QCD dense matter and color superconductor

Mei Huang 黄梅

University of Chinese Academy of Sciences,
中国科学院大学核科学与技术学院

2022年复旦大学粒子物理与核物理暑期学校

2022年8月13-21

I. A brief introduction on QCD dense matter

II. QCD critical end point

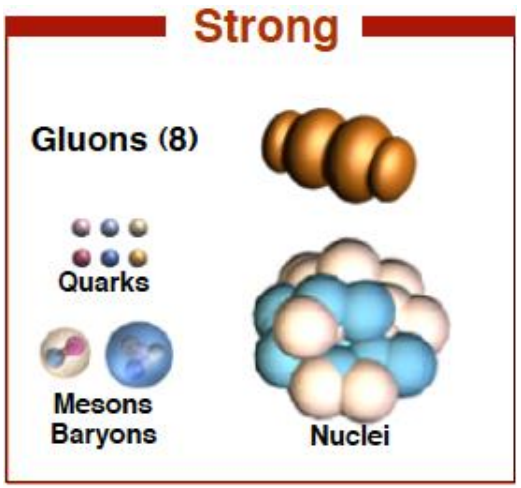
III. Quarkyonic matter and EOS for neutron star

IV. Color superconductor

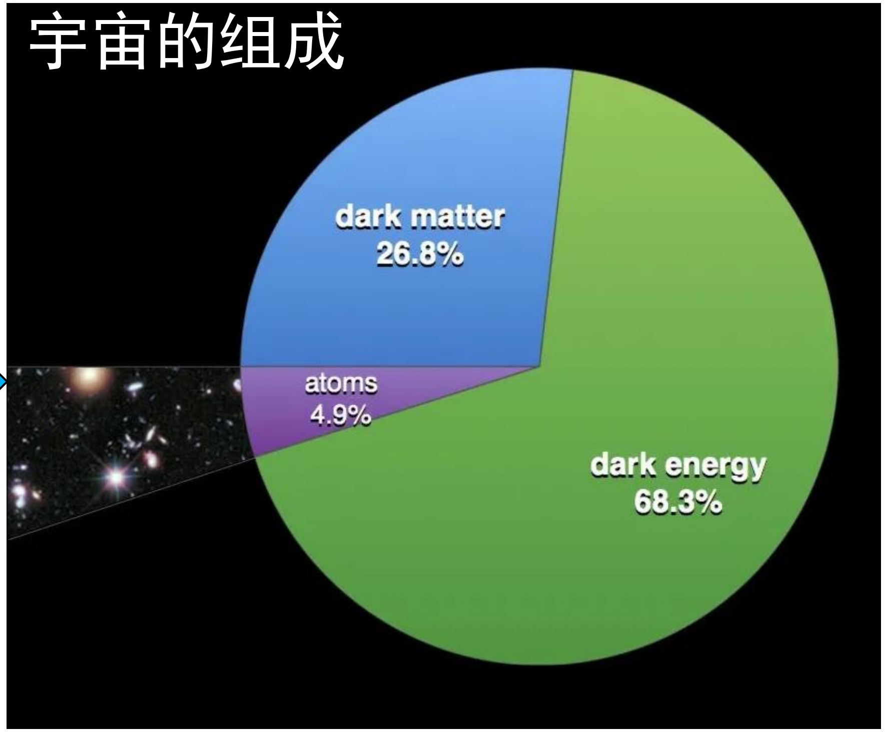
V. Summary and outlook

A brief Introduction

量子色动力学QCD



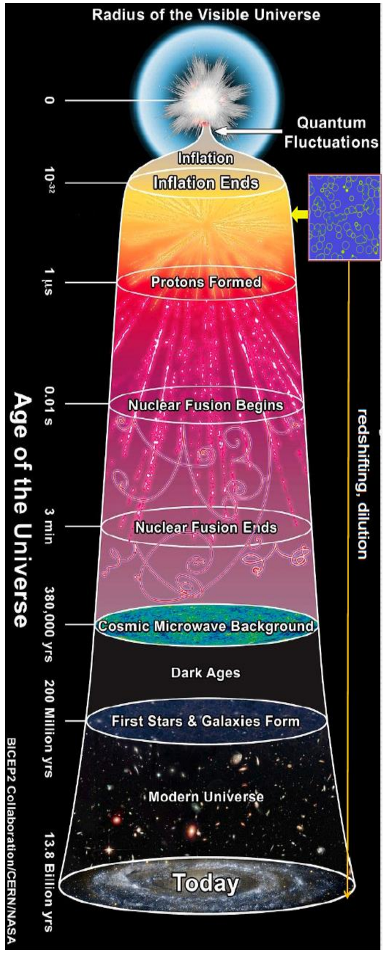
QCD两个重要非微扰性质：
色禁闭和手征对称性自发破缺

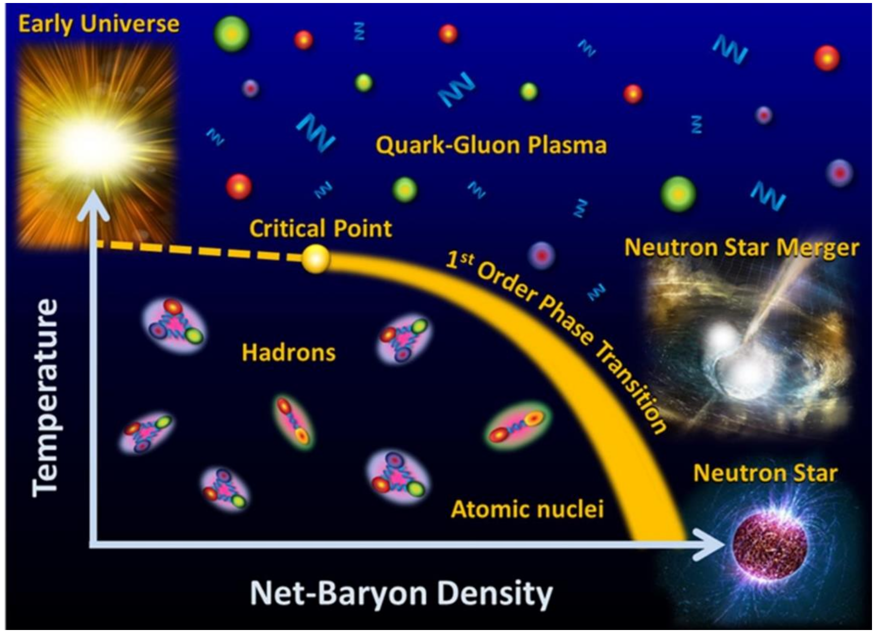


宇宙的可见物质占5%，这些可见物质的质量99%来自于强相互作用（QCD）的手征对称性自发破缺。

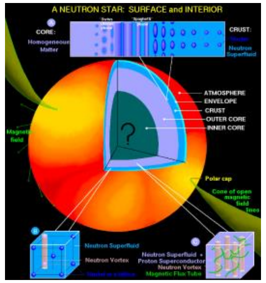
高温高密QCD相结构

宇宙早期



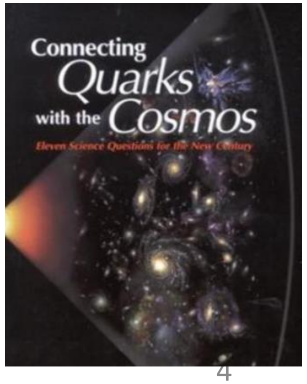


致密星体内部夸克物质



美国国家研究委员会由19名权威物理学家和天文学家联合执笔的2002年报告中列出了新世纪基础物理的最重要11个科学问题之一：

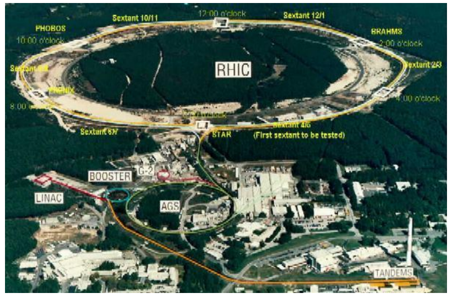
What are the new state of matter at exceedingly high density and temperature?
How were the elements from iron to uranium made?

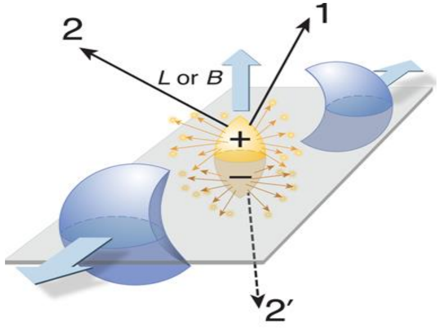


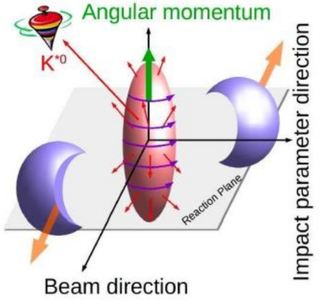
“多信使”时代

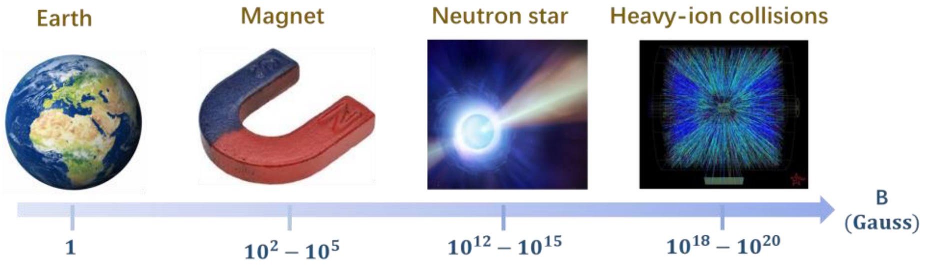
重离子碰撞实验：

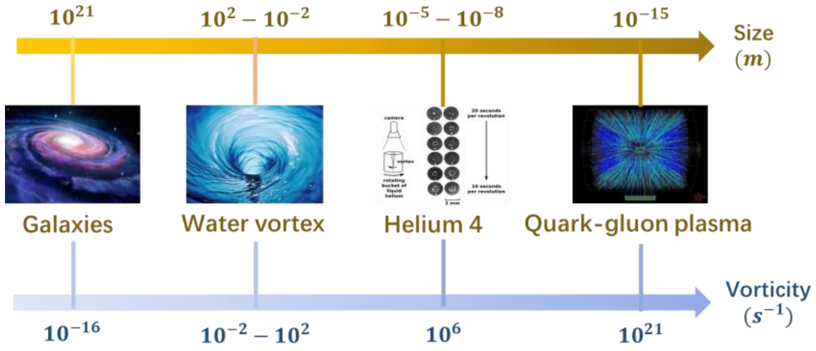
- RHIC@BNL 美国布鲁克海文国家实验室
- LHC@CERN 欧洲核子研究中心
- FAIR@GSI 德国亥姆霍兹重离子研究中心
- NICA@DUBNA，俄罗斯杜布纳联合核子研究所
- CSR @兰州、HIAF @惠州，中科院近代物理研究所









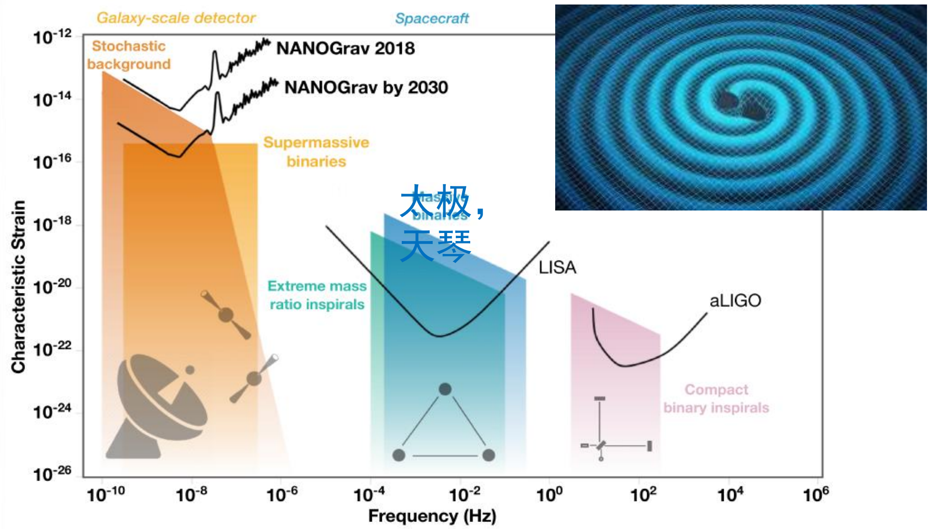


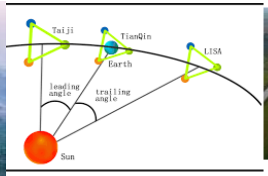
重离子碰撞不仅产生高温高密的环境，还可以产生强磁场强转动。

“多信使”时代

更多的引力波探测器及射电望远镜对引力波和致密星体（质量和半径）进行精确测量，一方面对理论进行约束，另一方面也需要理论对实验结果进行理解。

Pulsar Timing Array对PSRJ0348+0432和PSR J1624-2230的质量测量2倍太阳质量



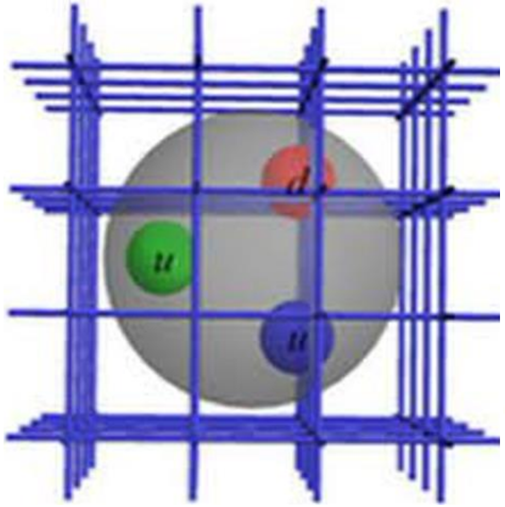


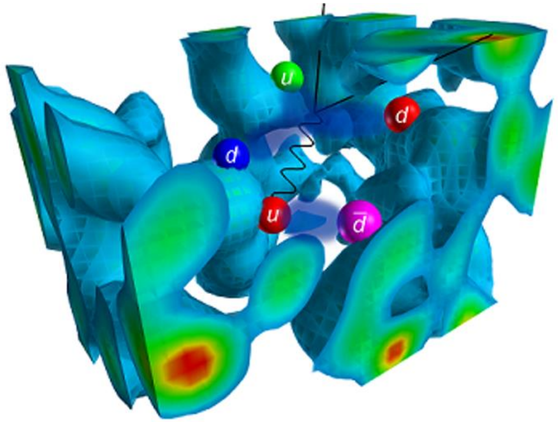
LIGO/VirgoGW170817，致密星体半径约12km

“多信使”时代

格点计算：QCD格点强子物理，QCD相变

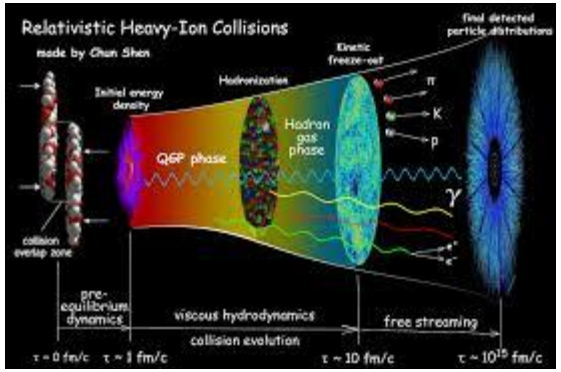
超级计算机，机器学习算法，量子计算方法的发展将提供更多的信息，需要理论物理学家揭示背后的原理或机制。



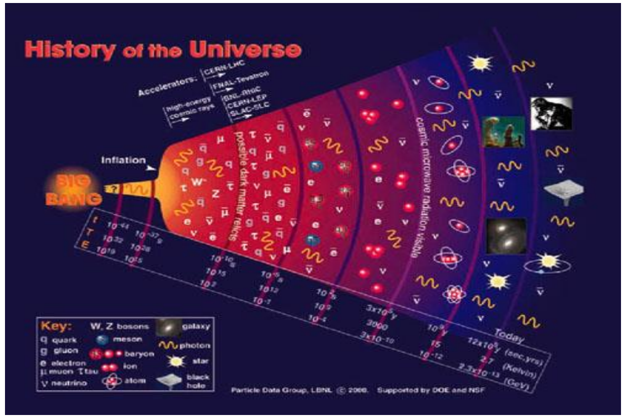


QCD matter under extreme conditions

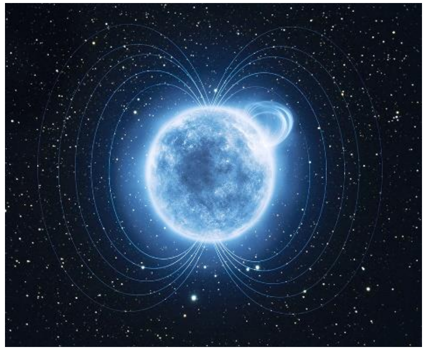
$$\mathrm{T}, \mu_B, \mathbf{B}, \mathbf{E}\cdot\mathbf{B}, \boldsymbol{\omega}, \mu_{\mathrm{I}}, \boldsymbol{L}$$



