Quark-Gluon Plasma (QGP) & Heavy Ion Collisions (HIC)

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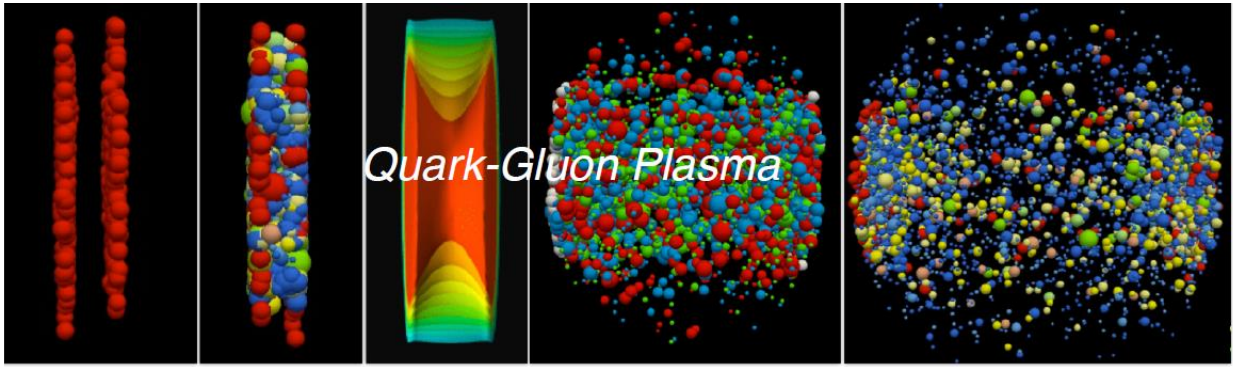


第1次课: *A new state of matter: QGP*
第2次课: *The only way to realize QGP: HIC*
第3次课: *Heavy flavor hadrons as a probe of QGP*

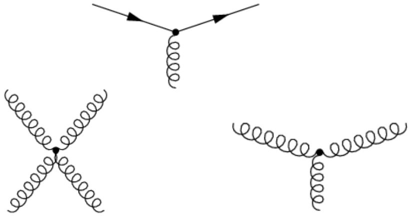
第1次课: *A new state of matter: QGP*

QCD

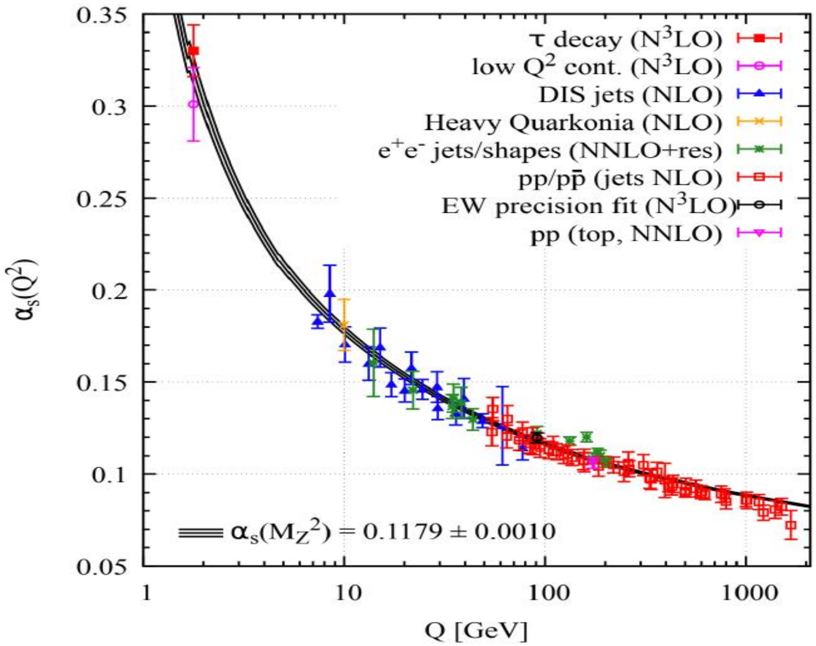
$$standard\ model \begin{cases} Electromagnetic\ initeraction \\ Weak\ interaction \\ Strong\ interaction, Quantum\ chromodynamics\ (QCD) \end{cases}$$

$$\mathcal{L}_{QCD} = \bar{\psi}_i \left(i\gamma_\mu D^\mu_{ij} - m\delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

$$D^\mu_{ij} = \delta_{ij}\partial^\mu + i\, g\, A^\mu_a T^a_{ij} \qquad G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\, f^{abc} A^b_\mu A^c_\nu$$



Non-Abelian interaction → QCD running coupling



Short range: weak coupling, asymptotic freedom, pQCD

2004 诺贝尔物理奖

David J. Gross

H. David Politzer

Frank Wilczek

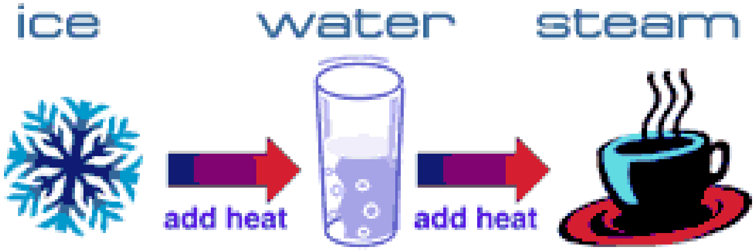
Long range: strong coupling, color confinement, non-perturbative properties !

物质的状态

可否通过热运动，即多体相互作用，来改变非微扰性质？

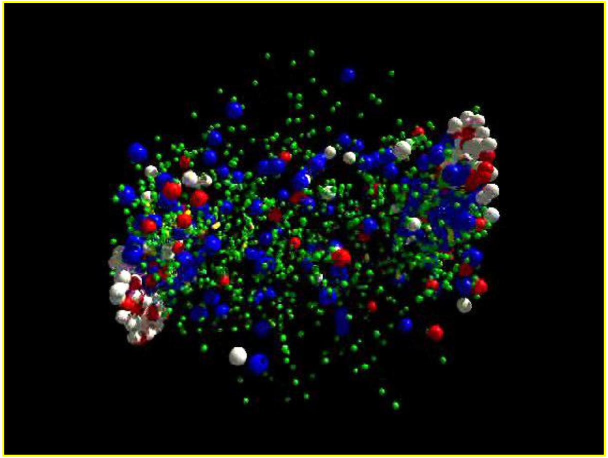
问题：看不见的夸克，质量起源，真空结构
　　　真空不知道，动动真空怎么样？

熟悉的物质三态：固态，液态，气态



第四态：原子弹爆炸后产生大范围的电磁等离子体

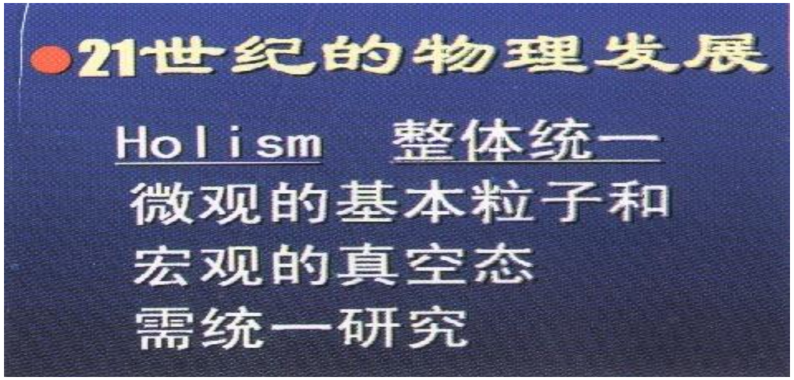
猜测：是否存在由夸克和胶子组成的QCD等离子体(QGP)？



统一研究物质的结构与状态

李政道【*Rev. Mod. Phys. 47, 267(1975)*】

In order to study the question of ‘vacuum’, we must turn to a different direction: we should investigate some ‘bulk’ phenomena by distributing high energy over a relatively large volume.”



●21世纪的物理发展

Holism 整体统一

微观的基本粒子和

宏观的真空态

需统一研究

from 1897 to present:

to comprehend the largest, we need only understand the smallest.

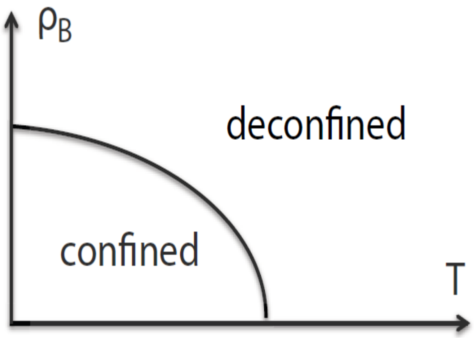
from present to 21st century:

to know the smallest, we need also the largest!

QCD Phase Diagrams

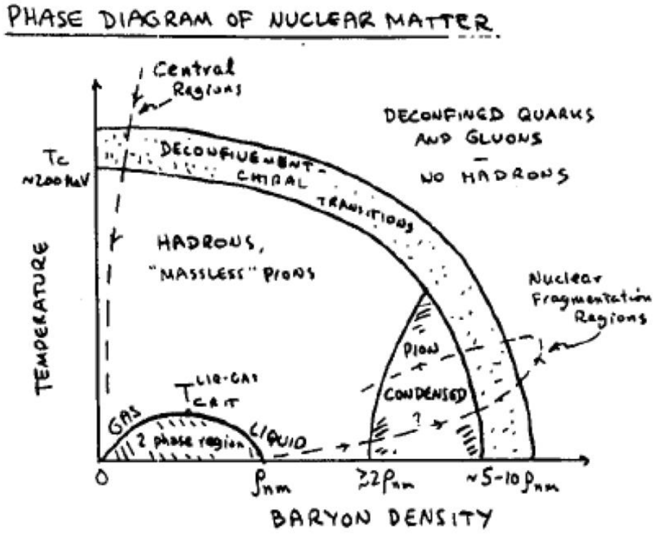
Deconfinement

N.Cabbibo and G.Parisi, PLB59, 67(1975)

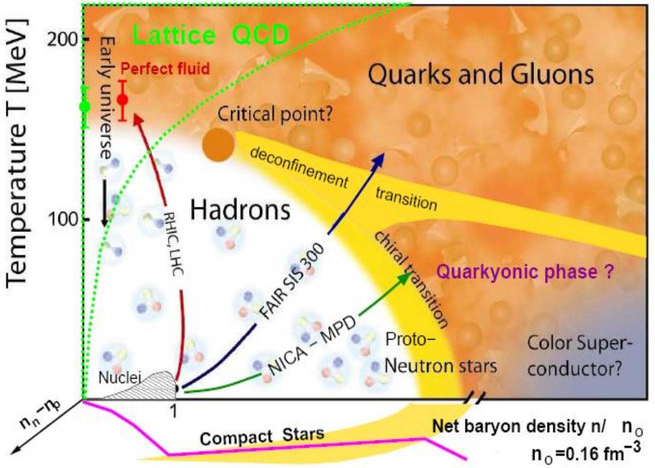


+ Chiral restoration

G.Baym, NSAC Long Rang Plan, 1983



+ CSC, quarkyonic phase and critical point, ……



有哪些QCD相变？

相变是体系对称性质的改变

Deconfinement：	*Z(3) symmetry*
Chiral restoration:	*SU(3) chiral symmetry*
Color superconductivity:	*SU(3) color symmetry*
Pion superfluidity:	*SU(2) isospin symmetry*

研究相变的方法：粒子（场）的统计力学

【详见侯德富课程有限温度场论】

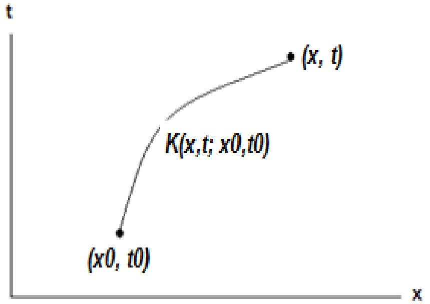
波函数的时间演化：$|\psi,t\rangle=\hat{U}(t,0)|\psi,0\,\rangle$

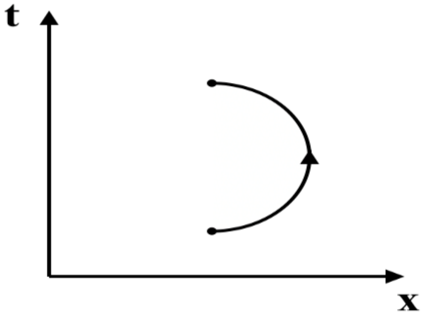
坐标表象： $\psi(x,t)=\langle x|\psi,t\rangle=\left\langle x\middle|\hat{U}(t,0)\middle|\psi,0\,\right\rangle$

$$=\int dx_0\left\langle x\middle|\hat{U}(t,0)\middle|x_0\right\rangle\langle x_0|\psi,0\,\rangle=\int dx_0 K(x,t;x_0,0)\psi(x,0\,)$$

时空中波函数的传播子$K(x,t;x_0,0)$：

空间封闭传播子$K(x,t;x,0)$：





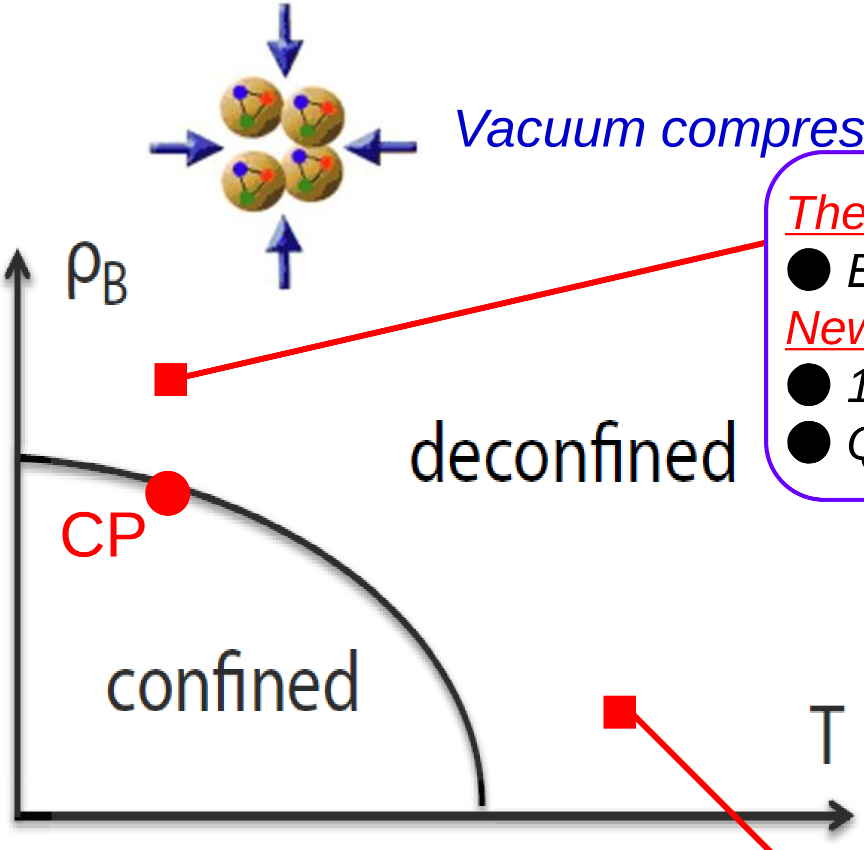
考虑所有的封闭传播子的集合：

$$\int dx\,K(x,t;x,0)=\int dx\,\langle x|e^{-\frac{i}{\hbar}\hat{H}t}|x\rangle=\sum_E\int dx\,\langle x|E\rangle\langle E|e^{-\frac{i}{\hbar}\hat{H}t}|x\rangle$$

$$=\sum_E\int dx\,\langle E|x\rangle\langle x|E\rangle\,e^{-\frac{i}{\hbar}Et}=\sum_E e^{-\frac{i}{\hbar}Et}\xrightarrow{\frac{i}{\hbar}t\to\frac{1}{T}}\sum_E e^{-\frac{E}{T}}=Z$$

封闭传播子的集合就是配分函数。用虚时动力学方法计算场的热力学。

Vacuum excitation VS Vacuum compression



Vacuum compression !

Theories:

- Effective models become more important

New phenomena:

- 1st order phase transitions
- Quantum phase transitions

Vacuum excitation !

