

pQCD 方法中 ϕ_{B_2} 对 $B \rightarrow PP$ 过程影响的研究

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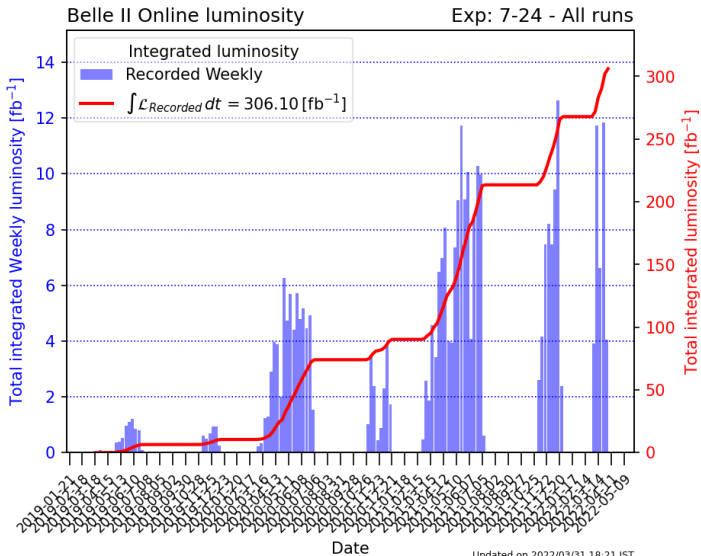
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May 14, 2022

1 $B \rightarrow PP$ 衰变过程的研究

- 研究背景
- 理论框架
- 形状因子讨论
- 数值结果与分析
- 总结与展望

2 致谢



- 我们以低能有效哈密顿量为出发点:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} \{ V_{ub} V_{uq}^* \sum_{i=1}^2 C_i O_i - V_{tb} V_{tq}^* \sum_{j=3}^{10} C_j O_j \} + \text{H.c.}$$

- $B \rightarrow PP$ 过程衰变振幅的表达式可以表示为:

$$\begin{aligned} \mathcal{A}(B \rightarrow P_1 P_2) &= \langle P_1 P_2 | \mathcal{H}_{\text{eff}} | B \rangle \\ &= \frac{G_F}{\sqrt{2}} \sum_{i=1}^{10} V_i C_i(\mu) \langle P_1 P_2 | O_i(\mu) | B \rangle \end{aligned}$$

- 微扰 QCD 方法中，衰变振幅可以表示如下：

$$\mathcal{A}_i = \int dx_1 dx_2 dx_3 db_1 db_2 db_3 C_i(t_i) \mathcal{H}_i(x_1, x_2, x_3, b_1, b_2, b_3) \\ \Phi_B(x_1, b_1) e^{-S_B} \Phi_{P_1}(x_2, b_2) e^{-S_{P_1}} \Phi_{P_2}(x_3, b_3) e^{-S_{P_2}}$$

■ B 介子波函数:

$$\begin{aligned}
 & \langle 0 | \bar{q}_\alpha(z) b_\beta(0) | \bar{B}(p) \rangle \\
 = & +\frac{i}{4} f_B \int d^4 k e^{-ik_1 \cdot z} \{ (\not{p} + m_B) \gamma_5 [\frac{\not{h}_+}{\sqrt{2}} \phi_B^+ + \frac{\not{h}_-}{\sqrt{2}} \phi_B^-] \}_{\beta\alpha} \\
 = & -\frac{i}{4} f_B \int d^4 k e^{-ik_1 \cdot z} \{ (\not{p} + m_B) \gamma_5 [\phi_B^+ + \frac{\not{h}_-}{\sqrt{2}} (\phi_B^+ - \phi_B^-)] \}_{\beta\alpha} \\
 = & -\frac{i}{4} f_B \int d^4 k e^{-ik_1 \cdot z} \left\{ (\not{p} + m_B) \gamma_5 \left(\phi_{B1} + \frac{\not{h}_-}{\sqrt{2}} \phi_{B2} \right) \right\}_{\beta\alpha}
 \end{aligned}$$

