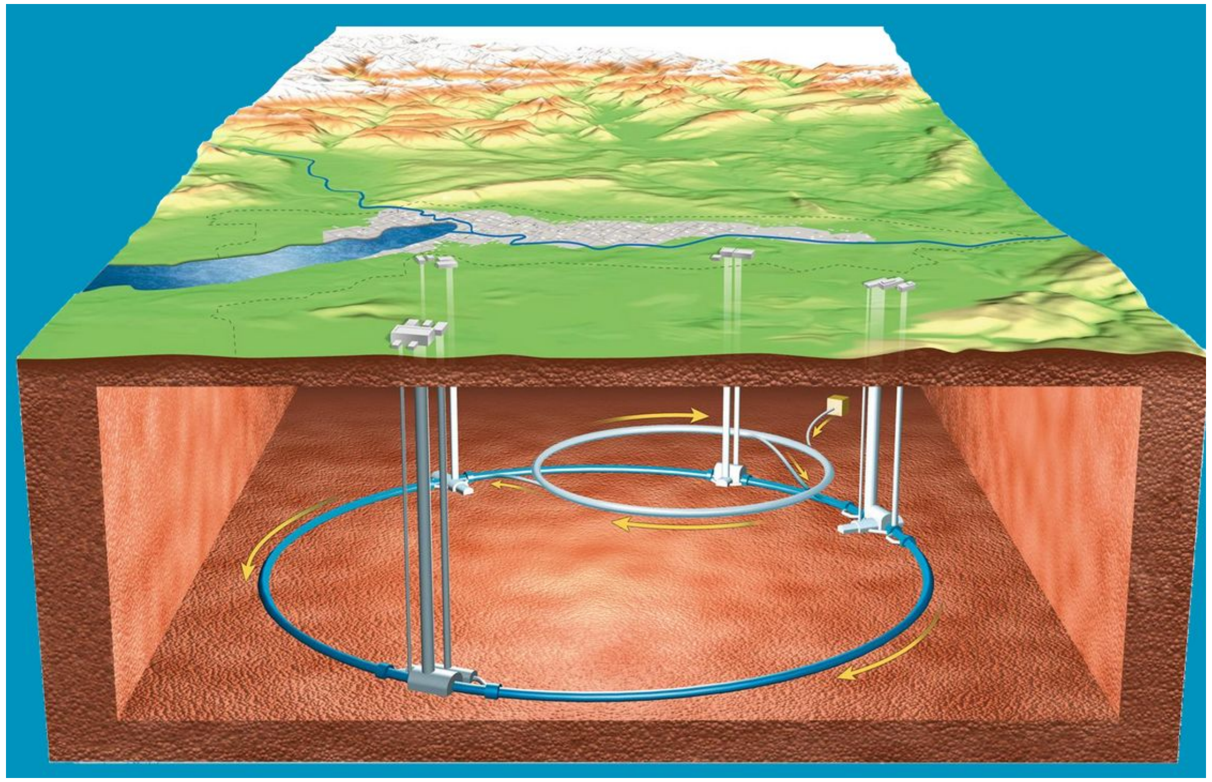
**同电荷W-W玻色子(极化)散射**
**~200(20) 事例 (2016-2018)!**




WW->WW behavior on scattering energy

# Introduction to HE-Colliders: LHC


**Large Hadron Collider: ALICE、ATLAS、CMS、LHCb**
**O(1000) person experiments**



## The Nobel Prize in Physics 2013

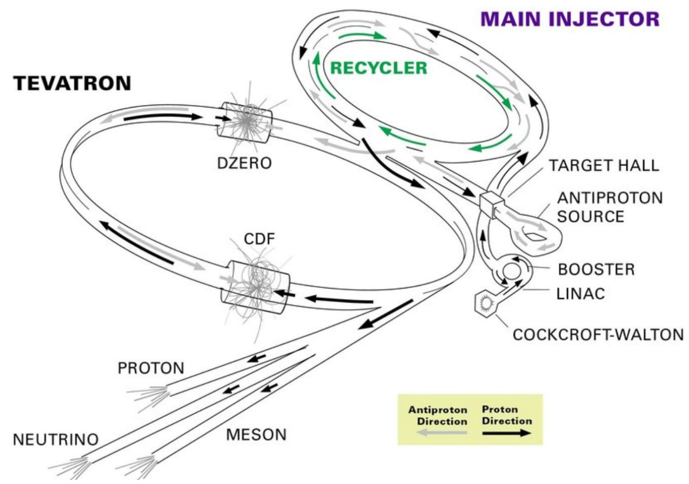
Photo: A. Mahmoud
François Englert
Prize share: 1/2

Photo: A. Mahmoud
Peter W. Higgs
Prize share: 1/2

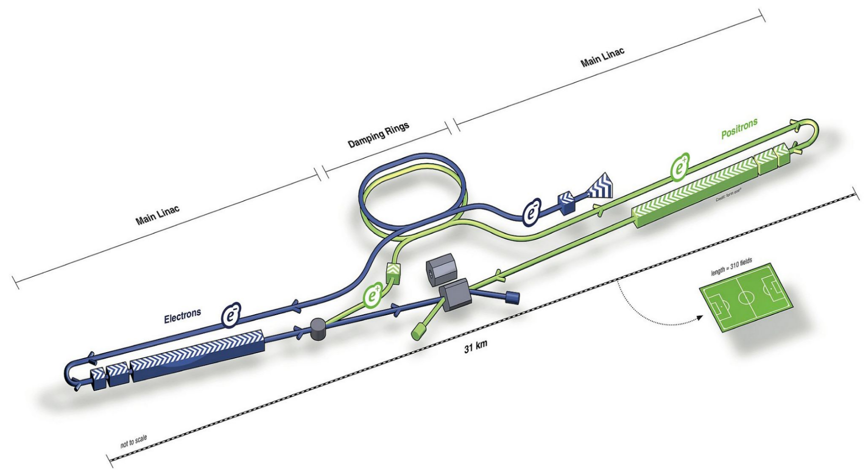
The Nobel Prize in Physics 2013 was awarded jointly to François Englert and Peter W. Higgs *"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"*

# Introduction to HE-Colliders

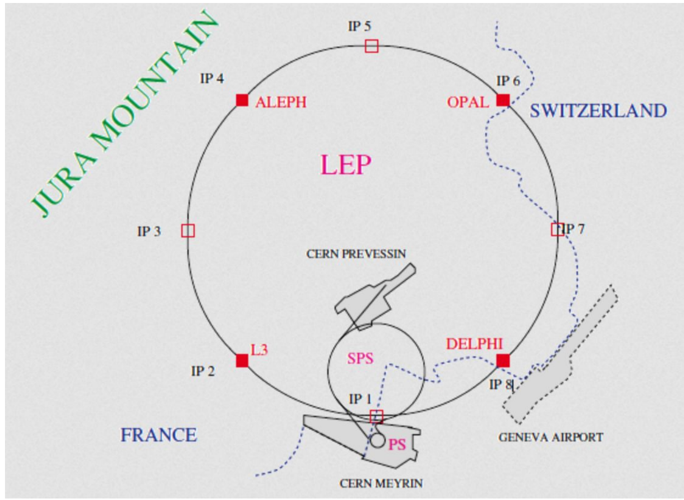
*Past (multiple colliders😉)*




**Tevatron: 1983 ~ 2011**

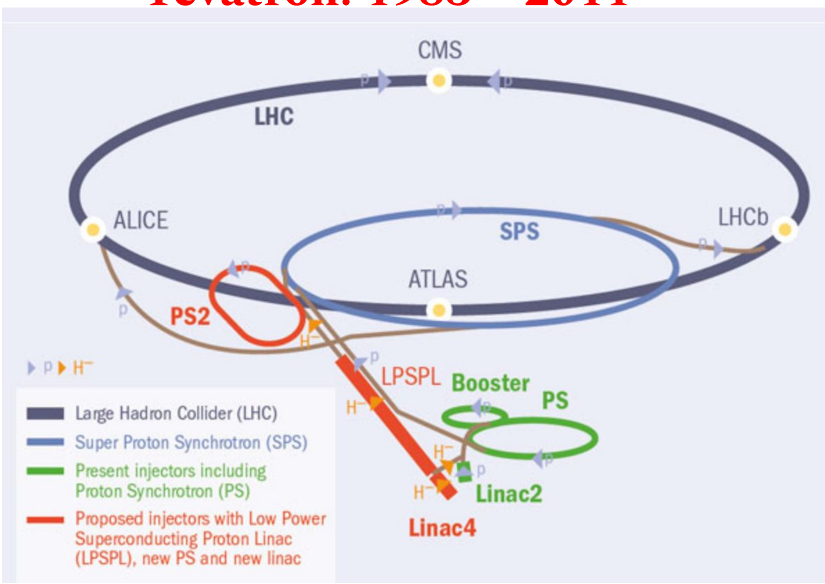



**SLC: 1987/9 ~ 1998**




**LEP: 1989 ~ 2000**

*Now*

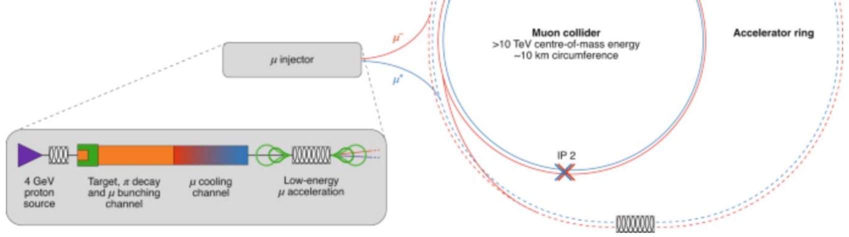



**(HL)LHC: 2008 ~**

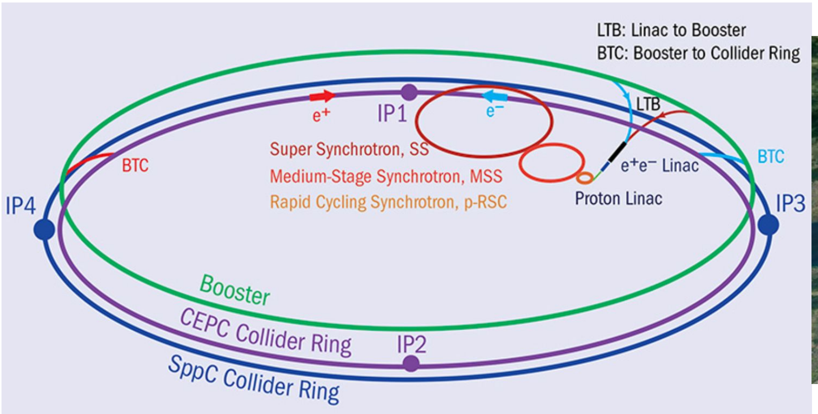
**Also ILC, CLIC …**

*Future?*

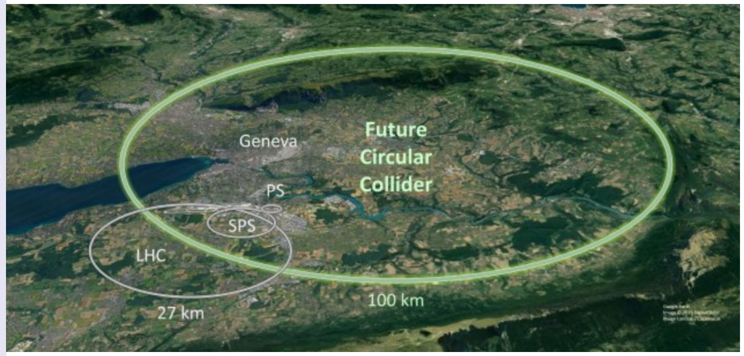
**Muon Collider**



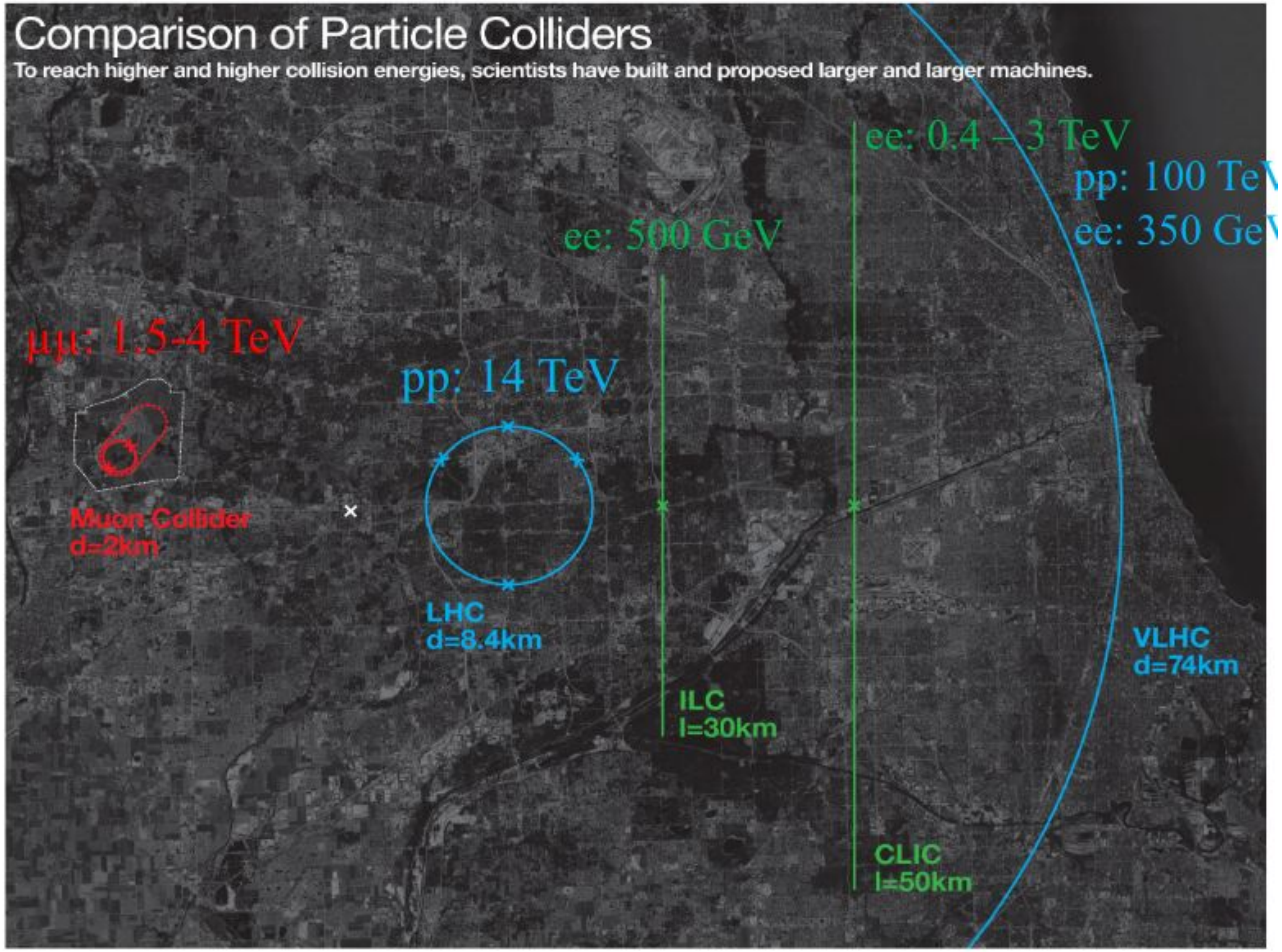

**CEPC/SPPC**



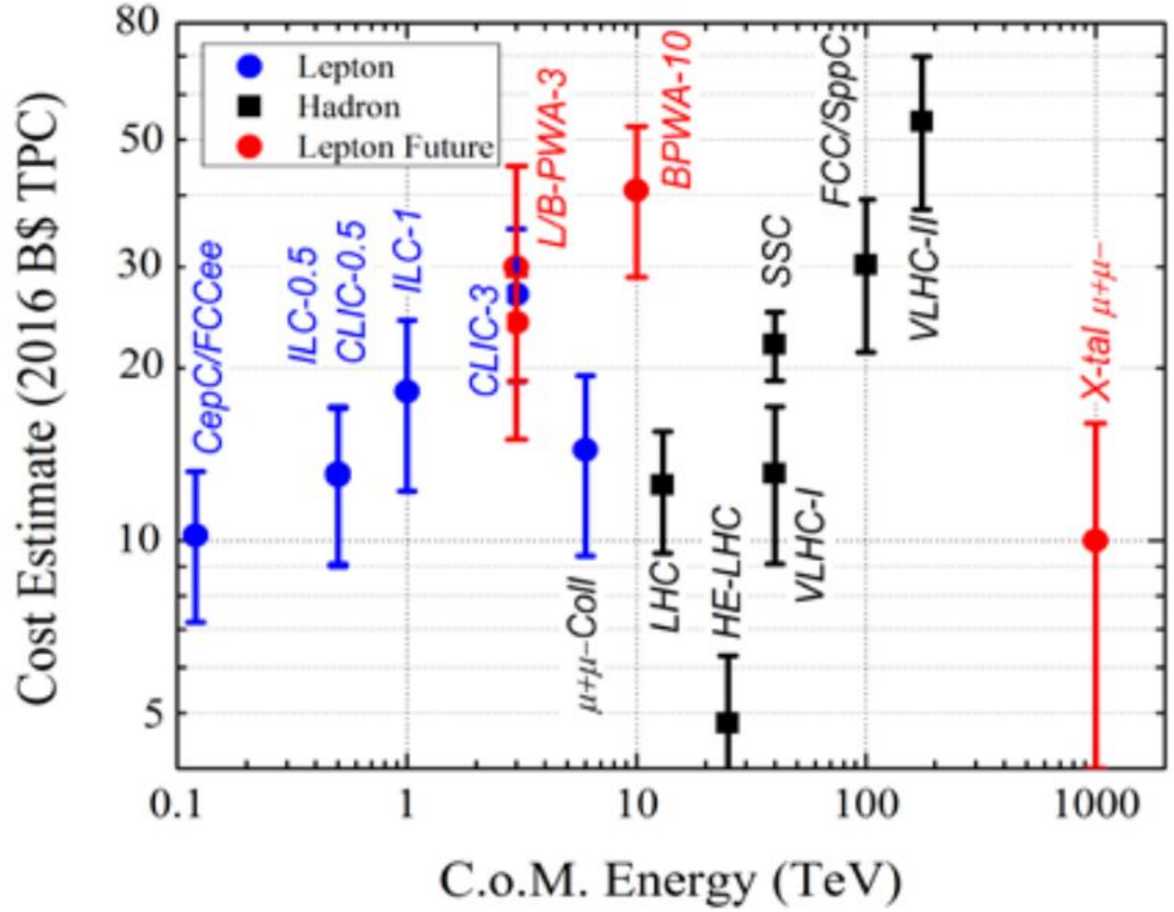

**FCC**



# Introduction to HE-Colliders

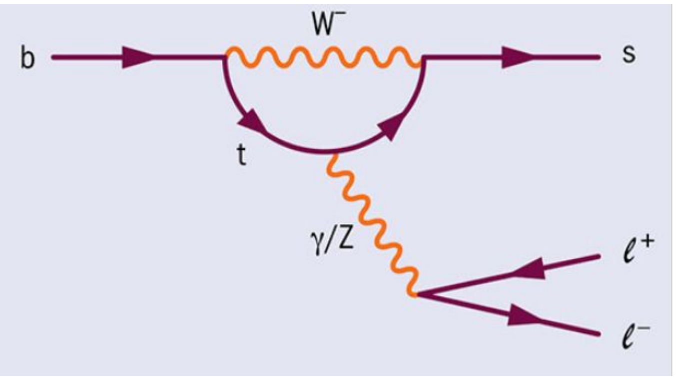



arXiv: [1705.02011]