Theories:

- (resumed) pQCD
- Lattice QCD
- FRG【详见付伟杰课程FRG】，DS equations, Effective models

极高温极高密：*pQCD Thermodynamics*

Kapusta and Gale:
Finite-Temperature Field Theory: Principles and Applications

Free QGP (封闭自由传播子):

$$\Omega_0(T,\mu) = -2N_cN_f\left[\frac{7\pi^2}{360}T^4 + \frac{\mu^2T^2}{12} + \frac{\mu^4}{24\pi^2}\right] - 2(N_c^2-1)\frac{\pi^2}{90}T^4$$

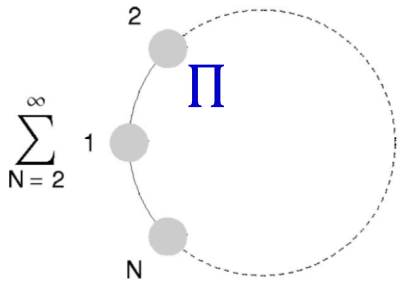
$\sim g$:

$$\Omega_1(T,\mu) = 0$$

$\sim g^2$:

$$\Omega_2^q(T,\mu) = \frac{N_f(N_c^2-1)}{8\pi}\alpha_s\left[\frac{5\pi^2}{18}T^4 + \mu^2T^2 + \frac{\mu^4}{2\pi^2}\right]$$

Summation over ring diagrams:



HTL:

$$V_{HTL}(r) = -\tilde{\alpha}_s\left[m_D + \frac{e^{-m_Dr}}{r} + iT\phi(m_Dr)\right] + \mathcal{O}(g^4)$$

$$\phi(x) = 2\int_0^\infty \mathrm{d}z\,\frac{z}{(z^2+1)^2}\left(1 - \frac{\sin(xz)}{xz}\right)$$

袋模型估计相变温度

将QGP看成一个大口袋，袋内是自由的QGP，袋外有压强作用在口袋上，平衡袋内的压强，形成稳定的体系。

袋外压强常数设为B，称为袋常数（非微扰效应）。

体系的压强为

$$P = P_{QGP} - B$$
$$= 2N_cN_f\left[\frac{7\pi^2}{360}T^4 + \frac{\mu^2T^2}{12} + \frac{\mu^4}{24\pi^2}\right] + 2(N_c^2-1)\frac{\pi^2}{90}T^4 - B$$

如果袋内为强子气体，只考虑π介子时的压强，

$$P = \frac{3\pi^2}{90}T^4$$

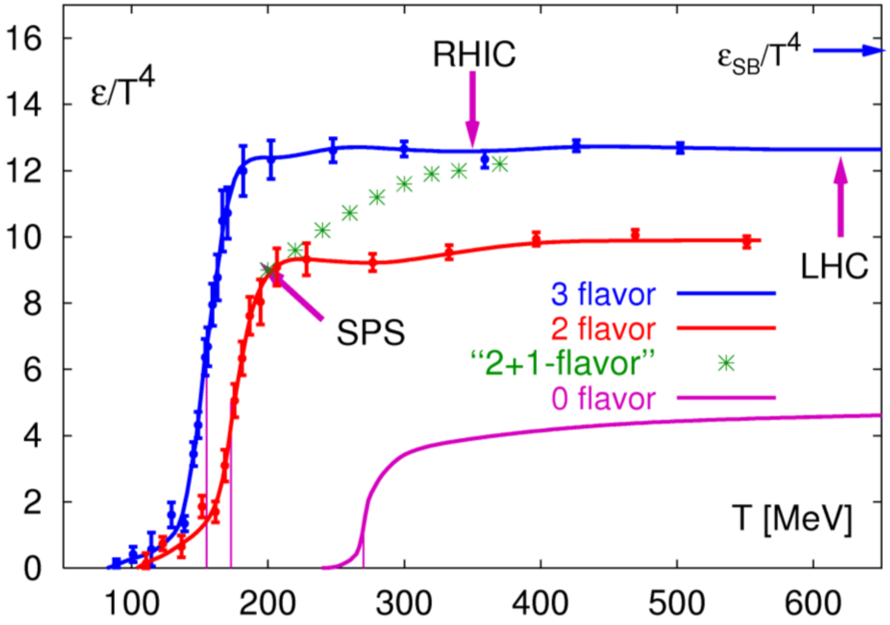
设体系在T_c时发生相变，由Gibbs相平衡条件，

$$2N_cN_f\left[\frac{7\pi^2}{360}T_c^4 + \frac{\mu^2T_c^2}{12} + \frac{\mu^4}{24\pi^2}\right] + 2(N_c^2-1)\frac{\pi^2}{90}T_c^4 - B = \frac{3\pi^2}{90}T_c^4$$

取$B^{1/4} = 200$ MeV，有

$$T_c = 144 \text{ MeV}$$

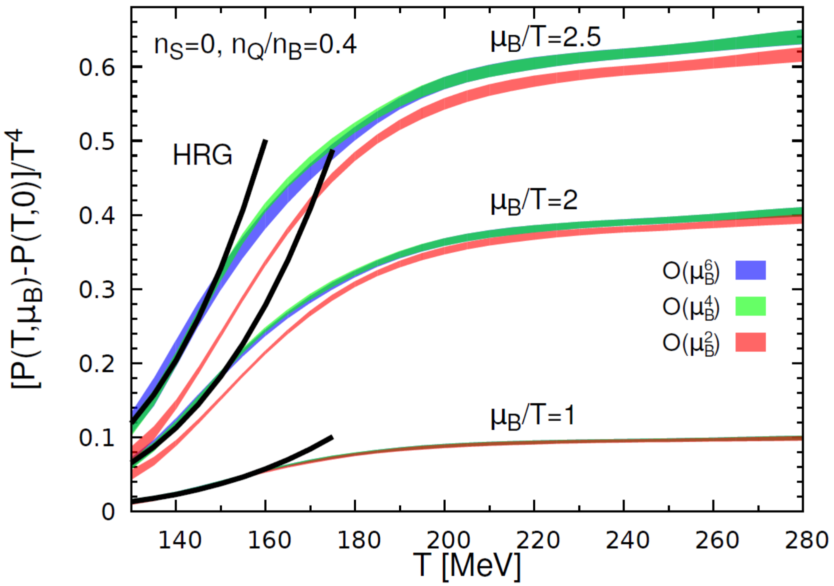
有限温度低密: *Lattice QCD*



- *Not a phase transition but a crossover around* $T_c \sim 160\ \mathrm{MeV}$
- *Still interacting QGP at* $T/T_c \sim 4$

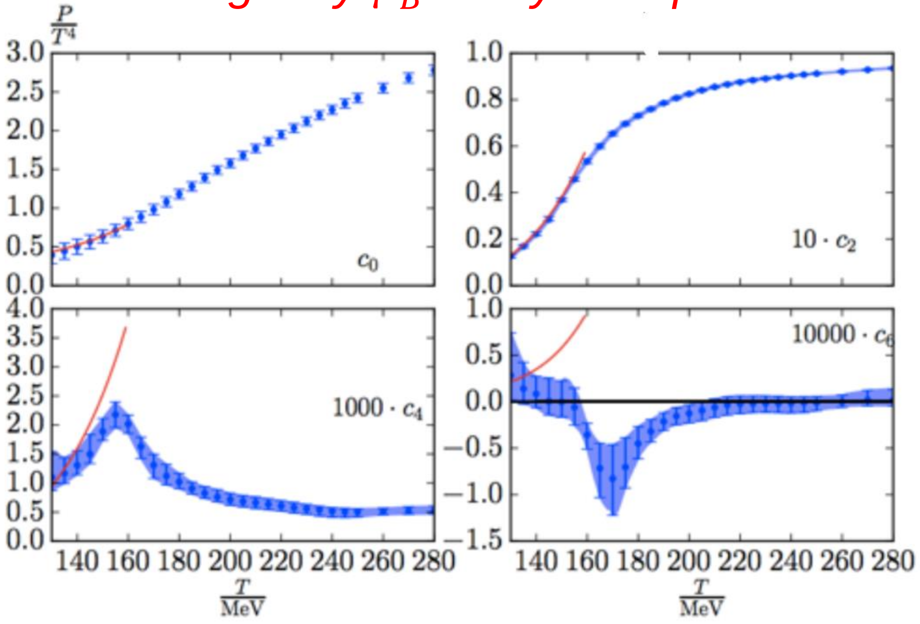
Lattice QCD: Equation of State at Nonzero μ_B

Taylor expansion $$\frac{P}{T^4}=\sum_n c_n\left(\frac{\mu_B}{T}\right)^n$$



Bielefeld-BNL-CCNU, Phys.Rev. D95 (2017) no.5, 05450

Imginary μ_B +.Taylor expansion



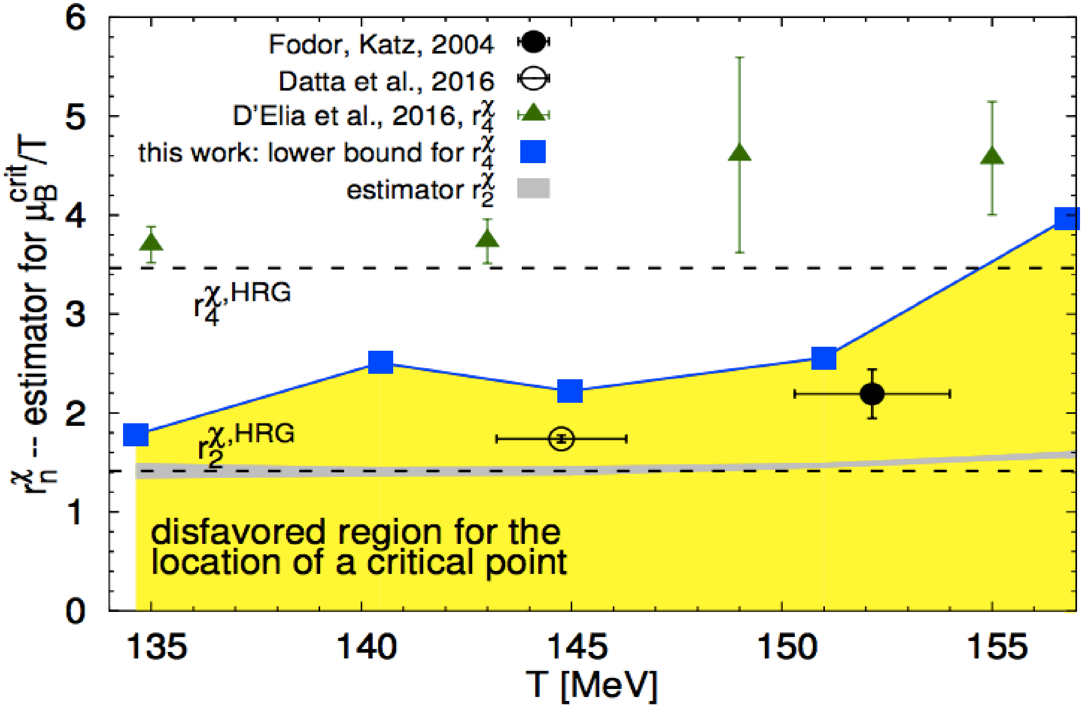
Wuppertal-Budapest-Houston:
EPJ Web Conf. 137 (2017) 07008

The EoS is well under control at $\mu_B/T \lesssim 2$ or $\sqrt{s_{NN}} \gtrsim 12$ GeV

Lattice QCD: Critical Point is Disfavored at $\mu_B/T \lesssim 2$

Critical point is a singularity of EoS,

Calculating the radius of convergence of EoS to 6th order at high T



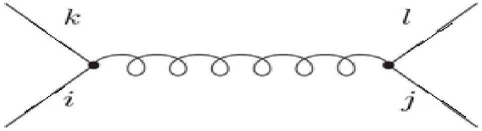
Bielefeld-BNL-CCNU, PRD 95 (2017) no.5, 054504
D'Elia et al., PRD 95 (2017) 094503
Datta et al., PRD 95 (2017) 054512
Fodor and Katz, JHEP 0404 (2004) 050

低温高密：色超导

【详见黄梅课程色超导】

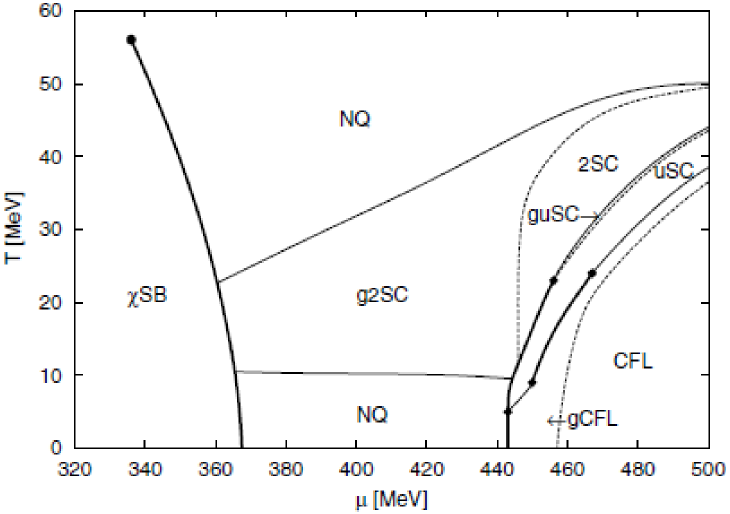
由*BCS*理论，在*QED*凝聚态中，两个电子通过交换光子是排斥相互作用，吸引相互作用是通过交换集体激发模式—声子来实现的(低温超导)。*Cooper*对的形成自发破缺了电磁规范对称性，光子获得了质量。

Frank Wilczek(2004诺贝尔奖): 色超导研究的开创者之一



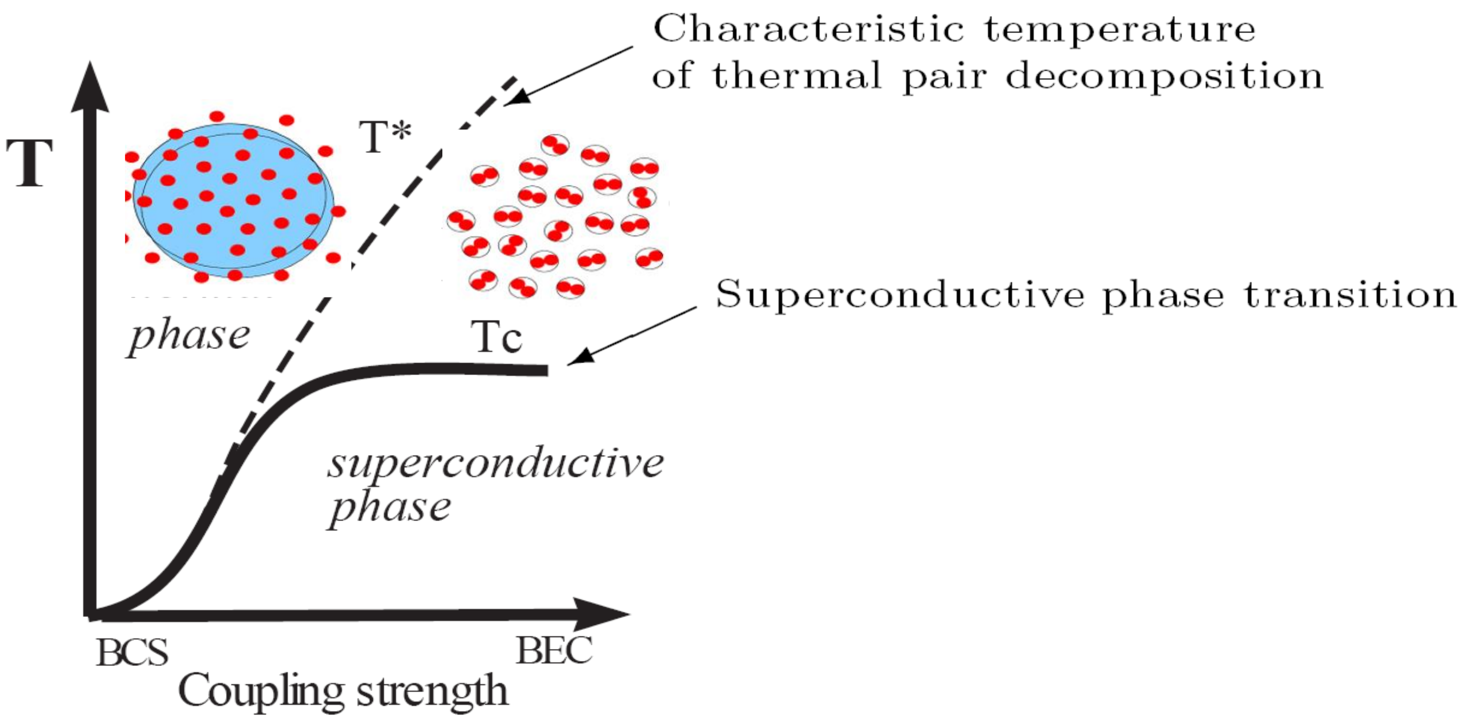
$$\sim (T_a)_{ki}(T_a)_{lj} = -\frac{N_c+1}{4N_c}\left(\delta_{jk}\delta_{il}-\delta_{ik}\delta_{jl}\right) + \frac{N_c-1}{4N_c}\left(\delta_{jk}\delta_{il}+\delta_{ik}\delta_{jl}\right)$$

第一项在交换初态或末态的两个夸克色指标时是反对称的，吸引相互作用。在单胶子交换的层次就使得两个夸克可以配对，*Diquark*凝聚，形成色超导 (高温超导)。由于色对称性自发破缺，胶子获得质量。



夸克的色味结构导致丰富的色超导相图

BCS-BEC Crossover



in BCS, T_c is determined by thermal excitation of fermions,
in BEC, T_c is controlled by thermal excitation of collective modes

BCS-BEC crossover is probably a way to understand deconfinement in QCD !

