QCD dense matter and color superconductor

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I. A brief introduction on QCD dense matter

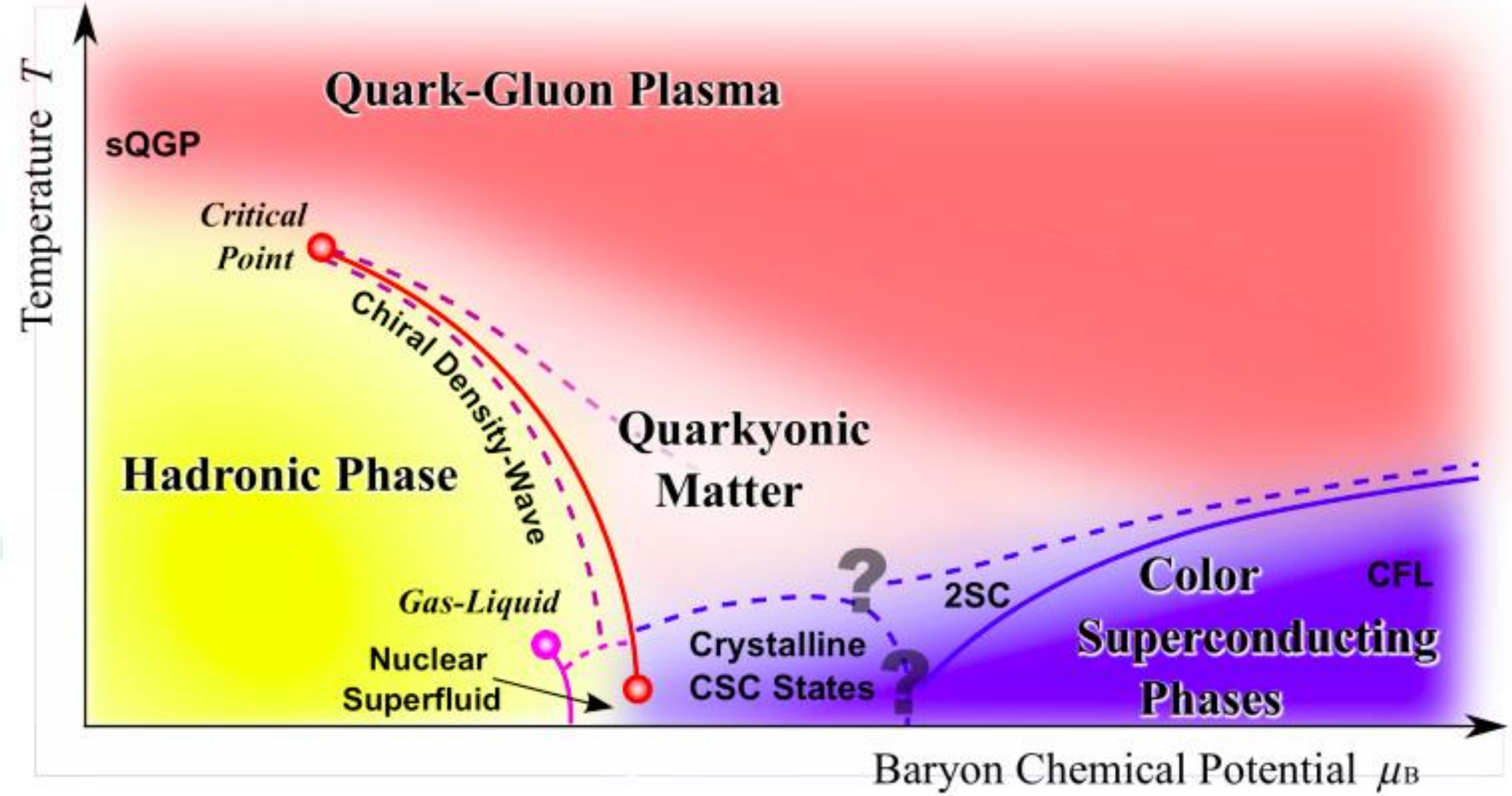
II. QCD critical end point

III. Quarkyonic matter and EOS for neutron star

IV. Color superconductor(CSC)

V. Summary and outlook

Dense Matter: QCD CEP, Quarkyonic matter, CSC

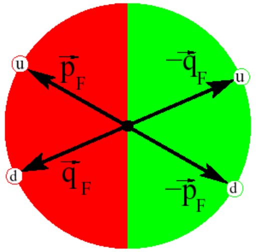


K. Fukushima and T. Hatsuda, Rept. Prog. Phys. ***74****, 014001(2011);*
arXiv: 1005.4814

Reviews on CSC:

K. Rajagopal and F. Wilczek, hep-ph/0011333;
D. K. Hong, Acta Phys.Polon. B32, 1253 (2001);
M. Alford, Ann. Rev.Nucl. Part.Sci. 51, 131 (2001);
T. Schaefer, hep-ph/0304281;
D.H. Rischke, Prog.Part. Nucl. Phys. 52, 197 (2004);
M. Buballa, Phys. Rept. 407, 205 (2005);
H.-C. Ren, hep-ph/0404074;
M. Huang, Int. J. Mod.Phys. E14, 675 (2005);
I.Shovkovy, Found. Phys. 35, 1309 (2005);
Qun Wang, Prog.Phys. 30 (2010) 173, e-Print: 0912.24855
Mark G. Alford, Andreas Schmitt, Krishna Rajagopal, Thomas Schäfer, Rev.Mod.Phys. 80 (2008) 1455-1515 • e-Print: 0709.4635

BCS Theorem



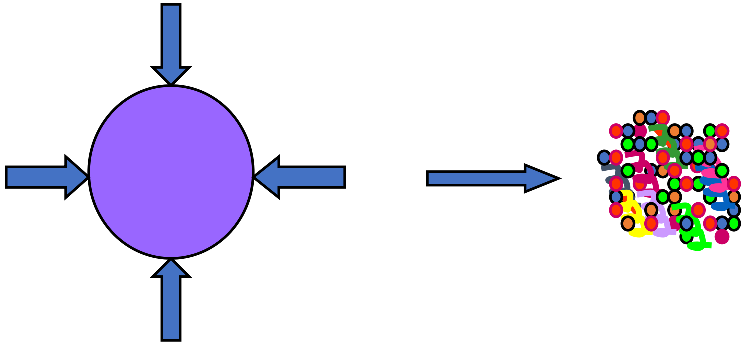
If there is an attractive interaction in a cold fermi sea, the system is unstable with respect to formation of a particle-particle condensate

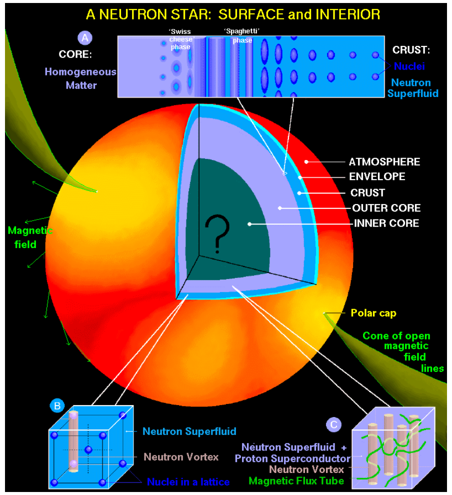
Cooper pair in momentum space.

QCD at high baryon density: Color superconductivity

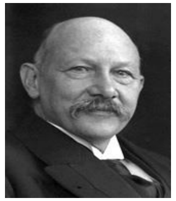
$$3_c \otimes 3_c = \boxed{\bar{3}_c} \oplus 6_c$$

$$\langle qq \rangle_{\bar{3}_c}$$

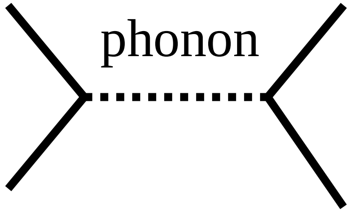




The birth of CSC

1911  **Superconductor**

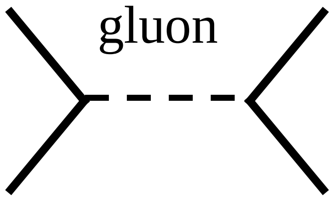
1957 



$\langle e^- e^- \rangle$ **BCS Theorem**

1973  **QCD**

1977-1984 $\Delta \propto 1 MeV$ **Barrois, NPB129 (1977),390; Bailin, Love, Phys. Rep. 107, 325(1984).**



$\langle qq \rangle_{\bar{3}_c}$

1998 $\Delta \propto 100 MeV$ **Rapp, Schaefer, Shuryak,Velkovsky, PRL81, 53 (1998); hep-ph/9711396**
Alford, Rajagopal, Wilczek, PLB422, 247 (1998), hep-ph/9711395

Color superconductivity

BCS超导理论：

Bogoliubov transformation

电子相互作用：

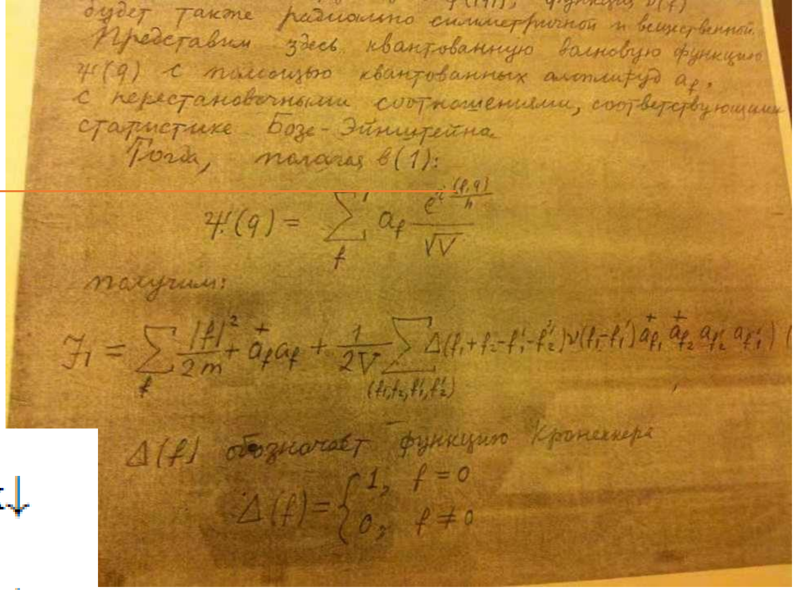
$$H = \sum_{\mathbf{k}\sigma} \xi_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k}\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow}$$

电子形成 Cooper pair：

$$\Delta_{\mathbf{k}} = -\frac{1}{N} \sum_{\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} \left\langle c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow} \right\rangle$$

$$H = \sum_{\mathbf{k}\sigma} \xi_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - \sum_{\mathbf{k}} \left(\Delta_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} + \Delta^{*}_{\mathbf{k}} c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \right) + \sum_{\mathbf{k}} \Delta_{\mathbf{k}} \left\langle c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} \right\rangle$$

BCS超导理论：



Bogoliubov transformation：

电子-空穴
组合成准粒子

$$c_{\mathbf{k}\uparrow} = u^*_{\mathbf{k}}\gamma_{\mathbf{k}\uparrow} + v_{\mathbf{k}}\gamma^\dagger_{-\mathbf{k}\downarrow}$$
$$c^\dagger_{-\mathbf{k}\downarrow} = u_{\mathbf{k}}\gamma^\dagger_{-\mathbf{k}\downarrow} - v^*_{\mathbf{k}}\gamma_{\mathbf{k}\uparrow}$$

对角化的准粒子
（自由准粒子系统）：

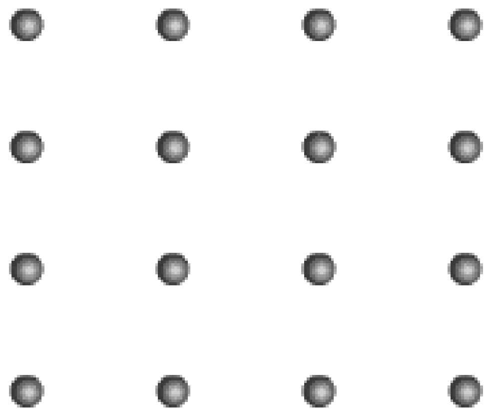
$$H = \sum_{\mathbf{k}\sigma} E_{\mathbf{k}}\gamma^\dagger_{\mathbf{k}\sigma}\gamma_{\mathbf{k}\sigma} + E_0$$

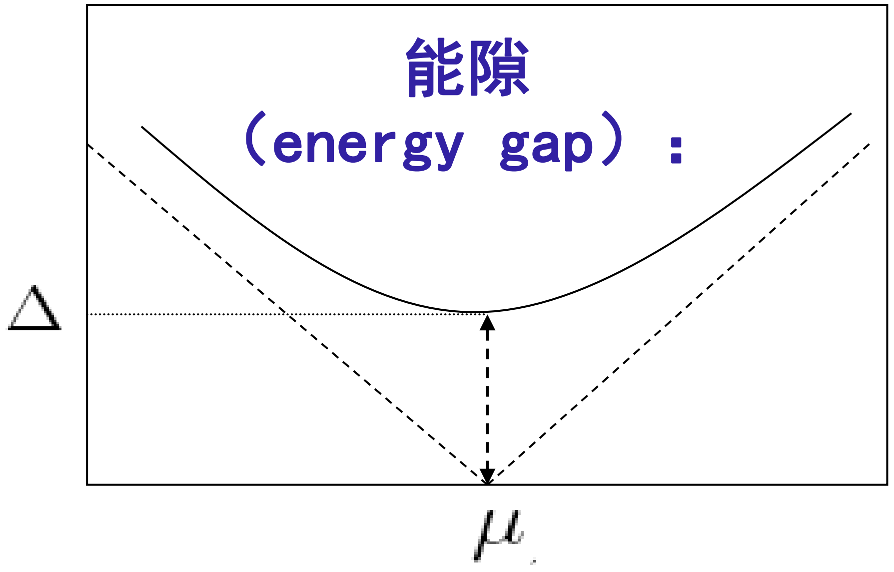
准粒子色散关系：

$$E_{\mathbf{k}} = \sqrt{\xi^2_{\mathbf{k}} + |\Delta_{\mathbf{k}}|^2}$$

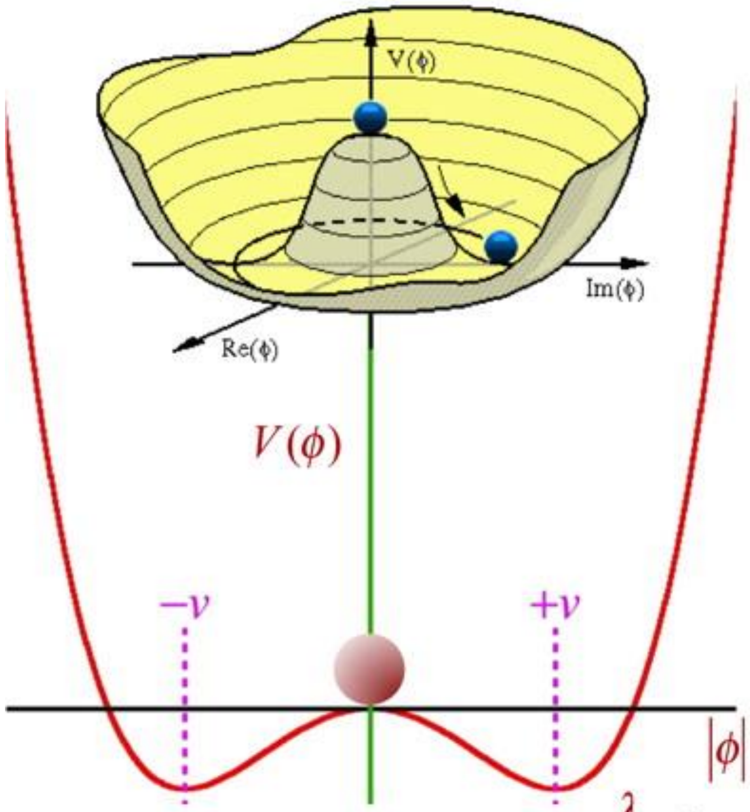
准粒子色散关系：

$$E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + |\Delta_{\mathbf{k}}|^2}$$





对称性破缺机制：BCS理论、手征对称性自发破缺、Higgs 机制



$$V(\phi)=\tfrac{1}{2}\mu^2\phi^\dagger\phi+\tfrac{1}{4}\lambda(\phi^\dagger\phi)^2$$

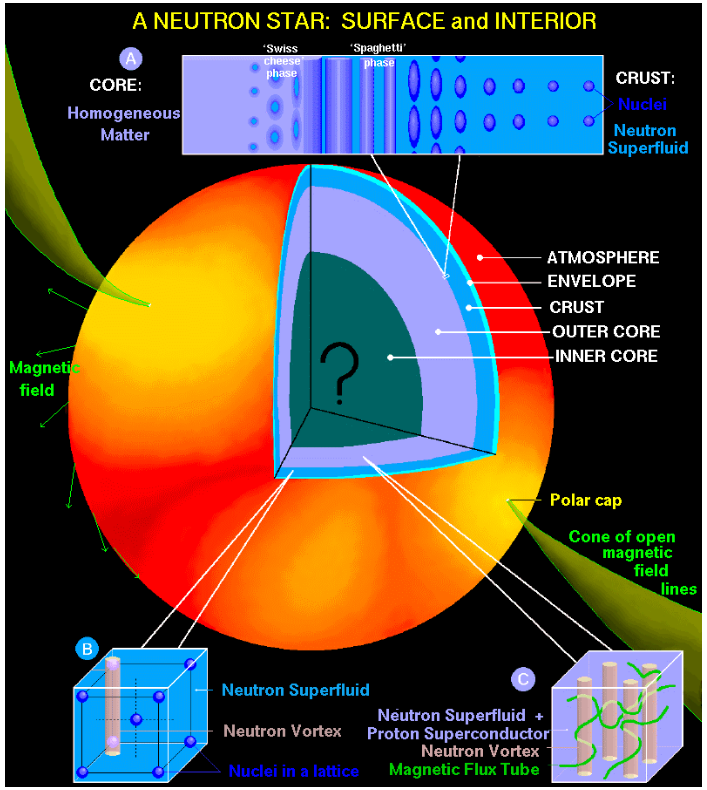
Groundstate at $|\phi_0|=\sqrt{\frac{-\mu^2}{\lambda}}\equiv\upsilon$

$$|\phi|=\sqrt{\phi^\dagger\phi}=\sqrt{\phi^{+\dagger}\phi^{+}+\phi^{0\dagger}\phi^{0}}$$

$$V(\phi_0)=-\frac{\lambda}{4}\upsilon^4$$

Where to find CSC?

CSC in Compact Stars?



inner core

$$\rho/\rho_0 \approx 5 - 10$$

It's natural to expect that CS exists in the core of compact stars

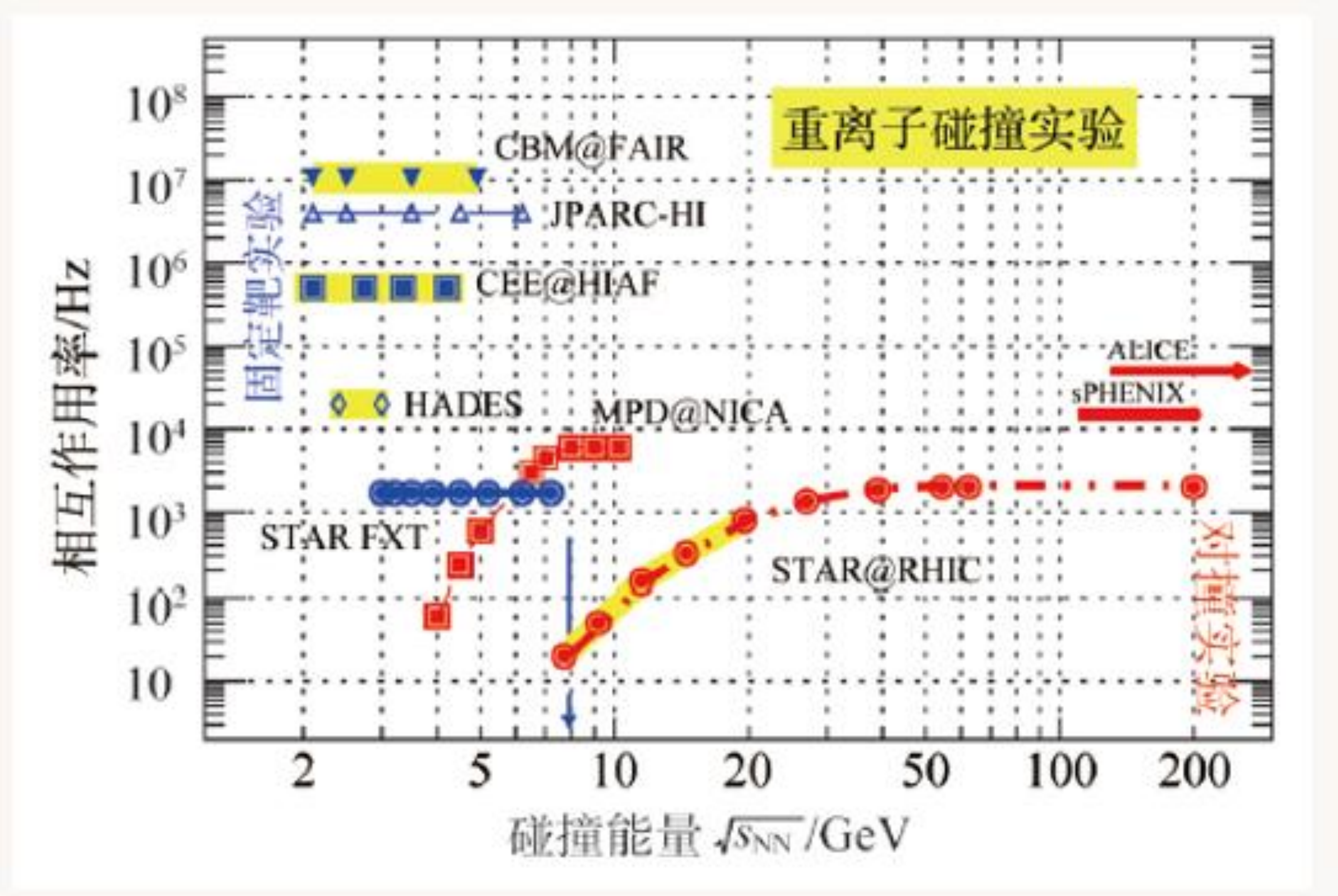
Where to find CSC?

