QCD dense matter and color superconductor

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I. A brief introduction on QCD dense matter

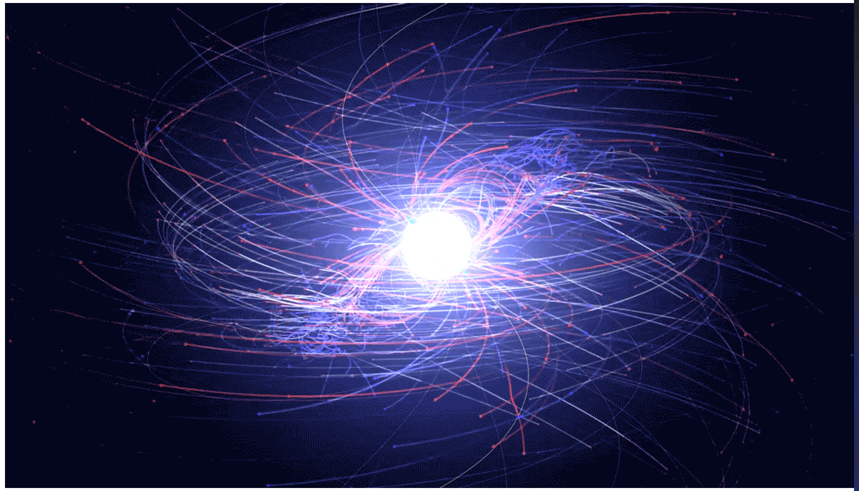
II. QCD critical end point

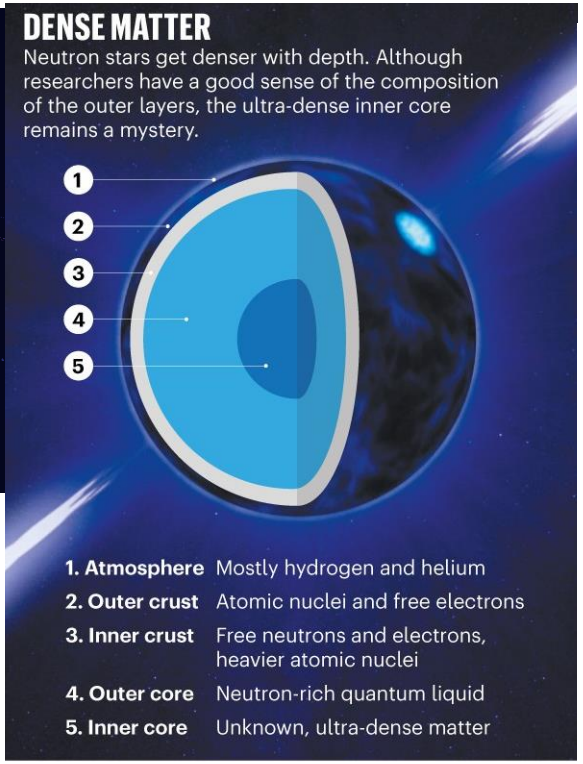
III. Quarkyonic matter and EOS for neutron star

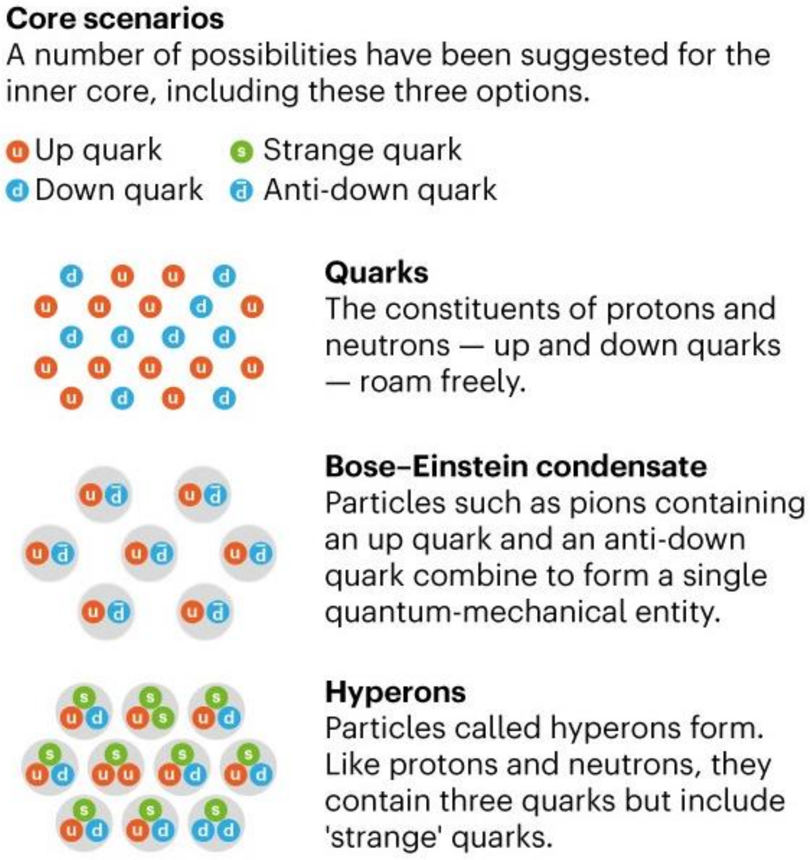
IV. Color superconductor

V. Summary and outlook

Neutron star (NS) is a kind of compact stars, which is the remnant after a massive super-giant star collapses. From August 1967 on, when the existence of NSs was confirmed by the discovery of radio pulsars, more than 2700 radio pulsars have been detected. It has been wondered for more than a half century what's the internal structure of NSs, whether quark matter exists inside the core of NSs.



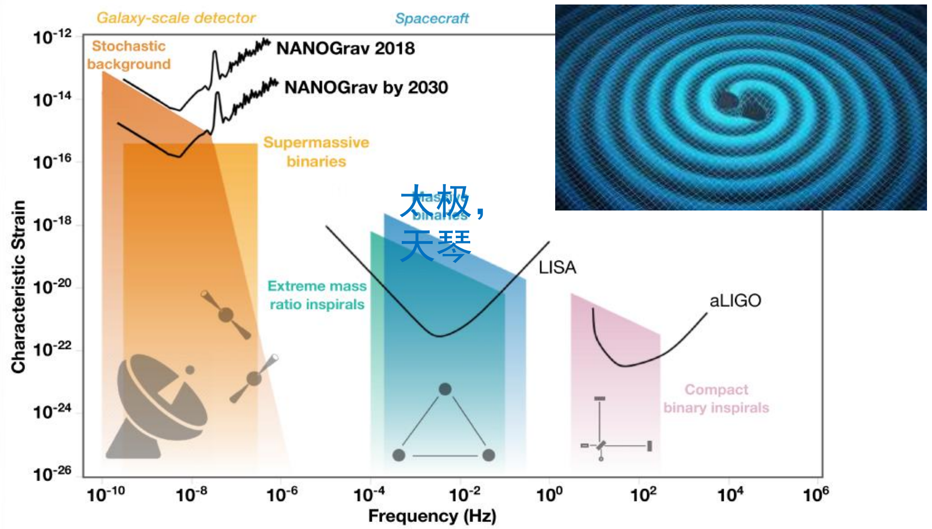


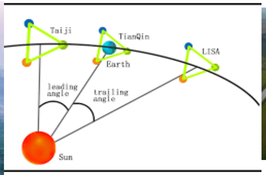


“多信使”时代

更多的引力波探测器及射电望远镜对引力波和致密星体（质量和半径）进行精确测量，一方面对理论进行约束，另一方面也需要理论对实验结果进行理解。

Pulsar Timing Array对PSRJ0348+0432和PSR J1624-2230的质量测量2倍太阳质量





LIGO/VirgoGW170817，致密星体半径约12km

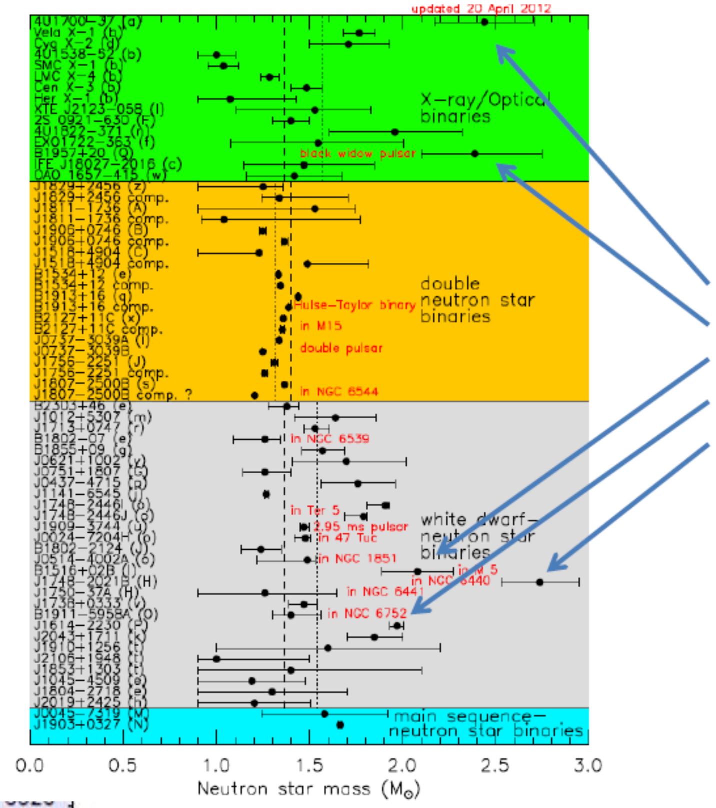
nature > news feature > article

Nature | Vol 579 | 5 March 2020 | 21

NEWS FEATURE | 04 March 2020

The golden age of neutron-star physics has arrived

These stellar remnants are some of the Universe's most enigmatic objects – and they are finally starting to give up their secrets.



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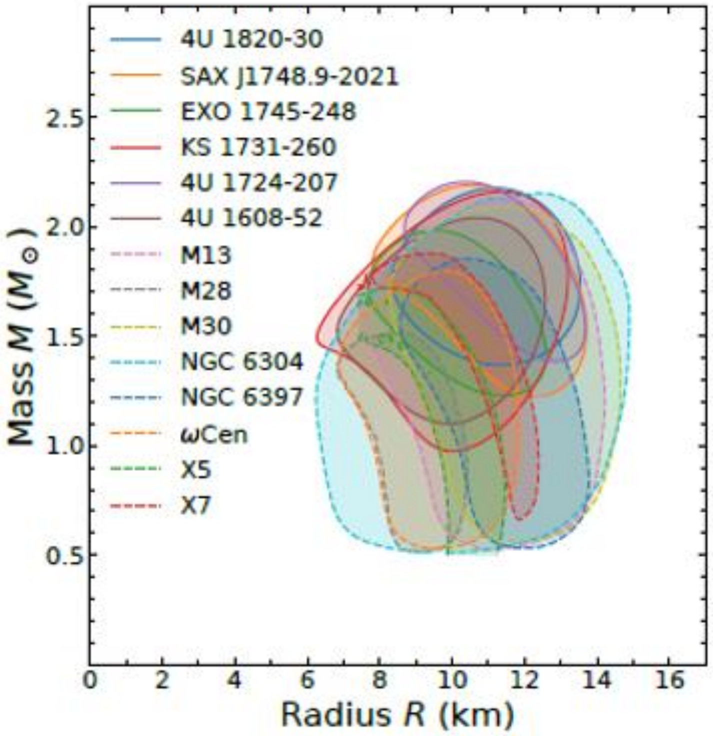
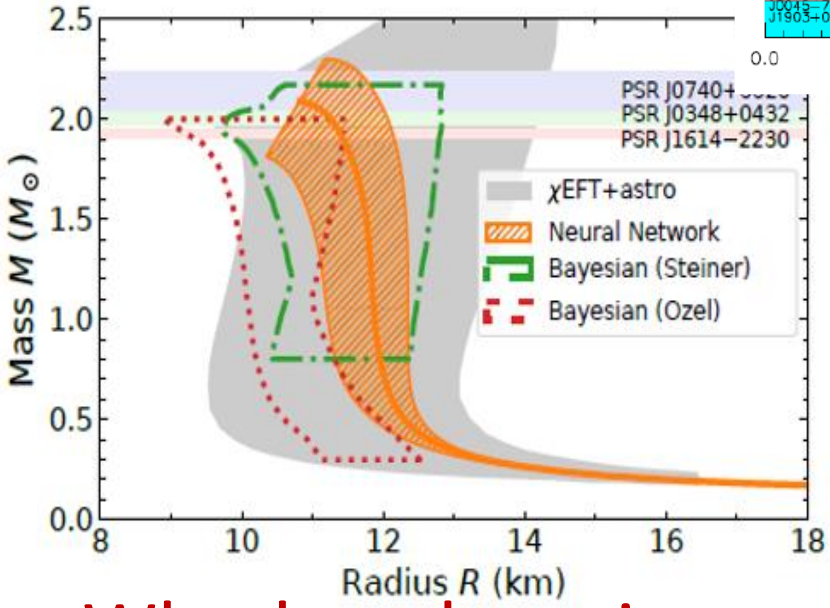




Whether there is quark matter inside NSs?

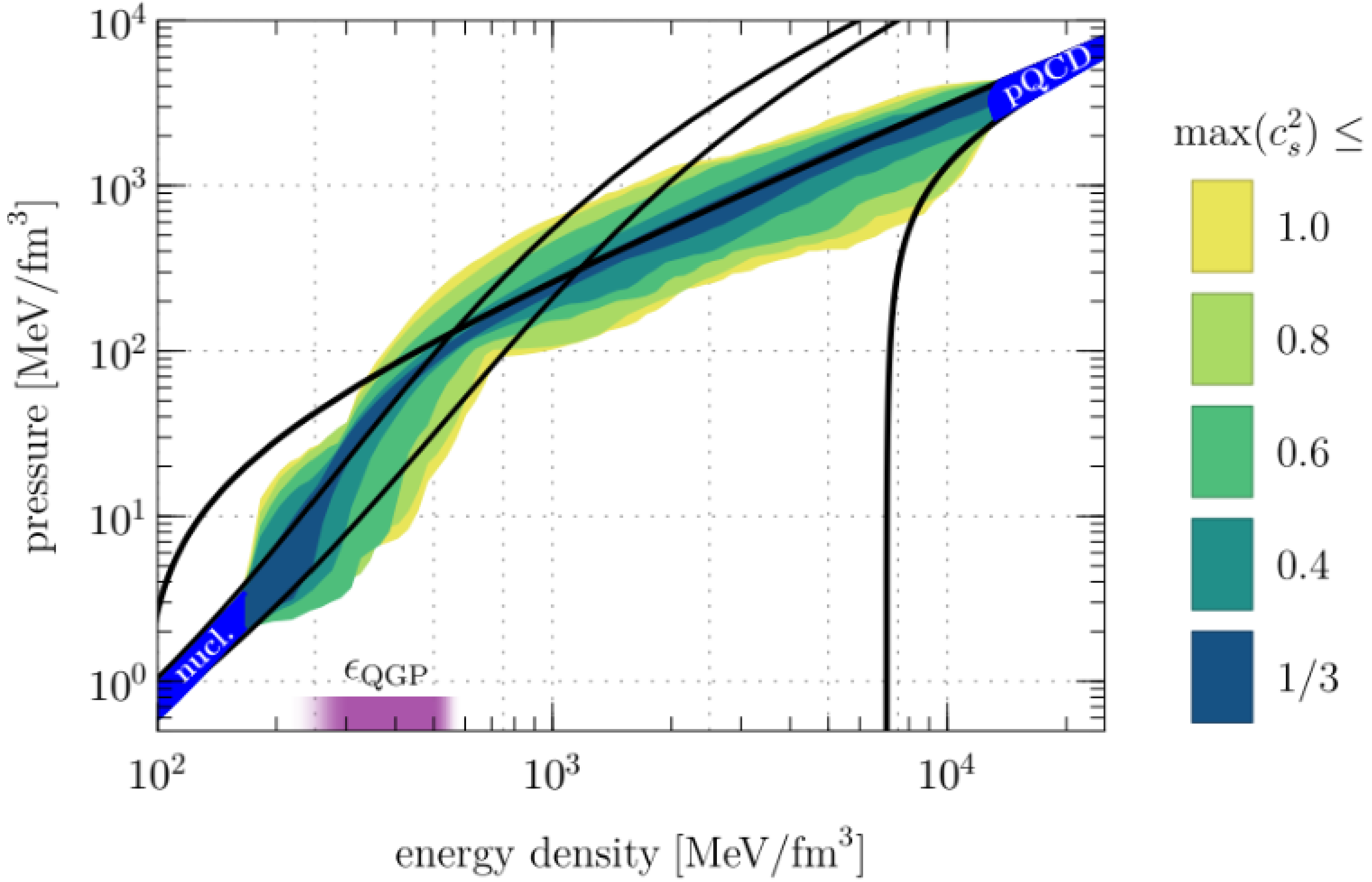
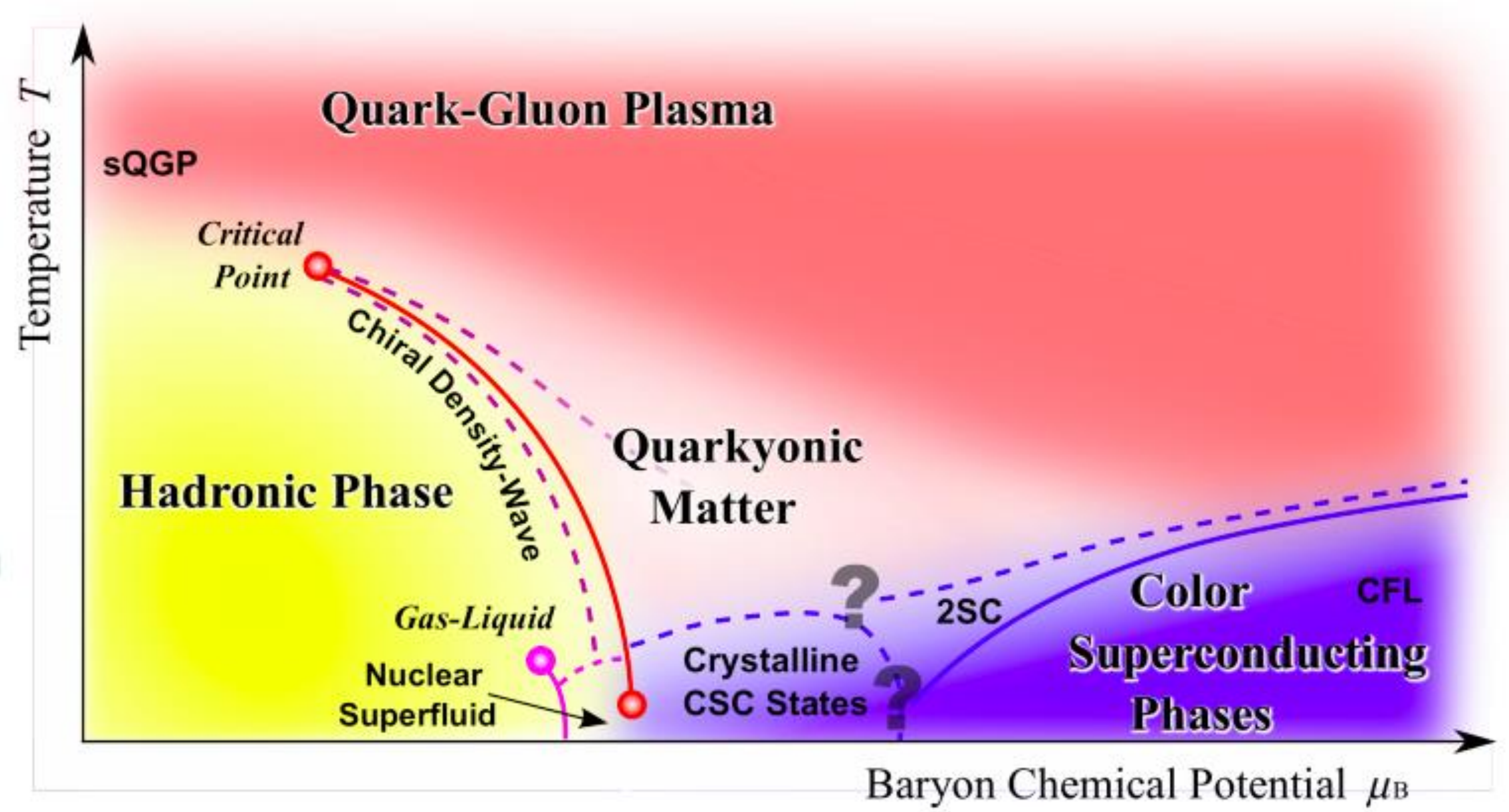


Fig 1: The relationship between energy density and pressure for possible neutron stars. The color relates to the speed of sound in the material, which is an indicator of likelihood for a star like that to exist. The dark blue part of the line indicates where most neutron stars probably lie. The black lines are extrapolations at high and low densities. The purple line at the bottom shows the rough location of the transition into quarks. (Source: Figure 2 in the paper)

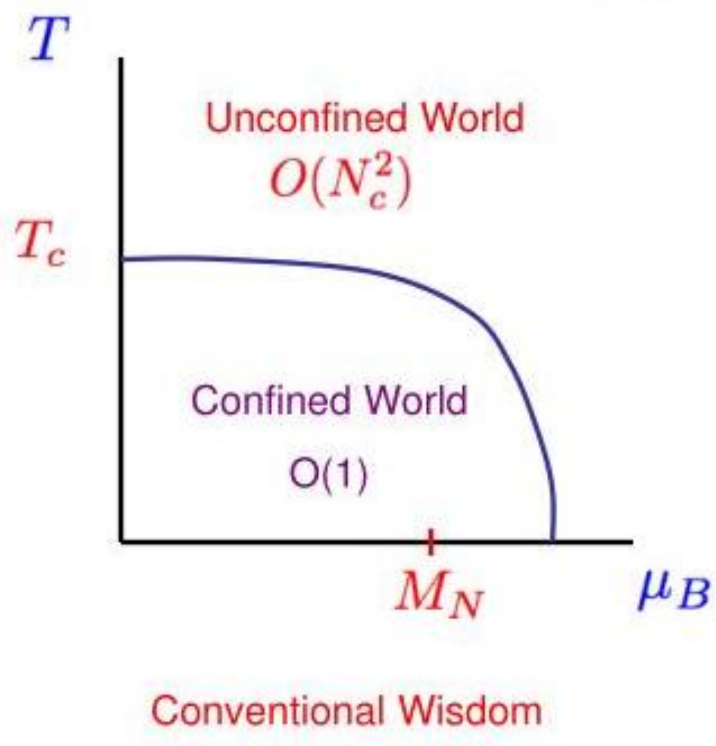
Dense Matter: QCD CEP, Quarkyonic matter, CSC

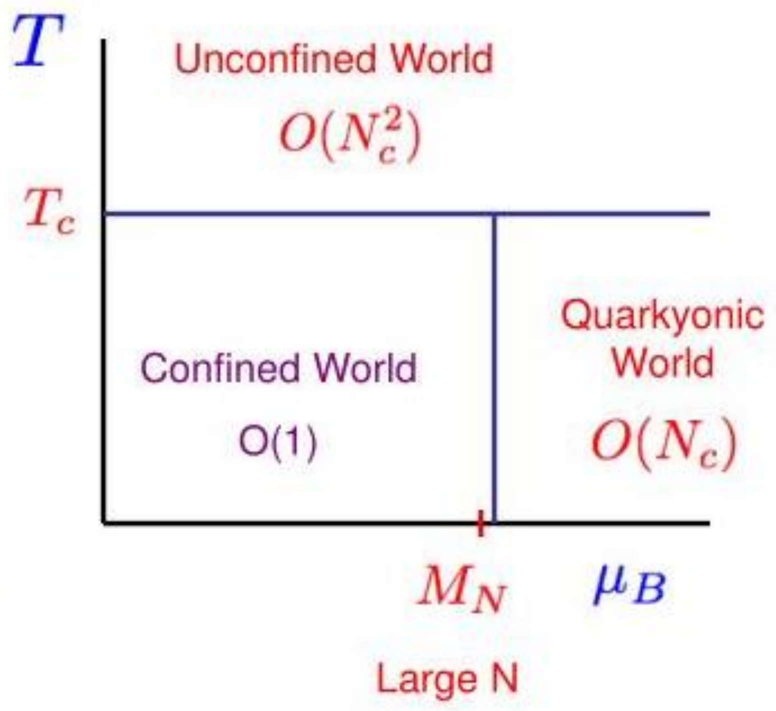


K. Fukushima and T. Hatsuda, Rept. Prog. Phys. 74, 014001(2011); arXiv: 1005.4814

Quarkyonic matter

Separation of quark dynamics and gluodynamics?





McLerran, Pisarski, Nucl. Phys. A 796 (2007) 83.