Quantum Phase Transitions

Symmetry of $\mathcal{L}$
(may be explicitly broken like the term $m\bar{\psi}\psi$ in QCD)

$T=0$: *spontaneously broken at classical level (mean field level), like chiral symmetry*

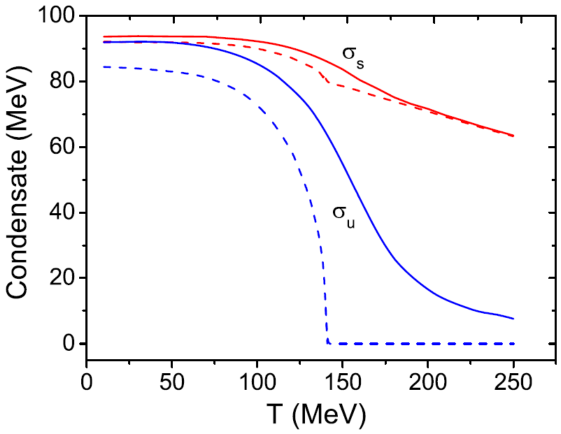
$T\neq 0$: *classical phase transition, like chiral restoration*

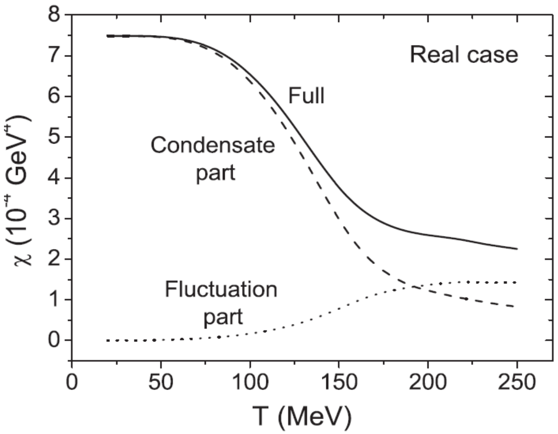
anomalously broken at quantum level (loop level), like $U_A(1)$ symmetry

quantum phase transition, like $U_A(1)$ restoration

- *$U_A(1)$ symmetry*

$$\chi=\int d^4x\langle T\big(Q(x)Q(0)\big)\rangle_{connected}\begin{cases}\neq 0 & U_A(1)\ breaking\\ =0 & U_A(1)\ symmetry\end{cases}\qquad Q(x)=\frac{g^2}{32\pi^2}F^a_{\mu\nu}(x)\tilde{F}_a^{\mu\nu}(x)$$





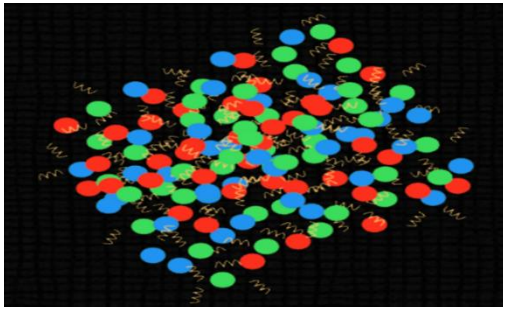
- *Chiral symmetry is restored at $T_c\sim 150$ MeV, but $U_A(1)$ symmetry is still broken above T_c !*

Lattice QCD:
HotQCD, PRD86, 094503(2012)
Effective models:
Jiang, Xia and PZ, PRD93, 074006(2016)

Classical Transport Equations

When quark matter is in non-equilibrium state, like the matter created in HIC, one should take transport theory to treat the phase-space evolution of the matter.

Boltzmann equations:



$$\left(p^\mu \partial_\mu^x + F^\mu \partial_\mu^p\right) f(x,p) = C$$

like QMD, AMPT, BAMPS,……

1) A single distribution f, no spin

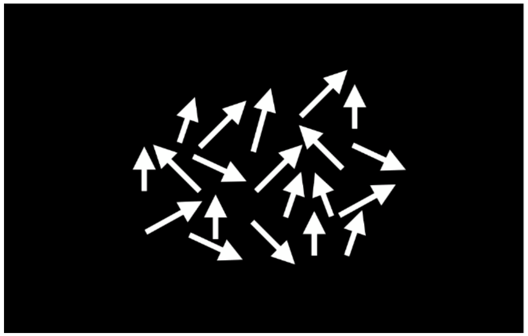
2) Quasi-particle approximation:

$$(p^2 - m^2) f(x,p) = 0$$

Quantum Kinetic Theory

【详见王群课程量子输运理论】

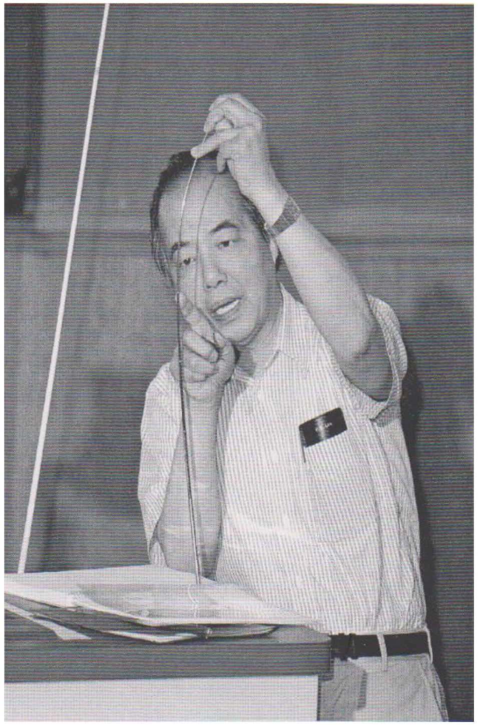
Spin is a pure quantum quantity. Many quantum anomalies in science are induced by spin, for instance the CME and CVE in high-energy nuclear collisions..



	Non-relativistic Quantum Mechanics	*Relativistic Quantum Mechanics*
Wave function:	*scalar* $\psi(x)$	*spinor* $\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \psi_3(x) \\ \psi_4(x) \end{pmatrix}$
Probability in x *space:*	$\psi(x)\psi^+(x), scalar$	$\psi(x)\bar{\psi}(x), \quad 4\times 4\, matrix$
Probability in (x,p) *space:*	$f(x,p) = \int d^4y e^{ipy}\psi\left(x+\frac{y}{2}\right)\psi^+\left(x-\frac{y}{2}\right)$	$W(x,p) = \int d^4y e^{ipy}\,\psi\left(x+\frac{y}{2}\right)\bar{\psi}\left(x-\frac{y}{2}\right)$ $Wigner\ function,\ \ 16\ distributions,$ *without on-shell constraint* $(p^2-m^2)W \neq 0$

要想认识最小的，需要知道最大的

李政道, 1996



Large things are made of small
And even smaller.
To know the smallest
We need also the largest

All lie in vacuum
Everywhen and everywhere.
How can the micro
Be separate from the macro?

Let vacuum be a condensate
Violating harmony
We can then penetrate
Through asymmetry into symmetry

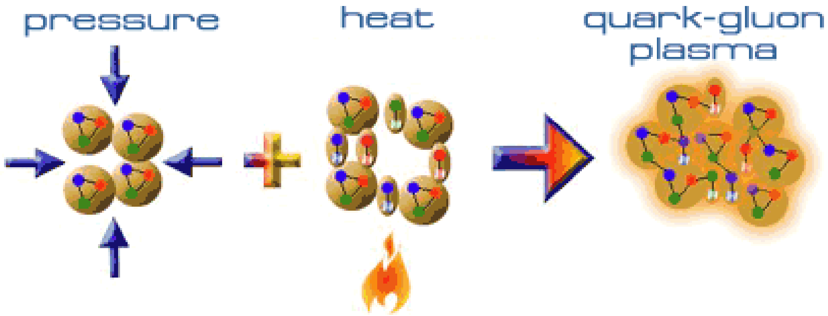
大事物由小事物组成
甚至是更小的。
要想认识最小的
我们也需要知道最大的。
一切都取决于真空
无论何时何地。
微观的事物怎能
与宏观相分离？
真空其实是一种凝聚
破坏了和谐。
如此我们方可洞穿
不对称中的对称。

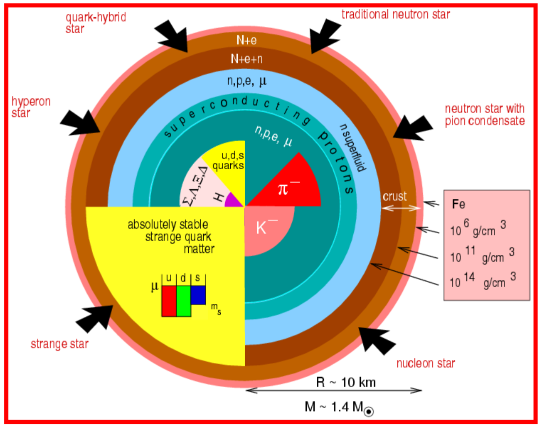
【杨振伟翻译】

第2次课: *The only way to realize QGP: HIC*

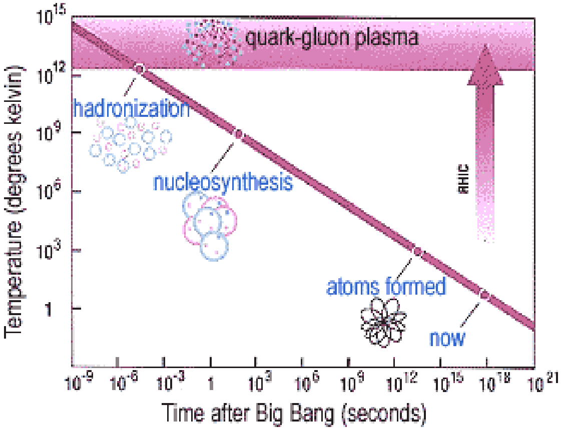
如何实现*QGP*

产生QGP的条件:

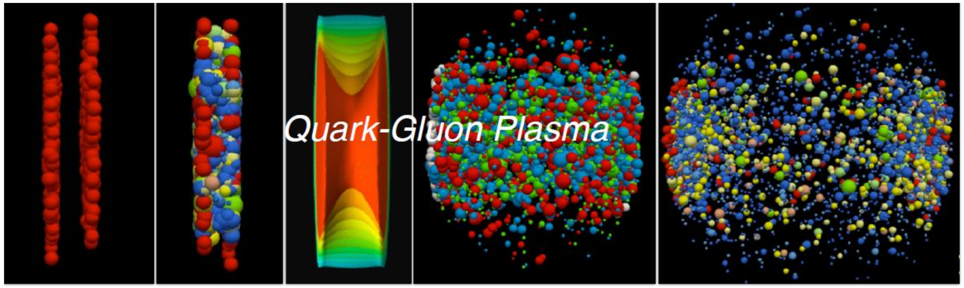




致密星体



早期宇宙



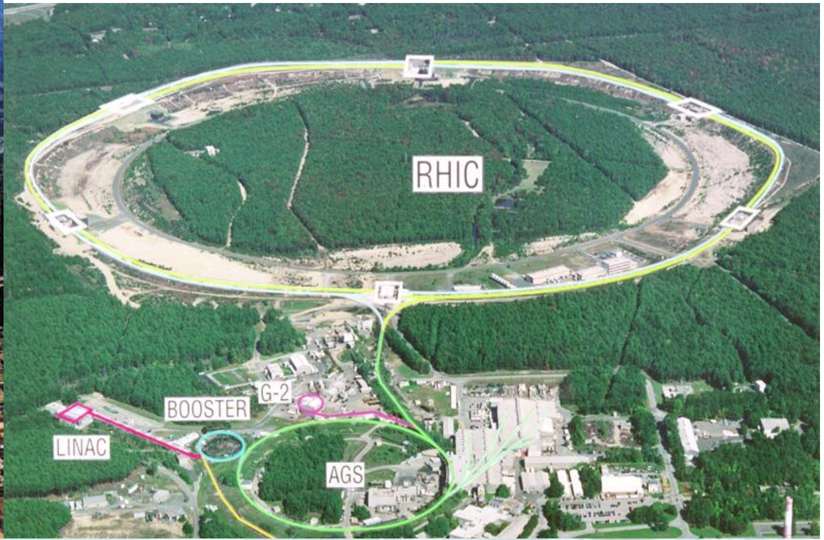
相对论重离子碰撞是在实验室产生QGP的唯一可能手段！

相对论重离子碰撞是在实验室实现QGP的唯一可能方式

【详见周代翠和刘峰课程重离子实验】

LHC, Pb+Pb@√s = 5500 GeV, ALICE,ATLAS,CMS,LHCb

RHIC, Au+Au@√s = 200 GeV, STAR, PHENIX



FAIR, U+U@Elab = 40A GeV, CBM

CSR, U+U@Elab = 0.6A GeV, CEE

常用物理量

核几何

碰撞参数$\vec{b}$：两个碰撞核中心的相对位置，矢量。$\vec{b}=0$,中心碰撞。

参与碰撞核子数N_{part}：软过程数$\sim N_{part}$

参与碰撞的核子对数N_{coll}: 硬过程数$\sim N_{coll}$

纵向快度（沿碰撞方向）$y=\frac{1}{2}ln\frac{E+p_z}{E-p_z}$, $\begin{cases}E=m_\perp Cosh(y)\\ p_z=m_\perp Sinh(y)\end{cases}$

优点：Lorentz变换时，快度相加 $y'=y+Y$

E_{lab}与$\sqrt{s}$的转换

$$\sqrt{s}=\sqrt{(E_1+E_2)^2-(\vec{p}_1+\vec{p}_2)^2}=\sqrt{(E_{lab}+m_N)^2-\vec{p}_1^{\,2}}=\sqrt{2m_N(m_M+E_{lab})}$$

正常核物质的能量密度

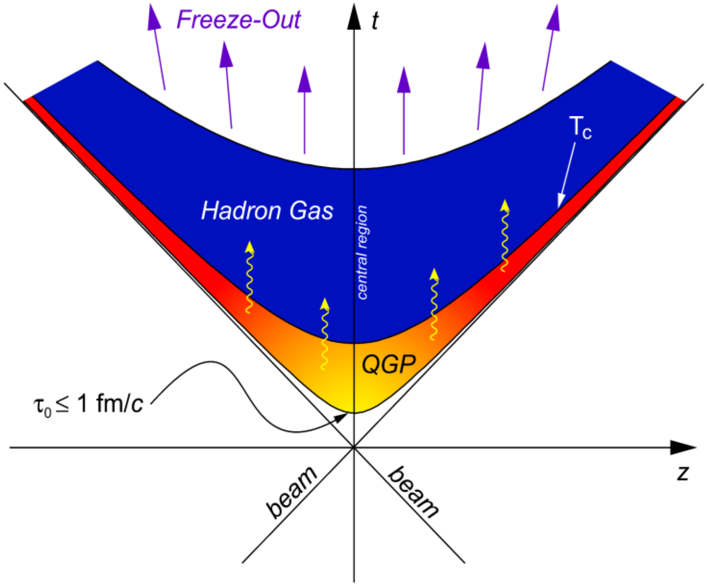
核内核子数密度 $n_0=0.17/fm^3$，核内能量密度$\varepsilon_0=m_N n_0\sim 0.17\text{GeV}/fm^3$