**[The updated strategy for particle physics in Europe] recommends pursuing an electron-positron Higgs factory as the highest-priority.**

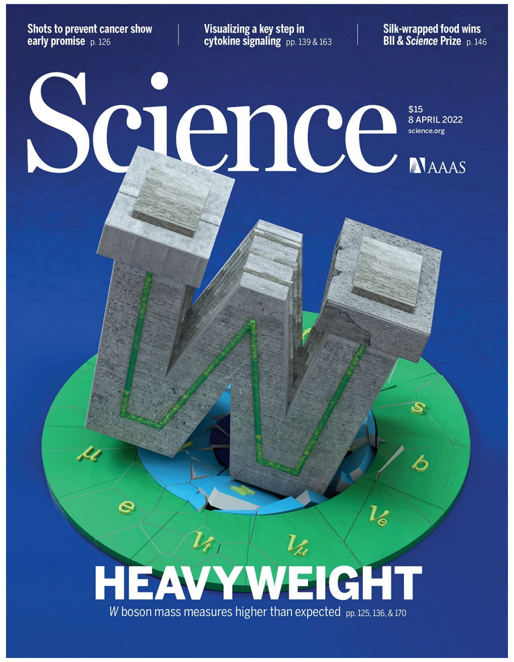
# Muon Collider



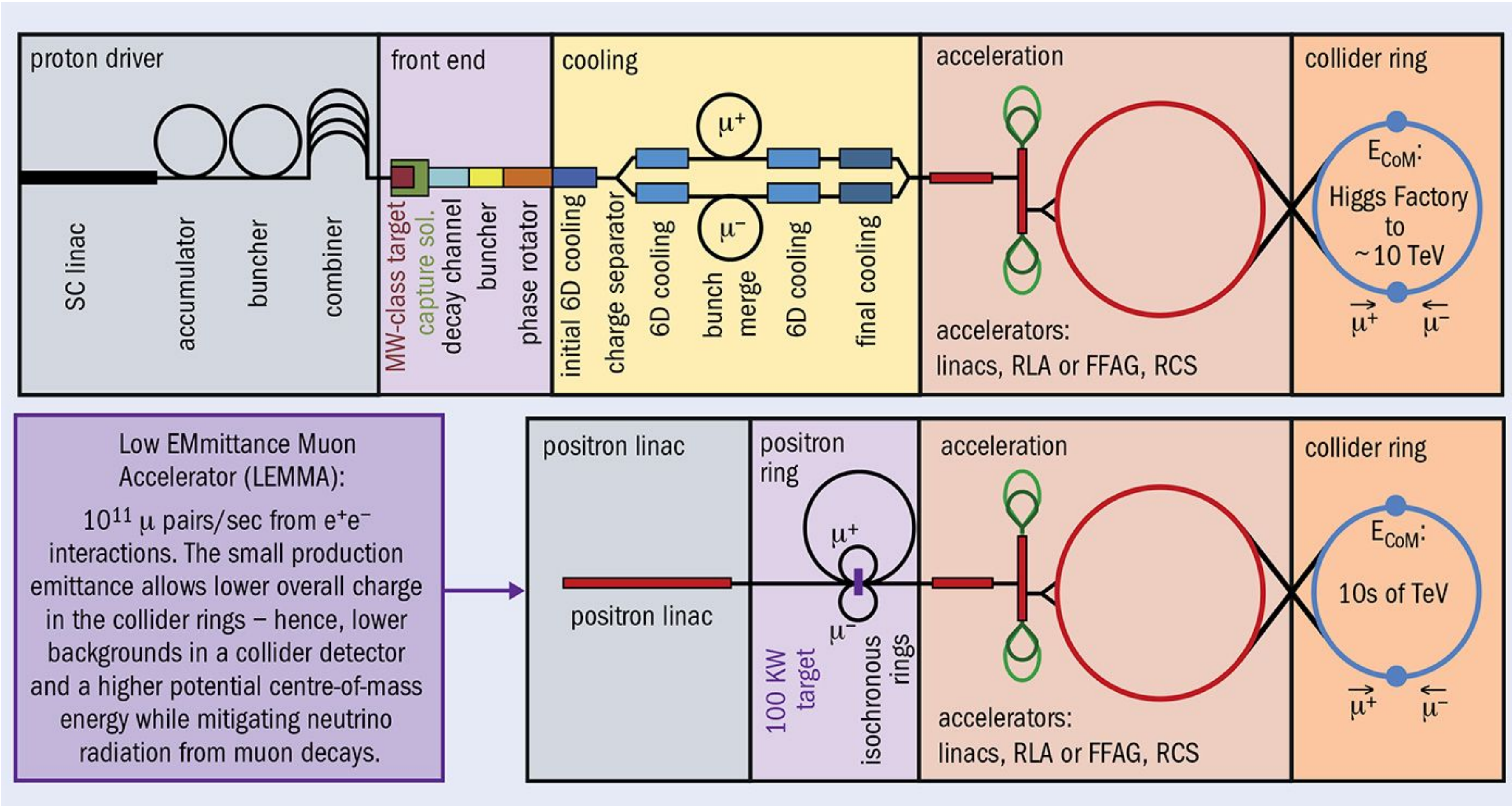

**Muon Collider interest Revived upon Muon Anomalies: lepton unitarity, muon g-2, w mass.**


**Pros:**
- Clean environment as electron collider
- Synchrotron radiation is smaller than electron O($10^8$)
- High energy as Hadron collider

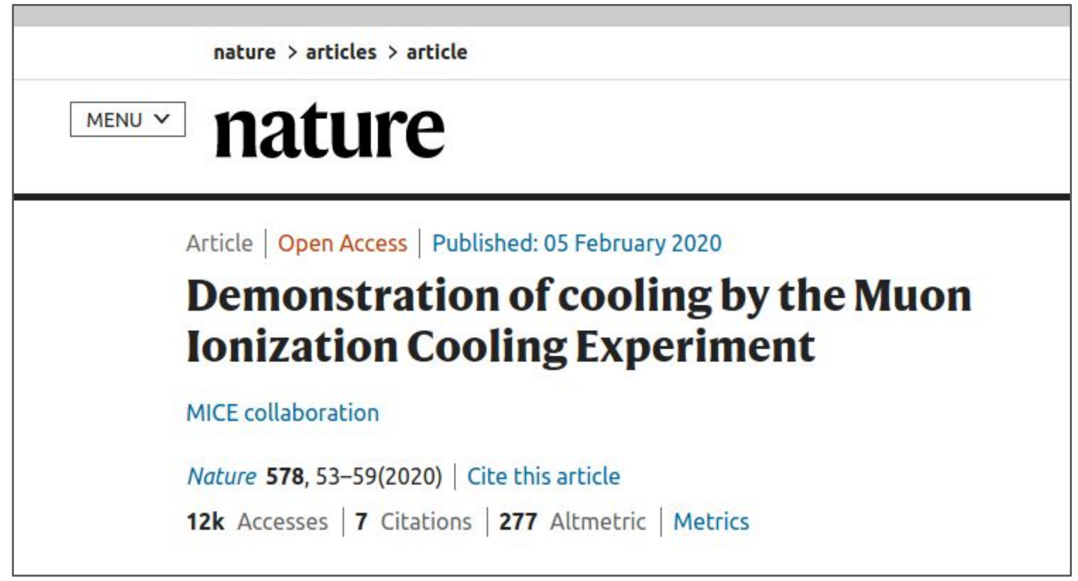
**Limits:**
- Muon is unstable particle, lifetime at rest only 2.2 μs, high quality beam source is challenging (both on lumi and emittance)
- Beam-induced background is challenging



# Muon Collider: Source




## Muon Ionisation Cooling Experiment (MICE)


nature > articles > article

MENU

nature

Article | Open Access | Published: 05 February 2020

**Demonstration of cooling by the Muon Ionization Cooling Experiment**

MICE collaboration

*Nature* **578**, 53–59(2020) | Cite this article

**12k** Accesses | **7** Citations | **277** Altmetric | Metrics

**Tertiary production** from **protons on target:** p + target→ π/K → μ
typically **$P_\mu$ ≈ 100 MeV/c** (π, K rest frame)
whatever is the boost $P_T$ will stay in Lab frame
→ **very high emittance** at production → ***cooling needed***
production Rate > $10^{13}$μ/sec   $N_\mu = 2{\cdot}10^{12}$/bunch

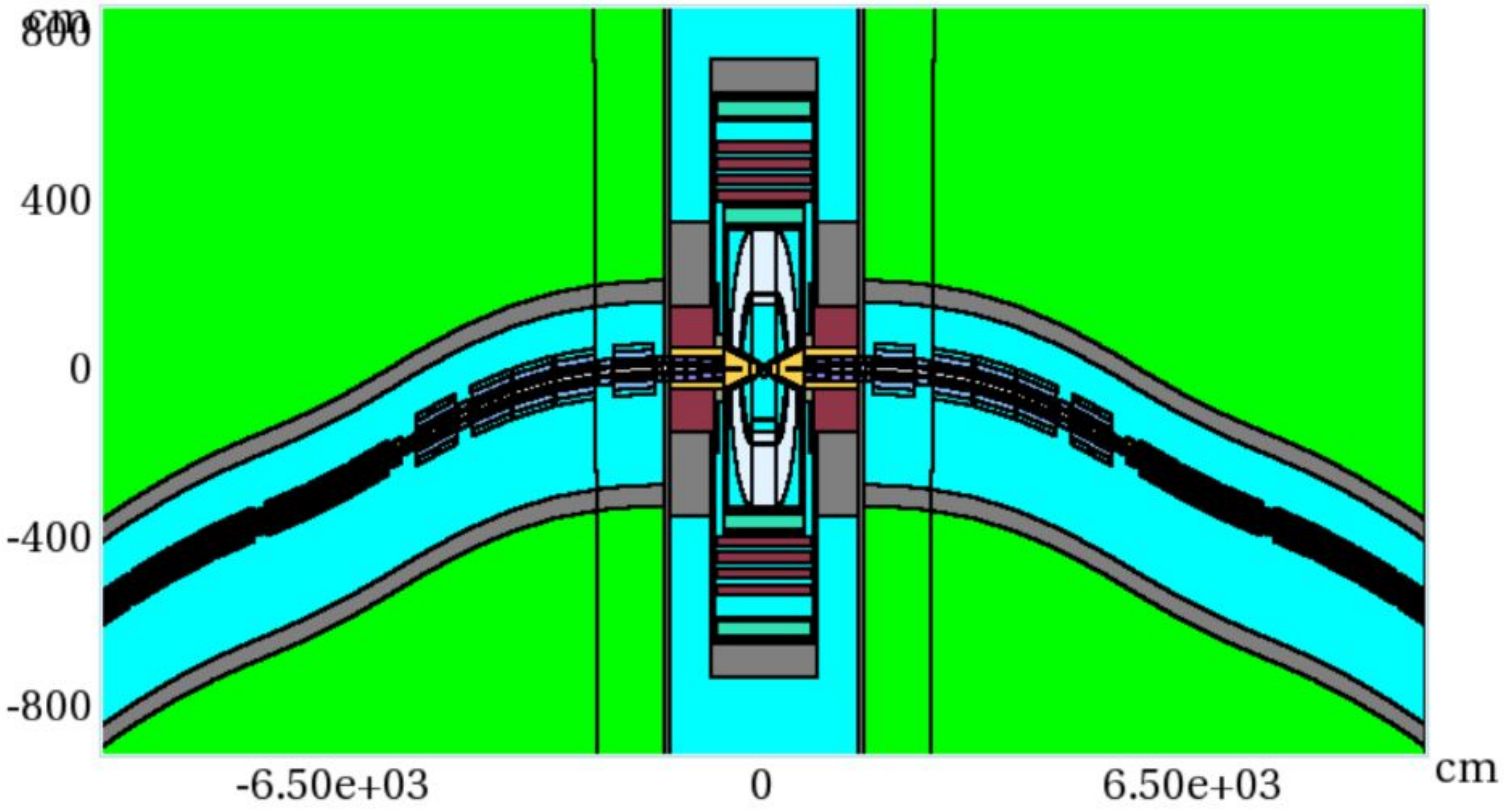
**MAP**

from **direct μ pair production**:
muons produced from **$e^+e^- \to \mu^+\mu^-$** at √s around the $\mu^+\mu^-$ threshold
**(√s ≈ 0.212GeV)** in asymmetric collisions (to collect $\mu^+$ and $\mu^-$ )
**$e^+e^-$ annihilation**: **e+ beam on target**
→ **cooled muon** beam with **low emittance** at production
Goal: production Rate ≈ $10^{11}$ μ/sec   $N_\mu \approx 6{\cdot}10^9$/bunch

**LEMMA**

| | **LEMMA** | **Protons on target (MAP)** |
|---|---|---|
| Physical process | *e+e-→ μ+μ−* | p N→ πX, KX→ μ X' |
| Luminosity [$cm^{-2}s^{-1}$] | ~$5\times10^{34}$ | ≳$10^{35}$ |
| $\varepsilon_N$ [μm-rad] | 0.04 | 25 |
| Rate $N_\mu$/s | $0.9\times10^{11}$ | $10^{13}$ |
| $N_\mu$/bunch | $6\times10^9$ | $2\times10^{12}$ |
| ΔE/E [%] | 0,07 | 0.1 |