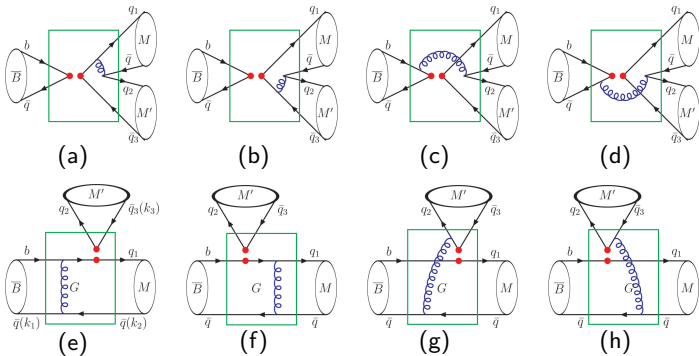


C. Lü, M. Yang, Eur. Phys. J. C 28, 515 (2003).

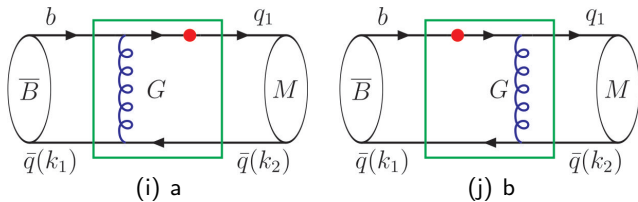


T. Kurimoto, Phys. Rev. D 74, 014027 (2006).

■ B 介子两体非轻衰变的费曼图



■ $B \rightarrow M$ 跃迁形状因子的最低阶费曼图

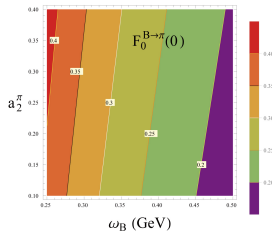
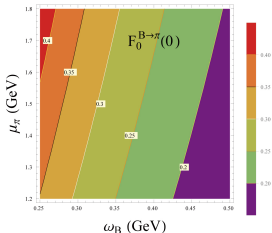
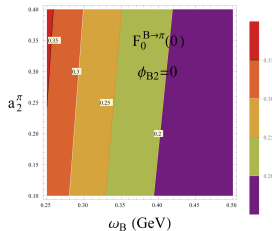
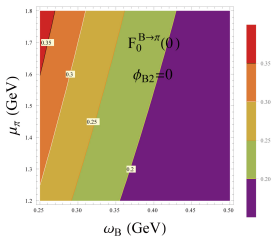


- $B \rightarrow M$ 过程的形状因子可以定义为

$$\begin{aligned} & \langle M(p_2) | (\bar{q} b)_{V-A} | \bar{B}(p_1) \rangle \\ &= \left\{ (p_1 + p_2)^\mu - \frac{m_B^2 - m_M^2}{q^2} q^\mu \right\} F_1(q^2) + \frac{m_B^2 - m_M^2}{q^2} q^\mu F_0(q^2) \end{aligned}$$

- 形状因子 F_i 可以写作夸克散射振幅 $\mathcal{T}$ 和强子波函数 Φ_i 的卷积的积分形式

$$\begin{aligned} F_i = & \int dx_1 dx_2 db_1 db_2 \Phi_B(x_1, b_1) e^{-S_B} \\ & \mathcal{T}(x_1, x_2, b_1, b_2) \Phi_M(x_2, b_2) e^{-S_M} \end{aligned}$$

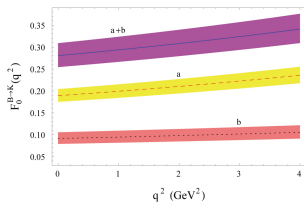
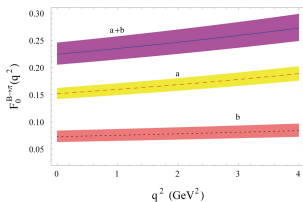
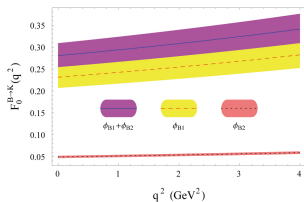
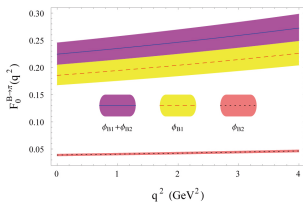
■ $F_0^{B \rightarrow \pi}(0)$ 等高线图

