Self-Resonant Dark Matter: Frameworks and Models

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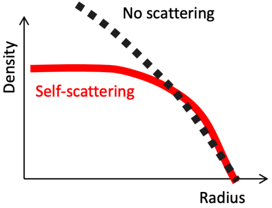
July 24, 2022

Motivation for self-resonant dark matter

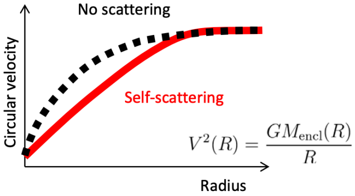
Motivations for self-interacting Dark Matter

There are some mismatches between ΛCDM and observations:

Core-Cusp Problem, Diversity Problem



Particles get scattered out of dense halo centers



Rotation curves reduced (apparent mass deficit in inner halo)

Building up Self-interacting Dark Matter

What scattering cross section value is needed?

- Rate equation:

$$R_{\mathrm{scat}} = \sigma v_{\mathrm{rel}} \rho_{\mathrm{dm}}/m \approx 0.1\mathrm{Gyr}^{-1} \times \left(\frac{\rho_{\mathrm{dm}}}{0.1 M_{\odot}/\mathrm{pc}^3} \right) \left(\frac{v_{\mathrm{rel}}}{50\ \mathrm{km/s}} \right) \left(\frac{\sigma/m}{1\ \mathrm{cm}^2/\mathrm{g}} \right)$$

- Figure-of-merit:

$$\sigma/m_{\chi} \sim 1\ \mathrm{cm}^2/\mathrm{g} \approx 2\mathrm{barns/GeV} \approx \left(\frac{1}{60\mathrm{MeV}} \right)^3$$

Astrophysics points to dark physics at the MeV-GeV scale Motivates searches for light dark states but doesn't say how they couple to SM

Particle physics of self-interactions

- WIMPs have self-interactions (weak interaction):

$$\sigma \sim \frac{g^4 m_\chi^2}{m_Z^4} \sim 10^{-36}\ \mathrm{cm}^2$$

- WIMP self-interaction cross section is way too small

$$\sigma / m_\chi \sim 10^{-14}\ \mathrm{cm}^2/\mathrm{g}$$

- Large cross section required

$$\sigma \sim \frac{g^4 m_\chi^2}{m_\phi^4}$$

- Mediator mass below than weak scale

$$m_\phi \sim 1 - 100 \mathrm{MeV}$$

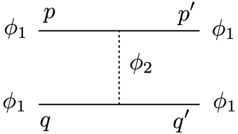
Self-interactions require new dark sector states < 1 GeV: Existence of light mediator

Conventional approach: t-channel resonance

Consider dark matter and mediator with masses, m_1, m_2. Triple coupling for dark matter and mediator:

$$\mathcal{L}_{\text{int}} = -2gm_1\phi_2 \left|\phi_1\right|^2$$

Elastic $2 \to 2$ scattering in t-channel



$$\tilde{\Gamma}_t(p,q;p',q') = \frac{4g^2m_1^2}{|\vec{p}-\vec{p}'|^2 + m_2^2 - \omega^2}$$

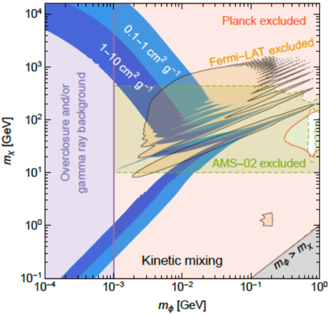
$w = p_0 - p_0' \simeq 0:$ "Instantaneous" interaction

Need of a small mass m_2 for enhanced t-channel! Resummation of ladder diagrams: Sommerfeld factor

Strong Bounds on light mediator

Light mediators enhance dark matter annihilations/detection from non-perturbative corrections:

Strong constraints from indirect detections and CMB ionizations.