# Muon Collider: BIB



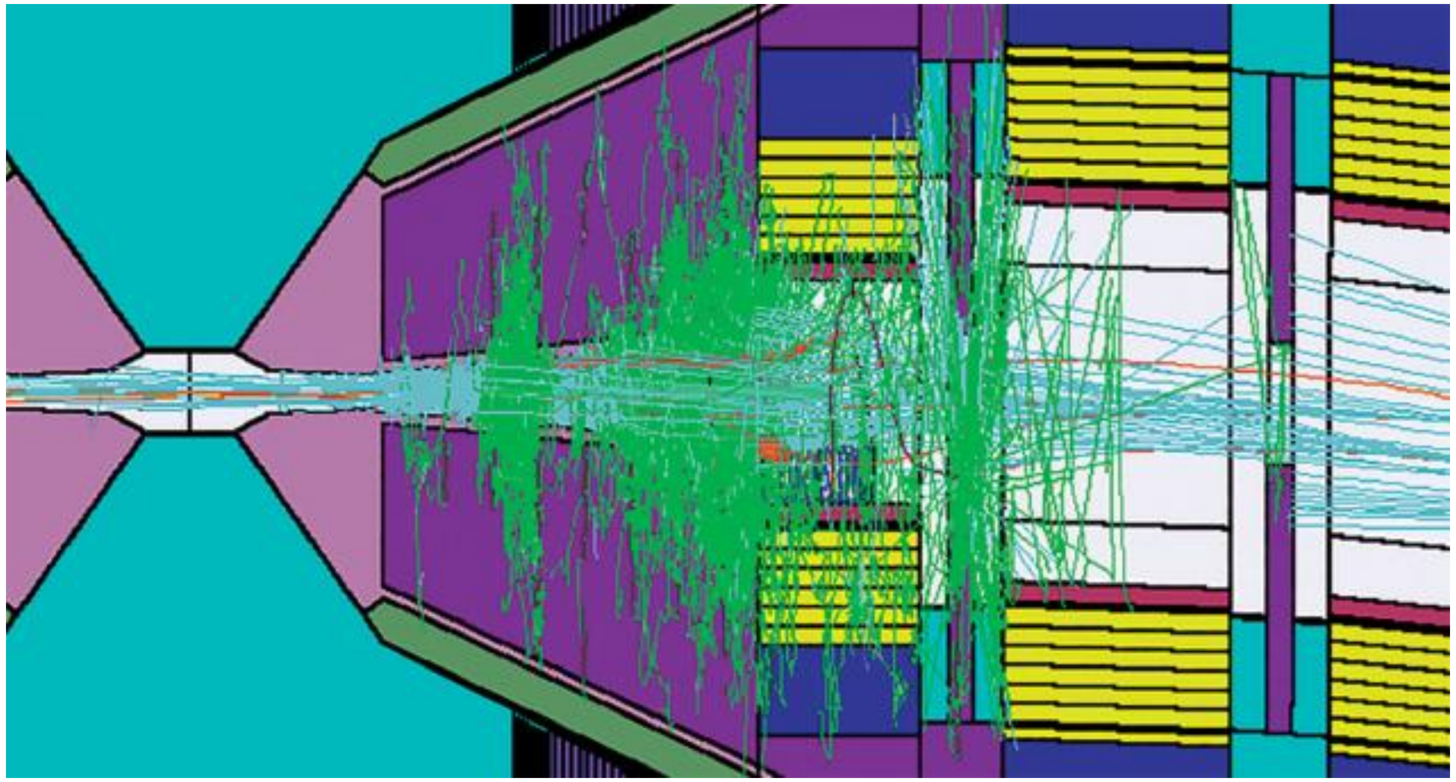



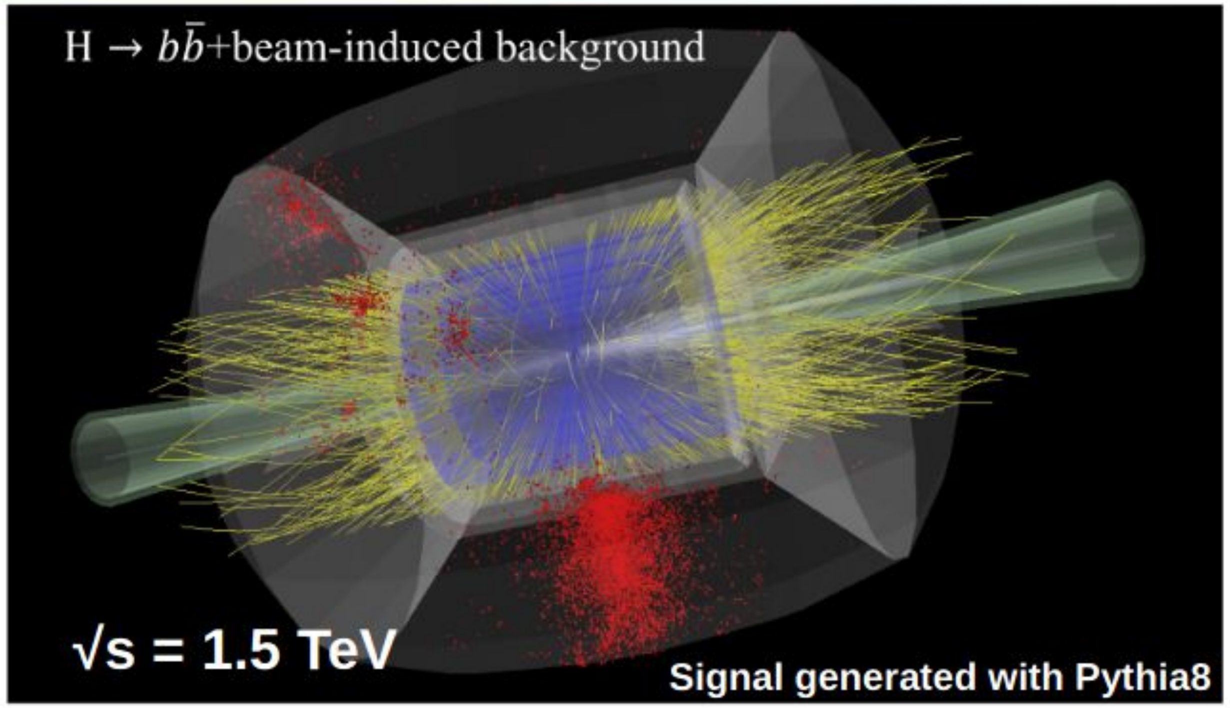
illustration of the model built for the MARS15 simulation in a range of ±100 m around the interaction point. It includes the machine components in the tunnel and the ILC 4th concept detector with the CMS-type tracker upgraded for the High-Luminosity LHC phase. **The shielding nozzles are represented in yellow inside the detector, which is used to suppress the BIB**. Link

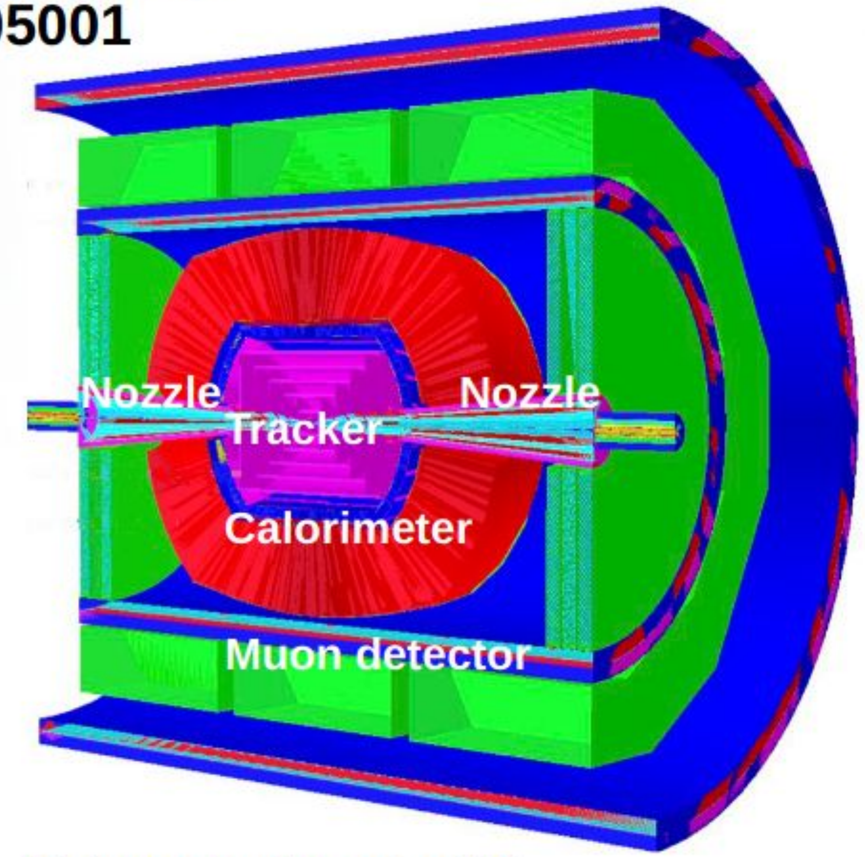
Zoomin in the IP ragion, tracker of charged particle from muon decay.

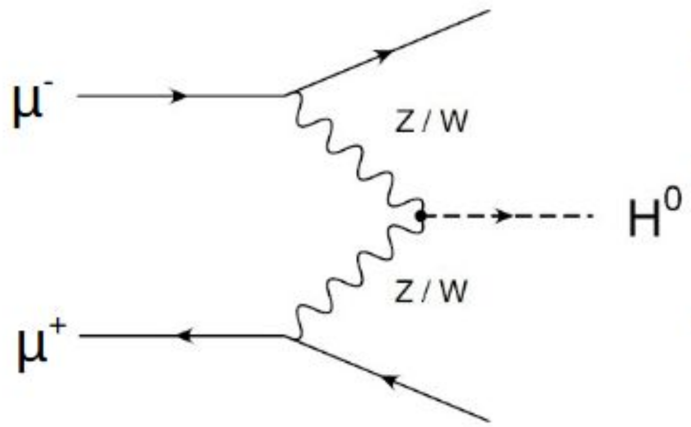
# Muon Collider: Higgs physics

## $\mu^+\mu^- \rightarrow \nu\bar{\nu}\, H(\rightarrow b\bar{b})$

2020 JINST 15 P05001




- We studied the μμ → νν H( → bb) production at a MC
- The goal is to determine the **sensitivity to the cross section measurement and to the Hbb couping determination**
- In the full simulation (Geant4) we used the detector developed by the MAP collaboration → not optimized for the full event reconstruction 7

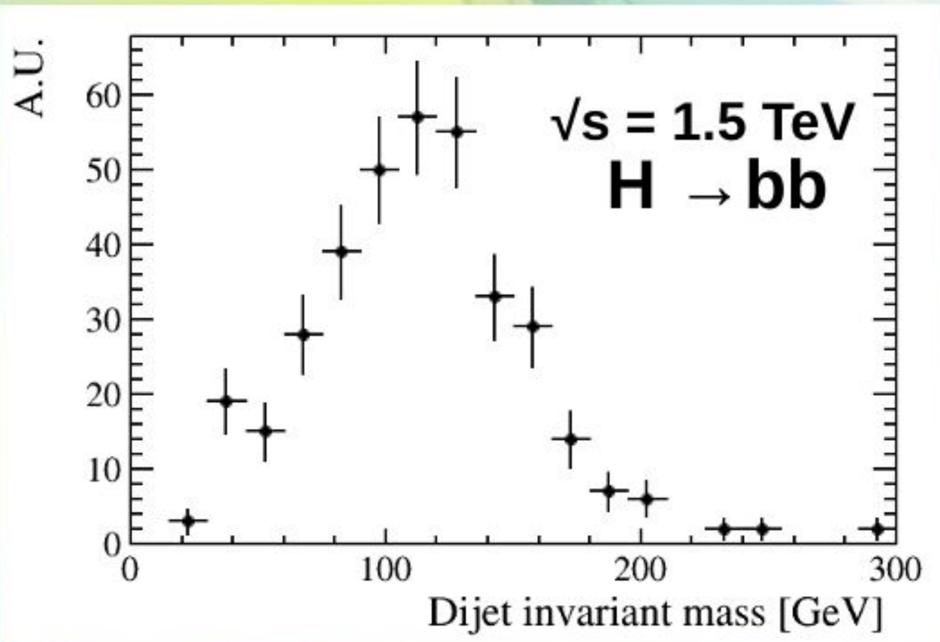
**At multi-TeV, a muon collider is basically a W+W- collider (VBS type): it will be possible to produce high yields of single H, HH and HHH events.**

Lorenzo Sestini @ICHEP2020, link

# Muon Collider: Higgs physics

## Cross section and Hbb coupling

2020 JINST 15 P05001




Two b-tagged jets with $p_T$>40 GeV, |η|<2.5 are selected

- **As a conservative approach we applied the efficiencies obtained at √s = 1.5 TeV to the 3.0 and 10 TeV case → BUT the BIB yield is expected to be lower at higher energies.**
- We assumed **4 Snowmass years of data taking**, at the luminosities expected by MAP.
- Cross section sensitivity obtained with $\frac{\Delta\sigma}{\sigma} \simeq \frac{\sqrt{N+B}}{N}$,
- Hbb coupling sensitivity $\frac{\Delta g_{Hbb}}{g_{Hbb}} = \frac{1}{2}\sqrt{\left(\frac{\Delta\sigma}{\sigma}\right)^2 + \left(\frac{\Delta \frac{g^2_{HWW}}{\Gamma_H}}{\frac{g^2_{HWW}}{\Gamma_H}}\right)^2}$, → Taken from CLIC expectation

| $\sqrt{s}$ [TeV] | $A$ [%] | $\epsilon$ [%] | $\mathcal{L}$ [cm$^{-2}$s$^{-1}$] | $\mathcal{L}_{int}$ [ab$^{-1}$] | $\sigma$ [fb] | $N$ | $B$ | $\frac{\Delta\sigma}{\sigma}$ [%] | $\frac{\Delta g_{Hbb}}{g_{Hbb}}$ [%] |
|---|---|---|---|---|---|---|---|---|---|
| 1.5 | 35 | 15 | $1.25 \cdot 10^{34}$ | 0.5 | 203 | 5500 | 6700 | 2.0 | 1.9 |
| 3.0 | 37 | 15 | $4.4 \cdot 10^{34}$ | 1.3 | 324 | 33000 | 7700 | 0.60 | 1.0 |
| 10 | 39 | 16 | $2 \cdot 10^{35}$ | 8.0 | 549 | 270000 | 4400 | 0.20 | 0.91 |

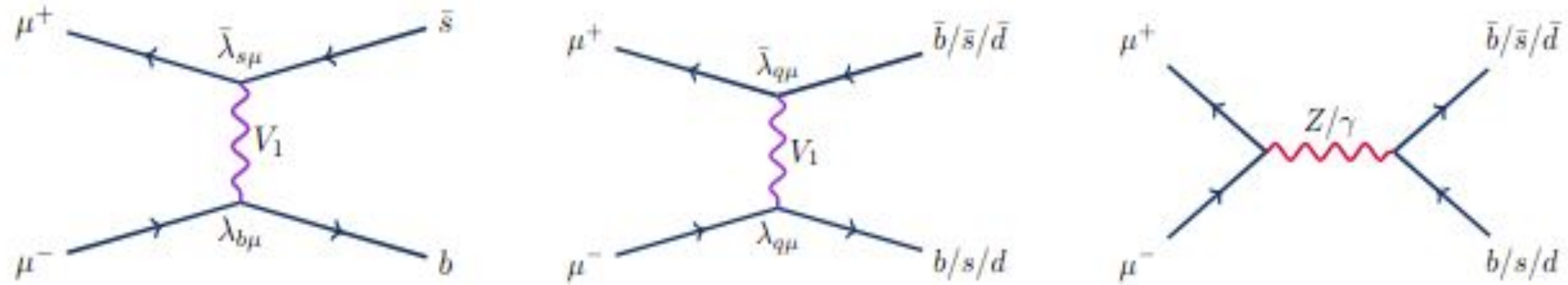
### Physics backgrounds

| Process |
|---|
| $\mu^+\mu^- \to \gamma^*/Z \to q\bar{q}$ |
| $\mu^+\mu^- \to \gamma^*/Z\gamma^*/Z \to q\bar{q}$ +X |
| $\mu^+\mu^- \to \gamma^*/Z\gamma \to q\bar{q}\gamma$ |

**At 3 TeV the Hbb coupling sensitivity is compatible with the one expected by CLIC, but very conservative assumptions have been done!**