- 通过最小 χ^2 方法，我们可以获得三种最优方案

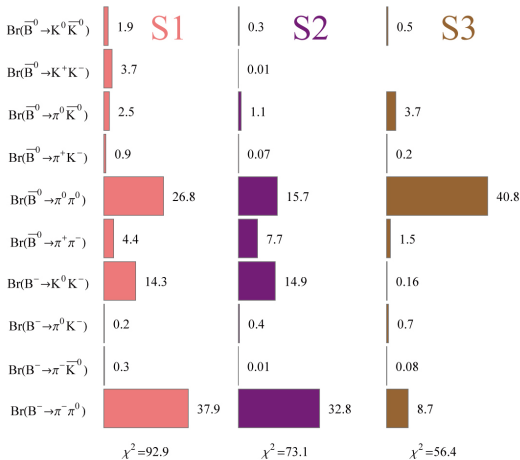
$$\chi^2 = \sum_i \frac{(\mathcal{B}r^{\text{theo.}} - \mathcal{B}r^{\text{exp.}})^2}{\sigma_{\mathcal{B}r^{\text{exp.}}}^2}.$$

- 方案一(S1) : $\omega_B = 0.45$ GeV , $\mu_M = 1.7$ GeV PDG ;
方案二(S2) : $\omega_B = 0.43$ GeV , $\mu_M = 1.6$ GeV Belle ;
方案三(S3) : $\omega_B = 0.41$ GeV , $\mu_M = 1.5$ GeV BABAR 。

- S2 参数方案下，形状因子 $F_0^{B \rightarrow \pi}(q^2)$ 和 $F_0^{B \rightarrow K}(q^2)$ 随 q^2 的分布



■ 三种方案下分支比的 χ^2 分布



■ $B_u \rightarrow PP$ 衰变过程的分支比（单位： 10^{-6} ）

	Mode	$B^- \rightarrow \pi^- \pi^0$	$B^- \rightarrow \pi^- \bar{K}^0$	$B^- \rightarrow \pi^0 K^-$	$B^- \rightarrow K^0 K^-$
S1	PDG	5.5 ± 0.4	23.7 ± 0.8	12.9 ± 0.5	1.31 ± 0.17
	$\phi_{B1} + \phi_{B2}$	$3.04^{+0.18+0.16+0.27}_{-0.17-0.16-0.25}$	$24.12^{+1.73+3.69+2.50}_{-1.60-3.32-2.36}$	$13.13^{+0.93+1.95+1.11}_{-0.87-1.76-1.07}$	$1.95^{+0.13+0.34+0.18}_{-0.13-0.30-0.16}$
	ϕ_{B1}	$2.16^{+0.14+0.15+0.14}_{-0.13-0.15-0.13}$	$17.89^{+1.36+2.97+1.30}_{-1.26-2.65-1.24}$	$9.71^{+0.73+1.57+0.58}_{-0.68-1.41-0.56}$	$1.45^{+0.11+0.27+0.07}_{-0.10-0.25-0.06}$
S2	Belle	5.86 ± 0.46	23.97 ± 0.89	12.62 ± 0.64	1.11 ± 0.20
	$\phi_{B1} + \phi_{B2}$	$3.23^{+0.19+0.18+0.29}_{-0.18-0.18-0.27}$	$23.87^{+1.72+3.83+2.51}_{-1.60-3.43-2.38}$	$13.03^{+0.93+2.03+1.12}_{-0.87-1.83-1.07}$	$1.88^{+0.13+0.35+0.17}_{-0.12-0.31-0.15}$
	ϕ_{B1}	$2.27^{+0.15+0.17+0.15}_{-0.14-0.16-0.14}$	$17.64^{+1.36+3.08+1.30}_{-1.25-2.74-1.25}$	$9.60^{+0.74+1.64+0.59}_{-0.68-1.46-0.57}$	$1.38^{+0.10+0.29+0.06}_{-0.10-0.25-0.05}$
S3	BABAR	5.02 ± 0.54	23.9 ± 1.5	13.6 ± 0.9	1.61 ± 0.45
	$\phi_{B1} + \phi_{B2}$	$3.42^{+0.21+0.20+0.32}_{-0.19-0.20-0.30}$	$23.49^{+1.71+3.96+2.51}_{-1.58-3.53-2.38}$	$12.87^{+0.93+2.11+1.12}_{-0.86-1.89-1.07}$	$1.79^{+0.12+0.36+0.15}_{-0.12-0.32-0.13}$
	ϕ_{B1}	$2.39^{+0.16+0.18+0.16}_{-0.14-0.18-0.15}$	$17.30^{+1.35+3.19+1.30}_{-1.24-2.81-1.24}$	$9.44^{+0.73+1.70+0.59}_{-0.67-1.51-0.57}$	$1.30^{+0.10+0.29+0.05}_{-0.09-0.26-0.04}$