LHC,RHIC,FAIR,NICA,HIAF



Early universe

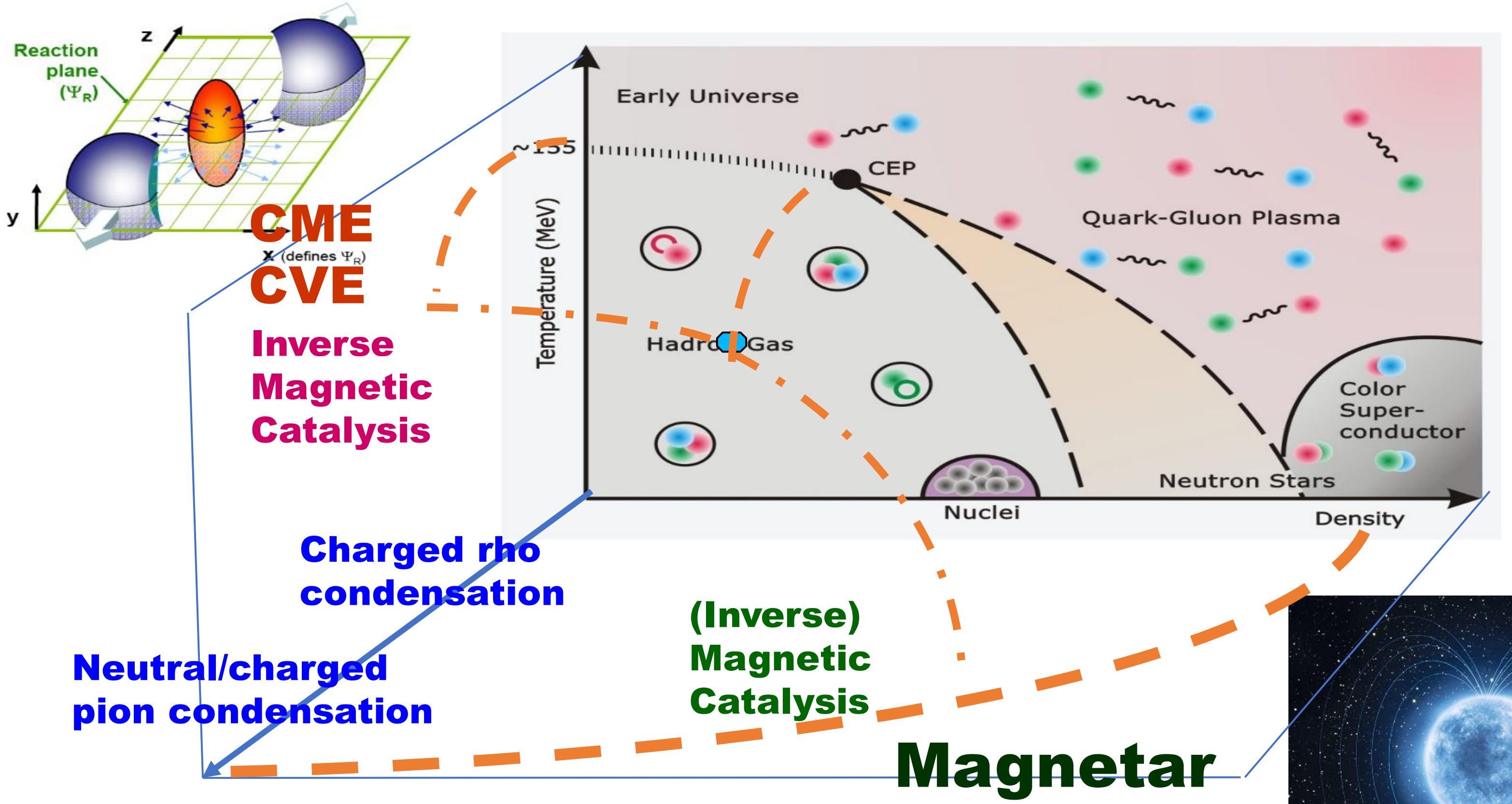


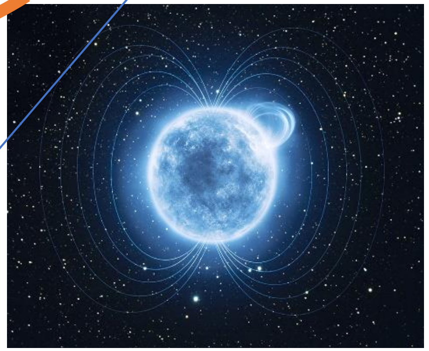
Neutron star

Neutron star merge→BH

Explored QCD phase diagram

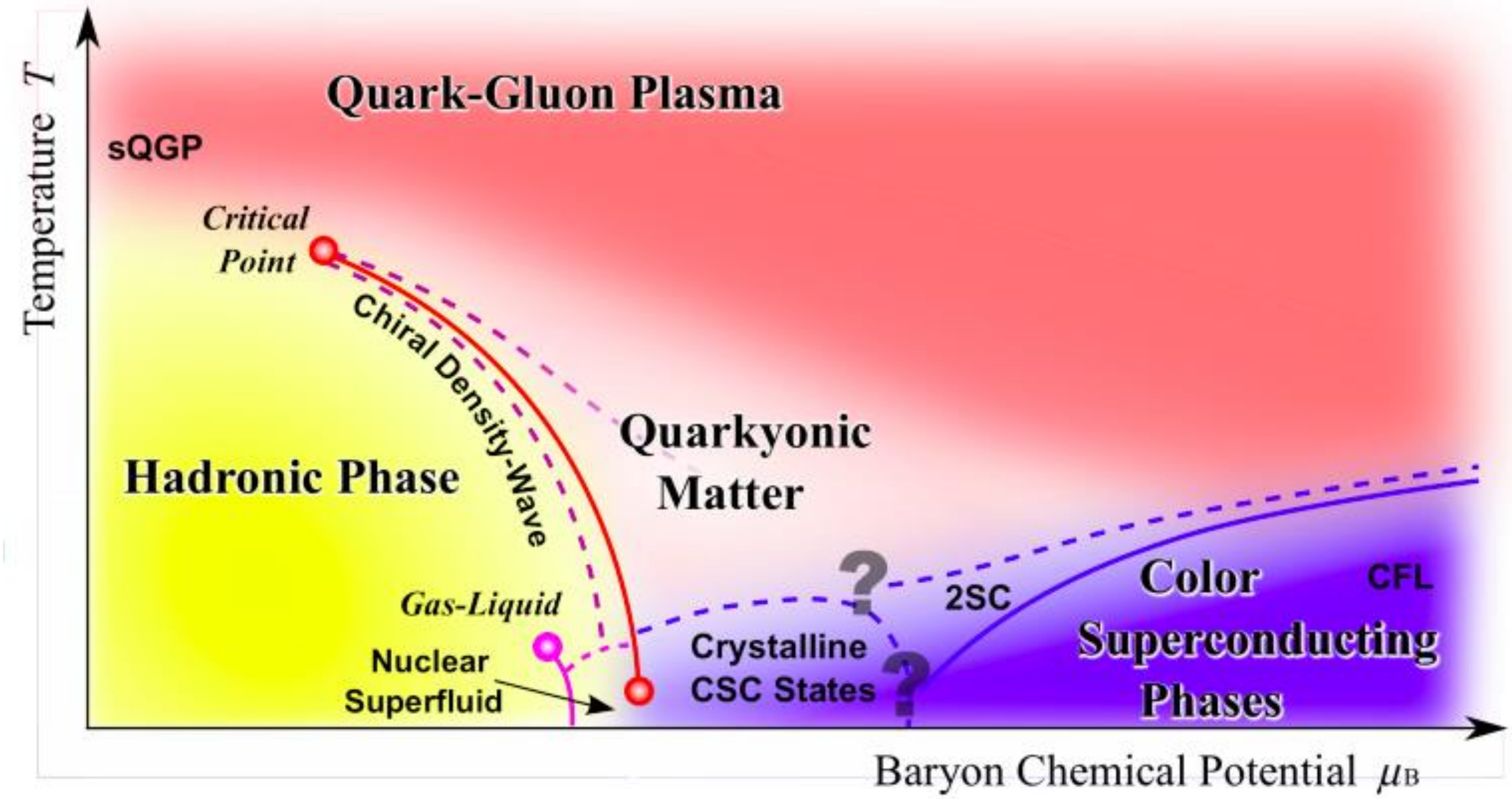
by theorists





$$\mathbf{B}, \mathbf{E}\cdot\mathbf{B}, \omega, \mu_I, L$$

Dense Matter: QCD CEP, Quarkyonic matter, CSC



K. Fukushima and T. Hatsuda, Rept. Prog. Phys. 74, 014001(2011); arXiv: 1005.4814

QCD properties in the vacuum

重要难题

I.Spontaneous Chiral symmetry breaking (quark dynamics)

Goldstone boson and chiral condensate

Chiral partners have different masses

II. Confinement (Gluodynamics)

手征模型

基于AdS/CFT对偶的全息方法（侯德富）

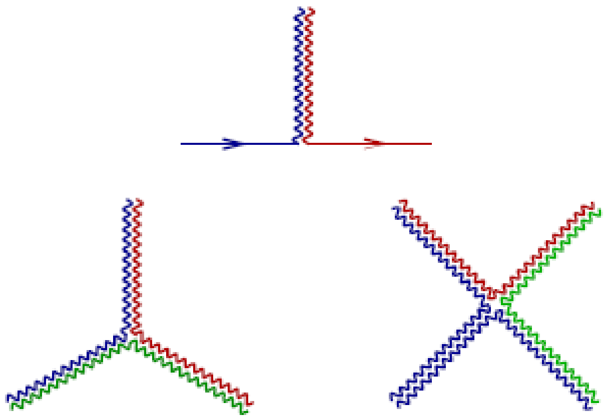
DSE，
泛函重整化群（FRG）（付伟杰）

QCD

$$\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$i\not{D} q = \gamma^\mu \left(i\partial_\mu + g A^a_\mu t^a \right) q$$



Elementary fields: Quarks Gluons

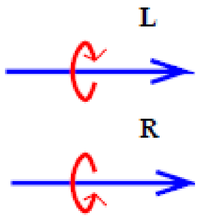
$$(q_\alpha)^a_f \begin{cases} \text{color} & a = 1, \ldots, 3 \\ \text{spin} & \alpha = 1, 2 \\ \text{flavor} & f = u, d, s, c, b, t \end{cases}$$

$$A^a_\mu \begin{cases} \text{color} & a = 1, \ldots, 8 \\ \text{spin} & \epsilon^\pm_\mu \end{cases}$$

Chiral Symmetry

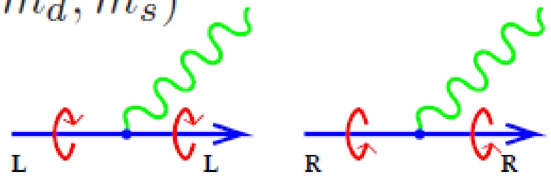
Define left and right handed fields

$$\psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi$$

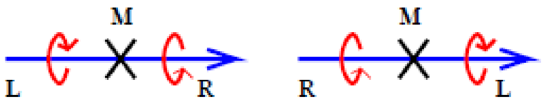


Fermionic lagrangian, $M = \mathrm{diag}(m_u, m_d, m_s)$

$$\mathcal{L} = \bar{\psi}_L(i\not{D})\psi_L + \bar{\psi}_R(i\not{D})\psi_R$$



$$+\bar{\psi}_L M \psi_R + \bar{\psi}_R M \psi_L$$



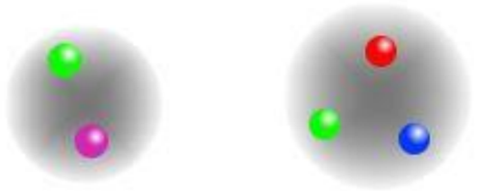
$M = 0$: Chiral symmetry $(L, R) \in SU(3)_L \times SU(3)_R$

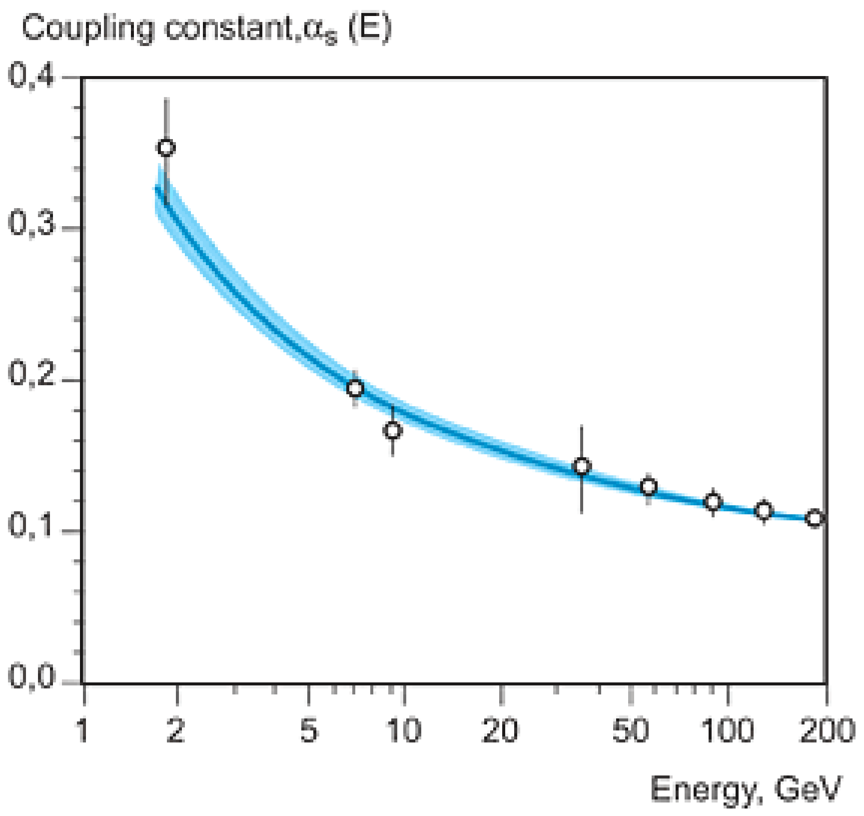
$$\psi_L \to L\psi_L, \qquad \psi_R \to R\psi_R$$

QCD

Strong coupling:

Confinement





Chiral Symmetry Breaking

Chiral symmetry is spontaneously broken

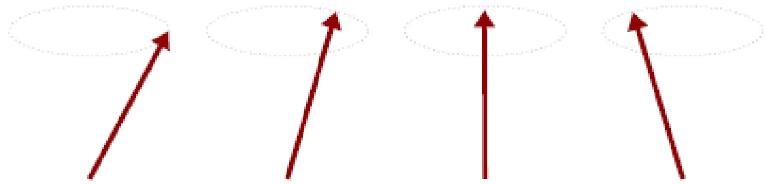
$$\langle \bar{\psi}_L^f \psi_R^g + \bar{\psi}_L^f \psi_R^g \rangle \simeq -(230\,\mathrm{MeV})^3\ \delta^{fg}$$

$$SU(3)_L \times SU(3)_R \rightarrow SU(3)_V \qquad\qquad (G \rightarrow H)$$

Consequences: dynamical mass generation $m_Q = 300\,\mathrm{MeV} \gg m_q$

$$m_N = 890\,\mathrm{MeV} + 45\,\mathrm{MeV} \qquad\qquad (\mathrm{QCD},\ 95\%) + (\mathrm{Higgs},\ 5\%)$$

Goldstone Bosons: Collective oscillations of order parameter



$$m_\pi = 139\,\mathrm{MeV}$$

Chiral and deconfinement phase transitions

CEP is for chiral phase transition!

Chiral phase transition:

quark-antiquark condensate (for m=0)

Chiral symmetry breaking: $\langle\bar{\psi}\psi\rangle \neq 0$

Chiral symmetry restoration: $\langle\bar{\psi}\psi\rangle = 0$

Deconfinement phase transition:

referring to the "permanent confinement"

Polyakov loop (for m= infinity)

$$L(\vec{x}) = \frac{1}{N_c}\mathrm{tr}\,\mathcal{P}(\vec{x}) \text{ with } \mathcal{P}(\vec{x}) = \mathrm{P}\,\mathrm{e}^{ig\int_0^\beta dt\, A_0(t,\vec{x})}$$

$$\langle L(\vec{x})\rangle \sim \exp(-\beta F_q)$$

Confinement: center symmetric $\langle L\rangle = 0 \quad F_q \to \infty$

Deconfinement: center symmetry breaking $\langle L\rangle \neq 0 \quad F_q < \infty$

Mechanism of spontaneous chiral symmetry breaking

Nobel prize 2008

Press Release
7 October 2008
The Royal Swedish Academy of Sciences has decided to award the
Nobel Prize in Physics for 2008 with one half to
Yoichiro Nambu
Enrico Fermi Institute, University of Chicago, IL, USA
"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

Spontaneous Symmetry Breaking in Particle Physics

The really bold assumption that Nambu now made in 1960 [44] was that spontaneous symmetry breaking could also exist in a quantum field theory for elementary particles. In magnetism or in superconductivity, the "vacuum" is really a ground state, in the first case of atoms and in the second case of electrons and atoms. It is possible to give a vacuum expectation value for a physical quantity like a spin. In a particle theory, the vacuum is an abstract state and was assumed to be empty apart from quantum fluctuations. Nambu now introduced vacuum expectation values for certain fields. In fact, he put forward a scheme for the theory of the strong interactions that mimicked superconductivity in the following way

Superconductivity	**Strong Interactions**
free electrons	hypothetical fermions with small mass
phonon interaction	unknown interaction
energy gap	observed mass of the nucleon
collective excitations	mesons, bound states
charge	chirality
gauge invariance	chiral invariance, possibly approximate

NJL model

Nambu--Jona-Lasnio Model

Mechanism for spontaneous chiral symmetry breaking

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi - m_0 \bar{\psi}\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2]$$

The same global symmetry as QCD

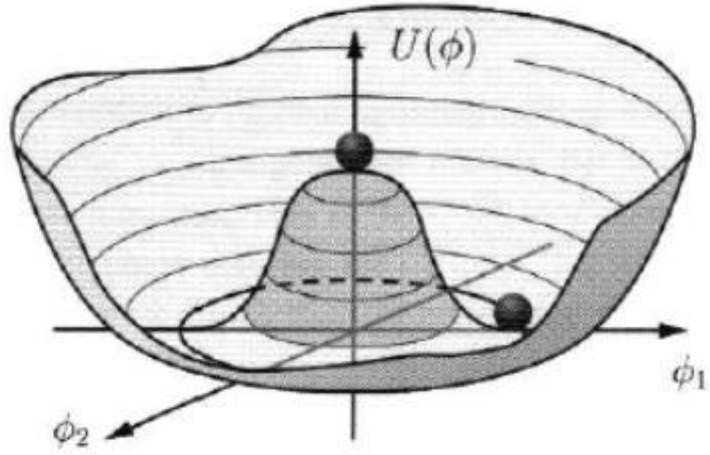
Symmetry	Transformation	Current	Name
$SU_V(2)$	$\psi \to e^{-i\tau\cdot\omega/2}\psi$	$J^k_\mu = \bar{\psi}\gamma_\mu \tau^k \psi$	isospin
$U_V(1)$	$\psi \to e^{-i\alpha}\psi$	$j_\mu = \bar{\psi}\gamma_\mu \psi$	baryonic
$SU_A(2)$	$\psi \to e^{-i\tau\cdot\theta\gamma_5/2}\psi$	$J^k_{5\mu} = \bar{\psi}\gamma_\mu\gamma_5\tau^k\psi$	chiral
$U_A(1)$	$\psi \to e^{-i\beta\gamma_5}\psi$	$j_{5\mu} = \bar{\psi}\gamma_\mu\gamma_5\psi$	axial

From QCD to NJL, heavy massive gluons in the propagator, four fermion interaction.