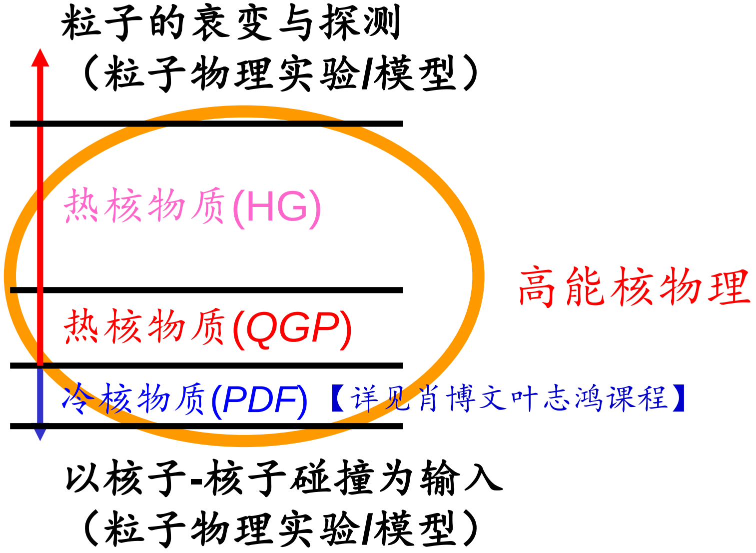
产生QGP的最低能量密度

即核子内的能量密度 $\varepsilon_{min}=\frac{m_N}{\frac{4}{3}\pi r_N^3}\xrightarrow{r_N=0.8\ fm}0.5\ \text{GeV}/fm^3$

*QGP*的信号

*QGP*只是重离子碰撞的中间态，即使在*RHIC*和*LHC*产生了*QGP*，也不能在末态直接观测到，只能通过携带*QGP*信号的末态粒子来判断。





粒子的衰变与探测
（粒子物理实验/模型）

热核物质(HG)

高能核物理

热核物质(QGP)

冷核物质(*PDF*)【详见肖博文叶志鸿课程】

以核子-核子碰撞为输入
（粒子物理实验/模型）

希望看到什么？

1）与$p+p$碰撞的差别 → 有新的物理

核修正因子：$R_{AA}=\frac{\sigma_{AA}}{N_{coll}\sigma_{pp}}$

2）与强子气体的差别 → QGP的产生！

热密物质的时空演化—流体力学

Landau: 用流体力学描述高能碰撞后至衰变前体系的时空演化。

忽略粘滞效应的理想流体力学方程

$$\begin{cases} \partial_\mu T^{\mu\nu} = 0 & (\text{能动量守恒}) \\ \partial_\mu n^\mu = 0 & (\text{重子数守恒}) \end{cases}$$

$$T^{\mu\nu} = -Pg^{\mu\nu} + (\varepsilon + P)u^\mu u^\nu, \qquad n^\mu = n_B u^\mu, \qquad u_\mu u^\mu = 1$$

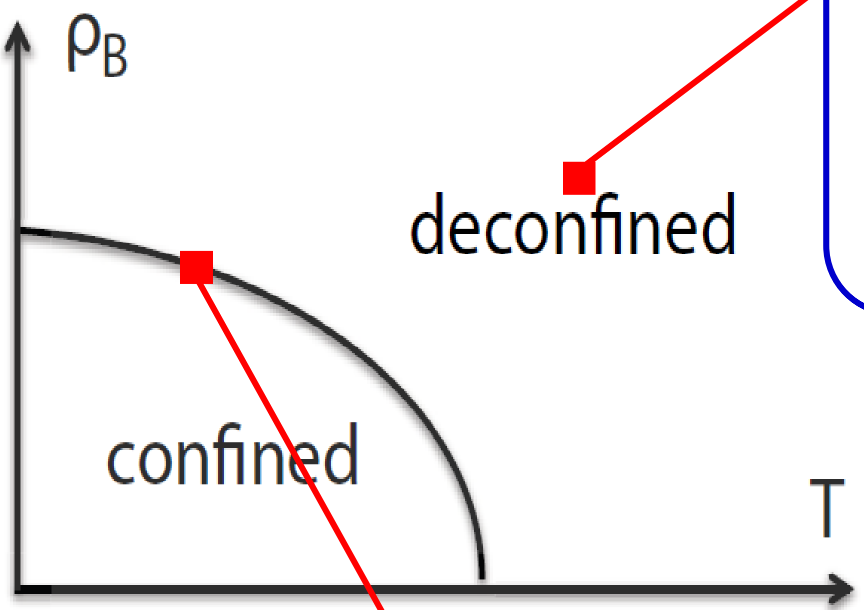
+ Equation of state $\epsilon(P)$ （QGP or Hadron gas from QCD thermodynamics）

$\rightarrow\ u_\mu(x), T(x), \mu_B(x), P(x)$

一维膨胀→快度中心区的能量密度(Bjorken估计)

$$\begin{cases} \varepsilon(\tau) = \epsilon(\tau_0)\left(\dfrac{\tau_0}{\tau}\right)^{1+c_s^2} \\ n(\tau) = n(\tau_0)\dfrac{\tau_0}{\tau} \\ T(\tau) = T(\tau_0)\left(\dfrac{\tau_0}{\tau}\right)^{c_s^2} \end{cases}$$

Probes of QCD Phases



- *Jet quenching*
- *Quarkonium suppression* 【详见第3次课】，
- *Dileptons*
- *Strangeness enhancement……*
- *Chiral magnetic effect (CME)*【详见王群课程】
- *Chiral vortical effect (CVE)* 【详见梁作堂课程】
- ……

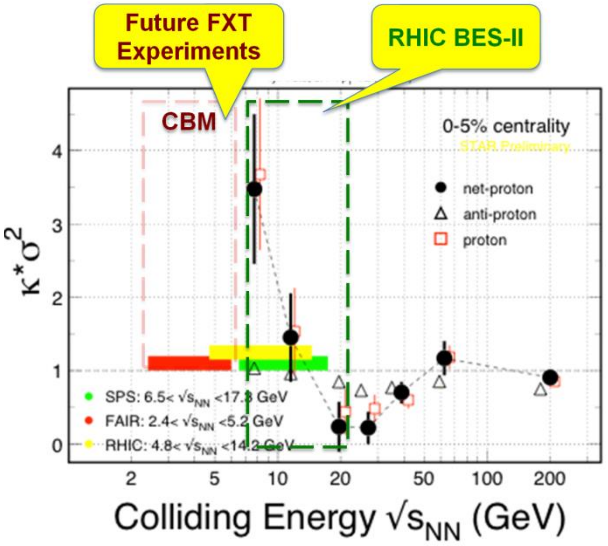
Quantum fluctuations

Probes of the chiral critical point:
Dynamical fluctuations of conserved charges.

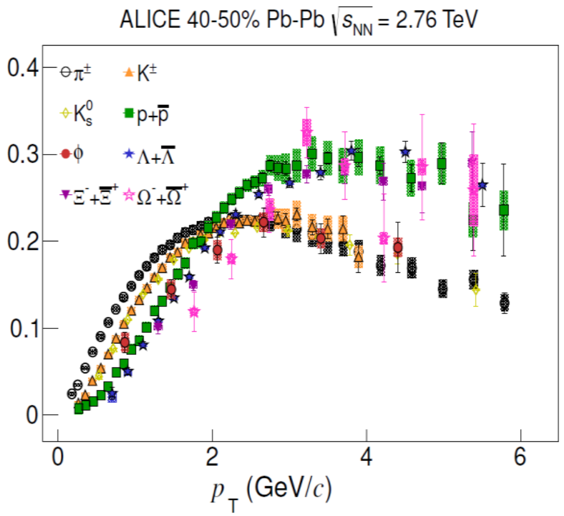
What We Learned from RHIC & LHC

【详见周代翠和刘峰课程】

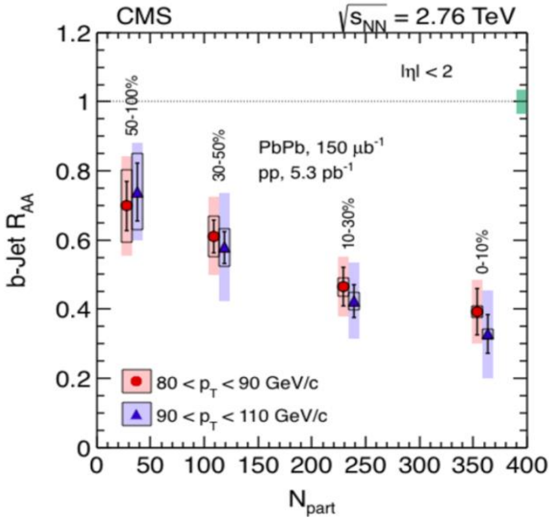
●*Large proton number fluctuations*



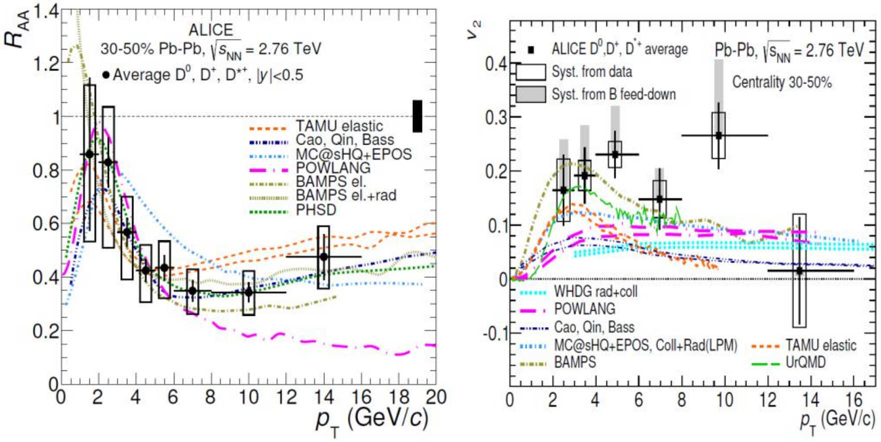
●*Large collective flow*



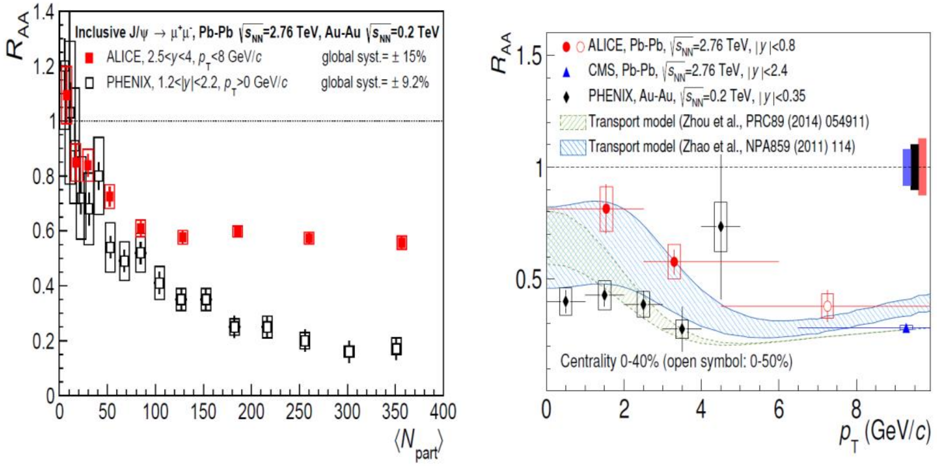
●*Strong jet suppression*



●*Heavy quark suppression and flow*



●*Quarkonium enhancement at LHC*



Dynamical Fluctuations around Critical Point

- *Correlation length $\xi \to \infty$ at a critical point, strong dynamical fluctuations!*
- *Order parameter field $\sigma(x)$*

  fluctuation distribution $P[\sigma] \sim e^{-\Omega(\sigma)/T}$

  Cumulants:

$$C_2 = \langle \sigma_V^2 \rangle \sim \xi^2, \qquad C_3 = \langle \sigma_V^3 \rangle \sim \xi^6, \qquad \ldots\ldots$$

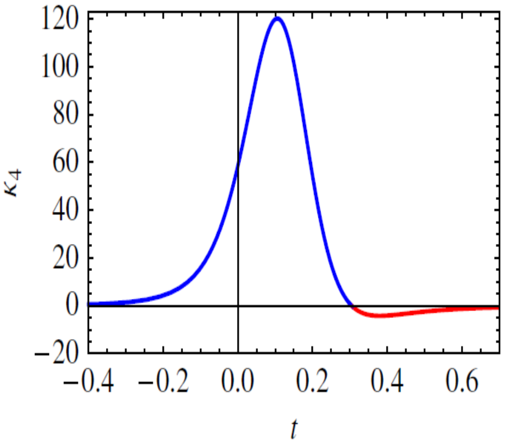
High order cumulants are more sensitive to ξ and cab be used to sensitively probe the critical point.

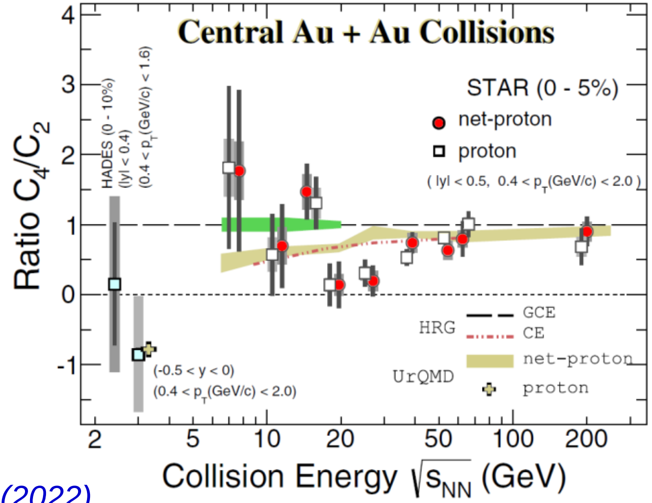
M.Stephanov, PRL102, 032301(2009)
M.Asakava, S.Ejiri, M.Kitazawa, PRL103, 262301(2009)

- *The sign of C_4 (C_4/C_2) depends on which side of the critical point we are.*

M.Stephanov, PRL107, 052301(201

Far from CP: $C_4 = 0$
Crossover side: $C_4 < 0$
First order side: $C_4 > 0$

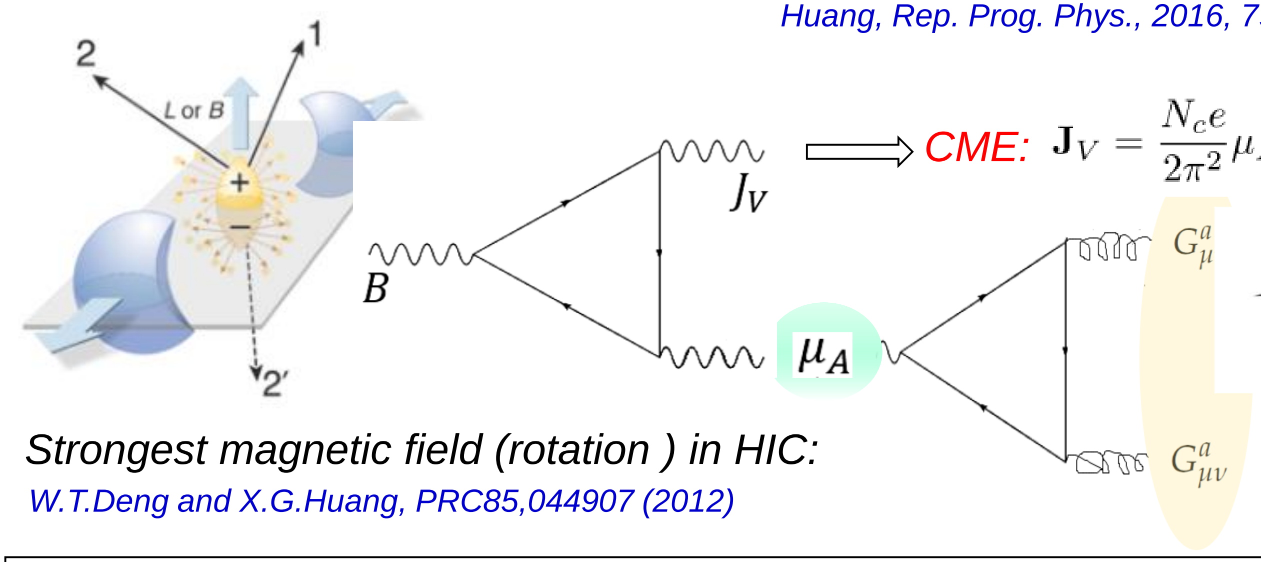




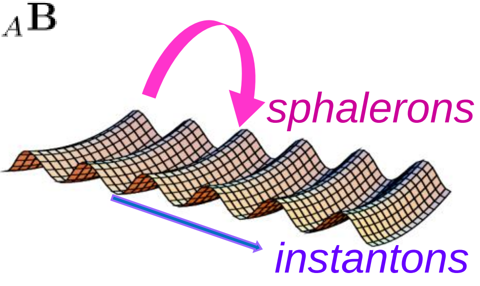
STAR（罗晓峰，许怒等）， PRL128, 202303(2022)

Chiral Magnetic Effect

Kharzeev, Warringa, McLarren, Fukushima, 2008
Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys., 2016, 88: 1
Huang, Rep. Prog. Phys., 2016, 79: 076302,……



CME: $\mathbf{J}_V = \frac{N_c e}{2\pi^2}\mu_A \mathbf{B}$



$$q = \frac{g^2}{32\pi^2}\int d^4x G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$

Strongest magnetic field (rotation) in HIC:

W.T.Deng and X.G.Huang, PRC85,044907 (2012)

A chirality imbalance induced electric current in magnetic field is a probe of nontrivial topology of QCD.