■ $B_d \rightarrow PP$ 衰变过程的分支比（单位： 10^{-6} ）

	Mode	$\bar{B}^0 \rightarrow \pi^+ \pi^-$	$\bar{B}^0 \rightarrow \pi^0 \pi^0$	$\bar{B}^0 \rightarrow \pi^+ K^-$
S1	PDG	5.12 ± 0.19	1.59 ± 0.26	19.6 ± 0.5
	$\phi_{B1} + \phi_{B2}$	$5.52^{+0.34+0.37+0.47}_{-0.32-0.36-0.43}$	$0.24^{+0.02+0.04+0.01}_{-0.02-0.03-0.01}$	$20.07^{+1.44+3.13+2.05}_{-1.33-2.81-1.94}$
	ϕ_{B1}	$3.81^{+0.25+0.31+0.23}_{-0.23-0.30-0.20}$	$0.19^{+0.01+0.03+0.02}_{-0.01-0.03-0.01}$	$15.00^{+1.14+2.53+1.07}_{-1.05-2.25-1.02}$
S2	Belle	5.04 ± 0.28	1.31 ± 0.27	20.00 ± 0.69
	$\phi_{B1} + \phi_{B2}$	$5.82^{+0.36+0.41+0.50}_{-0.34-0.39-0.46}$	$0.24^{+0.02+0.04+0.01}_{-0.02-0.04-0.01}$	$19.81^{+1.43+3.24+2.05}_{-1.33-2.89-1.95}$
	ϕ_{B1}	$3.99^{+0.26+0.34+0.24}_{-0.25-0.33-0.21}$	$0.18^{+0.01+0.03+0.02}_{-0.01-0.03-0.01}$	$14.76^{+1.14+2.62+1.07}_{-1.05-2.32-1.02}$
S3	BABAR	5.5 ± 0.5	1.83 ± 0.25	19.1 ± 0.8
	$\phi_{B1} + \phi_{B2}$	$6.12^{+0.39+0.45+0.54}_{-0.36-0.43-0.50}$	$0.23^{+0.02+0.04+0.01}_{-0.02-0.04-0.01}$	$19.45^{+1.42+3.34+2.04}_{-1.31-2.97-1.94}$
	ϕ_{B1}	$4.16^{+0.28+0.37+0.26}_{-0.26-0.36-0.23}$	$0.18^{+0.01+0.03+0.02}_{-0.01-0.03-0.01}$	$14.43^{+1.13+2.70+1.06}_{-1.04-2.38-1.01}$
	Mode	$\bar{B}^0 \rightarrow \pi^0 \bar{K}^0$	$\bar{B}^0 \rightarrow K^+ K^-$	$\bar{B}^0 \rightarrow K^0 \bar{K}^0$
S1	PDG	9.9 ± 0.5	0.078 ± 0.015	1.21 ± 0.16
	$\phi_{B1} + \phi_{B2}$	$9.11^{+0.66+1.48+1.15}_{-0.61-1.32-1.08}$	$0.107^{+0.003+0.005+0.062}_{-0.003-0.004-0.048}$	$1.43^{+0.10+0.25+0.05}_{-0.09-0.23-0.03}$
	ϕ_{B1}	$6.81^{+0.52+1.19+0.61}_{-0.48-1.05-0.57}$	$0.089^{+0.003+0.004+0.047}_{-0.003-0.004-0.037}$	$1.06^{+0.08+0.21+0.03}_{-0.07-0.19-0.01}$
S2	Belle	9.68 ± 0.68	0.10 ± 0.09	1.26 ± 0.20
	$\phi_{B1} + \phi_{B2}$	$8.95^{+0.65+1.53+1.15}_{-0.61-1.36-1.08}$	$0.109^{+0.003+0.005+0.065}_{-0.003-0.004-0.050}$	$1.37^{+0.10+0.26+0.03}_{-0.09-0.24-0.02}$
	ϕ_{B1}	$6.67^{+0.52+1.22+0.60}_{-0.48-1.08-0.57}$	$0.091^{+0.003+0.004+0.050}_{-0.003-0.004-0.039}$	$1.01^{+0.07+0.22+0.04}_{-0.07-0.19-0.02}$
S3	BABAR	10.1 ± 0.7		1.08 ± 0.30
	$\phi_{B1} + \phi_{B2}$	$8.75^{+0.64+1.57+1.15}_{-0.60-1.38-1.07}$	$0.111^{+0.003+0.005+0.068}_{-0.003-0.004-0.052}$	$1.30^{+0.09+0.27+0.02}_{-0.08-0.24-0.01}$
	ϕ_{B1}	$6.50^{+0.51+1.26+0.60}_{-0.47-1.10-0.57}$	$0.093^{+0.003+0.004+0.053}_{-0.003-0.004-0.040}$	$0.94^{+0.07+0.22+0.05}_{-0.06-0.20-0.03}$