Lorenzo Sestini @ICHEP2020, link

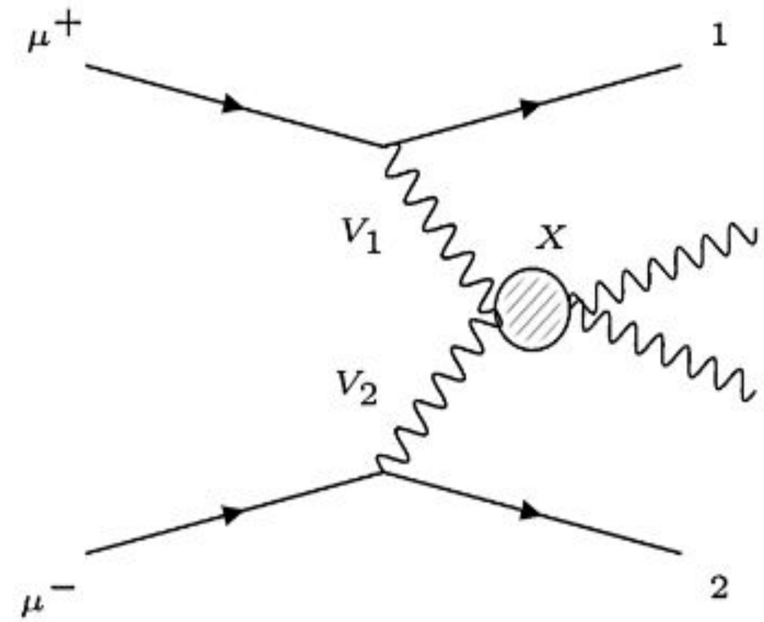
# More on Muon Collider

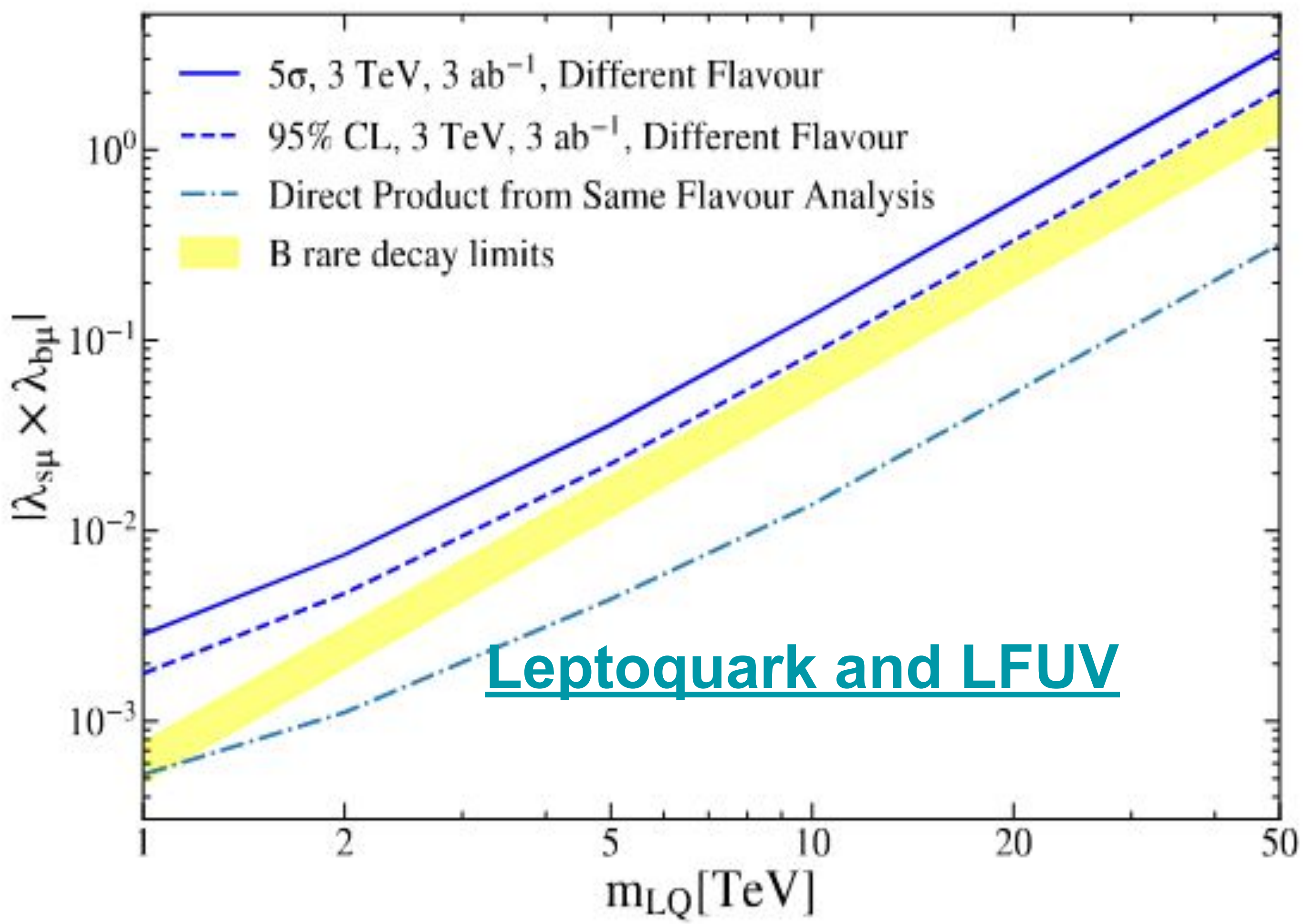



(a) Different Flavour $t$-channel Di-jet Final State

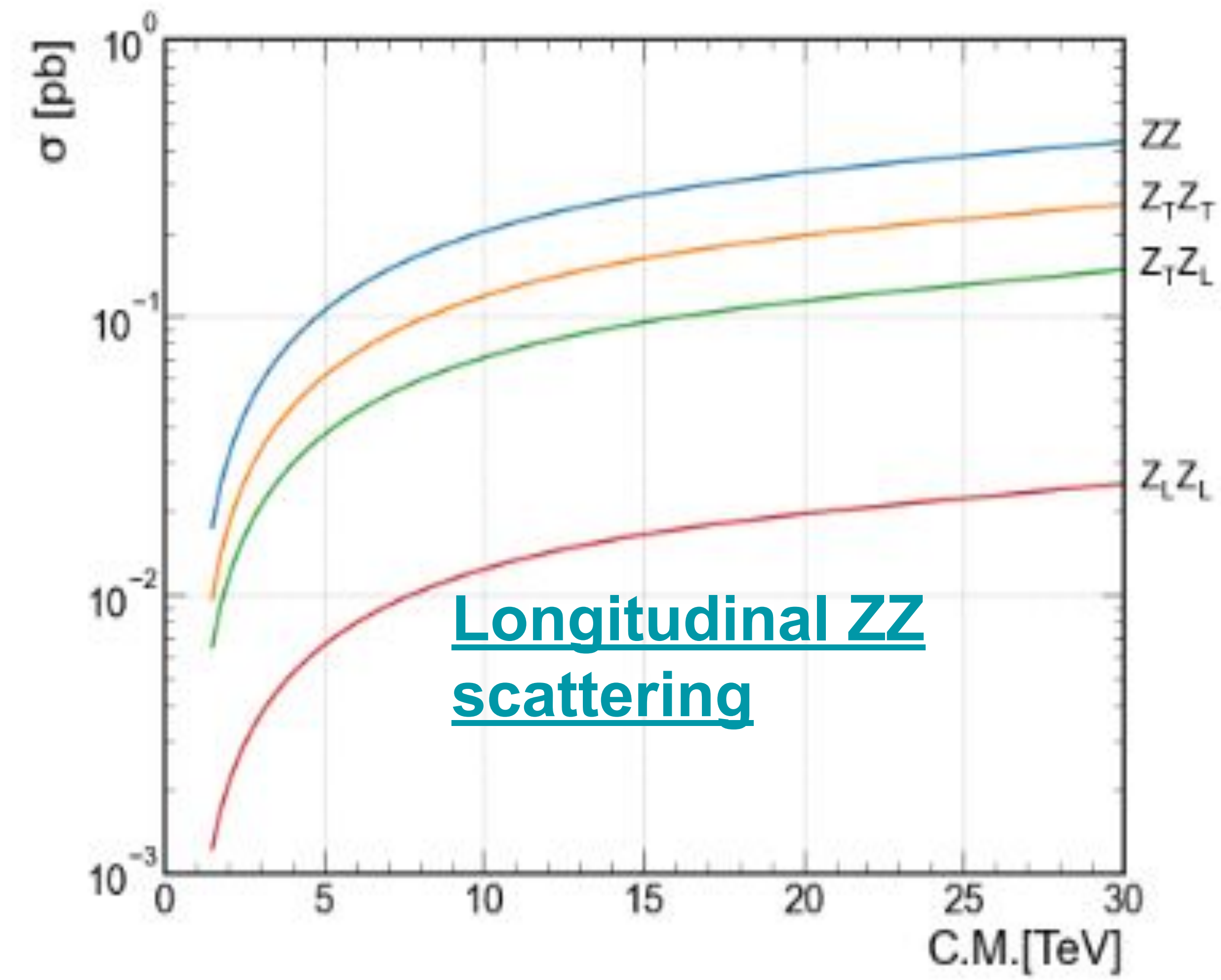
(b) Same Flavour $t$-channel Di-jet Final State

(c) SM $s$-channel Di-jet Production




Leptoquark and LFUV




Longitudinal ZZ scattering

# International Muon Collider Collaboration

https://ipac2022.vrws.de/talks/tuizsp2_talk.pdf

## MoC and Design Study Partners

| | |
|---|---|
| IEIO | **CERN** |
| FR | **CEA** |
| | CNRS-LNCMI |
| DE | DESY |
| | **Technical University of Darmstadt** |
| | **University of Rostock** |
| | KIT |
| IT | **INFN** |
| | University of Milano |
| | ***University of Padova*** |
| | University of Pavia |
| | University of Bologna |
| | ENEA |
| CH | PSI |
| | ***University of Geneva*** |

| | |
|---|---|
| UK | **STFC-RAL** |
| | UK Research and Innovation |
| | University of Lancaster |
| | University of Southampton |
| | University of Strathclyde |
| | University of Sussex |
| | Imperial College |
| | Royal Holloway |
| | **University of Huddersfield** |
| | University of London |
| | **JAI** |
| | University of Oxford |
| | **University of Warwick** |
| SE | ***ESS*** |
| | **University of Uppsala** |

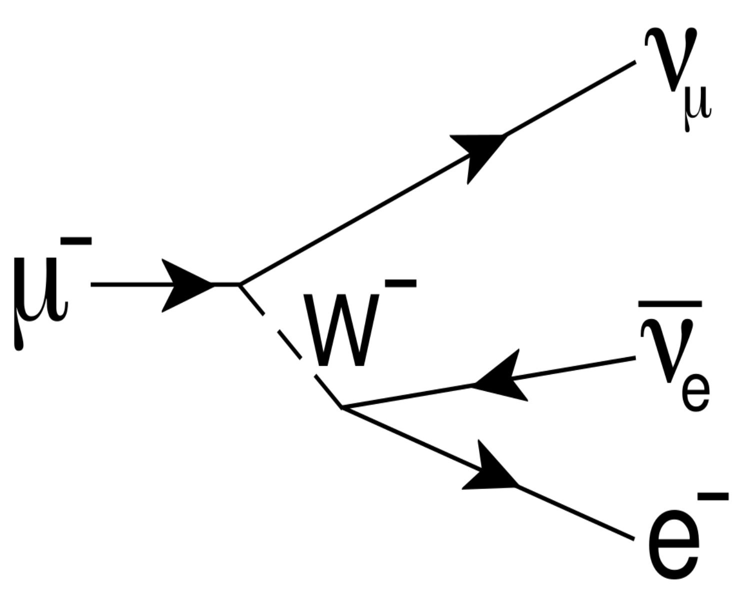
| | |
|---|---|
| PT | LIP |
| NL | University of Twente |
| FI | **Tampere University** |
| US | **Iowa State University** |
| | BNL |
| China | ***Sun Yat-sen University*** |
| | **IHEP** |
| | ***Peking University*** |
| EST | ***Tartu University*** |
| LAT | ***Riga Technical Univers.*** |
| AU | ***HEPHY*** |
| ES | ***I3M*** |

CHART is contributing (and EPFL)
Informal contributions (US, Japan)

Note: *some MoC still being prcessed*

D. Schulte    Muon Collider, IPAC, June 2022    24

# Neutrino beam




B. J. King hep-ex/0005007

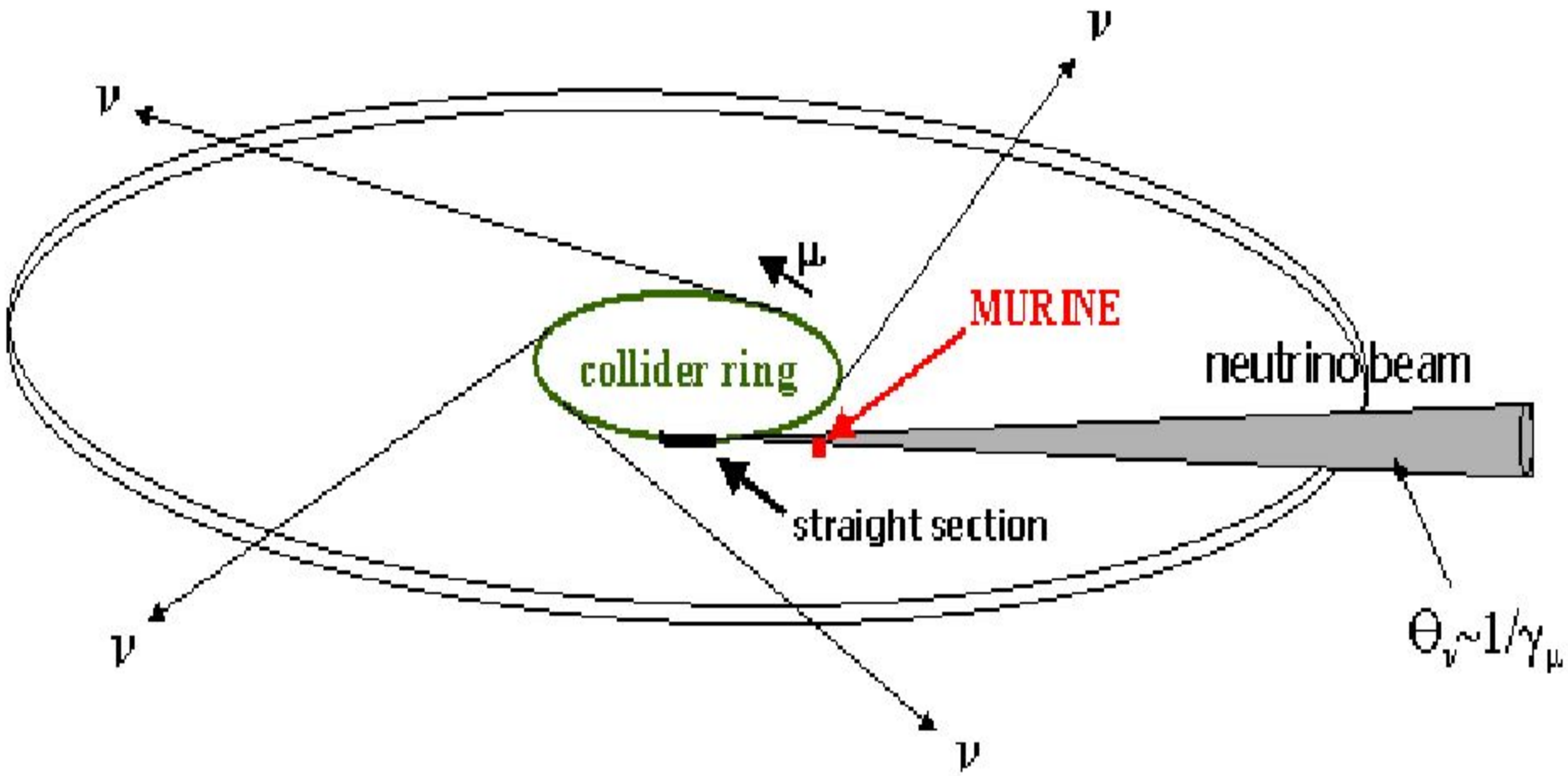
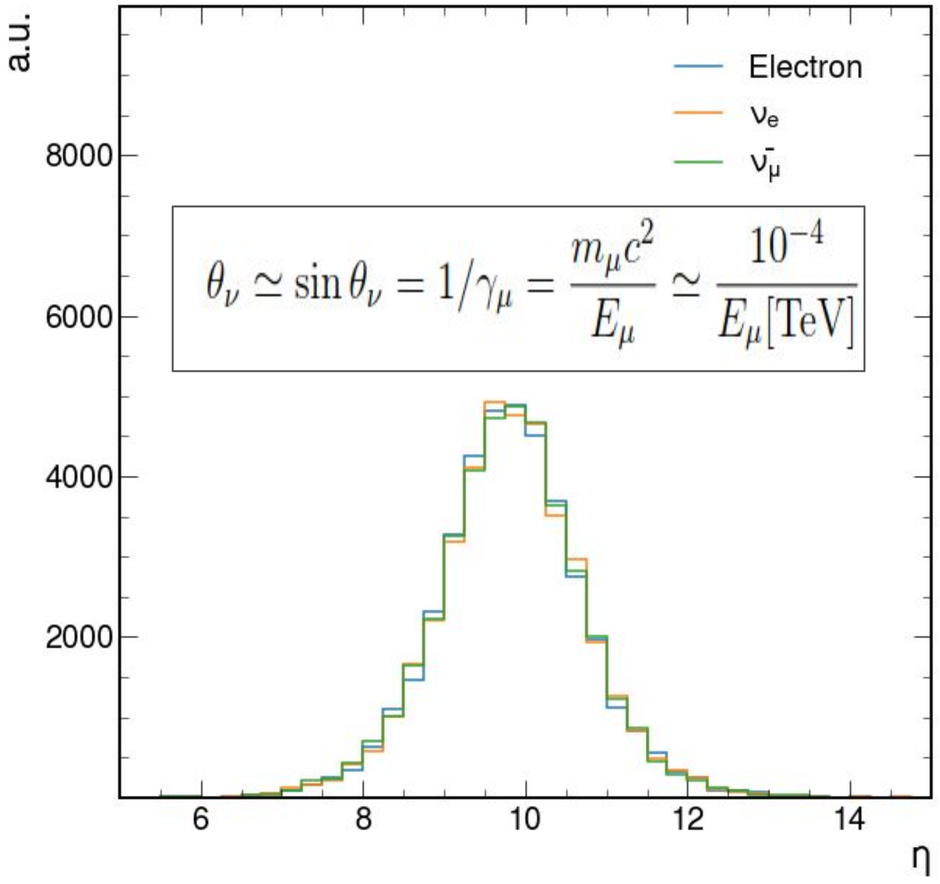


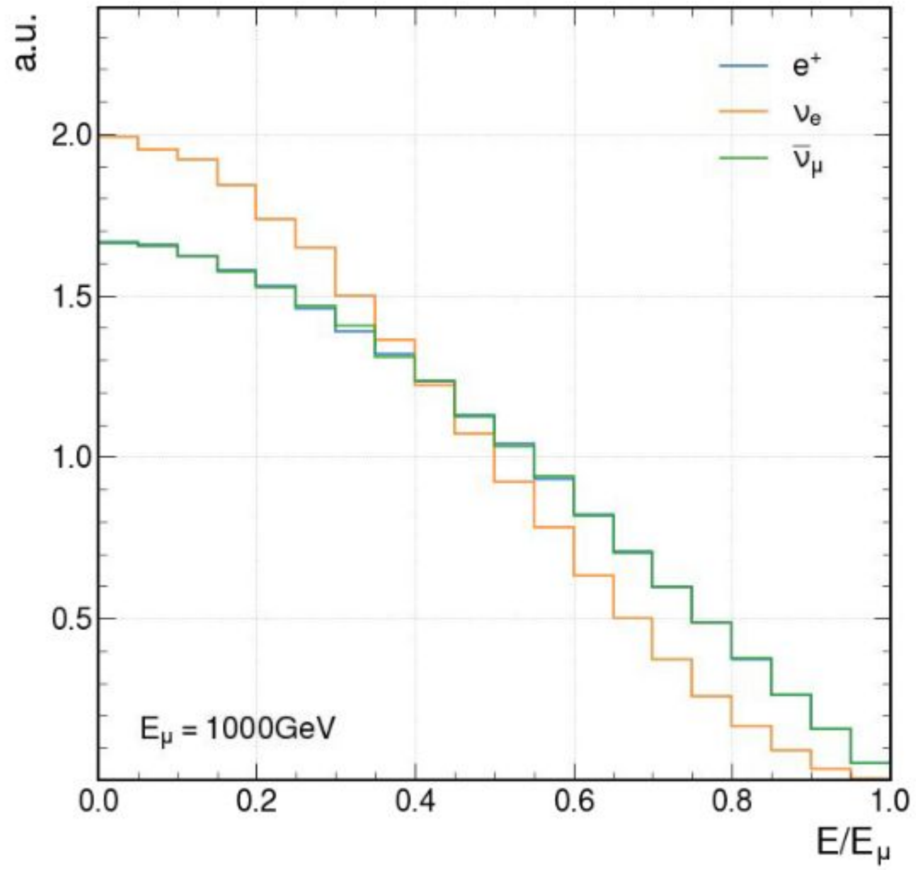

**FIGURE 1.** The decays of muons in a muon collider will produce a disk of neutrinos emanating out tangentially from the collider ring. The neutrinos from decays in straight sections will line up into beams suitable for experiments. The MURINEs will be sited in the center of the most intense beam and as close as is feasible to the production straight section.

