光前夸克模型下$P \to T$跃迁形状因子研究

学科、专业 ： 物理学、 粒子物理与原子核物理
研 究 方 向 ： 粒子物理理论
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Outline

- **由重夸克到轻夸克跃迁引起的B介子和D介子的衰变过程在检验标准模型和寻找新物理等方面具有重要意义。**

  B介子到张量介子的跃迁过程可能存在轻子味普适性破缺。

  与标量介子和矢量介子相比，张量介子的衰变具有更多的极化态。

  Y. S. Amhis *et al.* [HFLAV], Eur. Phys. J. C **81**, no.3, 226 (2021);

- **在介子弱衰变的研究中，形状因子是必不可少的非微扰输入参数。**

  光前夸克模型为非微扰物理量的计算提供了一个概念上简单，唯象上可行的理论框架。

  M. V. Terentev. Sov. J. Nucl. Phys. **24**, 106 (1976);

  V. B. Berestetsky and M. V. Terentev, Sov. J. Nucl. Phys. **25**, 347-354 (1977);

  W. Jaus, Phys. Rev. D **41**, 3394 (1990);

  W. Jaus and D. Wyler, Phys. Rev. D **41**, 3405 (1990).

主要任务：

$$\mathcal{B} \equiv \langle T(\epsilon^{\mu\nu*}, p'')|\bar{q}_1''(k_1'')\Gamma q_1'(k_1')|P(p')\rangle\,, \qquad \Gamma = \sigma_{\mu\nu},\ \sigma_{\mu\nu}\gamma_5,\ \ldots \tag{1}$$

标准光前夸克模型（SLF QM）：

介子束缚态：

$$\begin{aligned}|M(p,L,J)\rangle = \quad & \sum_{h_1,h_2}\int \frac{\mathrm{d}^3 k_1}{(2\pi)^3 2\sqrt{k_1^+}}\frac{\mathrm{d}^3 k_2}{(2\pi)^3 2\sqrt{k_2^+}}(2\pi)^3\delta^3(p-k_1-k_2) \\ & \Psi_{LS}^{JJ_z}(k_1,h_1,k_2,h_2)|q_1(k_1,h_1)\rangle|\bar{q}_2(k_2,h_2)\rangle\,,\end{aligned} \tag{2}$$

动量空间波函数$\Psi_{LS}^{JJ_z}(k_1,h_1,k_2,h_2)$:

$$\Psi_{LS}^{JJ_z}(k_1,h_1,k_2,h_2) = S_{h_1,h_2}(x,\mathbf{k}_\perp)\psi(x,\mathbf{k}_\perp) \tag{3}$$

自旋轨道波函数$S_{h_1,h_2}(x,\mathbf{k}_\perp)$: Phys. Rev. D **41**, 3405 (1990);Phys. Rev. D **69**, 074025 (2004);

$$S_{h_1,h_2}=\frac{\bar{u}(k_1,h_1)\Gamma' v(k_2,h_2)}{\sqrt{2}\hat{M}_0}\,, \tag{4}$$

其中，P和T介子的顶角算符Γ':

$$\Gamma'_P=\gamma_5\,, \tag{5}$$

$$\Gamma'_T=-\frac{1}{2}\hat{\epsilon}^{\mu\nu}\left[\gamma_\mu-\frac{(k_1-k_2)_\mu}{D_{T,LF}}\right](k_1-k_2)_\nu\,,\quad D_{T,\mathrm{LF}}=M_0+m_1+m_2\,, \tag{6}$$

径向波函数$\psi(x,\mathbf{k}_\perp)$:

$$\psi_s(x,\mathbf{k}_\perp)=4\frac{\pi^{\frac{3}{4}}}{\beta^{\frac{3}{2}}}\sqrt{\frac{\partial k_z}{\partial x}}\exp\left[-\frac{k_z^2+\mathbf{k}_\perp^2}{2\beta^2}\right]\,, \tag{7}$$

$$\psi_p(x,\mathbf{k}_\perp)=\frac{\sqrt{2}}{\beta}\psi_s(x,\mathbf{k}_\perp)\,, \tag{8}$$

矩阵元:

$$\mathcal{B}_{\mathrm{SLF}}=\sum_{h_1',h_1'',h_2}\int\frac{\mathrm{d}x\,\mathrm{d}^2\mathbf{k}_\perp'}{(2\pi)^3\,2x}\psi''^*(x,\mathbf{k}_\perp'')\psi'(x,\mathbf{k}_\perp')S''^\dagger_{h_1'',h_2}(x,\mathbf{k}_\perp'')\,C_{h_1'',h_1'}(x,\mathbf{k}_\perp',\mathbf{k}_\perp'')\,S'_{h_1',h_2}(x,\mathbf{k}_\perp') \tag{9}$$