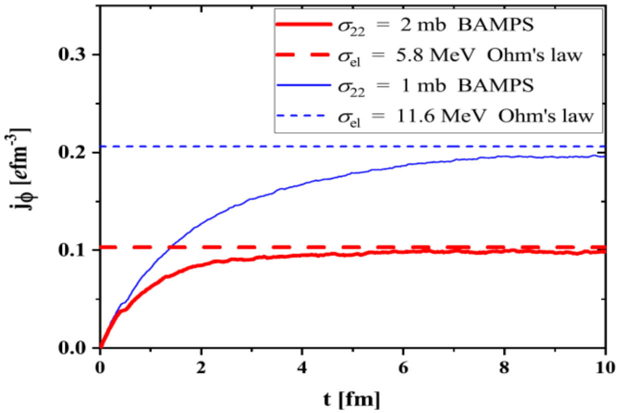
【CME的实验观测详见刘峰课程？】

A reason why it is hard to observe CME in HIC:

重离子碰撞中破缺的欧姆定律:

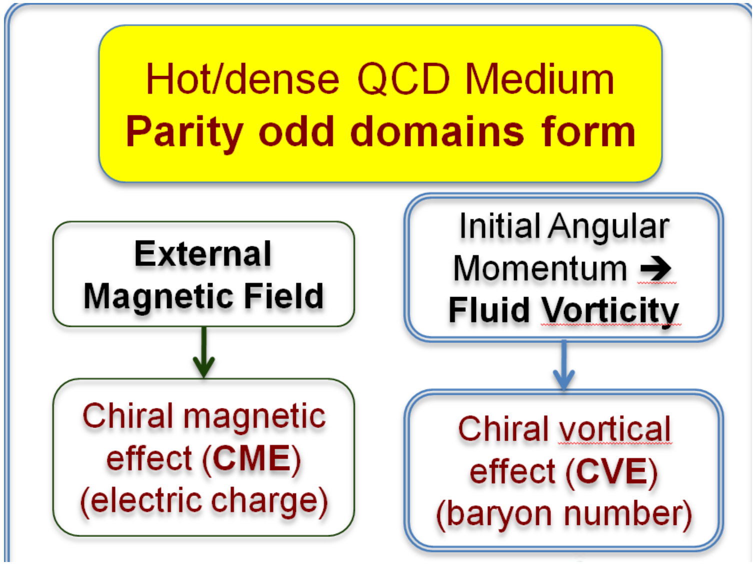
$$\vec{J}_{ohm} \leq \sigma_{el}(\vec{E} + \vec{v} \times \vec{B})$$

Wang, Zhao, Greiner, Xu and Zhuang,
PRC105, L041901(2022), Letter, Featured in Physics



Chiral Vortical Effect

【详见梁作堂理论课程和刘峰实验课程】



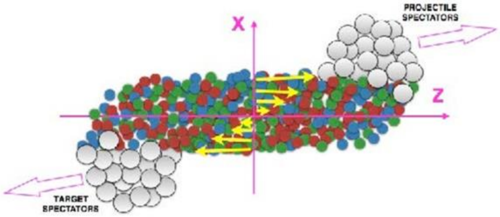
Hot/dense QCD Medium
Parity odd domains form

External Magnetic Field

Chiral magnetic effect (**CME**) (electric charge)

Initial Angular Momentum ➔ **Fluid Vorticity**

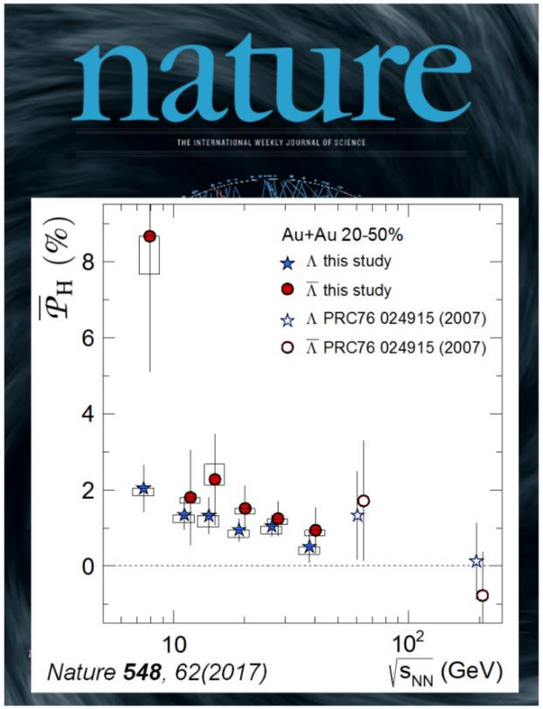
Chiral vortical effect (**CVE**) (baryon number)



Global collision angular momentum generates QGP vorticity

$$P_\pm \sim \exp\left[\pm\frac{\frac{1}{2}\hbar\omega+\mu B}{T}\right] \qquad (\mu_\Lambda=-\mu_{\bar{\Lambda}})$$

*The signal is consistent with vorticity **ω = (9 ± 1)x10²¹/s**, greater than previously observed in any system!*



Nature ***548****, 62(2017)*

Liang & Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)

Comments

- *Nuclear physics is complicated (strong interaction and many-body system, both are key problems in physics).*

- *Since QGP is an intermediate state created in HIC, we cannot directly measure it. At the moment there is no a unique signal observed in experiments, we need to characterize the QGP properties comprehensively.*

- *Some sensitive signals like high-order moments, CME and CVE are quantum fluctuations, we need precise measurement and carefully excluding the influence from the background.*

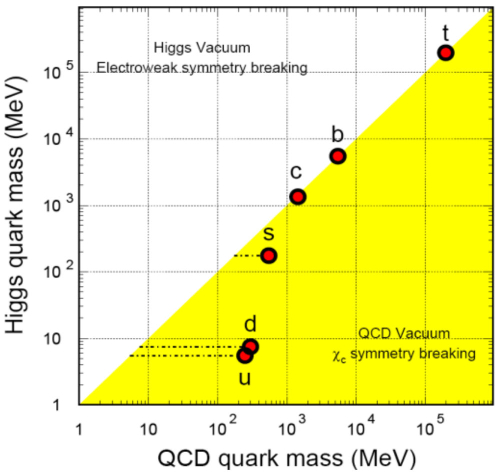
第3次课: *Heavy flavor hadrons as a probe of QGP*

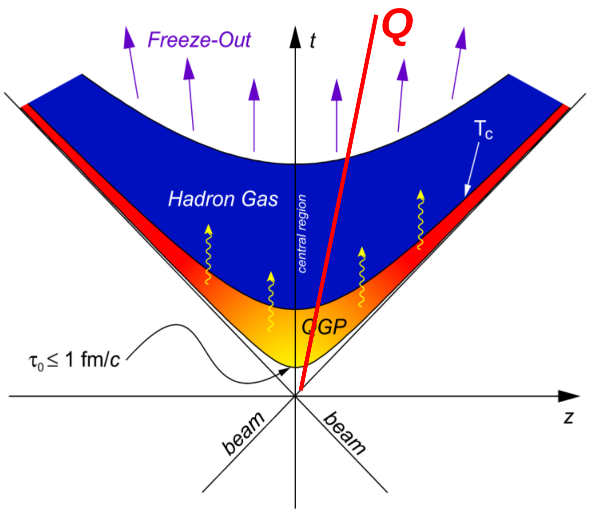
Why Heavy Flavors?

- *QCD → NRQCD → pNRQCD, a relatively solid calculation*

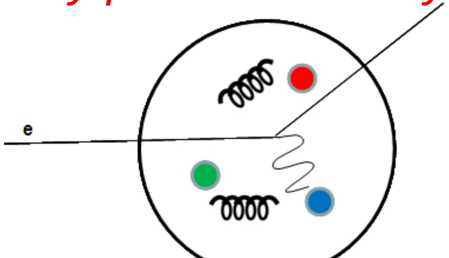
【详见贾宇课程*NRQCD*】

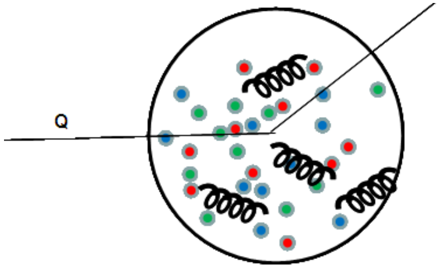
- *Medium-independent heavy-quark mass*





→ Initially produced heavy quarks pass through the QGP and carry its information.

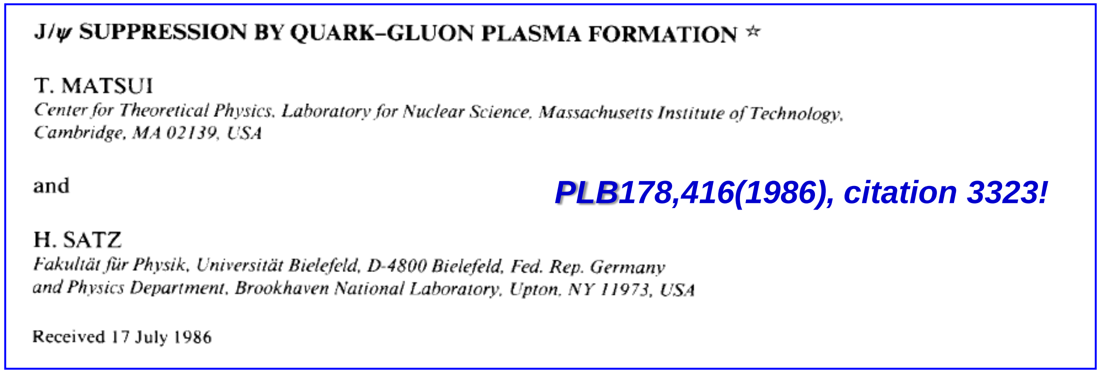




【详见肖博文叶志鸿课程】

electrons as a probe of nucleon structure *heavy quarks as a probe of QGP structure*

J/Ψ Suppression or Enhancement ?



J/ψ SUPPRESSION BY QUARK–GLUON PLASMA FORMATION ☆

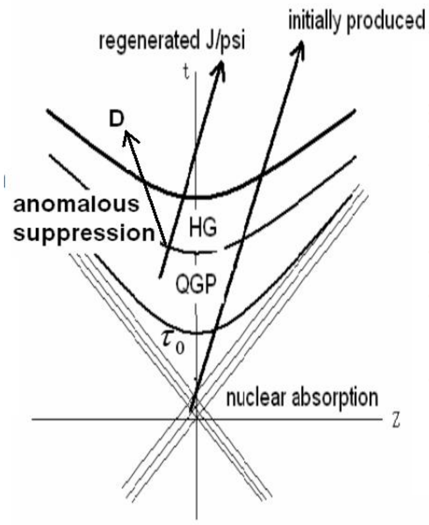
T. MATSUI
Center for Theoretical Physics, Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

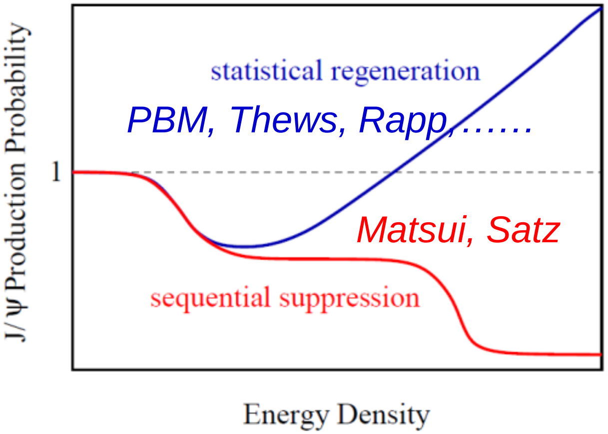
and

H. SATZ
Fakultät für Physik, Universität Bielefeld, D-4800 Bielefeld, Fed. Rep. Germany and Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

Received 17 July 1986

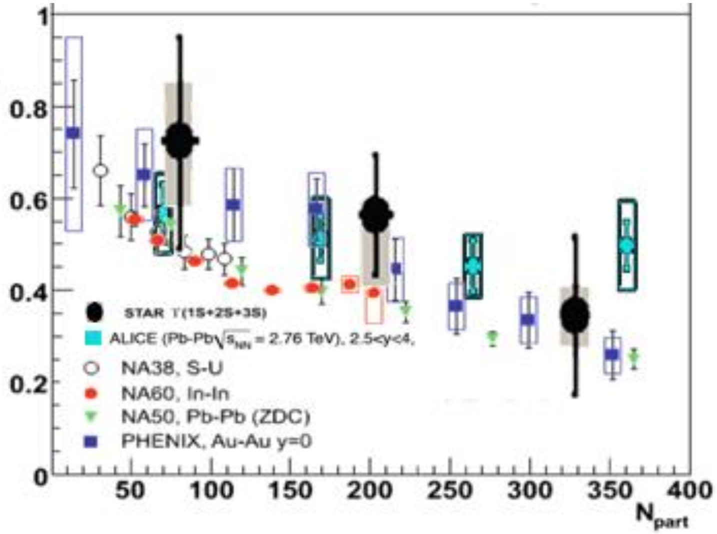
PLB178,416(1986), citation 3323!





PBM, Thews, Rapp,……

Matsui, Satz



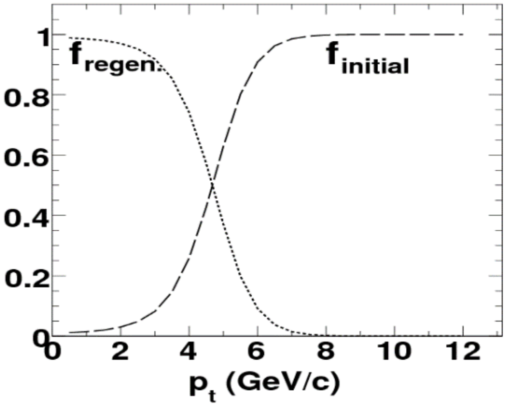
Should the competition between suppression and regeneration lead to $R_{AA}\begin{cases} <1 \ \ at\ SPS \\ \sim 1\ at\ RHIC \\ >1\ at\ LHC \end{cases}$ *?*

No, the data show $R_{AA} < 1$ *at SPS, RHIC and LHC !*

→ We need sensitive probes!