BCS pairing:

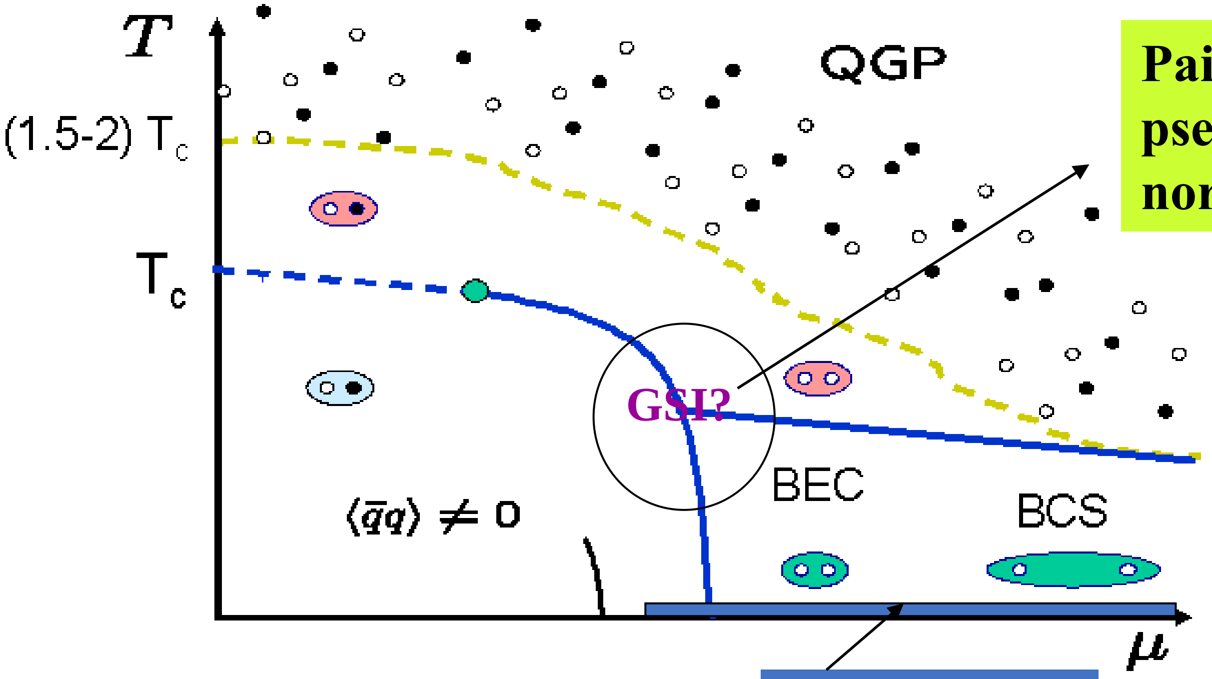
$$\Delta_{\mathrm{BCS}} \sim 100\,\mathrm{MeV}, \quad T^{C}_{\mathrm{BCS}} = 0.567\,\Delta_{\mathrm{BCS}}$$

Is it possible to find some signatures of diquark fluctuations in HIC?

Future HICs for CEP



Where to find CSC?

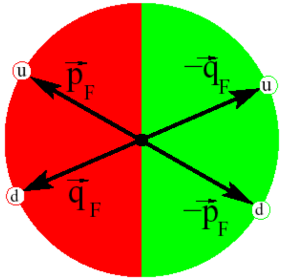


Pair fluctuation state: pseudo-gap phase (a normal state gap) ?

Compact Star

Some basic properties of standard BCS Superconductor

BCS Theorem



If there is an attractive interaction in a cold Fermi sea, the system is unstable with respect to formation of a particle-particle condensate

Cooper pair in momentum space.

The 2SC Phase-I

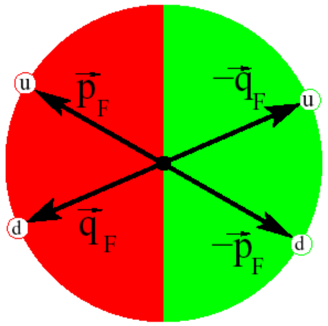
Spin-0: 2SC

$$m_{u,d} = 0, \quad m_s \gg m_{u,d}$$

$$\Delta^{ij}_{\alpha\beta} \propto \Delta \varepsilon^{ij} \varepsilon_{\alpha\beta 3}$$

$$\langle u_p d_{-p} \rangle = - \langle u_q d_{-q} \rangle \neq 0$$

Ideal BCS pairing



Symmetries

$$SU(3)_C \otimes U(1)_{EM} \otimes SU(f)_L \otimes SU(f)_R \otimes U(1)_B$$

$$Q_{EM} = \tfrac{1}{3}\, diag(2,-1,-1)$$
$$B = \tfrac{1}{3}\, diag(1,1,1)$$

in flavor space

$$Q_3 = \tfrac{1}{2}\, diag(1,-1,0)$$
$$Q_8 = \tfrac{1}{3}\, diag(1,1,-2)$$

in color space

Residual Symmetries

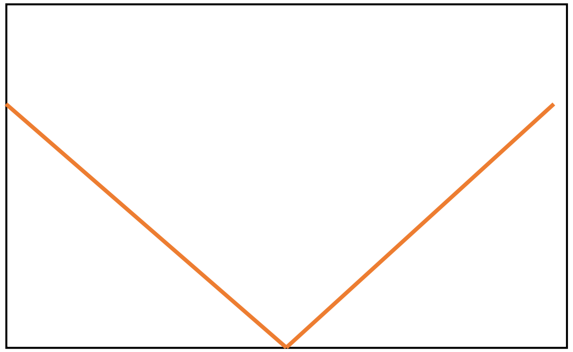
$$SU(2)_C \otimes \tilde{U}(1)_{EM} \otimes SU(2)_L \otimes SU(2)_R \otimes \tilde{U}(1)_B$$

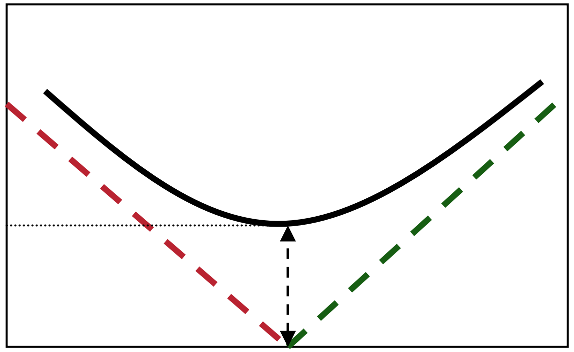
3 massless gluons + 5 massive gluons

$$\tilde{Q} = Q_{EM} - \frac{1}{2} Q_8$$

$$\tilde{B} = B - Q_8$$

Quasiparticle excitation

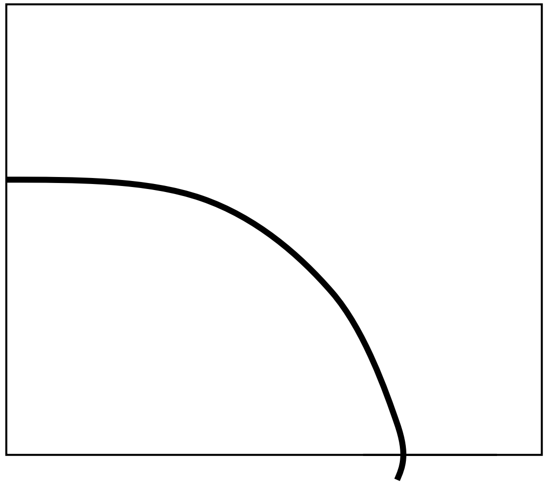




$$E_b^{\pm} = \pm|p - \mu|$$

$$E_{\Delta}^{\pm} = \pm\sqrt{(p - \mu)^2 + \Delta^2}$$

Finite temperature behavior



$$r_{\mathrm{BCS}} = \frac{T_c^{\mathrm{BCS}}}{\Delta_0^{\mathrm{BCS}}} = \frac{e^{\gamma_E}}{\pi} \approx 0.567,$$

A universal value for all conventional BCS SCs

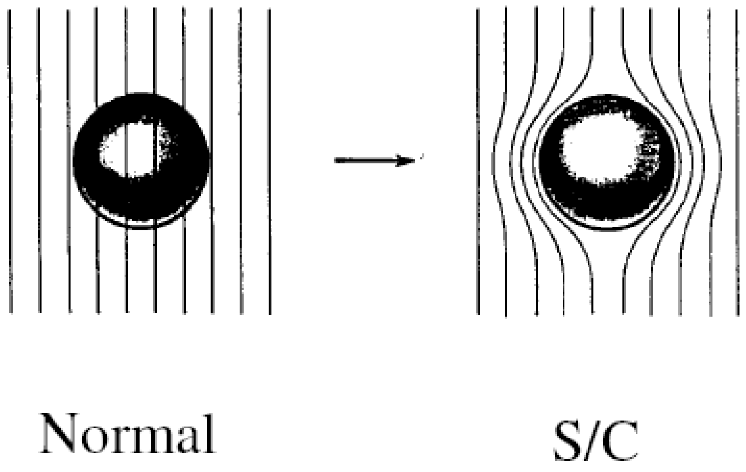
Properties of 2SC-III

Meissner effect

$$SU(3)_c \to SU(2)_c$$

a=1,2,3 massless
a=4,5,6,7 massive
a=8 massive

Rischke, PRD62:034007,2000



1933: Meissner & Ochsenfeld

Anderson-Higgs Mechanism

Rich structure of
Color Superconductor

Rich Structure of CSC-I

due to flavor, spin, and other defects

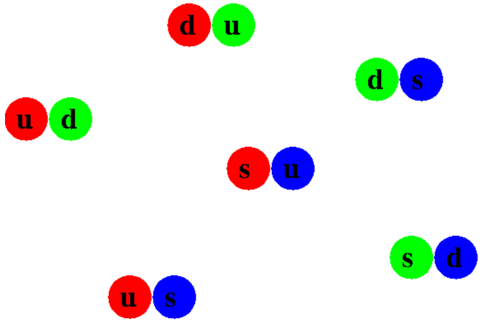
spin-0	2-flavor	3-flavor
Pairing without mismatch	2SC	CFL
Pairing with mismatch	g2SC	CFL+K, gCFL, uSC, dSC, sSC
	LOFF (Larkin Ovchinnikov Fulde Ferrell)	
spin-1	1-flavor	

Rich Structure of CSC-II

Pairing without mismatch

Spin-0: 2SC

Spin-0: CFL



Spin-1: CSL, Polar, …

Same flavor

Rich Structure of CSC-III

Pairing with mismatch

$$\delta\mu, \delta m \rightarrow \delta p_F$$

---- due to mass or chemical potential difference

Zero momentum pairing

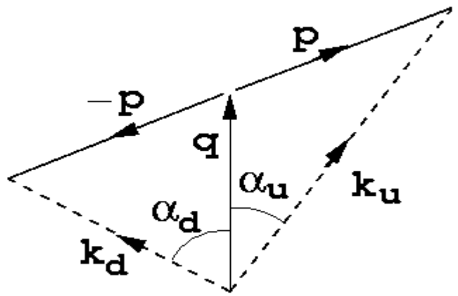
g2SC

gCFL CFL+K

uSC, dSC, sSC

Nonzero momentum pairing

Crystalline



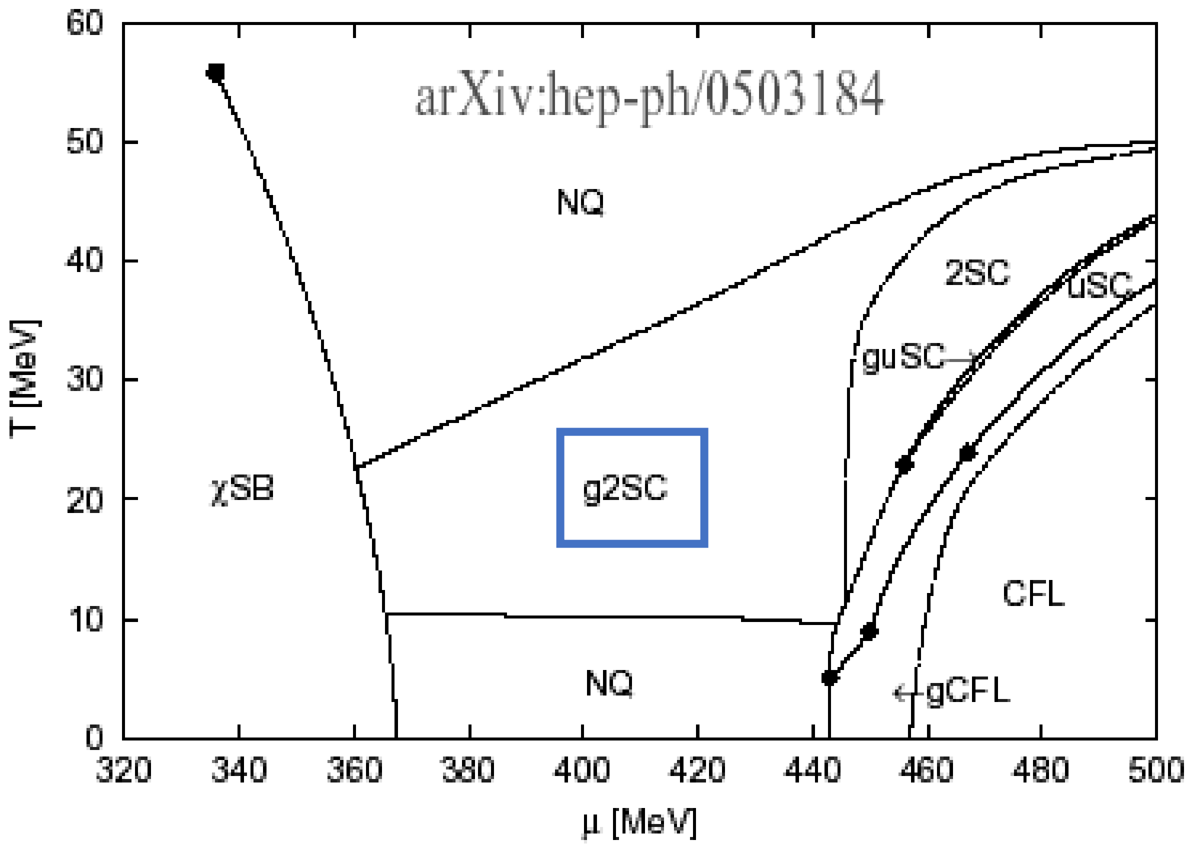
LOFF (Larkin Ovchinnikov Fulde Ferrell) State

LOFF → III.3

Rich Structure of CSC-IV

The phase diagram of neutral quark matter:
Self-consistent treatment of quark masses

Stefan B. Rüster,[1,*] Verena Werth,[2,†] Michael Buballa,[2,‡] Igor A. Shovkovy,[3,§] and Dirk H. Rischke[1,¶]

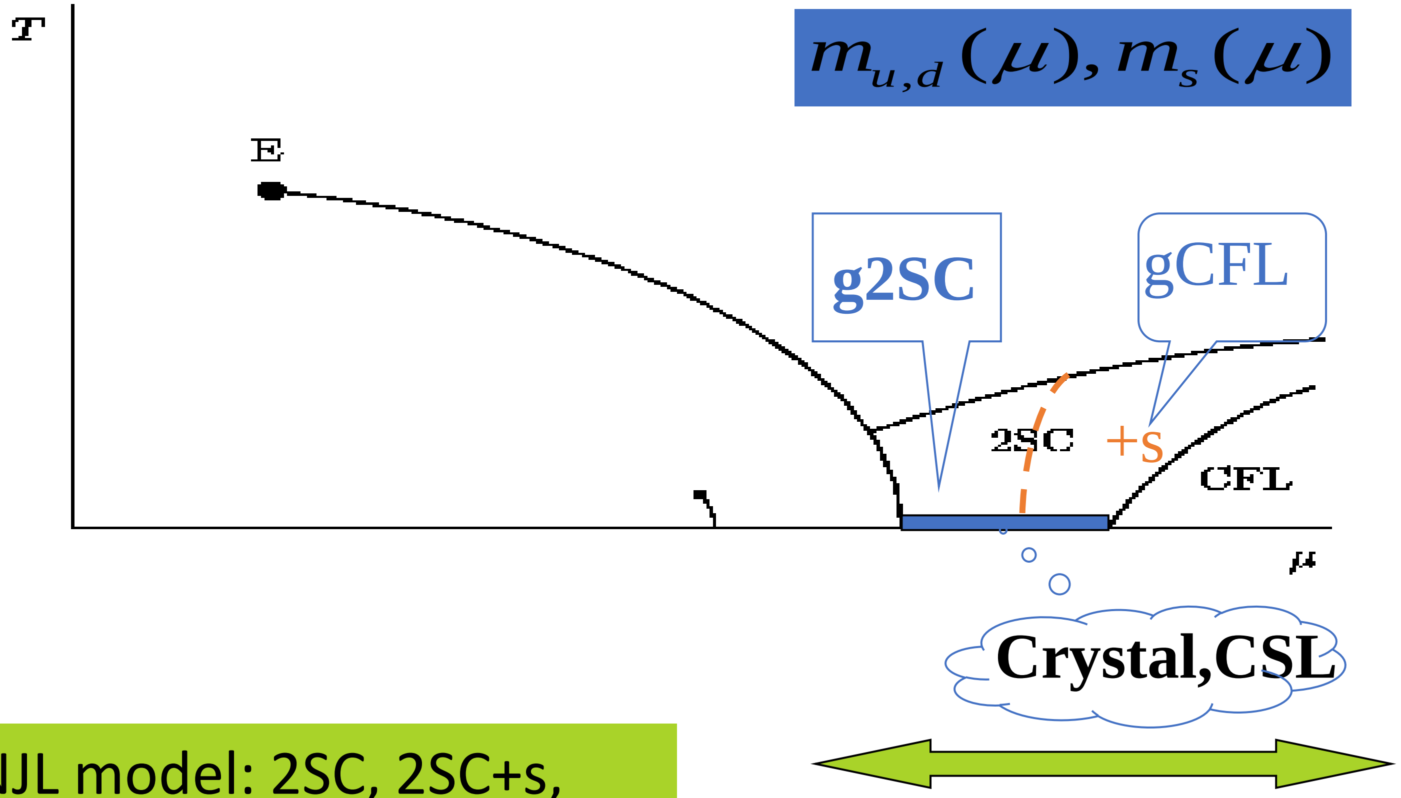


g2SC → III.1

II. Competition between chiral & diquark condensations

Why NJL model?

$$m_{u,d}(\mu), m_s(\mu)$$



NJL model: 2SC, 2SC+s, CFL

Bag model: only CFL

Nambu-Gorkov formalism with chiral condensate

To investigate the chiral restoration and color superconductivity phase transitions simultaneously

Nambu-Gorkov Formalism

Very convenient at high baryon density

Introducing charge-conjugate fields

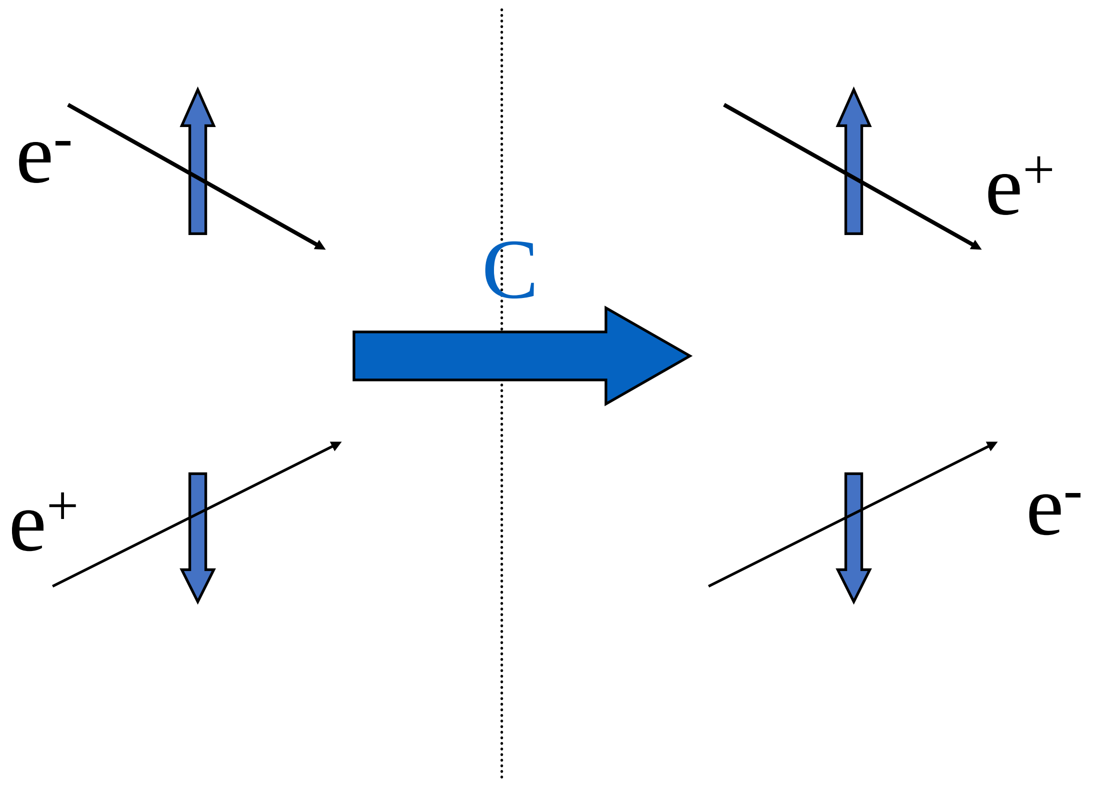
$$\psi_C(x) = C\,\bar{\psi}^T(x)\ ,\quad \bar{\psi}_C(x) = \psi^T(x)\,C$$

$$\psi(x) = C\,\bar{\psi}_C^T(x)\ ,\quad \bar{\psi}(x) = \psi_C^T(x)\,C$$

$$C = i\gamma^2\gamma_0 \qquad C\gamma_\mu C^{-1} = -\gamma_\mu^T$$

$$C = -C^{-1} = -C^T = -C^\dagger$$

Charge Conjugate



Nambu-Gorkov formalism

R.Pisarski,D.Rischke,PRD61(2000) 074017

$$\mathcal{L} = \bar{\psi}\,(i\gamma\cdot\partial - m)\,\psi - g\bar{\psi}\psi\,\phi + \frac{1}{2}\left(\partial_\mu\phi\,\partial^\mu\phi - M_s^2\phi^2\right)$$

$$\mathcal{Z} = \mathcal{N}\int \mathcal{D}\bar{\psi}\,\mathcal{D}\psi\,\mathcal{D}\phi\,\exp\left\{I[\bar{\psi},\psi,\phi]\right\}$$

$$I[\bar{\psi},\psi,\phi] = \int_{x,y}\left(\bar{\psi}(x)\;\left[G_0^+\right]^{-1}(x,y)\,\psi(y) - \frac{1}{2}\sum_{a,b=1}^{N}\phi^a(x)\,D_{ab}^{-1}(x,y)\,\phi^b(y)\right) - \int_x\sum_{a=1}^{N} g\,\bar{\psi}(x)\,\Gamma_a\,\psi(x)\,\phi^a(x)\;.$$

where $\left[G_0^{\pm}\right]^{-1}(x,y) \equiv -i\left[\,i\gamma\cdot\partial \pm \mu\,\gamma_0 - m\right]\delta^{(4)}(x-y)$

Introducing 8-component Spinors

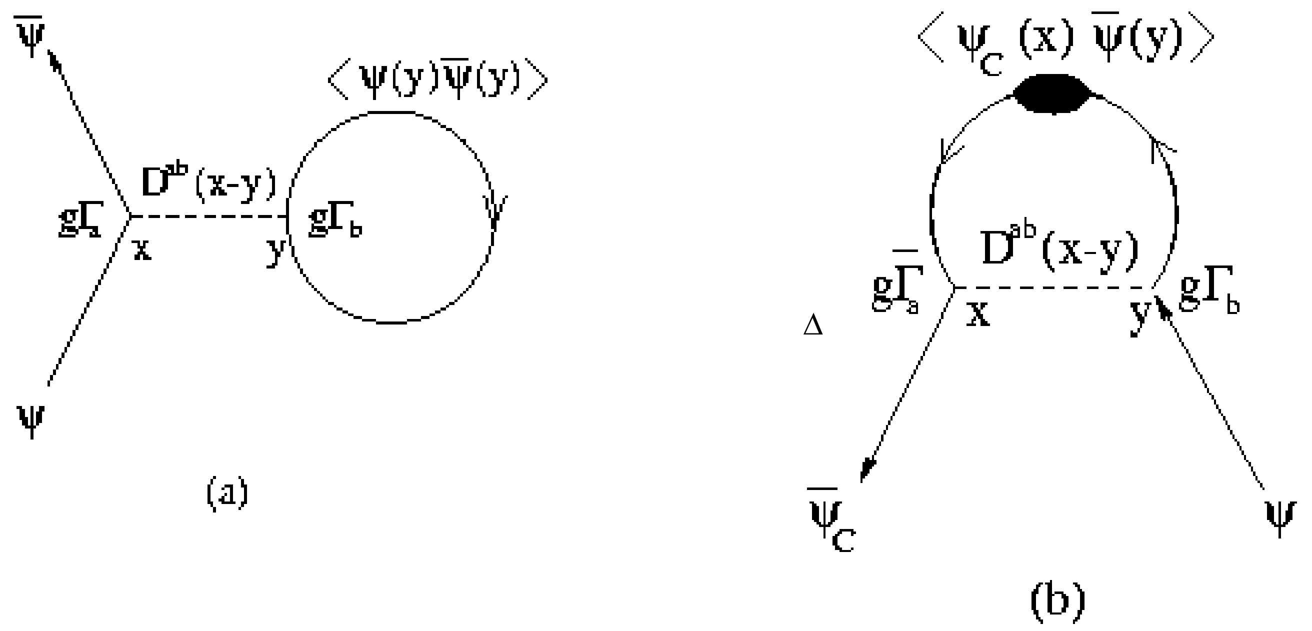
Nambu-Gorkov Spinors

$$\Psi \equiv \begin{pmatrix} \psi \\ \psi_C \end{pmatrix} \qquad \bar{\Psi} \equiv (\bar{\psi}\,,\, \bar{\psi}_C)$$

$$I[\bar{\Psi}, \Psi] = \frac{1}{2} \int_{x,y} \bar{\Psi}(x)\, \mathcal{S}^{-1}(x,y)\, \Psi(y)$$