■ $B_u \rightarrow PP$ 衰变过程的 CP 破坏 (单位: 10^{-2})

$\mathcal{A}_{CP}$	$\pi^- \pi^0$	$\pi^- \bar{K}^0$	$\pi^0 K^-$	$K^0 K^-$	$\pi^+ K^-$
PDG	3 ± 4	-1.7 ± 1.6	3.7 ± 2.1	4 ± 14	-8.3 ± 0.4
$\phi_{B1} + \phi_{B2}$	-0.004	$-0.67^{+0.02+0.04+0.14}_{-0.02-0.05-0.11}$	$-6.25^{+0.25+0.30+1.51}_{-0.26-0.33-1.56}$	$14.78^{+0.27+1.05+3.33}_{-0.27-0.90-3.87}$	$-7.33^{+0.29+0.41+1.98}_{-0.30-0.44-2.04}$
ϕ_{B1}	-0.02	$-0.74^{+0.02+0.04+0.14}_{-0.02-0.05-0.10}$	$-6.73^{+0.30+0.36+1.74}_{-0.31-0.40-1.88}$	$14.83^{+0.26+1.10+3.28}_{-0.26-0.93-3.71}$	$-8.14^{+0.33+0.49+2.38}_{-0.34-0.54-2.54}$
Belle	2.5 ± 4.4	-1.1 ± 2.2	4.3 ± 2.4	1.4 ± 16.8	-6.9 ± 1.6
$\phi_{B1} + \phi_{B2}$	-0.01	$-0.69^{+0.02+0.05+0.15}_{-0.02-0.05-0.12}$	$-6.05^{+0.25+0.30+1.43}_{-0.26-0.33-1.49}$	$15.28^{+0.28+1.21+3.53}_{-0.28-1.03-4.08}$	$-7.18^{+0.28+0.41+1.91}_{-0.29-0.45-1.97}$
ϕ_{B1}	-0.02	$-0.75^{+0.02+0.05+0.15}_{-0.02-0.05-0.12}$	$-6.51^{+0.29+0.36+1.66}_{-0.30-0.40-1.80}$	$15.39^{+0.27+1.30+3.55}_{-0.27-1.08-3.98}$	$-7.98^{+0.33+0.50+2.31}_{-0.34-0.54-2.47}$
BABAR	3 ± 8	-2.9 ± 4.0	3 ± 4	10 ± 26	-10.7 ± 1.7
$\phi_{B1} + \phi_{B2}$	-0.01	$-0.70^{+0.02+0.05+0.16}_{-0.02-0.06-0.13}$	$-5.86^{+0.24+0.30+1.36}_{-0.25-0.32-1.41}$	$15.90^{+0.29+1.41+3.78}_{-0.29-1.18-4.34}$	$-7.03^{+0.28+0.41+1.84}_{-0.29-0.44-1.89}$
ϕ_{B1}	-0.03	$-0.77^{+0.02+0.05+0.16}_{-0.02-0.06-0.13}$	$-6.30^{+0.29+0.36+1.57}_{-0.30-0.40-1.72}$	$16.12^{+0.29+1.56+3.89}_{-0.29-1.27-4.31}$	$-7.83^{+0.33+0.50+2.24}_{-0.34-0.55-2.40}$