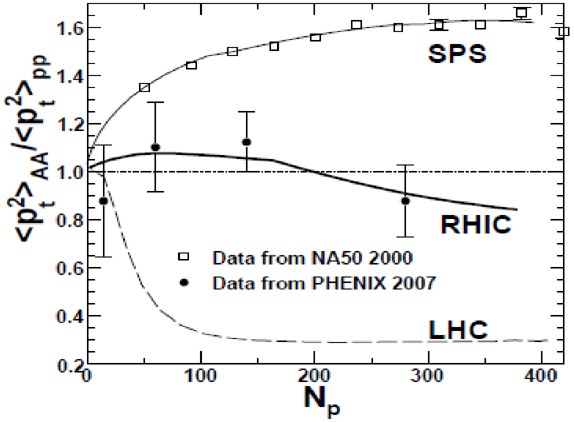
P_t Distributions

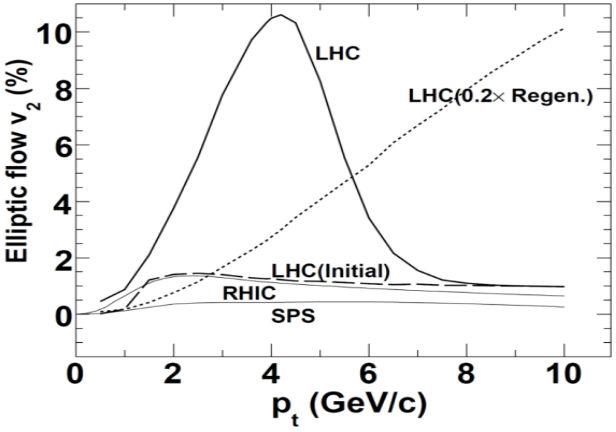
【Tsinghua Group, since 2006】



- *P_t distribution should be more sensitive to the QGP properties!*

$$r_{AA}=\frac{\langle p_t^2\rangle_{AA}}{\langle p_t^2\rangle_{pp}}\begin{cases}>1 & SPS\\ \approx 1 & RHIC\\ <1 & LHC\end{cases}$$





A Transport Approach

We need a transport approach to control J/ψ motion in medium, including self-consistently the competition between suppression and regeneration

Yan, Xu, Zhuang, PRL97, 232301(2006), PRC89, 054911(2014)

- *Transport equation for quarkonia:*

$$\partial f_\Psi/\partial\tau + \mathbf{v}_\Psi\cdot\nabla f_\Psi = -\alpha_\Psi f_\Psi + \beta_\Psi .$$

$$\alpha_\Psi(\mathbf{p}_t,\mathbf{x}_t,\tau|\mathbf{b}) = \frac{1}{2E_\Psi}\int\frac{d^3\mathbf{p}_g}{(2\pi)^3 2E_g}W_{g\Psi}^{c\bar{c}}(s)f_g(\mathbf{p}_g,\mathbf{x}_t,\tau)\Theta\left(T(\mathbf{x}_t,\tau|\mathbf{b})-T_c\right),$$

suppression: g+J/ψ → $c+\bar{c}$

$$\begin{aligned}\beta_\Psi(\mathbf{p}_t,\mathbf{x}_t,\tau|\mathbf{b}) &= \frac{1}{2E_\Psi}\int\frac{d^3\mathbf{p}_g}{(2\pi)^3 2E_g}\frac{d^3\mathbf{p}_c}{(2\pi)^3 2E_c}\frac{d^3\mathbf{p}_{\bar{c}}}{(2\pi)^3 2E_{\bar{c}}}W_{c\bar{c}}^{g\Psi}(s)f_c(\mathbf{p}_c,\mathbf{x}_t,\tau|\mathbf{b})f_{\bar{c}}(\mathbf{p}_{\bar{c}},\mathbf{x}_t,\tau|\mathbf{b})\\ &\quad\times(2\pi)^4\delta^{(4)}(p+p_g-p_c-p_{\bar{c}})\Theta\left(T\left(\mathbf{x}_t,\tau|\mathbf{b}\right)-T_c\right),\end{aligned}$$

regeneration: c+$\bar{c}$ →g+J/ψ

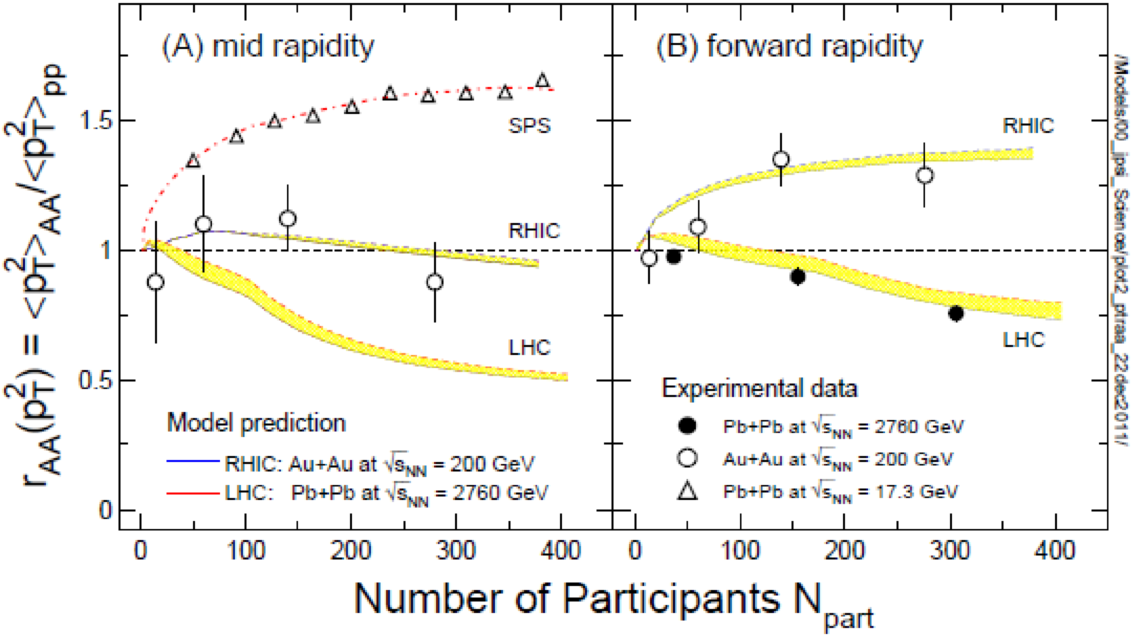
$$\begin{aligned}f_\Psi(\mathbf{p}_t,\mathbf{x}_t,\tau|\mathbf{b}) &= f_\Psi(\mathbf{p}_t,\mathbf{x}_t-\mathbf{v}_\Psi(\tau-\tau_0),\tau_0|\mathbf{b})e^{-\int_{\tau_0}^{\tau}d\tau'\alpha_\Psi(\mathbf{p}_t,\mathbf{x}_t-\mathbf{v}_\Psi(\tau-\tau'),\tau'|\mathbf{b})}\\ &\quad+\int_{\tau_0}^{\tau}d\tau'\beta_\Psi(\mathbf{p}_t,\mathbf{x}_t-\mathbf{v}_\Psi(\tau-\tau'),\tau'|\mathbf{b})e^{-\int_{\tau'}^{\tau}d\tau''\alpha_\Psi(\mathbf{p}_t,\mathbf{x}_t-\mathbf{v}_\Psi(\tau-\tau''),\tau''|\mathbf{b})}.\end{aligned}$$

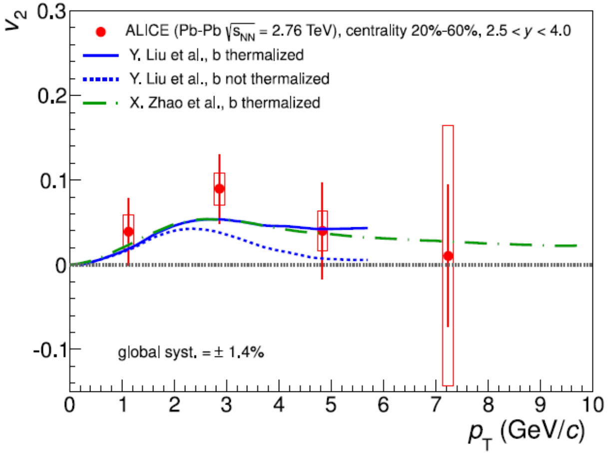
- *Hydrodynamic equations for QGP:*

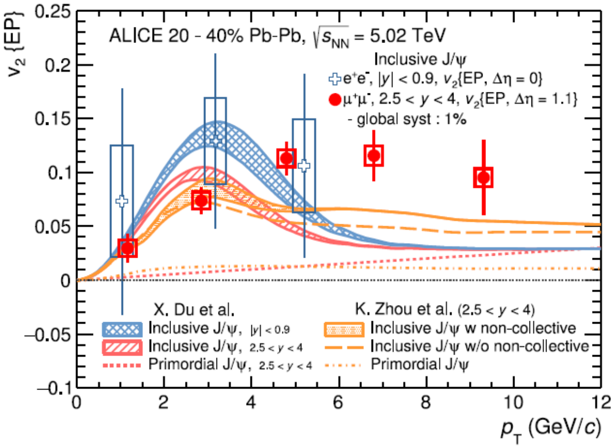
$$\partial_\mu T^{\mu\nu}=0,\qquad \partial_\mu n^\mu=0$$

- *QCD equation of state*
- *Initial distribution from p+p data.*
- *Cold nuclear matter effects:* *shadowing effect, Cronin effect, nuclear absorption.*

r_{AA} and v_2


(A) mid rapidity
(B) forward rapidity
$r_{AA}(p_T^2) = \langle p_T^2 \rangle_{AA} / \langle p_T^2 \rangle_{pp}$
SPS
RHIC
LHC
RHIC
LHC
Model prediction
RHIC: Au+Au at $\sqrt{s_{NN}}$ = 200 GeV
LHC: Pb+Pb at $\sqrt{s_{NN}}$ = 2760 GeV
Experimental data
● Pb+Pb at $\sqrt{s_{NN}}$ = 2760 GeV
○ Au+Au at $\sqrt{s_{NN}}$ = 200 GeV
△ Pb+Pb at $\sqrt{s_{NN}}$ = 17.3 GeV
0
0.5
1
1.5
0
100
200
300
400
Number of Participants N_{part}
/Models/00_jpsi_Science/plot2_ptraa_22dec2011/

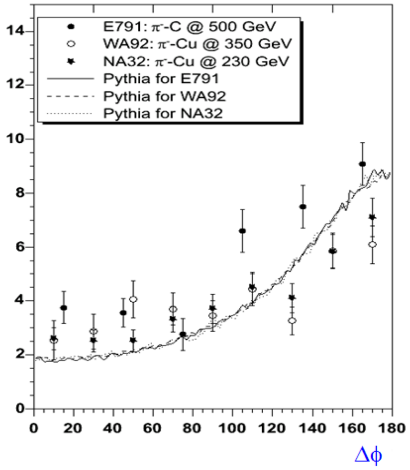

v_2
ALICE (Pb-Pb $\sqrt{s_{NN}}$ = 2.76 TeV), centrality 20%-60%, 2.5 < y < 4.0
Y. Liu et al., b thermalized
Y. Liu et al., b not thermalized
X. Zhao et al., b thermalized
global syst. = ± 1.4%
0.3
0.2
0.1
0
-0.1
0
1
2
3
4
5
6
7
8
9
10
p_T (GeV/c)


v_2 {EP}
ALICE 20 - 40% Pb-Pb, $\sqrt{s_{NN}}$ = 5.02 TeV
Inclusive J/ψ
e^+e^-, $|y|$ < 0.9, v_2\{EP, Δη = 0\}
$\mu^+\mu^-$, 2.5 < y < 4, v_2\{EP, Δη = 1.1\}
- global syst : 1%
X. Du et al.
Inclusive J/ψ, $|y|$ < 0.9
Inclusive J/ψ, 2.5 < y < 4
Primordial J/ψ, 2.5 < y < 4
K. Zhou et al. (2.5 < y < 4)
Inclusive J/ψ w non-collective
Inclusive J/ψ w/o non-collective
Primordial J/ψ
0.25
0.2
0.15
0.1
0.05
0
−0.05
−0.1
0
2
4
6
8
10
12
p_T (GeV/c)

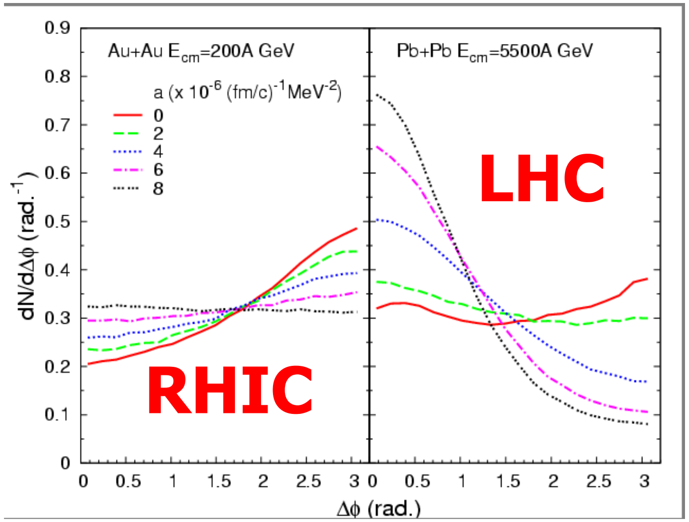
Partonic Wind

Zhu, Xu, Zhuang, PRL97, 232301(2008)

- *Strong* $c\,\bar{c}\ (D\,\bar{D})$ *back-to-back correlation in p+p collisions!*



- *How does the QGP modify the angular correlation?*



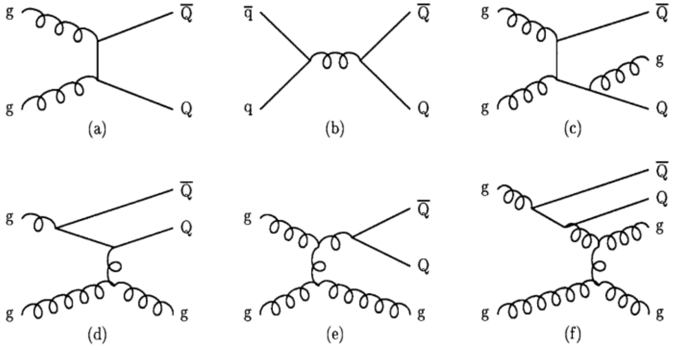
- *at RHIC, the back-to-back correlation is washed away by partonic collectivity.*

- *At LHC, larger partonic density, higher temperature, and stronger collective expansion lead to a near side correlation!*

Heavy Quark Production in QGP at FCC

Zhou, Chen, Greiner, Zhuang, PLB758, 434(2016)

Charm production in QGP

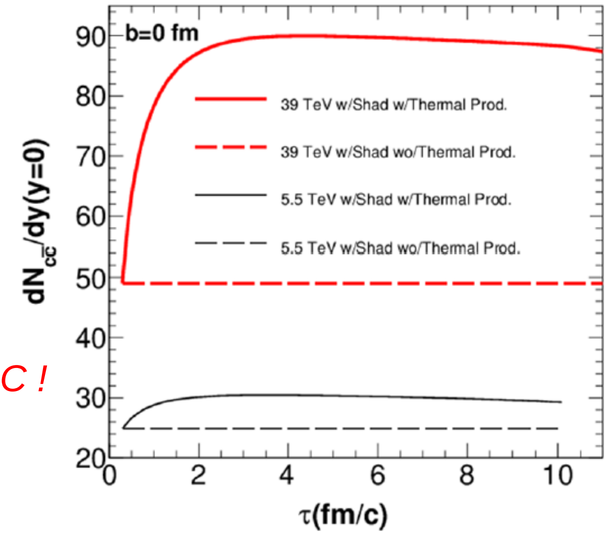


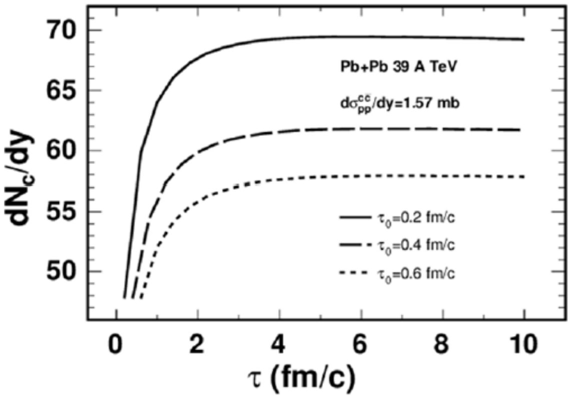
Levai, Muller, Wang, PRC51, 3326(1995);
Kaempfer, Pavlenko, PLB391, 185(1997);
Uphoff, Fochler, Xu, Greiner, PRC82, 044906(2010);
Zhang, Ko, Liu, PRC77, 024901(2008),……

FCC physics opportunities, [FCC collaboration], EPJC79, 474(2019)

Fig. 16.4 Time-evolution of the $c\bar{c}$ yield (per unit of rapidity at midrapidity) for central Pb–Pb collisions at $\sqrt{s_{NN}} = 39$ TeV [395, 396]

80% enhancement at FCC !