NJL model

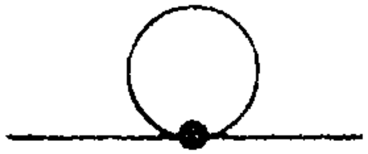
In the vacuum:

$$\langle \bar{\psi}\psi \rangle \neq 0$$



Quark obtains a dynamical mass:

$-i\Sigma$ =



The Nambu–Jona-Lasinio model of quantum chromodynamics

S. P. Klevansky

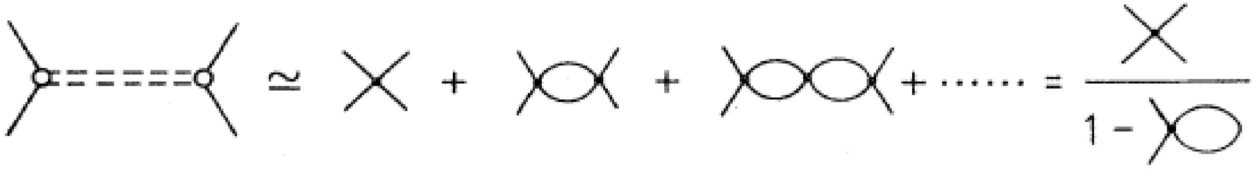
Institut für Theoretische Physik, 6900 Heidelberg, Federal Republic of Germany

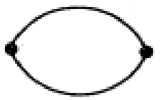
Reviews of Modern Physics, Vol. 64, No. 3, July 1992

Quark obtains a dynamical mass, and chiral symmetry is spontaneously broken.

NJL model

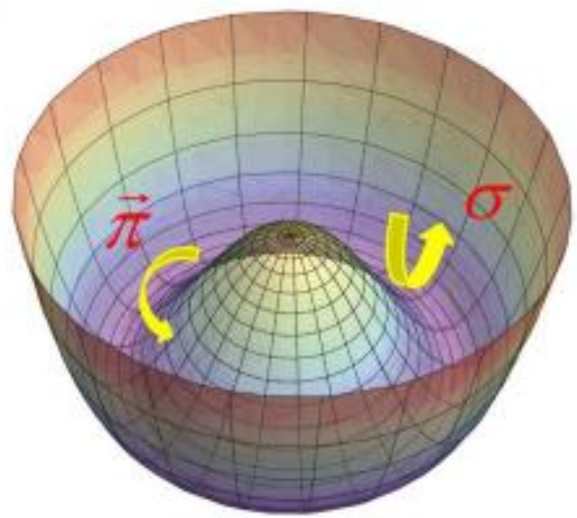
Nambu-Goldstone Theorem: Global symmetry breaking induces Massless Nambu-Goldstone bosons



$\frac{1}{i}\Pi_{ps}(k^2) = i\gamma_5 T$  $i\gamma_5 T$

$$\frac{1}{i}\Pi_{ps}(k^2) = -4N_cN_f \int \frac{d^4p}{(2\pi)^4} \frac{m^{*2} - p^2 + \frac{1}{4}k^2}{[(p+\frac{1}{2}k)^2 - m^{*2}][(p-\frac{1}{2}k)^2 - m^{*2}]}$$

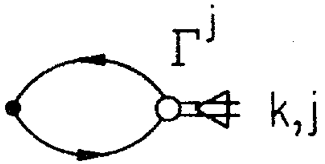
$$m_\pi^2 = -\frac{m_0}{m^*} \frac{1}{4iGN_cN_fI(m_\pi^2)}$$



Zero pion mass in the chiral limit

NJL model

$$\langle 0 | J^i_{5\mu}(x) | \pi^j \rangle = i\, \gamma_\mu\, \gamma_5\, \frac{\tau^i}{2}$$



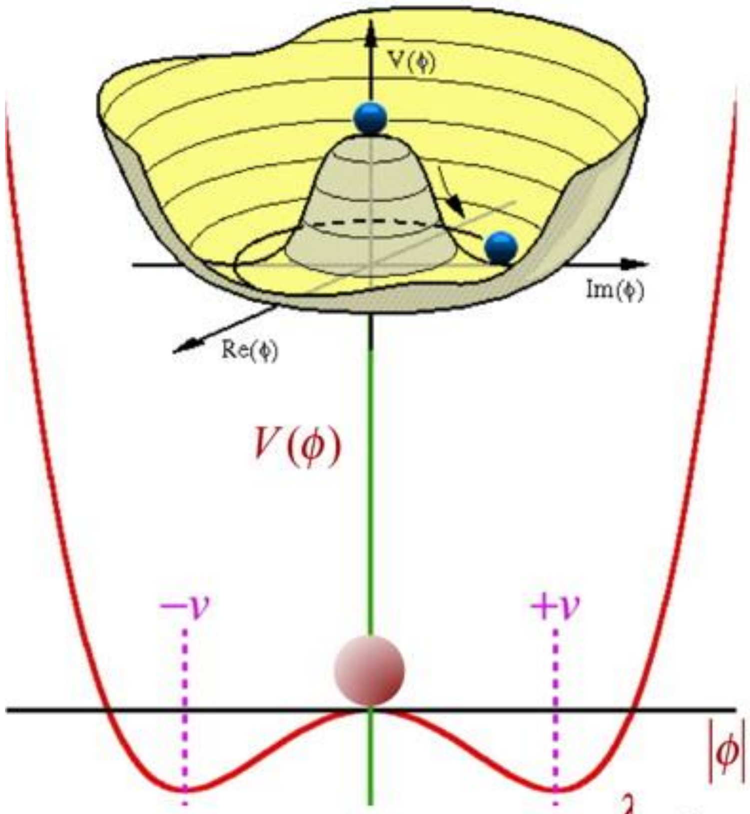
The Goldberger-Treiman relation:

$$f_\pi^2 g_{\pi qq}^2 = m^{*2}$$

The Gell-Mann—Oakes—Renner relation:

$$f_\pi^2 m_\pi^2 = -\tfrac{1}{2}(m_u + m_d)\langle \bar{u}u + \bar{d}d \rangle$$

对称性破缺机制：BCS理论、手征对称性自发破缺、Higgs 机制



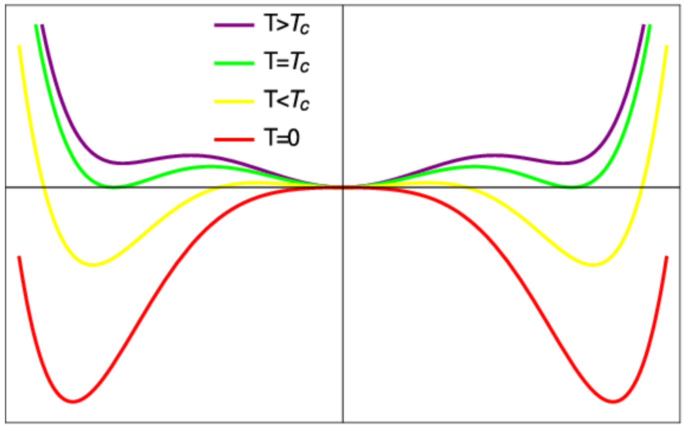
$$V(\phi) = \tfrac{1}{2}\mu^2\phi^\dagger\phi + \tfrac{1}{4}\lambda(\phi^\dagger\phi)^2$$

Groundstate at $|\phi_0| = \sqrt{\dfrac{-\mu^2}{\lambda}} \equiv \upsilon$

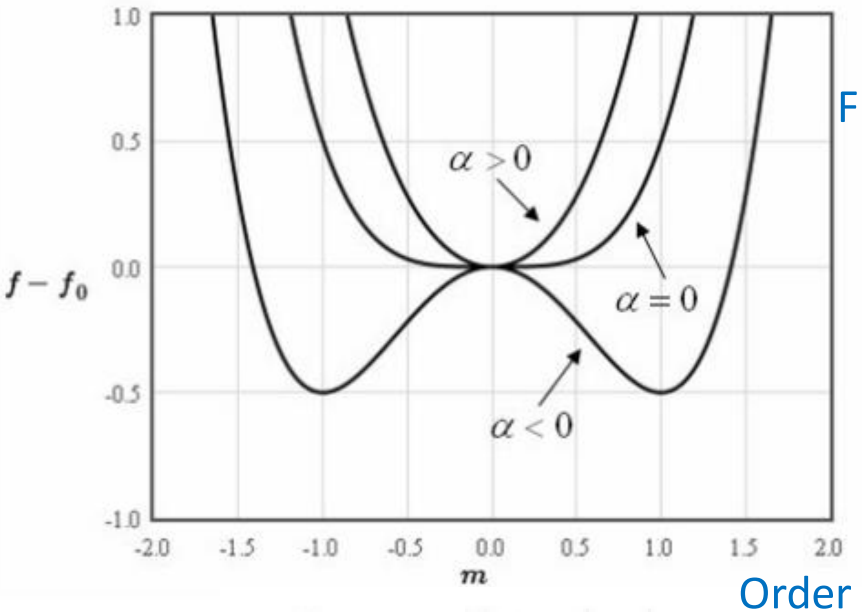
$$|\phi| = \sqrt{\phi^\dagger\phi} = \sqrt{\phi^{+\dagger}\phi^+ + \phi^{0\dagger}\phi^0}$$

$$V(\phi_0) = -\frac{\lambda}{4}\upsilon^4$$

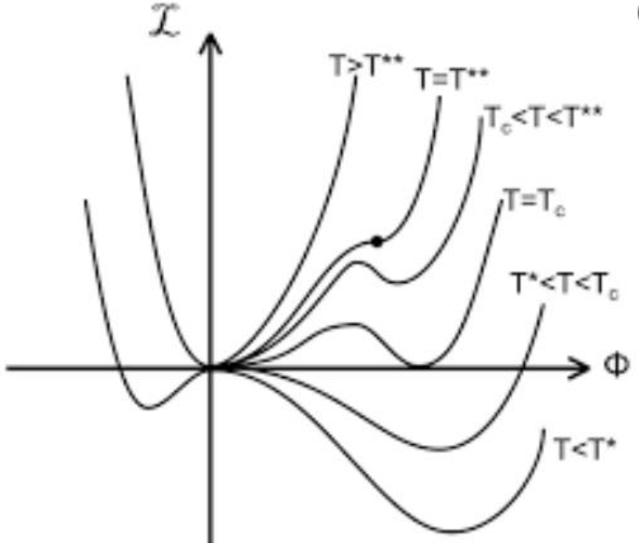
Phase transition



Landau theory for phase transition

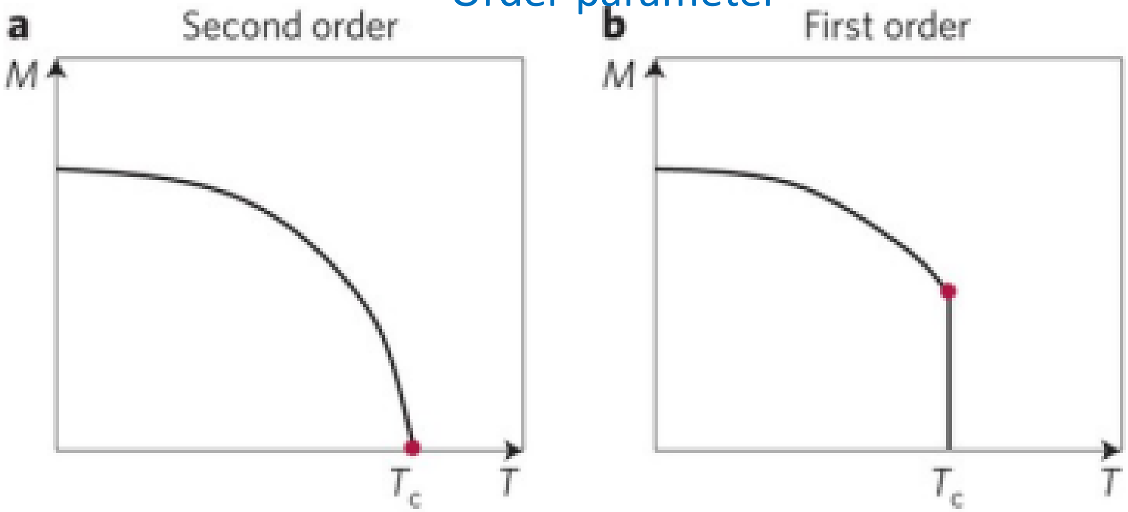


Free Energy



Lev Landau

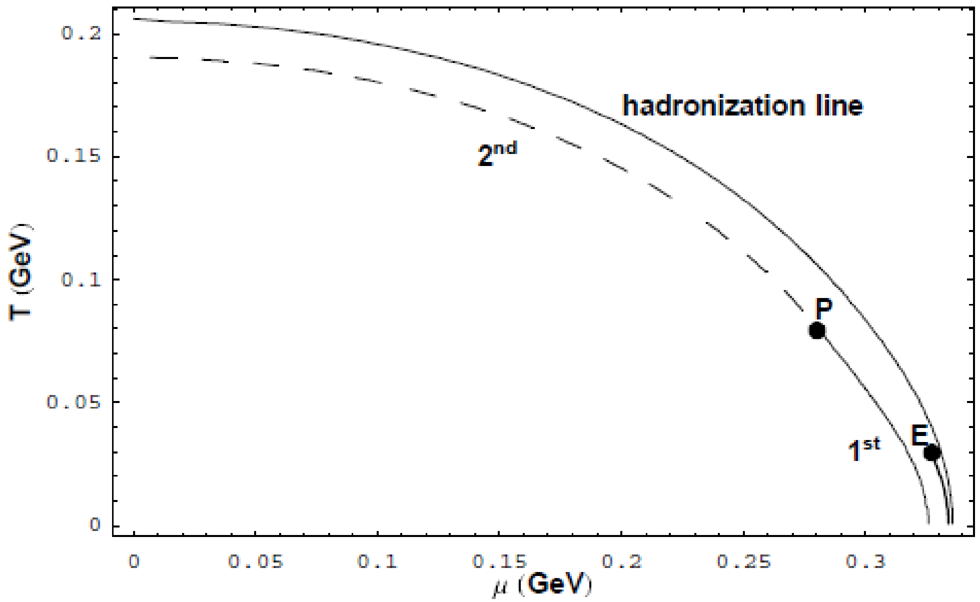
Order parameter



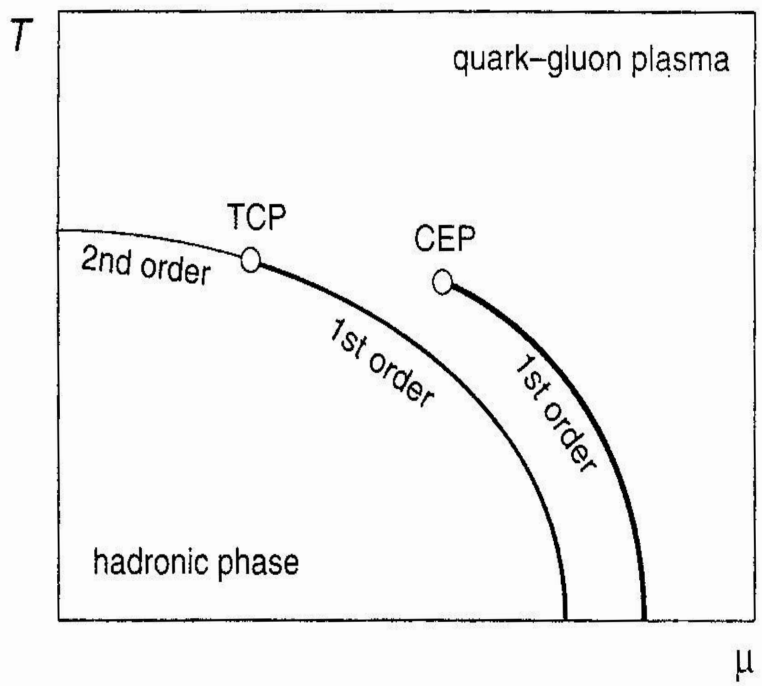
有限温度场论方法:

$$\mathcal{L} \rightarrow \Omega \begin{cases} p, s, \varepsilon, c_s^2 \\ \text{gap equations} \end{cases}$$