协变光前夸克模型（CLF QM）：

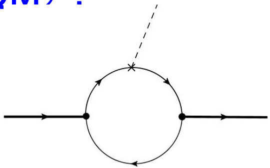


Figure: 矩阵元$\mathcal{B}$费曼图。

矩阵元：

$$\mathcal{B} = N_c \int \frac{\mathrm{d}^4 k_1'}{(2\pi)^4} \frac{H_P H_T}{N_1' N_1'' N_2} i S_{\mathcal{B}} \cdot \epsilon^* \,, \tag{10}$$

内线费米子圈求迹部分$S_{\mathcal{B}}$：

$$S_{\mathcal{B}} = \mathrm{Tr}\left[\Gamma\,(\not{k}_1' + m_1')\,(i\Gamma_P)\,(-\not{k}_2 + m_2)\,(i\gamma^0\Gamma_T^\dagger\gamma^0)(\not{k}_1'' + m_1'')\right] \,, \tag{11}$$

顶角算符$\Gamma_{P,T}$：

$$i\Gamma_P = -i\gamma_5 \,, \tag{12}$$

$$i\Gamma_T = i\frac{1}{2}\left[\gamma_\mu - \frac{(k_1'' - k_2)_\mu}{D_{T,\mathrm{con}}}\right](k_1'' - k_2)_\nu \,, \quad D_{T,\mathrm{con}} = M + m_1'' + m_2 \,, \tag{13}$$

假设：$H_{P,T}$在k_1^-的复平面上是解析的。则，在对k_1^-积分后，q_2在壳，且有以下替换：

$$N_1'^{(\prime\prime)} \to \hat{N}_1'^{(\prime\prime)} = x\left(M'^{(\prime\prime)2} - M_0'^{(\prime\prime)2}\right) \tag{14}$$

$$\chi_{P(T)} = H_{P(T)}/N'^{(\prime\prime)} \to h_{P(T)}/\hat{N}'^{(\prime\prime)}\,, \qquad D_{T,\mathrm{con}} \to D_{T,\mathrm{LF}}\,, \qquad \text{(type-I)} \tag{15}$$

其中，顶角$h_{P(T)}$为

$$h_{P(T)}/\hat{N}'^{(\prime\prime)} \quad = \quad \frac{1}{\sqrt{2N_c}}\sqrt{\frac{\bar{x}}{x}}\frac{\psi_{s(p)}}{\hat{M}_0'^{(\prime\prime)}}\,, \tag{16}$$

$$\chi_{P(T)} = H_{P(T)}/N'^{(\prime\prime)} \to h_{P(T)}'^{(\prime\prime)}/\hat{N}'^{(\prime\prime)}\,, \qquad M \to M_0\,, \qquad \text{(type-II)} \tag{17}$$

矩阵元：

$$\hat{\mathcal{B}} = N_c \int \frac{\mathrm{d}x\mathrm{d}^2\mathbf{k}_\perp'}{2(2\pi)^3}\frac{h_P h_T}{\bar{x}\hat{N}_1'\hat{N}_1''}\hat{S}_{\mathcal{B}}\cdot\epsilon^*\,, \tag{18}$$

为了有效确定zero-mode贡献，我们需要以下的分解替换：

Jaus, Phys. Rev. D **60**, 054026 (1999); H. Y. Cheng, Phys. Rev. D **69**, 074025 (2004)

$$\hat{k}_1'^{\mu}\hat{k}_1'^{\nu} \to g^{\mu\nu}A_1^{(2)} + P^{\mu}P^{\nu}A_2^{(2)} + (P^{\mu}q^{\nu} + q^{\mu}P^{\nu})A_3^{(2)} + q^{\mu}q^{\nu}A_4^{(2)} + \frac{P^{\mu}\omega^{\nu} + \omega^{\mu}P^{\nu}}{\omega \cdot P}B_1^{(2)} + \ldots(\omega, \mathbf{C_i^{(j)}}), \tag{19}$$

CLF QM中形状因子完整结果：

$$[\mathcal{F}]^{\rm full} = [\mathcal{F}]^{\rm CLF} + [\mathcal{F}]^{\rm B}. \tag{20}$$