Considerably Simplify calculations at μ

Mean-field Approximation



$$\Delta^+(k) = g^2 \frac{T}{V} \sum_q \sum_{a,b} \bar{\Gamma}_a D^{ab}(k-q)\, G_0^-(q)\, \Delta^+(q)\, G^+(q)\, \Gamma_b \ .$$

Full Propagator if $<\bar{\psi}\psi>=0$

$$\mathcal{S}^{-1} = \begin{pmatrix} [G_0^+]^{-1} & \Delta^- \\ \Delta^+ & [G_0^-]^{-1} \end{pmatrix} \qquad [G_0^\pm]^{-1}(k) \equiv \gamma \cdot k \pm \mu\, \gamma_0 - m$$

$$\mathcal{S} = \begin{pmatrix} G^+ & -G_0^+\, \Delta^-\, G^- \\ -G_0^-\, \Delta^+\, G^+ & G^- \end{pmatrix}$$

$$G^\pm \equiv \left\{ [G_0^\pm]^{-1} - \Sigma^\pm \right\}^{-1} \quad , \quad \Sigma^\pm \equiv \Delta^\mp\, G_0^\mp\, \Delta^\pm$$

$$G_0^\mp\, \Delta^\pm\, G^\pm = G^\mp\, \Delta^\pm\, G_0^\pm$$

at m=0

$$\Lambda_\pm(\vec{p}) = \frac{1}{2}\left(1 \pm \frac{\gamma_0\vec{\gamma}\cdot\vec{p}}{|\vec{p}|}\right)$$

$$G^\pm = \frac{p_0 - E_p^\pm}{p_0^2 - \left(E_p^\pm\right)^2 - \Delta^2}\gamma_0\Lambda_+ + \frac{p_0 + E_p^\mp}{p_0^2 - \left(E_p^\mp\right)^2 - \Delta^2}\gamma_0\Lambda_- \,,$$

$$\Xi^\pm = \frac{\mp\Delta\gamma_5}{p_0^2 - \left(E_p^\pm\right)^2 - \Delta^2}\Lambda_+ + \frac{\mp\Delta\gamma_5}{p_0^2 - \left(E_p^\mp\right)^2 - \Delta^2}\Lambda_- \,.$$

If quark mass m is not zero

1). Perturbative expansion around

$$m = 0$$

M.Rho,E.Shuryak,A.Wirzba,I.zahed

hep-ph/0001104,NPA676(2000),273

Correction of $m \neq 0$

$$
\begin{aligned}
S_{11}(q) = & \Big\{ \Lambda^+(\mathbf{q}) \left[q_0^2 - (\mu - |\mathbf{q}|)^2 - \left(\mathbf{M}^\dagger \mathbf{M} + \frac{\mathbf{M}^\dagger m^2 \mathbf{M}}{(q_0-\mu)^2 - |\mathbf{q}|^2} \right) |G(q)|^2 \right] \Lambda^+(\mathbf{q}) \\
& + \Lambda^-(\mathbf{q}) \left[q_0^2 - (\mu + |\mathbf{q}|)^2 - \left(\mathbf{M}^\dagger \mathbf{M} + \frac{\mathbf{M}^\dagger m^2 \mathbf{M}}{(q_0-\mu)^2 - |\mathbf{q}|^2} \right) |\overline{G}(q)|^2 \right] \Lambda^-(\mathbf{q}) \\
& - \Lambda^+(\mathbf{q})\, \gamma^0 \left[q^0 - \mu + |\mathbf{q}| \right] \left(m - \mathbf{M}^\dagger m \mathbf{M} \frac{G^*(q)\overline{G}(q)}{(q_0-\mu)^2 - |\mathbf{q}|^2} \right) \Lambda^-(\mathbf{q}) \\
& - \Lambda^-(\mathbf{q})\, \gamma^0 \left[q^0 - \mu - |\mathbf{q}| \right] \left(m - \mathbf{M}^\dagger m \mathbf{M} \frac{\overline{G}^*(q) G(q)}{(q_0-\mu)^2 - |\mathbf{q}|^2} \right) \Lambda^+(\mathbf{q}) \Big\}^{-1} \\
& \times \gamma^0 (q_0 - \mu - \boldsymbol{\alpha} \cdot \mathbf{q}) \,. \qquad \text{(A7)}
\end{aligned}
$$

Complicated!

Perturbative expansion is not always appropriate!

2. An easy way to deal with quark mass

M.H, P.Zhuang,W.Chao,PRD65(2002)076012

Energy projectors with quark mass

$$\Lambda_\pm(\mathbf{p}) = \frac{1}{2}(1 \pm \frac{\gamma_0(\boldsymbol{\gamma}\cdot\mathbf{p}+m)}{E_p}), \qquad \tilde{\Lambda}_\pm(\mathbf{p}) = \frac{1}{2}(1 \pm \frac{\gamma_0(\boldsymbol{\gamma}\cdot\mathbf{p}-m)}{E_p}) \ ,$$

$$\Lambda_\pm(\vec{p})\Lambda_\pm(\vec{p}) = \Lambda_\pm(\vec{p})$$

$$\Lambda_\pm(\vec{p})\Lambda_\mp(\vec{p}) = 0$$

$$\Lambda_+(\vec{p}) + \Lambda_-(\vec{p}) = 1$$

$$\gamma_0\Lambda_\pm(\mathbf{p})\gamma_0 = \tilde{\Lambda}_\mp(\mathbf{p}), \quad \gamma_5\Lambda_\pm(\mathbf{p})\gamma_5 = \tilde{\Lambda}_\pm(\mathbf{p}).$$

Massive Quark Propagator

M.H, P.Zhuang,W.Chao,PRD65(2002)076012

$$G_0^{\pm} = \frac{\gamma_0 \tilde{\Lambda}_+}{p_0 + E_p^{\pm}} + \frac{\gamma_0 \tilde{\Lambda}_-}{p_0 - E_p^{\mp}},$$

$$G^{\pm} = \left(\frac{p_0 - E_p^{\pm}}{p_0^2 - E_\Delta^{\pm 2}} \gamma_0 \tilde{\Lambda}_+ + \frac{p_0 + E_p^{\mp}}{p_0^2 - E_\Delta^{\mp 2}} \gamma_0 \tilde{\Lambda}_-\right)(\delta_{\alpha\beta} - \delta_{\alpha b}\delta_{\beta b})\delta^{ij},$$

$$\Xi^{\pm} = \left(\frac{\Delta^{\pm}}{p_0^2 - E_\Delta^{\pm 2}} \tilde{\Lambda}_+ + \frac{\Delta^{\pm}}{p_0^2 - E_\Delta^{\mp 2}} \tilde{\Lambda}_-\right),$$

Competition Between Chiral & Diquark Condensate

M.H, P.Zhuang,W.Chao,PRD65(2002)076012

a) Using the massive quark propagator Deriving

$$\Omega(T,\mu,m,\Delta)$$

b) Competition between the chiral & diquark condensates

SU(2) NJL Model

$$\mathcal{L} = \bar{q}(i\gamma^\mu\partial_\mu - m_0)q + G_S\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right]$$
$$+G_D\left[(i\bar{q}^C\varepsilon\epsilon^b\gamma_5 q)(i\bar{q}\varepsilon\epsilon^b\gamma_5 q^C)\right]$$

bosonic fields

$$\Delta^b \sim i\bar{q}^C\varepsilon\epsilon^b\gamma_5 q, \quad \Delta^{*b} \sim i\bar{q}\varepsilon\epsilon^b\gamma_5 q^C, \quad \sigma \sim \bar{q}q, \quad \pi \sim i\bar{q}\gamma^5\tau q.$$

$$\tilde{\mathcal{L}} = \bar{q}(i\gamma^\mu\partial_\mu - m_0)q - \bar{q}(\sigma + i\gamma^5\tau\pi)q - \frac{1}{2}\Delta^{*b}(i\bar{q}^C\varepsilon\epsilon^b\gamma_5 q) - \frac{1}{2}\Delta^b(i\bar{q}\varepsilon\epsilon^b\gamma_5 q^C)$$
$$-\frac{\sigma^2+\pi^2}{4G_S} - \frac{\Delta^{*b}\Delta^b}{4G_D},$$

$$\mathcal{L} = \bar{q}(i\gamma^\mu\partial_\mu - m_0)q + G_S[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2] + G_D[(i\bar{q}^C\varepsilon\epsilon^b\gamma_5 q)(i\bar{q}\varepsilon\epsilon^b\gamma_5 q^C)].$$

$$q^C = C\bar{q}^T,\ \bar{q}^C = q^T C \qquad C = i\gamma^2\gamma^0$$

$$\tilde{\mathcal{L}} = \bar{q}(i\gamma^\mu\partial_\mu - m_0)q - \bar{q}(\sigma + i\gamma^5\tau\pi)q - \frac{1}{2}\Delta^{*b}(i\bar{q}^C\varepsilon\epsilon^b\gamma_5 q) - \frac{1}{2}\Delta^b(i\bar{q}\varepsilon\epsilon^b\gamma_5 q^C)$$
$$-\frac{\sigma^2 + \pi^2}{4G_S} - \frac{\Delta^{*b}\Delta^b}{4G_D},$$

$$\Delta^b \sim i\bar{q}^C\varepsilon\epsilon^b\gamma_5 q,\ \ \Delta^{*b} \sim i\bar{q}\varepsilon\epsilon^b\gamma_5 q^C,\ \ \sigma \sim \bar{q}q,\ \ \pi \sim i\bar{q}\gamma^5\tau q.$$

$$\mathcal{Z} = N' \int [d\bar{q}][dq] exp\{\int_0^\beta d\tau \int d^3\mathbf{x}\ (\tilde{\mathcal{L}} + \mu\bar{q}\gamma_0 q)\},$$

$$\mathcal{Z} = \mathcal{Z}_{const}\mathcal{Z}_b\mathcal{Z}_{r,g}.$$

$$\mathcal{Z}_{const} = N'\exp\{-\int_0^\beta d\tau \int d^3\mathbf{x}\ [\frac{\sigma^2}{4G_S} + \frac{\Delta^*\Delta}{4G_D}\}.$$

$$\mathcal{Z}_b = \int [d\bar{q}_b][dq_b]\exp\{\int_0^\beta d\tau \int d^3\mathbf{x}\ [\frac{1}{2}\bar{q}_b(i\gamma^\mu\partial_\mu - m + \mu\gamma_0)q_b$$
$$+\frac{1}{2}\bar{q}_b^C(i\gamma^\mu\partial_\mu - m - \mu\gamma_0)q_b^C]\}.$$

$$\mathcal{Z}_{r,g} = \int [d\bar{Q}][dQ]\exp\{\int_0^\beta d\tau \int d^3\mathbf{x}\ [\frac{1}{2}\bar{Q}(i\gamma^\mu\partial_\mu - m + \mu\gamma_0)Q +$$
$$\frac{1}{2}\bar{Q}^C(i\gamma^\mu\partial_\mu - m - \mu\gamma_0)Q^C + \frac{1}{2}\bar{Q}\Delta^- Q^C + \frac{1}{2}\bar{Q}^C\Delta^+ Q]\}.$$

$$\Psi_b = \begin{pmatrix} q_b \\ q_b^C \end{pmatrix}, \quad \bar{\Psi}_b = (\bar{q}_b \quad \bar{q}_b^C),$$

$$\Psi = \begin{pmatrix} Q \\ Q^C \end{pmatrix}, \quad \bar{\Psi} = (\bar{Q} \quad \bar{Q}^C),$$

$$q(x) = \frac{1}{\sqrt{V}} \sum_n \sum_{\mathbf{p}} e^{-i(\omega_n \tau - \mathbf{p}\cdot\mathbf{x})} q(\mathbf{p}),$$

$$\mathrm{G_0}^{-1} = \begin{pmatrix} [G_0^+]^{-1} & 0 \\ 0 & [G_0^-]^{-1} \end{pmatrix},$$

$$[G_0^{\pm}]^{-1} = (p_0 \pm \mu)\gamma_0 - \boldsymbol{\gamma} \cdot \mathbf{p} - m$$

$$\mathcal{Z}_b = \int [d\Psi_b] \exp\{\frac{1}{2} \sum_{n,\mathbf{p}} \bar{\Psi}_b \frac{G_0^{-1}}{T} \Psi_b\}$$
$$= \mathrm{Det}^{1/2}(\beta G_0^{-1}),$$

$$\mathcal{Z}_{r,g} = \int [d\Psi] \exp\{\frac{1}{2} \sum_{n,\mathbf{p}} \bar{\Psi} \frac{\mathrm{G}^{-1}}{T} \Psi\}$$
$$= \mathrm{Det}^{1/2}(\beta \mathrm{G}^{-1}).$$

$$\mathrm{G}^{-1} = \begin{pmatrix} \left[G_0^+\right]^{-1} & \Delta^- \\ \Delta^+ & \left[G_0^-\right]^{-1} \end{pmatrix}. \qquad \mathrm{G} = \begin{pmatrix} G^+ & \Xi^- \\ \Xi^+ & G^- \end{pmatrix}$$

$$G^\pm \equiv \left\{ \left[G_0^\pm\right]^{-1} - \Sigma^\pm \right\}^{-1} \;, \quad \Sigma^\pm \equiv \Delta^\mp \, G_0^\mp \, \Delta^\pm \;,$$

$$\Xi^\pm \equiv -G^\mp \, \Delta^\pm \, G_0^\pm = -G_0^\mp \, \Delta^\pm \, G^\pm.$$

$\Omega(T, \mu, m, \Delta)$

$$\Omega = -T\frac{\ln Z}{V} = \frac{\sigma^2}{4G_S} + \frac{\Delta^2}{4G_D} - 2N_f \int \frac{d^3p}{(2\pi)^3}[E_p + T\ln(1 + e^{-\beta E_p^+}) + T\ln(1 + e^{-\beta E_p^-})$$

$$+E_\Delta^+ + 2T\ln(1 + e^{-\beta E_\Delta^+}) + E_\Delta^- + 2T\ln(1 + e^{-\beta E_\Delta^-})]$$

$$E_p^\pm = E_p \pm \mu \qquad E_\Delta^{\pm 2} = E_p^{\pm 2} + \Delta^2$$

Gap Equations:

A. Variation Method

$$\frac{\partial \Omega}{\partial m} = \frac{\partial \Omega}{\partial \Delta} = 0.$$

B. Feynman Diagram

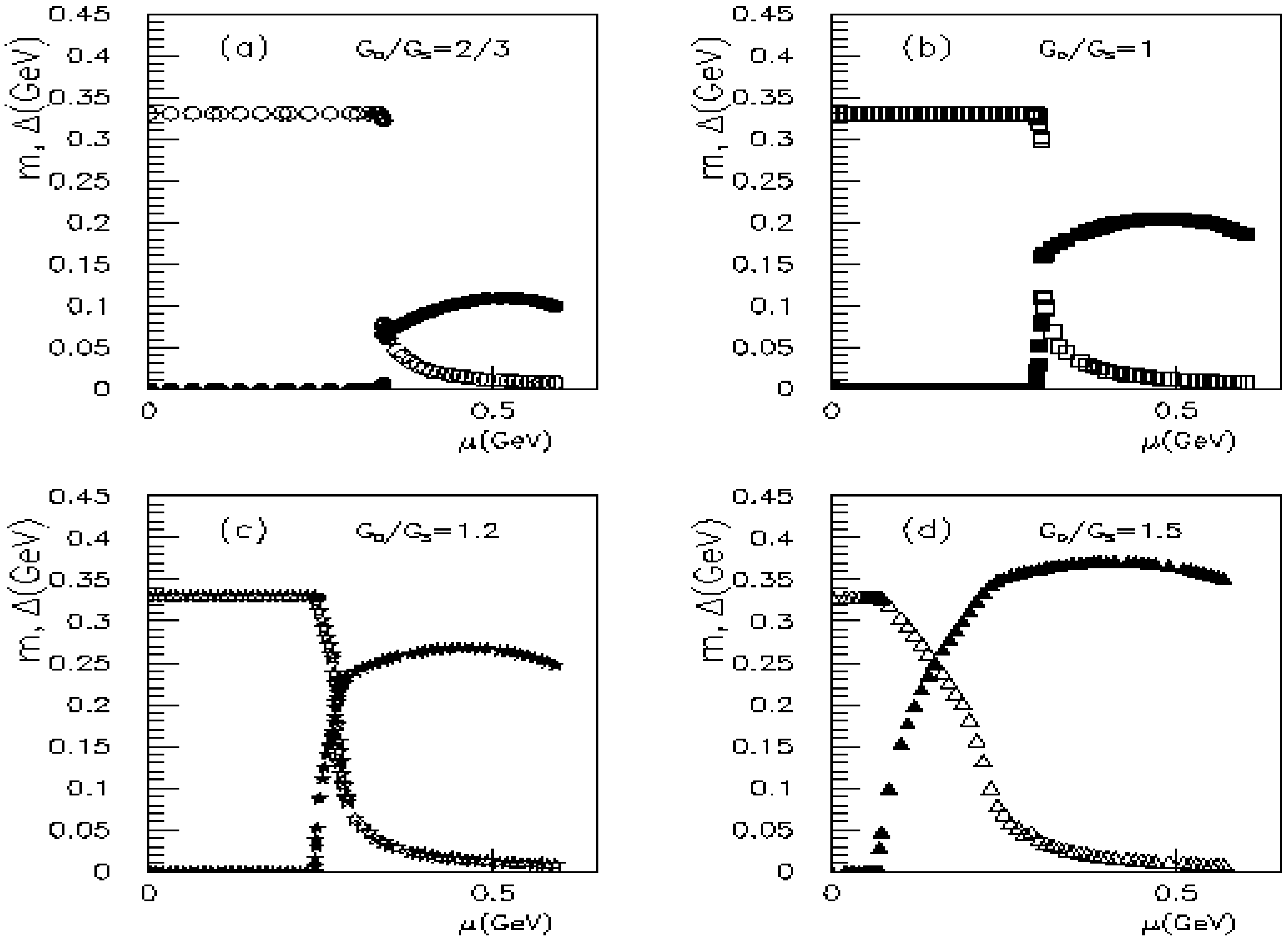
$$< \bar{q}q >= 2 < \bar{q}_1 q^1 > + < \bar{q}_3 q^3 >$$

$$< \bar{q}_1 q^1 >= -iT \sum_n \int \frac{d^3p}{(2\pi)^3} tr[G^+],$$

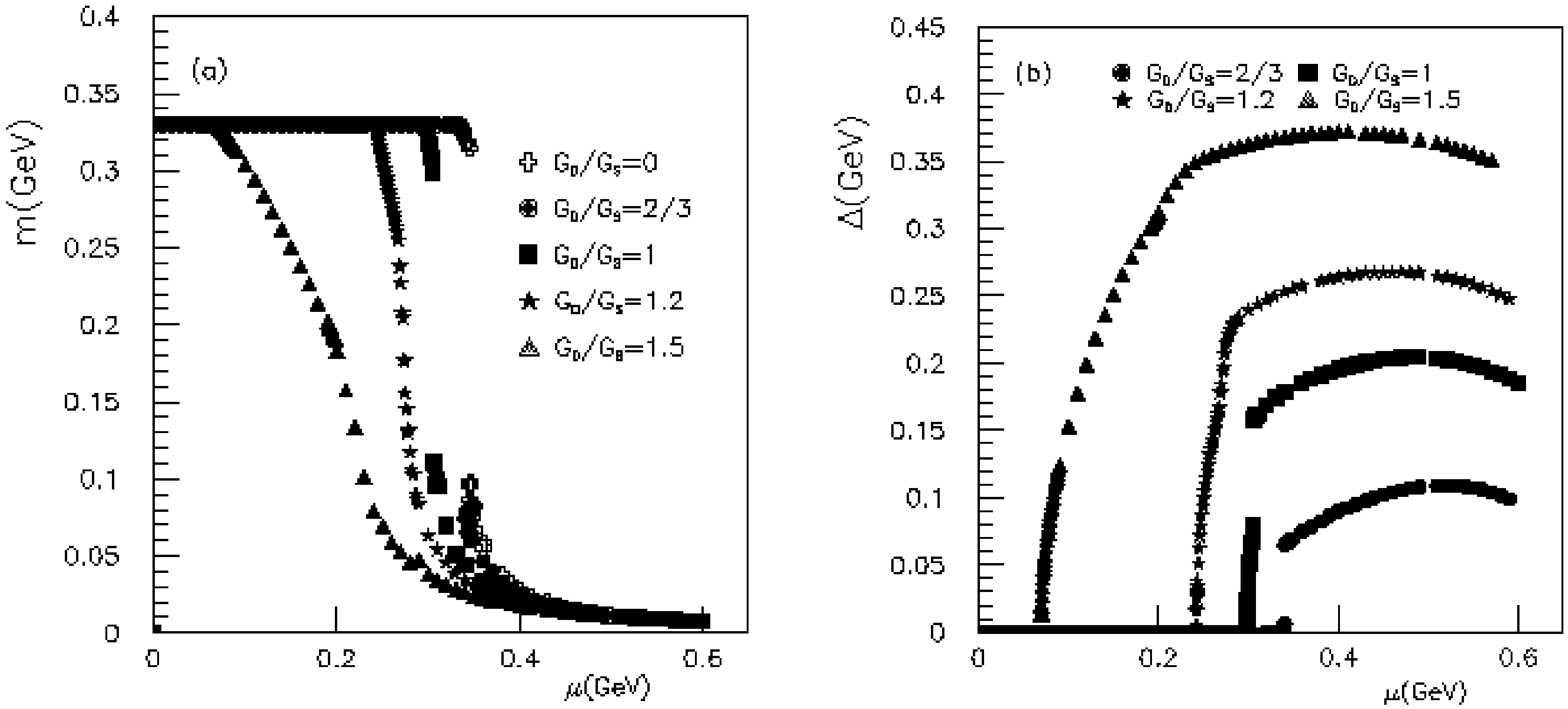
$$< \bar{q}_3 q^3 >= -iT \sum_n \int \frac{d^3p}{(2\pi)^3} tr[G_0^+],$$

$$\Delta = -2G_D < \bar{q}^C \gamma_5 q >,$$

$$< \bar{q}^C \gamma_5 q >= (iT \sum_n) \int \frac{d^3p}{(2\pi)^3} tr[\Xi^- \gamma_5].$$