NuTeV
Neutrino-Nucleon Scattering
NuMAX
NuSOnG
Neutrino Scattering on Glass
nuSTORM
"Neutrinos from STORed Muons," …for neutrino oscillation searches

**很多中微子打靶实验，但是没有head-on对撞构想**

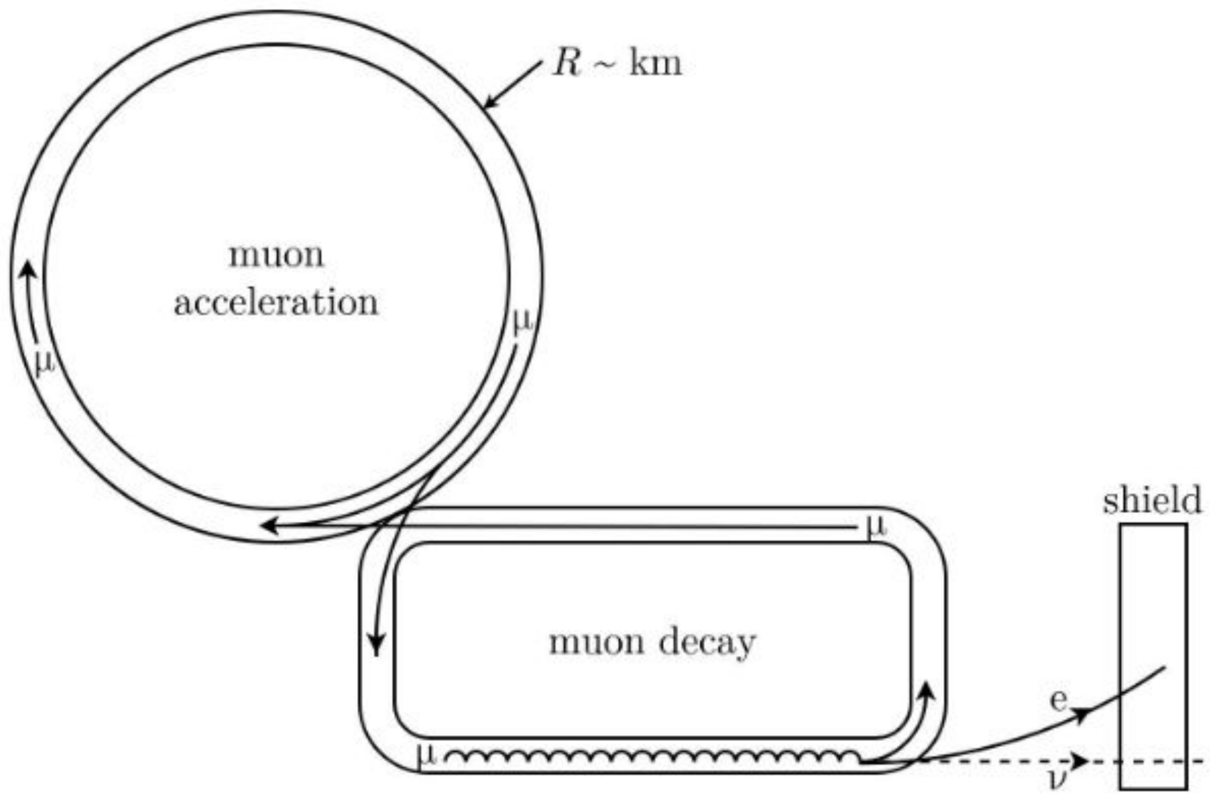
# Neutrino Beam from 1 TeV Muon beam



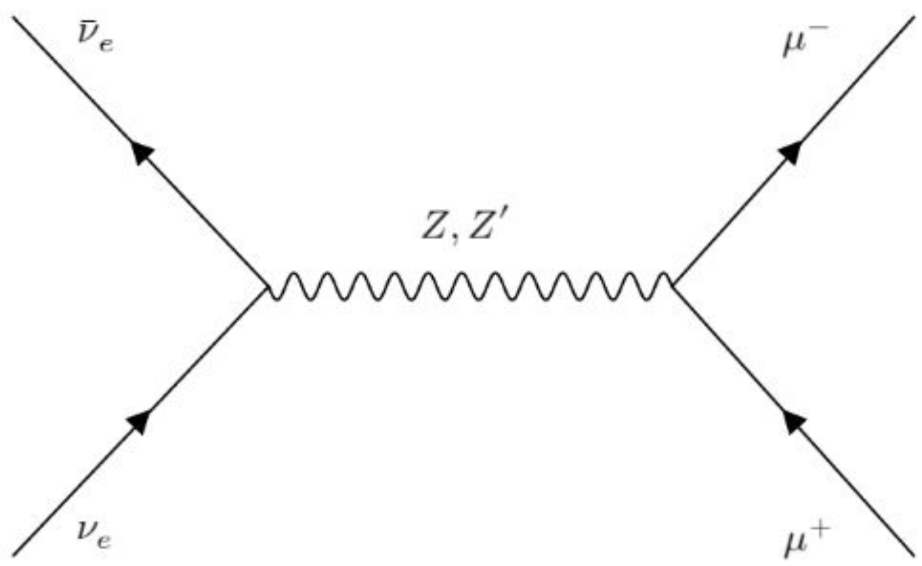

"Parton Distribution Function"
Included in MadGaph

Highly collimated in angle, yet widely distributed in Energy



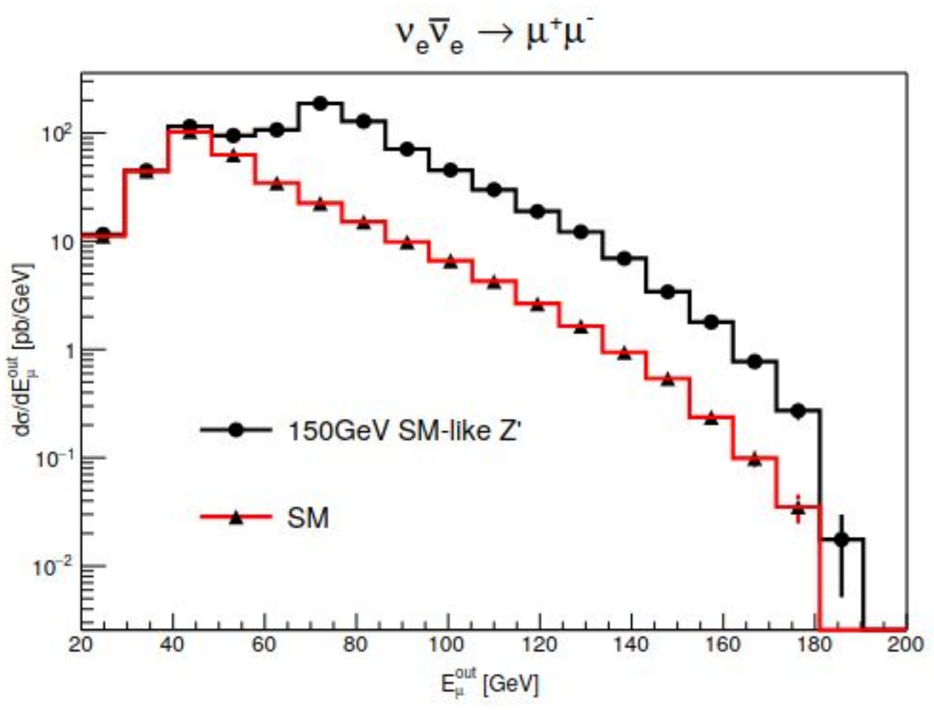

**A small modulation of the muon decay angle through vertical bending, symbolized by the squiggly line, is used to focus the neutrino beam.**
**Link**

# Neutrino collision: neutrino annihilation




Neutrino annihilation could be served as a vector boson collider.

$$\nu_e\nu_e \to \mathrm{HH}$$
$$\nu_e\nu_e \to \mathrm{ZZ}, \mathrm{ZH}$$
$$\nu_e\nu_e \to \nu_e\nu_e\mathrm{H},$$
$$\nu_e\nu_e \to \nu_e\nu_e\mathrm{ZZ}, \nu_e\nu_e\mathrm{WW},$$
$$\nu_e\nu_e \to \nu_e\nu_e\mathrm{ZH}, \nu_e\nu_e\mathrm{HH},$$
$$\nu_e\nu_e \to e^-e^-\mathrm{W}^+\mathrm{W}^+,$$

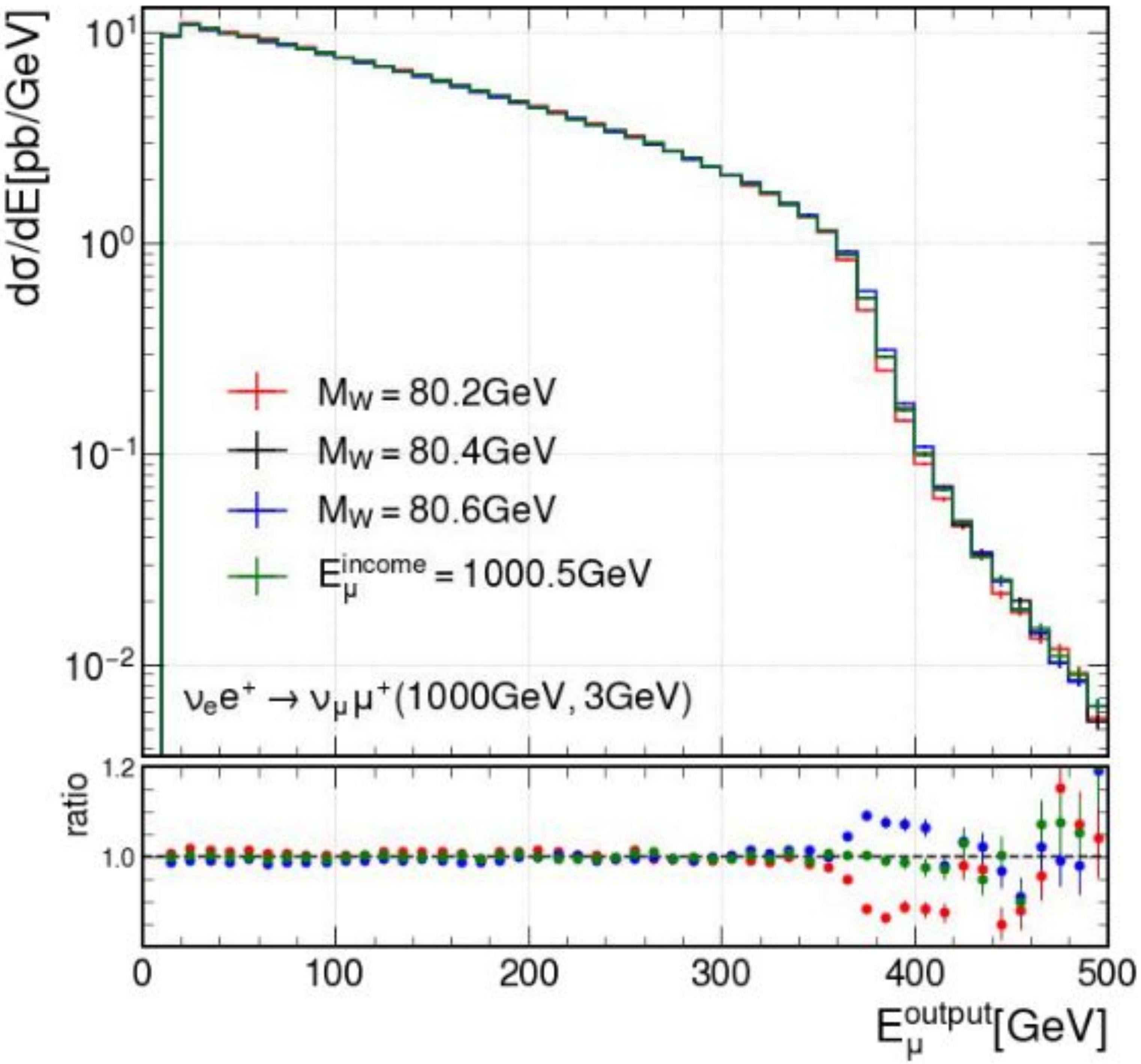


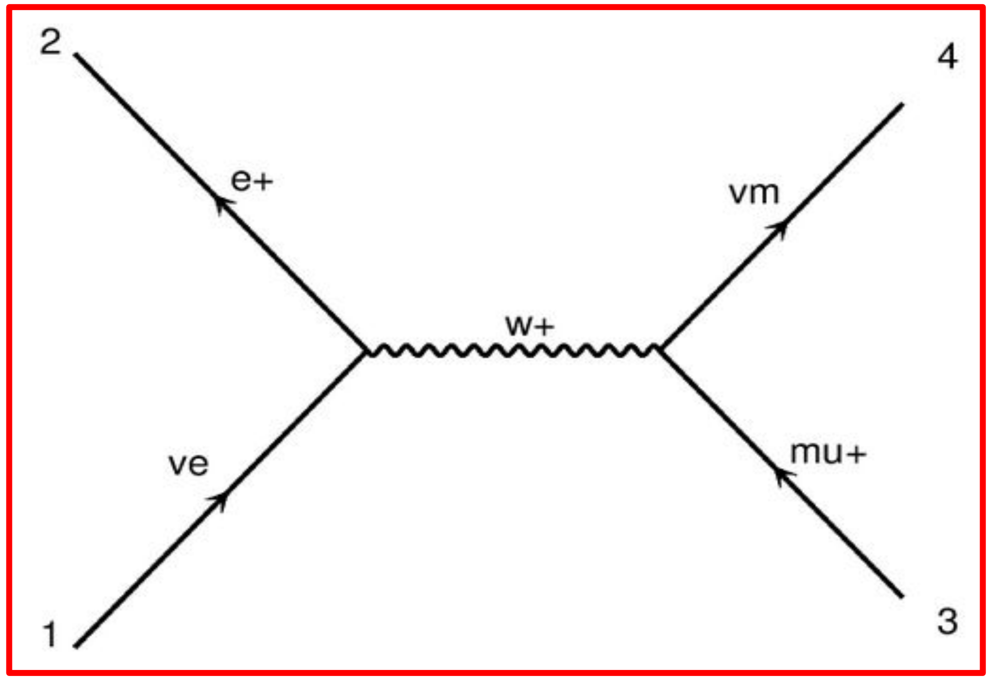

Simplest neutrino mass origination: dimension-5 Weinberg operator, which lead Majorana neutrino masses and introduce lepton number violation. So we can use vvHH (t-channel Majorana neutrino) to study the BSM.

$$\mathcal{L}_5 = \left( C_5^{\ell\ell'}/\Lambda \right) \left[\Phi \cdot \overline{L}_\ell^c\right] \left[L_{\ell'} \cdot \Phi\right],$$

$$m_{\ell\bar{\ell}} = C_5^{\ell\bar{\ell}} v^2/\Lambda$$

# Neutrino collision: neutrino-lepton collision

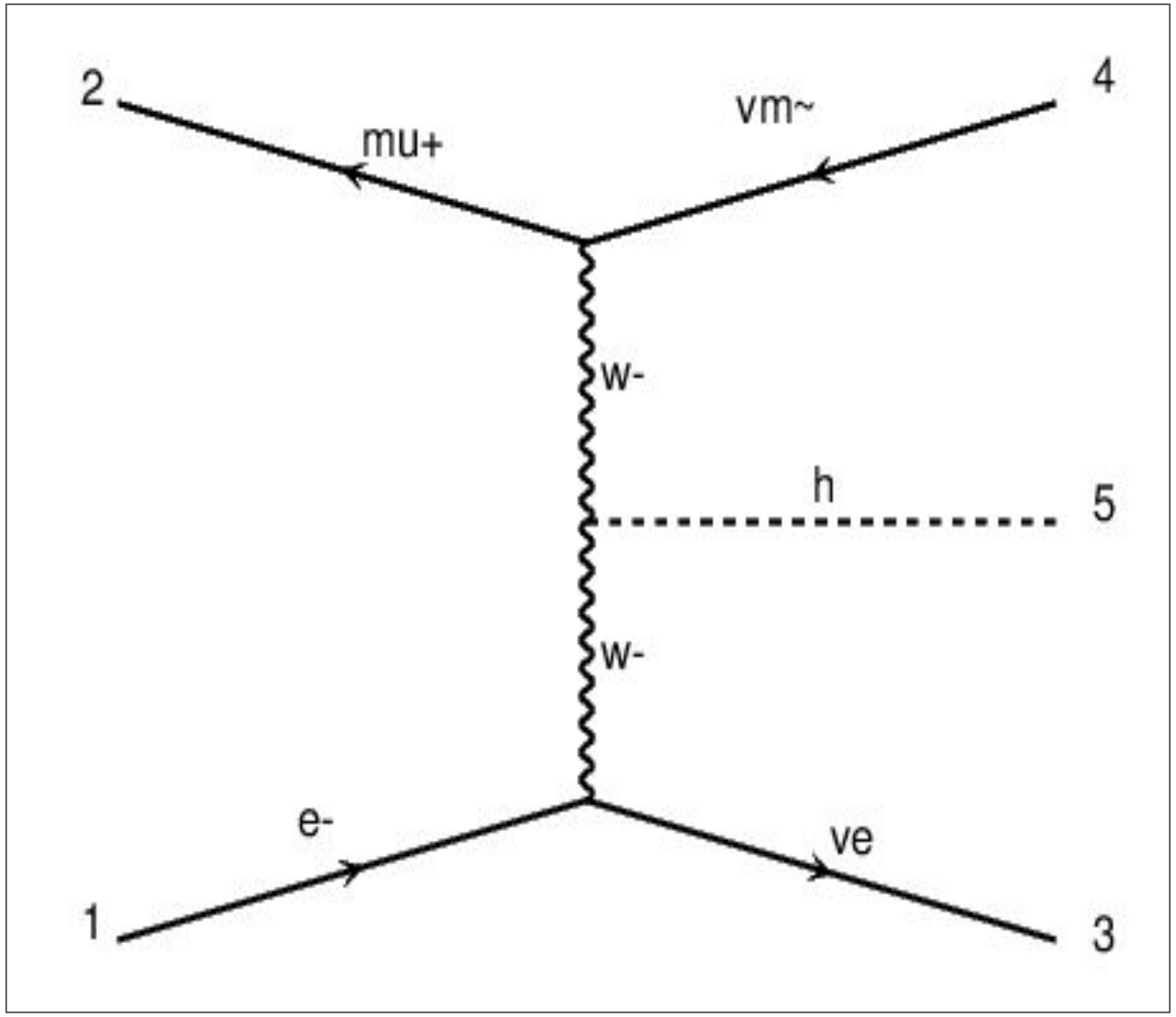



**If pT(outgoing muon) > 40 GeV**
the cross sections with **MW = 80.4 (80.41) are 166.2 (167.6) pb.**

Based on a simple counting experiment, **a 10 MeV accuracy on MW** can be achieved with an integrated luminosity of **only 0.1 fb−1.**