$P \to T$跃迁形状因子的定义：

$$\langle T(\epsilon^{\mu\nu}, p'')|\bar{q}_2\gamma_\mu q_1|P(p')\rangle = i\varepsilon_{\mu\nu\alpha\beta}e^{*\nu}P^\alpha q^\beta \frac{V(q^2)}{M'+M''}, \quad (21)$$

$$\begin{aligned}\langle T(\epsilon^{\mu\nu}, p'')|\bar{q}_2\gamma_\mu\gamma_5 q_1|P(p')\rangle =& -2M''\frac{e^*\cdot q}{q^2}q_\mu A_0(q^2)\\ &-\Big(M'+M''\Big)\Big(e^*_\mu-\frac{e^*\cdot q}{q^2}q_\mu\Big)A_1(q^2)\\ &+\frac{e^*\cdot q}{M'+M''}\left(P_\mu-\frac{M'^2-M''^2}{q^2}q_\mu\right)A_2(q^2), \end{aligned} \quad (22)$$

$$\begin{aligned}\langle T(\epsilon^{\mu\nu}, p'')|\bar{q}_2\sigma_{\mu\nu}q^\nu(1+\gamma_5)q_1|P(p')\rangle =& -\varepsilon_{\mu\nu\alpha\beta}e^{*\nu}P^\alpha q^\beta\, T_1(q^2)\\ &+i\Big[(M'^2-M''^2)e^*_\mu-(e^*\cdot q)P_\mu\Big]T_2(q^2)\\ &+i\Big(e^*\cdot q\Big)\Big[q_\mu-\frac{q^2}{M'^2-M''^2}P_\mu\Big]T_3(q^2), \end{aligned} \quad (23)$$

其中，$\varepsilon_{0123}=-1$，$P=p'+p''$，$q=p'-p''$，$e^{*\nu}\equiv\frac{\epsilon^{*\mu\nu}\cdot p'_\mu}{M'}$.

自洽性问题

$$\widetilde{T}_3^{\mathrm{B}} = \frac{2(M'^2 - M''^2)}{e^* \cdot q} \frac{\epsilon^{\lambda\delta *} q_\lambda \omega_\delta}{\omega \cdot P} \left\{ B_1^{(2)} \left[3 + \frac{4(m_1'' + 2m_2 - 2m_1'')}{D_{V,\mathrm{con}}} \right] + 4B_1^{(3)} \left(1 + \frac{m_1' - m_1'' - 2m_2}{D_{V,\mathrm{con}}} \right) + \frac{8(m_1' - m_2)B_2^{(3)}}{D_{V,\mathrm{con}}} \right\}, \quad (24)$$

$$\widetilde{T}_3^{\mathrm{B}} = \begin{cases} \frac{2M'(M'^2 - M''^2)(M'^2 - M''^2 + q_\perp^2)}{(M'^2 - M''^2)^2 + 2(M'^2 - 2M''^2)\mathbf{q}_\perp^2 + \mathbf{q}_\perp^4} \left\{ B_1^{(2)} \left[3 + \frac{4(m_1'' + 2m_2 - 2m_1')}{D_{V,\mathrm{con}}} \right] + 4B_1^{(3)} \left(1 + \frac{m_1' - m_1'' - 2m_2}{D_{V,\mathrm{con}}} \right) + \frac{8(m_1' - m_2)B_2^{(3)}}{D_{V,\mathrm{con}}} \right\}, & \lambda'' = 0 \\ \frac{M'(M'^2 - M''^2)}{M'^2 - M''^2 + q_\perp^2} \left\{ B_1^{(2)} \left[3 + \frac{4(m_1'' + 2m_2 - 2m_1')}{D_{V,\mathrm{con}}} \right] + 4B_1^{(3)} \left(1 + \frac{m_1' - m_1'' - 2m_2}{D_{V,\mathrm{con}}} \right) + \frac{8(m_1' - m_2)B_2^{(3)}}{D_{V,\mathrm{con}}} \right\}, & \lambda'' = \pm 1 \\ 0\,. & \lambda'' = \pm 2 \end{cases} \quad (25)$$

显然，这依赖于螺旋度λ''的选择。

$$[T_3]^{\mathrm{full}}_{\lambda=0} \neq [T_3]^{\mathrm{full}}_{\lambda=\pm 1} \neq [T_3]^{\mathrm{full}}_{\lambda=\pm 2}$$