Chiral symmetry restoration in the NJL model



P. Zhuang, M.Huang,Z.Yang, Phys.Rev.C62:054901,2000



CEP (predicted 40 years ago)

Remarks on the Chiral Phase Transition in Chromodynamics

Robert D. Pisarski (Santa Barbara, KITP), Frank Wilczek (Santa Barbara, KITP) (Dec, 1983)

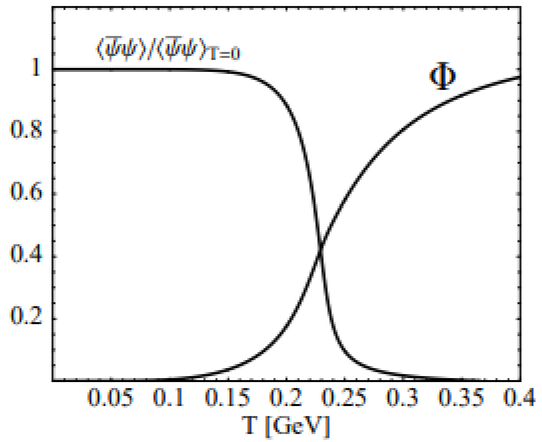
Published in: *Phys.Rev.D* 29 (1984) 338-341

Chiral restoration and deconfinement
Polyakov loop NJL model

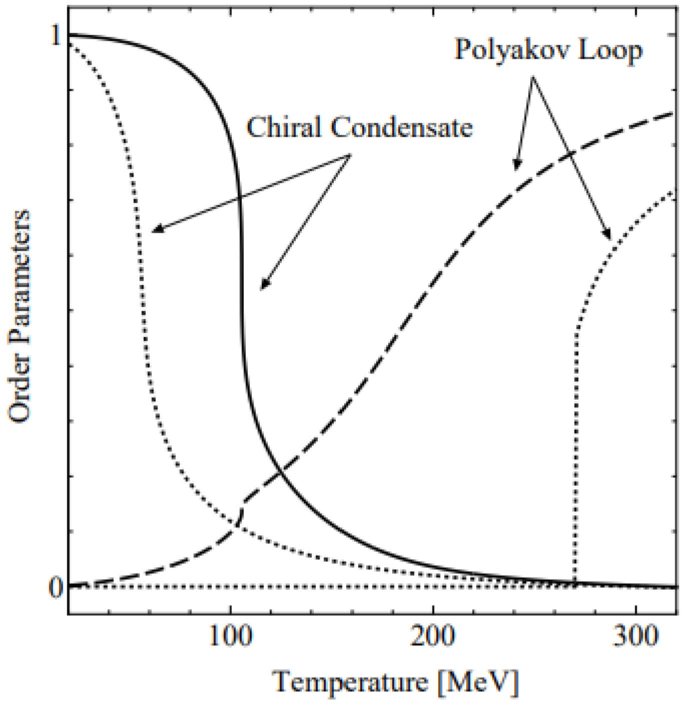
$$\mathcal{L}_{PNJL} = \bar{\psi}\left(i\gamma_\mu D^\mu - \hat{m}_0\right)\psi + \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right] - \mathcal{U}\left(\Phi[A], \bar{\Phi}[A], T\right),$$

$$\frac{\mathcal{U}\left(\Phi, \bar{\Phi}, T\right)}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3 + \bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2 \quad \Phi = (\mathrm{Tr}_c L)/N_c$$

$$L(\vec{x}) = \mathcal{P}\exp\left[i\int_0^\beta d\tau\, A_4(\vec{x}, \tau)\right]$$



Claudia Ratti, Michael A. Thaler, Wolfram Weise, hep-ph/0506234

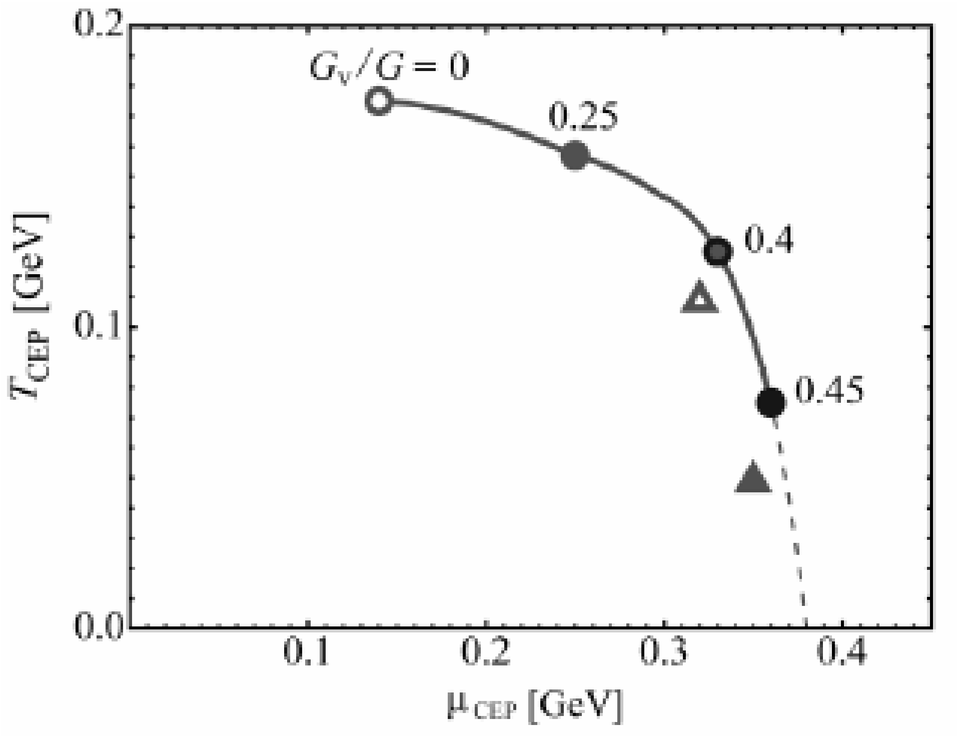


Kenji Fukushima, Phys.Lett.B 591 (2004) 277-284, hep-ph/0310121

Location of CEP: NJL

NJL, PNJL, Nonlocal NJL, ……

P.F Zhuang,M.Huang, M.Hong
Y.X.Liu,W.J.Fu, Z.Zhang
H.S.Zong, …..
J.Deng, J.W.Chen,……
Weise,
Klevansky,
Hatsuda,Kunihiro,
Fukushima,
……

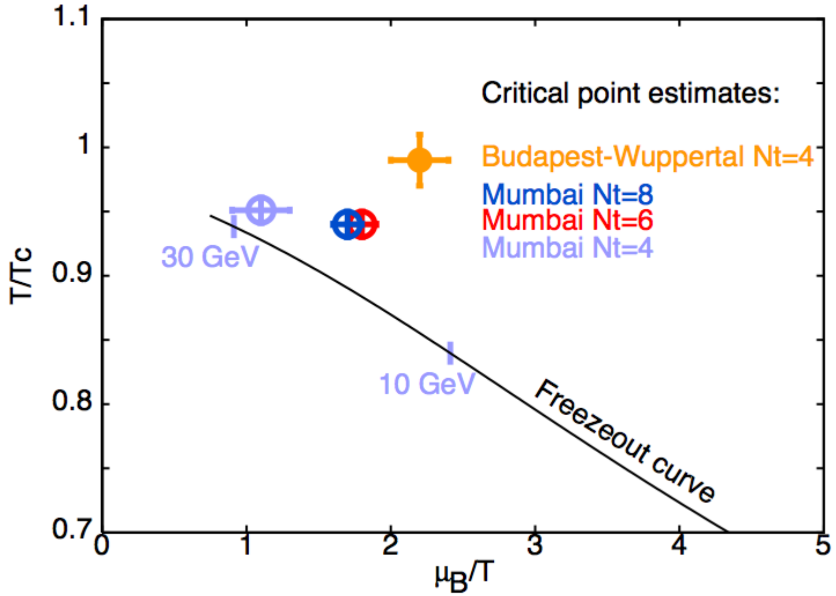


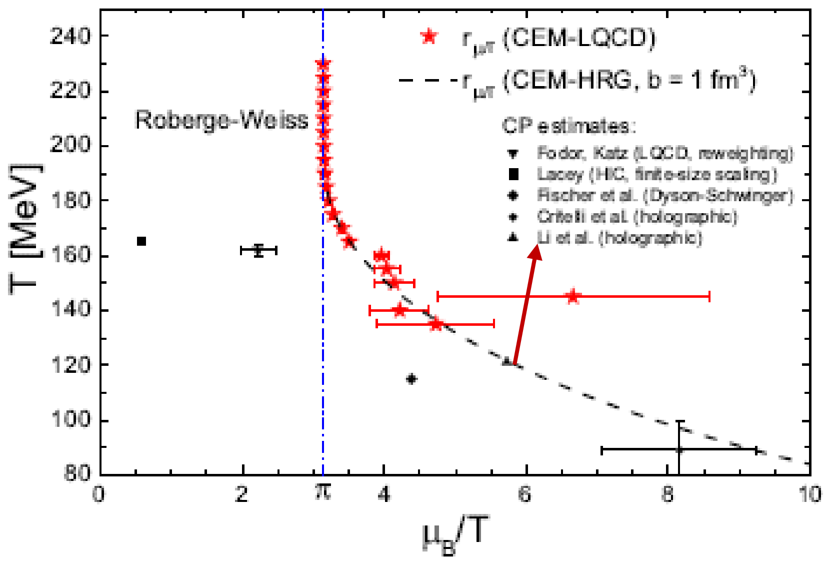
Hell, Kashiwa, Weise

Journal of Modern Physics, 2013, 4, 644-650

from small to high baryon number density region ……

Location of CEP from Lattice QCD





1) Fodor&Katz, JHEP 0404,050 (2004).

$(\mu^E_B, T_E) = (360, 162)$ MeV

2) Gavai&Gupta, NPA 904, 883c (2013)

$(\mu^E_B, T_E) = (279, 155)$ MeV

3) F. Karsch (CPOD2016)

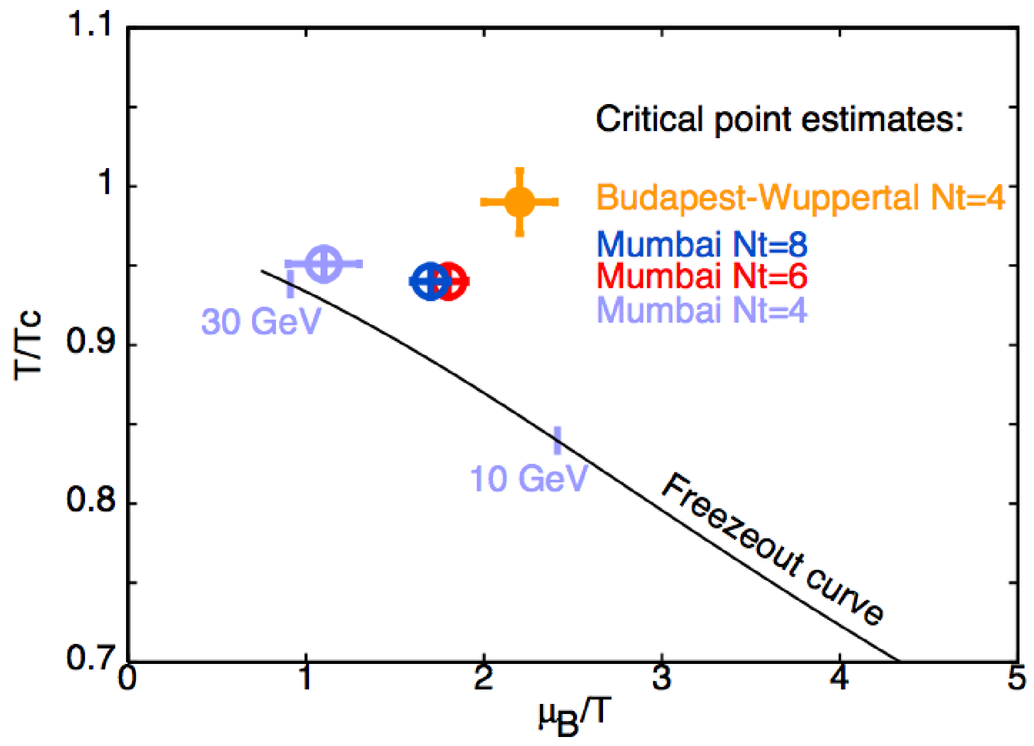
$\mu^E_B / T_E > 2$

4) V. Vovchenko, J. Steinheimer, O. Philipsen,H.Stoecker, arXiv:1711.01261

$$\mu_B^E / T_E > \pi$$

Latest lattice calculation shows that small baryon number density region for CEP is ruled out!

Location of CEP: Lattice QCD



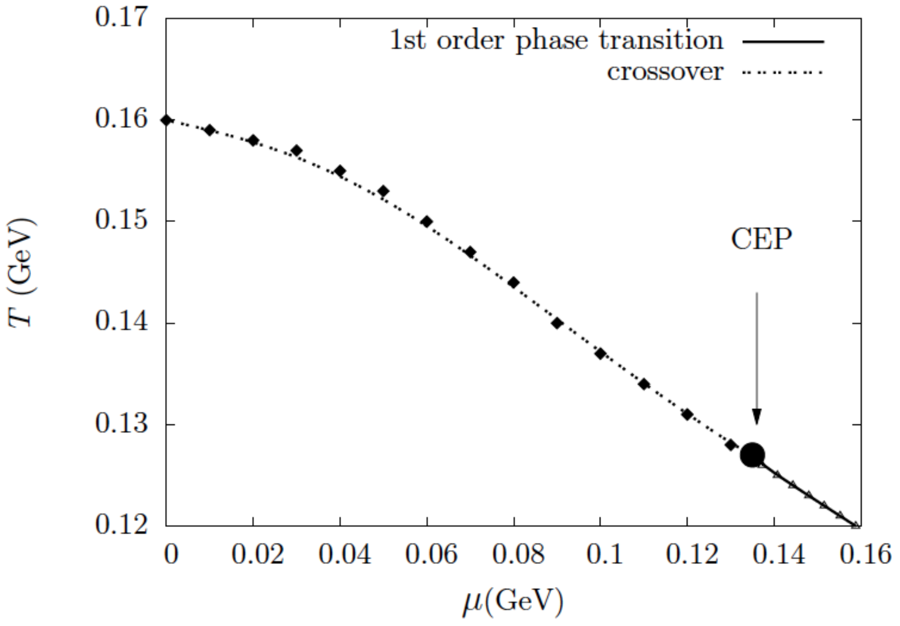
1): Fodor&Katz, JHEP 0404,050 (2004).
$(\mu^E_B, T_E) = (360, 162)$ MeV

2): Gavai&Gupta, NPA 904, 883c (2013)
$(\mu^E_B, T_E) = (279, 155)$ MeV

3): F. Karsch ($\mu^E_B/ T_E > 2$, CPOD2016)

Small baryon number density region

Location of CEP: DSE



1): Y. X. Liu, et al., PRD90, 076006 (2014).
$(\mu^E_B, T^E) = (372, 129)$ MeV

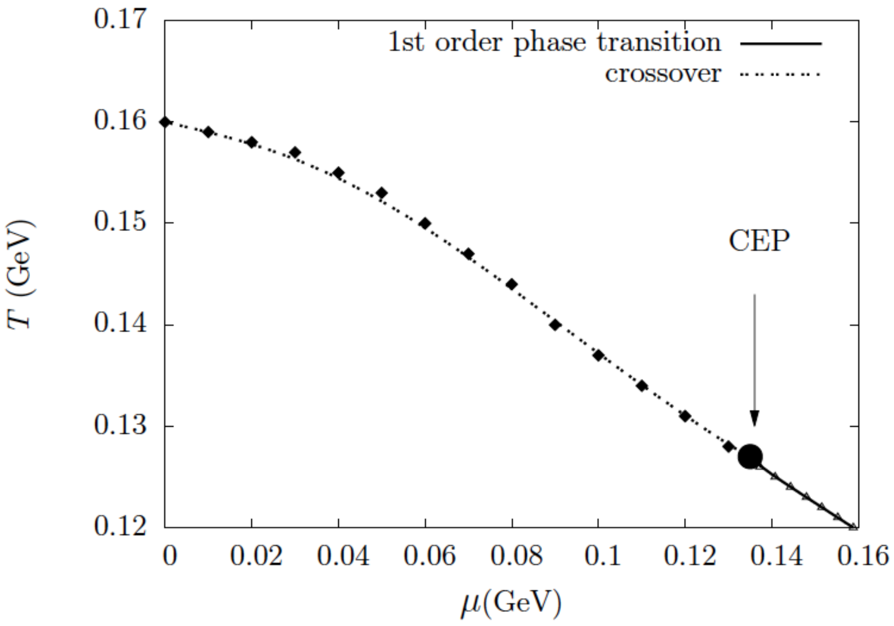
2): Hong-shi Zong et al., JHEP 07, 014 (2014).
$(\mu^E_B, T_E) = (405, 127)$ MeV