QCD properties in the vacuum

重要难题

I.Spontaneous Chiral symmetry breaking (quark dynamics)

手征模型

Goldstone boson and chiral condensate

Chiral partners have different masses

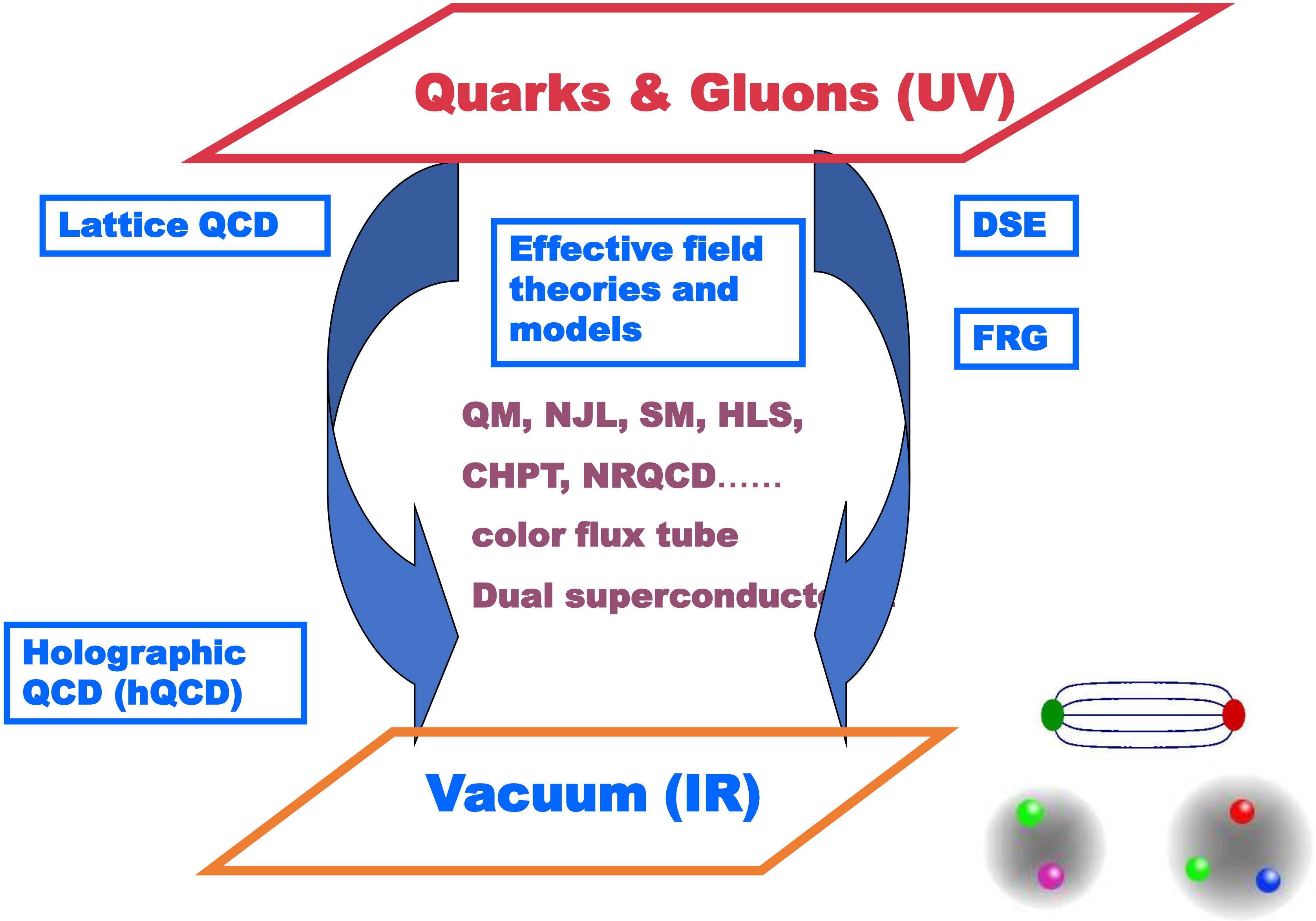
基于AdS/CFT对偶的全息方法

II. Confinement (Gluodynamics)

DSE，

泛函重整化群（FRG）

Strong QCD



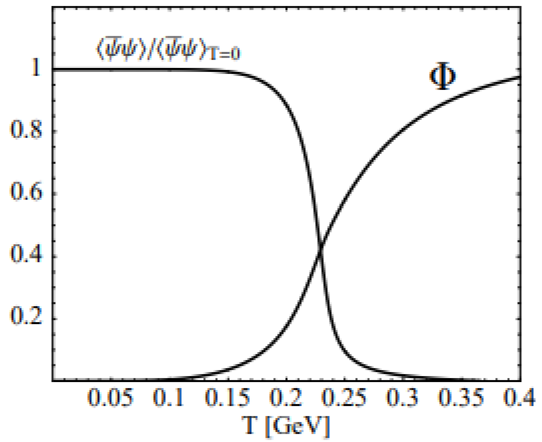
Quarks & Gluons (UV)
Lattice QCD
Effective field theories and models
DSE
FRG
QM, NJL, SM, HLS,
CHPT, NRQCD……
color flux tube
Dual superconduct
Holographic QCD (hQCD)
Vacuum (IR)

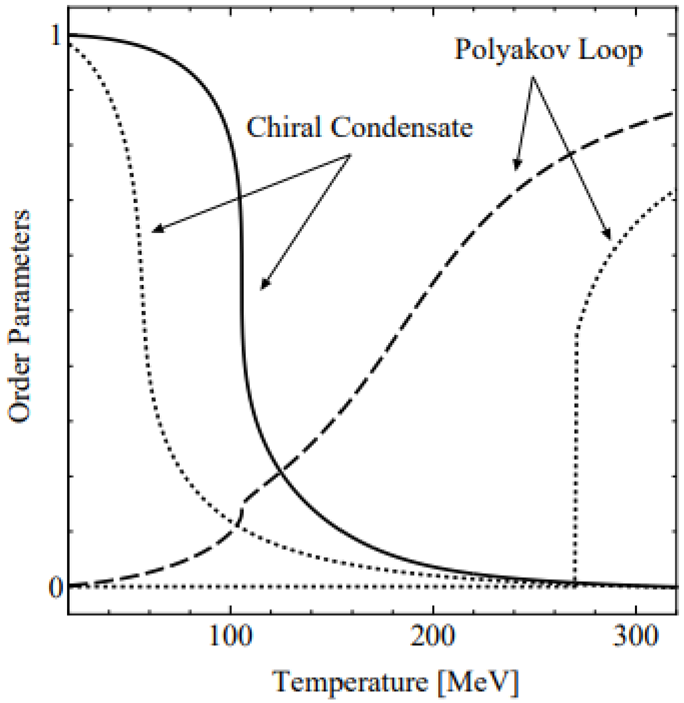
Chiral restoration and deconfinement
Polyakov loop NJL model

$$\mathcal{L}_{PNJL} = \bar{\psi}\left(i\gamma_\mu D^\mu - \hat{m}_0\right)\psi + \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right] - \mathcal{U}\left(\Phi[A], \bar{\Phi}[A], T\right),$$

$$\frac{\mathcal{U}\left(\Phi, \bar{\Phi}, T\right)}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3 + \bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2 \qquad \Phi = (\mathrm{Tr}_c\, L)/N_c$$

$$L(\vec{x}) = \mathcal{P}\exp\left[i\int_0^\beta d\tau\, A_4(\vec{x}, \tau)\right]$$





Kenji Fukushima, Phys.Lett.B 591 (2004) 277-284, hep-ph/0310121

Claudia Ratti, Michael A. Thaler, Wolfram Weise, hep-ph/0506234

Chiral dynamics and Gluodynamics in Dynamical hQCD model

Holographic Duality: Gravity/QFT

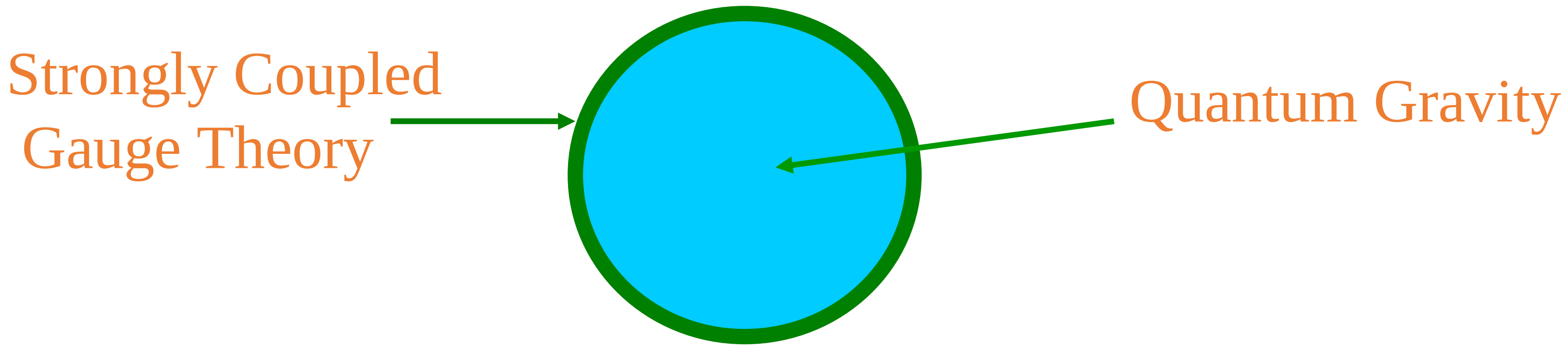
AdS/CFT : Original discovery of duality

J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998)

Supersymmetry and conformality are required for AdS/CFT.

In general, supersymmetry and conformality are not necessary

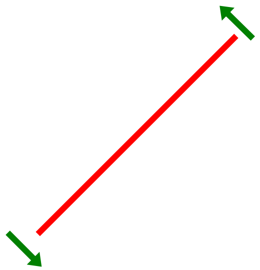
General Gravity/QFT:



QCD and string theory: 1968-1974

String theory was born out of attempts to understand the strong interactions:
Veneziano model,
string model: Nambu,Nielsen, Susskind

In the sixties many new mesons and hadrons were discovered. It was suggested that these might not be new fundamental particles. Instead they could be viewed as different oscillation modes of a string.



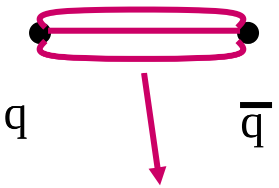
1, String model & "Regge trajectories"

$$J_{\max} \sim \alpha' m^2 + const$$

$$m^2 \sim T J_{\max} + const$$

QCD and string theory: 1968-1974

2, String model & confinement



$V = T\ R$

Dual superconductor picture

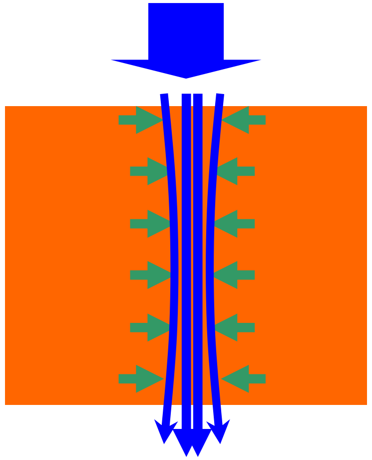
Type-II superconductor
Abrikosov vortex in U(1) theory

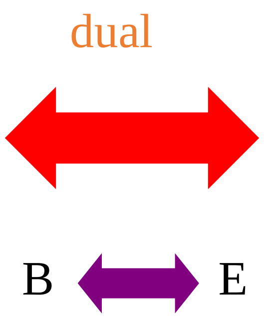
A.A.Abrikosov, Soviet Phys.JTEP 5, 1174(1957)

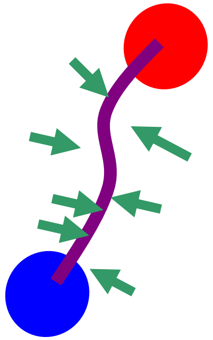
Color flux tube in QCD

Y.Nambu, PRD.122,4262(1974)
't Hooft , Nucl.Phys.B190.455(1981)
Mandelstam, Phys.Rep.C23.245(1976)

electric
Cooper-pair
condensation







magnetic monopole
condensation

squeeze
magnetic field

squeeze
color electric flux

QCD and string theory: 1968-1974

3, Effective theory in terms of strings

t' Hooft '74

t' Hooft large Nc limit

take Nc colors instead of 3, SU(Nc)

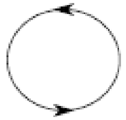
$$S = \frac{1}{4\, g_{\rm YM}^2} \int \mathrm{d}^4 x \ \mathrm{Tr}\left(F_{\mu\nu} F^{\mu\nu} \right)$$

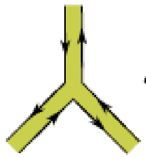
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

$$(A_\mu)_{ij} = A_\mu^a \, (T^a)_{ij}$$

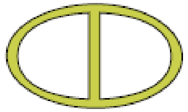
QCD and string theory: 1968-1974

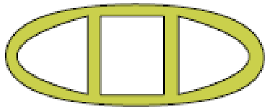
Gluon propagator  $\sim g_{YM}^2$

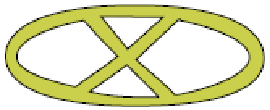
 $\sim N_c$

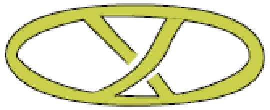
Interactions  $\sim \frac{1}{g_{YM}^2}$  $\sim \frac{1}{g_{YM}^2}$

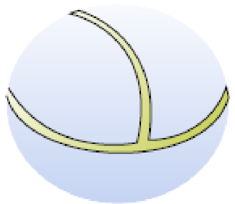
't Hooft coupling $\lambda = g_{YM}^2 N_c$

 $\sim (g_{YM}^2)^{3-2} N_c^3 = \lambda N_c^2$

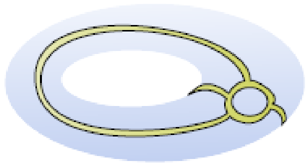
 $\sim (g_{YM}^2)^{6-4} N_c^4 = \lambda^2 N_c^2$

 $\sim (g_{YM}^2)^{8-5} N_c^5 = \lambda^3 N_c^2$

 $\sim (g_{YM}^2)^{6-4} N_c^2 = \lambda^2$



Planar diagram most dominant



Non-planar diagram 1/Nc^2 suppressed

QCD and string theory: 1968-1974

QCD at low energies, when the coupling is large,
dual of a weakly coupled string theory

Vacuum-to-vacuum amplitude in large Nc gauge theory

$$\log Z = \sum_{h=0}^{\infty} N_c^{2-2h} f_h(\lambda) = N_c^2 f_0(\lambda) + f_1(\lambda) + \frac{1}{N_c^2} f_2(\lambda) + \cdots ,$$

Vacuum-to-vacuum amplitude in string theory

$$\mathcal{A} = \sum_{h=0}^{\infty} g_s^{2h-2} F_h(\alpha') = \frac{1}{g_s^2} F_0(\alpha') + F_1(\alpha') + g_s^2 F_2(\alpha') + \cdots ,$$

where g_s is the string coupling, $2\pi\alpha'$ is the inverse string tension, and $F_h(\alpha')$ is the contribution of 2d surfaces with h holes.

The string coupling constant gs is of order 1/Nc,

Closed strings would be glueballs.
Open strings would be the mesons.

QCD and string theory: 1968-1974

Problems:

1) Strings do not make sense in 4 (flat) dimensions

Trying to quantize a string in four dimension leads to tacyons.

2) Strings always include a graviton, ie., a particle with m=0, s=2

For this reason strings are normally studied as a model for quantum gravity.

QCD and string theory: 1974-1997

QCD: pQCD is confirmed by DIS
non-perturbative QCD region, challenging in
describing hadrons in terms of quark and gluon DOF.

String theory: trying to make itself a theory of everything.

Holographic Duality: Gravity/QFT

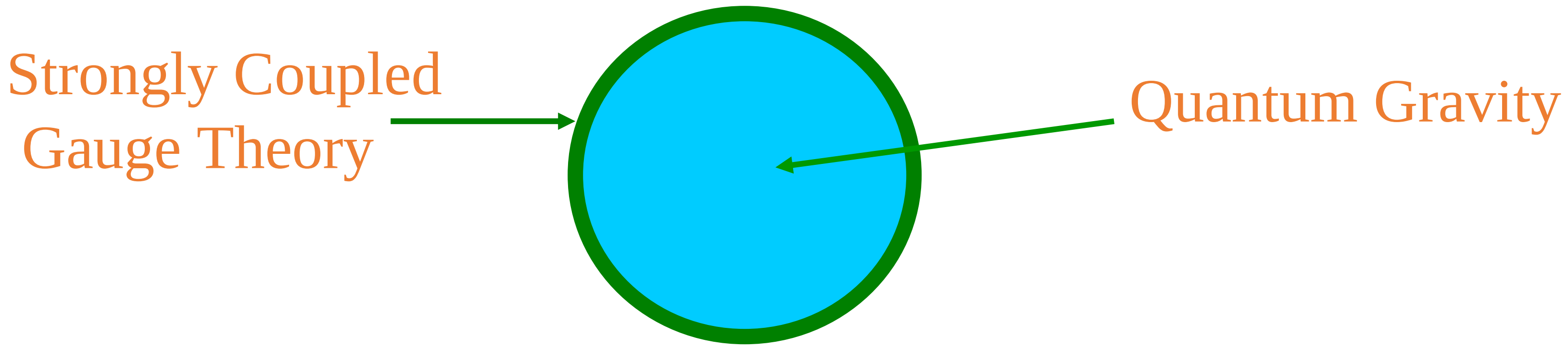
AdS/CFT : Original discovery of duality

J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998)

Supersymmetry and conformality are required for AdS/CFT.

In general, supersymmetry and conformality are not necessary

General Gravity/QFT:



Holographic Duality: (d+1)-Gravity/ (d)-QFT

Holography & Emergent critical phenomena:

When system is strongly coupled, new weakly-coupled degrees of freedom dynamically emerge.

The emergent fields live in a dynamical spacetime with an extra spatial dimension.

The extra dimension plays the role of energy scale in QFT, with motion along the extra dimension representing a change of scale, or renormalization group (RG) flow.

arXiv:1205.5180 Allan Adams,[1] Lincoln D. Carr,[2,3] Thomas Schäfer,[4] Peter Steinberg[5] and John E. Thomas[4]

Holographic QCD or gravity dual of QCD

holographic QCD (5D)

String theorists' business: whether it can be deduced from 10D string theory ?

Build the connection between QCD dynamics and geometry

3rd step: gravity dual
systematic framework

2nd step: deformed AdS_5
intelligent guess

1st step: just AdS_5
naïve try

Real QCD world:

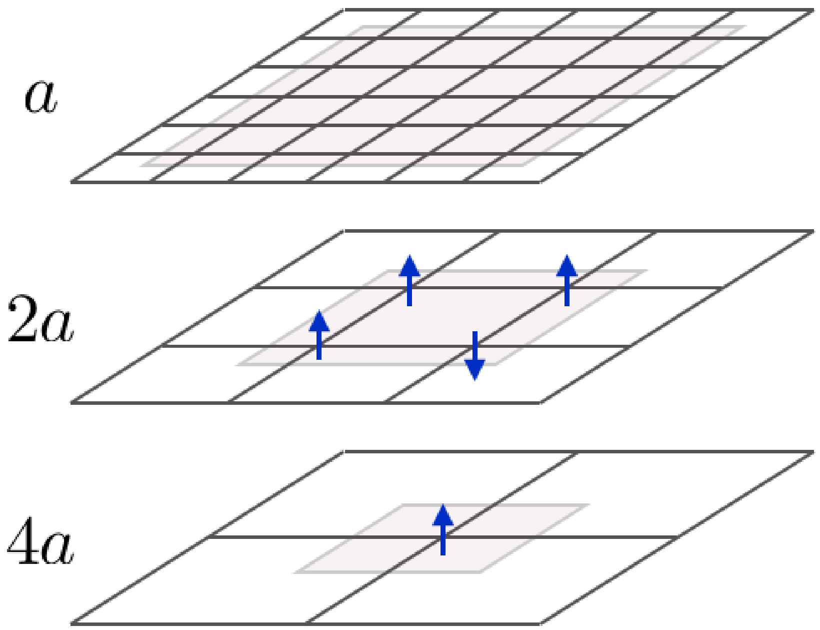
Rich experimental data and lattice data

Holographic Duality & RG flow

Coarse graining spins on a lattice: Kadanoff and Wilson

$$H = \sum_{x,i} J_i(x)\mathcal{O}^i(x)$$

J(x): coupling constant or source for the operator



$$H = \sum_i J_i(x,a)\,\mathcal{O}^i(x)$$

$$H = \sum_i J_i(x,2a)\,\mathcal{O}^i(x)$$

$$H = \sum_i J_i(x,4a)\,\mathcal{O}^i(x)$$

$$u\frac{\partial}{\partial u}J_i(x,u) = \beta_i(J_j(x,u),u)$$

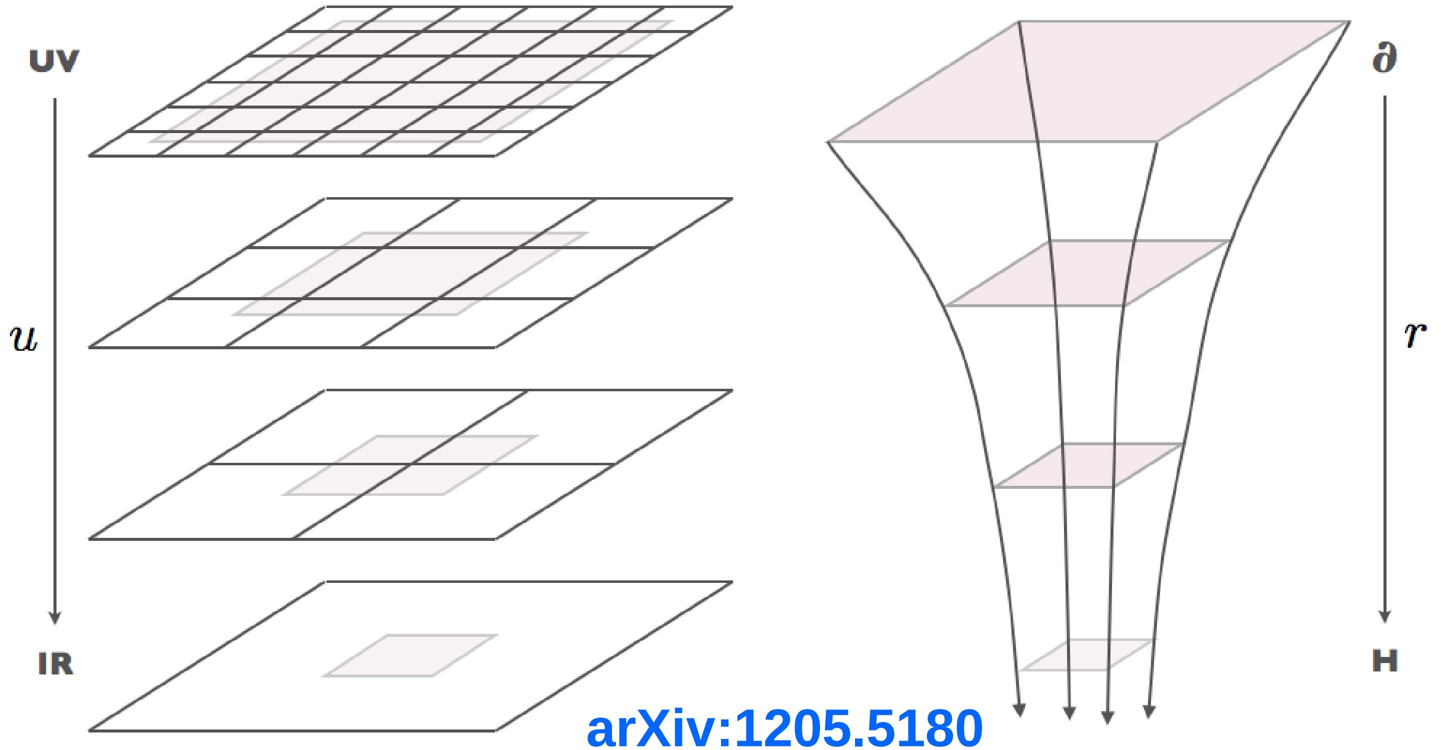
arXiv:1205.5180

Holographic Duality & RG flow

QFT on lattice   equivalent to   GR problem from Gravity

RG scale -> an extra spatial dimension
Coupling constant -> dynamical filed

$$J_i|_{UV} \quad = \quad \Phi_i|_{\partial}$$



arXiv:1205.5180

A systematic framework: Graviton-dilaton system

$$S_G = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\Phi}\left(R_s + 4\partial_M\Phi\partial^M\Phi - V_G^s(\Phi)\right)$$

N=4 Super YM

conformal

AdS_5

$$ds^2 = \frac{L^2}{z^2}\left(dt^2 + d\vec{x}^2 + dz^2\right)$$

$$V_E(\phi) = -\frac{12}{L^2}$$

QCD

nonconformal

deformed AdS_5

$$ds^2 = \frac{h(z)L^2}{z^2}\left(dt^2 + d\vec{x}^2 + dz^2\right)$$

Dilaton field breaks conformal symmetry

Input: QCD dynamics at IR

Solve: Metric structure, dilaton potential

Pure gluon system:

$$\mathscr{L}_G = -\frac{1}{4} G^a_{\mu\nu}(x) G^{\mu\nu,a}(x),$$

Gluon condensate at IR: $\mathrm{Tr}\langle G^2 \rangle$

5D action: graviton-dilaton

$$S_G = \frac{1}{16\pi G_5} \int d^5x \sqrt{g_s} e^{-2\Phi} \left(R_s + 4\partial_M \Phi \partial^M \Phi - V^s_G(\Phi) \right)$$

$\mathrm{Tr}\langle G^2 \rangle$ **dual to** $\Phi(z)$

A systematic framework: Graviton-dilaton system

$$S_G = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\Phi}\left(R_s + 4\partial_M\Phi\partial^M\Phi - V_G^s(\Phi)\right)$$