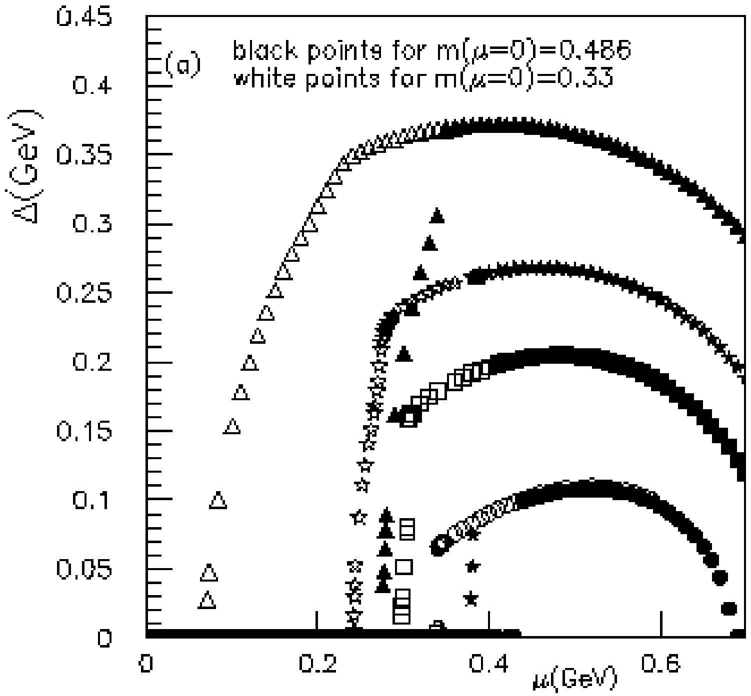
$\mu_\chi - \mu_\Delta$ double broken regime

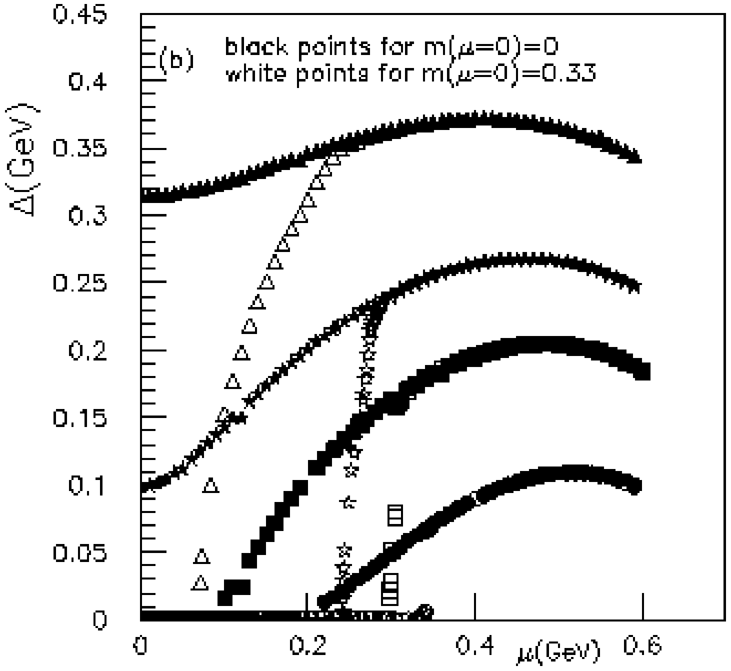


$\frac{G_D}{G_S}\uparrow$

$\mu_\chi \downarrow$ $\mu_\Delta \downarrow$ $\mu_\chi - \mu_\Delta \uparrow$

First order $\longrightarrow$ Second order





$m(\mu=0)\uparrow$

$m(\mu=0)\downarrow$

$\mu_\Delta\uparrow$

$\mu_\Delta\downarrow$

Summary I

Chiral breaking phase	$\mu < \mu_\Delta$
Mixed broken phase	$\mu_\Delta < \mu < \mu_\chi$
Color superconductivity	$\mu > \mu_\chi$

$$\frac{G_D}{G_S}\uparrow$$

$$\mu_\chi - \mu_\Delta \uparrow$$

$$\mu_\chi - \mu_\Delta$$

Competition regime

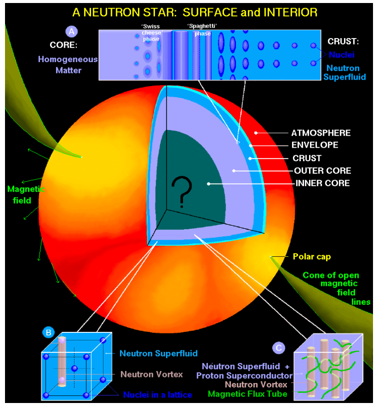
Diquark pair smoothes quark's Fermi surface

II. Pairing with mismatched Fermi surfaces

Unconventional CSC (since 2002)

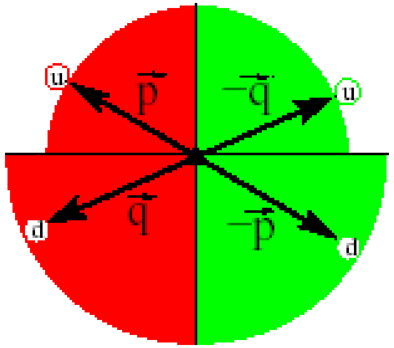
Reviews:

M. Huang, Int. J. Mod.Phys. E14, 675 (2005)



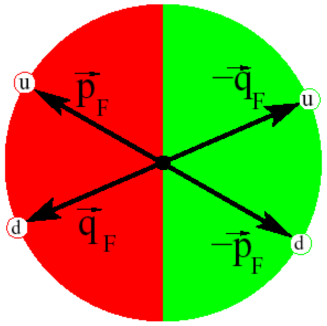
beta-equilibrium,
charge neutrality

$$\delta\mu, \delta m \rightarrow \delta p_F$$

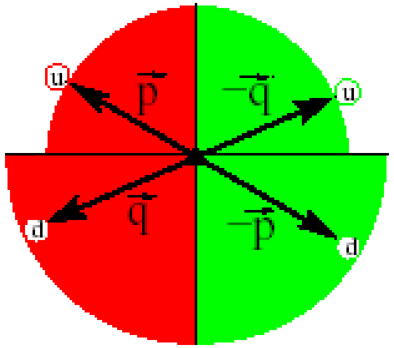


Many puzzles! Unsolved problem !

$$\delta\mu, \delta m \rightarrow \delta p_F$$



$$\langle u_p d_{-p} \rangle = - \langle u_q d_{-q} \rangle \neq 0$$



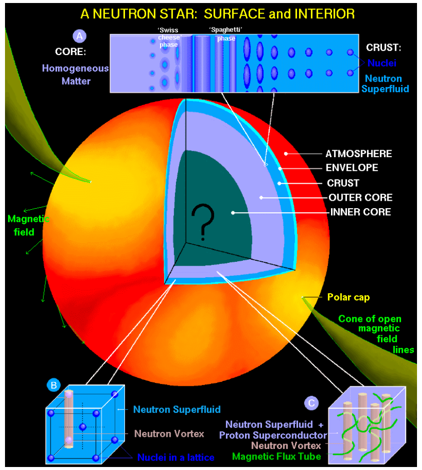
Pair breaking?

Neutral dense quark matter
Imbalanced cold atom system,
Asymmetric nuclear matter,
Electric SC under external magnetic field
……

Charge neutrality condition on 2SC

$$n_Q^{\rm el} = 0, \qquad n_Q^{\rm color} = 0$$

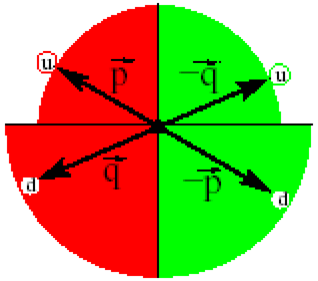
$$E_{\rm Coulomb} \sim n_Q^2 R^5$$



Color neutrality: easily satisfied

Electric neutrality:

$$n_d \approx 2 n_u$$



$$\mu_e \approx \mu / 4$$

Pair breaking?

Gapless 2SC: result from BCS at Mean-field

$$\mathcal{L} = \bar{q}(i\gamma^\mu\partial_\mu - m_0)q + G_S\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] + G_D\left[(i\bar{q}^C\varepsilon\epsilon^b\gamma_5 q)(i\bar{q}\varepsilon\epsilon^b\gamma_5 q^C)\right]$$

Beta-equilibrium:

$$\mu_{ij,\alpha\beta} = (\mu\delta_{ij} - \mu_e Q_{ij}) + \frac{2}{\sqrt{3}}\mu_8\delta_{ij}(T_8)_{\alpha\beta}$$

$$\bar{\mu} = \mu - \mu_e/6 + \mu_8/3$$
$$\delta\mu = \mu_e/2$$

Mean-field(MF):

$$\Delta = |\Delta|$$

$$\Omega = \Omega_0 - \frac{1}{12\pi^2}\left(\mu_e^4 + 2\pi^2T^2\mu_e^2 + \frac{7\pi^4}{15}T^4\right) + \frac{(m-m_0)^2}{4G_S} + \frac{\Delta^2}{4G_D} - \sum_a\int\frac{d^3p}{(2\pi)^3}\left[E_a + 2T\ln\left(1+e^{-E_a/T}\right)\right]$$

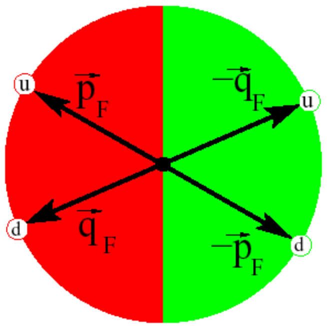
$$E_{ub}^{\pm} = E(p) \pm \mu_{ub}, \quad E_{db}^{\pm} = E(p) \pm \mu_{db},$$

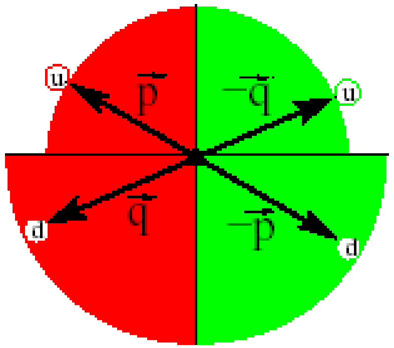
$$E_{\Delta^\pm}^{\pm} = E_{\Delta}^{\pm}(p) \pm \delta\mu.$$

Pairing with mismatch

$$\delta\mu, \delta m \rightarrow \delta p_F$$

---- due to mass or chemical potential difference





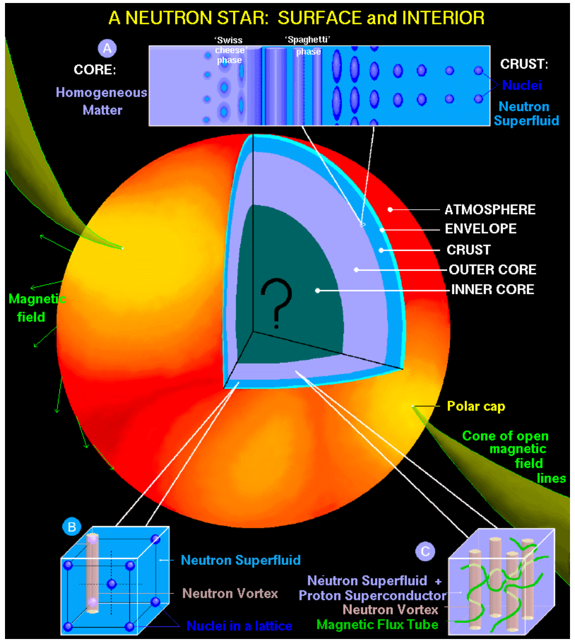
$$\langle u_p d_{-p} \rangle = - \langle u_q d_{-q} \rangle \neq 0$$

Pairing breaking?

Charge Neutrality in Compact Stars

$$n_Q^{\text{el}} = 0, \qquad n_Q^{\text{color}} = 0$$

$$E_{\text{Coulomb}} \sim n_Q^2 R^5$$



Otherwise, bulk matter cannot be formed inside stars because of the repulsive Coulomb interaction

1. Global charge neutrality: mixed phase

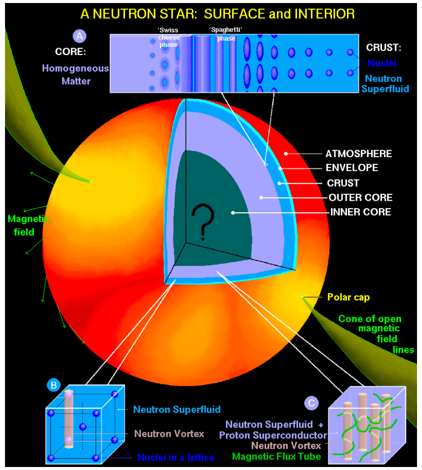
small surface tension

I. Shovkovy, M. Hanauske, M.H, Phys.Rev.D67:103004,2003
S. Reddy and G. Rupak, nucl-th/0405054

$$\rho / \rho_0 \approx 5-10$$

2. Local charge neutrality: homogeneous phase

How charge neutrality affects 2SC?



$$n_Q^{\rm el} = 0, \qquad n_Q^{\rm color} = 0$$

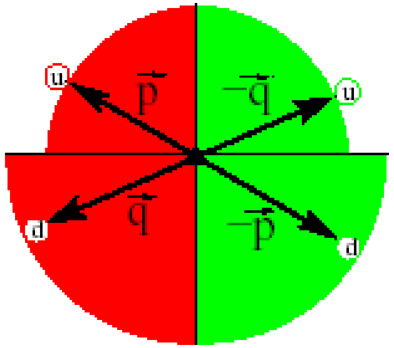
$$E_{\rm Coulomb} \sim n_Q^2 R^5$$

Color neutrality: easily satisfied

Electric neutrality:

$$n_d \approx 2 n_u$$

$$\mu_e \approx \mu / 4$$



Destroy Cooper pairing ?

Results in SU(2) NJL Model

The Model

$$\begin{aligned}\mathcal{L} &= \bar{q}(i\gamma^\mu\partial_\mu - m_0)q + G_S\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] \\ &+G_D\left[(i\bar{q}^C\varepsilon\epsilon^b\gamma_5 q)(i\bar{q}\varepsilon\epsilon^b\gamma_5 q^C)\right]\end{aligned}$$

Model Parameters

$$m_0 = 0, G_S = 5.0163GeV^{-2}, \Lambda = 653.3MeV, \eta = G_D/G_S$$

Results in SU(2) NJL Model

SU(2) NJL Model

$$\mathcal{L} = \bar{q}(i\not{D} + \hat{\mu}\gamma^0)q + G_S[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]$$
$$+ G_D[(i\bar{q}^C\varepsilon\epsilon^b\gamma_5 q)(i\bar{q}\varepsilon\epsilon^b\gamma_5 q^C)],$$

$$G_S = 5.0163\text{GeV}^{-2}, \Lambda = 0.6533\text{GeV}.$$
$$m = 0.314\text{GeV} \qquad \eta = G_D/G_S$$

chiral limit

Beta-equilibrium:

$$\mu_{ij,\alpha\beta} = (\mu\delta_{ij} - \mu_e Q_{ij}) + \frac{2}{\sqrt{3}}\mu_8\delta_{ij}(T_8)_{\alpha\beta}$$

$$\bar{\mu} = \mu - \mu_e/6 + \mu_8/3$$
$$\delta\mu = \mu_e/2$$

Requirement of beta-equilibrium-I

β-equilibrium:

$$\mu_{i\alpha} = \mu - \mu_e Q_{ii} + \mu_3 (Q_3)_{\alpha\alpha} + \mu_8 (Q_8)_{\alpha\alpha}$$

Electric charge chemical potential

Electron chemical potential

$$i = u, d; \alpha = 1,2,3$$

Charges

$$U(1)_Q : Q = diag\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$$

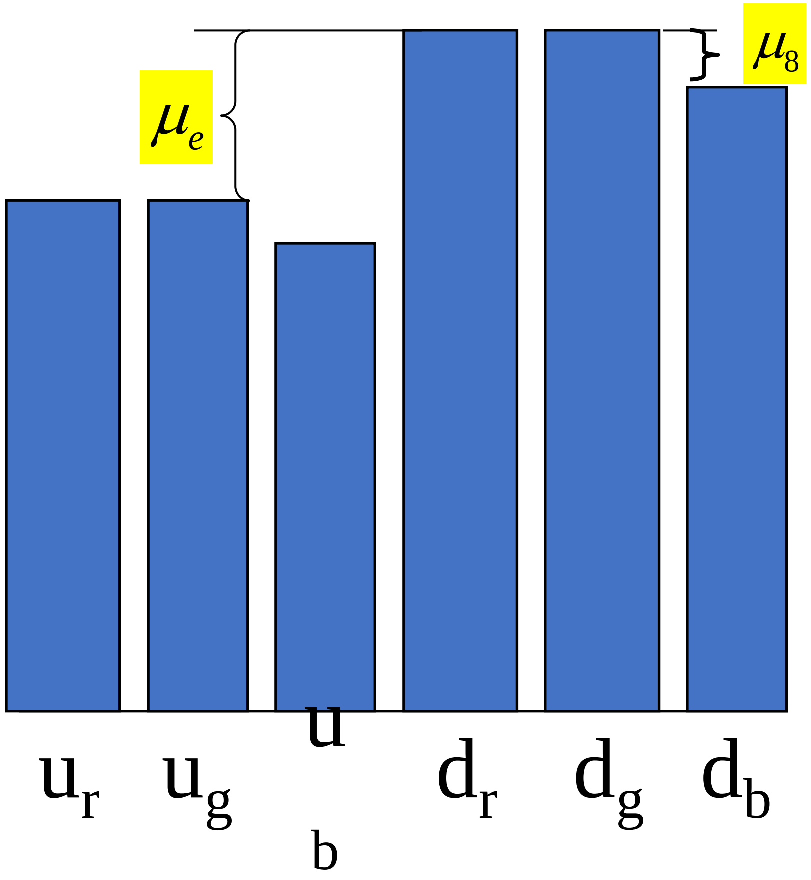
$$U(1)_{3c} : Q_3 = diag\left(\frac{1}{2}, -\frac{1}{2}, 0\right)$$

$$U(1)_{8c} : Q_8 = diag\left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right)$$

Requirement of beta-equilibrium-II

$$\mu_{ij,\alpha\beta} = (\mu\delta_{ij} - \mu_e Q_{ij}) + \frac{2}{\sqrt{3}}\mu_8 \delta_{ij}(T_8)_{\alpha\beta}$$

$$\bar{\mu} = \mu - \mu_e/6 + \mu_8/3$$
$$\delta\mu = \mu_e/2$$



$$\mu_3 = 0, SU(2)_C$$

Thermodynamic potential

$$\Omega_{u,d,e} = -\frac{1}{12\pi^2}\left(\mu_e^4 + 2\pi^2 T^2 \mu_e^2 + \frac{7\pi^4}{15}T^4\right) + \frac{m^2}{4G_S} + \frac{\Delta^2}{4G_D} - \sum_a \int \frac{d^3p}{(2\pi)^3}\left[E_a + 2T\ln\left(1 + e^{-E_a/T}\right)\right],$$

Dispersion relation

$$E_{ub}^{\pm} = E(p) \pm \mu_{ub}, \qquad [\times 1]$$
$$E_{db}^{\pm} = E(p) \pm \mu_{db}, \qquad [\times 1]$$
$$E_{\Delta^{\pm}}^{\pm} = E_{\Delta}^{\pm}(p) \pm \delta\mu. \qquad [\times 2]$$

The ground state: Balance of energy gain and loss

Charge Neutrality Condition

$$\frac{\partial \Omega}{\partial \mu_8} = 0, \frac{\partial \Omega}{\partial \mu_e} = 0$$

Diquark Gap Equation

$$\frac{\partial \Omega}{\partial \Delta} = 0$$

Charge Neutrality

Electric Charge Neutrality:

$$\frac{2}{3}n_u - \frac{1}{3}n_d - n_e = 0$$

Color Charge Neutrality:

$$n_{ur} + n_{dr} = n_{ug} + n_{dg} = n_{ub} + n_{db}$$

M.H., P.Zhuang, W.Chao PRD67:0650152,2003

$$\eta = \frac{G_D}{G_S} = \frac{3}{4}$$

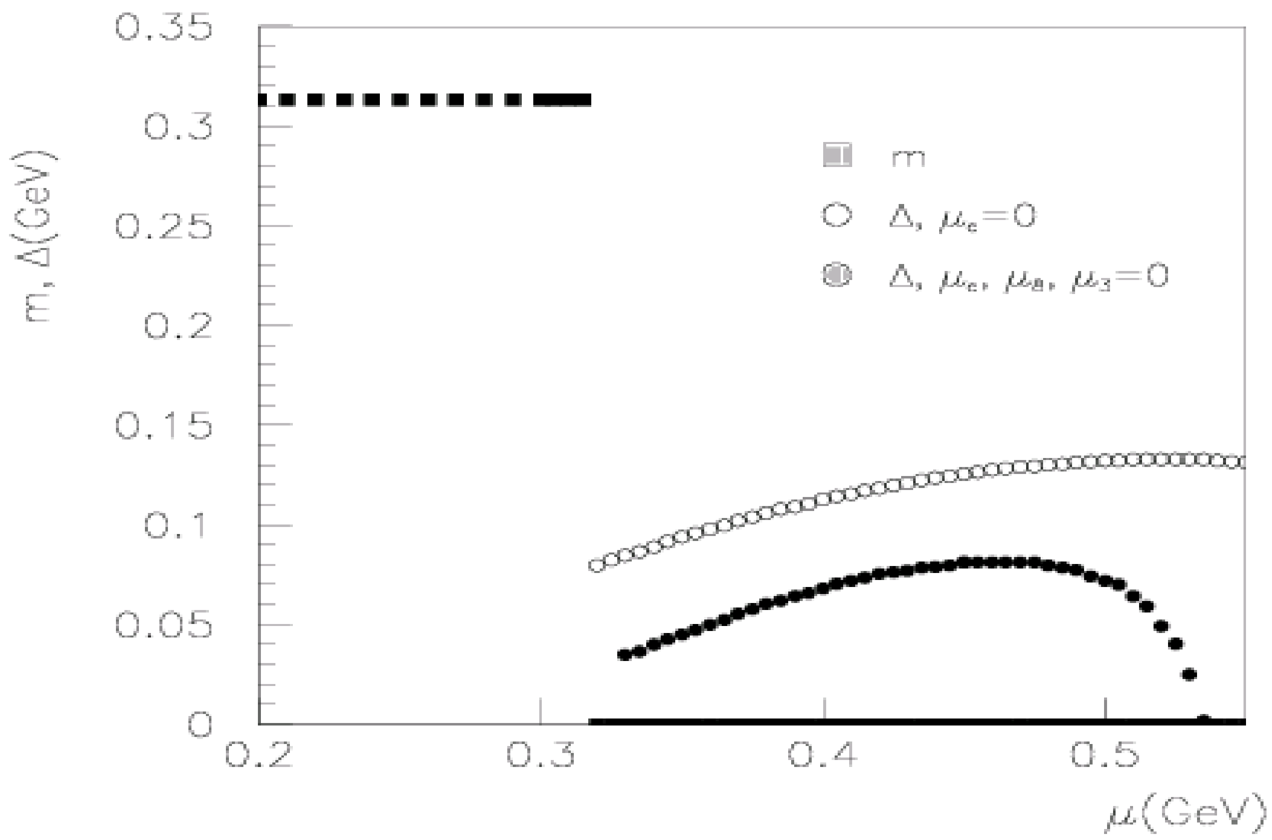
We focus on the region

$$m_{u,d}(\mu) < \mu < m_s(\mu)$$

$$330MeV < \mu < 550MeV$$

Phase diagram

M.H., P.Zhuang, W.Chao PRD67:0650152,2003

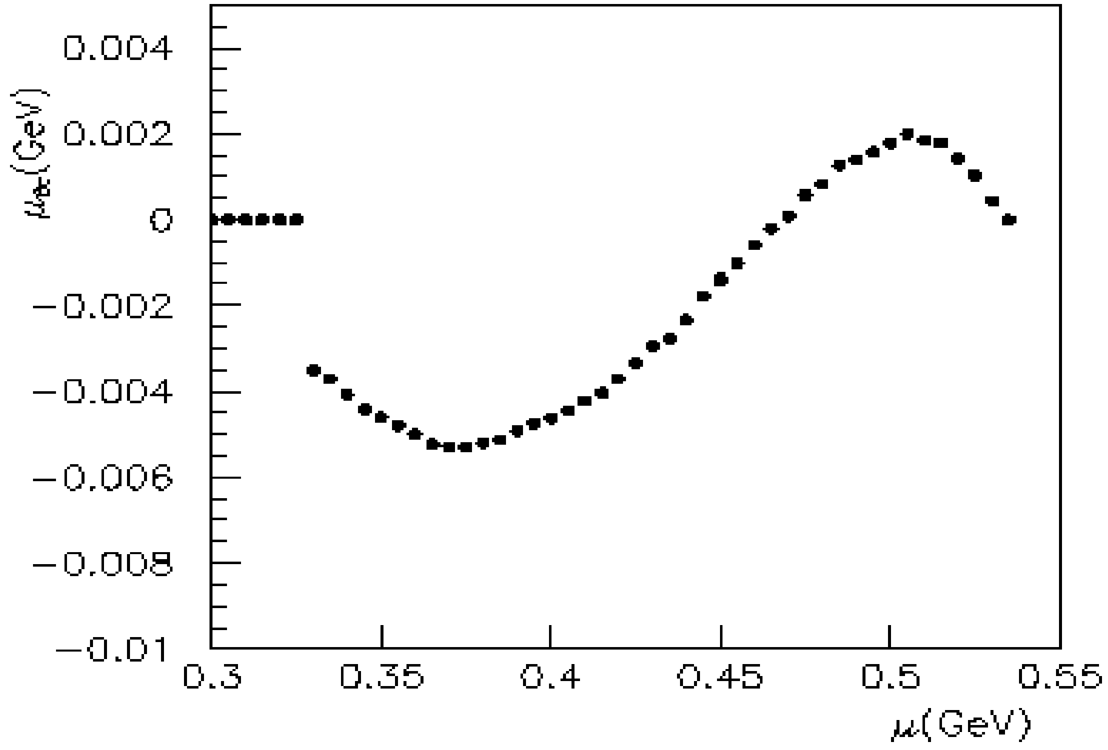


$$\eta = \frac{G_D}{G_S} = \frac{3}{4}$$

Gap is largely suppressed

Chemical Potential of Color charge

M.H., P.Zhuang, W.Chao PRD67:0650152,2003

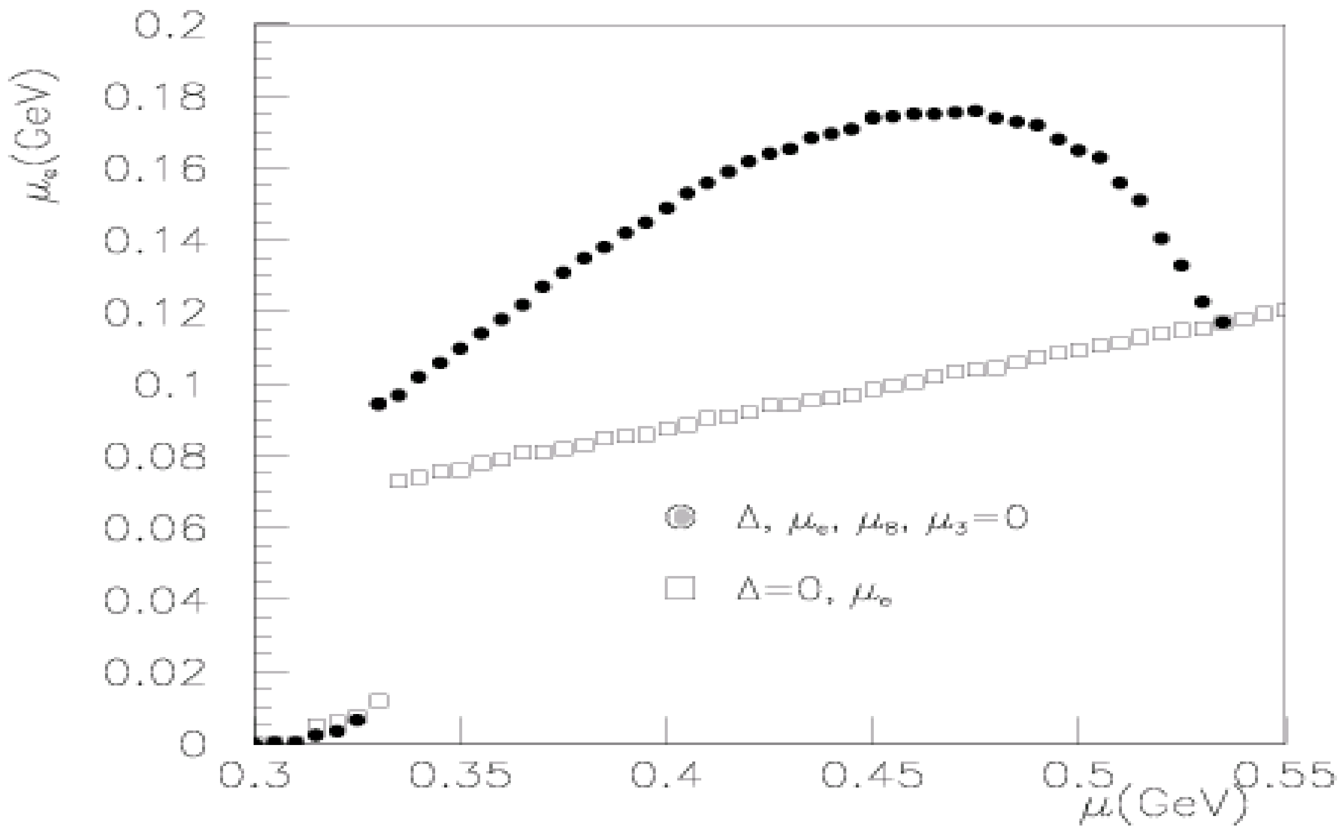


$$\eta = \frac{G_D}{G_S} = \frac{3}{4}$$

$$\mu_8 \approx (-5 \sim 2) MeV$$

Chemical Potential of Electron

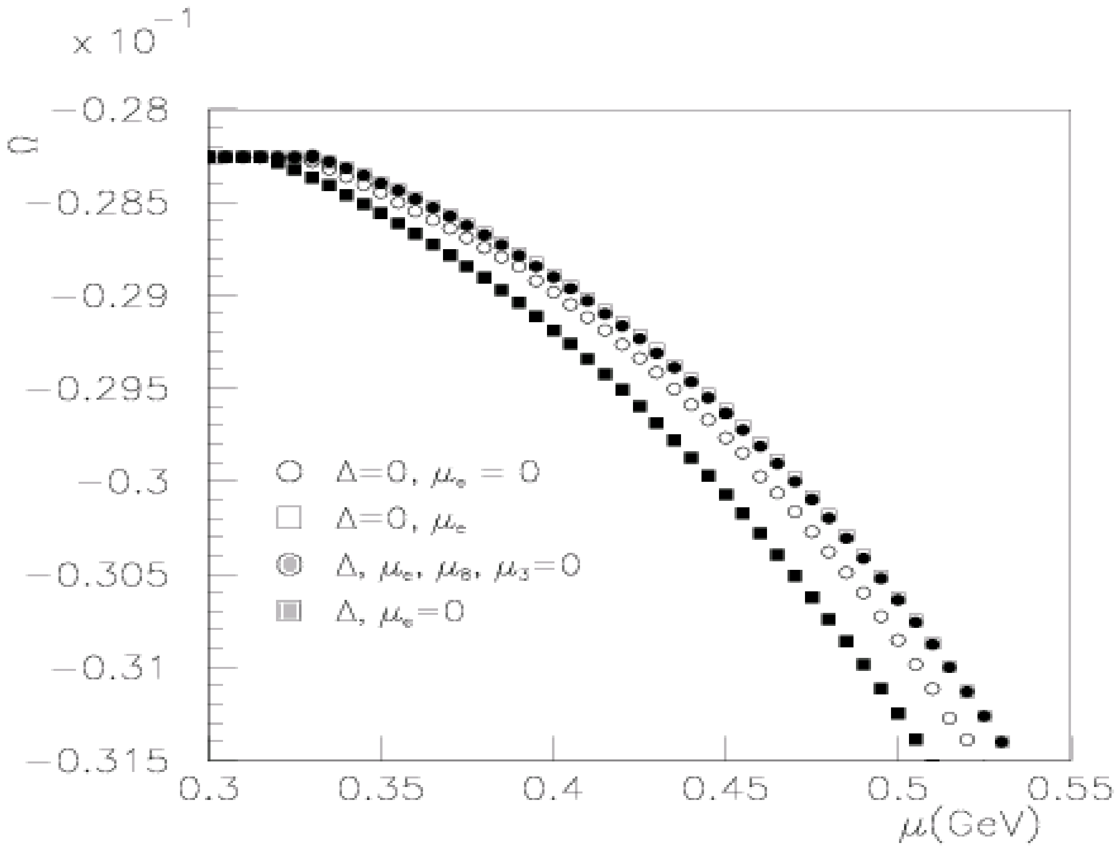
M.H., P.Zhuang, W.Chao PRD67:0650152,2003



$$\eta = \frac{G_D}{G_S} = \frac{3}{4}$$

Thermodynamic Potential

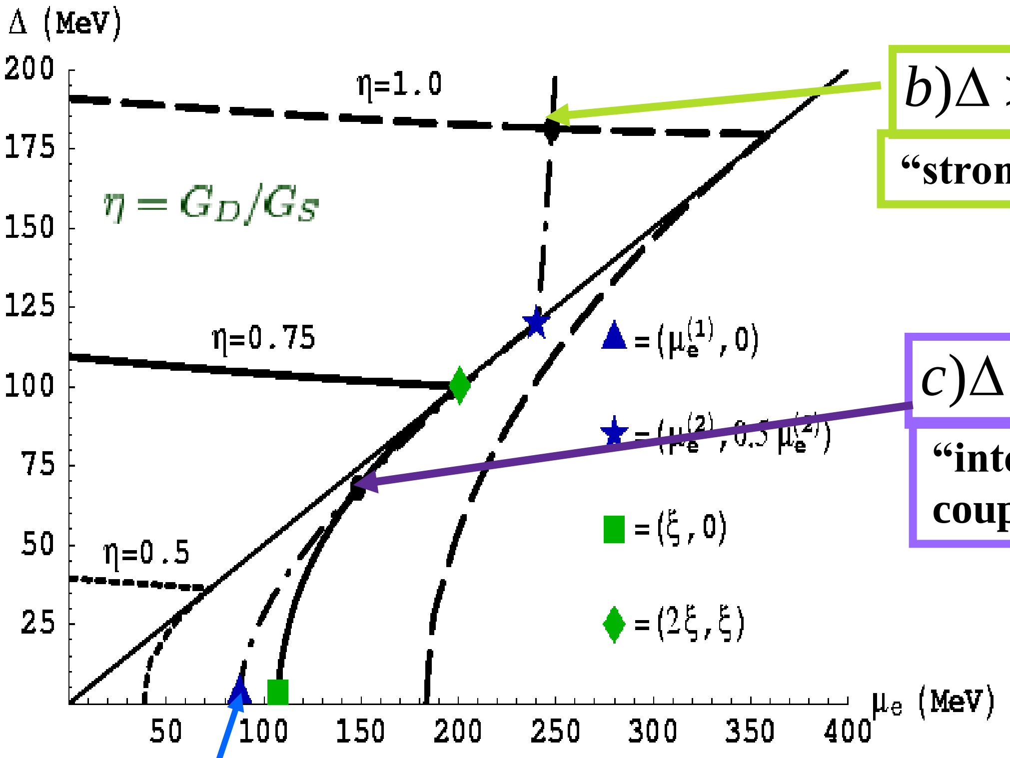
M.H., P.Zhuang, W.Chao PRD67:0650152,2003



$$\eta = \frac{G_D}{G_S} = \frac{3}{4}$$

Destroy Cooper pairing ?

The ground state: Balance of energy gain and loss



$b)\Delta > \delta\mu$ **2SC**

"strong" coupling

$c)\Delta < \delta\mu$ **g2SC**

"intermediate" coupling

$a)\Delta = 0$ **NM**

"weak" coupling

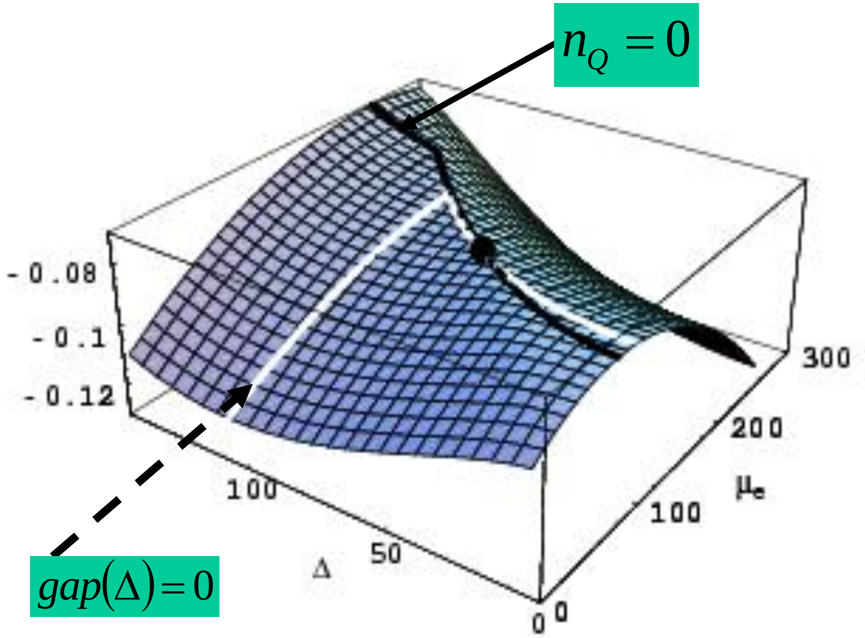
M.H, P.Zhuang, W.Chao, hep-ph/0207008

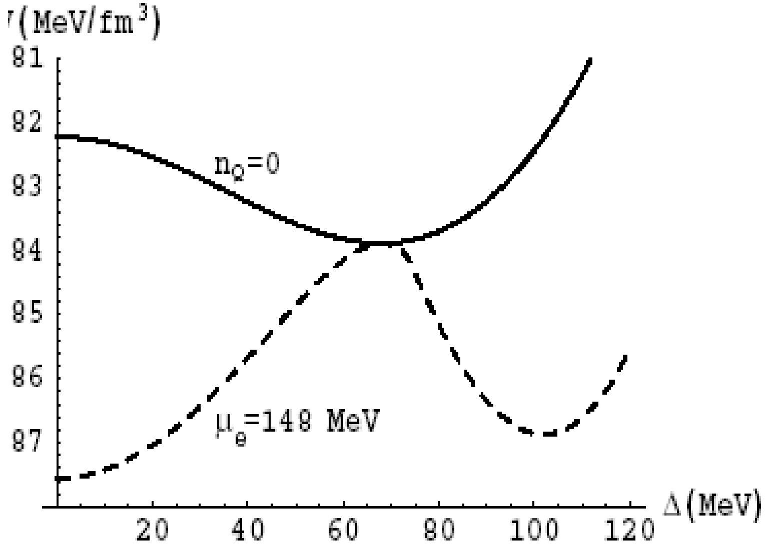
I. Shovkovy, M.H, Phys.Lett.B564:205,2003

Thermal Stability of g2SC phase

$$E_{\mathrm{Coulomb}} \sim n_Q^2 R^5$$

I. Shovkovy, M.H, Phys.Lett.B564:205,2003

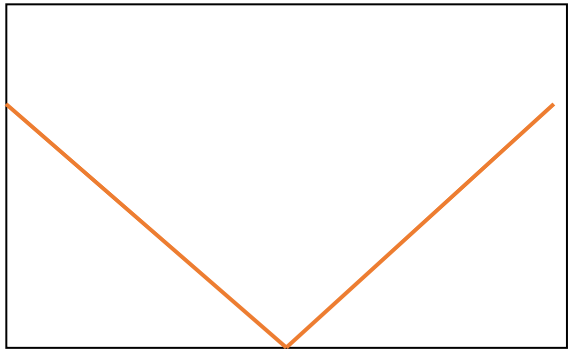




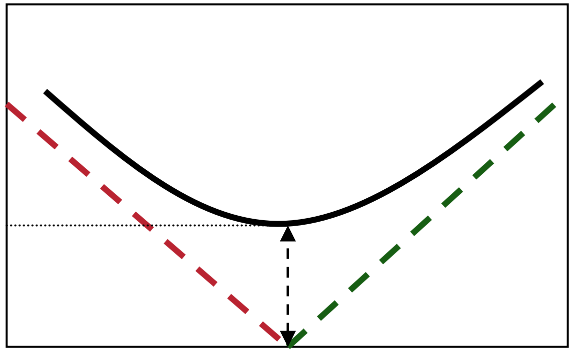
g2SC phase is a thermal stable state under the restriction of local neutrality condition !!

Remind: Properties of 2SC-I

Quasiparticle excitation



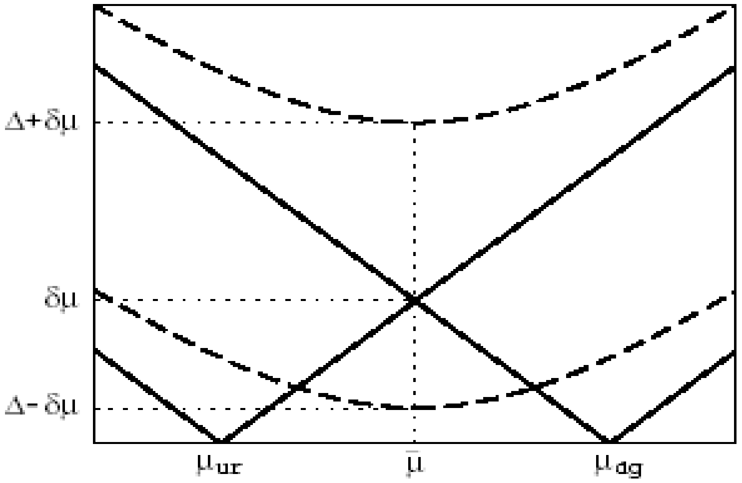
$$E_b^{\pm} = \pm|p-\mu|$$



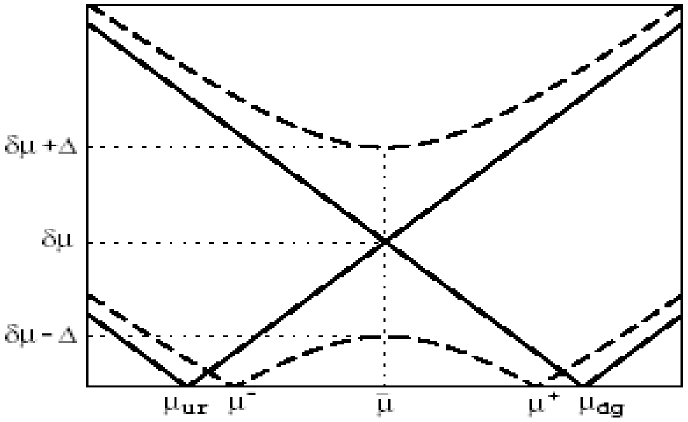
$$E_{\Delta}^{\pm} = \pm\sqrt{(p-\mu)^2 + \Delta^2}$$

gapless mode

$\delta\mu < \Delta$



$\delta\mu > \Delta$



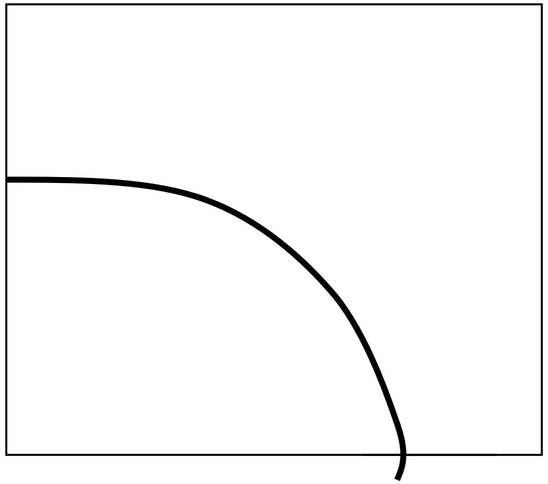
Gapless Mode !

g2SC at Finite Temperature

2SC BCS pairing:

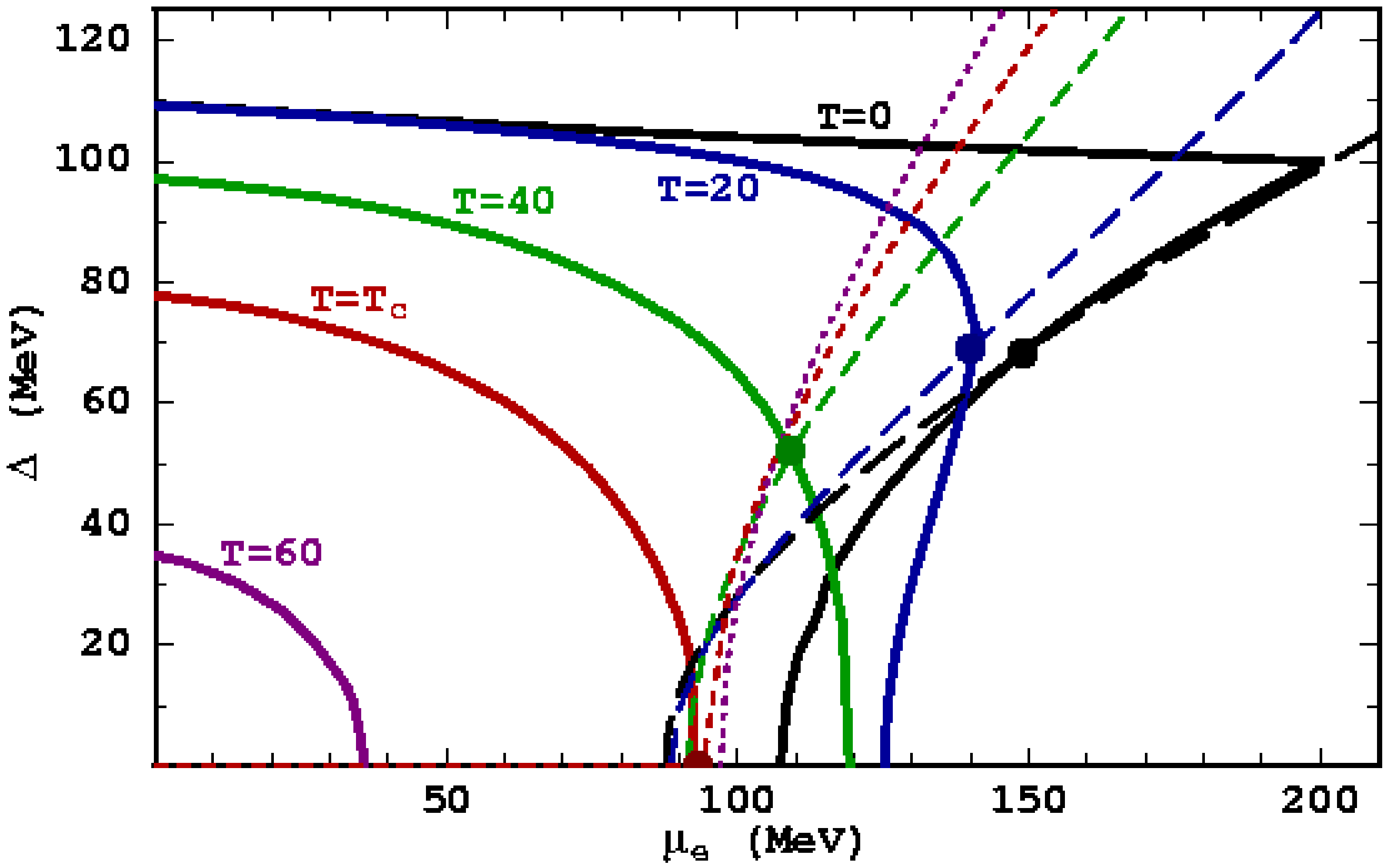
$$r_{BCS} = T_c / \Delta = 0.567$$

Finite temperature behavior



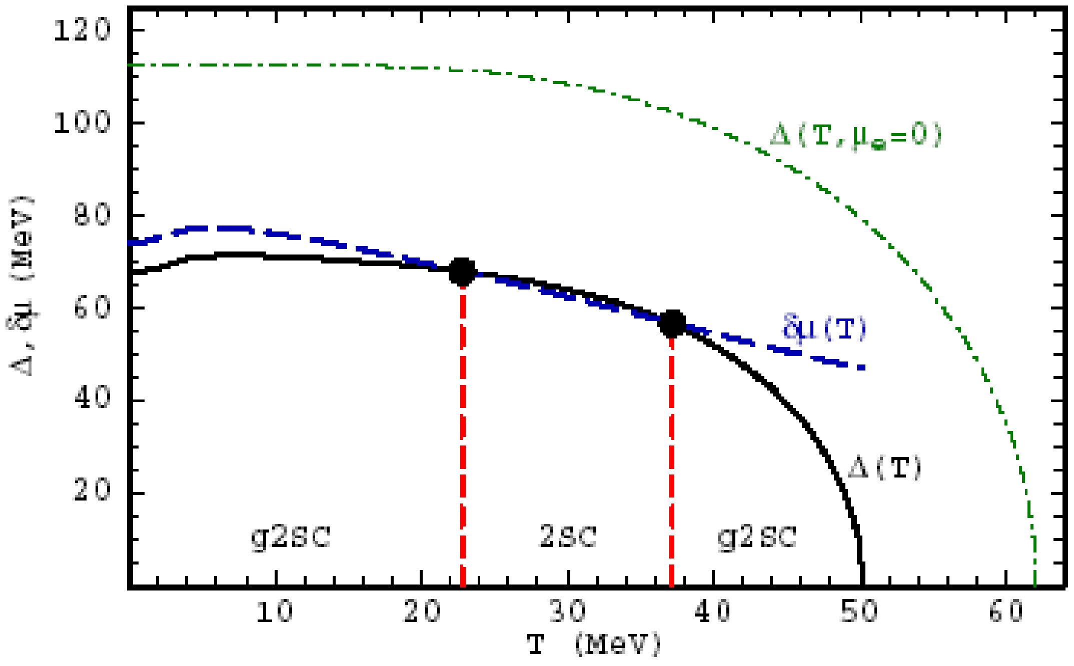
$$r_{\mathrm{BCS}} = \frac{T_c^{\mathrm{BCS}}}{\Delta_0^{\mathrm{BCS}}} = \frac{\mathrm{e}^{\gamma_E}}{\pi} \approx 0.567,$$