# electron-muon collider arXiv:2010.15144




- A novel kind of collider from 0 -> 1
  - low to high collision energy
  - linear/circular/hybrid
  - various beam combinations:
    $e^-\mu^+$, $e^+\mu^-$, $e^+\mu^+$,$e^-\mu^-$, polarization
- An important intermediate step
  - between e-e and mu-mu
  - Robust under muon beam induced background
- Rich physics with economical budget
  - Charged Lepton Flavor violation
  - Higgs precision measurement
  - majorana neutrino, heavy lepton
  - ~ 1-2 billion $ in total

**Flexibility to extend to various options!**

# electron-muon collider arXiv:2010.15144

CERNCOURIER

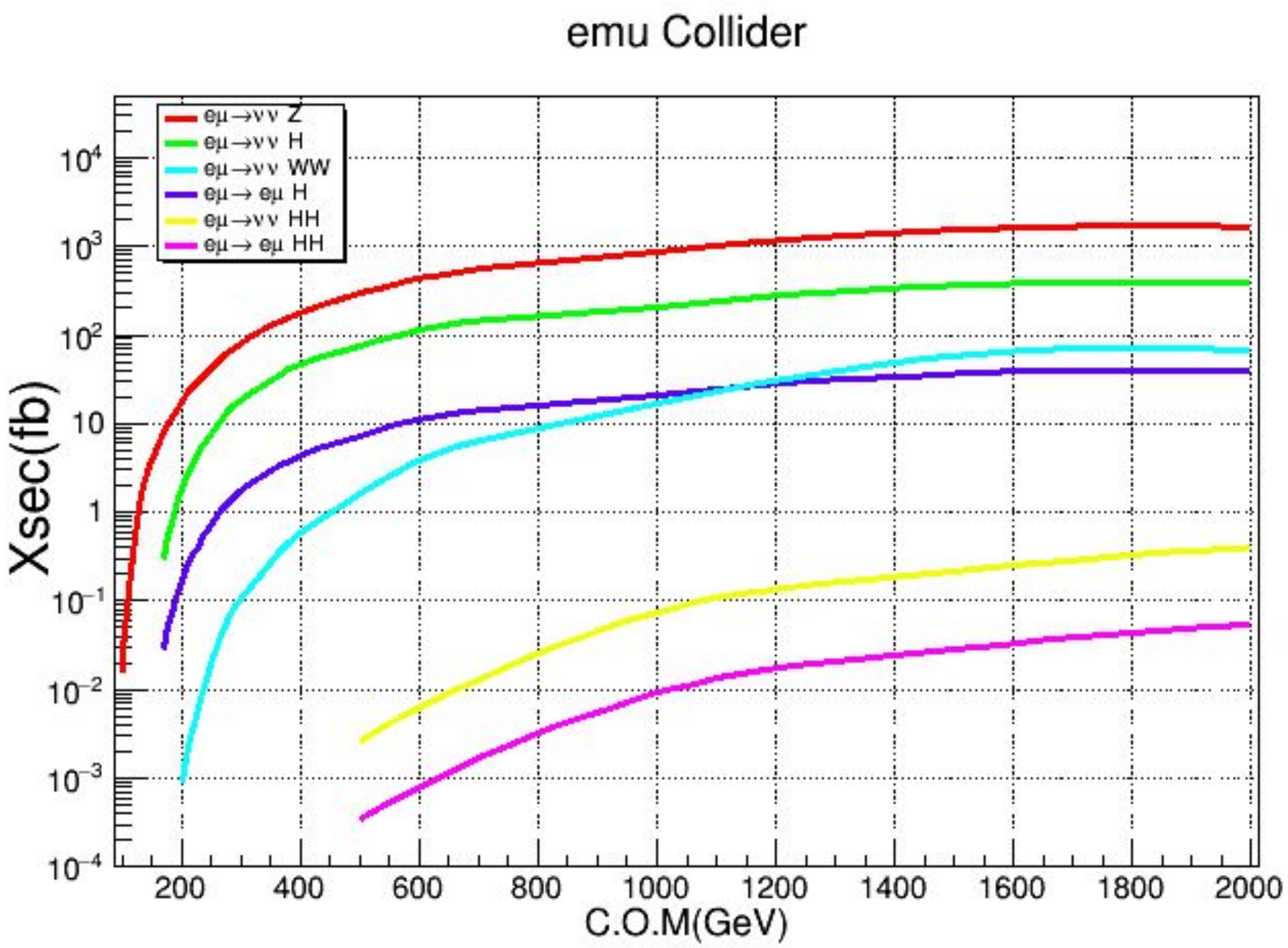
March/April 2021 cerncourier.com Reporting on international high-energy physics

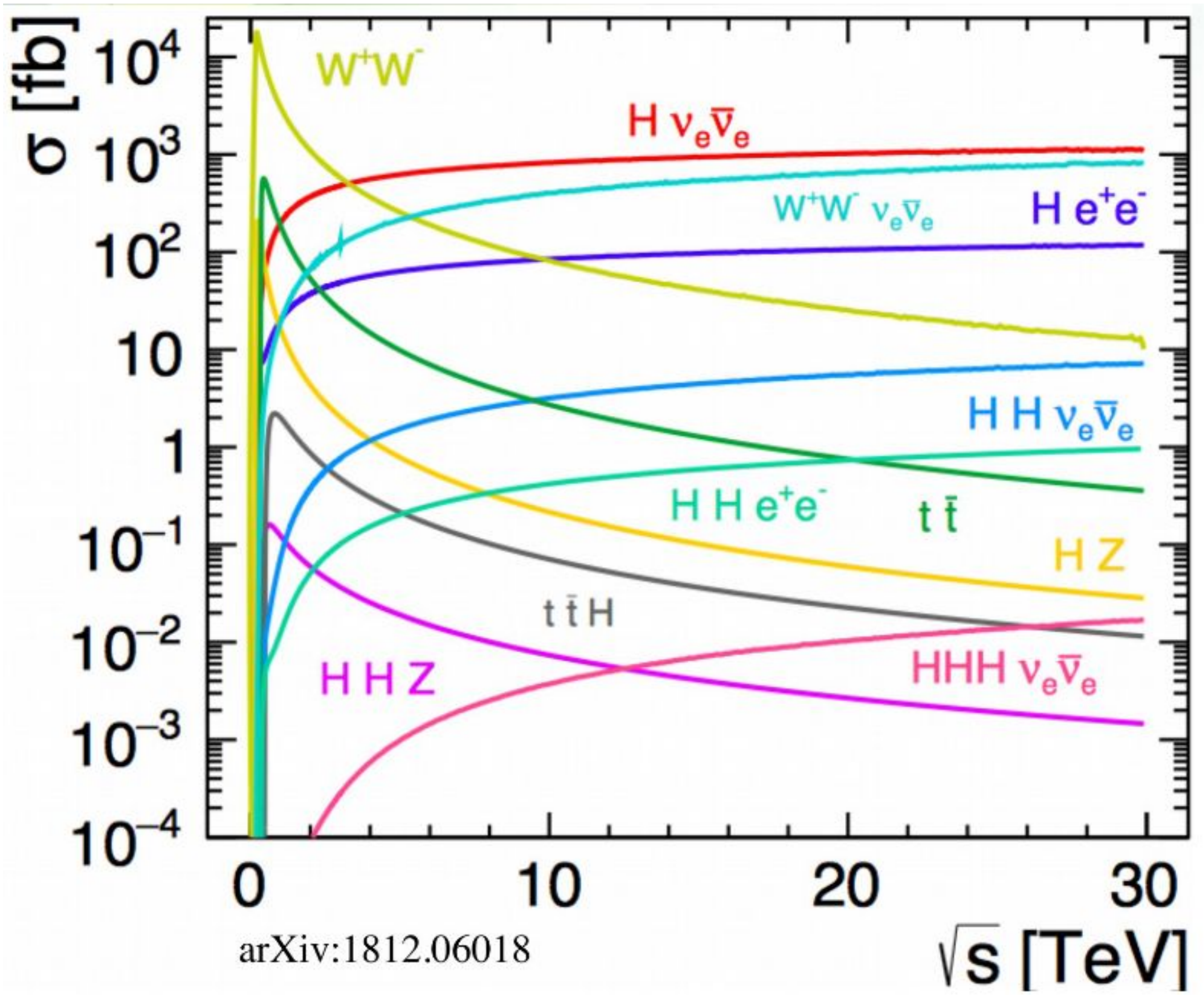
《欧洲核子研究中心快报》创刊于1959年，是欧洲核子研究中心报道全球高能物理重要进展的期刊。


**Novel collider concept**
Peking University physicists urge the community to consider the merits of a novel electron–muon collider (arXiv:2010.15144). Collisions between different species of lepton could reduce physics backgrounds for studies of charged-lepton flavour violation and Higgs-boson properties, and the asymmetric nature of the collisions could be used to control troublesome backgrounds caused by muon decays inside the accelerator, argue the authors. The preprint proposes 10 GeV electron and muon beams initially, and upgrades culminating in a TeV-scale muon–muon collider.

“**北京大学物理学家**呼吁学界考虑一种**新型**的电子缪子对撞机及其带来的好处（arXiv:2010.15144）。**不同种类轻子的对撞，可压低物理本底，从而可以（更好地）研究带电轻子味道破坏以及希格斯玻色子的性质。**作者认为，**不对称的对撞能量构型，可控制加速器中缪子衰变带来的讨厌的本底。该预印本提议可从10GeV能级的电子和缪子束流开始，最终升级直至TeV能级的缪子—缪子对撞机。**”

# electron-muon collider: physics




- **A vector boson scattering/fusion machine**
- **No s-channel SM background**

**mu-mu collider**

# electron-muon collider: Energy benchmark

| (Benchmark\GeV) | e- | mu+ | COM | Comments |
|---|---|---|---|---|
| A | 10 | 10 | 20 | Lepton Flavor Violation |
| B | 50 | 50 | 100 | Lepton Flavor Violation |
| C | 20 | 200 | 126.5 | H->emu |
| D | 50 | 1000 | 447.21 | LFV, Higgs, Top **H ~60fb** |
| E | 100 | 1000 | 632.46 | LFV, Higgs, Top **H ~115fb** |
| F | 100 | 3000 | 1095.4 | Higgs Top **H ~300fb** |

**Mostly background free, or at most from VBS processes, e.g. e mu > v v Z.**

**Higgs xsec~210fb at CEPC@250GeV.**

# electron-muon collider: Higgs property

$$\sigma = \sigma(\nu\nu\mathrm{H}) \cdot BR(\mathrm{H} \to \mathrm{b}\bar{\mathrm{b}}) = \frac{g^2_{\mathrm{HWW}} g^2_{\mathrm{Hbb}}}{\Gamma_{\mathrm{H}}}$$

$$\frac{\Delta g_{\mathrm{Hbb}}}{g_{\mathrm{Hbb}}} = \frac{1}{2}\sqrt{\left(\frac{\Delta\sigma}{\sigma}\right)^2 + \left(\frac{\Delta \frac{g^2_{\mathrm{HWW}}}{\Gamma_{\mathrm{H}}}}{\frac{g^2_{\mathrm{HWW}}}{\Gamma_{\mathrm{H}}}}\right)^2}. \qquad \frac{\Delta\sigma}{\sigma} \simeq \frac{\sqrt{N+B}}{N},$$

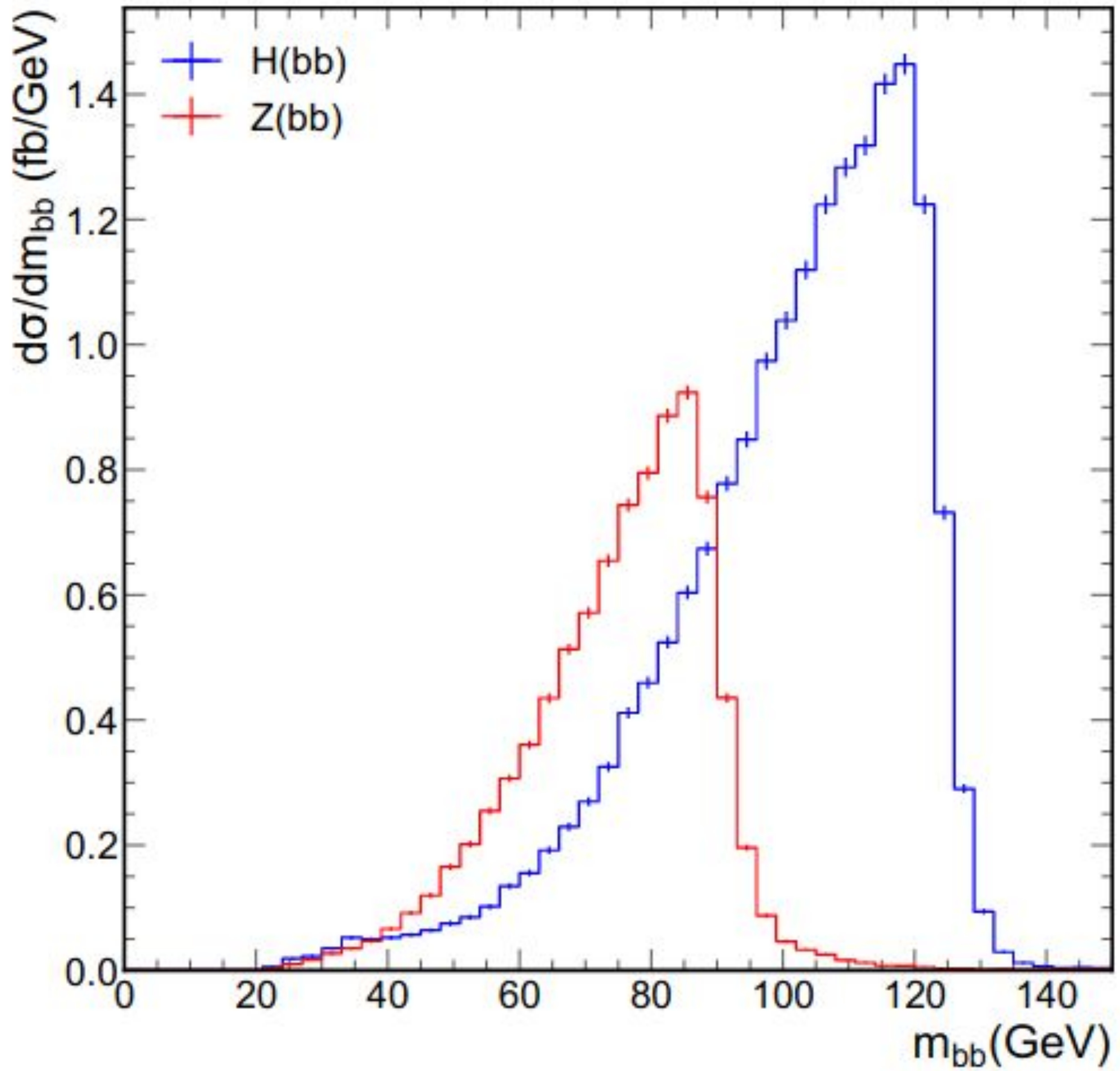
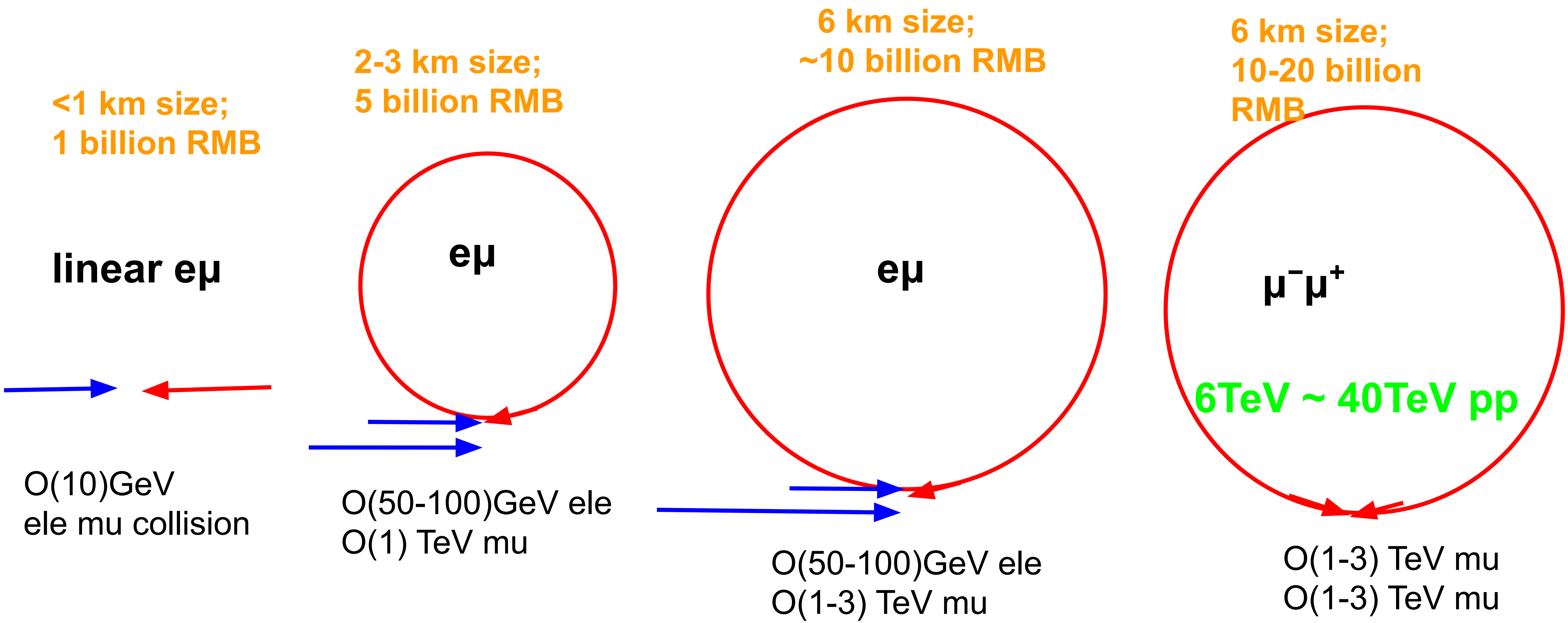