N=4 Super YM

conformal

AdS$_5$

$$ds^2 = \frac{L^2}{z^2}\left(dt^2 + d\vec{x}^2 + dz^2\right)$$

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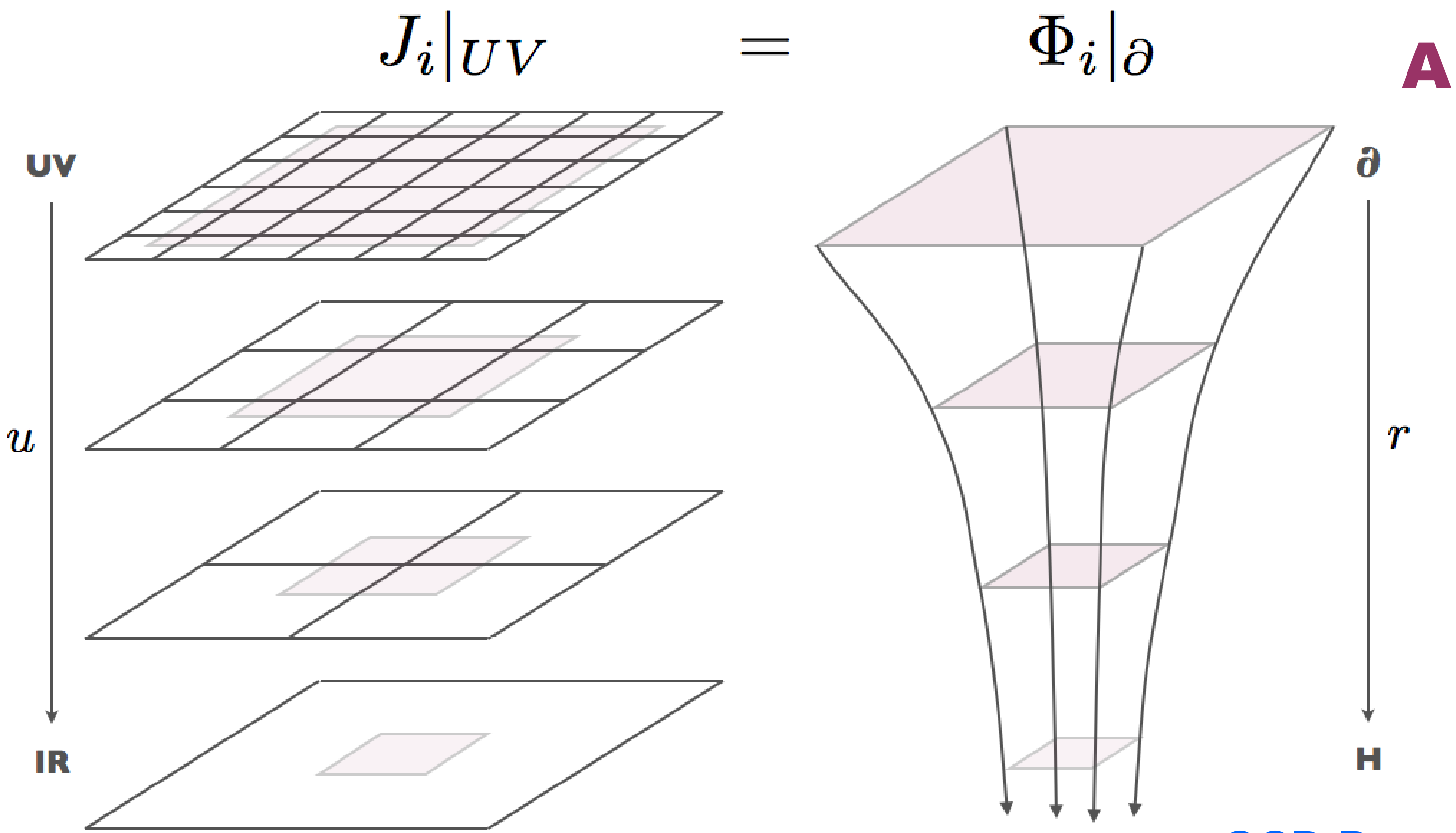
Dilaton field breaks conformal symmetry

Input: QCD dynamics at IR

Solve: Metric structure, dilaton potential

Dynamical hQCD & RG



A AdS$_5$

QCD Dynamics at IR

deformed AdS$_5$

Gluodynamics

Glueball spectra
EOS for pure gluon system

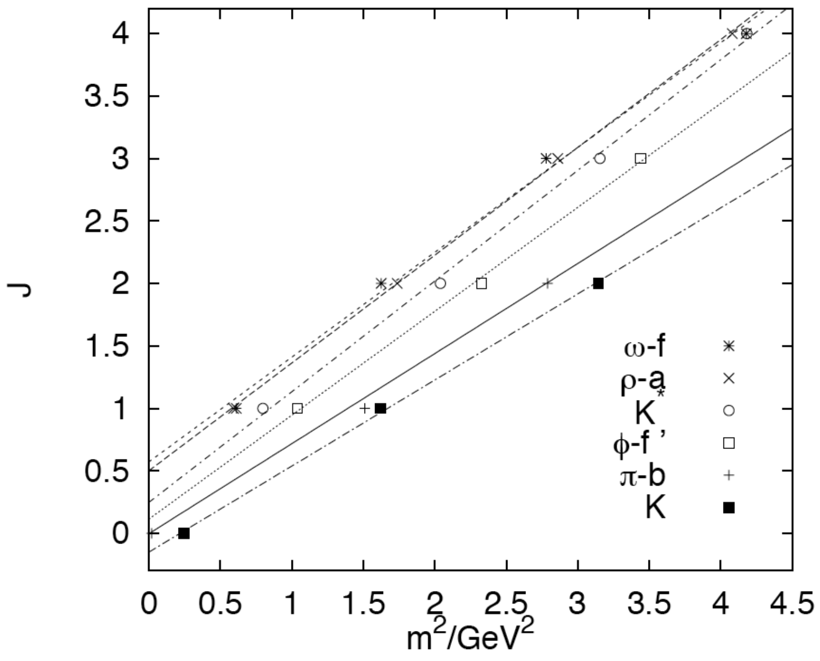
Confinement and deconfinement in graviton-dilaton system

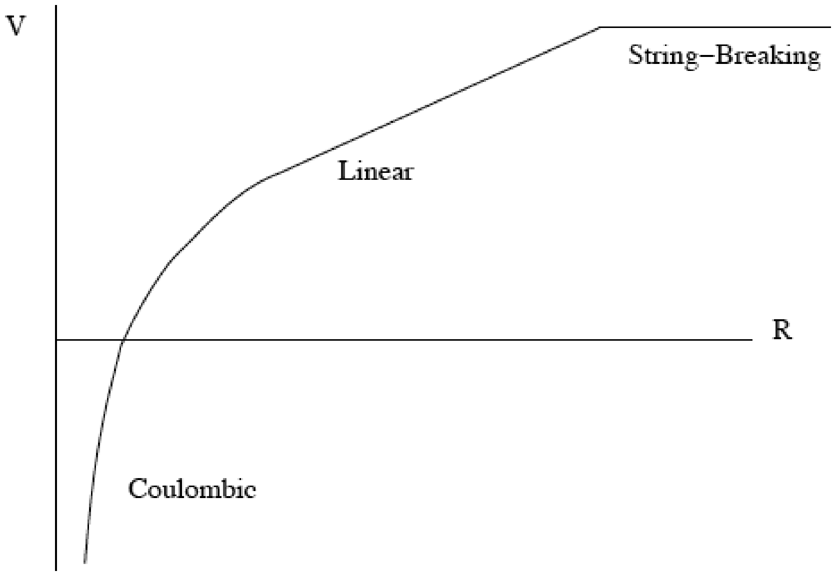
For pure gluon system

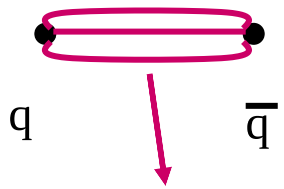
S. He, M. H., Q. S. Yan, arXiv:1004.1880, PRD2011
D.N, Li, S. He, M. H., Q. S. Yan, arXiv:1103.5389, JHEP2011

What's confinement?

Confinement: Regge behavior and linear quark potential







$$V \sim \sigma_s R$$

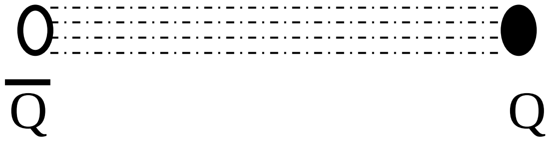
Flux tubes of color field = glue

QCD and string theory I:

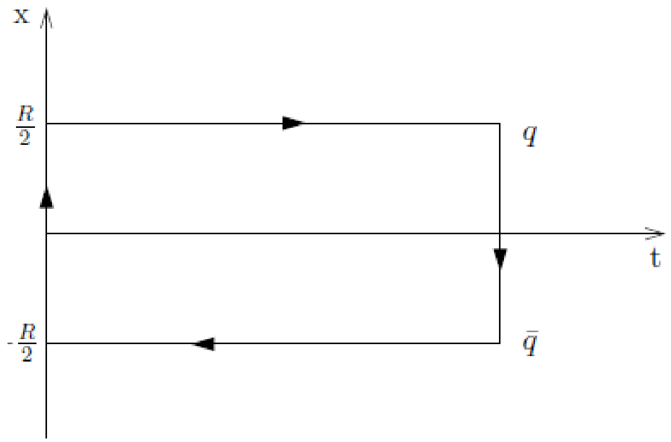
String model & confinement

Confinement for pure glue system

Confinement potential $V_{Q\bar{Q}}(R) = -\frac{\kappa}{R} + \sigma_{str} R + V_0$



$$W[C] = \frac{1}{N} Tr P \exp[i \oint_C A_\mu dx^\mu]$$



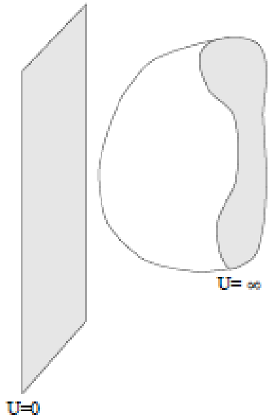
$$\langle W(C) \rangle \propto e^{-TV_{Q\bar{Q}}}$$

$$T \to \infty.$$

Holographic dictionary:

J. M. Maldacena, Phys. Rev. Lett. 80, 4859 (1998), hep-th/9803002.

$$\langle W^{4d}[C]\rangle = Z^{5d}_{string}[C] \simeq e^{-S_{NG}[C]}$$



$$V_{Q\overline{Q}}(r) = \lim_{T\to\infty} \frac{1}{T} S_{NG}[C]$$

Metric structure determines the quark potential !

1, AdS_5 only gives Coulomb potential !

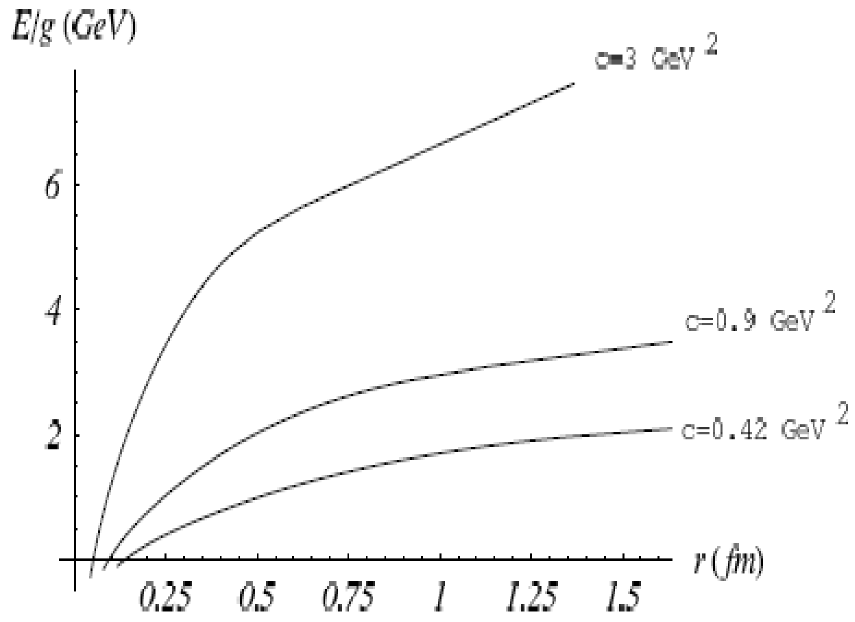
2, Deformed metric structure is needed to produce the linear potential!

Deformed AdS_5 models I:

Andreev-Zakharov model: quadratic correction

O. Andreev, V.Zakharov, hep-ph/0604204

$$ds^2 = G_{nm} dX^n dX^m = R^2 \frac{h}{z^2}\left(dx^i dx^i + dz^2\right) \qquad h = e^{\frac{1}{2}cz^2}$$



Holographic Duality: Dictionary

Boundary QFT | **Bulk Gravity**

Local operator $\mathcal{O}_i(x)$ | **Bulk field** $\Phi_i(x,r)$

$$\Delta(d-\Delta) = m^2 L^2$$

Strongly coupled | **Semi-classical**

$$Z_{\mathrm{QFT}}[J_i] = Z_{\mathrm{QG}}[\Phi[J_i]]$$

$$Z_{\mathrm{QFT}}[J] \simeq e^{-I_{\mathrm{GR}}[\Phi[J]]}$$

$$\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle = \frac{\delta^n I_{\mathrm{GR}}[\Phi[J_i]]}{\delta J_1(x_1)\ldots\delta J_n(x_n)}\Big|_{J_i=0}$$

Pure gluon system:

D.N. Li, M.H., JHEP2013, arXiv:1303.6929

$$\mathscr{L}_G = -\frac{1}{4} G^a_{\mu\nu}(x) G^{\mu\nu,a}(x),$$

IR: Gluon condensate $\mathrm{Tr}\langle G^2\rangle$

Effective gluon mass $\langle g^2 A^2\rangle$ String tension, linear confinement

5D action: graviton-dilaton

$$S_G = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\Phi}\left(R_s + 4\partial_M\Phi\partial^M\Phi - V^s_G(\Phi)\right)$$

$\mathrm{Tr}\langle G^2\rangle$ $\langle g^2 A^2\rangle$ **dual to** $\Phi(z)$

$$\Phi(z) = \mu_G^2 z^2 \tanh(\mu_{G2}^4 z^2/\mu_G^2)$$

$$\Phi(z) \overset{z\to 0}{\to} \mu_{G2}^4 z^4, \qquad \Phi(z) \overset{z\to\infty}{\to} \mu_G^2 z^2.$$

However, the dual gluon operator of dimension-2 dilaton field is not known!

$$\langle g^2 A^2 \rangle \longleftrightarrow \Phi(z) \qquad \mathrm{Tr}\langle G^2 \rangle \longleftrightarrow \Phi^2(z)$$

Gauge invariant & Local operator

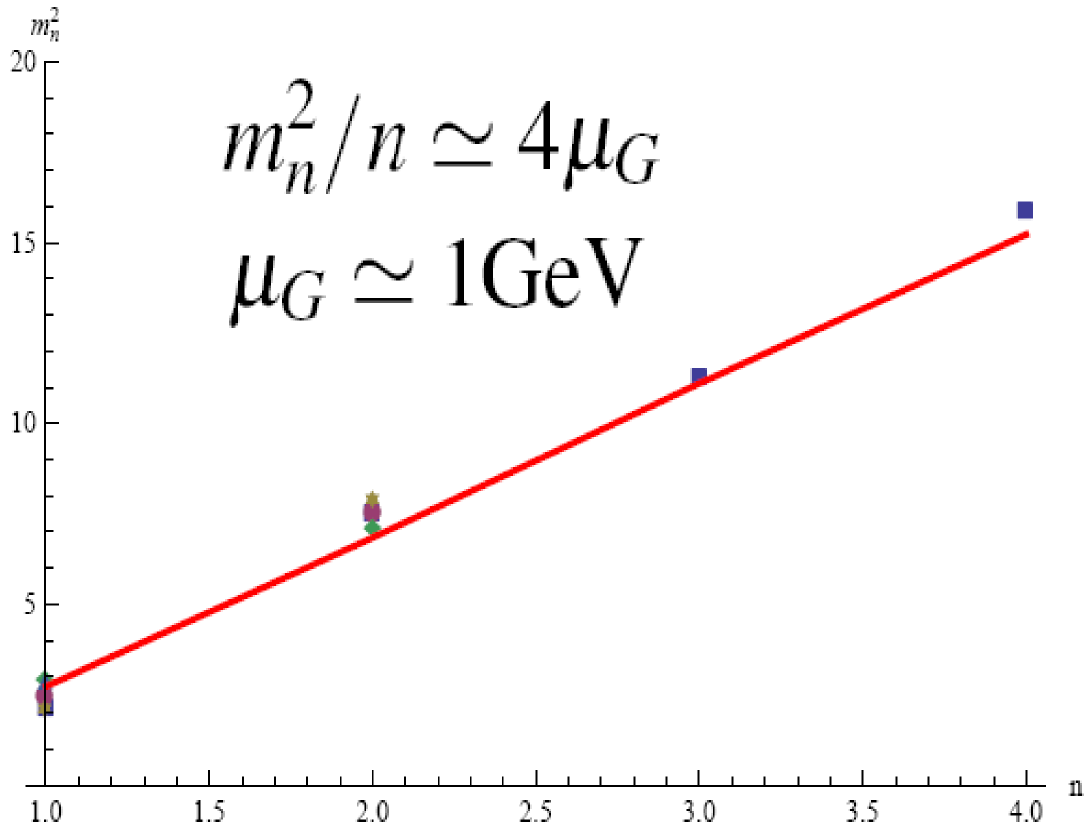
4) Dilaton field: quartic at UV and quadratic at IR

$$\Phi(z) = \mu_G^2 z^2 \tanh(\mu_{G^2}^4 z^2 / \mu_G^2)$$

$$\Phi(z) \stackrel{z \to 0}{\to} \mu_{G^2}^4 z^4,$$

$$\Phi(z) \stackrel{z \to \infty}{\to} \mu_G^2 z^2.$$

Scalar glueball D.N. Li, M.H., JHEP2013, arXiv:1303.6929



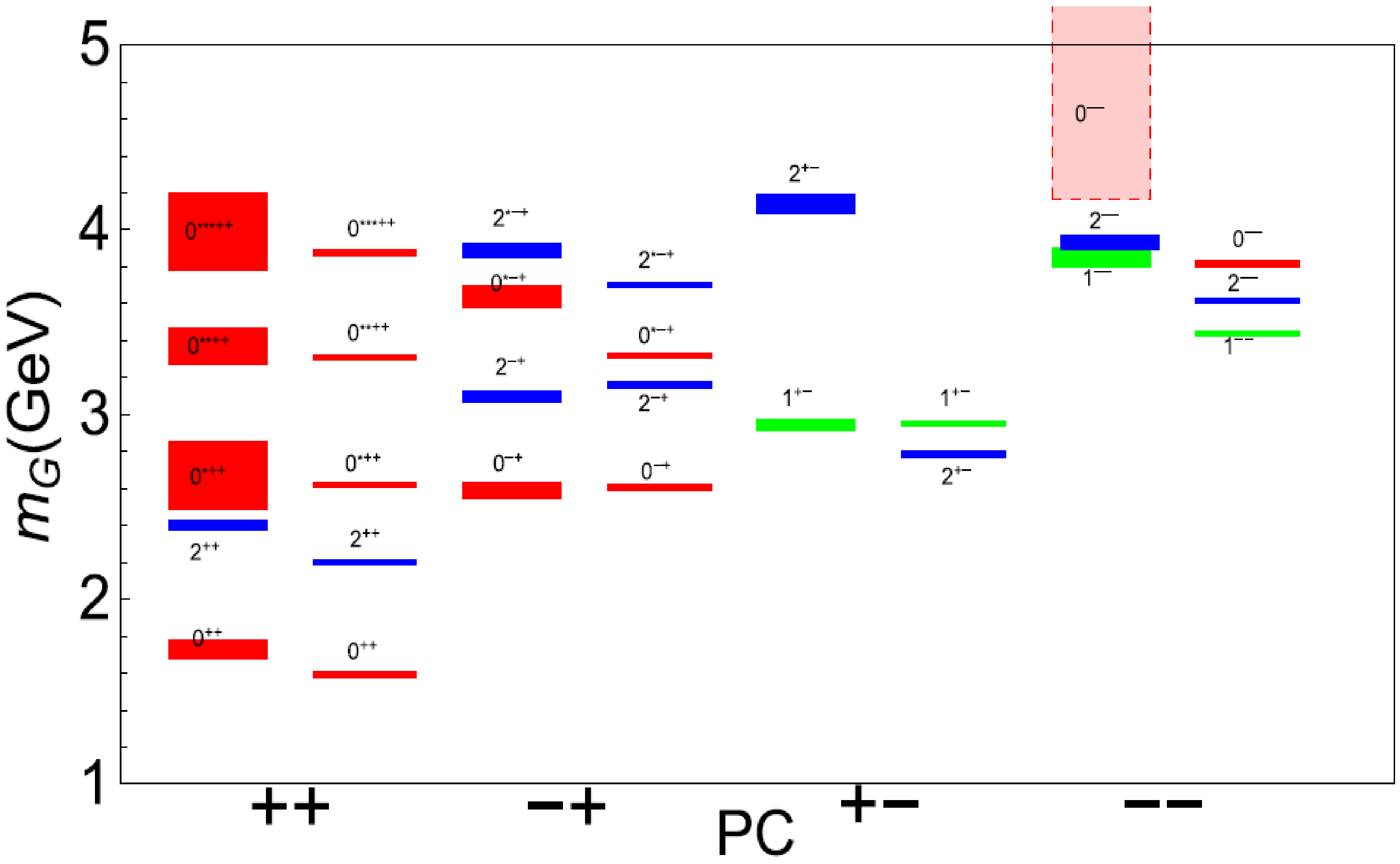
hep-lat/0508002.
[hep-lat/0510074].
[hep-lat/0103027].
[hep-lat/9901004]

Glueball spectra:

Yidian Chen, M.H., 1511.07018

J^{PC}	Operator	Dimension	Supergravity	M_5^2
0^{++}	$Tr(G^2)$	4	ϕ	0
0^{-+}	$Tr(G\tilde{G})$	4	C_τ	0
$1^{\pm-}$	$Tr(G\{G,G\})$	6	$B_{ij}, C_{ij\tau}$	15
2^{++}	$Tr(G_{\mu\alpha}G_{\alpha\nu} - \frac{1}{4}\delta_{\mu\nu}G^2)$	4	G_{ij}	4
2^{++}	$E_i^aE_j^a - B_i^aB_j^a - trace$	4	Absent	4
2^{-+}	$E_i^aB_j^a + B_i^aE_j^a - trace$	4	Absent	4
$2^{\pm-}$	$Tr(G\{G,G\})$	6	$B_{ij}, C_{ij\tau}$	16

Glueball spectra: **Yidian Chen, M.H., 1511.07018**



Agree well with lattice result except 0^{--} and 2^{+-} but …

J^{PC}	4-dimensional operator: $\mathcal{O}(x)$	Δ	p	M_5^2
0^{++}	$Tr(G^2) = \vec{E}^a \cdot \vec{E}^a - \vec{B}^a \cdot \vec{B}^a$	4	0	0
0^{-+}	$Tr(G\tilde{G}) = \vec{E}^a \cdot \vec{B}^a$	4	0	0
0^{+-}	$\mathrm{Tr}\left(\{(D_\tau G_{\mu\nu}), (D_\tau G_{\rho\nu})\}(D_\mu G_{\rho\alpha})\right)$	9	0	45
0^{--}	$\mathrm{Tr}\left(\{(D_\tau G_{\mu\nu}), (D_\tau G_{\rho\nu})\}\left(D_\mu \tilde{G}_{\rho\alpha}\right)\right)$	9	0	45
1^{-+}	$f^{abc}\partial_\mu \left[G^a_{\mu\nu}\right]\left[G^b_{\upsilon\rho}\right]\left[G^c_{\rho\alpha}\right]$, $f^{abc}\partial_\mu \left[G^a_{\mu\nu}\right]\left[\tilde{G}^b_{\upsilon\rho}\right]\left[\tilde{G}^c_{\rho\alpha}\right]$, $f^{abc}\partial_\mu \left[\tilde{G}^a_{\mu\nu}\right]\left[G^b_{\upsilon\rho}\right]\left[\tilde{G}^c_{\rho\alpha}\right]$, $f^{abc}\partial_\mu \left[\tilde{G}^a_{\mu\nu}\right]\left[\tilde{G}^b_{\upsilon\rho}\right]\left[G^c_{\rho\alpha}\right]$	7	1	24
1^{+-}	$d^{abc}\left(\vec{E}_a \cdot \vec{E}_b\right)\vec{B}_c$	6	1	15
1^{--}	$d^{abc}\left(\vec{E}_a \cdot \vec{E}_b\right)\vec{E}_c$	6	1	15
2^{++}	$E^a_i E^a_j - B^a_i B^a_j - trace$	4	2	4
2^{-+}	$E^a_i B^a_j + B^a_i E^a_j - trace$	4	2	4
2^{+-}	$d^{abc}\mathcal{S}\left[E^i_a \left(\vec{E}_b \times \vec{B}_c\right)^j\right]$	6	2	16
2^{--}	$d^{abc}\mathcal{S}\left[B^i_a \left(\vec{E}_b \times \vec{B}_c\right)^j\right]$	6	2	16
3^{+-}	$d^{abc}\mathcal{S}\left[B^i_a B^j_b B^k_c\right]$	6	3	15
3^{--}	$d^{abc}\mathcal{S}\left[E^i_a E^j_b E^k_c\right]$	6	3	15

$$S_{\mathscr{G}} = -\frac{1}{2}\int d^5x\sqrt{g_s}e^{-p\Phi}(\partial_M \mathscr{G}\partial^M \mathscr{G} + M^2_{\mathscr{G},5}(z)\mathscr{G}^2)$$

$$S_V = -\frac{1}{2}\int d^5x\sqrt{g_s}e^{-p\Phi}(\frac{1}{2}F^{MN}F_{MN} + M^2_{\mathscr{V},5}(z)\mathscr{V}^2),$$

$$S_T = -\frac{1}{2}\int d^5x\sqrt{g_s}e^{-p\Phi}(\nabla_L h_{MN}\nabla^L h^{MN} - 2\nabla_L h^{LM}\nabla^N h_{NM} + 2\nabla_M h^{MN}\nabla_N h - \nabla_M h\nabla^M h + M^2_{h,5}(z)(h^{MN}h_{MN} - h^2)),$$

$$-\mathscr{G}_n^{''} + V_{\mathscr{G}}\mathscr{G}_n = m^2_{\mathscr{G},n}\mathscr{G}_n,$$

$$V_{\mathscr{G}} = \frac{3A_s^{''} + \frac{3}{z^2} - p\Phi^{''}}{2} + \frac{\left[3A_s^{'} - \frac{3}{z} - p\Phi^{'}\right]^2}{4} + \frac{1}{z^2}e^{2A_s}e^{-c_{r.m.}\Phi}M^2_{\mathscr{G},5}.$$