A universal value for all conventional BCS SCs



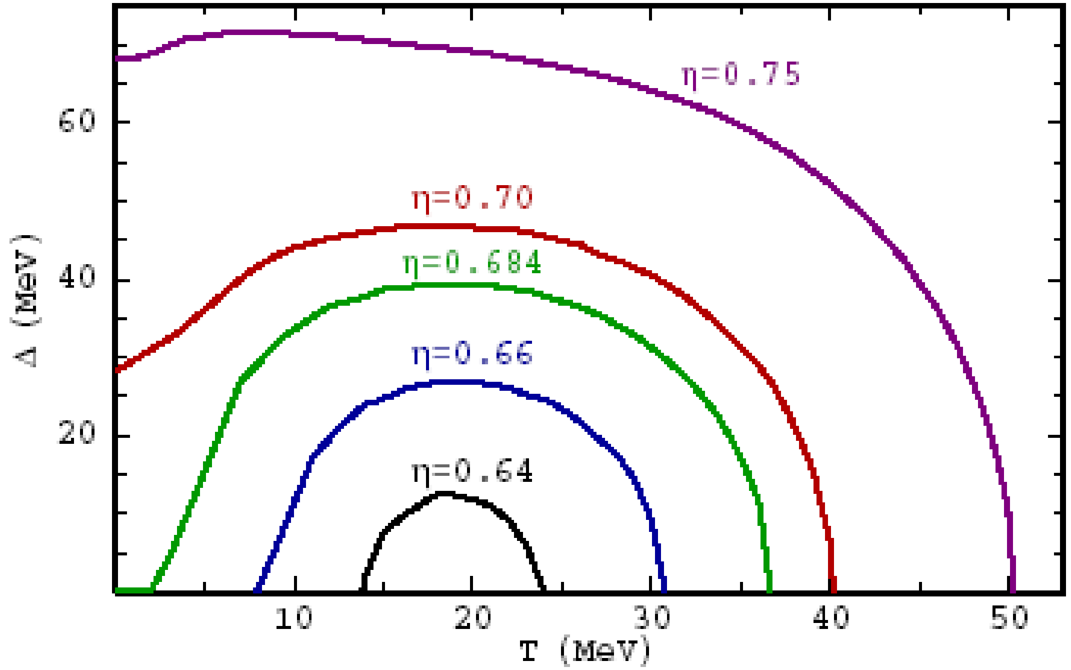
120
100
80
60
40
20
Δ (MeV)
50
100
150
200
μ_e (MeV)
T=0
T=20
T=40
T=T$_c$
T=60

Temperature dependence of the gap. I.



- *Nonmonotonic* temperature dependence
- Transitional behavior: g2SC $\rightarrow$ 2SC $\rightarrow$ g2SC $\rightarrow$ normal phase

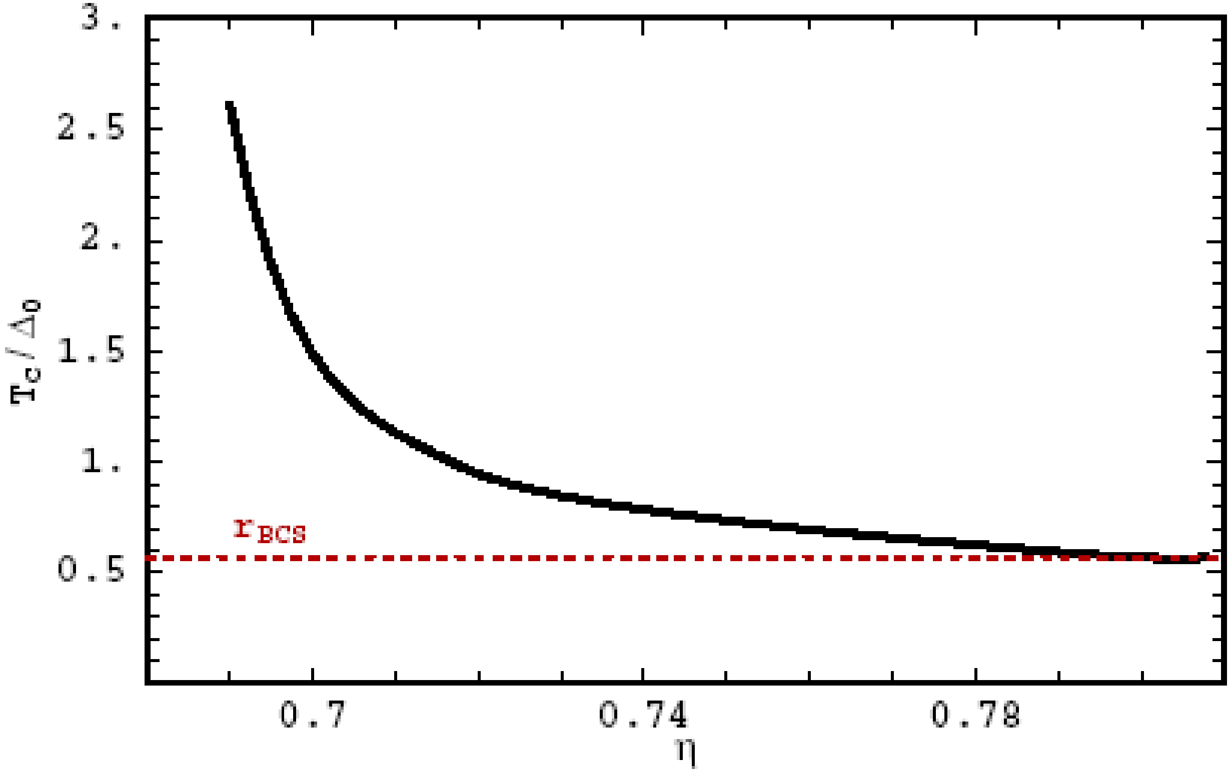
Temperature dependence of the gap. II.



- Extreme *nonmonotonic* temperature dependence
- Transitional behavior: **normal phase !** $\rightarrow$ g2SC $\rightarrow$ normal phase

also got by J. Liao and P. Zhuang independently

Nonuniversal ratio T_c/Δ_0



- The ratio is *not universal* (unlike in BCS), T_c/Δ_0 can be *arbitrarily* large for small η, and approaches r_{BCS} at large η!

Gapless mode in other systems

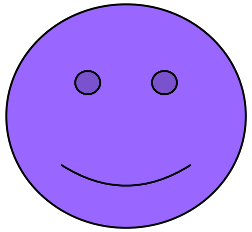
Cold atomic system — W. Liu, F. Wilczek 2002, 2004

Asymmetric Nuclear Matter — A. Sedrakian, U. Lombardo, 2000

u, s or d, s quark Matter — E. Gubankova, W. Liu, F. Wilczek 2003

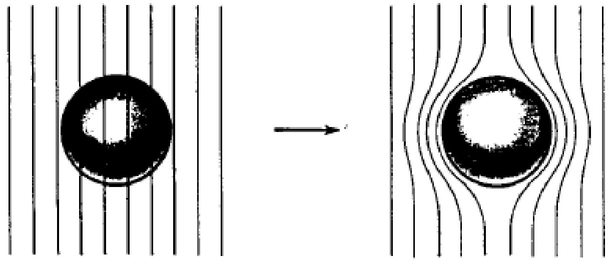
Charge neutral 3-flavor quark matter

M. Alford, C. Kouvaris, K. Rajagopal 2003, 2004
S. Ruster, I. Shovkovy, D. Rischke 2004



Meissner effect in g2SC ?

1933: Meissner & Ochsenfeld



Normal

S/C

Linear response theory

$$J_i^{ind}(\omega,\vec{q}) = \Pi_{ij}(\omega,\vec{q})A^j(\omega,\vec{q})$$

$$\Pi_{ij} = \Pi_{ij}^p + \Pi_{ij}^d$$

$\Pi_{ij}^p(0,\vec{0}) + \Pi_{ij}^d(0,\vec{0}) = 0$

$\Pi_{ij}^p(0,\vec{0}) < \Pi_{ij}^d(0,\vec{0})$

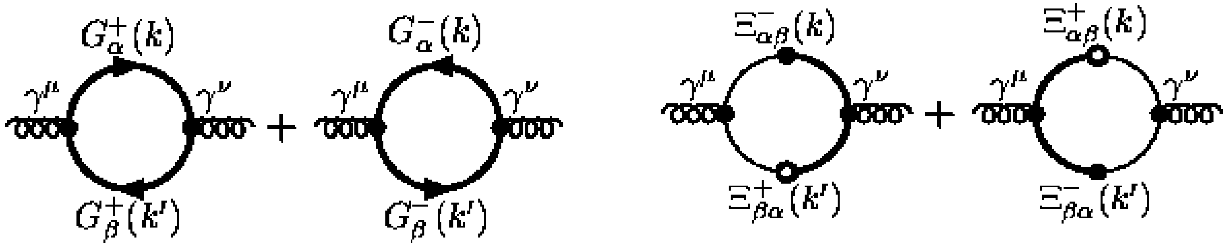
Perfect Diamagnet

Meissner effect in 2SC!

D. Rischke, PRD 62:034007, 2000

$$\Pi^{\mu\nu}_{ab}(P) = \frac{1}{2} g_1 g_2 \frac{T}{V} \sum_K \mathrm{Tr}_{s,c,f,NG} \left[\hat{\Gamma}^\mu_a \, \mathcal{S}(K) \, \hat{\Gamma}^\nu_b \, \mathcal{S}(K-P) \right]$$

$$\mathcal{S}(K) = \begin{pmatrix} G^+(K) & \Xi^-(K) \\ \Xi^+(K) & G^-(K) \end{pmatrix} \qquad \hat{\Gamma}^\mu_a \equiv \begin{pmatrix} \Gamma^\mu_a & 0 \\ 0 & \bar{\Gamma}^\mu_a \end{pmatrix}$$



$$M^2_{M,a} = \Pi^{ij}_{aa}(0,\vec{0})\delta_{ij}$$

Remind: Properties of 2SC-III

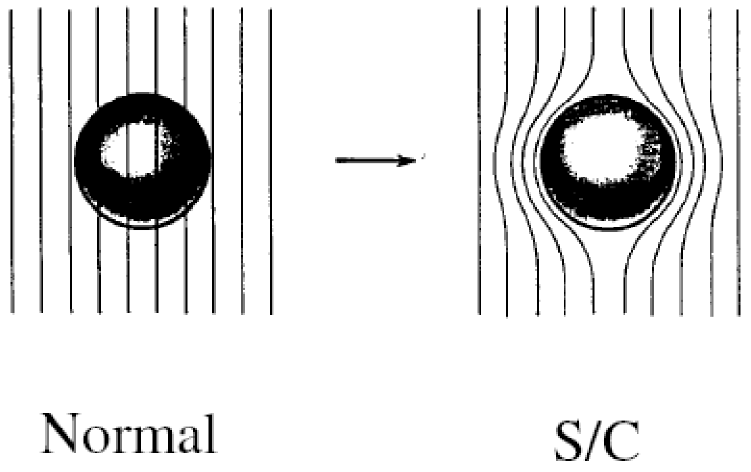
Meissner effect

$$SU(3)_c \rightarrow SU(2)_c$$

a=1,2,3	massless
a=4,5,6,7	massive
a=8	massive

Rischke, PRD62:034007,2000

Anderson-Higgs Mechanism



1933: Meissner & Ochsenfeld

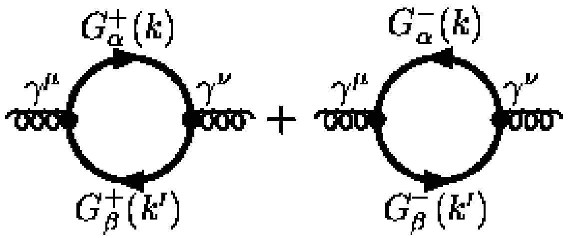
Meissner effect in g2SC?

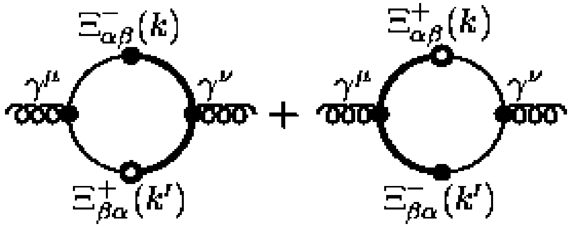
Including mismatch $\delta\mu$

M. H, I. Shovkovy, Phys. Rev. D70 (2004), 051501
M. H, I. Shovkovy, Phys. Rev. D70 (2004) 094030

$$\Pi^{\mu\nu}_{ab}(P) = \frac{1}{2} g_1 g_2 \frac{T}{V} \sum_K \mathrm{Tr}_{s,c,f,NG} \left[\hat{\Gamma}^{\mu}_{a} \mathcal{S}(K) \hat{\Gamma}^{\nu}_{b} \mathcal{S}(K-P) \right]$$

$$\mathcal{S}(K) = \begin{pmatrix} G^{+}(K) & \Xi^{-}(K) \\ \Xi^{+}(K) & G^{-}(K) \end{pmatrix} \qquad \hat{\Gamma}^{\mu}_{a} \equiv \begin{pmatrix} \Gamma^{\mu}_{a} & 0 \\ 0 & \bar{\Gamma}^{\mu}_{a} \end{pmatrix}$$





$$M^2_{M,a} = \Pi^{ij}_{aa}(0,\vec{0})\delta_{ij}$$

Propagator in explicit form

$$G_u^{\pm} = \frac{p_0 - E_d^{\pm}}{(p_0 \mp \delta\mu)^2 - E_\Delta^{\pm 2}}\gamma^0\tilde{\Lambda}_+ + \frac{p_0 + E_d^{\mp}}{(p_0 \mp \delta\mu)^2 - E_\Delta^{\mp 2}}\gamma^0\tilde{\Lambda}_-$$

$$G_d^{\pm} = \frac{p_0 - E_u^{\pm}}{(p_0 \pm \delta\mu)^2 - E_\Delta^{\pm 2}}\gamma^0\tilde{\Lambda}_+ + \frac{p_0 + E_u^{\mp}}{(p_0 \pm \delta\mu)^2 - E_\Delta^{\mp 2}}\gamma^0\tilde{\Lambda}_-$$

$$G_{bu}^{\pm} = \frac{1}{p_0 + E_{bu}^{\pm}}\gamma^0\tilde{\Lambda}_+ + \frac{1}{p_0 - E_{bu}^{\mp}}\gamma^0\tilde{\Lambda}_-$$

$$G_{bd}^{\pm} = \frac{1}{p_0 + E_{bd}^{\pm}}\gamma^0\tilde{\Lambda}_+ + \frac{1}{p_0 - E_{bd}^{\mp}}\gamma^0\tilde{\Lambda}_-$$

$$\Lambda_p^{\pm} = \frac{1}{2}\left(1 \pm \gamma^0 \frac{\vec{\gamma}\cdot\vec{p} + m}{E(p)}\right);$$

$$\tilde{\Lambda}_p^{\pm} = \frac{1}{2}\left(1 \pm \gamma^0 \frac{\vec{\gamma}\cdot\vec{p} - m}{E(p)}\right).$$

$$\Xi_{ud}^{+} = -i\Delta^*\left[\frac{1}{(p_0 + \delta\mu)^2 - E_\Delta^{+2}}\gamma^5\tilde{\Lambda}_+ + \frac{1}{(p_0 + \delta\mu)^2 - E_\Delta^{-2}}\gamma^5\tilde{\Lambda}_-\right]$$

$$\Xi_{du}^{+} = -i\Delta^*\left[\frac{1}{(p_0 - \delta\mu)^2 - E_\Delta^{+2}}\gamma^5\tilde{\Lambda}_+ + \frac{1}{(p_0 - \delta\mu)^2 - E_\Delta^{-2}}\gamma^5\tilde{\Lambda}_-\right]$$

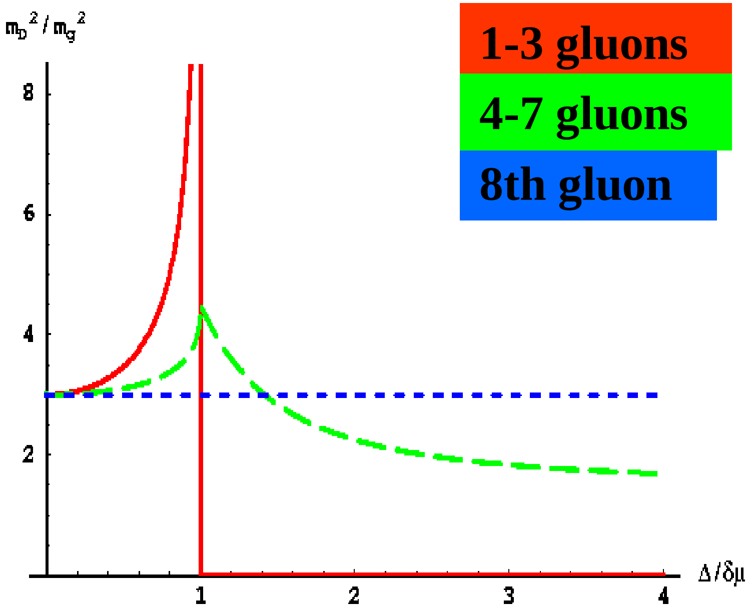
$$\Xi_{ud}^{-} = -i\Delta\left[\frac{1}{(p_0 - \delta\mu)^2 - E_\Delta^{-2}}\gamma^5\tilde{\Lambda}_+ + \frac{1}{(p_0 - \delta\mu)^2 - E_\Delta^{+2}}\gamma^5\tilde{\Lambda}_-\right]$$

$$\Xi_{du}^{-} = -i\Delta\left[\frac{1}{(p_0 + \delta\mu)^2 - E_\Delta^{-2}}\gamma^5\tilde{\Lambda}_+ + \frac{1}{(p_0 + \delta\mu)^2 - E_\Delta^{+2}}\gamma^5\tilde{\Lambda}_-\right]$$

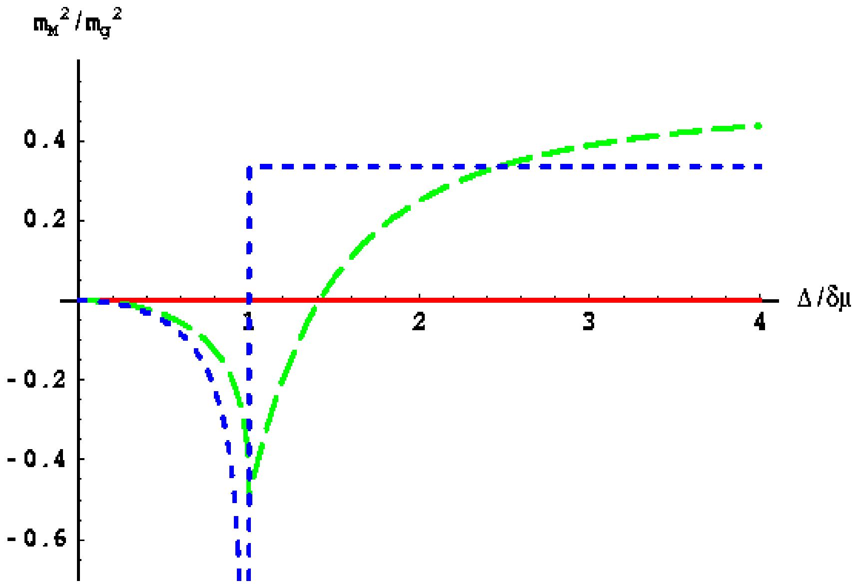
M. H, I. Shovkovy, Phys. Rev. D70 (2004), 051501
M. H, I. Shovkovy, Phys. Rev. D70 (2004) 094030

Paramagnetic -Meissner effect in g2SC!

$$M_D^2 = -\Pi^{00}(0,\vec{0})$$



$$M_M^2 = \Pi^{ij}(0,\vec{0})\delta_{ij}$$



Meissner mass square for the gluons of color 8 and 4-7 are negative!!!