3): C. S. Fischer et al., PRD90, 034022 (2014).
$(\mu^E_B, T^E) = (504, 115)$ MeV

$$\mu_B = 3\,\mu_q$$

Rather mall baryon number density region

Location of CEP from DSE



1): Y. X. Liu, et al., PRD90, 076006 (2014).
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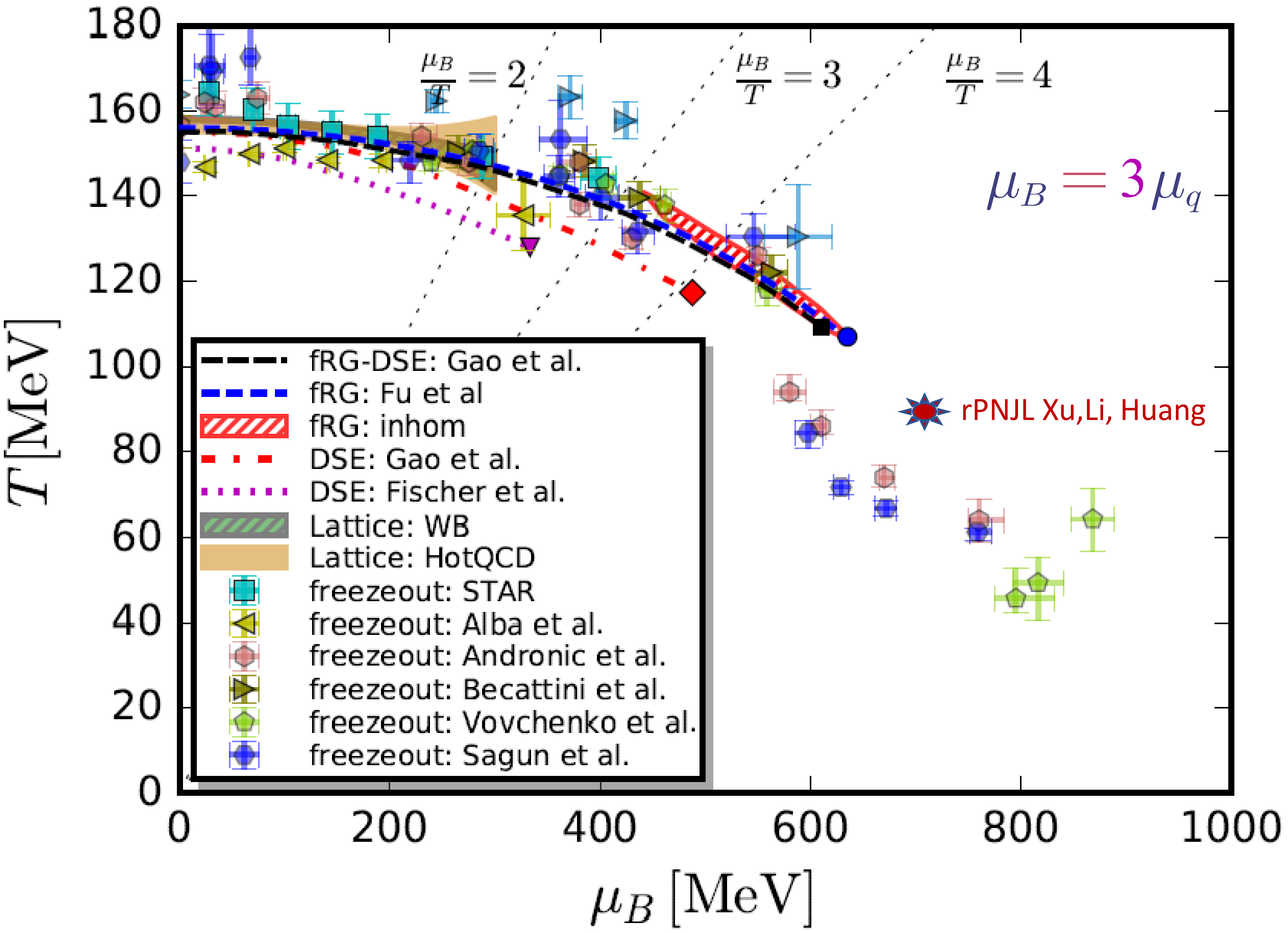
2): Hong-shi Zong et al., JHEP 07, 014 (2014).
$(\mu^E_B, T_E) = (405, 127)$ MeV

3): C. S. Fischer et al., PRD90, 034022 (2014).
$(\mu^E_B, T^E) = (504, 115)$ MeV

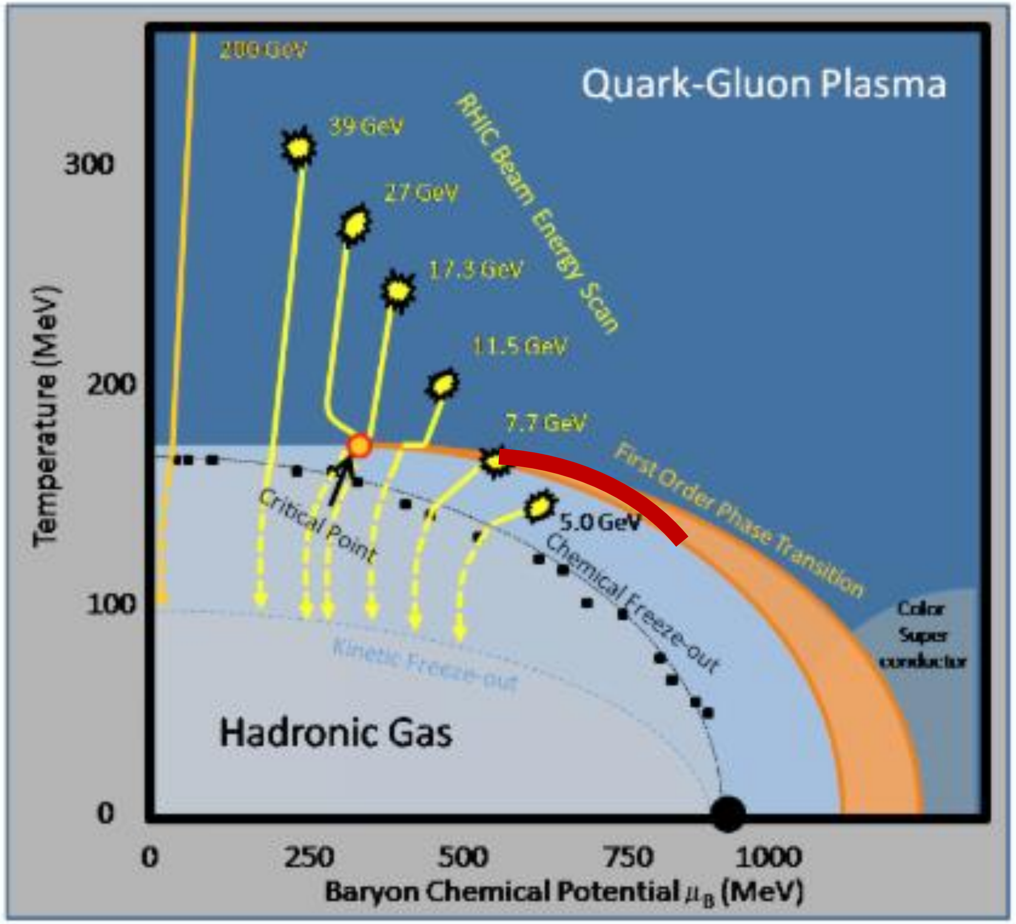
$$\mu_B = 3\,\mu_q$$

baryon number density region 300-500 MeV



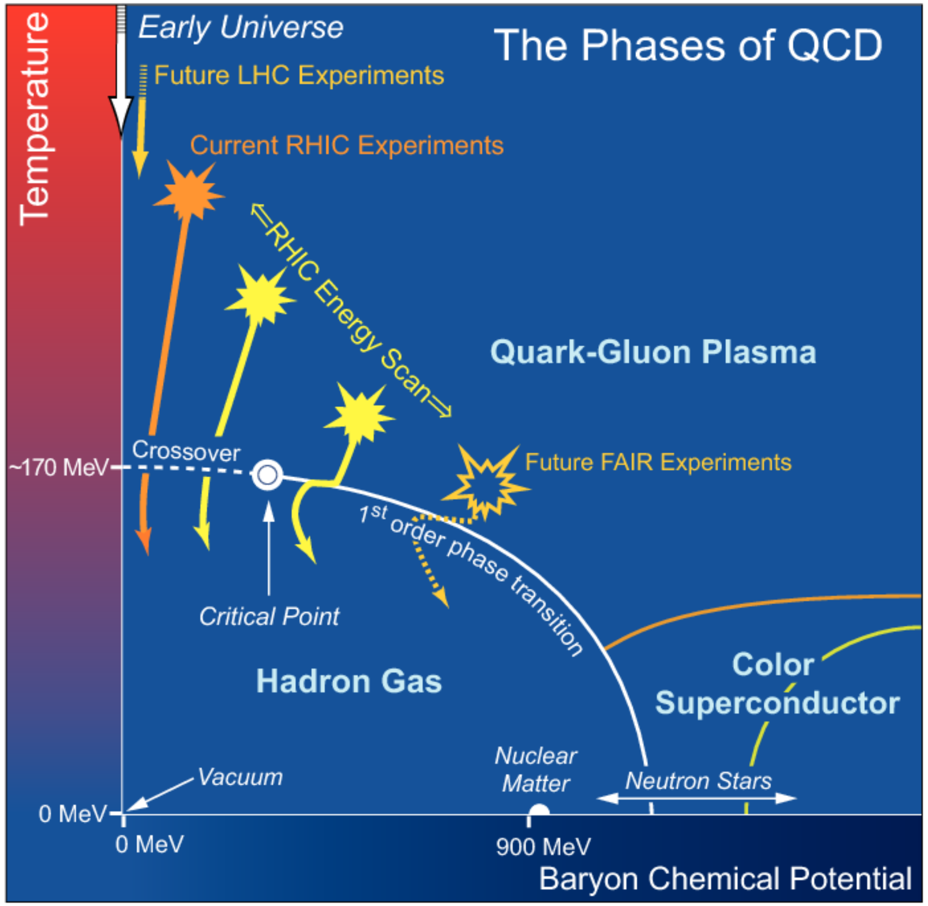
Wei-jie Fu, Jan M. Pawlowski, and Fabian Rennecke. QCD phase structure at finite temperature and density. Phys. Rev. D, 101(5):054032, 2020, 1909.02991.

Locating the QCD CEP



- ❑ BES @ RHIC
- ❑ NICA @Dubna
- ❑ CBM@FAIR
- ❑ HIAF@IMP

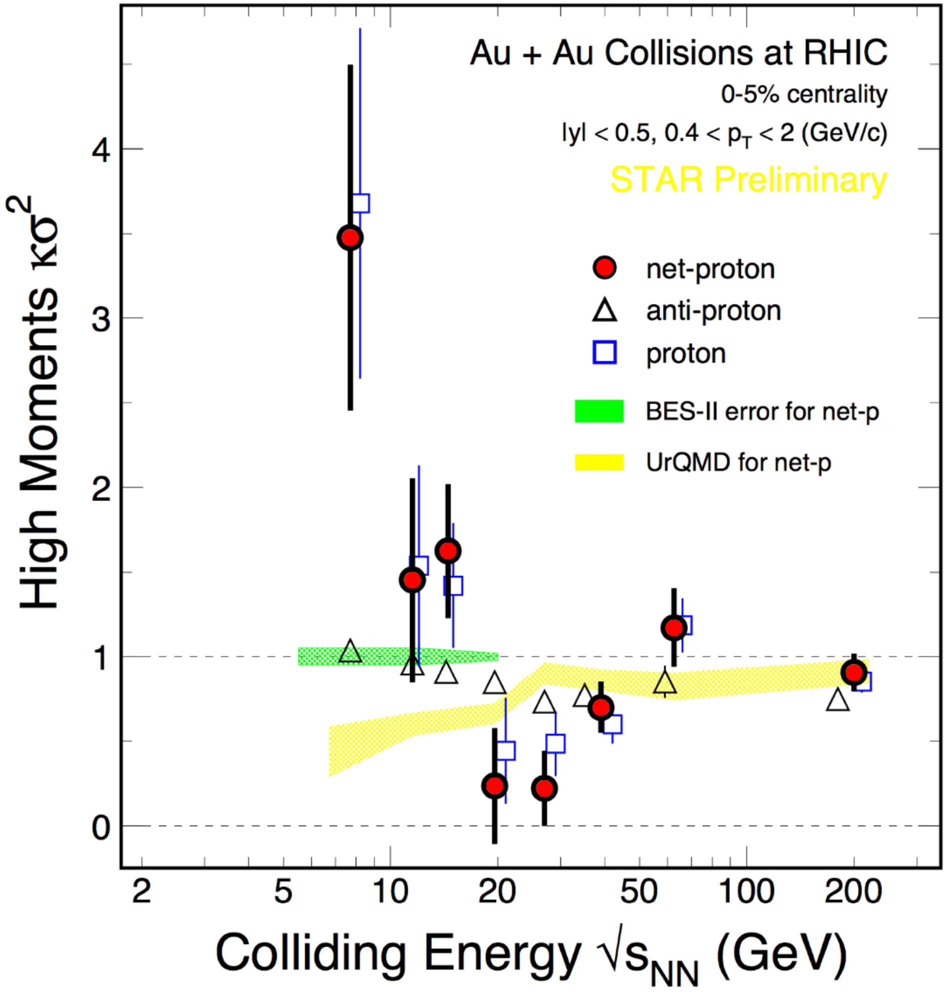
Searching for the QCD CEP



BES Phase-I

$\sqrt{s_{NN}}$ (GeV)	Events (10^6)	Year	*μ_B (MeV)	*T_{CH} (MeV)
200	350	2010	25	166
62.4	67	2010	73	165
39	39	2010	112	164
27	70	2011	156	162
19.6	36	2011	206	160
14.5	20	2014	264	156
11.5	12	2010	316	152
7.7	4	2010	422	140

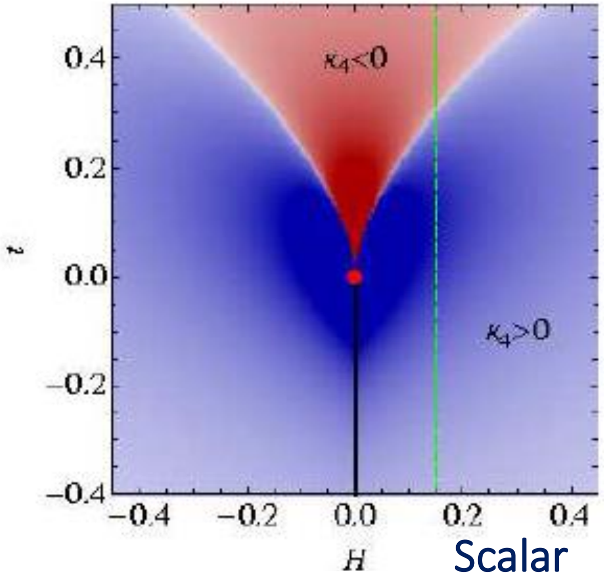
Measurement of Higher Order Fluctuations of Conserved Quantities



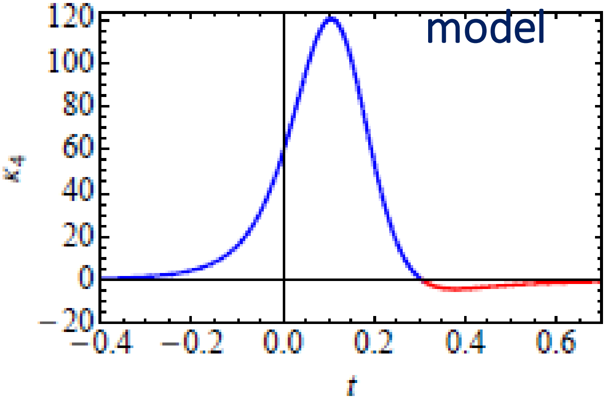
Non-monotonic trend is observed for the 0-5% most central Au+Au collisions. Dip structure is observed around 19.6 GeV.

STAR: **PRL112**, 32302(14); **PRL113**,092301(14);
X.F.Luo, N.Xu, arXiv:1701.02105

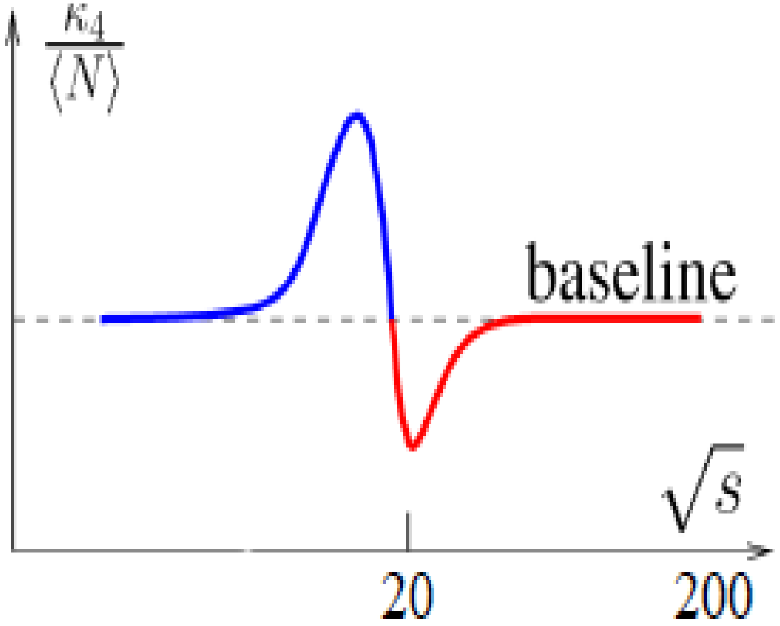
How to determine the location of CEP?