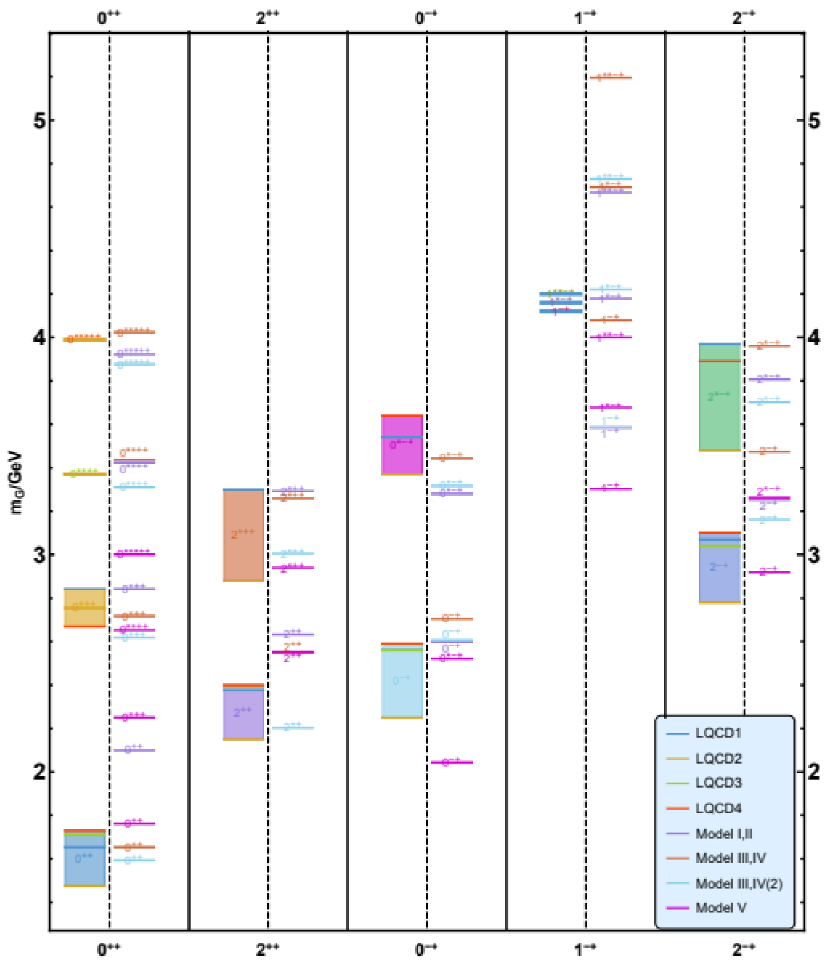
$$-\mathscr{V}_n^{''} + V_{\mathscr{V}}\mathscr{V}_n = m^2_{\mathscr{V},n}\mathscr{V}_n,$$

$$V_{\mathscr{V}} = \frac{A_s^{''} + \frac{1}{z^2} - p\Phi^{''}}{2} + \frac{\left[A_s^{'} - \frac{1}{z} - p\Phi^{'}\right]^2}{4} + \frac{1}{z^2}e^{2A_s}e^{-c_{r.m.}\Phi}M^2_{\mathscr{V},5}.$$

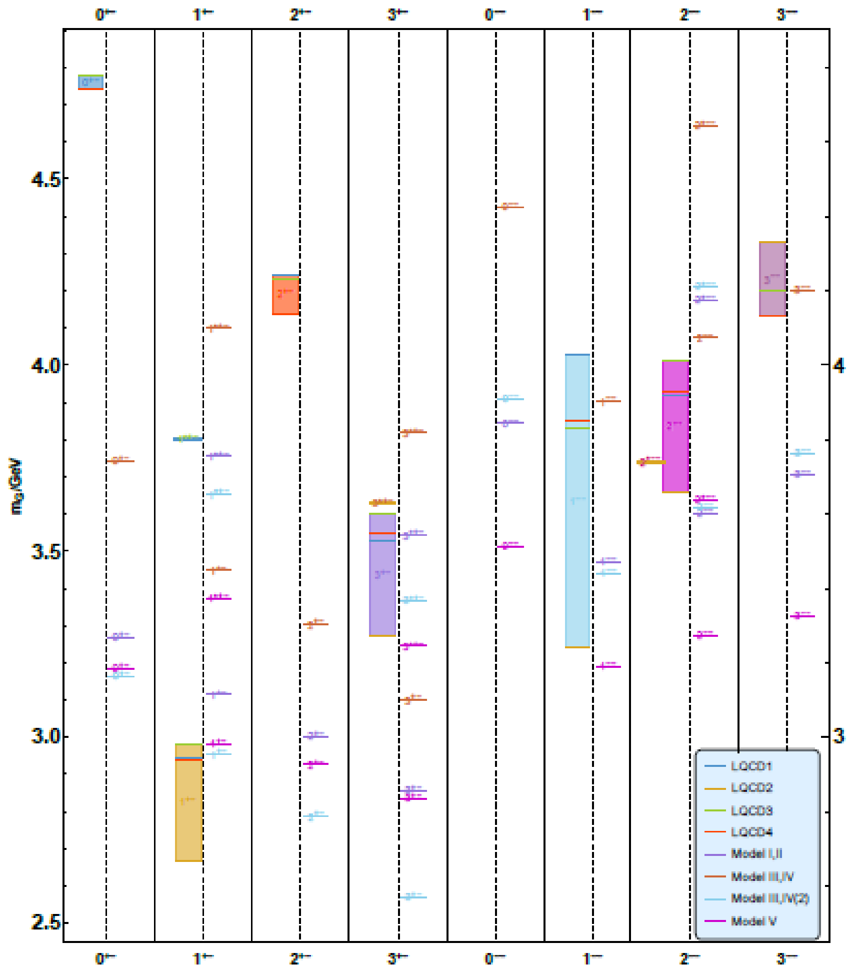
$$-\mathscr{T}_n^{''} + V_{\mathscr{T}}\mathscr{T}_n = m^2_{\mathscr{T},n}\mathscr{T}_n,$$

$$V_{\mathscr{T}} = \frac{3A_s^{''} + \frac{3}{z^2} - p\Phi^{''}}{2} + \frac{\left[3A_s^{'} - \frac{3}{z} - p\Phi^{'}\right]^2}{4} + \frac{1}{z^2}e^{2A_s}e^{-c_{r.m.}\Phi}M^2_{\mathscr{T},5}.$$

Lin Zhang, Chutian Chen, Yidian Chen, M.H.
Phys.Rev.D 105 (2022) 2, 026020



Glueball



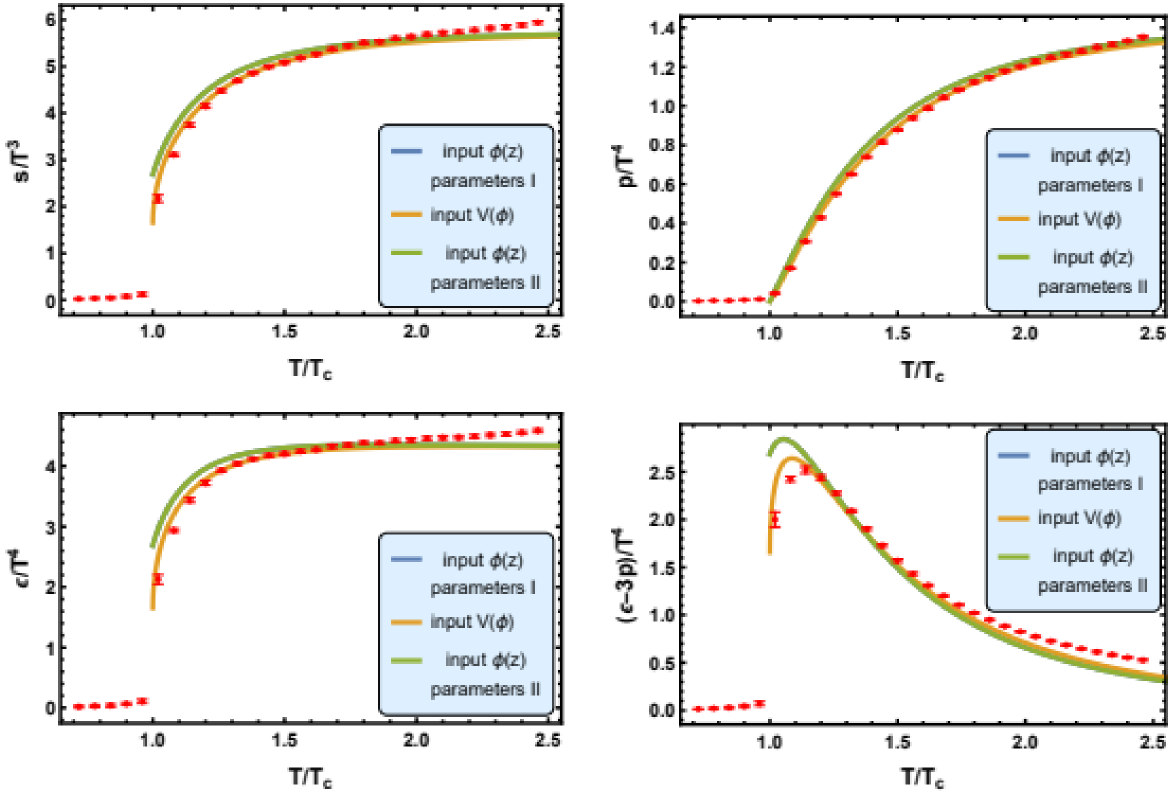
Odd ball

Lin Zhang, Chutian Chen, Yidian Chen, M.H.
Phys.Rev.D 105 (2022) 2, 026020

Model III： gluon background

$\phi(z) = c_1 z^2,$

Agree well with lattice results on EOS for pure gluon system



Lin Zhang, Chutian Chen, Yidian Chen, M.H.
Phys.Rev.D 105 (2022) 2, 026020

$\phi(z) = c_1 z^2,$

Quardratic dilaton field describes pure gluon system reasonably well.

Soft-wall AdS5 model or KKSS model

A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006)

AdS$_5$ metric

$$g_{MN}\, dx^M dx^N = e^{2A(z)}(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu)$$

$$A(z) = -\ln z, \;\; \Phi(z) = z^2$$

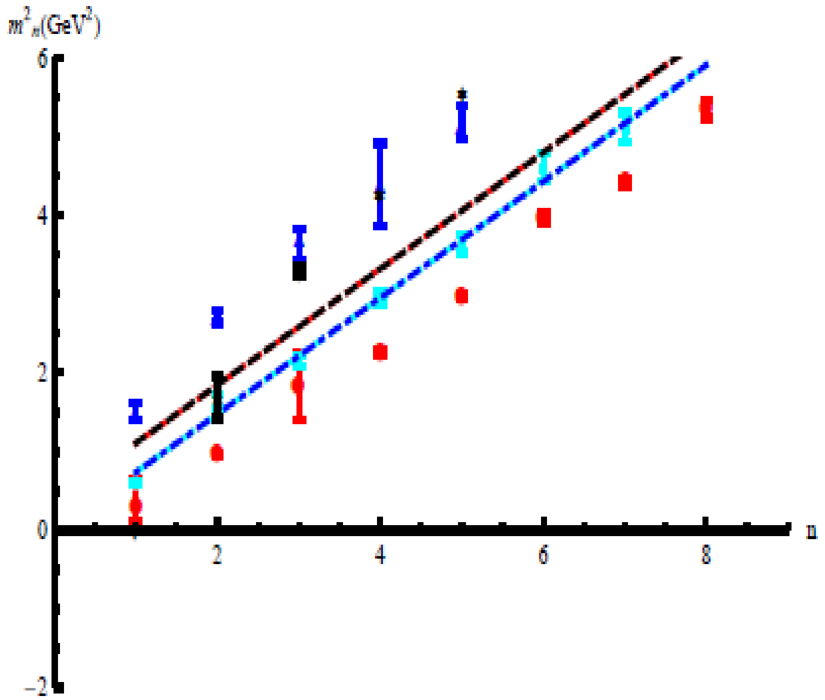
A dilaton field to restore Regge behavior

$$I = \int d^5x\, e^{-\Phi(z)}\sqrt{g}\left\{-|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\}$$

$$\boxed{M_{n,S}^2 = 4n + 4S}$$

However: only Coulomb potential, no linear quark potential

Degeneration of chiral partners in KKSS model



Light flavor meson spectra:

D.N. Li, M.H., JHEP2013, arXiv:1303.6929

Action for pure gluon system: Graviton-dilaton coupling

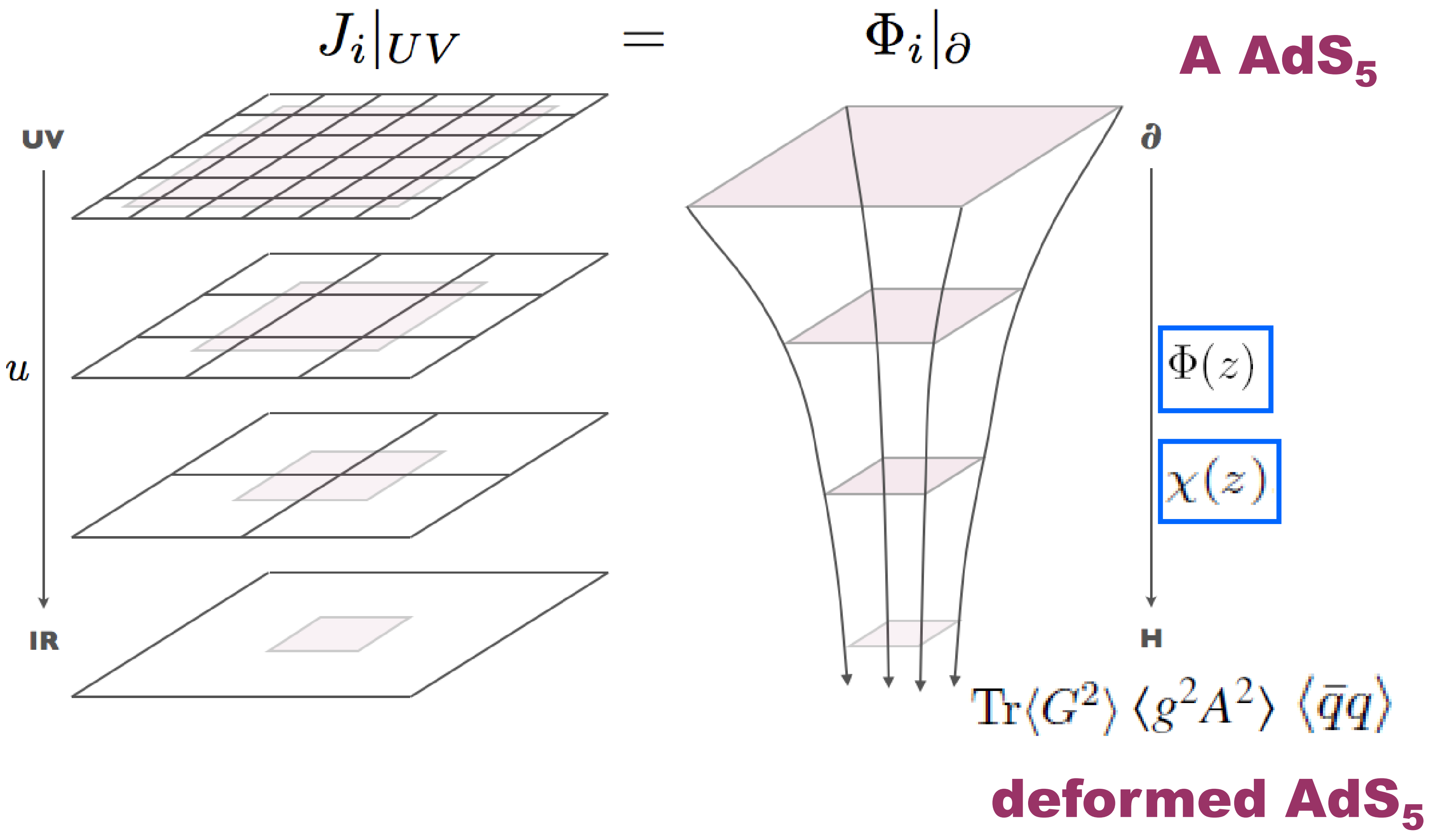
$$S_G = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\Phi}\left(R + 4\partial_M\Phi\partial^M\Phi - V_G(\Phi)\right)$$

Action for light hadrons: KKSS model

$$S_{KKSS} = -\int d^5x\sqrt{g_s}e^{-\Phi}Tr(|DX|^2 + V_X(X^+X,\Phi) + \frac{1}{4g_5^2}(F_L^2 + F_R^2)).$$

Total action: $S = S_G + \frac{N_f}{N_c}S_{KKSS}.$

Graviton-dilaton-scalar system



$$\textit{Dilaton in Mod I}: \quad \Phi(z) = \mu_G^2 z^2$$

$$\textit{Dilaton in Mod II}: \quad \Phi(z) = \mu_G^2 z^2 \tanh(\mu_{G^2}^4 z^2 / \mu_G^2)$$

	Mod IA	Mod IB	Mod IIA	Mod IIB
G_5/L^3	0.75	0.75	0.75	0.75
m_q (MeV)	5.8	5.0	8.4	6.2
$\sigma^{1/3}$ (MeV)	180	240	165	226
μ_G	0.43	0.43	0.43	0.43
μ_{G^2}	-	-	0.43	0.43

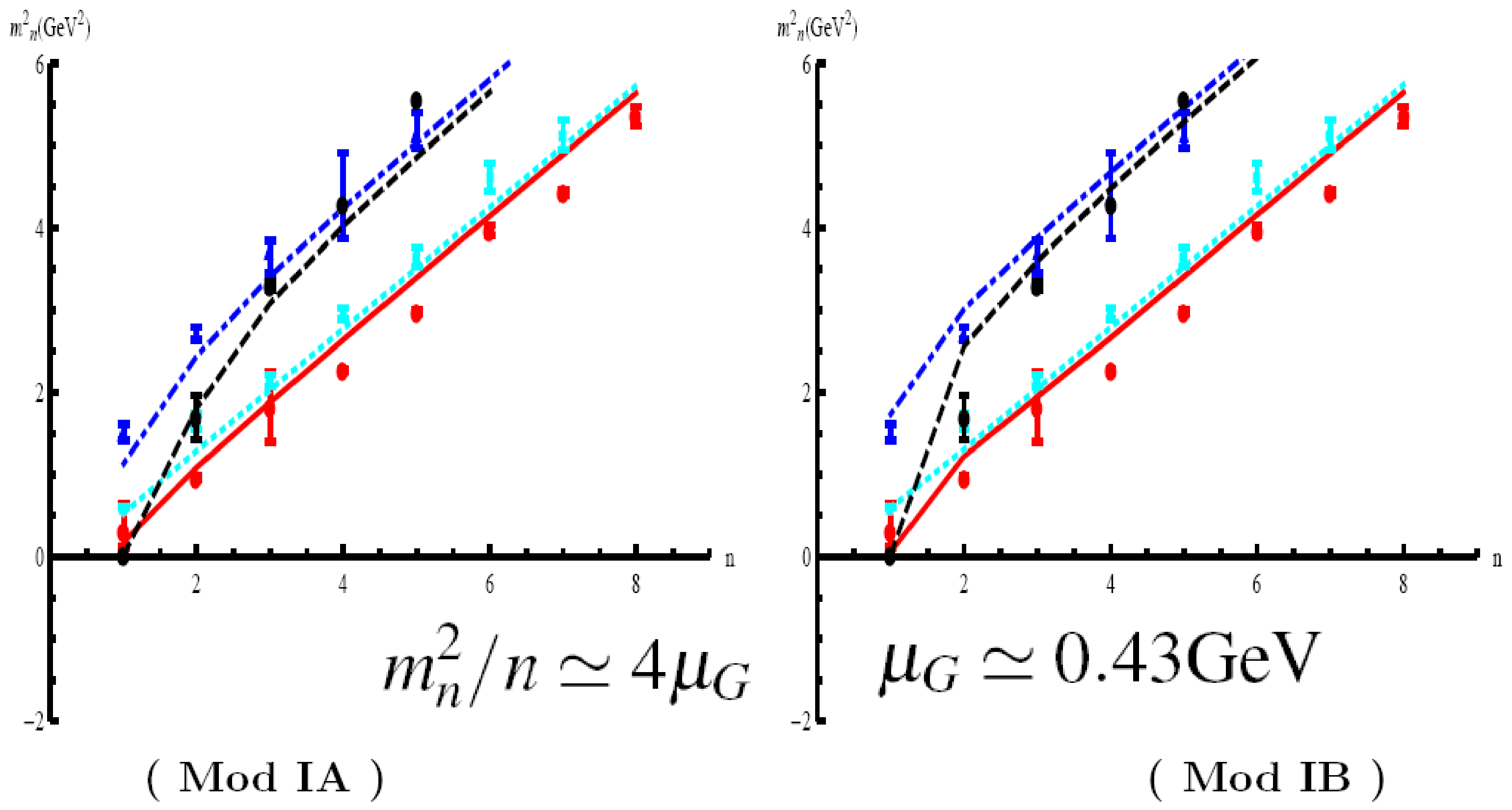
Table 7. Two sets of parameters.

$$-s_n'' + V_s(z)s_n = m_n^2 s_n,$$
$$-\pi_n'' + V_{\pi,\varphi}\pi_n = m_n^2(\pi_n - e^{A_s}\chi\varphi_n),$$
$$-\varphi_n'' + V_\varphi\varphi_n = g_5^2 e^{A_s}\chi(\pi_n - e^{A_s}\chi\varphi_n),$$
$$-v_n'' + V_v(z)v_n = m_{n,v}^2 v_n,$$
$$-a_n'' + V_a a_n = m_n^2 a_n,$$

$$V_s = \frac{3A_s'' - \phi''}{2} + \frac{(3A_s' - \phi')^2}{4} + e^{2A_s}V_{C,\chi\chi},$$
$$V_{\pi,\varphi} = \frac{3A_s'' - \phi'' + 2\chi''/\chi - 2\chi'^2/\chi^2}{2} + \frac{(3A_s' - \phi' + 2\chi'/\chi)^2}{4},$$
$$V_\varphi = \frac{A_s'' - \phi''}{2} + \frac{(A_s' - \phi')^2}{4},$$
$$V_v = \frac{A_s'' - \phi''}{2} + \frac{(A_s' - \phi')^2}{4},$$
$$V_a = \frac{A_s' - \phi'}{2} + \frac{(A_s' - \phi')^2}{4} + g_5^2 e^{2A_s}\chi^2.$$

Produced hadron spectra compared with data

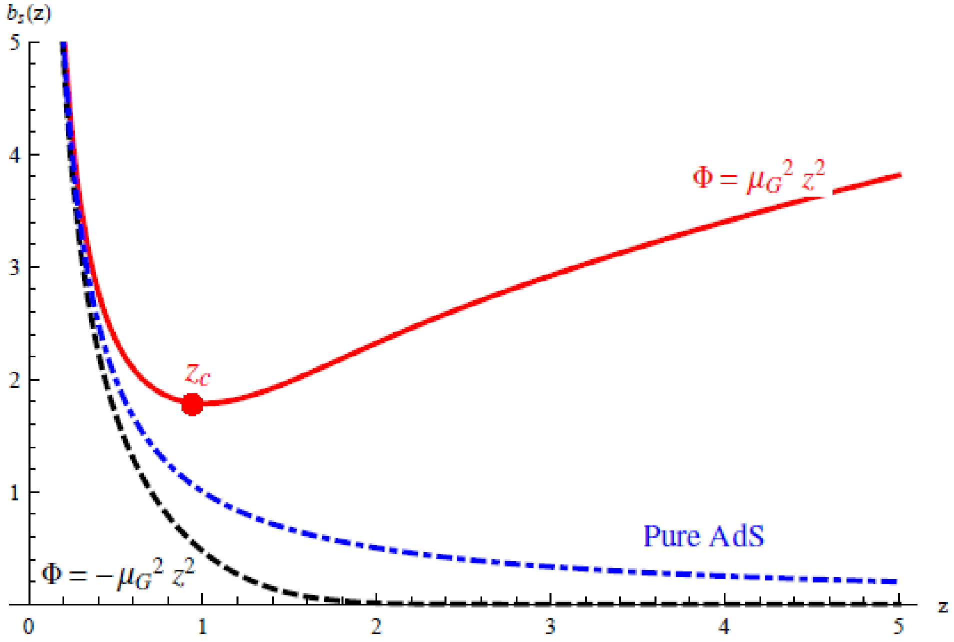
D.N. Li, M.H., JHEP2013, arXiv:1303.6929