定义$\Delta_{\mathrm{B}}(x)$为

$$\Delta_{\mathrm{B}}(x) \equiv \frac{\mathrm{d}[\mathcal{F}^{\mathrm{B}}]_{\lambda''}}{\mathrm{d}x}, \tag{26}$$

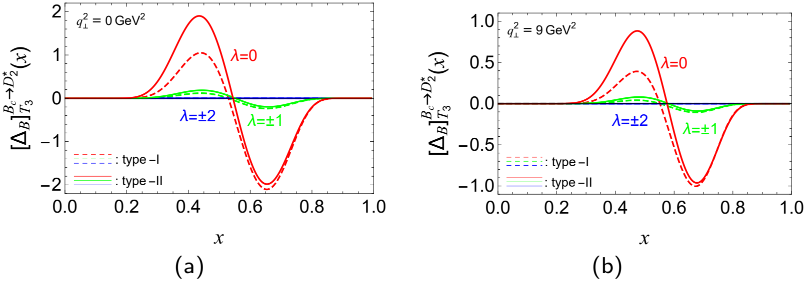


Figure: $B_c \to D_2^*$ 在$\mathbf{q}_\perp^2 = (0, 0.2)\,\mathrm{GeV}^2$时$[\Delta_B]_{T_3}(x)$对$x$的依赖性。

Table: $B_c \to D_2^*$ 过程形状因子 $T_3(\mathbf{q}_\perp^2)$ 在 $\mathbf{q}_\perp^2 = (0,1,4,9)\,\mathrm{GeV}^2$ 数值结果。

$B_c \to D_2^*$		$[T_3]^{\rm SLF}_{\lambda''=\pm2}$	$[T_3]^{\rm full}_{\lambda''=0}$	$[T_3]^{\rm full}_{\lambda''=\pm1}$	$[T_3]^{\rm full}_{\lambda''=\pm2}$	$[T_3]^{\rm val.}$	$[T_3]^{\rm CLF}$
$\mathbf{q}_\perp^2 = 0$	type-I	0.03	−0.14	−0.04	0.05	0.10	0.05
	type-II	0.08	0.08	0.08	0.08	0.08	0.08
$\mathbf{q}_\perp^2 = 1$	type-I	0.03	−0.13	−0.04	0.05	0.09	0.05
	type-II	0.07	0.07	0.07	0.07	0.07	0.07
$\mathbf{q}_\perp^2 = 4$	type-I	0.02	−0.11	−0.04	0.03	0.07	0.03
	type-II	0.05	0.05	0.05	0.05	0.05	0.05
$\mathbf{q}_\perp^2 = 9$	type-I	0.02	−0.08	−0.03	0.02	0.05	0.02
	type-II	0.04	0.04	0.04	0.04	0.04	0.04

$$[T_3]^{\rm full}_{\lambda=0} \neq [T_3]^{\rm full}_{\lambda=\pm1} \neq [T_3]^{\rm full}_{\lambda=\pm2} \qquad (\text{type}-\text{I}) \tag{27}$$

$$[T_3]^{\rm full}_{\lambda=0} \doteq [T_3]^{\rm full}_{\lambda=\pm1} \doteq [T_3]^{\rm full}_{\lambda=\pm2} \qquad (\text{type}-\text{II}) \tag{28}$$

■ “新”的自洽性问题

H. Y. Cheng and C. K. Chua, Phys. Rev. D **69**, 094007 (2004)

$$
\begin{aligned}
\mathrm{CC}: 2ig_{\nu\lambda}g_{\alpha\mu}g_{\beta\sigma}(P+q)^{\beta}\hat{k}_1'^{\sigma}\hat{k}_1'^{\alpha}\hat{k}_{1\delta}' =& 2ig_{\nu\lambda}g_{\alpha\mu}g_{\beta\sigma}(P+q)^{\beta}\Big[(g^{\alpha\sigma}P_{\delta}+g_{\delta}^{\alpha}P^{\sigma} \\
&+g_{\delta}^{\sigma}P^{\alpha})A_1^{(3)}+(g^{\alpha\sigma}q_{\delta}+g_{\delta}^{\alpha}q^{\sigma}+g_{\delta}^{\sigma}q^{\alpha})A_2^{(3)} \\
&+P^{\sigma}P^{\alpha}P_{\delta}A_3^{(3)}+\ldots\Big] \\
\mathrm{ours}: 2ig_{\nu\lambda}g_{\alpha\mu}g_{\beta\sigma}(P+q)^{\beta}\hat{k}_1'^{\sigma}\hat{k}_1'^{\alpha}\hat{k}_{1\delta}' =& 2ig_{\nu\lambda}\hat{k}_{1\mu}'\hat{k}_{1\delta}'\hat{k}_1'\cdot(P+q)
\end{aligned}
\tag{29}
$$