Anti-Meissner effect / chromomagnetic instability in g2SC $SU(3)_c \to SU(2)_c$

1, Ideal 2SC

a=1,2,3 massless
a=4,5,6,7 massive
a=8 massive

2, Normal phase

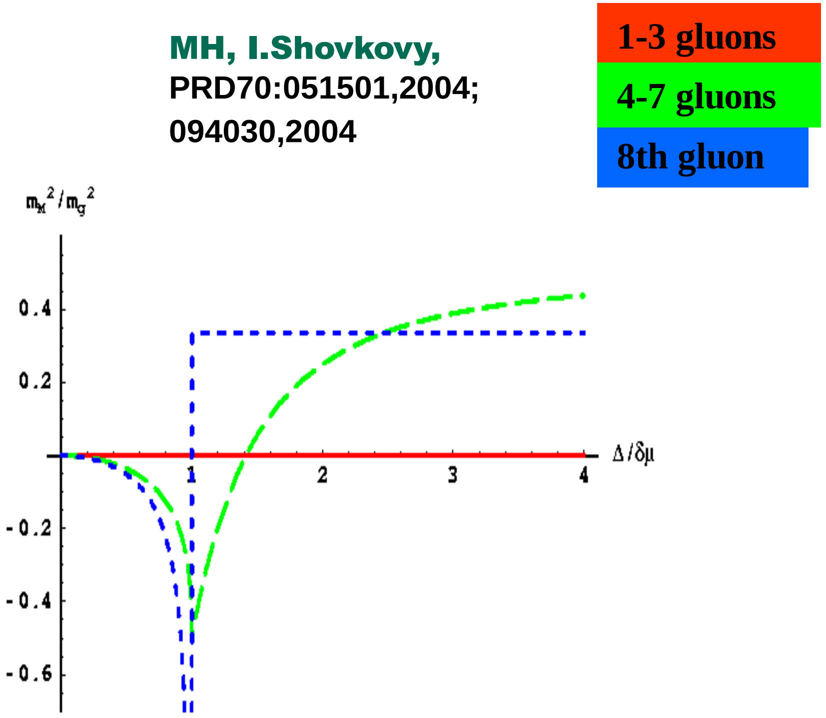
a=1,...,8 massless

3, 2SC with mismatch

$1 < \Delta/\delta\mu < \sqrt{2}$

a=1,2,3 massless
a=4,5,6,7 negative
a=8 positive

MH, I.Shovkovy,
PRD70:051501,2004;
094030,2004

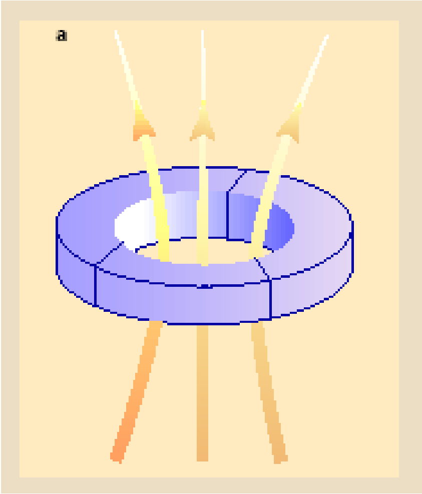


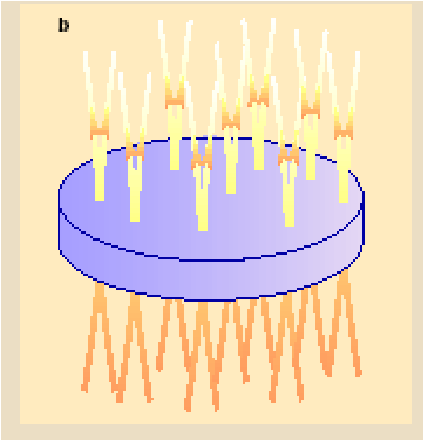
4, g2SC

a=1,2,3 massless
a=4,5,6,7 negative
a=8 negative

Paramagnetic Meissner Effect!

$$\Pi_{ij}^{p}(0,\vec{0}) > \Pi_{ij}^{d}(0,\vec{0})$$



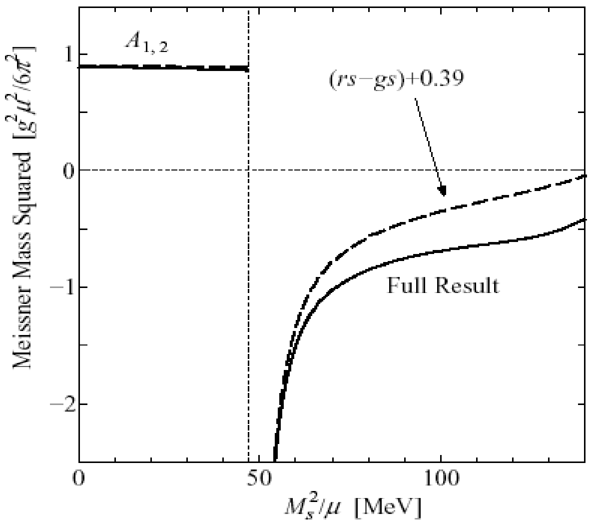


Mesoscopic superconductors: a) A loop made of three crystals of HTSC; b) Aluminium disc

Nature **396, 144 - 146 (1998); doi:10.1038/24110**

(Chromo)Magnetic instability in other gapless phases

gCFL



Casalbuoni, et.al., PLB605:362-368,2005

Alford, Wang, J.Phys.G31:719-738,2005

K. Fukushima, hep-ph/0506080

BP: superfluid density is negative

Wu, Yip, PRA67: 053603, 2003

Is this a universal property for gapless phase?

Chromo-magnetic instability in the gCFL phase!

Casalbuoni, Gatto, Mannarelli, Nardulli, Ruggieri, hep-ph/0410401

M.Alford, Qinghai Wang, hep-ph/0501078

Superfluid density instability in the gapless atomic system!

Y.Wu, S.Yip, Phys.Rev.A67(2003) **053603**

What's going wrong in gapless phases ?

Debye and Meissner masses for gluons with color 1,2,3

	$m^2_{D,1}(g^2\bar{\mu}^2)/\pi^2$	P-A, G	P-A, Ξ	P-H, G	P-H, Ξ
NQM	1	0	0	1	0
2SC	0	0	0	$1/2$	$-1/2$
g2SC	$\frac{\delta\mu}{\sqrt{\delta\mu^2-\Delta^2}}$	0	0	$\frac{1}{2}+\frac{1}{2}\frac{\delta\mu}{\sqrt{\delta\mu^2-\Delta^2}}$	$-\frac{1}{2}+\frac{1}{2}\frac{\delta\mu}{\sqrt{\delta\mu^2-\Delta^2}}$

0 → 1   0   opposite   same

	$m^2_{M,1}(g^2\bar{\mu}^2/\pi^2)$	P-A, G	P-A, Ξ	P-H, G	P-H, Ξ
NQM	0	$+1$	0	-1	0
2SC	0	$+1$	0	$-1/2$	$-1/2$
g2SC	0	$+1$	0	$-\frac{1}{2}-\frac{1}{2}\frac{\delta\mu}{\sqrt{\delta\mu^2-\Delta^2}}$	$-\frac{1}{2}+\frac{1}{2}\frac{\delta\mu}{\sqrt{\delta\mu^2-\Delta^2}}$

Right limit at NQM and 2SC

Debye and Meissner masses for the 8th gluon

	$\tilde{m}^2_{D,8}(g^2\bar{\mu}^2/\pi^2)$	P-A, G	P-A, Ξ	P-H, G	P-H, Ξ
NQM	1	0	0	1	0
2SC	1	0	0	1/2	1/2
g2SC	1	0	0	$\frac{1}{2}+\frac{1}{2}\frac{\delta\mu}{\sqrt{\delta\mu^2-\Delta^2}}$	$\frac{1}{2}-\frac{1}{2}\frac{\delta\mu}{\sqrt{\delta\mu^2-\Delta^2}}$

0 → 1 (P-A, G); 0 (P-A, Ξ); opposite (P-H, G); same (P-H, Ξ)

	$m^2_{M,8}(g^2\bar{\mu}^2/9\pi^2)$	P-A, G	P-A, Ξ	P-H, G	P-H, Ξ
NQM	0	+1	0	−1	0
2SC	+1	+1	0	−1/2	+1/2
g2SC	$1-\frac{\delta\mu}{\sqrt{\delta\mu^2-\Delta^2}}$	+1	0	$-\frac{1}{2}-\frac{1}{2}\frac{\delta\mu}{\sqrt{\delta\mu^2-\Delta^2}}$	$\frac{1}{2}-\frac{1}{2}\frac{\delta\mu}{\sqrt{\delta\mu^2-\Delta^2}}$

Right limit at NQM and 2SC

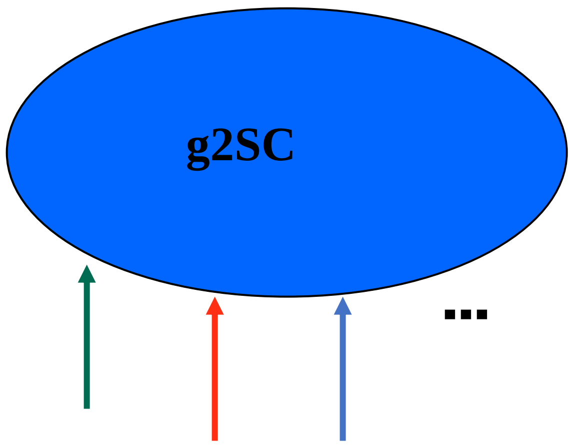
III. 3. Understanding and resolving instabilities

III.3.1 Probing g2SC phase using other external sources

Color neutral current instability

M. H., hep-ph/0504235, to appear in PRD

What's going wrong in g2SC phase ?
---- Are there any other instabilities?



different external sources

g2SC phase is not stable with respect to a baryon current !!!

$$\bar{\psi}\vec{\gamma}\psi$$

Spontaneous baryon current generation

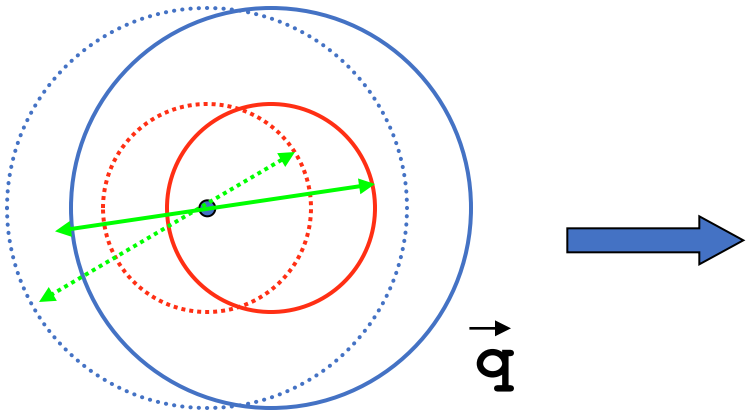
$$[\mathcal{S}(P)]^{-1} = \begin{pmatrix} [G_0^+(P)]^{-1} & \Delta^- \\ \Delta^+ & [G_0^-(P)]^{-1} \end{pmatrix}$$

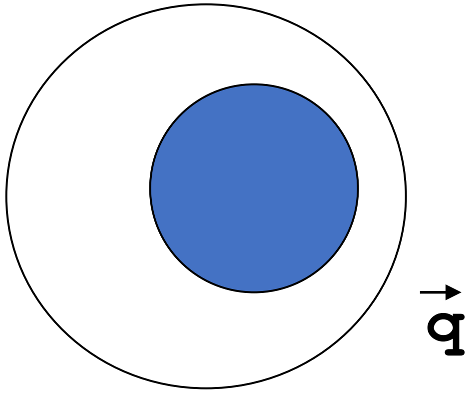
$$[G_0^\pm(P)]^{-1} = \gamma^0(p_0 \pm \hat{\mu}) - \vec{\gamma}\cdot\vec{p},$$

$$[\mathcal{S}_V(P)]^{-1} = \begin{pmatrix} \left[G_{0,V}^+(P)\right]^{-1} & \Delta^- \\ \Delta^+ & \left[G_{0,V}^-(P)\right]^{-1} \end{pmatrix}$$

$$\left[G_{0,V}^\pm(P)\right]^{-1} = \gamma^0(p_0 \pm \hat{\mu}) - \vec{\gamma}\cdot\vec{p} \mp \vec{\gamma}\cdot\vec{\Sigma}_V\,,$$

Baryon current generation state resmebles LOFF state





Rotation symmetry broken

FF state ?

$\Delta(r) = |\Delta_q| e^{(iqr)}$ (FF)

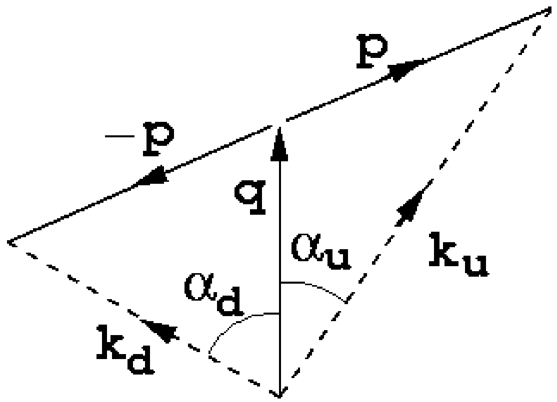
P. Fulde and A. Ferrell Phys. Rev. 135, A550 (1964).

$$\chi(\vec{r}) = e^{-i\vec{q}\cdot\vec{r}}\psi(\vec{r}).$$

Baryon current generation

I. Giannakis, H. Ren, hep-ph/0412015, hep-ph/0504053

LOFF state



(Larkin Ovchinnikov Fulde Ferrell)

LOFF in CSC, J. Bowers Ph.D thesis
hep-ph/0305301

Chromomagnetic instability in g2SC implies LOFF state is favored

I. Giannakis, H. Ren, hep-ph/0412015

No chromomagnetic instability in LOFF

I. Giannakis, H. Ren, hep-ph/0504053

III. 3. Understanding and resolving instabilities

III.3.2 Understanding the origin of instabilities

III.3.2 Understanding the origin of instability

Proposals for resolving magnetic instability

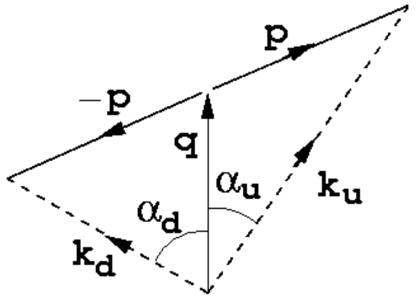
Proposal 1) **Single-plane wave LOFF**

$$< \bar{\psi}(\vec{r})\gamma_5\lambda_2\tau_2\psi_C(\vec{r}) >= \Phi e^{2i\vec{q}\cdot\vec{r}}$$ **(2SC)**

Giannakis, Ren, PLB611:137-146,2005; NPB723:255-280,2005

$$< \psi_{i\alpha} C \gamma_5 \psi_{\beta j} >= \sum_{I=1}^{3} \Delta_I(\mathbf{r})\, \epsilon^{\alpha\beta I}\, \epsilon_{ijI}$$ **(CFL)**

Casalbuoni, Gatto, Ippolito, Nardulli, Ruggieri, hep-ph/0507247



A "bad/good" news from Gorbar, Hashimoto, Miransky hep-ph/0509334:

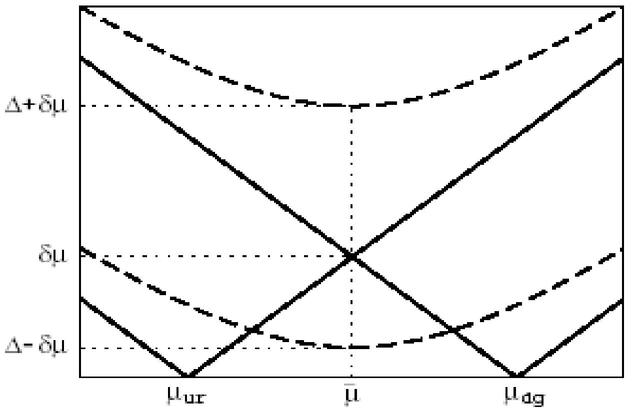
Charge neutral single-plane wave LOFF state cannot solve the instability related to the 4-7 gluons.

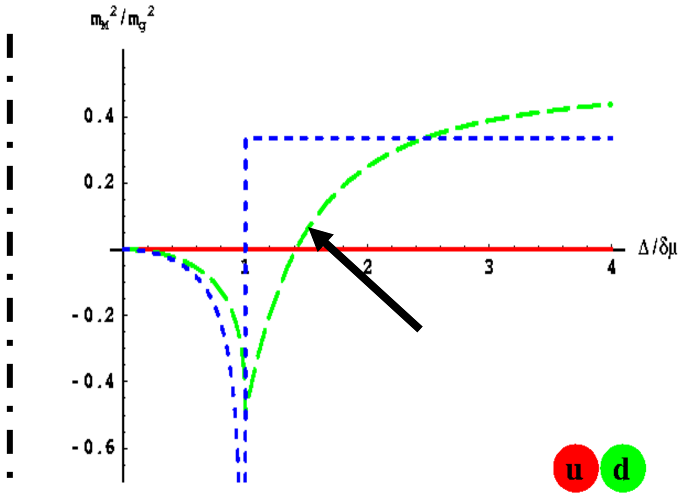
Other Proposals

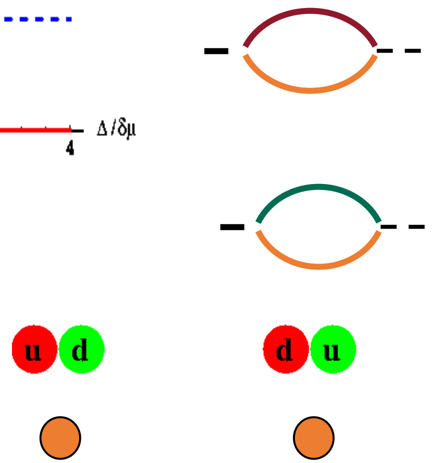
1. **Baryon current(?):** MH, hep-ph/0504235
2. **Goldstone current:** Hong, hep-ph/0506097; Kryjevski, hep-ph/0508180; Schaefer, hep-ph/0508190
3. **Gluon condensate:** Gorbar, Hashimoto, Miransky, hep-ph/0507303
4. **Multi-plane wave LOFF** Rajagopal, et.al
5. ......

What's really happening?

Low energy DOF changes with increase of mismatch!







BCS system: 1) broken gauge bosons are heavy and decouple from the system, 2) quasiparticles are difficult to be excited;

Mismatch increases: 1)broken gauge bosons' mass decreases and becomes low energy DOF, 2) one branch of quasi-particles becomes much easier to be excited.

Possible ground states:

1. LOFF (Larkin Ovchinnikov Fulde Ferrell)

$\Delta(r) = |\Delta_q| e^{(iqr)}$ (FF)

P. Fulde and A. Ferrell Phys. Rev. 135, A550 (1964).

$\Delta(r) = |\Delta_q| \cos(qr)$ (LO)

A.I. Larkin and Yu.N. Ovchinnikov Zh. Eksp. Teor. Fiz. 47, 1136 (1964).

also:

multi-plane wave

2. Phase separation

M.Alford, K.Rajagopal, S.Reddy, F.Wilczek,PRD64(2001), 074017;
F. Neumann, M. Buballa, M. Oertel, NPA 714, 2003;
I. Shovkovy, M. Hanauske, M.H, PRD67:103004,2003;
S. Reddy and G. Rupak, nucl-th/0405054, ……

Imbalanced Cold Atom System

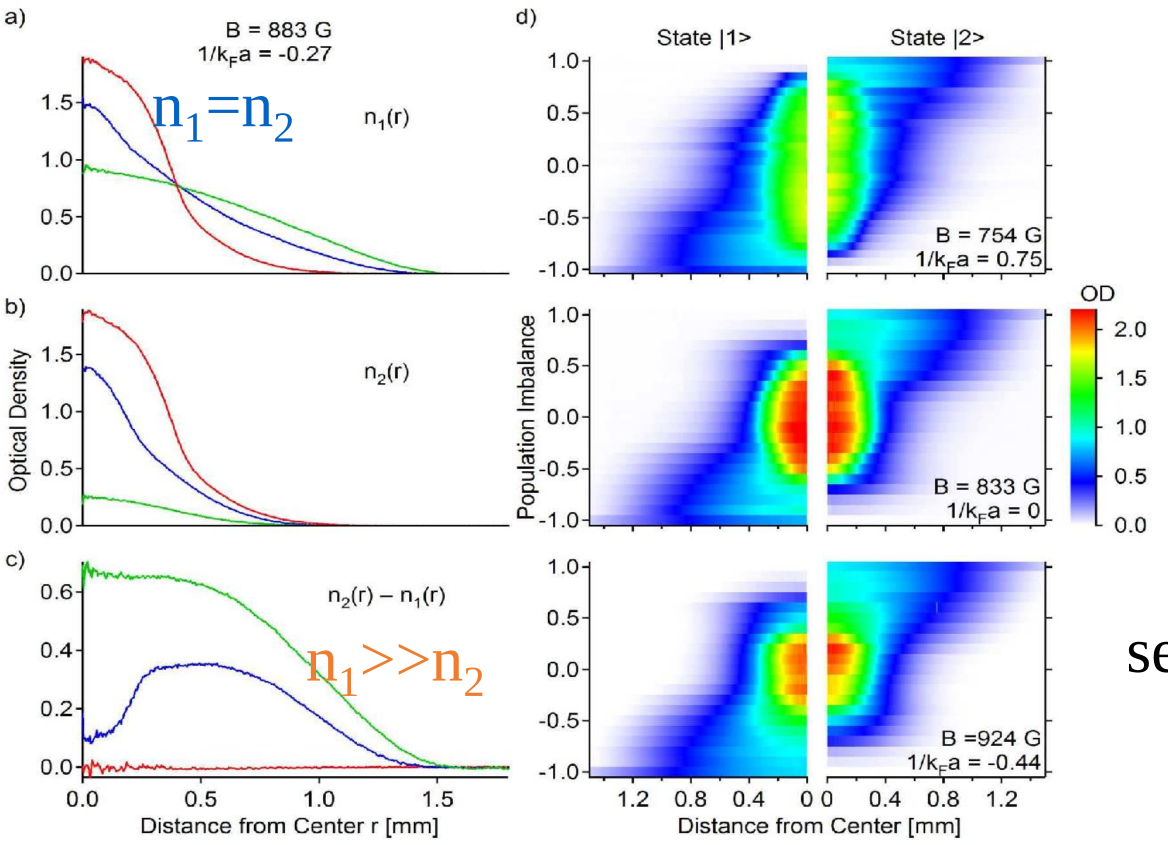
BP (gapless phase in cold atom system)

Liu, Wilczek 2003

Superfluid density is negative

Wu, Yip, PRA67: 053603, 2003

$\delta = 0\%$ (red), $\delta = 46\%$ (blue) and $\delta = 86\%$ (green)



phase separation

Zwierlein, Schirotzek, Schunck, & Ketterle, Science 2005, cond-mat/0511197
Partridge, Li, Kamar, Liao, & Hulet, Science 2005, cond-mat/0511752.