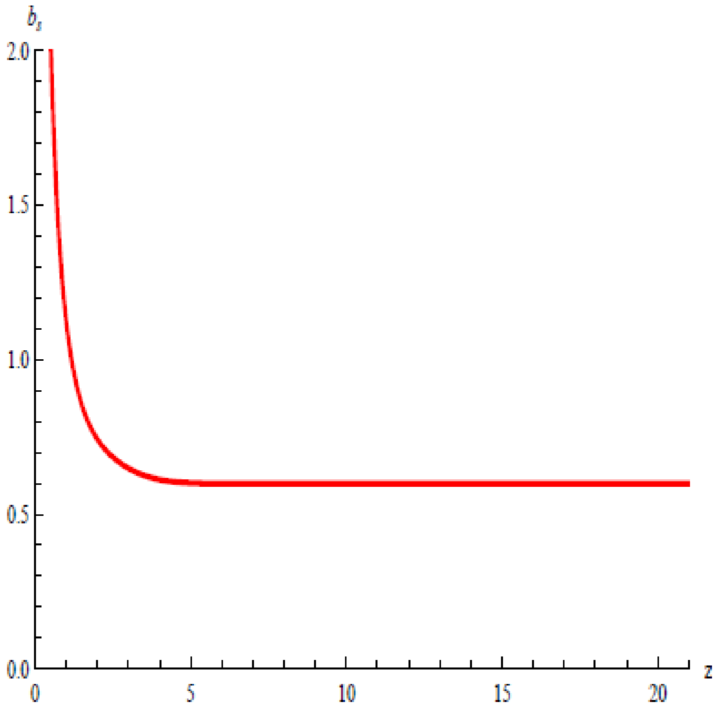
Ground states: chiral symmetry breaking

Excitation states: linear confinemnt

Quenched background



Unquenched background



$$-A_s'' + A_s'^2 + \frac{2}{3}\Phi'' - \frac{4}{3}A_s'\Phi' - \frac{\lambda}{6}e^{\Phi}\chi'^2 = 0,$$

$$\Phi'' + (3A_s' - 2\Phi')\Phi' - \frac{3\lambda}{16}e^{\Phi}\chi'^2 - \frac{3}{8}e^{2A_s - \frac{4}{3}\Phi}\partial_\Phi\left(V_G(\Phi) + \lambda e^{\frac{7}{3}\Phi}V_C(\chi,\Phi)\right) = 0,$$

$$\chi'' + (3A_s' - \Phi')\chi' - e^{2A_s}V_{C,\chi}(\chi,\Phi) = 0.$$

Quenched gluodynamics +flavor dynamics

$$S = S_b + S_m,$$

$$S_b = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g^s}e^{-2\phi}[R^s + 4\partial_\mu\phi\partial^\mu\phi - V_s(\phi) - \frac{h(z)}{4}e^{\frac{4\phi}{3}}F_{\mu\nu}F^{\mu\nu}],$$

Gluon Background

$$S_m = -\int d^5x\sqrt{-g^s}e^{-\phi}Tr[\nabla_\mu X^\dagger\nabla^\mu X + V_X(|X|, F_{\mu\nu}F^{\mu\nu})].$$

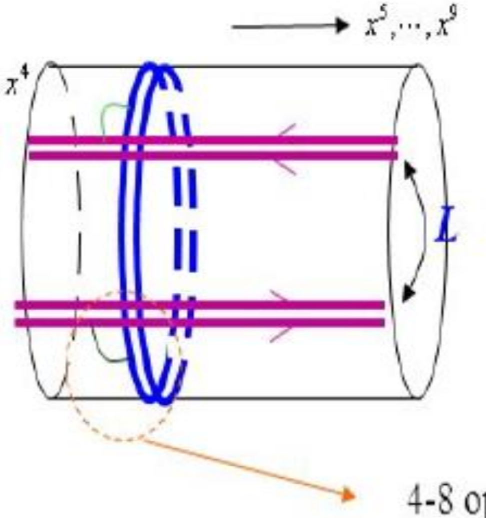
Matter part

Dynamical holographic QCD   Graviton-dilaton-scalar system

	Gluodynamics	Quark dynamics
DhQCD	Dilaton background	Flavor background
SS:D4-D8 D3-D7	Dp brane: D4, D3	Dq brane: D8, D7
PNJL	Polyakov-loop potential	NJL model

Interplay between gluodynamics and quark dynamics!!!

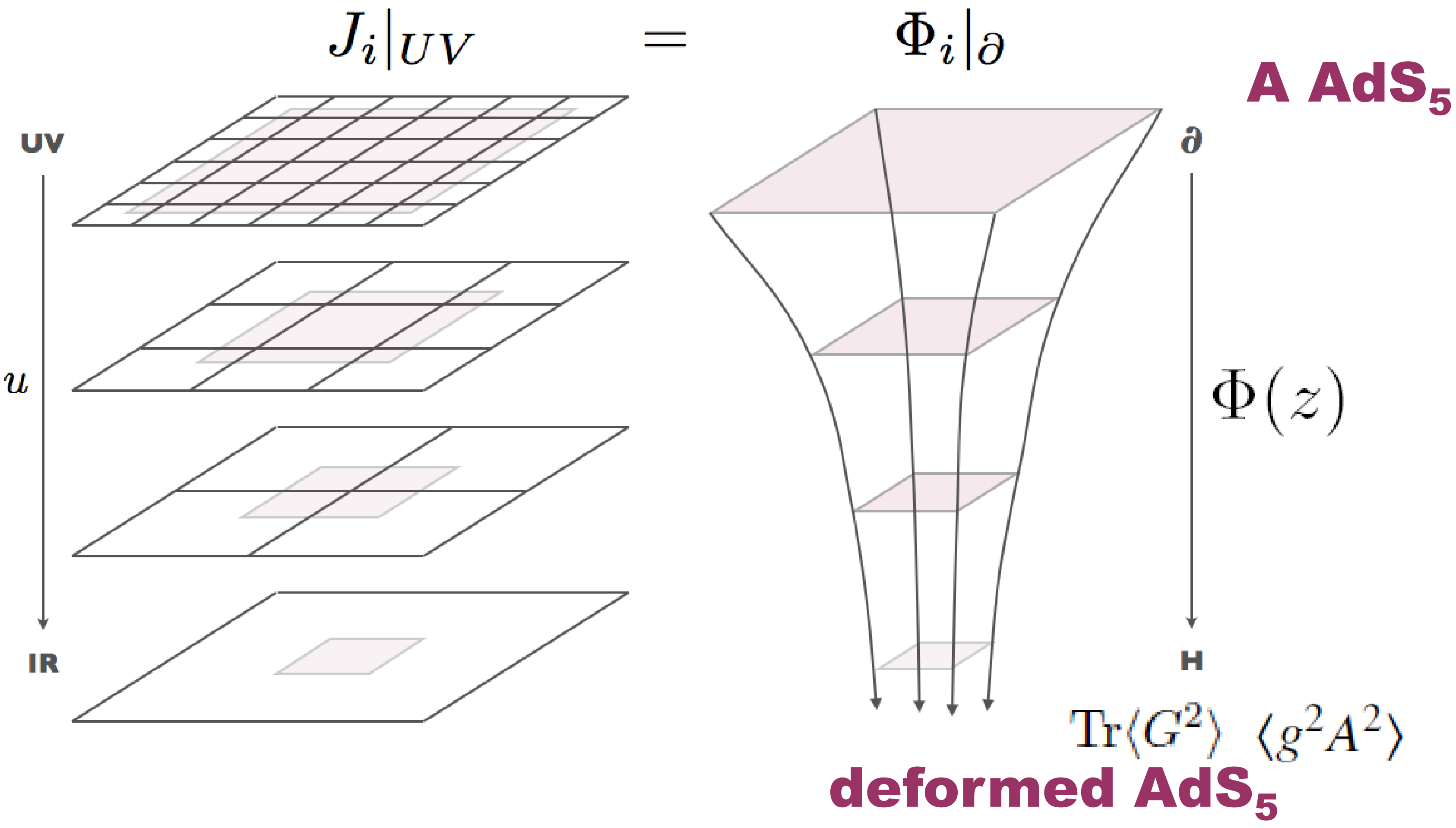
Comparing with the Witten-Sakai-Sugimoto model



	0	1	2	3	4	5	6	7	8	9
N_c D4	o	o	o	o	o					
N_f $D8 - \overline{D8}$	o	o	o	o		o	o	o	o	o

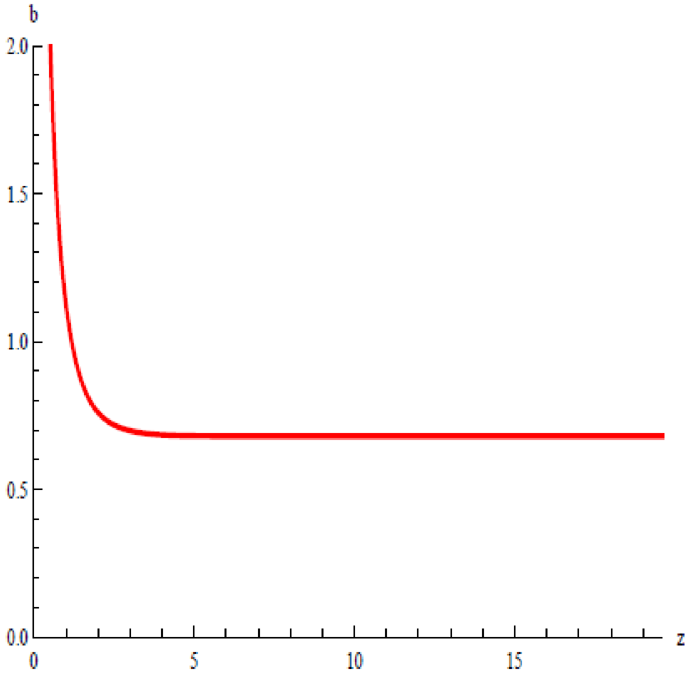
4-8 open strings give chiral (from D8) and anti-chiral (from anti-D8) fermions in the fundamental representation.

Graviton-dilaton system

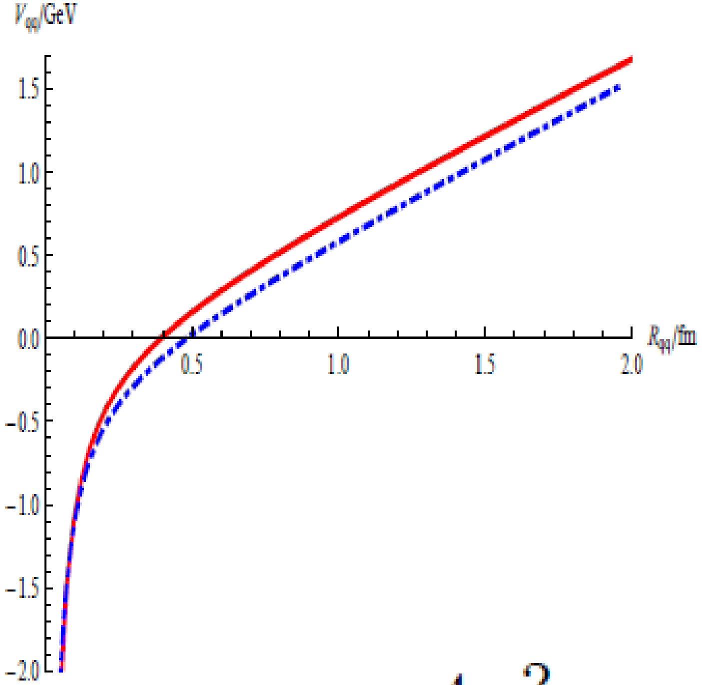


$$g^s_{MN} = b_s^2(z)(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu), \quad b_s(z) \equiv e^{A_s(z)}$$

Solved Metric

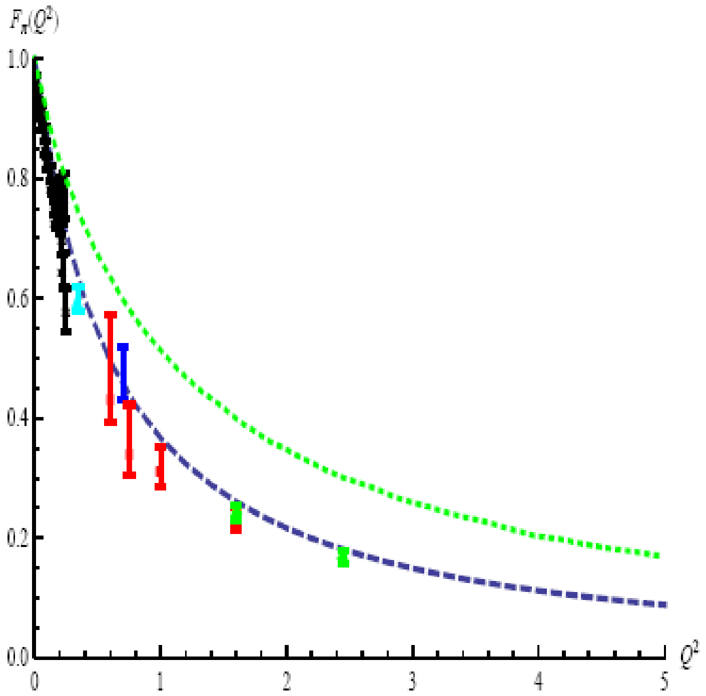


Produced quark potential compared with Cornell potential

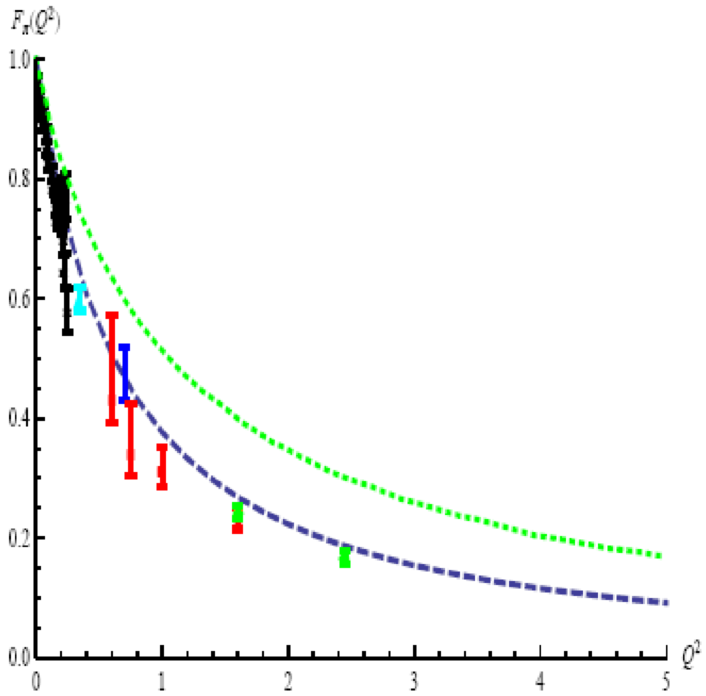


$$\sigma_s \sim 4\mu^2$$

Smaller chiral condensate, smaller pion decay constant, better pion form factor



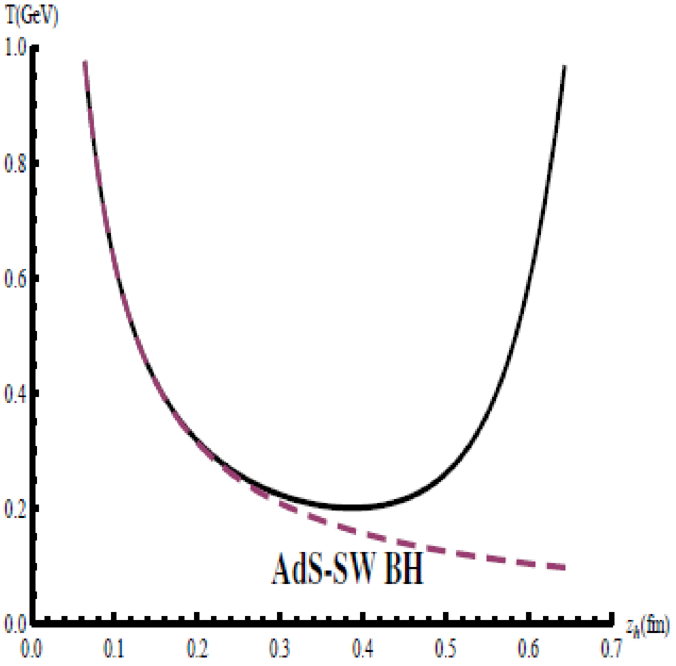
(Mod IA)

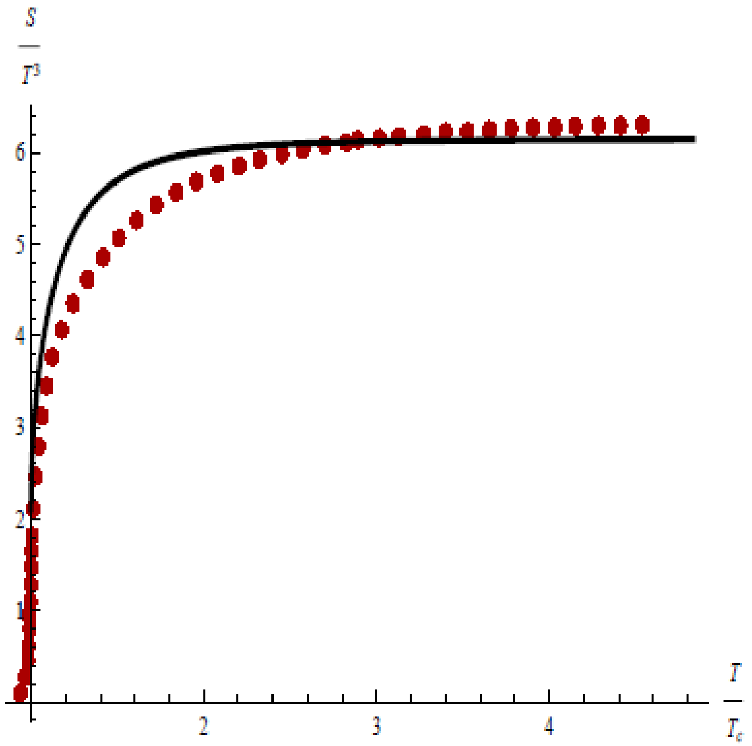


(Mod IIA)

$$T = \frac{|f'(z_h)|}{4\pi}.$$

$$s = \frac{A_{area}}{4G_5 V_3} = \frac{L^3}{4G_5}\left(\frac{e^{A_s - \frac{2}{3}\phi}}{z}\right)^3.$$



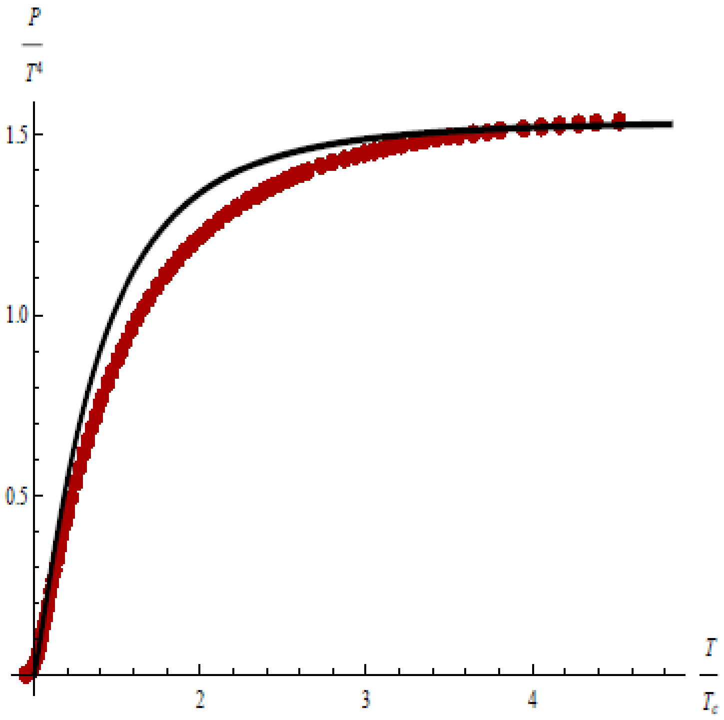


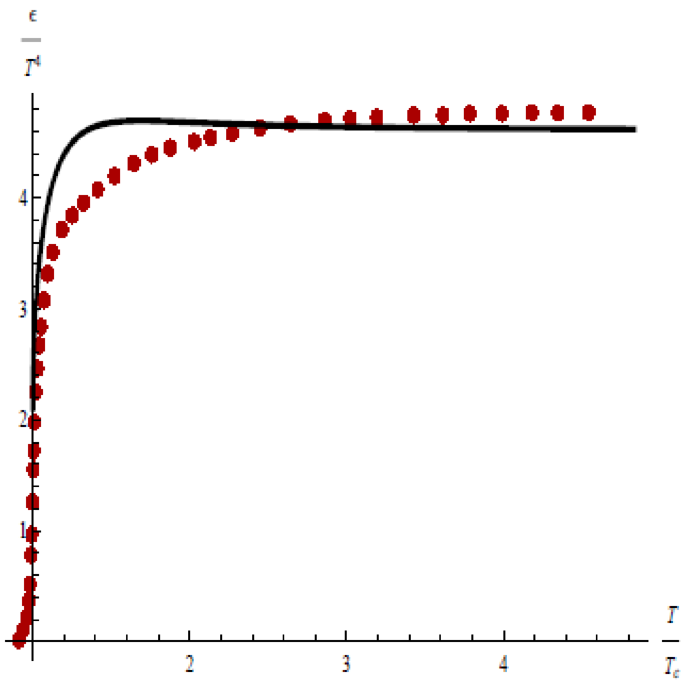
$$T_c = 201\text{MeV}$$

D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

$$\frac{dp(T)}{dT} = s(T).$$

$$\epsilon = -p + sT.$$

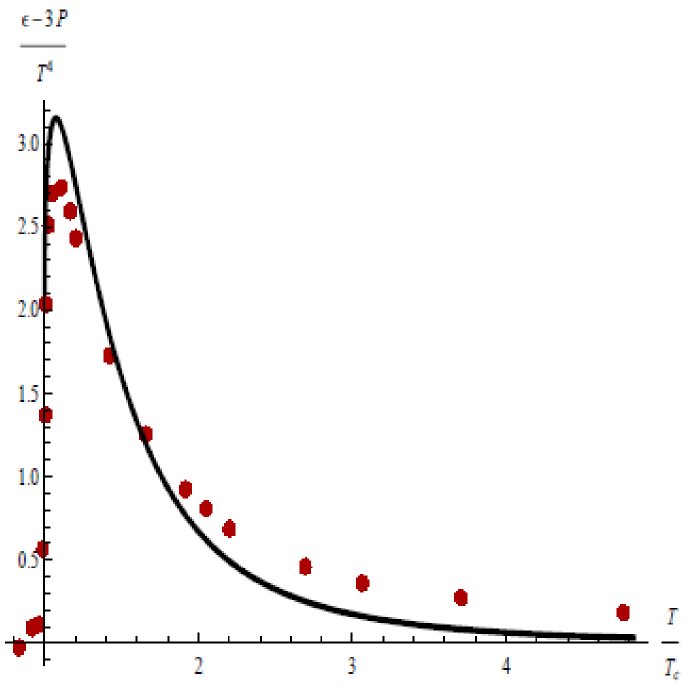


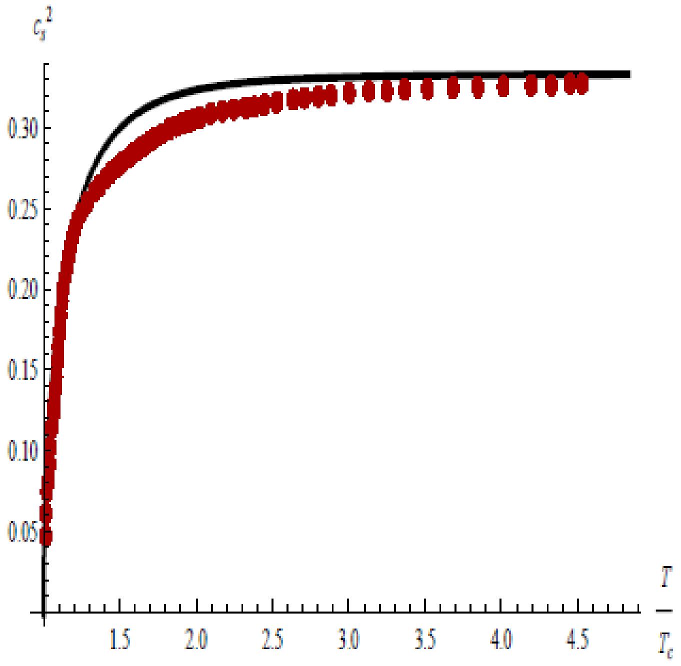


D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

Trace anomaly

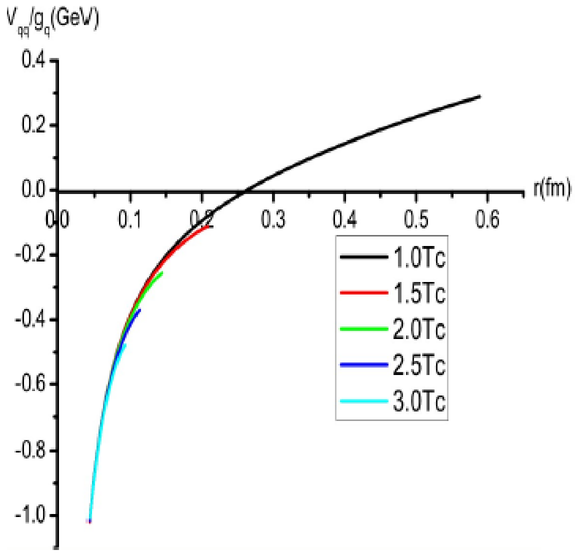
$$c_s^2 = \frac{d \log T}{d \log s} = \frac{s}{T ds/dT},$$