TABLE I: Summary of the parameters used in the estimation of the Higgs boson coupling to b-quarks in different collision schemes. The uncertainty on $g^2_{\mathrm{HWW}}/\Gamma_{\mathrm{H}}$ is set to 3% in all collision schemes. $\sqrt{s} = 447.2(1095.3)$ GeV corresponds to a 50(100) GeV electron beam and a 1(3) TeV muon beam. The ISR effect is not included as its effect is validated to be small.

| $\mathcal{L}_{\mathrm{int}}[ab^{-1}]$ | $\sqrt{s}$[GeV] | $\frac{\Delta\sigma}{\sigma}$[%] | $\frac{\Delta \frac{g^2_{\mathrm{HWW}}}{\Gamma_{\mathrm{H}}}}{\frac{g^2_{\mathrm{HWW}}}{\Gamma_{\mathrm{H}}}}$[%] | $\frac{\Delta g_{\mathrm{Hbb}}}{g_{\mathrm{Hbb}}}$[%] |
|---|---|---|---|---|
| 0.5 | 447.2 | 2.5 | 3 | 2.0 |
| | 1095.4 | 1.4 | | 1.7 |
| 1.5 | 447.2 | 1.4 | 3 | 1.7 |
| | 1095.4 | 0.8 | | 1.6 |
| 2.0 | 447.2 | 1.2 | 3 | 1.6 |
| | 1095.4 | 0.7 | | 1.6 |

**The measured precision of gHbb in the electron-muon collider can reach to a few percent level with order ab−1 of data and is dominated by the uncertainty on gHWW.**

# electron-muon collider: Facility and cost




**LFV, Higgs, majorana neutrino ....**

**~10-20 billion RMB in total to reach physics hopefully ~ CEPC + half-SPPC**

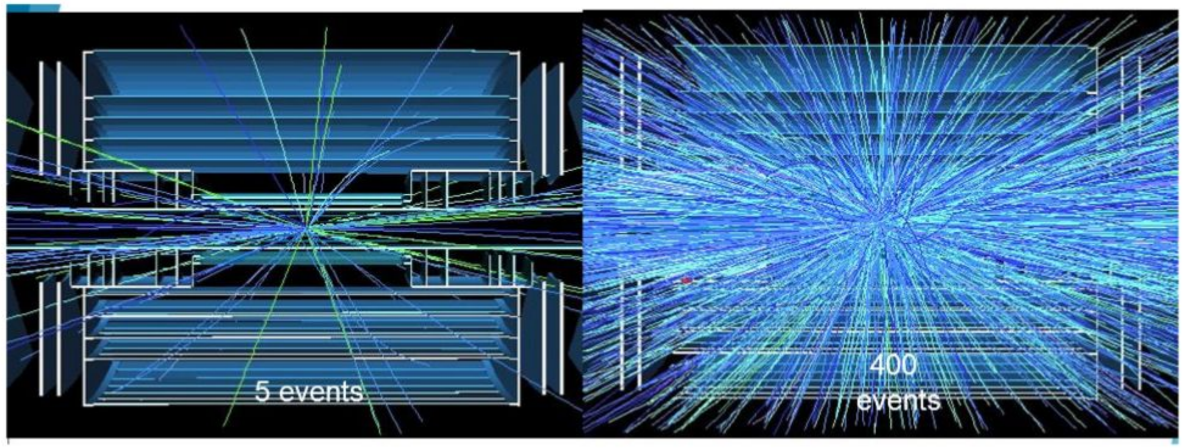
# Summary

- **An neutrino-neutron collider is quite sensitive to neutrino physics**
  - Several days of run to observe neutrino annihilation
- **An neutrino-lepton collider is quite sensitive to W mass**
  - 10MeV accuracy with 0.1/fb!
- **An electron-muon collider is sensitive to CLFV and Higgs Physics**

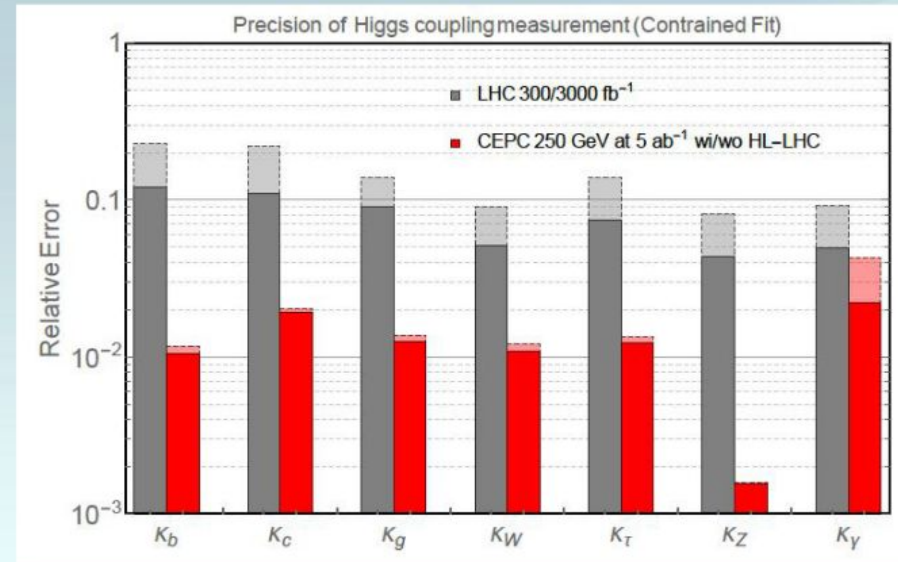

- **These colliders are both novel ideas in themselves, and may also be useful intermediate steps, with less muon cooling required, towards the muon-muon collider.**

# Additional slides

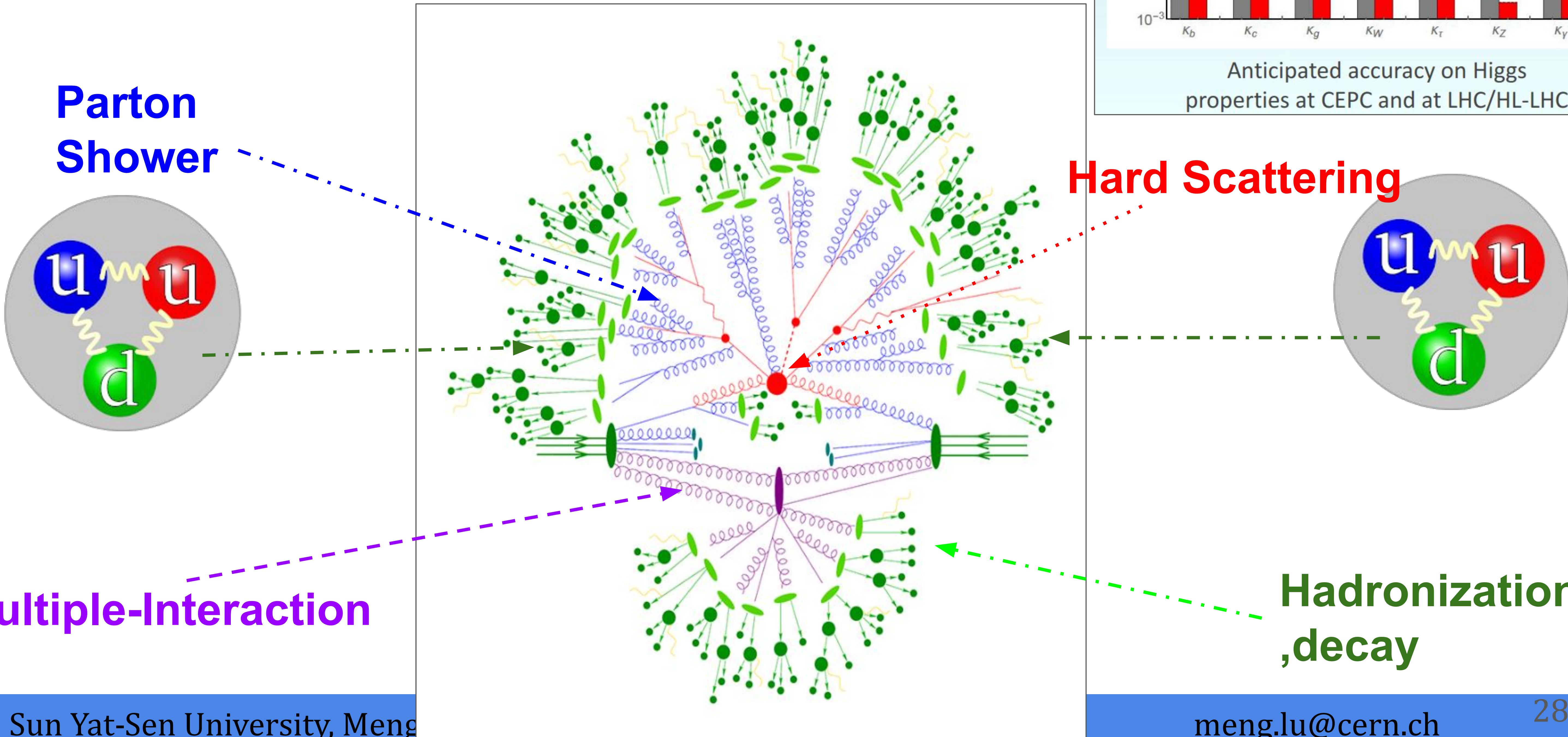
# (HL-)LHC: busy environment




Challenging yet exciting tasks on Detector, Computing, Analysis……




Anticipated accuracy on Higgs properties at CEPC and at LHC/HL-LHC



# Luminosity Estimation

$$\mathcal{L} = \frac{N_{\text{beam1}} N_{\text{beam2}}}{4\pi\sigma_x\sigma_y} f_{\text{rep}},$$

where $f_{\text{rep}}$ is the rate of collisions and is typically 100 kHz (40 MHz) for lepton colliders (hadron colliders), and $N_{\text{beam1,2}}$ are the number of particles in each bunch which can be taken as $\sim 10^{11}$–$10^{12}$, $\sigma_x$ and $\sigma_y$ are the beam sizes.

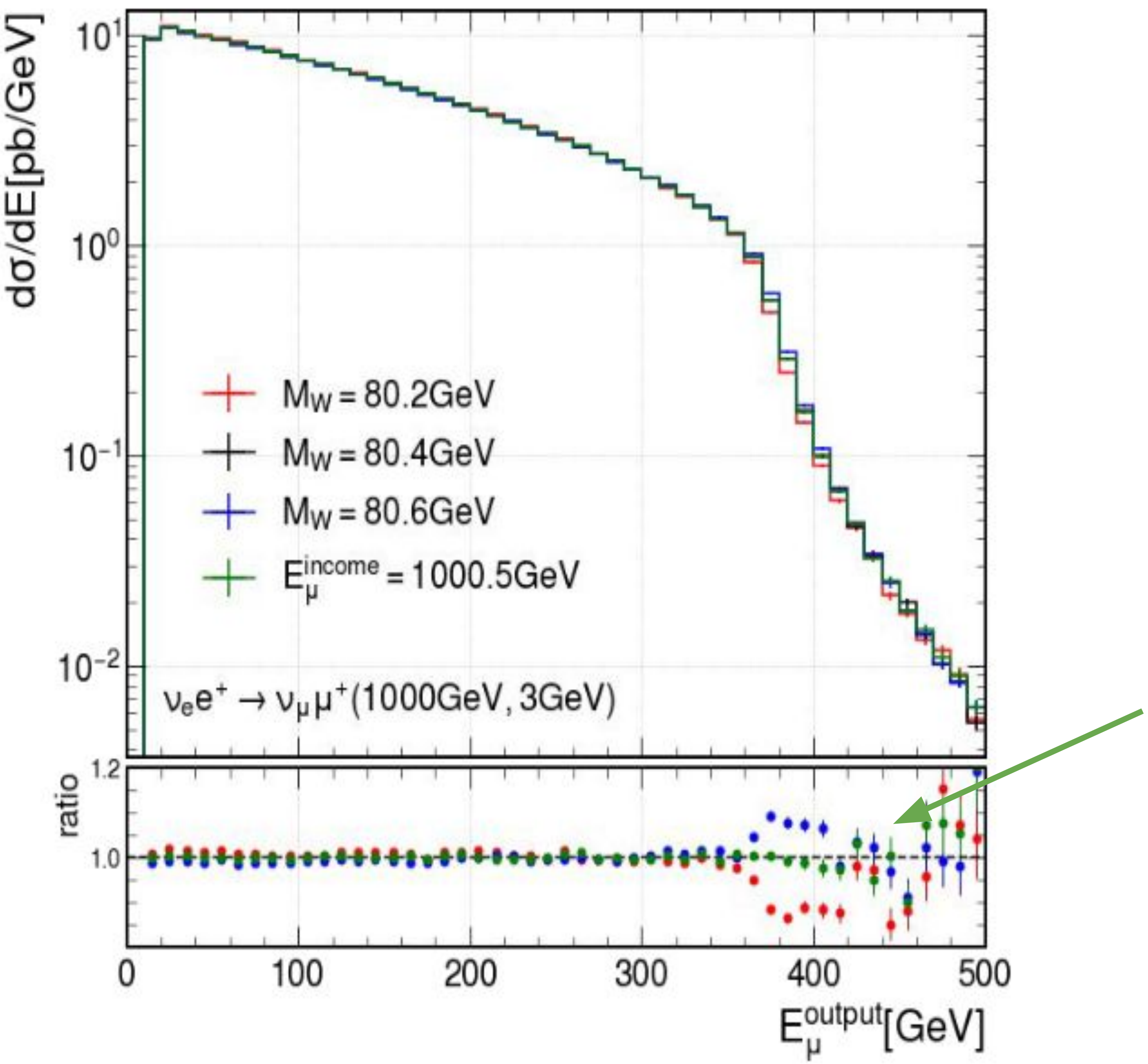
Take the LHC as an example, with $f_{\text{rep}} = 40\,\text{MHz}$, $\sigma_{x,y} = 16$ microns, and $N_{\text{beam1,2}} = 10^{11}$, one can get $\mathcal{L} = 10^{34}$ $\text{cm}^{-2}\text{s}^{-1}$.

As for TeV muon colliders, with $f_{\text{rep}} = 100\,\text{KHz}$, $\sigma_{x,y} \lesssim 10$ microns, and $N_{\text{beam1,2}} = 10^{12}$, then $\mathcal{L} = 10^{33}$–$10^{34}$ $\text{cm}^{-2}\text{s}^{-1}$.

As for the neutrino neutrino collisions discussed above, there are further suppression factors from linear over arc ratio ($L_l^2/L_c^2 \sim 1/100$) with the exact value depending on the realistic design, and the neutrino beam spread which can be around 1000 microns for $L_l \sim 10$ to 100 meters. Taking all these into account, a realistic instantaneous luminosity for neutrino neutrino collisions can reach around $\mathcal{L} = 10^{28}$ $\text{cm}^{-2}\text{s}^{-1}$ level. Although it is a small number, however, to reach the discovery threshold of neutrino antineutrino annihilation process $\nu_e\tilde{\nu}_e \to Z$, a tiny integrated luminosity of about $10^{-5}$ $\text{fb}^{-1}$ is needed, i.e., several days of data taking.

**正反中微子湮灭可以较快地被观测到！ 对束流质量要求可以放松。**

# Neutrino collision: neutrino-lepton collision



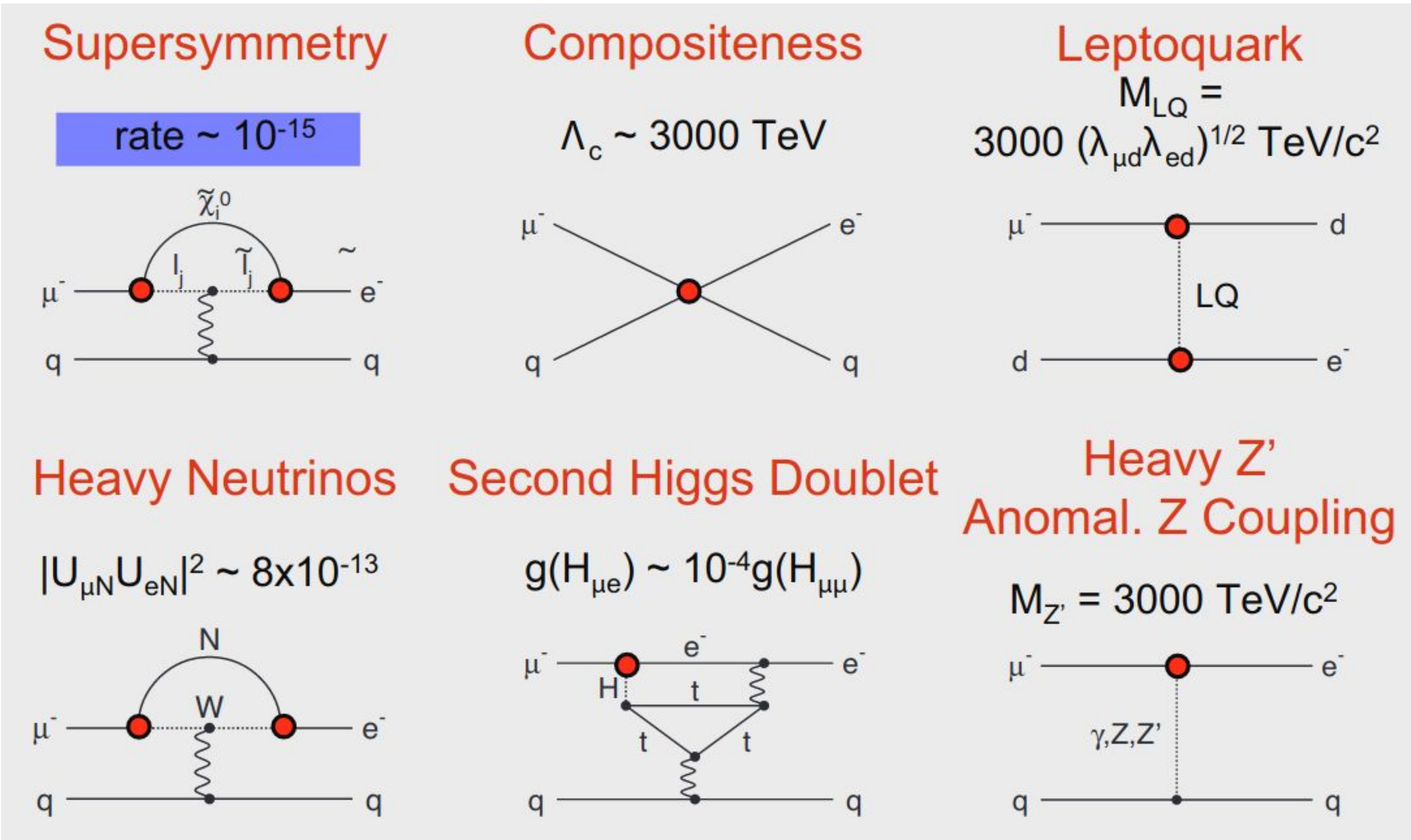

**We varied the incoming muon and electron beam energy by 0.5 GeV and 10 MeV,** respectively, which are quite conservative following previous refs.