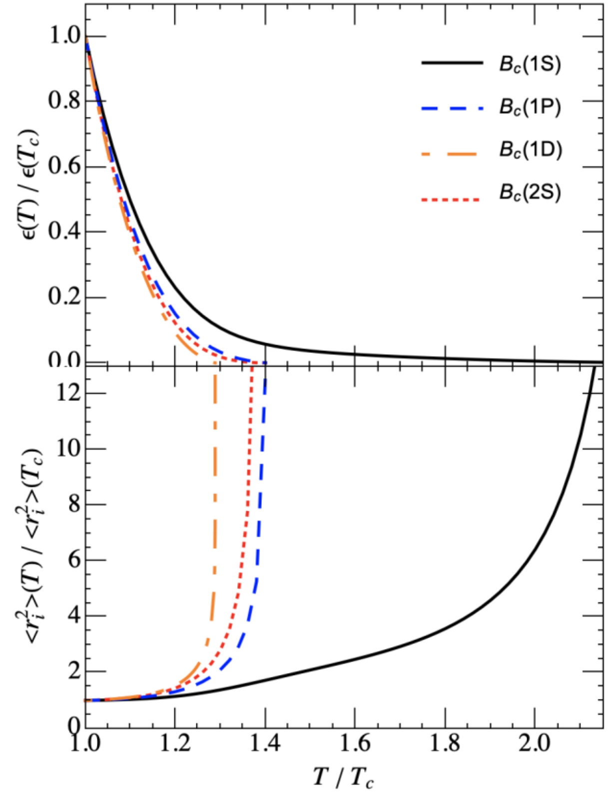


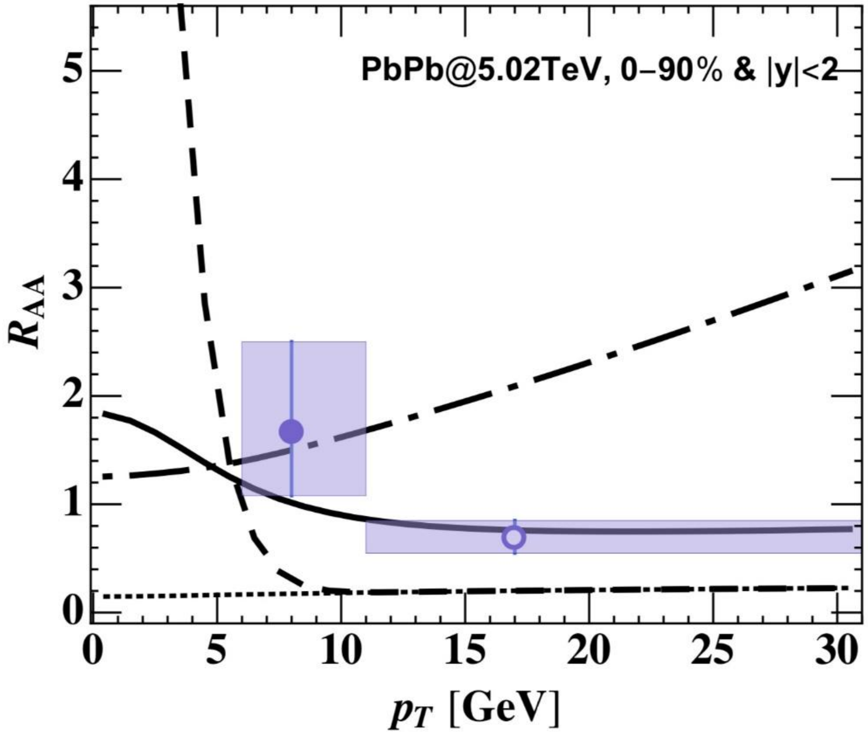
B_c enhancement

J.Zhao and PZ, arXiv: 202208******

- J/ψ and Υ production in p+p needs only a pair of $c\bar{c}$ or $b\bar{b}$ in one event.
- B_c production in p+p needs both a pair of $c\bar{c}$ and $b\bar{b}$ in one event !

→ from J/ψ suppression to B_c enhancement !

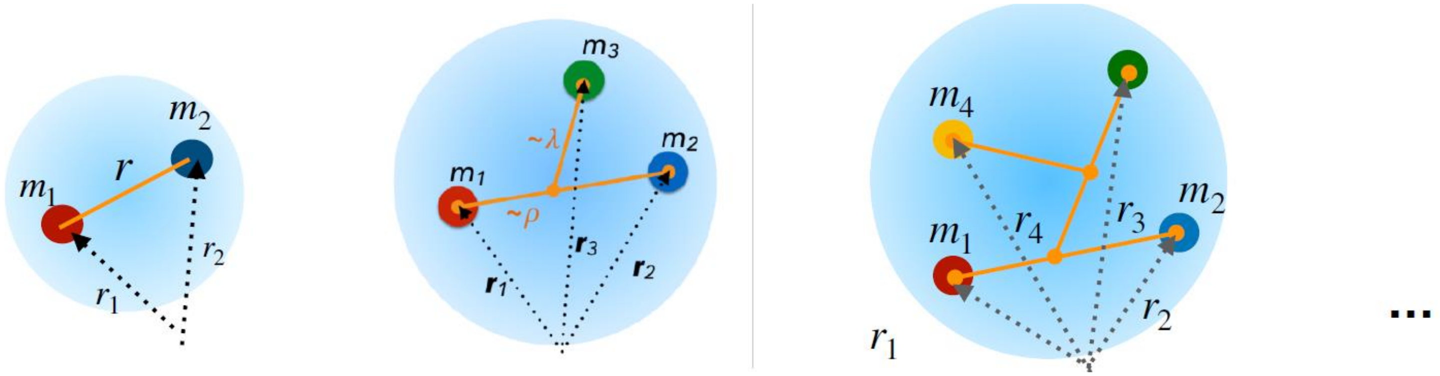




A challenge to experimentalists: Discovery of B_c family !

N-body Potential Model for Fully Heavy-flavor Hadrons

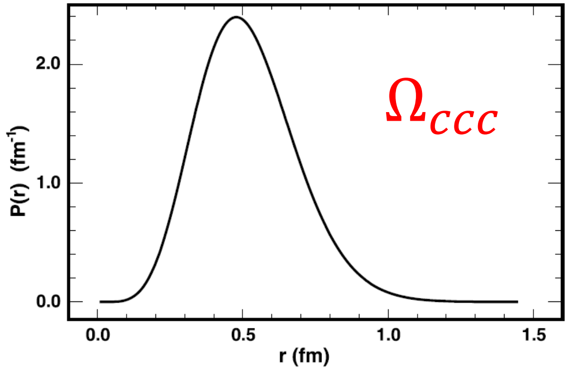
J.Zhao, S.Shi and PZ, PRD102, 114001 (2020)



$$\left[\sum_{i=1}^{N}\left(-\frac{\vec{\nabla}_i^2}{2m_i}\right)+V(\vec{r}_1,\ldots\ldots\vec{r}_N,T)\right]\Psi(\vec{r}_1,\ldots\ldots\vec{r}_N)=E_N\Psi(\vec{r}_1,\ldots\ldots\vec{r}_N)$$

$$V(\vec{r}_1,\ldots\ldots\vec{r}_N,T)=\sum_{i\neq j}^{N}\left[V_{ij}^{c}(|\vec{r}_{ij}|,T)+V_{ij}^{s}(|\vec{r}_{ij}|)\right]$$

$V_{ij}^{c}(|\vec{r}_{ij}|,T)$ *from Lattice QCD,* $V_{ij}^{s}(|\vec{r}_{ij}|)=\beta e^{-\gamma|\vec{r}_{ij}|}\vec{s}_i\cdot\vec{s}_j$



- *(Center−of−mass motion) · (relative motion)*

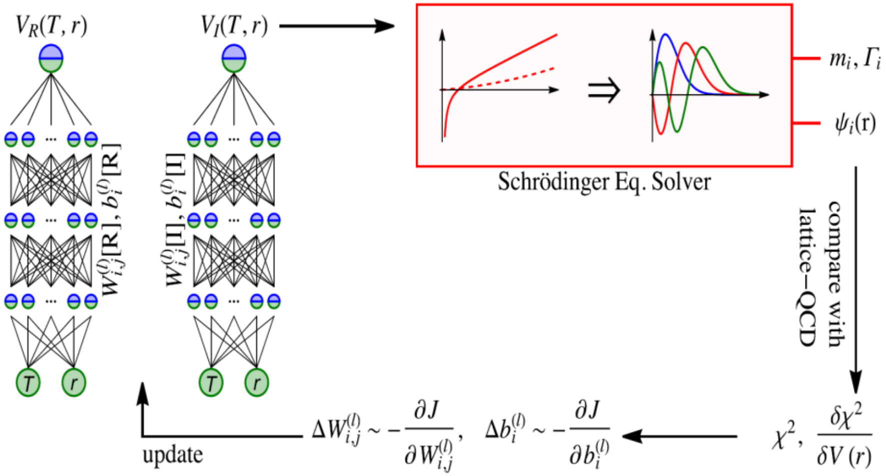
$$\Psi(\vec{r}_1,\ldots\ldots\vec{r}_N)=\Theta(\vec{R})\Phi(\vec{x}_1,\ldots\ldots\vec{x}_{N-1})$$

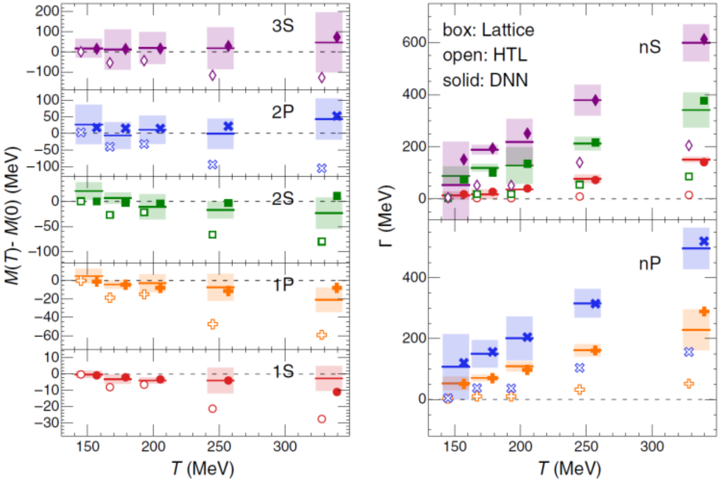
Machine Learning Meets LQCD

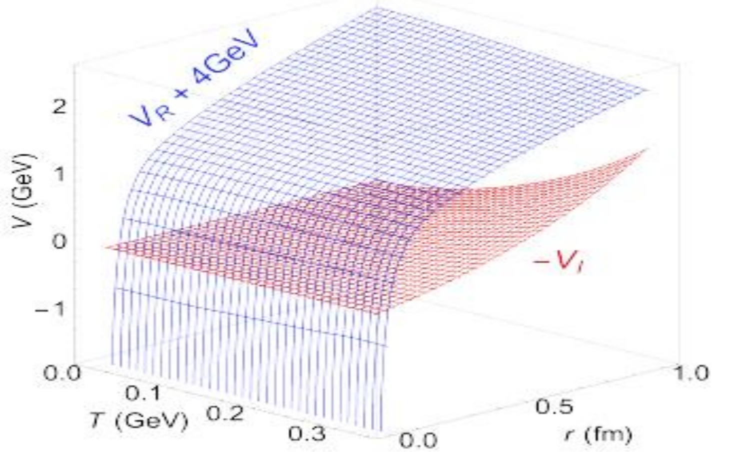
S.Shi, K.Zhou, J.Zhao, S.Mukherjee and PZ, PRD105, 014017(2021)

Inversely solving the Schroedinger equation

$$m(T),\ \ \Gamma(T) \rightarrow\ \ V(r,T) = Re\ V(r,T) + i\ Im\ V(r,T)$$







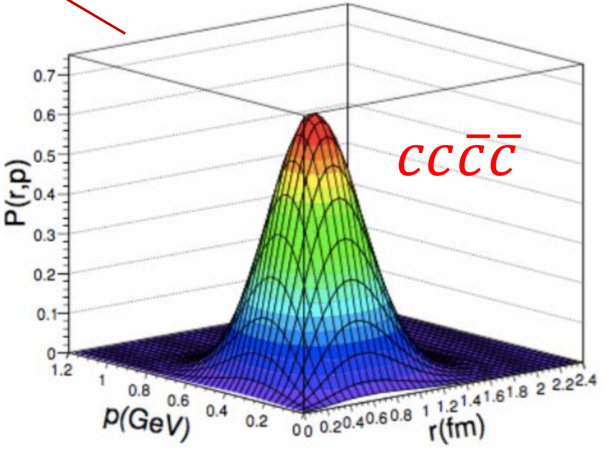
Heavy Quark Coalescence Mechanism

$$\frac{dn}{d^3\vec{p}} = C\int_{\Sigma} \frac{p^\mu d\sigma_\mu}{(2\pi)^3}\int \frac{d^{N-1}\vec{x}\,d^{N-1}\vec{y}}{(2\pi)^9} W(\vec{x},\vec{p})\, f_1(\vec{x}_1,\vec{p}_1) \dots\dots f_N(\vec{x}_N,\vec{p}_N)$$

controlled by eigen value of Schroedinger Eq.

controlled by eigen function of Schroedinger Eq.

heavy quark distribution



- Hadronization hypersurface is the production surface

$$\Sigma(\vec{x},t):\ T(\vec{x},t) = T_D\ \ (> T_c)$$

Temperature $T(\vec{x},t)$ can be calculated by hydrodynamics

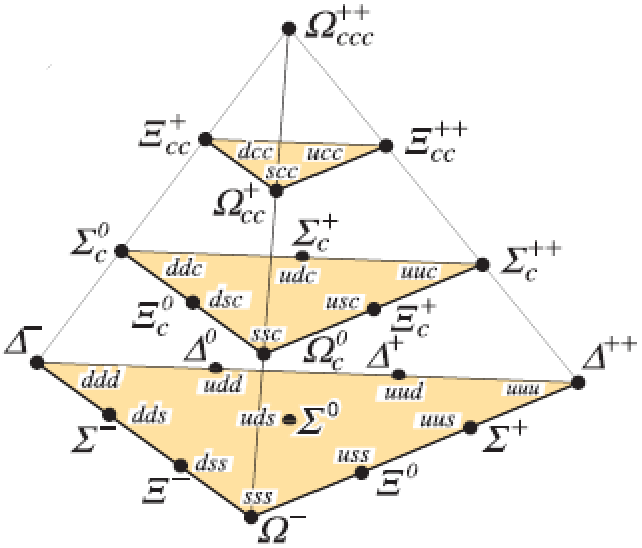
$$\partial^\mu T_{\mu\nu} = 0$$

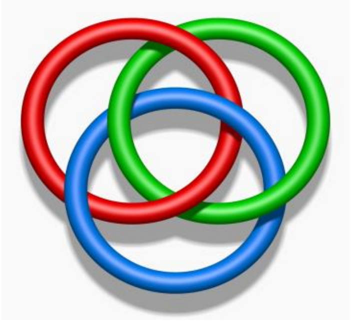
- $f_i(\vec{x}_i,\vec{p}_i)$ is controlled by heavy quark transport. If thermalized with QGP, Fermi-Dirac distribution