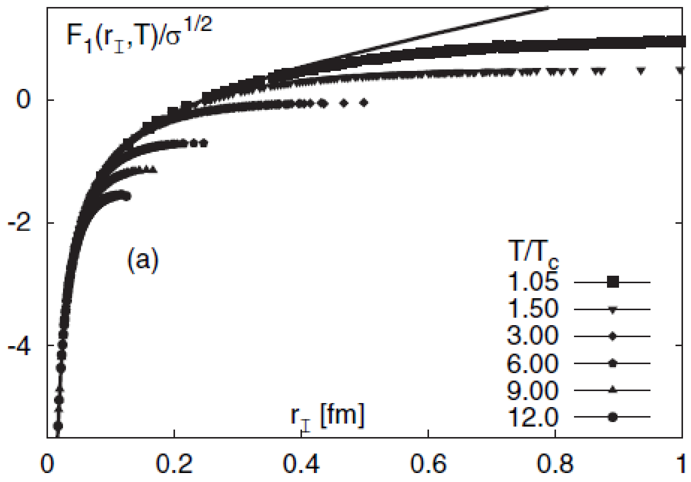


D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

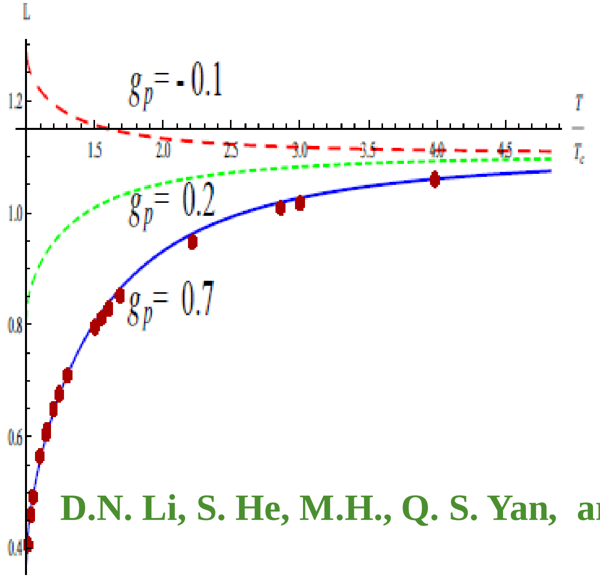
Electric screening



Heavy quark potential

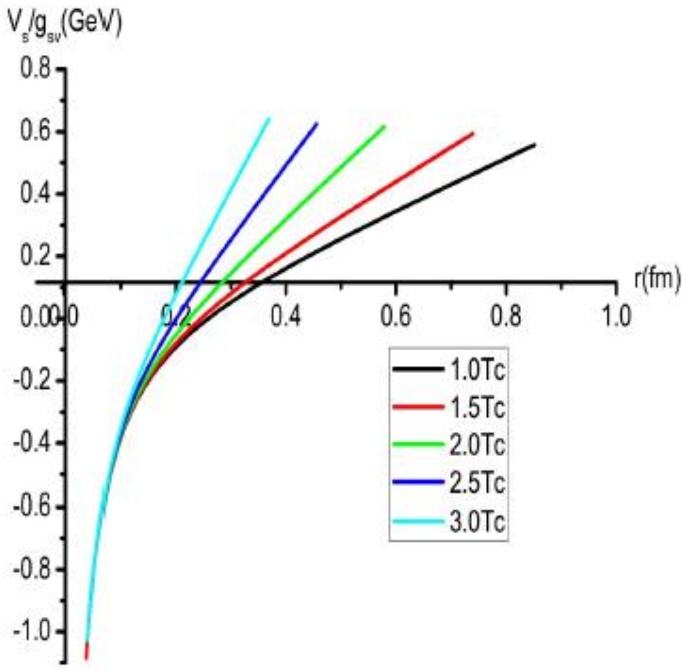


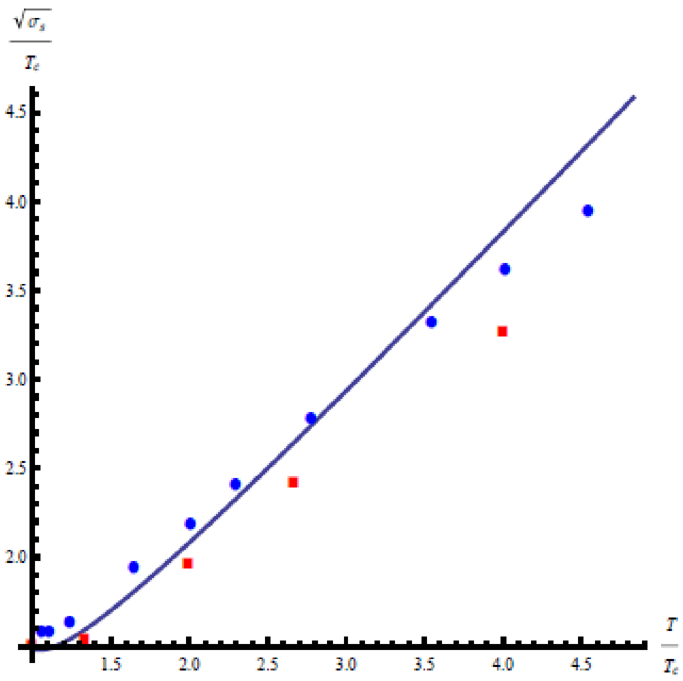
Polyakov loop: color electric deconfinement



D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

Magnetic screening and magnetic confinement

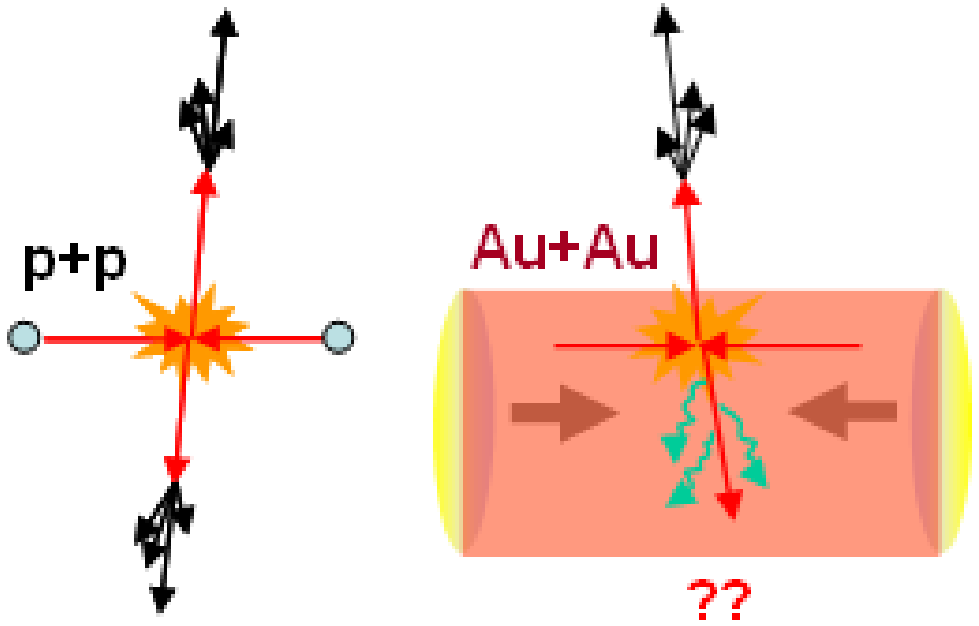




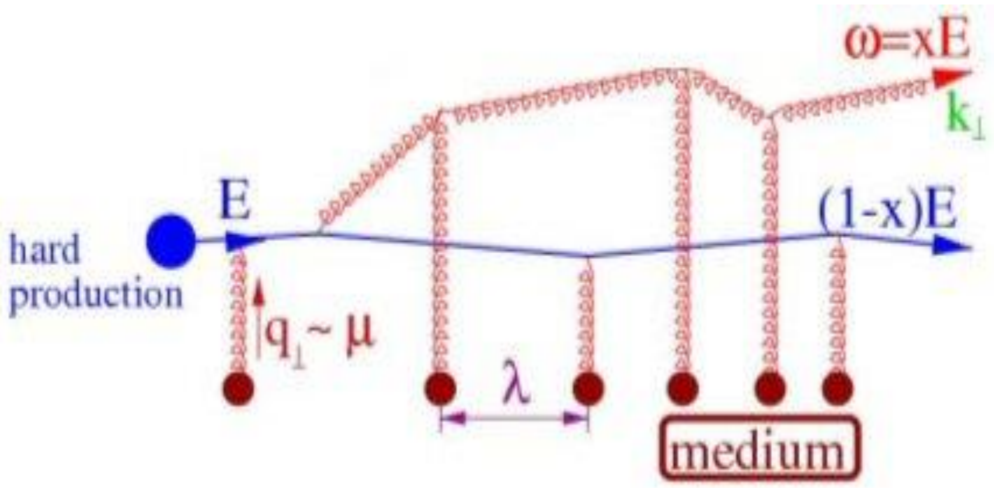
spatial Wilson loop spatial string tension

D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

V. HQCD and Jet quenching



Parton energy loss in QGP



The dominant effect of the medium on a high energy parton is medium-induced Bremsstrahlung.

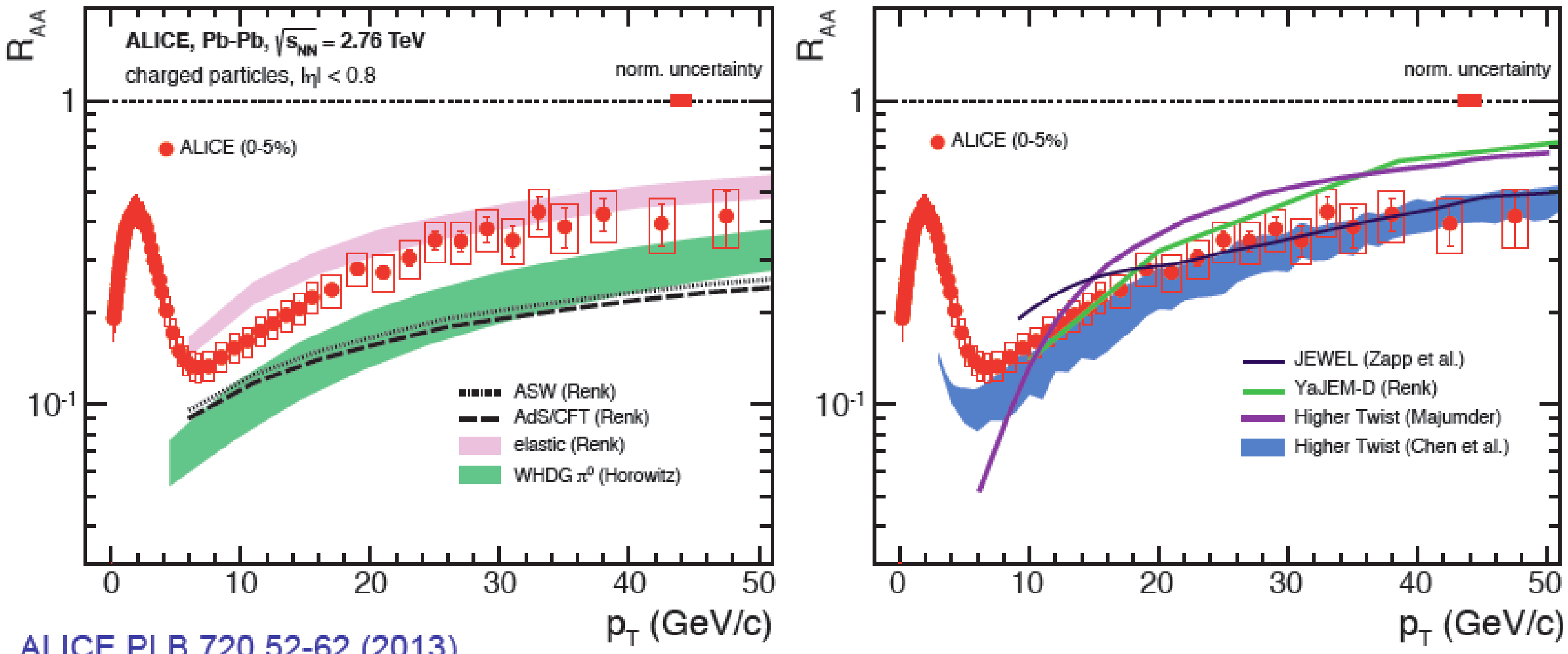
$$\Delta E \approx -\frac{\alpha_S}{2\pi} N_C \, \hat{q} \, L^2$$

Baier, Dokshitzer, Mueller, Peigne, Schiff (1996):

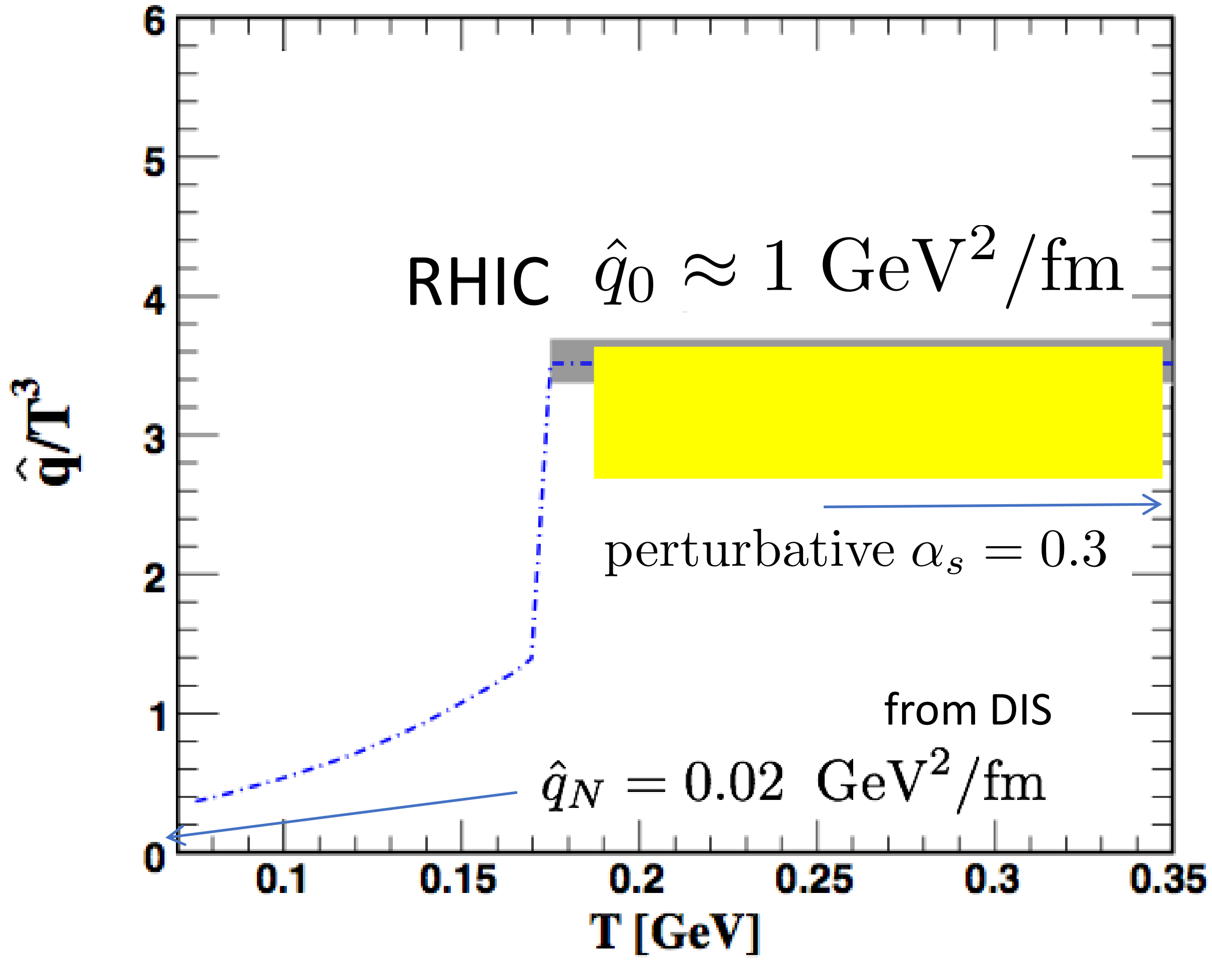
$\hat{q}$: reflects the ability of the medium to “quench” jets.

$$\hat{q} = \frac{\langle k_T^2 \rangle}{L} \approx \frac{\mu^2}{\lambda}$$

μ : Debye mass λ : mean free path



ALICE PLB 720 52-62 (2013)

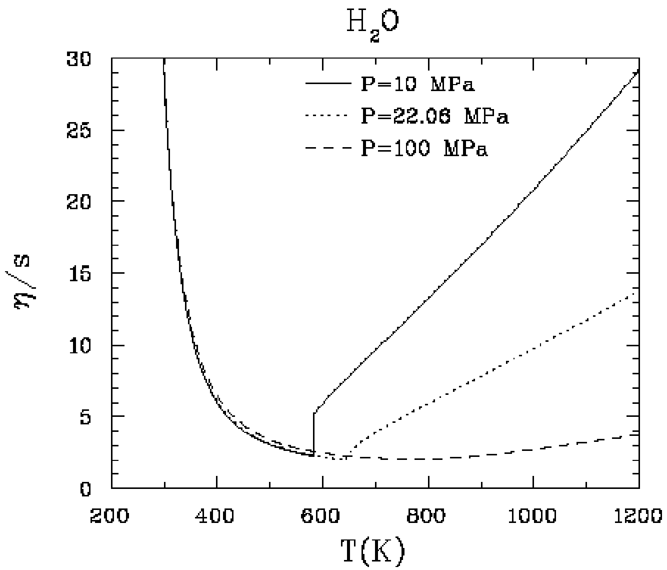


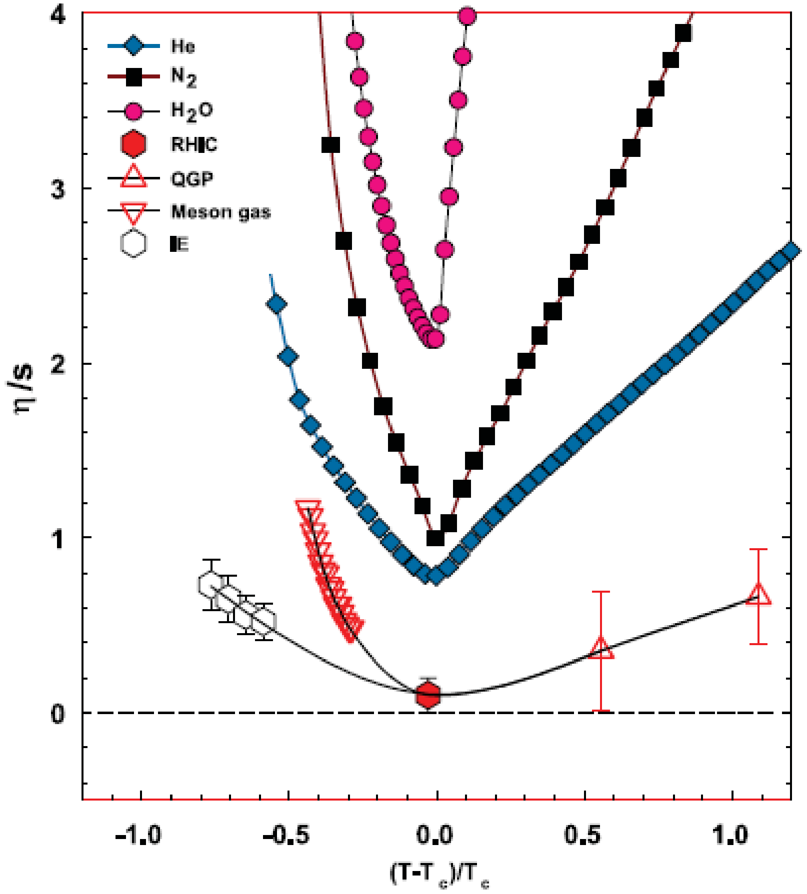
Chen, Greiner, Wang, XNW, Xu, PRC 81 (2010) 064908

Jet quenching characterizing phase transition?

$$\frac{\eta_A}{s} = \frac{8\pi^2}{63}\frac{T^3}{\hat{q}}$$

Majumder,Muller,Wang, PRL



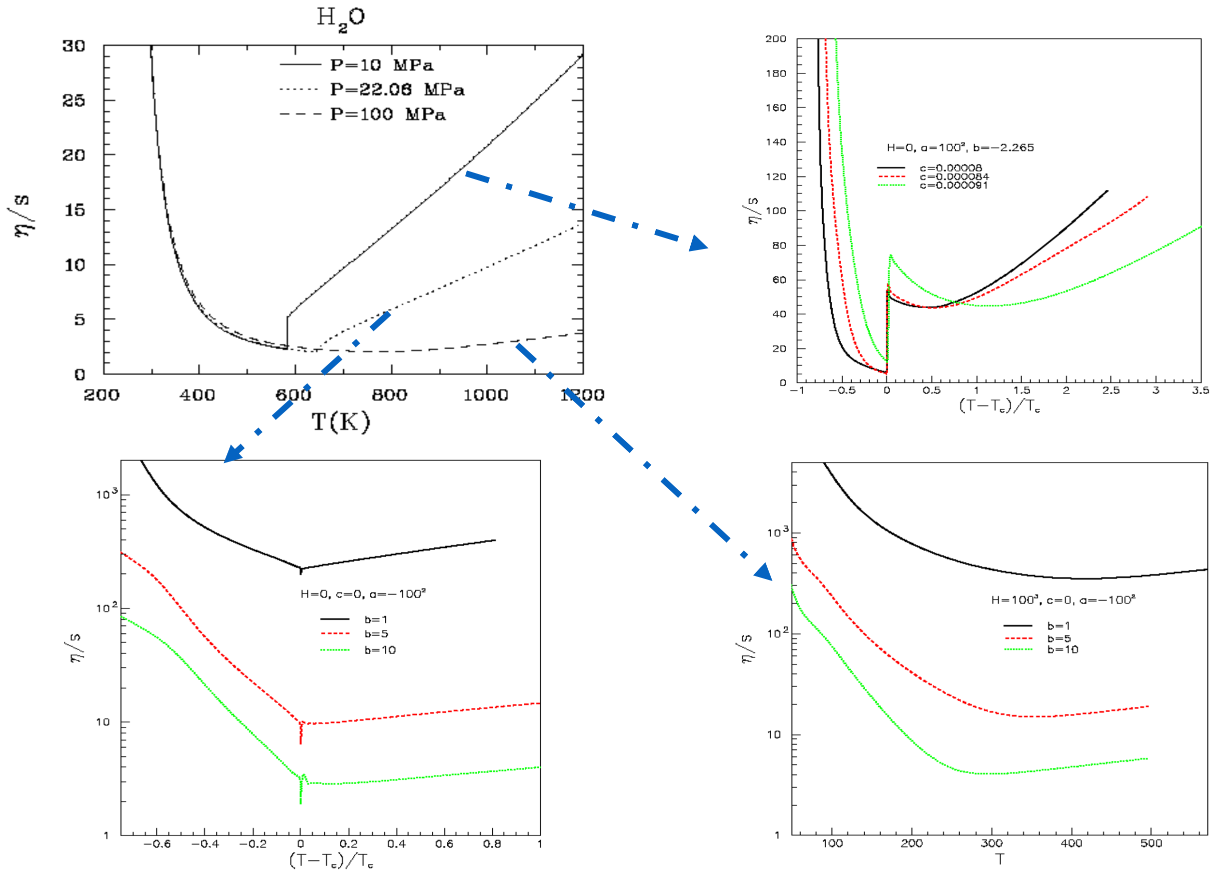


Csernai et al, Phys.Rev.Lett.97:152303,2006

Lacey et al., PRL 98:092301,2007

η/s characterizes phase transitions CJT+Boltzmann Eq

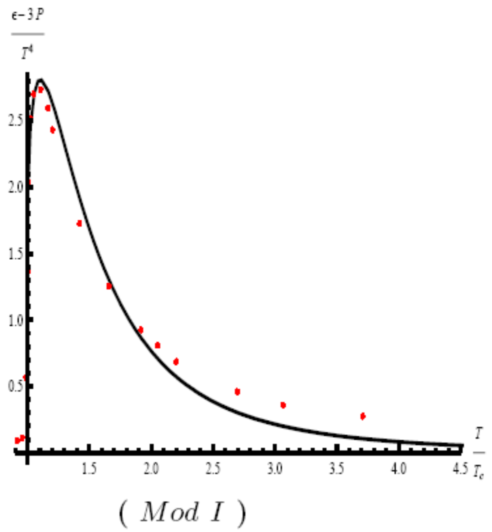
J.W Chen, MH, Y.H. Li, E. Nakano, D.L.Yang, Phys.Lett.B670:18-21,2008, arXiv: 0709.3434

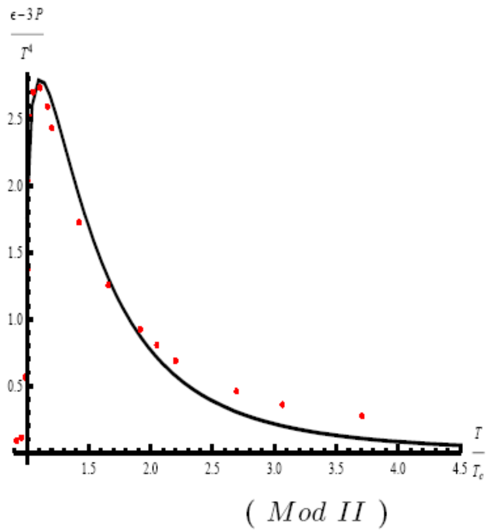


1, Minimum at Tc, most difficult condition for momentum transportation.