[Bringmann et al, 2016]

Way out: generate non-perturbative correction without light mediator

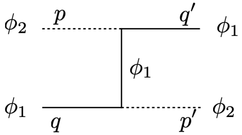
General Framework for self-resonant dark matter

Our approach: u-channel resonance

Take two component dark matter with masses m_1, m_2. Triple coupling between dark matter:

$$\mathcal{L}_{\rm int} = -2gm_1\phi_2\left|\phi_1\right|^2$$

Elastic $2 \to 2$ scattering in u-channel



$$\tilde{\Gamma}_u(p,q;p',q') = \frac{4g^2m_1^2}{|\vec{p}-\vec{q}'|^2 + m_1^2 - \omega^2}$$

[S. Kim, HML, B. Zhu, 2021]

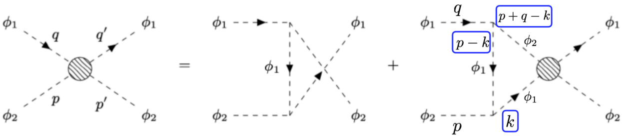
$\omega = p_0 - q_0' \approx m_2 - m_1 \neq 0$ "Delayed" interaction

$m_2 = 2m_1 \longrightarrow$ "Effectively massless" off-shell DM

Resummation of ladder diagrams is also needed. The particle itself became a mediator and present resonance. That is why we call it 'Self-Resonant'!

Non-perturbative scattering

Non-perturbative scattering in u-channel:



$$i\Gamma\left(p,q;p',q'\right)=i\tilde{\Gamma}\left(p,q;p',q'\right)-\int\frac{d^4k}{(2\pi)^4}\widetilde{\Gamma}(p,q;p+q-k,k)G_1(k)G_2(p+q-k)\Gamma\left(p+q-k,k;p',q'\right)$$

Bethe-Salpeter wave function: (Tree-level contribution ignored)

$$\chi\left(p,q;p',q'\right)\equiv G_2(p)G_1(q)\Gamma\left(p,q;p',q'\right)\equiv\chi(p,q)$$

Bethe-Salpeter equation

- Change of variables:

$$P = \frac{1}{2}(p+q), \quad Q = \mu\left(\frac{p}{m_2} - \frac{q}{m_1}\right) \rightarrow \begin{cases} \chi(p,q) = \widetilde{\chi}(P,Q) \\ \chi(p+q-k,k) = \widetilde{\chi}\left(P, \frac{2\mu}{m_2}P - k\right) \end{cases}$$

- BS function in terms of new variables:

$$i\widetilde{\chi}(P,Q) = -G_2\left(Q + \frac{2\mu}{m_1}P\right) G_1\left(-Q + \frac{2\mu}{m_2}P\right) \int \frac{d^4k'}{(2\pi)^4} \widetilde{\Gamma}(p,q;p+q-k,k)\widetilde{\chi}\left(P,k'\right)$$

- Tree-level amplitude:

$$\widetilde{\Gamma}(p,q;p+q-k,k) = \frac{4g^2m_1^2}{\left(\sqrt{\frac{m_1}{m_2}}\vec{Q} + \sqrt{\frac{m_2}{m_1}}k'\right)^2 + m_2\left(2m_1 - m_2\right)} \equiv U$$

- BS wavefunction in momentum space:

$$\widetilde{\psi}_{BS}(\vec{Q}) = \int \frac{dQ_0}{2\pi} \widetilde{\chi}(P,Q)$$

Finally, we obtain BS equation

$$i\widetilde{\psi}_{BS}(\vec{Q}) = -\int \frac{dQ_0}{2\pi} G_2\left(Q + \frac{2\mu}{m_1}P\right) G_1\left(-Q + \frac{2\mu}{m_2}P\right) \int \frac{d^3k'}{(2\pi)^3} U\widetilde{\psi}_{BS}\left(\vec{k}'\right)$$