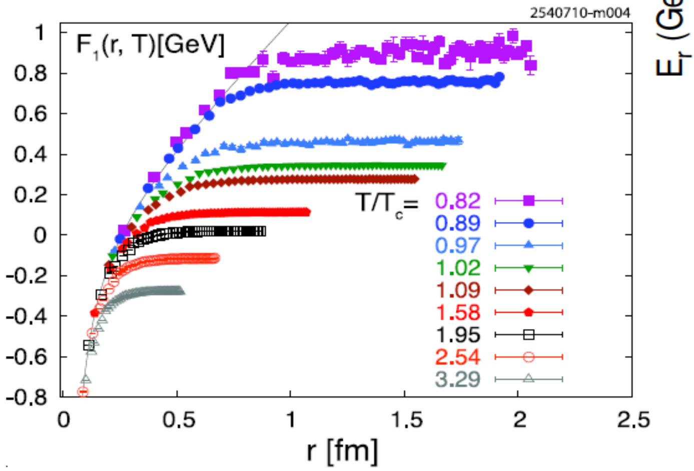
$$f_i(\vec{x}_i,\vec{p}_i) = \frac{1}{e^{p_i^\mu u_\mu/T(\vec{x}_i)}+1}$$

Ω_{ccc} as a Borromean Ring and an Efimov State

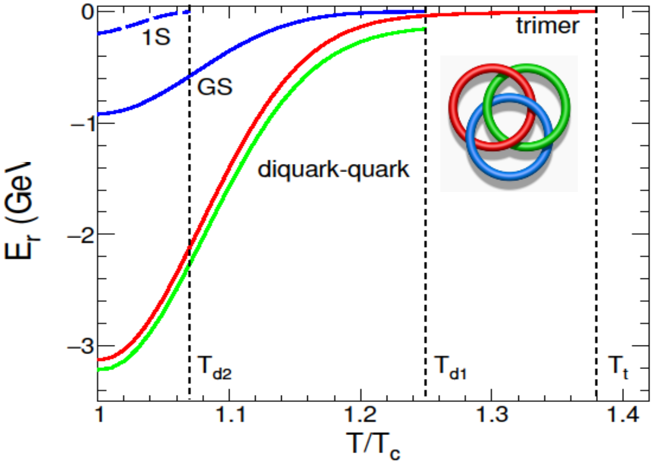
J.Zhao and PZ, PLB775,84(2017)

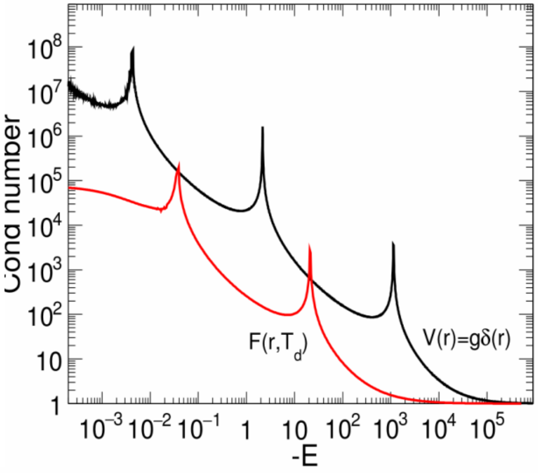






short range potential at high T !





$$\frac{E_n}{E_{n+1}} = e^{2\pi/s_0} = 515,$$ *Efimov state !*

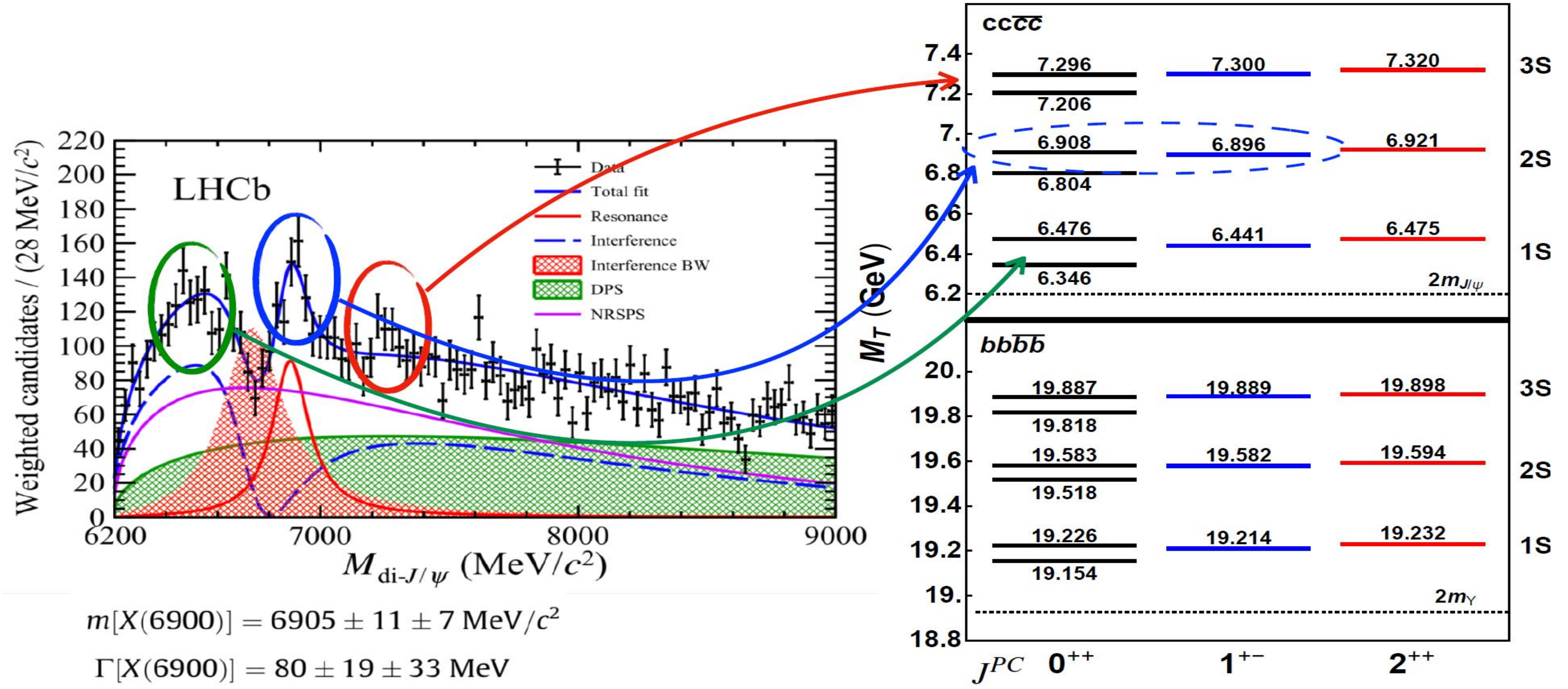
A challenge to experimentalists:

Discovery of Ω_{ccc} and its Borromean state in A+A !

Exotic Hadrons

J.Zhao, S.Shi and PZ, PRD102, 114001 (2020)

- *2022： ATLAS, CMS and LHCb announced the discovery of fully-heavy tetraquarks in p+p!*
- *HIC: Plenty of charm quarks and coalescence → exotic hadron enhancement in A+A!*



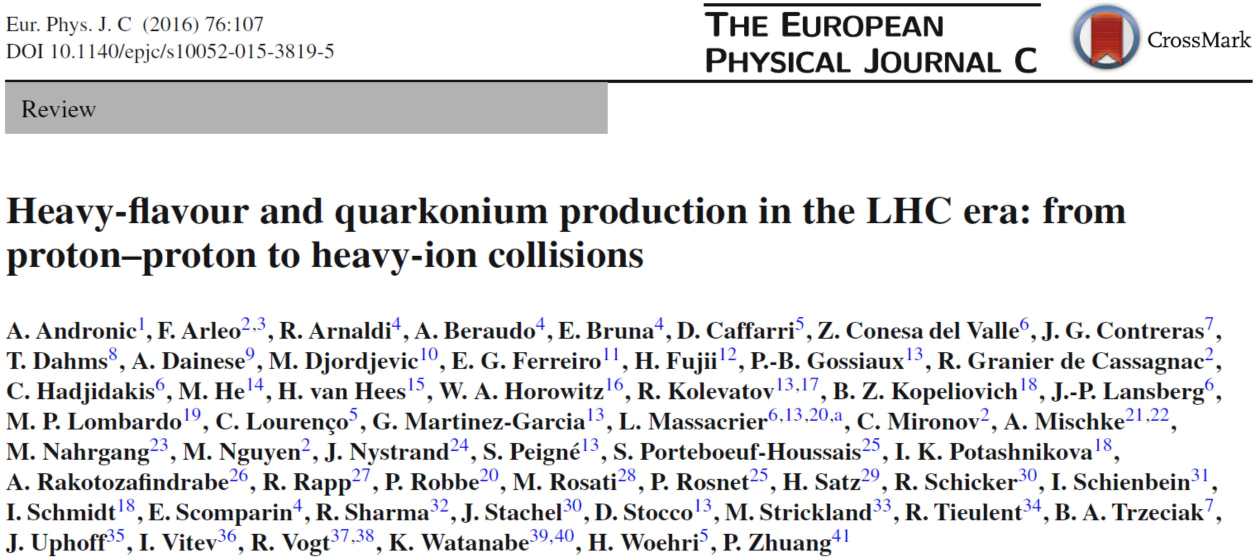
$m[X(6900)] = 6905 \pm 11 \pm 7$ MeV/c^2

$\Gamma[X(6900)] = 80 \pm 19 \pm 33$ MeV

$$\left(\frac{d\sigma}{N_c dy}\right)_{AA} \sim 770\, pb \gg \left(\frac{d\sigma}{dy}\right)_{pp} \sim 78\, pb$$

A challenge to experimentalists: Discovery of exotic hadrons and their structure !

2篇综述文章



Eur. Phys. J. C (2016) 76:107
DOI 10.1140/epjc/s10052-015-3819-5

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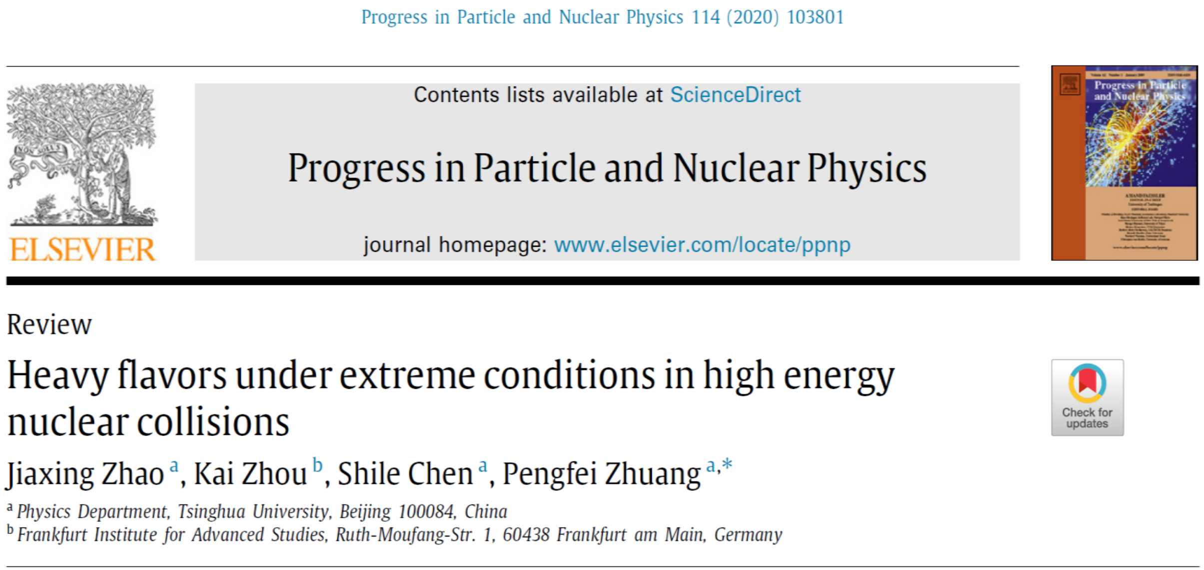
Review

Heavy-flavour and quarkonium production in the LHC era: from proton–proton to heavy-ion collisions

A. Andronic[1], F. Arleo[2,3], R. Arnaldi[4], A. Beraudo[4], E. Bruna[4], D. Caffarri[5], Z. Conesa del Valle[6], J. G. Contreras[7], T. Dahms[8], A. Dainese[9], M. Djordjevic[10], E. G. Ferreiro[11], H. Fujii[12], P.-B. Gossiaux[13], R. Granier de Cassagnac[2], C. Hadjidakis[6], M. He[14], H. van Hees[15], W. A. Horowitz[16], R. Kolevatov[13,17], B. Z. Kopeliovich[18], J.-P. Lansberg[6], M. P. Lombardo[19], C. Lourenço[5], G. Martinez-Garcia[13], L. Massacrier[6,13,20,a], C. Mironov[2], A. Mischke[21,22], M. Nahrgang[23], M. Nguyen[2], J. Nystrand[24], S. Peigné[13], S. Porteboeuf-Houssais[25], I. K. Potashnikova[18], A. Rakotozafindrabe[26], R. Rapp[27], P. Robbe[20], M. Rosati[28], P. Rosnet[25], H. Satz[29], R. Schicker[30], I. Schienbein[31], I. Schmidt[18], E. Scomparin[4], R. Sharma[32], J. Stachel[30], D. Stocco[13], M. Strickland[33], R. Tieulent[34], B. A. Trzeciak[7], J. Uphoff[35], I. Vitev[36], R. Vogt[37,38], K. Watanabe[39,40], H. Woehri[5], P. Zhuang[41]



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Review

Heavy flavors under extreme conditions in high energy nuclear collisions

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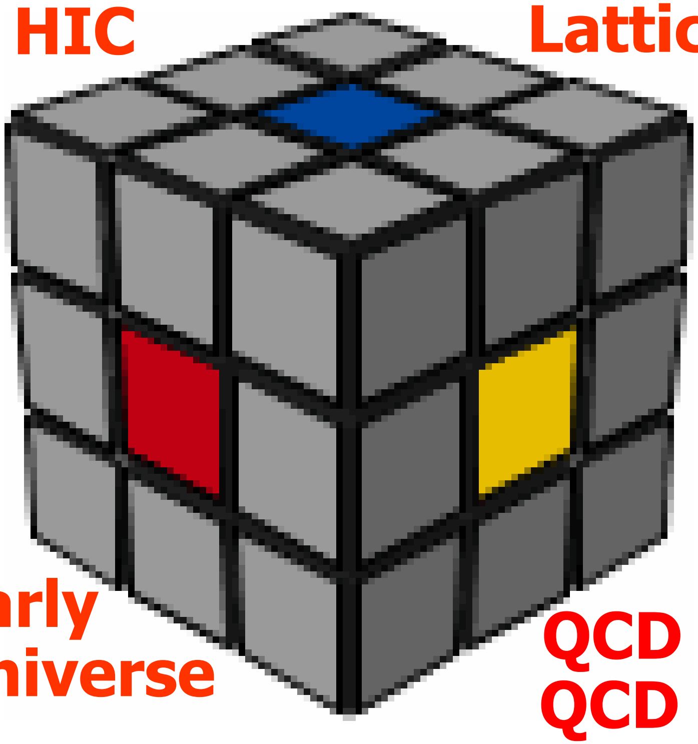
[b] Frankfurt Institute for Advanced Studies, Ruth-Moufang-Str. 1, 60438 Frankfurt am Main, Germany

QGP：A Rubik's Cube



HIC

Lattice

Compact Stars

FRG, AdS/CFT, Effective Models

Early Universe

QCD Condensed Matter
QCD Spin Electronics

革命尚未成功，同志仍需努力！