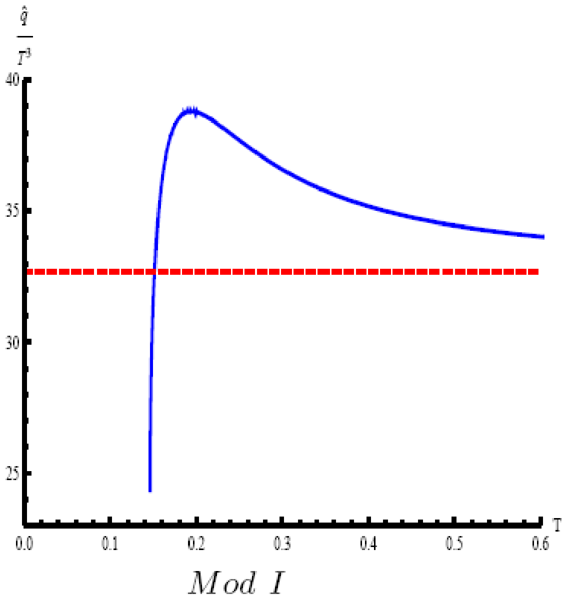
2. The value of η/s at phase transition decreases with increases of coupling strength

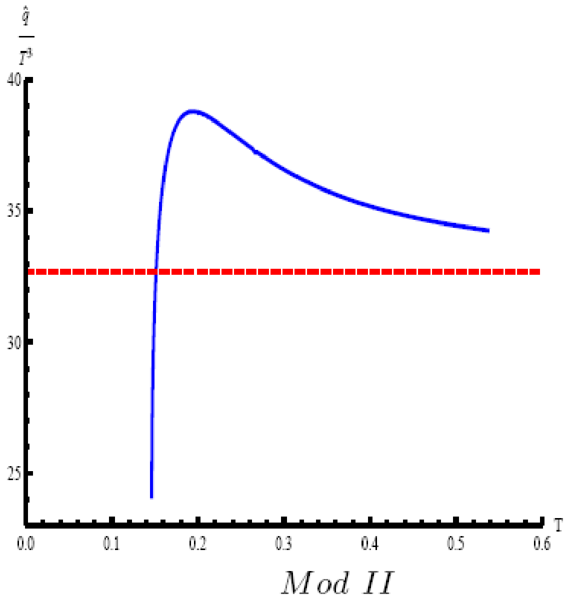
Jet quenching characterizing phase transition!



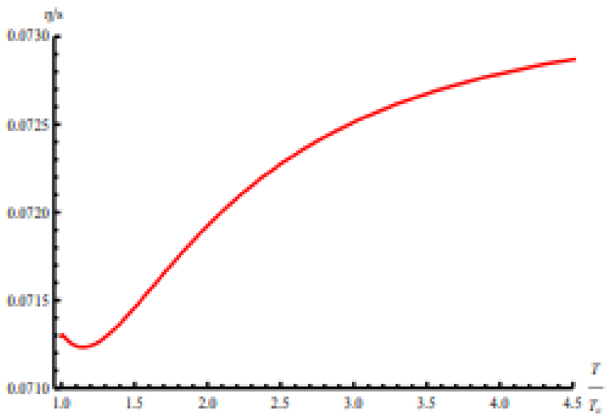


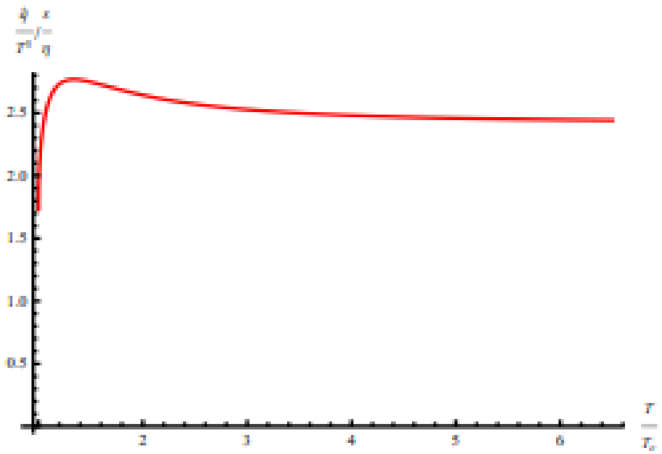
Danning Li, J.F.Liao, M.H.
Phys.Rev.D 89 (2014) 12, 126006

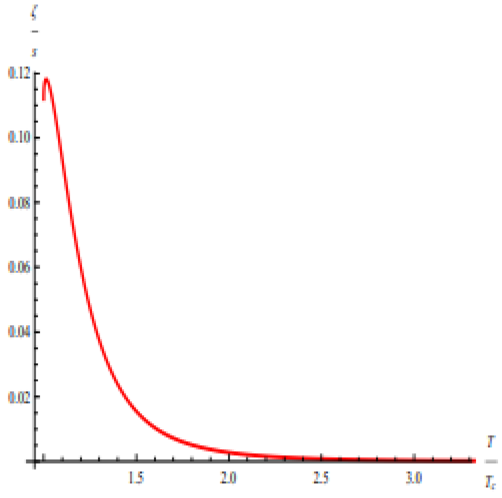




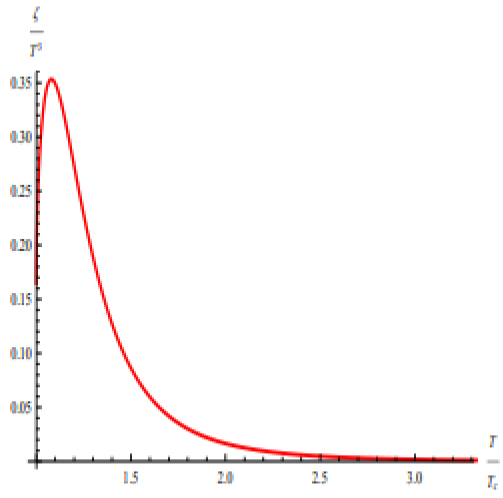
Shear/bulk viscosity characterizing phase transition!







(a)



(b)

D.N.Li, S.He, M. H. JHEP 06 (2015) 046

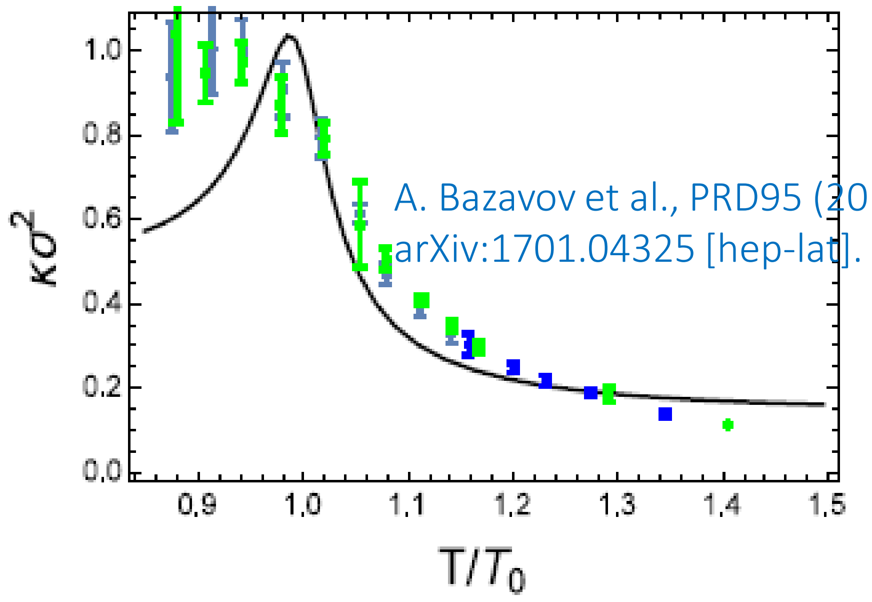
Quenched gluodynamics +flavor dynamics

$$
\begin{aligned}
S &= S_b + S_m, \\
S_b &= \frac{1}{16\pi G_5}\int d^5x\sqrt{-g^s}e^{-2\phi}[R^s + 4\partial_\mu\phi\partial^\mu\phi - V_s(\phi) - \frac{h(z)}{4}e^{\frac{4\phi}{3}}F_{\mu\nu}F^{\mu\nu}], \\
S_m &= -\int d^5x\sqrt{-g^s}e^{-\phi}Tr[\nabla_\mu X^\dagger\nabla^\mu X + V_X(|X|, F_{\mu\nu}F^{\mu\nu})].
\end{aligned}
$$

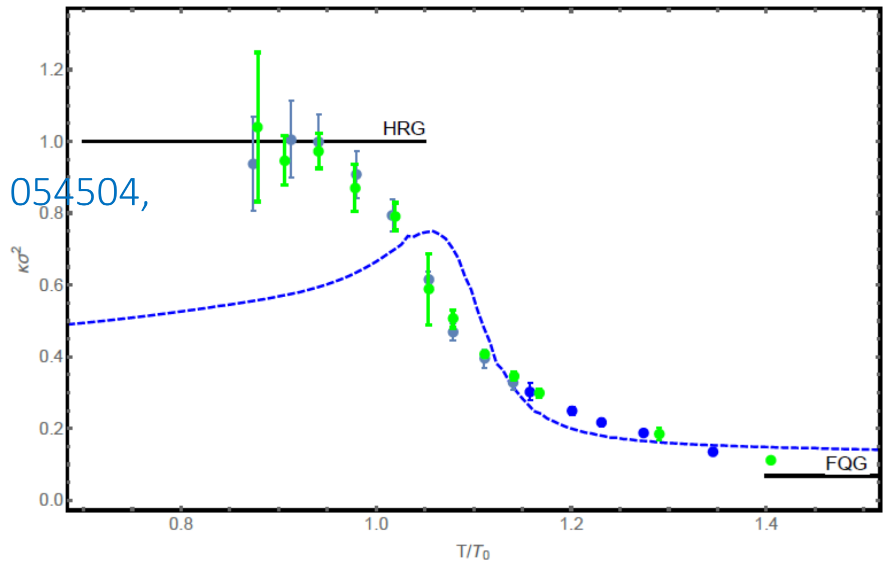
Gluon Background

Matter part

Baryon number fluctuations at mu=0



A. Bazavov et al., PRD95 (2017) 054504,
arXiv:1701.04325 [hep-lat].



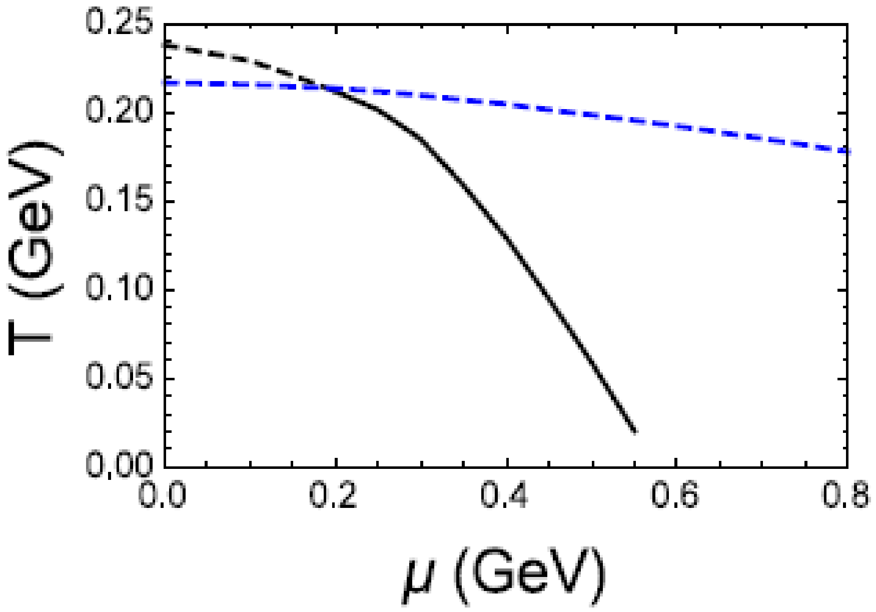
DhQCD

PNJL

Xun Chen, Danning Li, M.H,
JHEP 03 (2020) 073

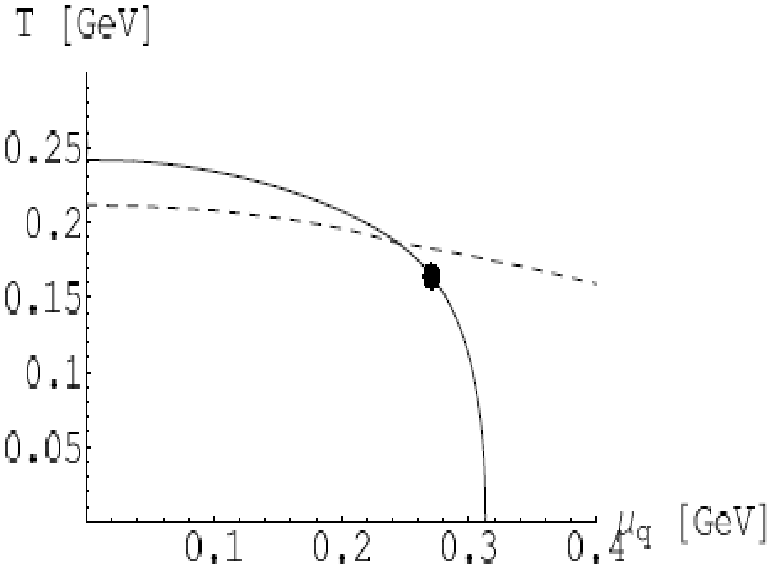
Z.B Li, K.Xu,X.Y.Wang, M.H.
arXiv:1801.09215

Quenched result: Quarkyonic phase



DhQCD

Xun Chen, Danning Li, M.H,
JHEP 03 (2020) 073



PNJL

Sasaki, Friman, Redlich,
hep-ph/0611147

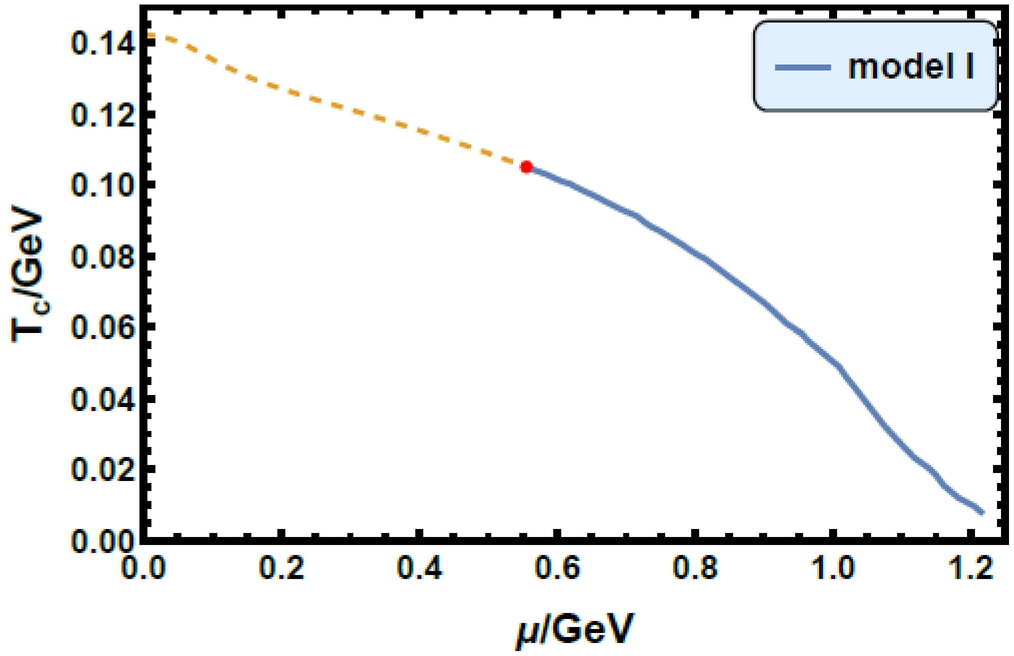
EOS of dense QCD matter

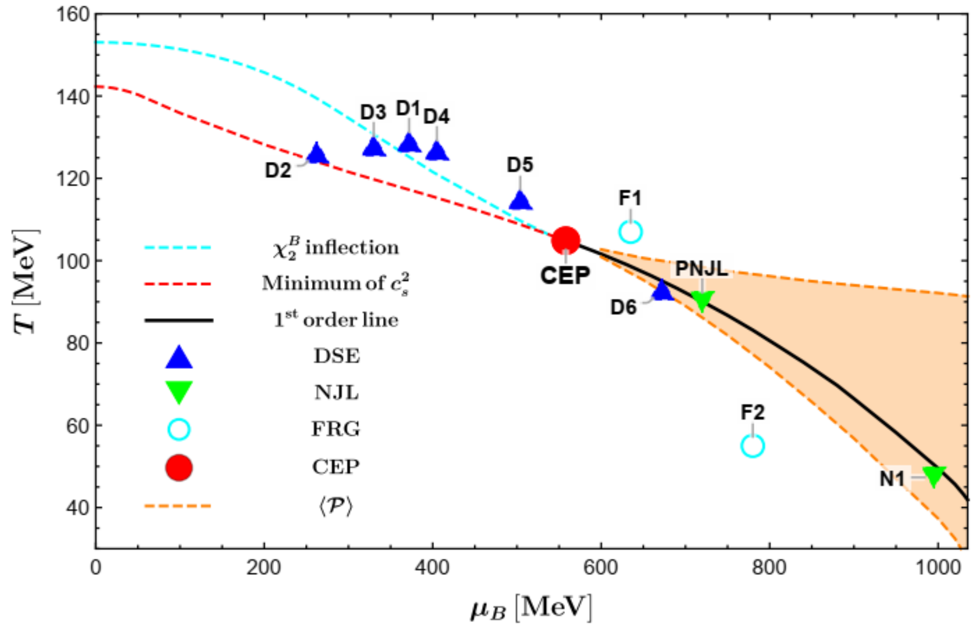
Model I: Gubser Model, extended by Song He et.al

$$V_\phi(\phi) = -12\cosh(c_1\phi) + (6c_1^2 - \frac{3}{2})\phi^2 + c_2\phi^6,$$

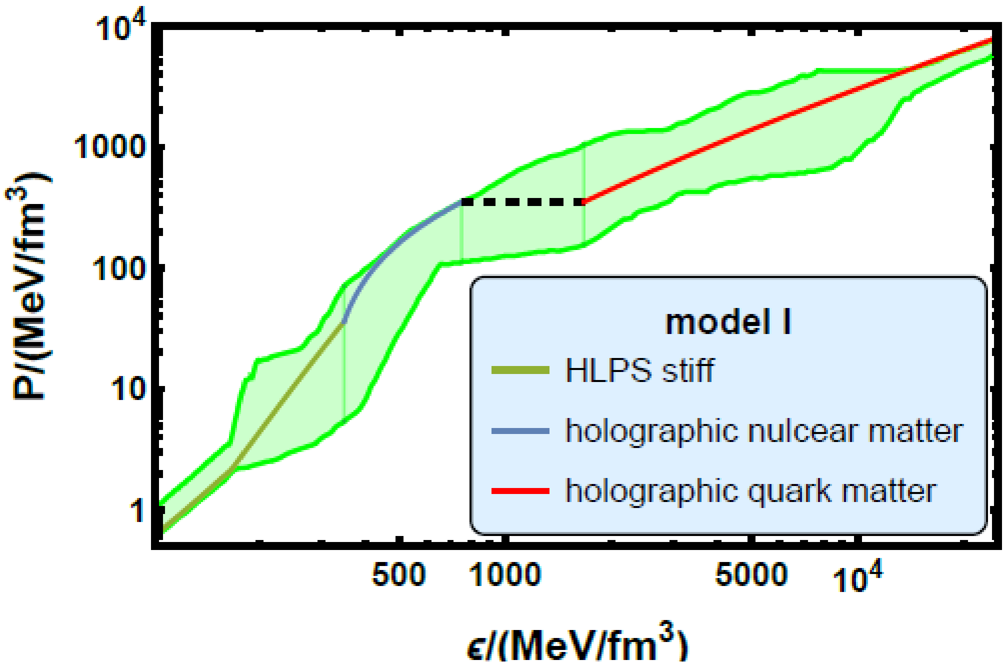
$$h_\phi(\phi) = \frac{1}{1+c_3}\mathrm{sech}(c_4\phi^3) + \frac{c_3}{1+c_3}\mathrm{e}^{-c_5\phi},$$

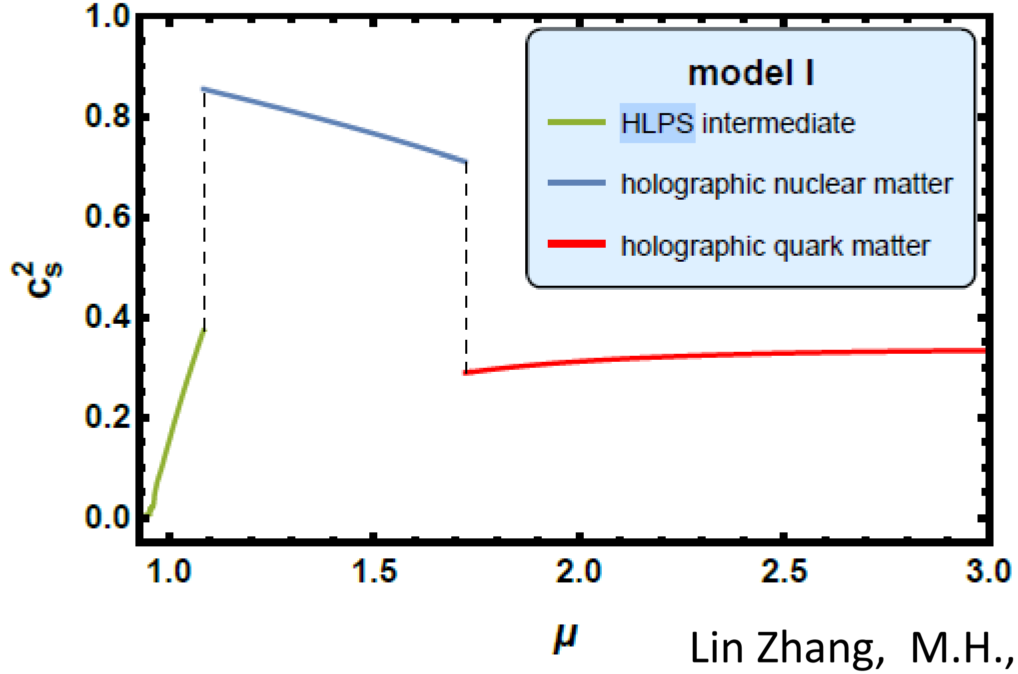
Model parameters are fixed by lattice result Nf=2+1 at \mu=0



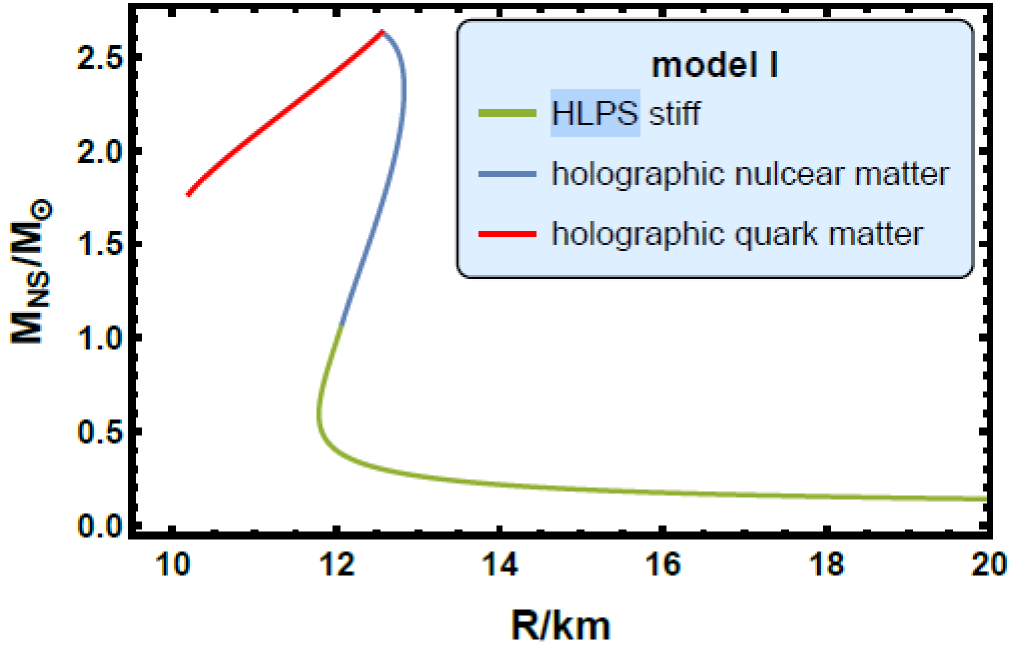


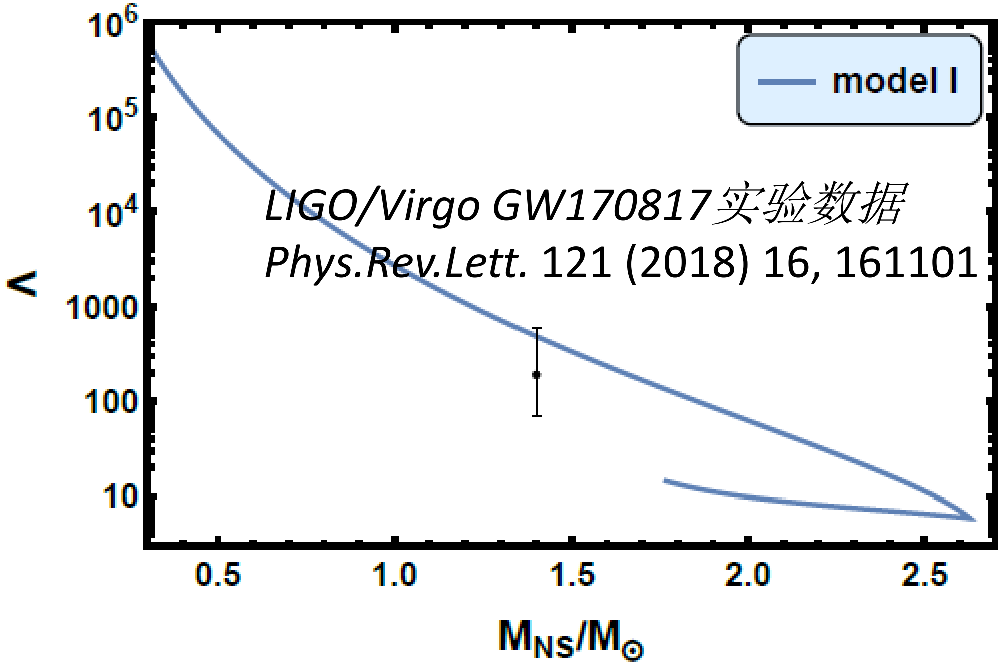
Rong-Gen Cai, Song He, Li Li, Yuan-Xu Wang, arXiv:2201.02004