Reduce Bethe-Salpeter equation into Schrodinger equation

Non-relativistic limit $P = \frac{1}{2}(m_1 + m_2 + E, 0), Q = \left(Q_0, \vec{Q}\right)$:

$$\int \frac{dQ_0}{2\pi} G_2\left(Q + \frac{2\mu}{m_1}P\right) G_1\left(-Q + \frac{2\mu}{m_2}P\right) = \frac{i}{4m_1 m_2\left(E - \frac{\vec{Q}^2}{2\mu}\right)}$$

Then, BS equation becomes:

$$\left(\frac{\vec{Q}^2}{2\mu} - E\right)\widetilde{\psi}_{BS}(\vec{Q}) = -\frac{1}{4m_1 m_2}\int \frac{d^3k'}{(2\pi)^3} U \widetilde{\psi}_{BS}\left(\vec{k}'\right)$$

or in coordinate space:

$$\left(-\frac{1}{2\mu}\nabla^2 - E\right)\psi_{BS}(\vec{x}) = -V(\vec{x})\psi_{BS}\left(-\frac{m_2}{m_1}\vec{x}\right)$$

Notice a huge difference in wave-function with t-channel:

$\psi(-m_2/m_1 x)$ filpes sign and has additional dependence

Delay differential equation

Expand Bethe-Salpeter in spherical coordinate:

$$\psi_{\mathrm{BS}}(\vec{x}) = R_l(r) Y_l^m(\theta,\phi) \to \psi_{\mathrm{BS}}\left(-\frac{m_2}{m_1}\vec{x}\right) = (-1)^l R_l\left(\frac{m_2}{m_1} r\right) Y_l^m(\theta,\phi)$$

Focus on the radial part: $R_l(x) = u_l(x)/x, \quad a = \frac{2v_{\mathrm{rel}}}{\alpha}, b = \frac{m_2}{m_1}$ and $c = \frac{2M}{\mu\alpha}$

$$\left(\frac{d^2}{dx^2} - \frac{l(l+1)}{x^2}\right) u_l(x) + \frac{4e^{-cx}}{bx}(-1)^l u_l(bx) + a^2 u_l(x) = 0$$

- Attractive for $l = \mathrm{even}$, repulsive for $l = \mathrm{odd}$
- Effective mediator mass: $M \equiv m_2\sqrt{2 - \frac{m_2}{m_1}} \to 0, \quad m_2 \to 2m_1$
- Make use of $x = \exp(-\rho)$ to obtain the canonical delay differential equation:

$$\tilde{u}_0''(\rho) + \tilde{u}_0'(\rho) + 2e^{-\rho}\tilde{u}_0(\rho - \ln 2) + a^2 e^{-2\rho}\tilde{u}_0(\rho) = 0$$

Implications after solving delay differential equation: Sommerfeld enhancement

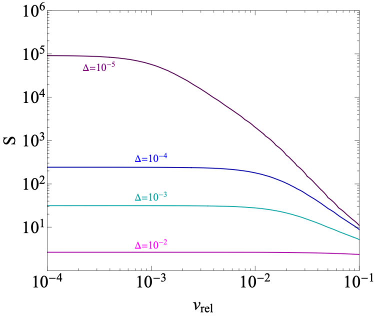
Imposing the boundary conditions and considering s-wave:

$$\tilde{u}_0(\rho) \longrightarrow \frac{1}{a}\sin\left(ae^{-\rho}+\delta_0\right), \quad \rho \to -\infty,$$
$$\tilde{u}_0(\rho) \longrightarrow Ae^{-\rho}, \quad \rho \to +\infty$$