Scalar model



- M. Stephanov, *PRL107*, 052301(2011)



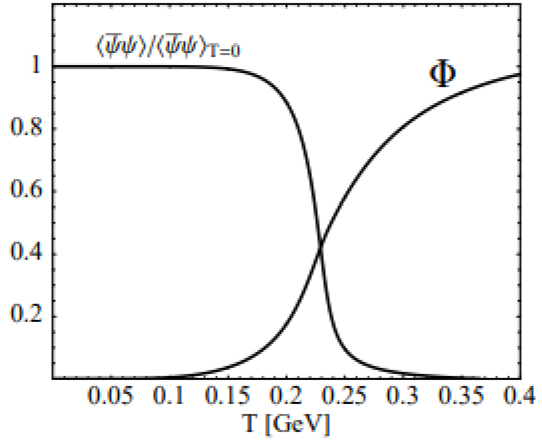
- Characteristic "Oscillating pattern" is expected for the QCD critical point but *the exact shape depends on the location of freeze-out with respect to the location of CP*
- Critical Region (CR)

Chiral restoration and deconfinement
Polyakov loop NJL model

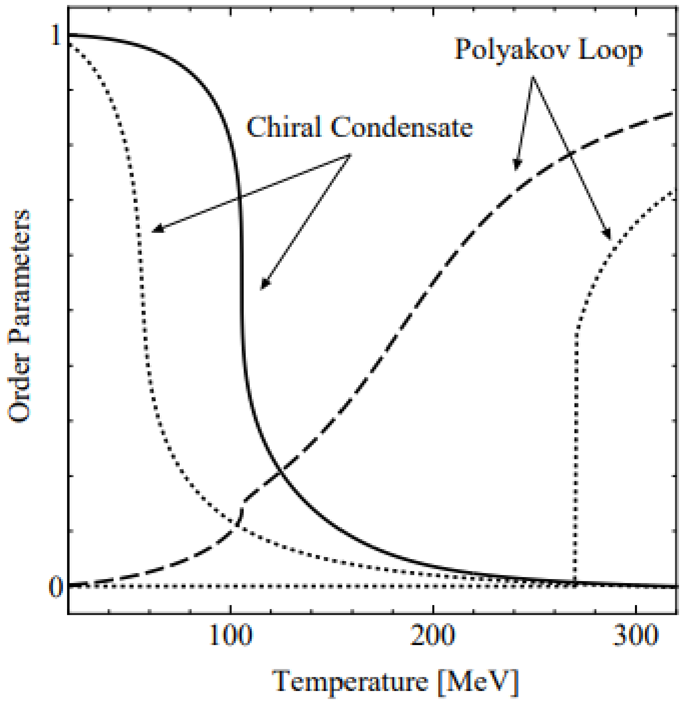
$$\mathcal{L}_{PNJL} = \bar{\psi}\left(i\gamma_\mu D^\mu - \hat{m}_0\right)\psi + \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right] - \mathcal{U}\left(\Phi[A], \bar{\Phi}[A], T\right),$$

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi},T\right)}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3 + \bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2 \qquad \Phi = (\mathrm{Tr}_c\, L)/N_c$$

$$L(\vec{x}) = \mathcal{P}\exp\left[i\int_0^\beta d\tau\, A_4(\vec{x},\tau)\right]$$



Claudia Ratti, Michael A. Thaler, Wolfram Weise, hep-ph/0506234



Kenji Fukushima, Phys.Lett.B 591 (2004) 277-284, hep-ph/0310121

A realistic PNJL model

A. Bhattacharyya,S. K. Ghosh, S. Maity, S. Raha,
B. R. Ray, K. Saha and S. Upadhaya,arXiv:1609.07882.

NJL part:

$$\Omega = g_S \sum_f \sigma_f^2 - \frac{g_D}{2}\sigma_u\sigma_d\sigma_s + 3\frac{g_1}{2}(\sum_f \sigma_f^2)^2 + 3g_2\sum_f \sigma_f^4 - 6\sum_f \int \frac{d^3p}{(2\pi)^3} E_f \Theta(\Lambda - |\vec{p}|)$$
$$- 2T\sum_f \int \frac{d^3p}{(2\pi)^3} \ln[1 + 3(\Phi + \bar{\Phi}e^{-(E_f-\mu_f)/T})e^{-(E_f-\mu_f)/T} + e^{-3(E_f-\mu_f)/T}]$$
$$- 2T\sum_f \int \frac{d^3p}{(2\pi)^3} \ln[1 + 3(\Phi + \bar{\Phi}e^{-(E_f+\mu_f)/T})e^{-(E_f+\mu_f)/T} + e^{-3(E_f+\mu_f)/T}]$$
$$+ U'(\Phi, \bar{\Phi}, T)$$

Z.B Li, K.Xu,X.Y.Wang, M.H,
arXiv:1801.09215,EPJC2019
arXiv:arXiv:1810.03524

Polyakov Loop:

$$\frac{U'}{T^4} = \frac{U}{T^4} - \kappa \ln[J(\Phi, \bar{\Phi})] \qquad \frac{U}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}(\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4}(\Phi\bar{\Phi})^2$$

$$J = (\frac{27}{24\pi^2})(1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2)$$

$$b_2(T) = a_0 + a_1\frac{T_0}{T}\exp(-a_2\frac{T}{T_0})$$

Parameters are fitted to lattice result at mu=0,

1) Tc=154 MeV;
2) EOS: p,e,s, trace anomaly;
3) Baryon number fluctuations

$m_{u,d}$(MeV)	m_s(MeV)	Λ(MeV)	$g_S\Lambda^2$	$g_D\Lambda^5$	g_1(MeV^{-8})	g_2(MeV^{-8})
5.5	183.468	637.720	2.914	75.968	2.193×10^{-21}	-5.890×10^{-22}

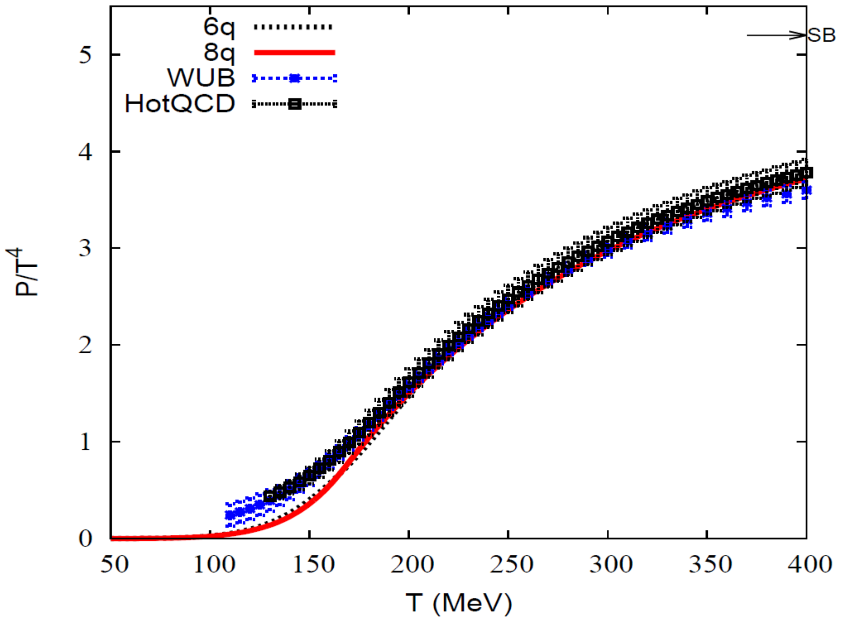
TABLE IV: Parameters for the NJL part in the realistic PNJL model.

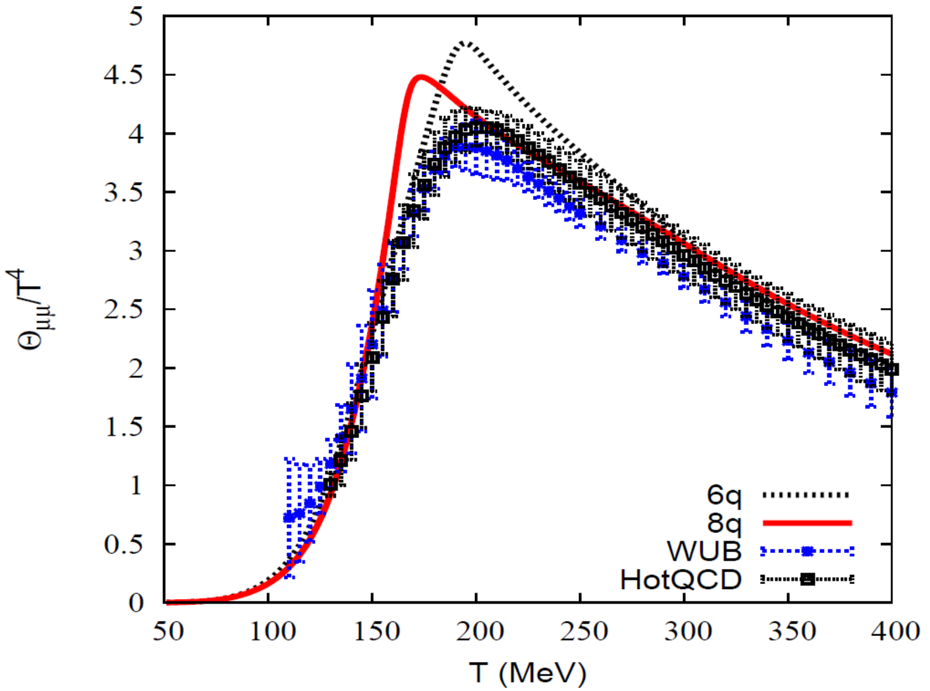
T_0 (MeV)	a_0	a_1	a_2	b_3	b_4	κ
175	6.75	-9.8	0.26	0.805	7.555	0.1

TABLE V: Parameters for the Polyakov loop part in the realistic PNJL model.

A. Bhattacharyya,S. K. Ghosh, S. Maity, S. Raha,
B. R. Ray, K. Saha and S. Upadhaya,arXiv:1609.07882.

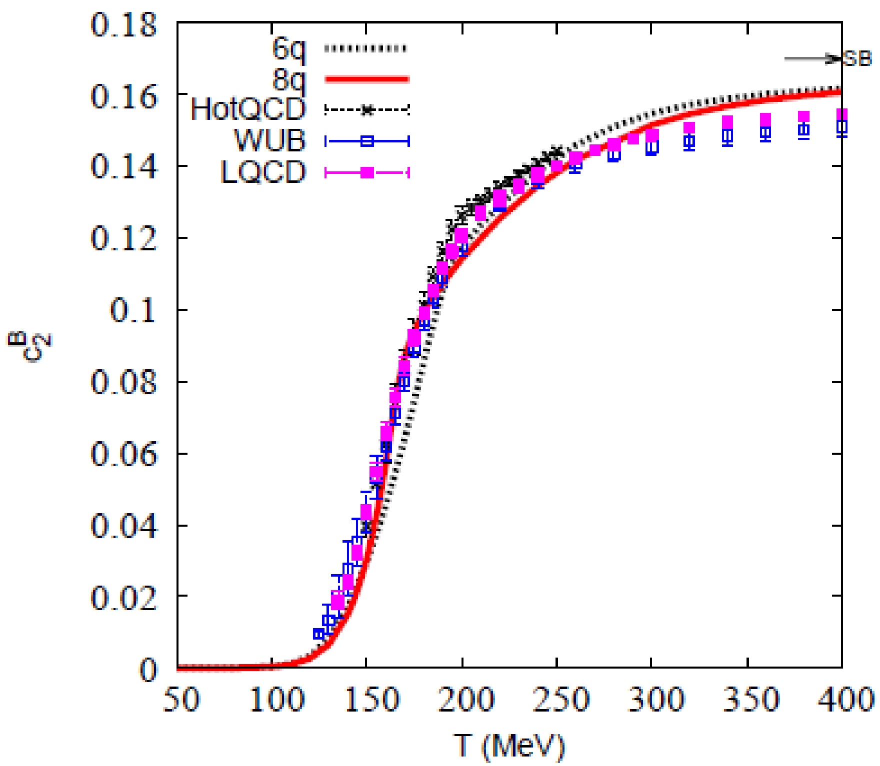
Equation of state at mu=0
model vs LQCD

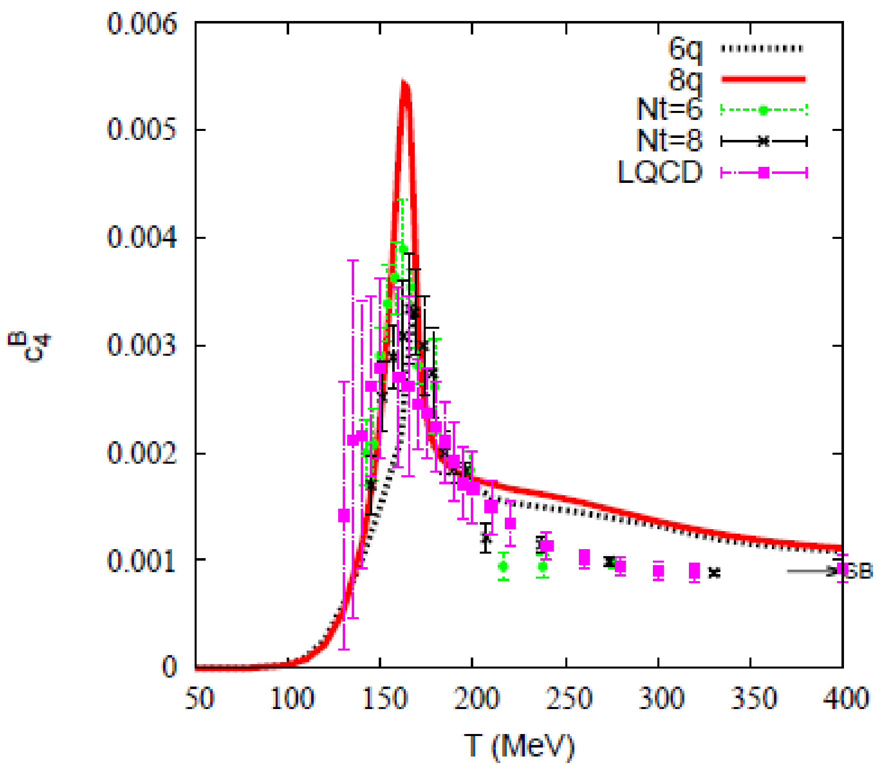




A. Bhattacharyya,S. K. Ghosh, S. Maity, S. Raha,
B. R. Ray, K. Saha and S. Upadhaya,arXiv:1609.07882.

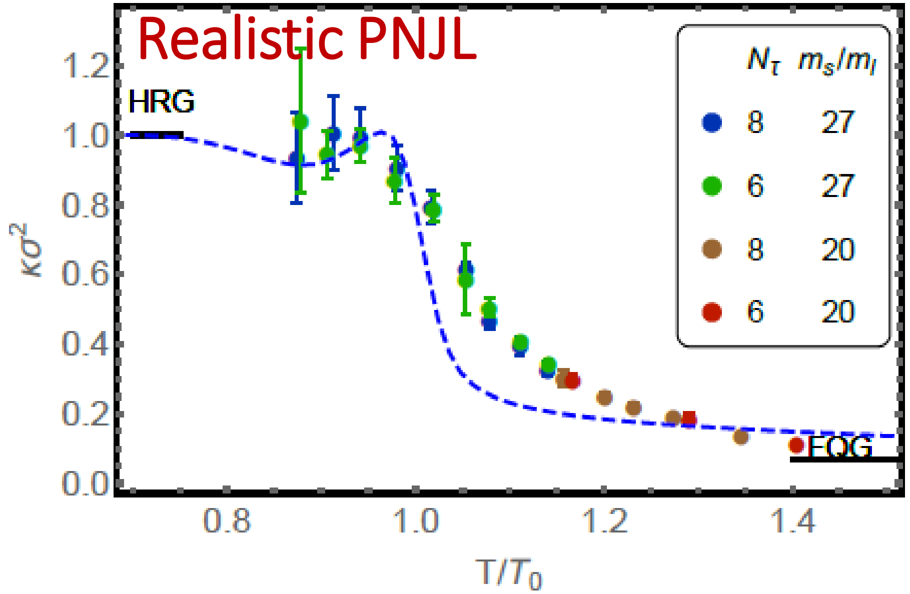
Baryon number fluctuation at mu=0
model vs LQCD