We found that the **cross sections changed by about 0.6 pb for both variations.**

This uncertainty could be **mitigated by using the shape of the outgoing muon energy**, by scanning different incoming beam energies, or by calibrating the incoming muon beam energy with the electron decay products.

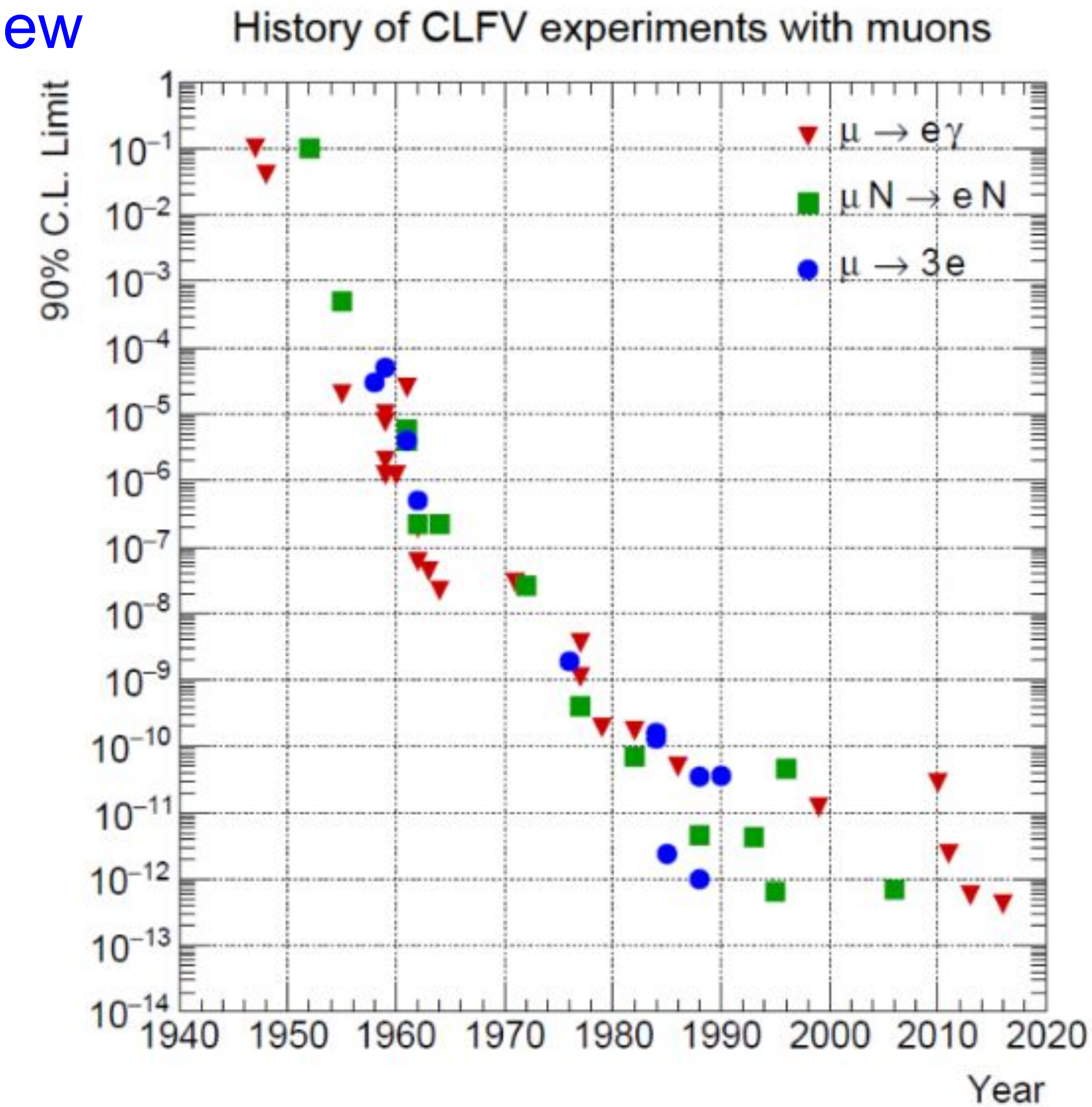
# Lepton Flavor Violation

Discovery of Charged Lepton Flavor Violation is New Physics! violation of a (so-far) conservation law.





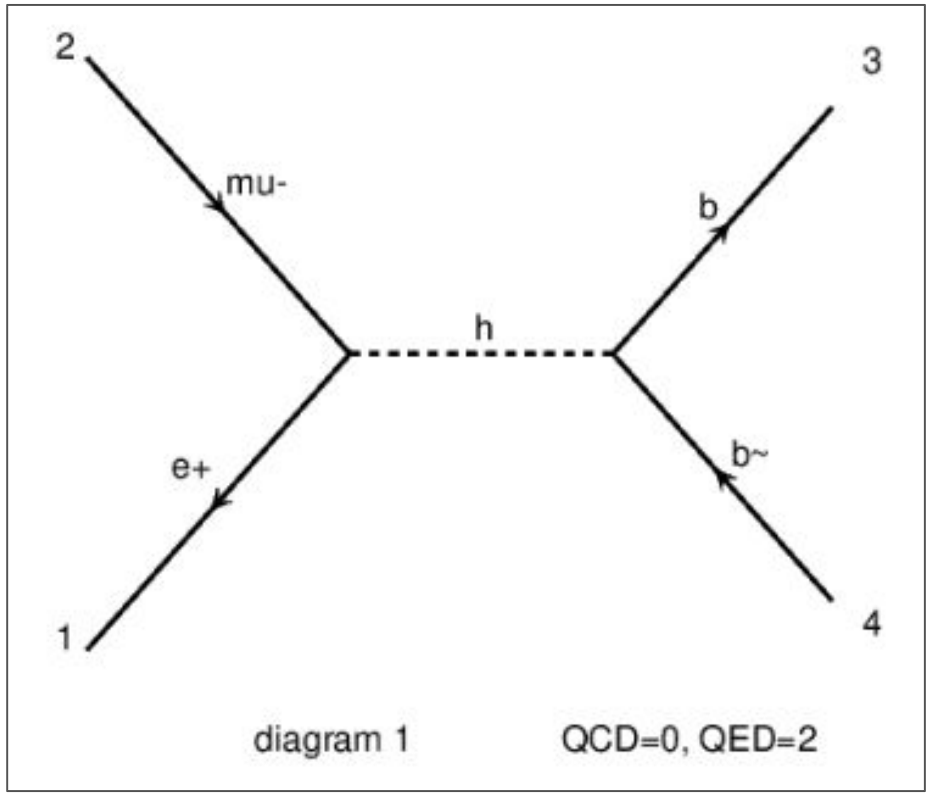
CLFV2019




https://arxiv.org/abs/1801.04688

# Emu: Lepton Flavor Violation arXiv:2010.15144

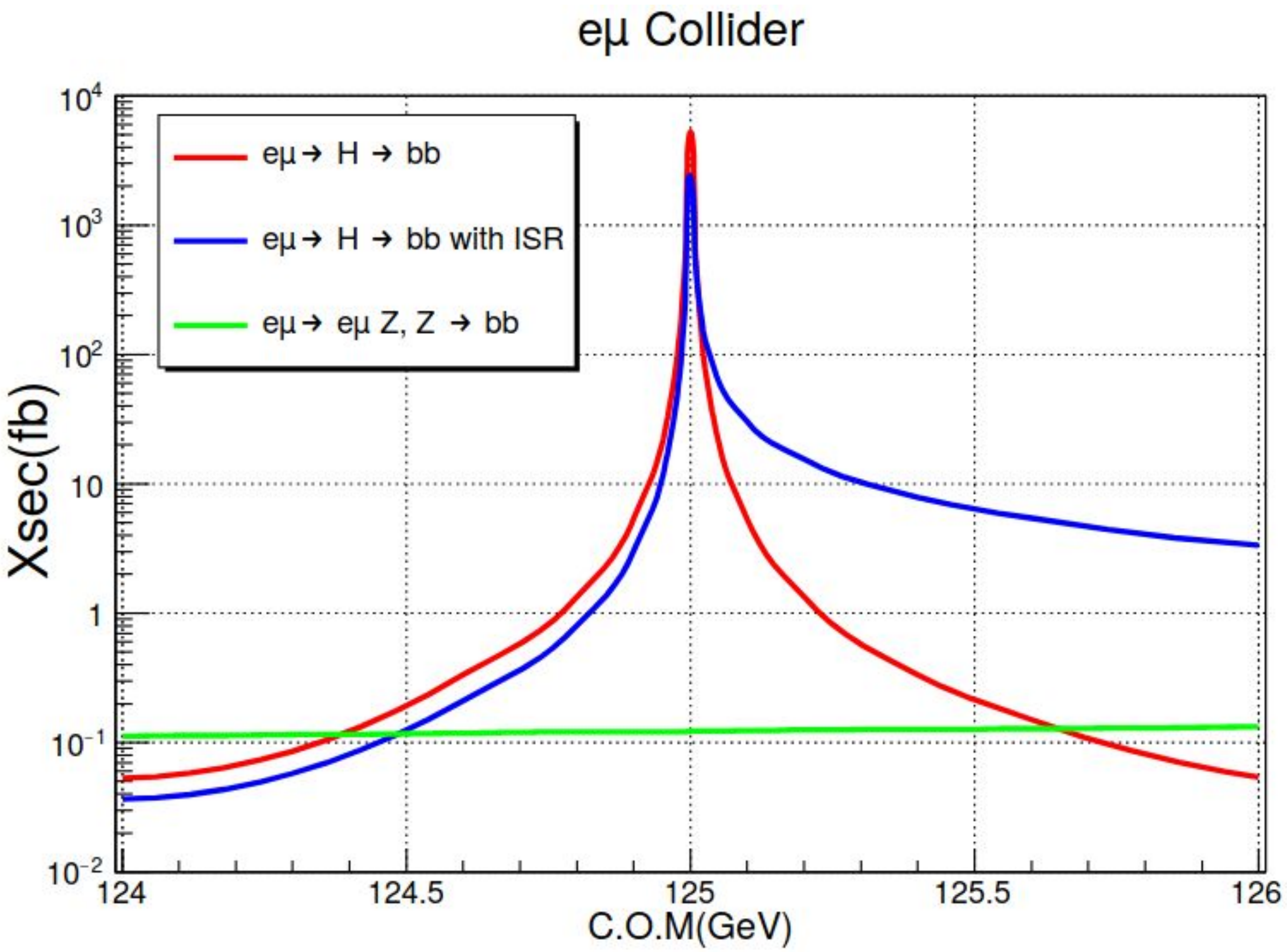
$$L_V \equiv -Y_{e\mu}\bar{e}_L\mu_R H - Y_{\mu e}\bar{\mu}_L e_R H - Y_{e\tau}\bar{e}_L\tau_R H - Y_{\tau e}\bar{\tau}_L e_R H - Y_{\mu\tau}\bar{\mu}_L\tau_R H - Y_{\tau\mu}\bar{\tau}_L\mu_R H$$




$$\Gamma(H \to \ell^\alpha \ell^\beta) = \frac{M_H}{8\pi}\left(|Y_{\ell^\beta \ell^\alpha}|^2 + |Y_{\ell^\alpha \ell^\beta}|^2\right)$$

$$\mathcal{B}(H \to \ell^\alpha \ell^\beta) = \frac{\Gamma(H \to \ell^\alpha \ell^\beta)}{\Gamma(H \to \ell^\alpha \ell^\beta) + \Gamma_{SM}}$$

Current best limit from ATLAS: $\mathrm{Br}(H \to e\mu) < 6.2 \times 10^{-5}$
CEPC Projection: $\mathrm{Br}(H \to e\mu) < 1.2 \times 10^{-5}$




**Model implemented in MG, w or wo ISR;**

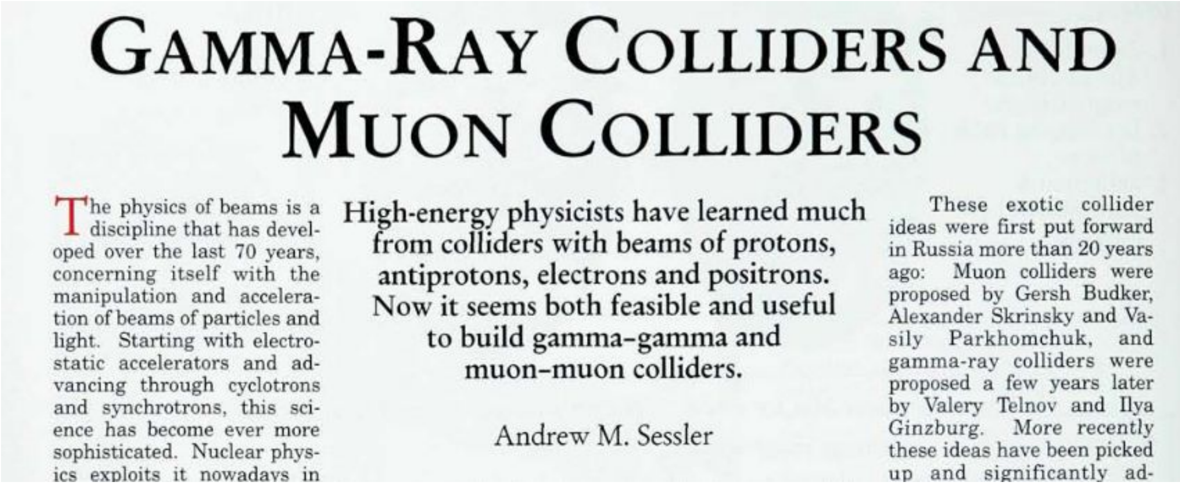
**Signal at peak ~ 5.3pb, while bkg ~0.1fb**

**A simple estimation gives 100 times better limit than ATLAS, with only 1/fb.**

# Facility and cost estimation

**Total Project Cost (TPC) model** in US accounting (EU accounting might be 2-3 times lower):
"civil construction", "accelerator components", "site power infrastructure"

$$TPC \approx \alpha \times (Length/10km)^{1/2} + \beta \times (Energy/TeV)^{1/2} + \gamma \times (Power/100MW)^{1/2} \quad , \quad (1)$$



GAMMA-RAY COLLIDERS AND MUON COLLIDERS

High-energy physicists have learned much from colliders with beams of protons, antiprotons, electrons and positrons. Now it seems both feasible and useful to build gamma-gamma and muon-muon colliders.

Andrew M. Sessler

The physics of beams is a discipline that has developed over the last 70 years, concerning itself with the manipulation and acceleration of beams of particles and light. Starting with electrostatic accelerators and advancing through cyclotrons and synchrotrons, this science has become ever more sophisticated. Nuclear physics exploits it nowadays in

These exotic collider ideas were first put forward in Russia more than 20 years ago: Muon colliders were proposed by Gersh Budker, Alexander Skrinsky and Vasily Parkhomchuk, and gamma-ray colliders were proposed a few years later by Valery Telnov and Ilya Ginzburg. More recently these ideas have been picked up and significantly ad-

**1TeV Muon beam:**
2B$*(4km/10km)^0.5+2B$*(2)^0.5+2B$*(100MW/100MW)^0.5
~6B$

(3TeV Muon ~8B $)

**If 3 times larger**

**2B$ in total**

**Note:**

- **electron part cost is relatively small**
- **O(100)GeV e-mu collider cost much less**

Physics Today 51, 3, 48 (1998)

"But the result might well be a machine that is less expensive than an ee linear collider with the same final energy, though a TeV muon collider would still be a **billion-dollar** undertaking."

## B Luminosities at Neutrino Experiments

For a cylindrical experimental target extending out from the beam center to an angle $\theta_\mu = 1/\gamma_\mu$, the luminosity, $\mathcal{L}$, is proportional to the product of the mass depth of the target, $l$, and the number of muon decays per second in the beam production straight section, according to:

$$\mathcal{L}[\mathrm{cm}^{-2}.\mathrm{s}^{-1}] = \mathrm{N}_{\mathrm{Avo}} \times \mathrm{f}_{\mathrm{ss}} \times \mathrm{n}_\mu\,[\mathrm{s}^{-1}] \times l[\mathrm{g.cm}^{-2}], \tag{3}$$

where $\mathrm{f}_{\mathrm{ss}}$ is the fraction of the collider ring circumference occupied by the production straight section, $n_\mu$ is the rate at which each sign of muons is injected into the collider ring (assuming they all circulate until decay rather than being eventually extracted and dumped) and the appropriate units are given in square brackets in this equation and all later equations in this paper. The proportionality constant is Avagadro's number, $\mathrm{N}_{\mathrm{Avo}} = 6.022 \times 10^{23}$, since exactly one neutrino per muon is emitted on average into the boosted forward hemisphere, i.e. each muon decay produces two neutrinos and half of them travel forwards in the muon rest frame.