Sommerfeld fator is

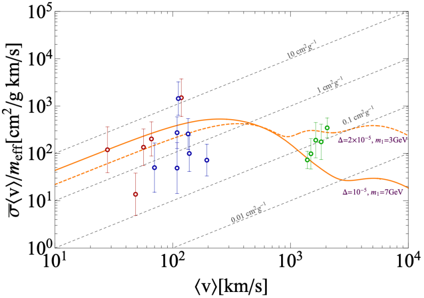
$$S = \frac{|\psi_{\rm BS}(0)|^2}{|\psi_{\sf pert}(0)|^2} = A^2$$

with Effective mass $\Delta = 1 - \frac{m_2}{2m_1} \geq 0$



Implications after solving delay differential equation: u-channel Self-scattering

Self-scattering for SRDM is velocity-dependent



$\phi_1\phi_2 \to \phi_1\phi_2$: $\sigma = \frac{4\pi}{k^2}\sin^2\delta_0$, s-wave cross section

Models for u-channel resonances

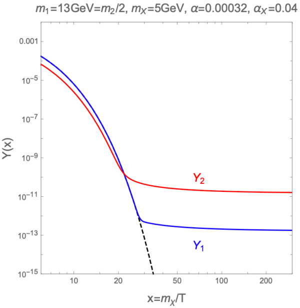
Classification of dark matter annihilation

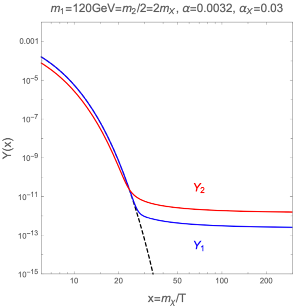
- $2 \to 2$ annihilation:
  - $\phi_2\phi_2 \to \phi_1\phi_1$, no Sommerfeld enhancement at initial state.
  - $\phi_1\phi_1 \to XX$, X is extra mediator for ϕ_1, in order not the threaten BBN and CMB
  - $\phi_1\phi_2 \to \phi_1 X$, Sommerfeld enhanced!
- $3 \to 2$ annihilation
  - $\phi_1\phi_1\phi_1 \to \phi_1\phi_2$, s and u-channel enhanced.
  - $\phi_1\phi_1\phi_1 \to \phi X$, phase space suppressed.

Good enough to derive the Boltzmann equation

$$
\begin{aligned}
\dot{n}_{\phi_1} + 3Hn_{\phi_1} =& \langle\sigma v\rangle_{\phi_2\phi_2\to\phi_1\phi_1^*}\left(n_{\phi_2}^2 - n_{\phi_1}^2\right) \\
&- \langle\sigma v\rangle_{\phi_1\phi_1^*\to XX}\left(n_{\phi_1}^2 - \left(n_{\phi_1}^{\rm eq}\right)^2\right) \\
&- 2\left\langle\sigma v^2\right\rangle_{\phi_1\phi_1\phi_1^*\to\phi_1\phi_2} n_{\phi_1}\left(n_{\phi_1}^2 - \frac{\left(n_{\phi_1}^{\rm eq}\right)^2}{n_{\phi_2}^{\rm eq}} n_{\phi_2}\right) \\
&- 2\left\langle\sigma v^2\right\rangle_{\phi_1\phi_1\phi_1^*\to\phi_1 X} n_{\phi_1}\left(n_{\phi_1}^2 - \left(n_{\phi_1}^{\rm eq}\right)^2\right) \\
\dot{n}_{\phi_2} + 3Hn_{\phi_2} =& -2\langle\sigma v\rangle_{\phi_2\phi_2\to\phi_1\phi_1^*}\left(n_{\phi_2}^2 - n_{\phi_1}^2\right) \\
&- \langle\sigma v\rangle_{\phi_1\phi_2\to\phi_1 X} n_{\phi_1}\left(n_{\phi_2} - n_{\phi_2}^{\rm eq}\right) \\
&+ \left\langle\sigma v^2\right\rangle_{\phi_1\phi_1\phi_1^*\to\phi_1\phi_2} n_{\phi_1}\left(n_{\phi_1}^2 - \frac{\left(n_{\phi_1}^{\rm eq}\right)^2}{n_{\phi_2}^{\rm eq}} n_{\phi_2}\right)
\end{aligned}
$$

Dark matter relic density





Two-component dark matter can be equally abundant.