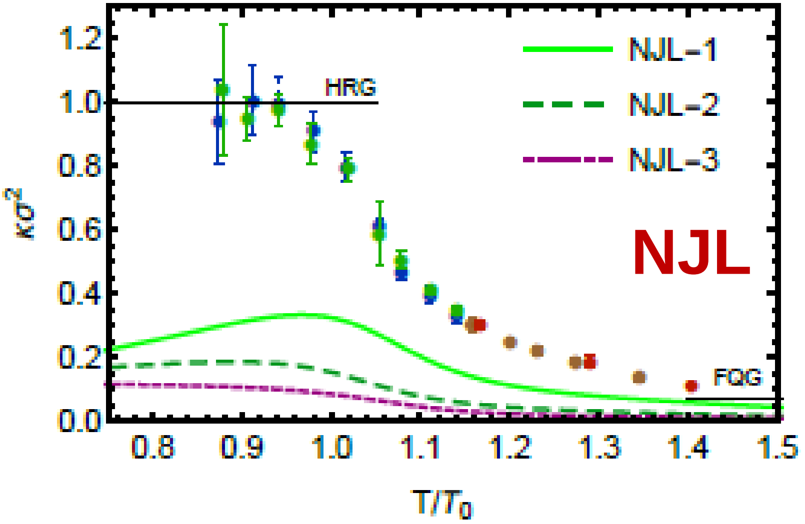




A. Bhattacharyya,S. K. Ghosh, S. Maity, S. Raha,
B. R. Ray, K. Saha and S. Upadhaya,arXiv:1609.07882.

Kurtosis of baryon number fluctuation at mu=0

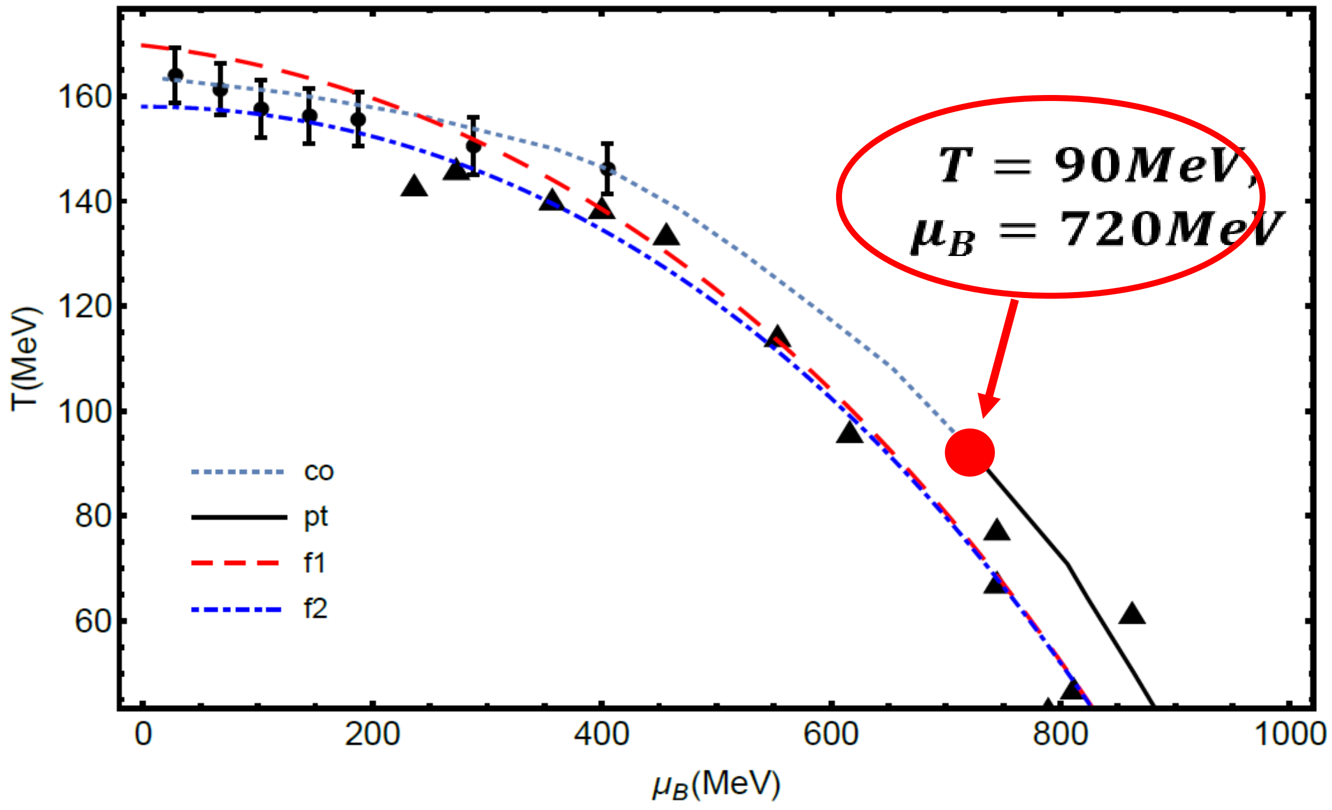




Gluodynamics is essential for C4/C2 !

Z.B Li, K.Xu,X.Y.Wang, M.Huang, arXiv:1801.09215

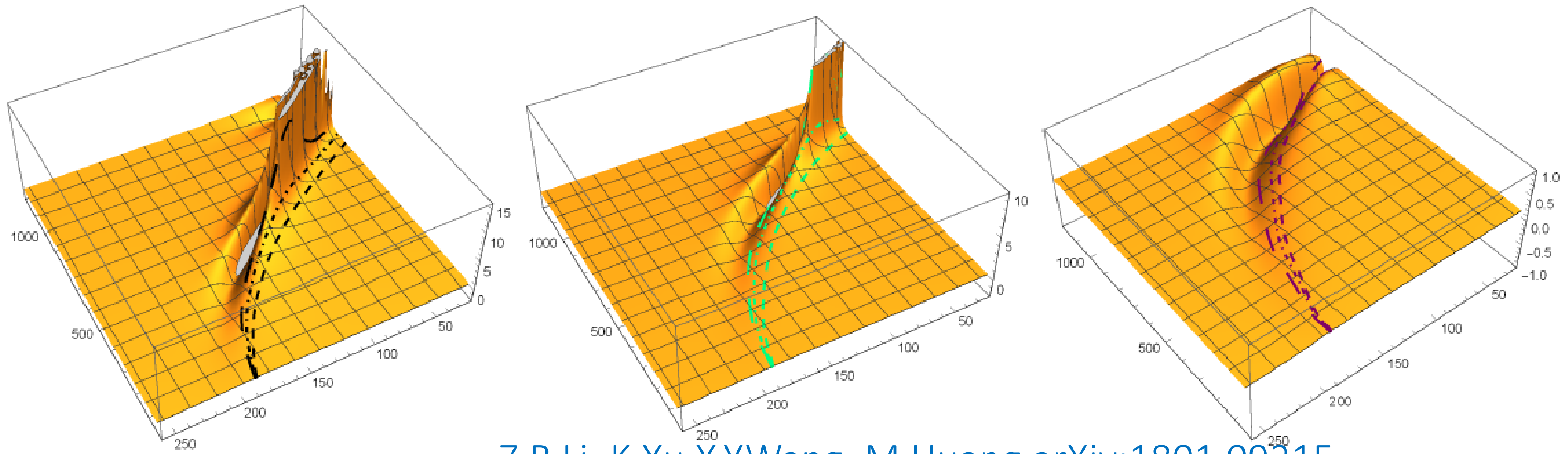
Phase boundary and CEP (mu_B^E=720 MeV, T^E=90 MeV)



Phase boundary is very close to the freeze-out data!!!

Z.B Li, K.Xu,X.Y.Wang, M.Huang,arXiv:1801.09215

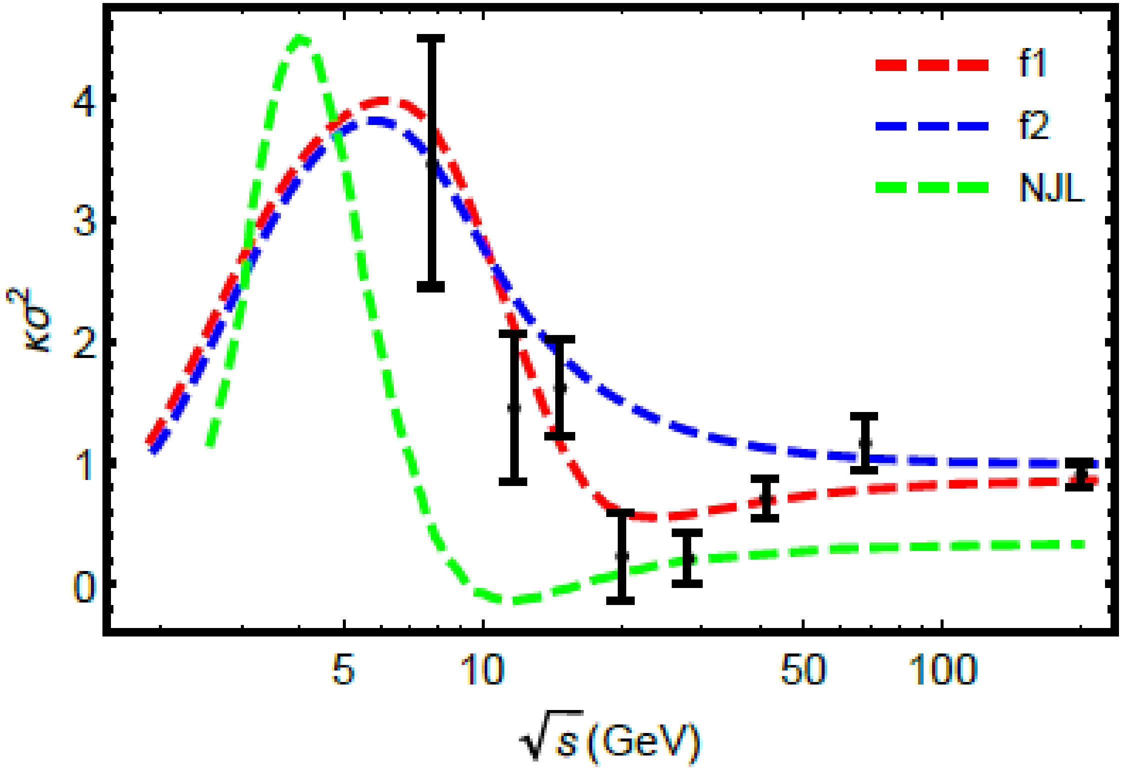
CEP location determines the location of the peak of kurtosis along the freeze-out line (close to the phase boundary) !



Z.B Li, K.Xu,X.Y.Wang, M.Huang,arXiv:1801.09215

BES-I measurement rules out the small baryon number density region for CEP!

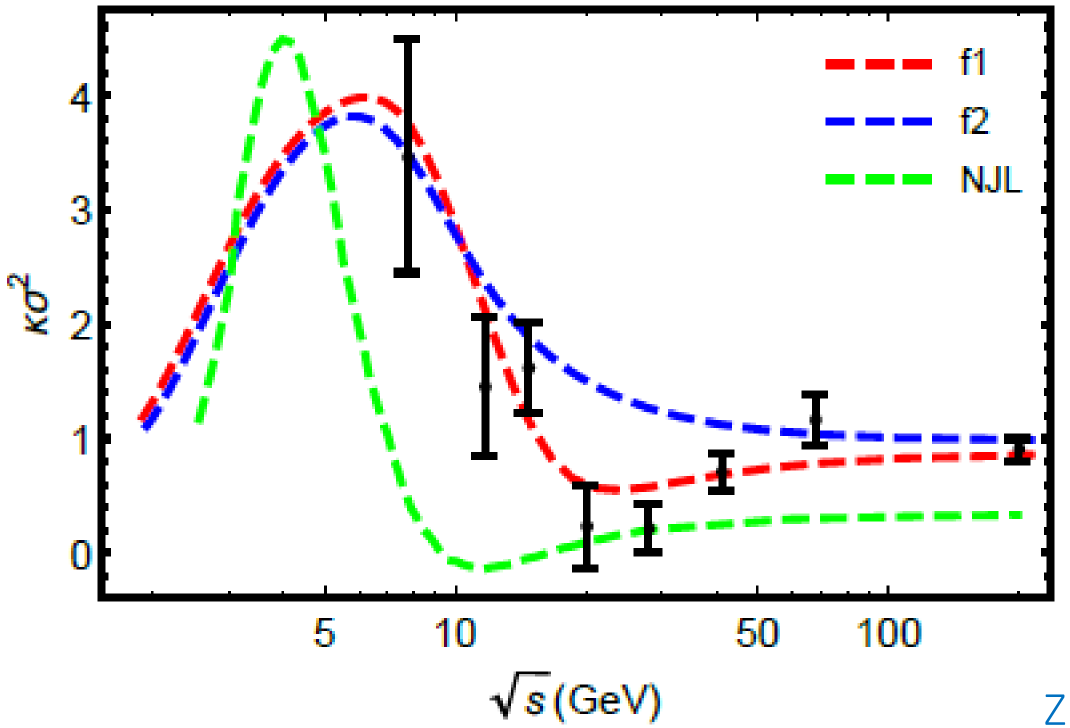
Kurtosis along experimental freeze-out lines

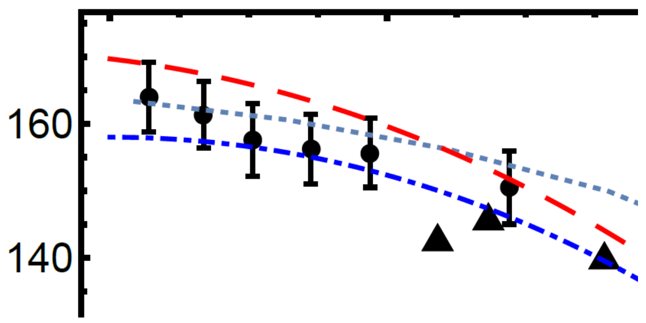


Realistic PNJL model results agree well with BES-I data!
Equilibrium result can describe the experimental data!!!

Z.B Li, K.Xu,X.Y.Wang, M.Huang, arXiv:1801.09215

Dip structure



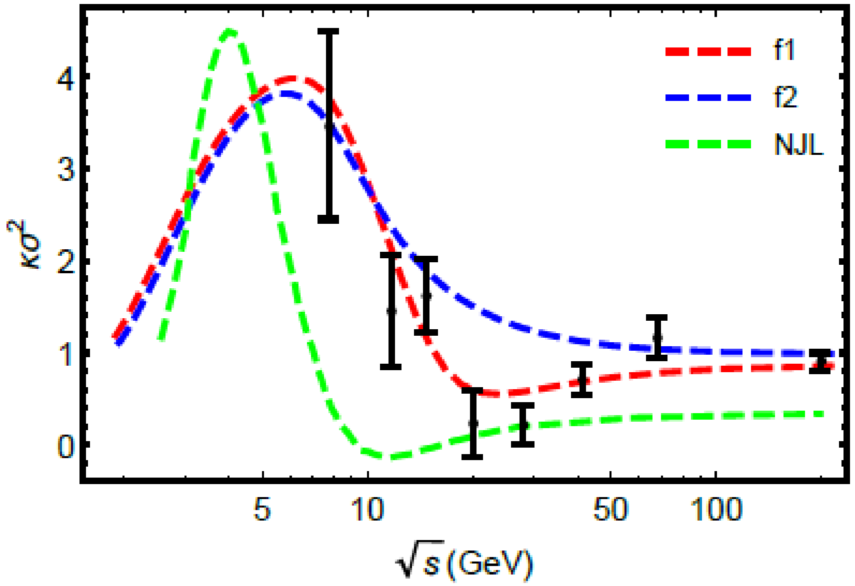


f1 cross the phase boundary while f2 not!

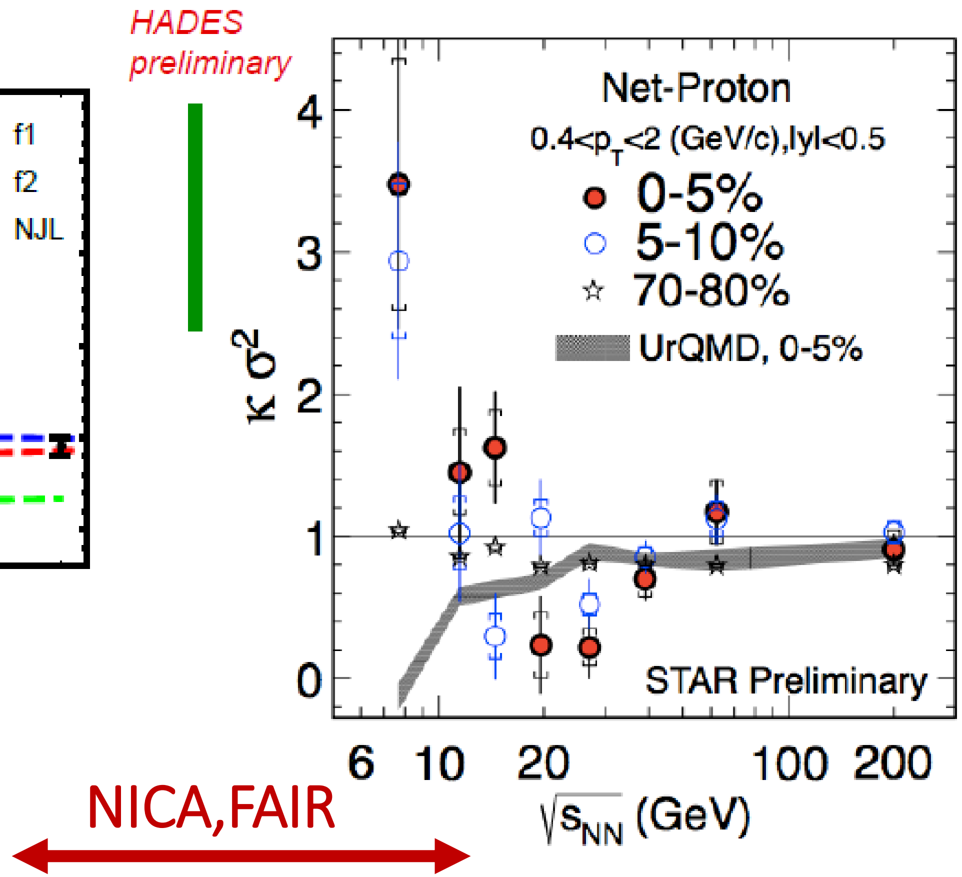
Z.B Li, K.Xu,X.Y.Wang, M.Huang
arXiv:1801.09215

The dip structure is sensitive to the relation between the freeze-out line and the phase boundary !