III. A new framework for mismatched systems

III. Nonlinear realization framework (beyond MF)

III.1. Instability of NG bosons & (LO)FF-like state

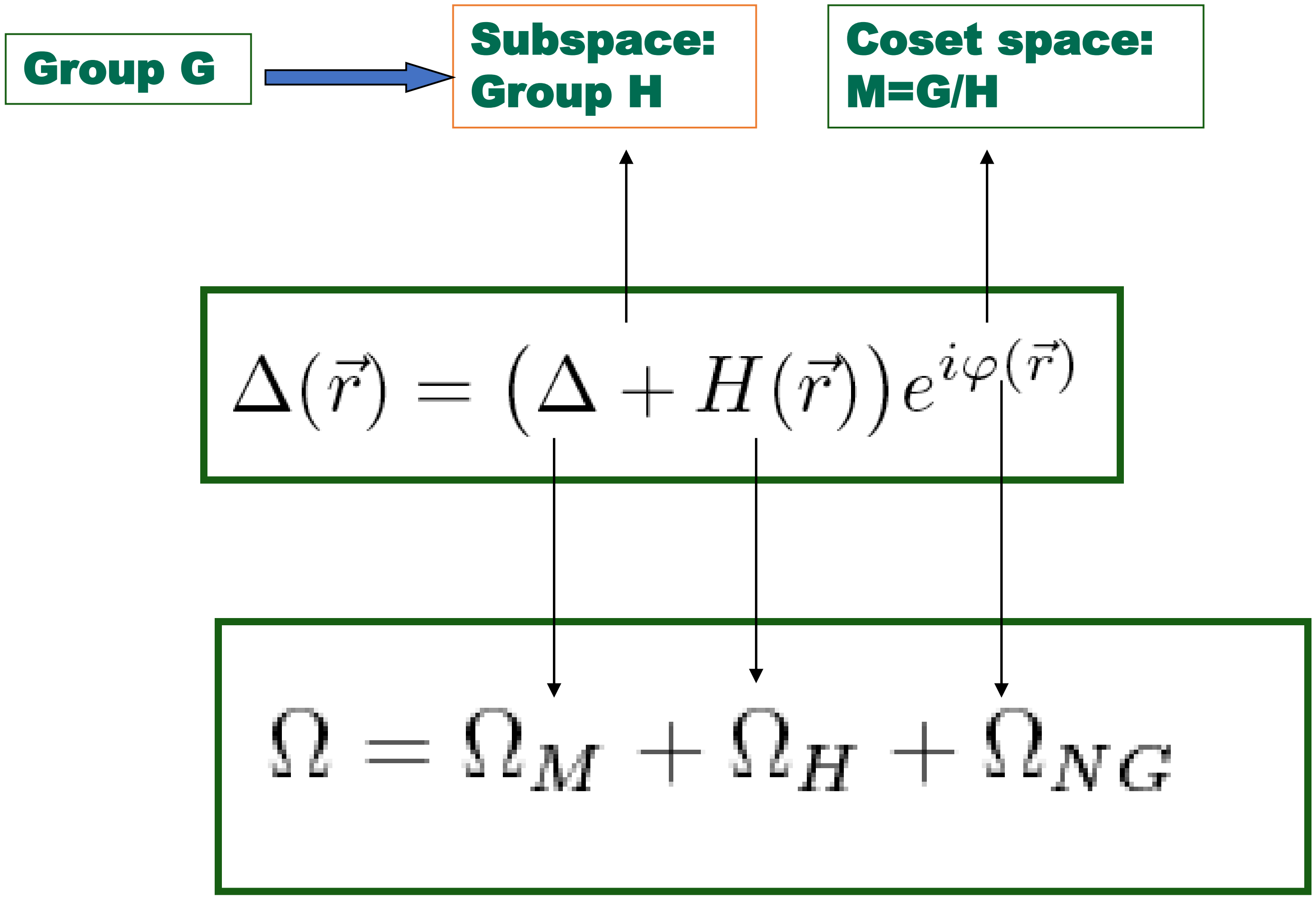
III.2. Higgs instability & spatial inhomogeneity

Based on the work:

M.H. PRD73:045007, 2006; Int.J.Mod.Phys.A21, 910, (2006)

I. Giannakis, D.F.Hou, M.H., H.C.Ren,

PRD75:011501,2007, PRD75:014015,2007


Group G
Subspace: Group H
Coset space: M=G/H
$\Delta(\vec{r}) = (\Delta + H(\vec{r}))e^{i\varphi(\vec{r})}$
$\Omega = \Omega_M + \Omega_H + \Omega_{NG}$

III.1. Instability of NG bosons & (LO)FF-like state

U(1): Hong, hep-ph/0506097 **SU(3)->SU(2): M. H.** PRD73:045007, 2006

2SC phase:

$$\begin{pmatrix} \Delta^1(\vec{r}) \\ \Delta^2(\vec{r}) \\ \Delta^3(\vec{r}) \end{pmatrix} = \exp\left[i \sum_{a=4}^{8} \varphi_a(\vec{r}) T_a \right] \begin{pmatrix} 0 \\ 0 \\ \Delta + H(\vec{r}) \end{pmatrix}$$

nonlinear realization framework

new quark field:

$$q = \mathcal{V}\chi \quad , \quad \bar{q} = \bar{\chi}\,\mathcal{V}^\dagger$$

$$X \equiv \begin{pmatrix} \chi \\ \chi_C \end{pmatrix} \quad , \quad \bar{X} \equiv (\bar{\chi}\,,\,\bar{\chi}_C)$$

$$\mathcal{L}_{nl} \equiv \bar{X}\,\mathcal{S}_{nl}^{-1}\,X - \frac{\Phi^+\Phi^-}{4G_D}$$

$$\mathcal{S}_{nl}^{-1} \equiv \begin{pmatrix} [G_{0,nl}^+]^{-1} & \Phi^- \\ \Phi^+ & [G_{0,nl}^-]^{-1} \end{pmatrix}$$

$$[G_{0,nl}^+]^{-1} = i\,\not{D} + \hat{\mu}\,\gamma_0 + \gamma_\mu\,V^\mu,$$

$$V^\mu \simeq -\sum_{a=4}^{8} (\partial^\mu \varphi_a)\,T_a$$

Shovkovy, Rischke, Phys.Rev.D66:054019,2002
M.H. PRD73:045007

NG sector

$$\Omega_{NG} = \frac{1}{2}\int d^3\vec{r}\sum_{a=1}^{8} m_a^2(\vec{\mathbf{A}}^a - \frac{1}{g}\vec{\nabla}\varphi^a)(\vec{\mathbf{A}}^a - \frac{1}{g}\vec{\nabla}\varphi^a) + higher\,orders$$

NG currents & (LO)FF-like state

$$(m_a)^2 < 0, \quad a = 4, 5, 6, 7 \qquad \sum_{a=4}^{7} < \vec{\mathbf{A}}^a - \frac{1}{g}\vec{\nabla}\varphi^a > \neq 0$$

Gluon phase, Gorbar, Hashimoto, Miransky, hep-ph/0507303

$$(m_8)^2 < 0 \qquad < \vec{\mathbf{A}}^8 - \frac{1}{g}\vec{\nabla}\varphi^8 > \neq 0.$$

U(1) (LO)FF-state, Giannakis, Ren, hep-ph/0412015

For the ground state, multi-plane wave might be more favorable

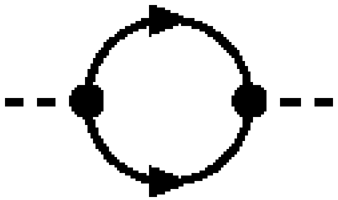
III.2. Higgs instability & spatial inhomogeneity

I. Giannakis, D.F.Hou, M.H., H.C.Ren, hep-ph/0606178; hep-ph/0609098

Higgs sector

$$\Omega_M = -\frac{T}{2}\sum_n \int \frac{d^3\vec{p}}{(2\pi)^3} \mathrm{Tr}\ln([\mathcal{S}_M(P)]^{-1}) + \frac{\Delta^2}{4G_D}$$

$$\Omega_H = \frac{T}{2}\sum_{k_0} \int \frac{d^3\vec{k}}{(2\pi)^3} H^*(\vec{k})\Pi_H(k)H(\vec{k}).$$

$\Pi_H =$ 

inhomogeneous field

$$\delta\mathcal{F} = \frac{1}{2}\left(\frac{\partial^2\mathcal{F}}{\partial\Delta^2}\right)_n \delta\Delta^2 + \frac{1}{2}\sum_{\vec{k}\neq 0}\left(\frac{\partial^2\mathcal{F}}{\partial\Delta^*_{\vec{k}}\partial\Delta_{\vec{k}}}\right)_n \delta\Delta^*_{\vec{k}}\delta\Delta_{\vec{k}}$$

$$\Pi(k) = A_H + B_H k^2$$

for $k << \Delta$

In gapless region

$$A_H = \left(\frac{\partial^2\Omega}{\partial\Delta^2}\right)_{\mu_i} = \frac{4\bar{\mu}^2}{\pi^2}\left[1 - \frac{\delta\mu}{\sqrt{(\delta\mu)^2 - \Delta^2}}\right]$$

Sarma Instability

$$B_H = \frac{2\bar{\mu}^2}{9\pi^2\Delta^2}\left[1 - \frac{(\delta\mu)^3}{((\delta\mu)^2 - \Delta^2)^{\frac{3}{2}}}\right]$$

Higgs Instability:
induce spatial inhomogeneity

Inhomogeneous Higgs field induces inhomogeneous charge distribution

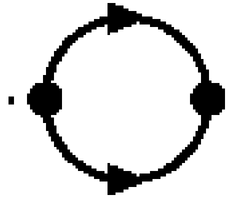
Coulomb energy

$$\delta \mathcal{F} = \frac{1}{2}\left(\frac{\partial^2 \mathcal{F}}{\partial \Delta^2}\right)_n \delta\Delta^2 + \frac{1}{2}\sum_{\vec{k}\neq 0}\left(\frac{\partial^2 \mathcal{F}}{\partial \Delta^*_{\vec{k}} \partial \Delta_{\vec{k}}}\right)_n \delta\Delta^*_{\vec{k}}\delta\Delta_{\vec{k}}$$

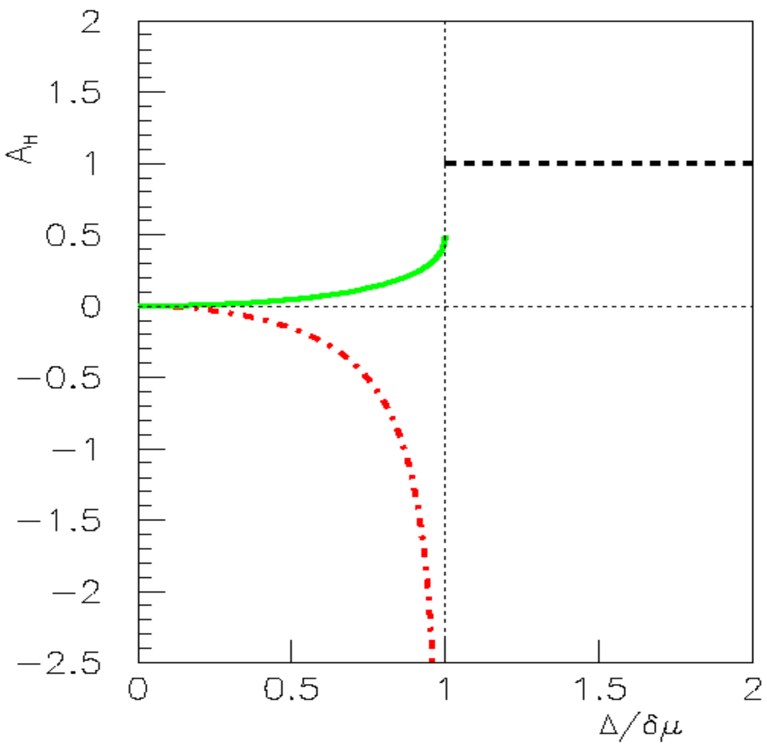
$$\delta\rho(\vec{k}) = \kappa(k) H(\vec{k})$$

$$E_{\mathrm{coul.}} = \frac{1}{2V}\sum_{\vec{k}\neq 0}\frac{\delta\rho(\vec{k})^*\delta\rho(\vec{k})}{k^2 + m_D^2(k)}$$

$$\tilde{\Pi}(k) \equiv \left(\frac{\partial^2 \mathcal{F}}{\partial H^*(\vec{k})\partial H(\vec{k})}\right)_{n_Q} = \Pi(k) + \frac{\kappa^*(k)\kappa(k)}{k^2 + m_D^2(k)}$$



Sarma Instability can be removed by Coulomb energy



$$\left(\frac{\partial^2\Omega}{\partial\Delta^2}\right)_\nu < 0$$

$$\left(\frac{\partial^2\mathcal{F}}{\partial\Delta^2}\right)_n = \left(\frac{\partial^2\Omega}{\partial\Delta^2}\right)_\nu + \frac{\left(\frac{\partial n}{\partial\Delta}\right)_\nu^2}{\left(\frac{\partial n}{\partial\nu}\right)_\Delta}$$

$$\tilde{A}_H = \frac{4(b^2-3a^2)\bar{\mu}^2(\delta\mu - \sqrt{\delta\mu^2-\Delta^2})}{\pi^2[3a^2\sqrt{\delta\mu^2-\Delta^2} + b^2(2\delta\mu + \sqrt{\delta\mu^2-\Delta^2})]} > 0$$

Electric Coulomb energy is not strong enough to compete the Higgs Instability in g2SC

numerical results in whole momentum space

results in whole momentum space

$E_{\mathrm{coul.}}$

$\tilde{\Pi}(k)$

$l \simeq k_{min}^{-1}$

$\Pi(k)$

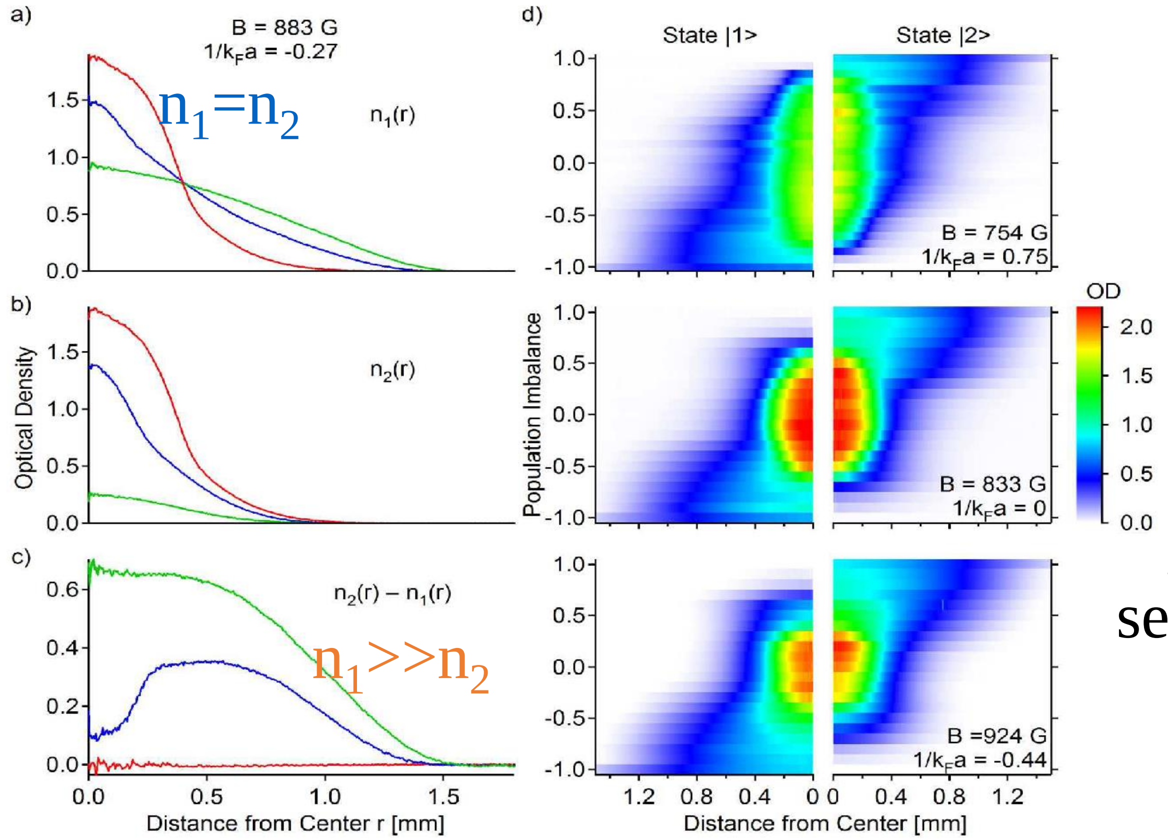
$l/\xi < 1$,

phase separation

$\Delta/\delta\mu = 1/2$ and $\alpha_e \bar{\mu}^2/\Delta^2 = 1$

Giannakis, D.F.Hou, M.H., H.C.Ren, hep-ph/0609098

For gapless superfluid systems, Higgs instability remains, phase separation is favored



phase separation

Zwierlein, Schirotzek, Schunck, & Ketterle, Science 2005, cond-mat/0511197
Partridge, Li, Kamar, Liao, & Hulet, Science 2005, cond-mat/0511752.

Summary

I. Two instabilities in gapless phases: NG-current & (LO)FF state, Higgs instability & spatial inhomogeneity

II. Gapless superfluidity (BP) state, no other mechanism compete with Higgs instability, phase separation is more favored.

III. G2SC phase, the Sarma instability can be removed, but electric Coulomb energy is not strong enough to compete the Higgs instability in the whole momentum space.

IV. gCFL phase, whether color Coulomb energy is strong enough to remove Higgs instability?

Rich Structure of CSC-I

$$3_c \otimes 3_c = \boxed{\bar{3}_c} \oplus 6_c$$

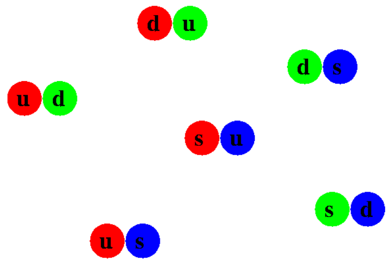
spin-0 CSC

2-flavor

2SC

3-flavor

CFL



spin-1 CSC

CSL, Polar, …

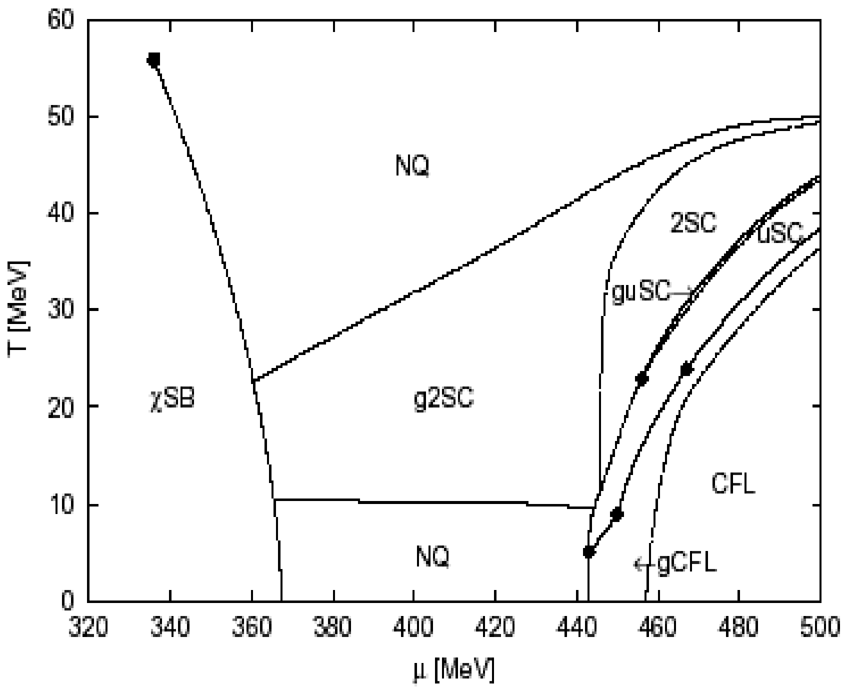
Rich Structure of CSC-II

$$3_c \otimes 3_c = \boxed{\bar{3}_c} \oplus 6_c$$

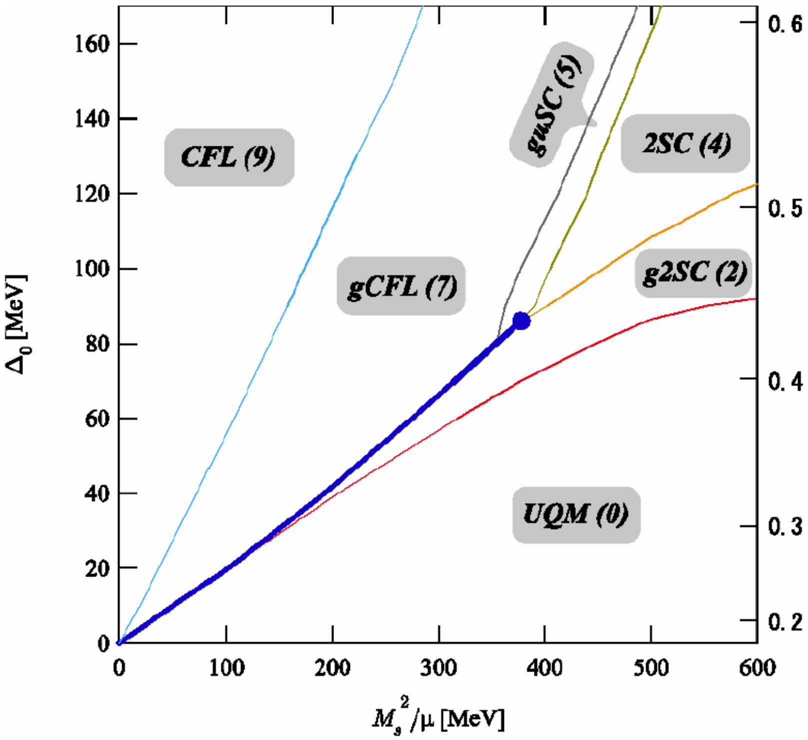
	2-flavor	3-flavor
Pairing without mismatch	2SC	CFL
Pairing with mismatch	g2SC	CFL+K, gCFL, uSC, dSC, sSC

LOFF (Larkin Ovchinnikov Fulde Ferrell)

Rich Structure of CSC



Darmstadt and Frankfurt CSC group
Phys.Rev.D72:034004,2005



Abuki, Kitazawa, & Kunihiro,
PLB 615, 102 (2005)

III. 3.3 Spontaneous Nambu-Goldstone currents generation driven by mismatch

$$\mathcal{V}(x) \equiv \exp\left[i \left(\sum_{a=4}^{7} \varphi_a(x) T_a + \frac{1}{\sqrt{3}} \varphi_8(x) B \right) \right]$$

$$q = \mathcal{V}\chi \quad , \quad \bar{q} = \bar{\chi}\,\mathcal{V}^\dagger \qquad\qquad q_C = \mathcal{V}^*\chi_C \quad , \quad \bar{q}_C = \bar{\chi}_C\,\mathcal{V}^T$$

new Nambu-Gor'kov spinors

$$X \equiv \begin{pmatrix} \chi \\ \chi_C \end{pmatrix} \quad , \quad \bar{X} \equiv (\bar{\chi}\,,\,\bar{\chi}_C)\,.$$

$$\Gamma = -\frac{T}{2}\sum_n \int \frac{d^3\vec{p}}{(2\pi)^3} \mathrm{Tr}\ln([\mathcal{S}_{nl}(P)]^{-1}) + \frac{\Phi^2}{4G_D}.$$

$$\mathcal{L}_{nl} \equiv \bar{X}\,\mathcal{S}_{nl}^{-1}\,X - \frac{\Phi^+\Phi^-}{4G_D} \qquad \mathcal{S}_{nl}^{-1} \equiv \begin{pmatrix} [G_{0,nl}^+]^{-1} & \Phi^- \\ \Phi^+ & [G_{0,nl}^-]^{-1} \end{pmatrix}$$

$$V^\mu \equiv \mathcal{V}^\dagger\ (i\,\partial^\mu)\ \mathcal{V}, \qquad [G_{0,nl}^+]^{-1} = i\,\not{D} + \hat{\mu}\,\gamma_0 + \gamma_\mu\,V^\mu,$$

$$V_C^\mu \equiv \mathcal{V}^T\ (i\,\partial^\mu)\ \mathcal{V}^* \qquad [G_{0,nl}^-]^{-1} = i\,\not{D}^T - \hat{\mu}\,\gamma_0 + \gamma_\mu\,V_C^\mu.$$

$$V^\mu \simeq -\sum_{a=4}^{7}(\partial^\mu\varphi_a)\ T_a - \frac{1}{\sqrt{3}}\ (\partial^\mu\varphi_8)\ B$$

$$V_C^\mu \simeq \sum_{a=4}^{7}(\partial^\mu\varphi_a)\ T_a^T + \frac{1}{\sqrt{3}}\ (\partial^\mu\varphi_8)\ B^T$$

Nambu-Goldstone currents driven by mismatch

$$< \sum_{a=4}^{7} \vec{\nabla}\varphi_a > \neq 0$$

gluon phase

Gorbar, Hashimoto, Miransky, hep-ph/0507303

$$< \vec{\nabla}\varphi_8 > \neq 0$$

LOFF

Gluon condensate

I. A brief introduction on CSC

II. Competition between chiral and diquark condensations

III. Pairing with mismatch

III. 1. The gapless 2SC (g2SC) phase

III. 2. Instabilities driven by mismatch

III. 3. Resolving instabilities in g2SC

IV. Summary