■ $B_d \rightarrow PP$ 衰变过程的 CP 破坏 (单位: 10^{-2})

	$C_{\pi^+ \pi^-}$	$S_{\pi^+ \pi^-}$	$C_{\pi^0 \pi^0}$	$S_{\pi^0 \pi^0}$
PDG	-32 ± 4	-65 ± 4	-33 ± 22	
$\phi_{B1} + \phi_{B2}$	$-17.85^{+0.55+0.05+3.84}_{-0.54-0.02-3.42}$	$-81.91^{+0.17+1.57+2.43}_{-0.17-1.52-1.96}$	$38.34^{+1.39+1.78+5.37}_{-1.37-1.57-5.40}$	$89.03^{+0.49+1.13+0.97}_{-0.51-1.38-1.21}$
ϕ_{B1}	$-21.52^{+0.71+0.13+4.96}_{-0.69-0.10-4.52}$	$-80.15^{+0.15+1.65+2.94}_{-0.16-1.60-2.30}$	$39.06^{+1.60+1.86+4.49}_{-1.58-1.62-4.17}$	$85.90^{+0.53+1.32+0.35}_{-0.54-1.59-0.25}$
Belle	-33 ± 7	-64 ± 9	-14 ± 37	
$\phi_{B1} + \phi_{B2}$	$-16.75^{+0.51+0.05+3.52}_{-0.52-0.02-3.11}$	$-83.12^{+0.16+1.55+2.34}_{-0.16-1.50-1.91}$	$37.29^{+1.41+1.91+5.29}_{-1.40-1.68-5.09}$	$88.69^{+0.50+1.31+0.83}_{-0.51-1.63-1.09}$
ϕ_{B1}	$-20.24^{+0.66+0.12+4.57}_{-0.68-0.09-4.13}$	$-81.46^{+0.15+1.64+2.86}_{-0.15-1.59-2.28}$	$37.65^{+1.63+2.01+4.49}_{-1.62-1.75-3.86}$	$85.41^{+0.54+1.54+0.34}_{-0.55-1.88-0.28}$
BABAR	-25 ± 8	-68 ± 10	-43 ± 26	
$\phi_{B1} + \phi_{B2}$	$-15.72^{+0.48+0.05+3.23}_{-0.49-0.02-2.82}$	$-84.33^{+0.15+1.53+2.24}_{-0.15-1.48-1.85}$	$36.32^{+1.43+2.08+5.21}_{-1.42-1.80-4.74}$	$88.11^{+0.50+1.55+0.70}_{-0.51-1.96-1.00}$
ϕ_{B1}	$-19.04^{+0.63+0.12+4.20}_{-0.64-0.09-3.76}$	$-82.79^{+0.14+1.62+2.77}_{-0.14-1.58-2.24}$	$36.33^{+1.66+2.22+4.47}_{-1.63-1.89-3.51}$	$84.65^{+0.55+1.83+0.37}_{-0.56-2.27-0.37}$
	$C_{\pi^0 \bar{K}^0}$	$S_{\pi^0 \bar{K}^0}$	$C_{K^+ K^-}$	$S_{K^+ K^-}$
PDG	0 ± 13	58 ± 17		
$\phi_{B1} + \phi_{B2}$	$-0.54^{+0.01+0.05+0.25}_{-0.01-0.05-0.21}$	$68.74^{+0.02+0.16+0.20}_{-0.02-0.13-0.16}$	$-81.17^{+0.45+0.79+0.13}_{-0.45-0.70-2.04}$	$-28.21^{+0.94+1.59+0.95}_{-0.93-1.58-0.53}$
ϕ_{B1}	$-0.85^{+0.01+0.06+0.36}_{-0.01-0.06-0.31}$	$69.29^{+0.03+0.18+0.13}_{-0.03-0.15-0.14}$	$-82.90^{+0.46+0.67+0.27}_{-0.45-0.59-1.24}$	$-28.35^{+1.02+1.25+0.35}_{-1.01-1.27-1.24}$
Belle	-14 ± 14	67 ± 32		
$\phi_{B1} + \phi_{B2}$	$-0.60^{+0.01+0.05+0.27}_{-0.00-0.06-0.23}$	$68.86^{+0.02+0.18+0.20}_{-0.02-0.15-0.16}$	$-81.26^{+0.44+0.88+0.11}_{-0.43-0.78-1.96}$	$-27.97^{+0.94+1.66+0.24}_{-0.92-1.65-0.12}$
ϕ_{B1}	$-0.93^{+0.01+0.07+0.39}_{-0.01-0.07-0.34}$	$69.40^{+0.04+0.21+0.15}_{-0.04-0.18-0.16}$	$-83.11^{+0.44+0.76+0.25}_{-0.43-0.67-1.24}$	$-27.65^{+1.01+1.35+0.75}_{-0.99-1.37-1.80}$
BABAR	13 ± 13	55 ± 20		
$\phi_{B1} + \phi_{B2}$	$-0.67^{+0.00+0.06+0.30}_{-0.00-0.07-0.25}$	$68.99^{+0.02+0.21+0.20}_{-0.02-0.18-0.16}$	$-81.24^{+0.42+0.97+0.08}_{-0.41-0.87-1.81}$	$-27.80^{+0.93+1.71+0.35}_{-0.92-1.70-0.56}$
ϕ_{B1}	$-1.02^{+0.01+0.08+0.43}_{-0.00-0.09-0.36}$	$69.53^{+0.04+0.24+0.16}_{-0.04-0.20-0.17}$	$-83.19^{+0.41+0.86+0.25}_{-0.40-0.76-1.24}$	$-27.06^{+1.00+1.45+1.18}_{-0.99-1.46-2.44}$