EFT for self-resonant DM

dark matter	scalar	pseudo-scalar	fermion	vector	axial-vector
scalar(ϕ)	$+4g^2m_\phi^2$	$+4g^2m_\phi^2$	$\pm 2y_\chi^2 m_\chi(2m_\chi - m_\phi)$	NA	NA
pseudo-scalar(a)	–	–	$\mp 2\lambda_\chi^2 m_\chi m_a$	NA	NA
fermion(χ)	–	–	NA	$\mp 2g_{Z'}^2 m_\chi m_{Z'}$	$\pm 2g_{A'}^2 m_\chi(2m_\chi - m_{A'})$
vector(Z')	NA	NA	–	$-6g_X^2 m_X(2m_X - m_{X_3})$	NA
axial-vector(A')	NA	NA	–	–	NA

Table 1: Tree-level amplitudes for elastic co-scattering for multi-component dark matter with minimal couplings, divided by the u-channel propagator. When there are two signs, the upper (lower) sign denotes the fermion (anti-fermion) scattering. "NA" implies that the processes are disallowed or velocity-suppressed. For vector (X)-vector (X_3) scattering, we consider gauge bosons in a non-abelian gauge theory in the dark sector. Here, X is a complex gauge boson and X_3 is a real gauge boson. We remark that the overall positive sign implies an attractive Yukawa potential for the s-wave scattering.

Scalar-(pseudo)scalar, fermion-pseudoscalar(vector) lead to u-channel enhanced self-scattering

Conclusions

- Non-perturbative effects are important for understanding dark matter self-scattering and annihilation.
- Dark matter self-scattering can be delayed due to an u-channel exchange of dark matter, mimicking a long-range interaction without a light mediator.
- Non-perturbative scattering amplitude for two- component dark matter is obtained a la Bethe- Salpeter.
- Multi-component dark matter with u-channel resonances identified; concrete model building anticipated.

Back up

Notations

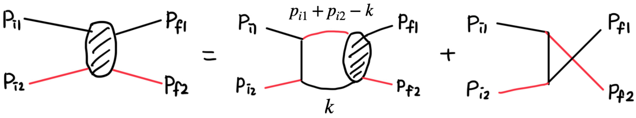
- $G_i(q), (i = 1, 2)$: particle i propagator with momentum transfer q, $\frac{i}{q^2 - m_i^2}$
- $i\tilde{\Gamma}$: 4 point vertex via single vertex exchange for $q = p_{f2} - p_{i1}$

$$-\frac{4im_1^2 g^2}{q^2 - m_1^2} \simeq 4im_1^2 g^2 \left[m_2 \left(2m_1 - m_2\right) + \left(\sqrt{\frac{m_1}{m_2}} \vec{p}_{f2} - \sqrt{\frac{m_2}{m_1}} \vec{p}_{i1} \right)^2 \right]^{-1}$$

  Note that ϕ_1 always mediate u-channel interaction.
- $i\Gamma$: total 4 point vertex

Bethe-Salpeter Equation

This is What we are going to calculate. To confirm whether enhancement occurs or not.



$$i\Gamma(p_{i1},p_{i2};p_{f1},p_{f2}) = -\int \frac{d^4k}{(2\pi)^4}\tilde{\Gamma}(p_{i1},p_{i2};k,p_{i1}+p_{i2}-k)G_1(k)G_2(p_{i1}+p_{i2}-k)\Gamma(k,p_{i1}+p_{i2}-k;p_{f1},p_{f2})$$

$$\cancel{+i\tilde{\Gamma}(p_{i1},p_{i2},p_{f1},p_{f2})}$$

Seems like Integral Form of Schrodinger equation

Approximation. First order contribution is much smaller than higher-order terms