Lin Zhang, M.H., to appear

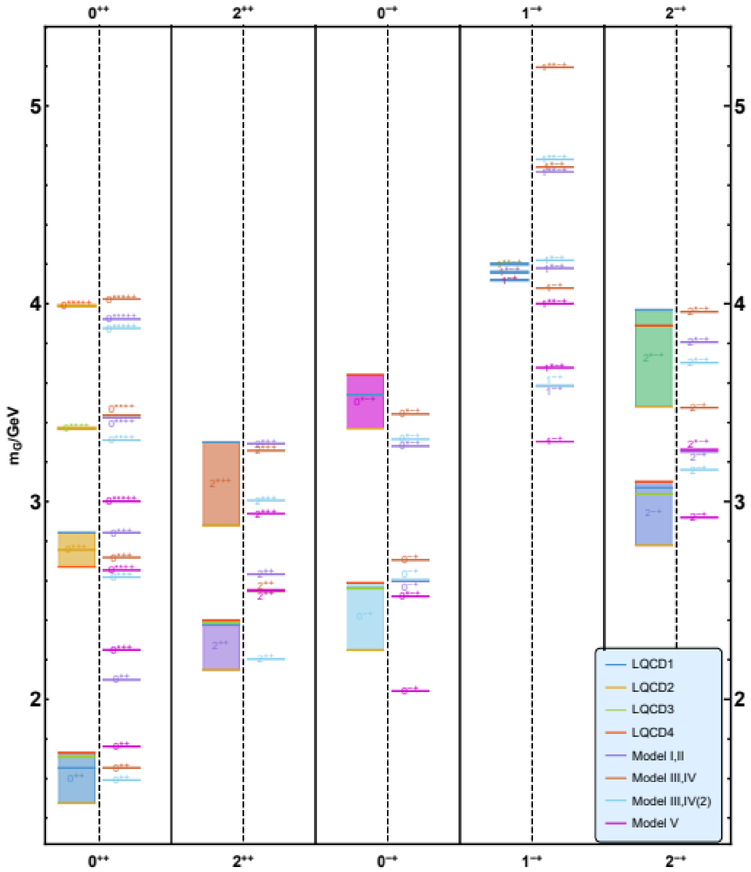


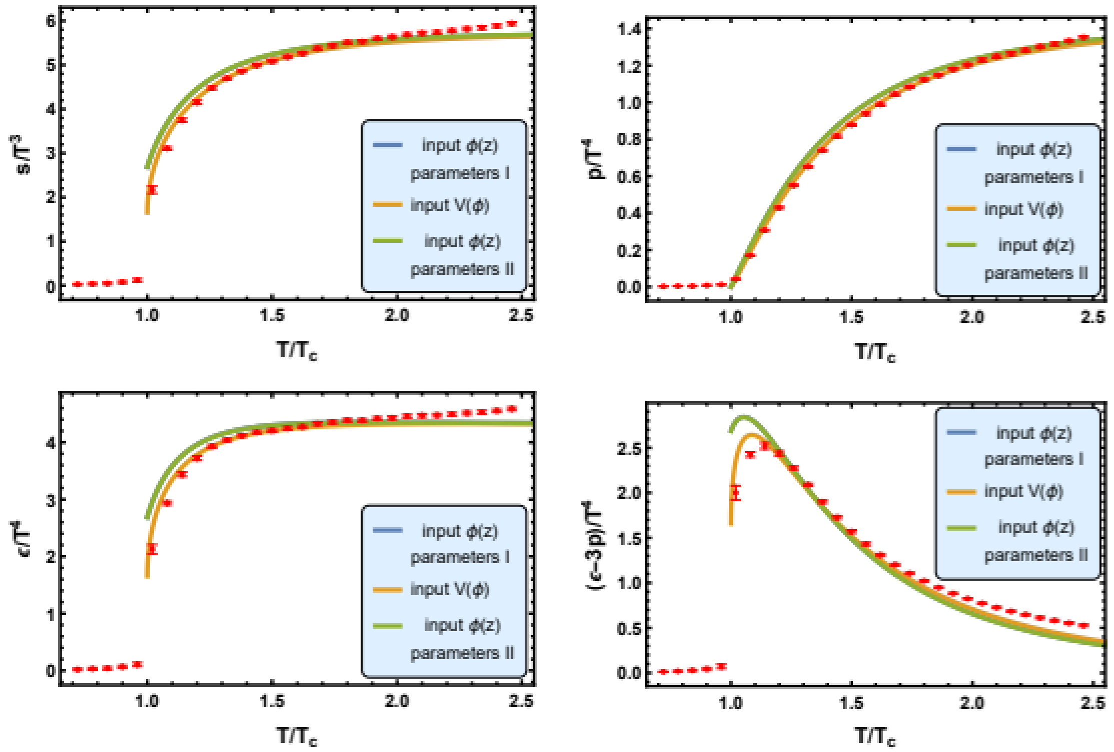


Model III： gluon background

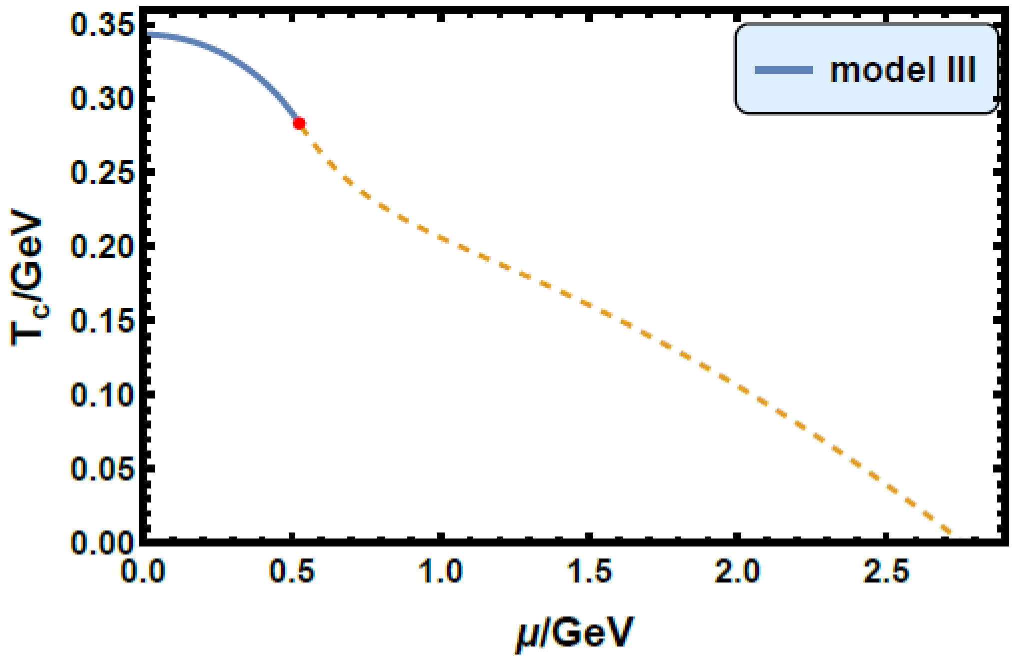
$$\phi(z)=c_1 z^2,$$

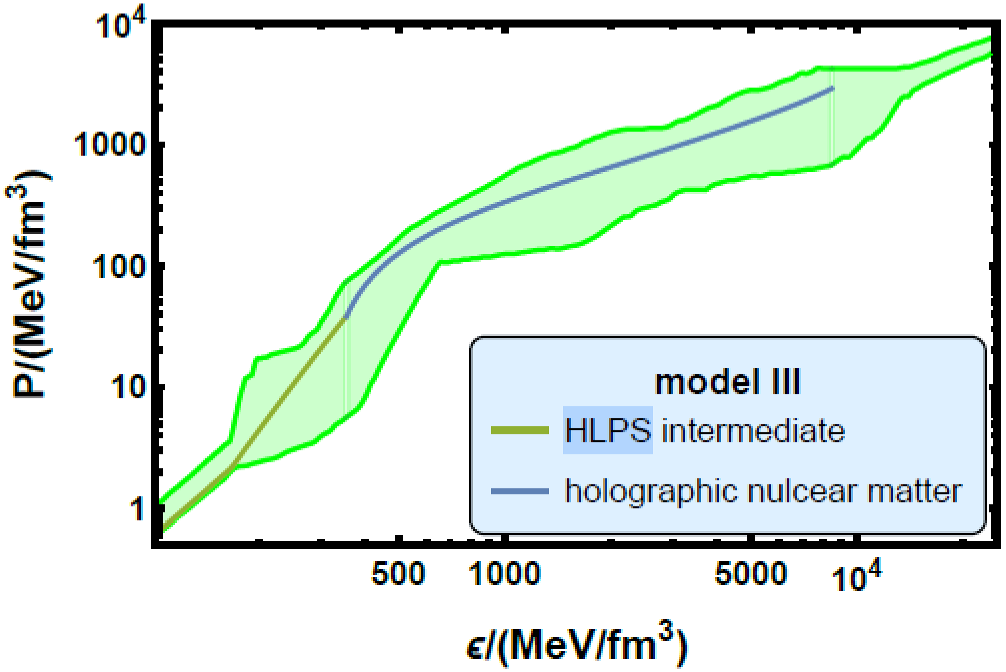
Parameters fixed by lattice results of EOS for pure gluon system



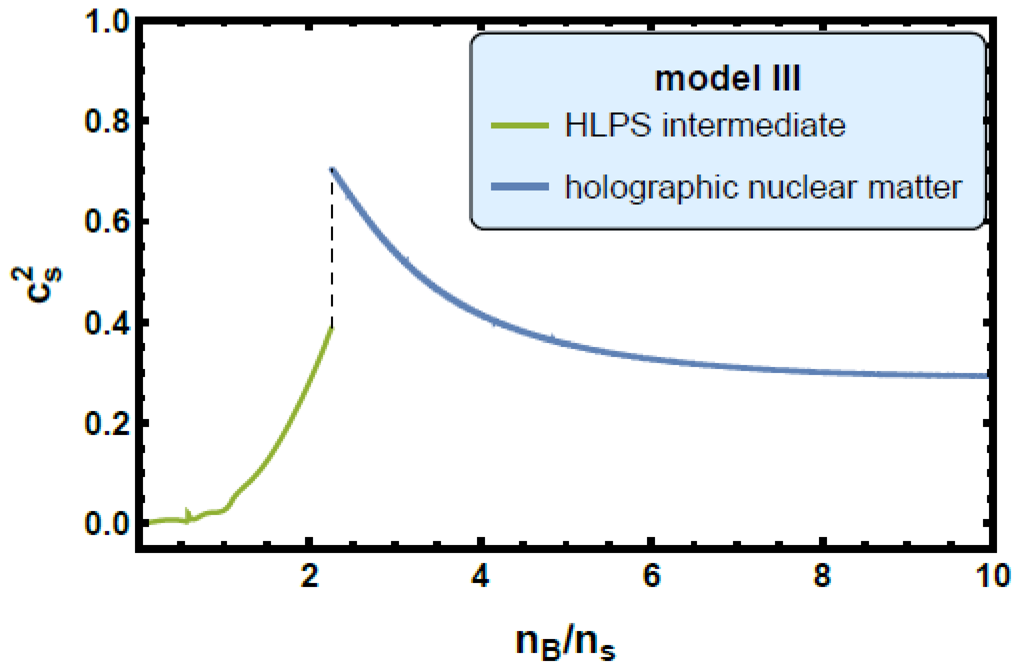


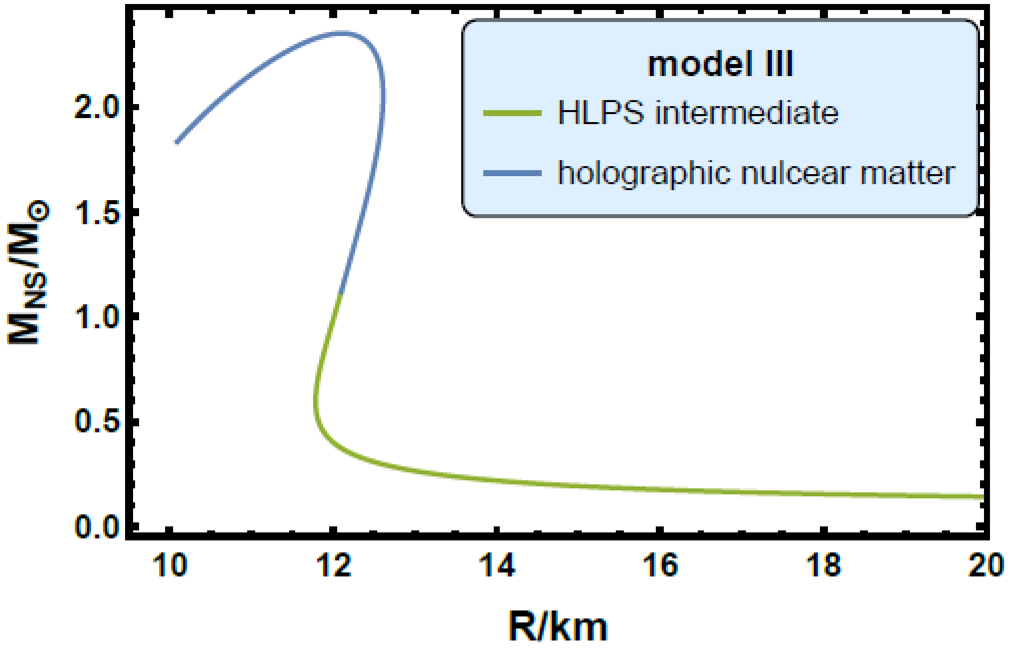
Lin Zhang, Chutian Chen, Yidian Chen, M.H. *Phys.Rev.D* 105 (2022) 2, 026020

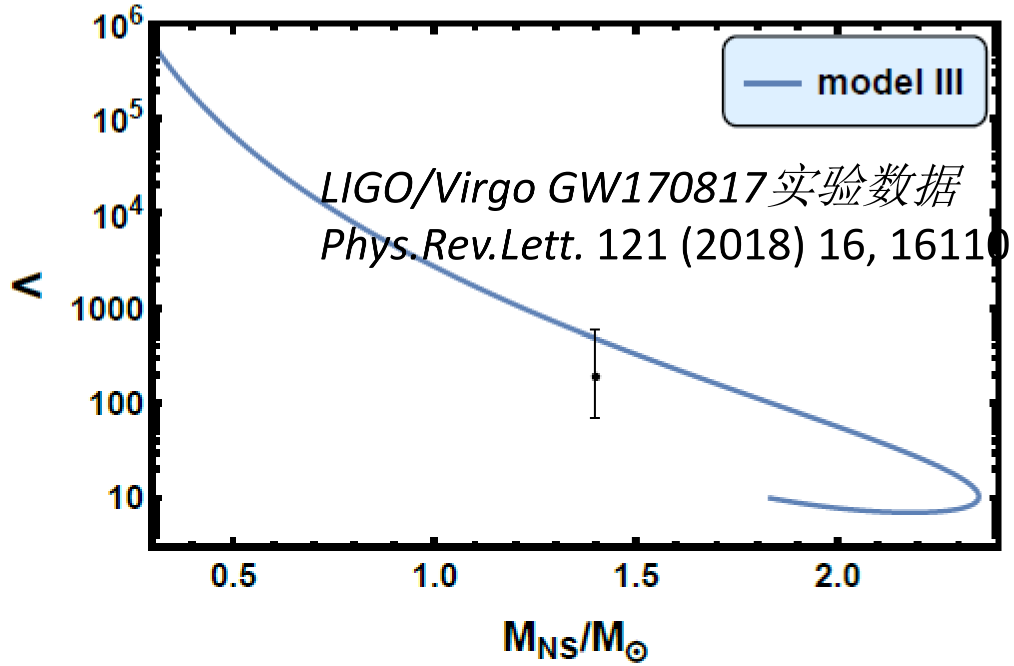




Lin Zhang, M.H., to appear



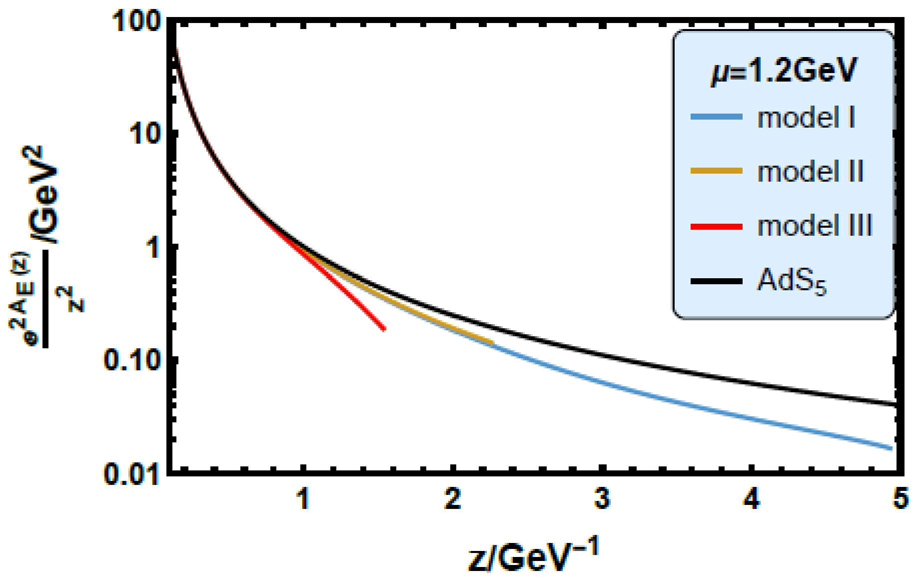


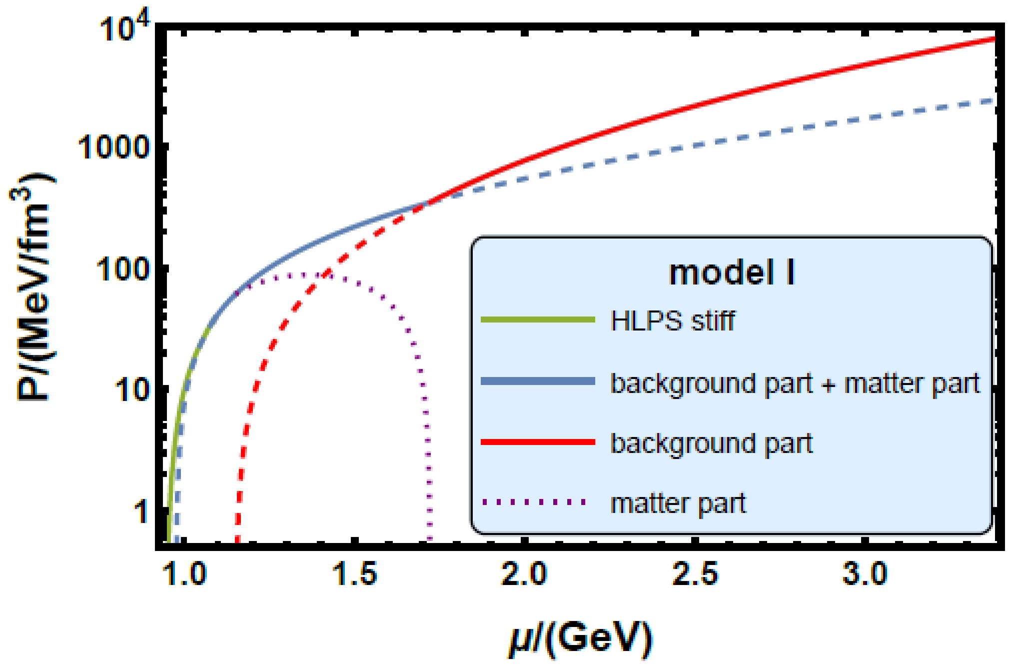


LIGO/Virgo GW170817实验数据
Phys.Rev.Lett. 121 (2018) 16, 161101

Lin Zhang, M.H., to appear

1, For stable NS, confined matter is favored; most probably, a quarkyonic state.
2,pure gluon part gives a stiff EOS, matter part soften the EOS,
2, quark matter will give unstable NSs.
3, a peak for sound velocity





Relation between chiral symmetry breaking and confinement and whether there is quarkyonic matter can only be answered when gluodynamics and chiral dynamics are fully solved!

Light flavor Hadron spectra

	Effective models	hQCD models
Ground states:	Easy	Easy
Excitation states:	Hard	Easy

Heavy flavor Hadron spectra?

Phase structure

	Effective models	hQCD models
Chiral restoration	Easy	Not easy
Deconfinement	Hard	Easy

Hot QCD matter properties

Which **properties of hot QCD matter** can we hope to determine ?

Easy for LQCD

$$T_{\mu\nu} \quad \Leftrightarrow \quad \varepsilon, p, s$$

$$c_S^2 = \partial p / \partial \varepsilon$$

Equation of state: spectra, coll. flow, fluctuations

Speed of sound: correlations

$$\eta = \frac{1}{T} \int d^4x \left\langle T_{xy}(x) T_{xy}(0) \right\rangle$$

Shear viscosity: anisotropic collective flow

Hard for LQCD

$$\hat{q} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \left\langle U^\dagger F^{a+i}(y^-) U F_i^{a+}(0) \right\rangle$$

$$\hat{e} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \left\langle i U^\dagger \partial^- A^{a+}(y^-) U A^{a+}(0) \right\rangle$$

$$\kappa = \frac{4\pi \alpha_s}{3N_c} \int d\tau \left\langle U^\dagger F^{a0i}(\tau) t^a U F^{b0i}(0) t^b \right\rangle$$

Momentum/energy diffusion: parton energy loss, jet fragmentation

Easy for LQCD

$$m_D = -\lim_{|x|\to\infty} \frac{1}{|x|} \ln \left\langle U^\dagger E^a(x) U E^a(0) \right\rangle$$

Color screening: Quarkonium states

BROOKHAVEN

Berndt Mueller

Which **properties of hot QCD matter** can we hope to determine ?

$T_{\mu\nu} \Leftrightarrow \varepsilon, p, s$ **Equation of state**: spectra, coll. flow, fluctuations

$c_S^2 = \partial p / \partial \varepsilon$ **Speed of sound**: correlations

Hard for Effective QCD

$\eta = \frac{1}{T}\int d^4x \langle T_{xy}(x) T_{xy}(0) \rangle$ **Shear viscosity**: anisotropic collective flow

$$\hat{q} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle U^\dagger F^{a+i}(y^-) U F_i^{a+}(0) \rangle$$

$$\hat{e} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle i U^\dagger \partial^- A^{a+}(y^-) U A^{a+}(0) \rangle$$

$$\kappa = \frac{4\pi \alpha_s}{3N_c} \int d\tau \langle U^\dagger F^{a0i}(\tau) t^a U F^{b0i}(0) t^b \rangle$$

Momentum/energy diffusion: parton energy loss, jet fragmentation

$m_D = -\lim_{|x|\to\infty} \frac{1}{|x|} \ln \langle U^\dagger E^a(x) U E^a(0) \rangle$ **Color screening**: Quarkonium states

Which **properties of hot QCD matter** can we hope to determine ?

Easy for hQCD

$T_{\mu\nu} \Leftrightarrow \varepsilon, p, s$ — **Equation of state**: spectra, coll. flow, fluctuations

$c_S^2 = \partial p / \partial \varepsilon$ — **Speed of sound**: correlations

$\eta = \frac{1}{T}\int d^4x \langle T_{xy}(x) T_{xy}(0) \rangle$ — **Shear viscosity**: anisotropic collective flow

$$\hat{q} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle U^\dagger F^{a+i}(y^-) U F_i^{a+}(0) \rangle$$

$$\hat{e} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle i U^\dagger \partial^- A^{a+}(y^-) U A^{a+}(0) \rangle$$

$$\kappa = \frac{4\pi \alpha_s}{3N_c} \int d\tau \langle U^\dagger F^{a0i}(\tau) t^a U F^{b0i}(0) t^b \rangle$$

Momentum/energy diffusion: parton energy loss, jet fragmentation

$m_D = -\lim_{|x|\to\infty} \frac{1}{|x|} \ln \langle U^\dagger E^a(x) U E^a(0) \rangle$ — **Color screening**: Quarkonium states

But we need a hQCD close to QCD!

Dynamical hQCD model is one of the candidates!