■ 之前研究中 $B \rightarrow \pi K$ 衰变的 LO 和 NLO 的 pQCD 计算结果

(分支比和直接 CP 破坏的单位分别为 10^{-6} 和 10^{-2})

Mode	LO [1]	NLO [1]	NLOG ^a [2]	NLO [3]	NLOG [3]
$Br(B^- \rightarrow \pi^- \bar{K}^0)$	17.0	$24.5^{+13.6}_{-8.1}$	21.1	$27.2^{+9.3}_{-6.7}$	$24.1^{+8.3}_{-6.0}$
$Br(B^- \rightarrow \pi^0 K^-)$	10.2	$13.9^{+10.0}_{-5.6}$	12.9	$15.3^{+5.2}_{-3.8}$	$14.0^{+4.7}_{-3.5}$
$Br(\bar{B}^0 \rightarrow \pi^+ K^-)$	14.2	$20.9^{+15.6}_{-8.3}$	17.7	$23.3^{+7.8}_{-5.7}$	$21.7^{+7.4}_{-5.3}$
$Br(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0)$	5.7	$9.1^{+5.6}_{-3.3}$	7.2	$10.2^{+3.4}_{-2.5}$	$9.3^{+3.2}_{-2.3}$
$\mathcal{A}_{CP}(B^- \rightarrow \pi^0 K^-)$	-8	-1^{+3}_{-5}	10	$-0.8^{+1.3}_{-1.4}$	2.1 ± 1.6
$\mathcal{A}_{CP}(\bar{B}^0 \rightarrow \pi^+ K^-)$	-12	-9^{+6}_{-8}	-11	-7.6 ± 1.7	-8.1 ± 1.7



[1] H. Li, S. Mishima, A. Sanda, Phys. Rev. D 72, 114005 (2005).



[2] H. Li, S. Mishima, Phys. Rev. D 90, 074018 (2014).



[3] X. Liu, H. Li, Z. Xiao, Phys. Rev. D 93, 014024 (2016).

- ϕ_{B2} 部分虽然在 pQCD 方法下对可因子化湮灭图无贡献，但可以对发射图振幅和不可因子化的湮灭图振幅做出贡献
- 对于 $B \rightarrow PP$ 非轻衰变过程， ϕ_{B2} 部分对形状因子与分支比都有一定的影响，对于分支比的贡献相当于不考虑 ϕ_{B2} 部分时次领头阶对分支比的修正。

感谢各位专家老师批评指正！