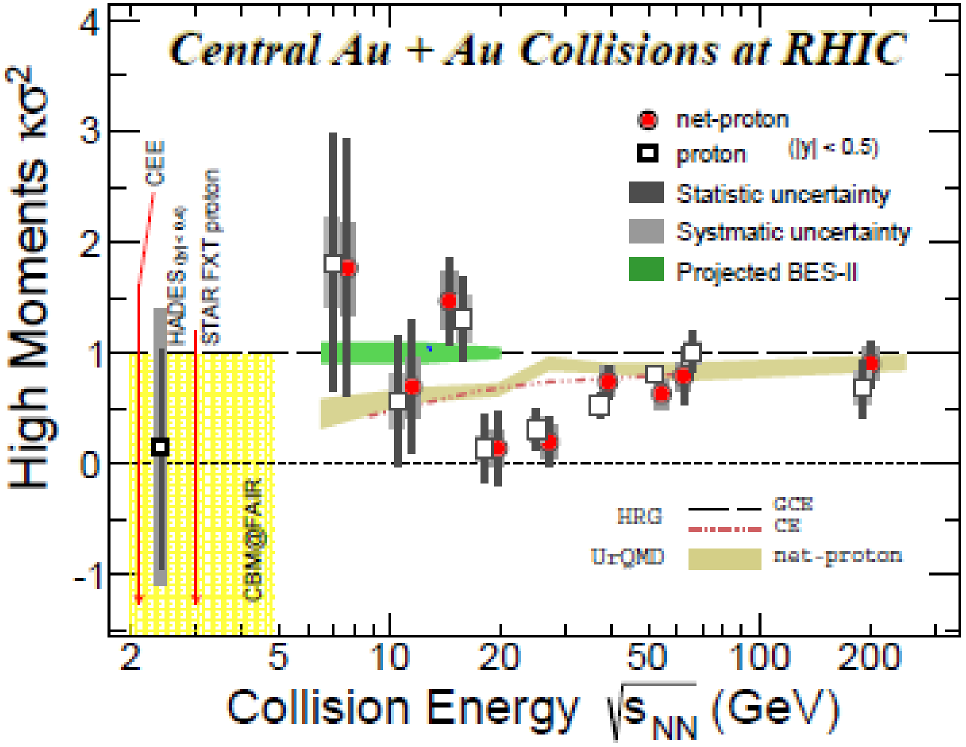
Peak structure is expected to show up in CBM and NICA

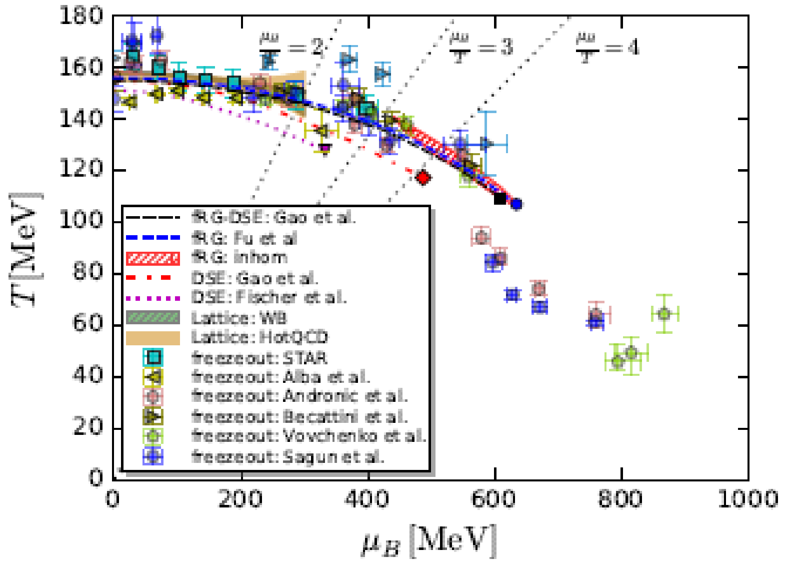


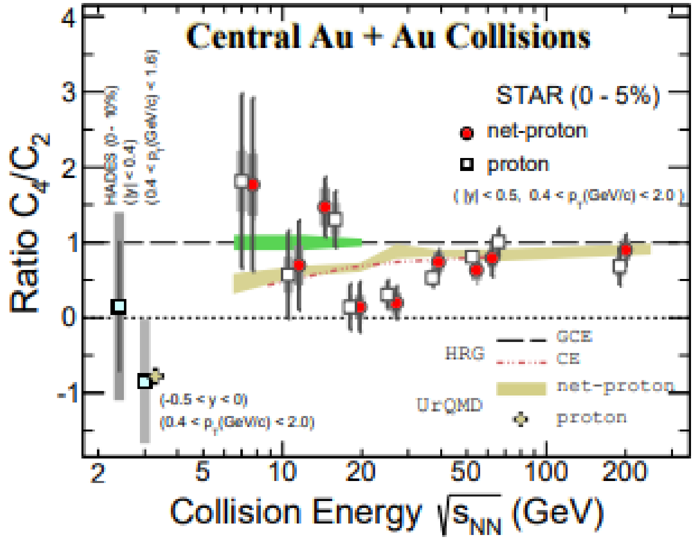
Z.B Li, K.Xu,X.Y.Wang, M.Huang
arXiv:1801.09215



The peak structure along the freeze-out line is the residue of the divergence of CEP along phase boundary! Unique structure for CEP!





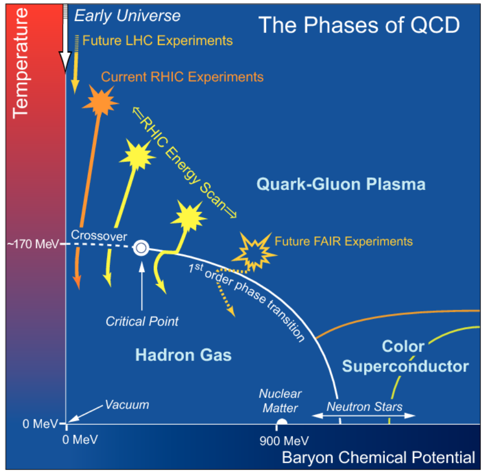


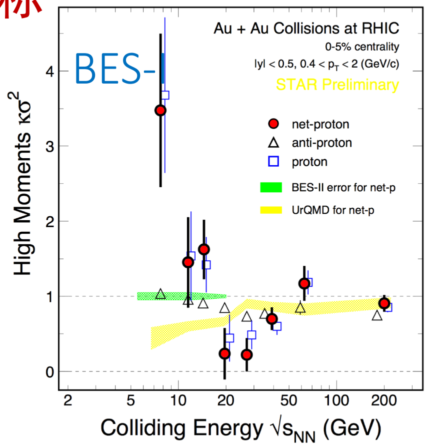
STAR: Phys. Rev. Lett. 128 (2022) 202303

3-7GeV 区间有峰？

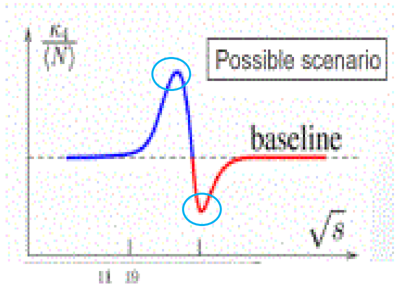
1，确定QCD的CEP位置及在高密区描述致密星体的状态方程

重离子碰撞实验的重要目标



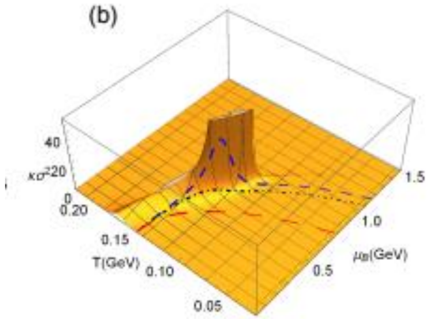


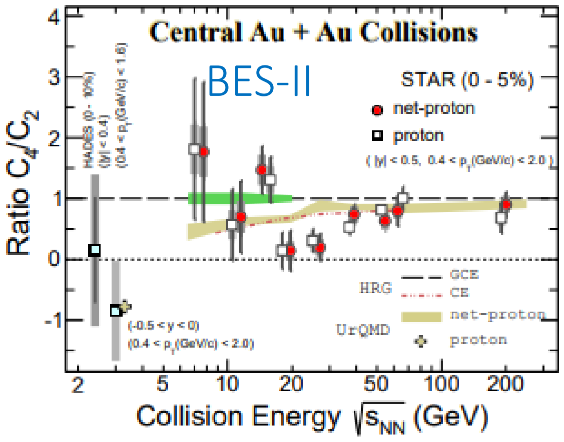
STAR: **PRL112**, 32302(14);



M.A. Stephanov，Phys.Rev.Lett. 107 (2011) 052301

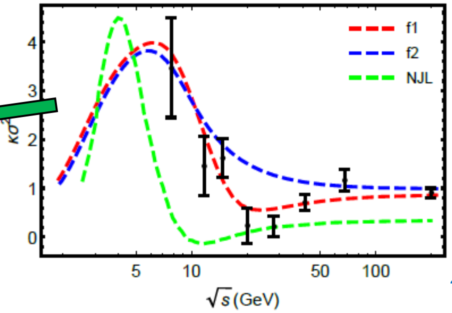
我们通过全息模型和夸克动力学模型论证峰值对应CEP位置并预言CEP在3-7GeV附近。





STAR: Phys. Rev. Lett. 128 (2022) 202303

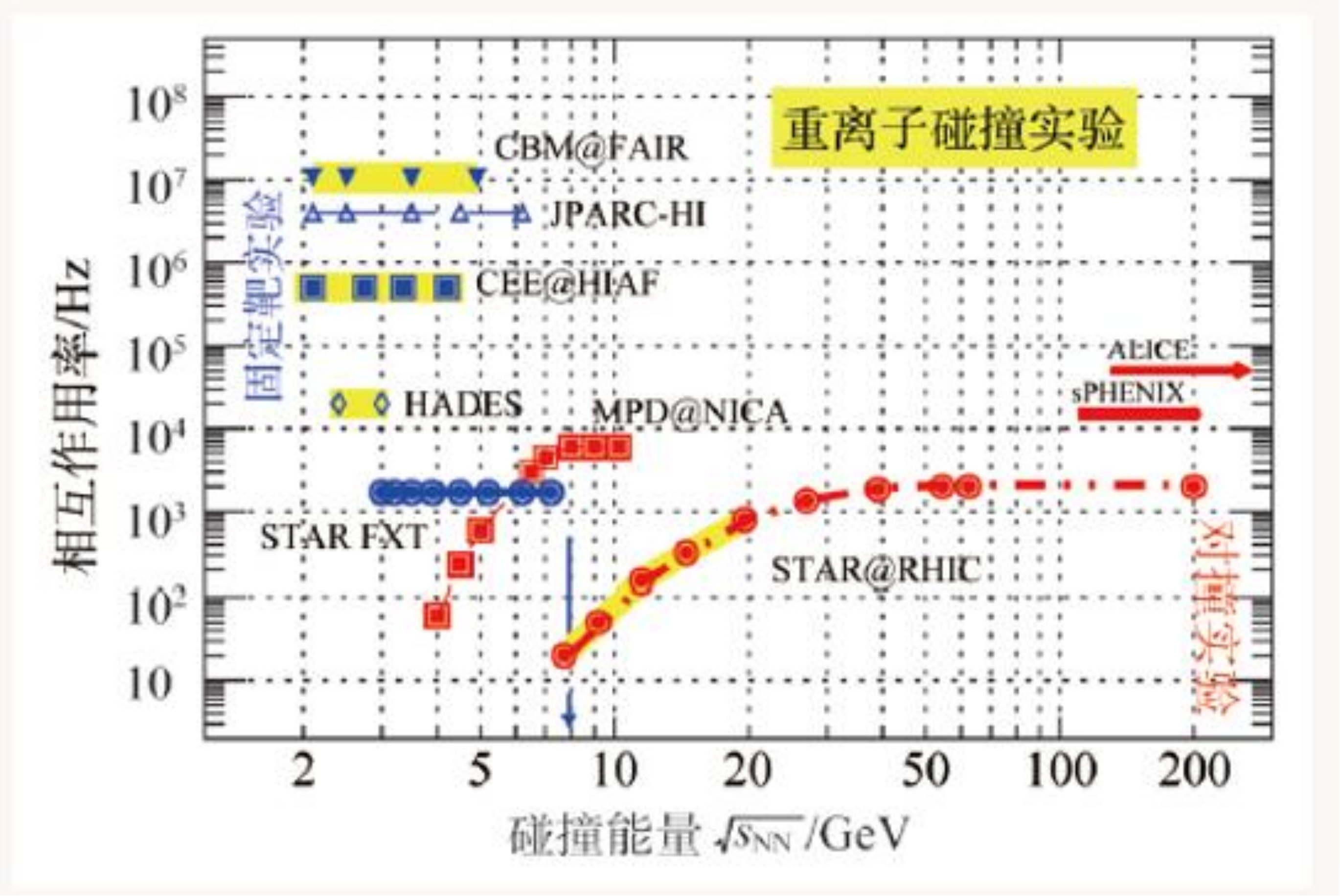
3-7GeV 区间有峰？



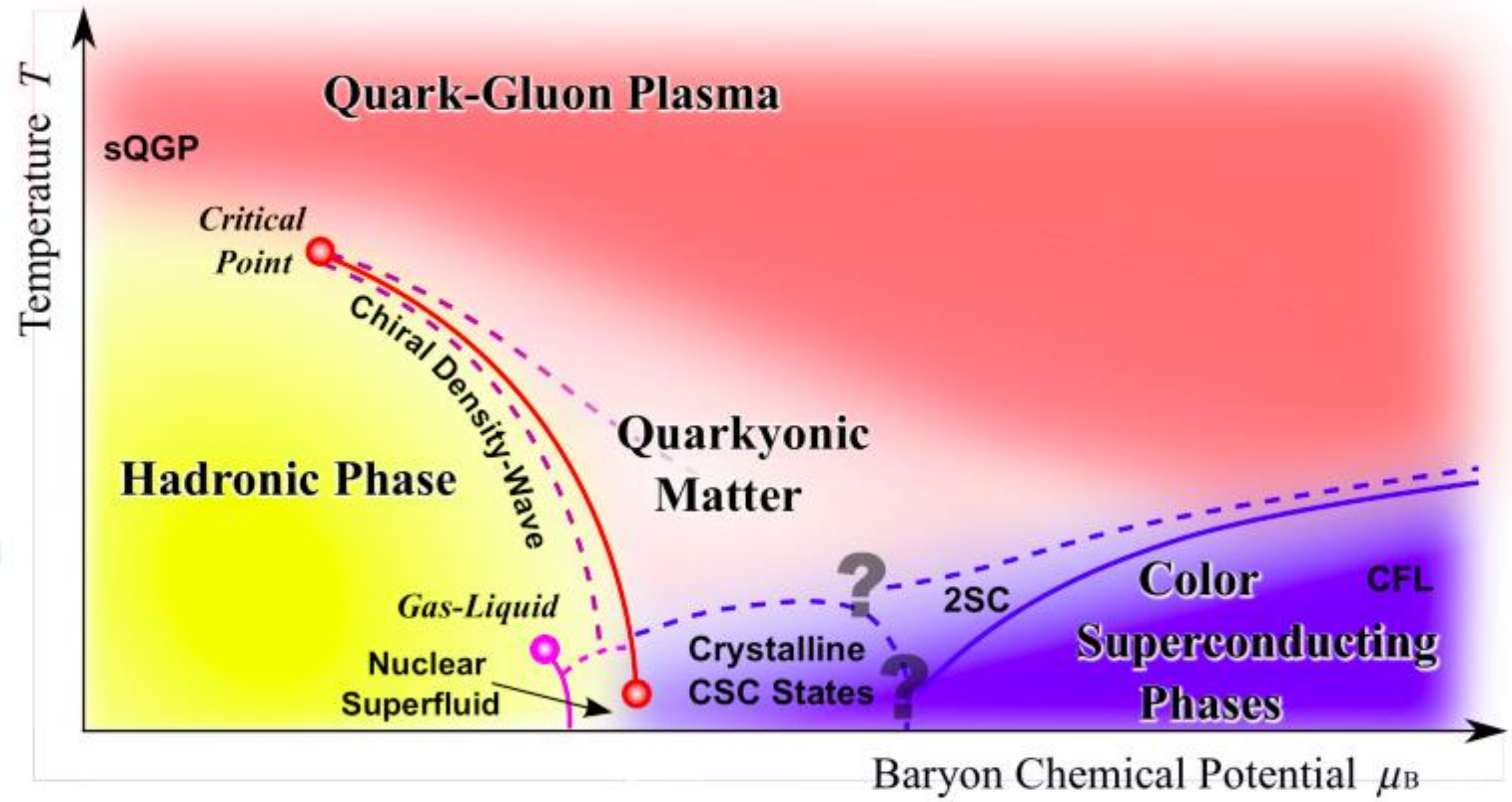
Z.B.Li et.al, Chin.Phys.C 42 (2018) 1, 013103； EPJC 79，2019

未来将考察流体力学演化对平衡态动力学模型结果的修正，跟实验结果比较确定CEP位置

Future HICs for CEP



Dense Matter: QCD CEP, Quarkyonic matter, CSC



K. Fukushima and T. Hatsuda, Rept. Prog. Phys. 74, 014001(2011); arXiv: 1005.4814