si

定义：

$$
\Delta_{\mathcal{F}}^{\mathrm{CLF}}(x,\mathbf{q}_{\perp}^2)\equiv\frac{\mathrm{d}[\mathcal{F}]_{\mathrm{ours}}^{\mathrm{CLF}}}{\mathrm{d}x}-\frac{\mathrm{d}[\mathcal{F}]_{\mathrm{CC}}^{\mathrm{CLF}}}{\mathrm{d}x}\,, \tag{30}
$$

其中，$\mathcal{F}=T_{2,3}$。

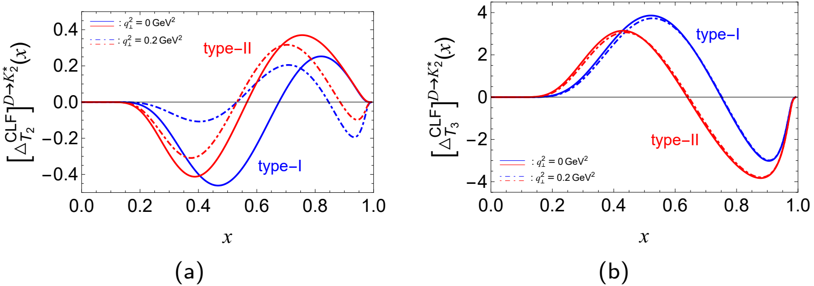


Figure: $D \to K_2^*$ 在$\mathbf{q}_\perp^2 = (0, 0.2)\,\mathrm{GeV}^2$时$\Delta_{T_{2,3}}^{\mathrm{CLF}}$对$x$的依赖性。

- 协变性问题

$$\widetilde{\mathcal{B}}_{\mathrm{B}}(\Gamma=\sigma_{\mu\nu}\gamma_5 q^{\nu}) = 4i\omega_{\mu}\epsilon^{*\lambda\delta}q_{\lambda}q_{\delta}\Big\{\frac{B_1^{(2)}}{\omega\cdot P}\Big[2M'^2-M''^2+(m_1'-m_2)(m_2-m_1'')+q^2 - \frac{2}{D_{V,\mathrm{con}}}\Big((M'^2-M''^2)(m_1'+m_1'')-q^2(m_1'-m_1''-2m_2)\Big)\Big] + \dots \tag{31}$$

type-I：矩阵元的协变性被破坏。

type-II：协变性问题得以解决。

- 零模贡献

$$[\mathcal{F}]^{\rm CLF} = [\mathcal{F}]^{\rm val.} + [\mathcal{F}]^{\rm z.m.}$$

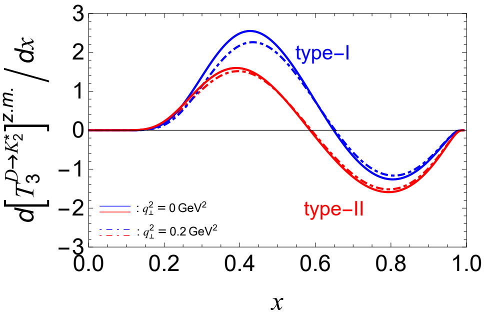


Figure: $D \to K_2^*$ 在$\mathbf{q}_\perp^2 = (0, 0.2)\,\mathrm{GeV}^2$ 时$\mathrm{d}[T_3]^{\rm z.m.}/\mathrm{d}x$ 对x的依赖性。

■ 数值结果

拟合方式：

$$\mathcal{F}(q^2) = \frac{\mathcal{F}(0)}{1 - q^2/m_{i,pole}^2}\left\{1 + \sum_{k=1}^{N} b_k \left[z(q^2, t_0)^k - z(0, t_0)^k\right]\right\}, \tag{32}$$

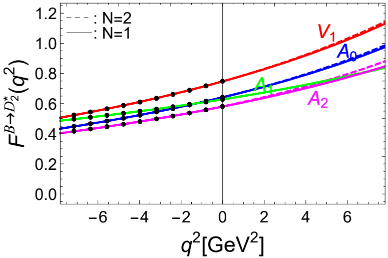
其中，$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$，$t_\pm = (M' \pm M'')^2$，$t_0 = (M' + M'')(\sqrt{M'} - \sqrt{M''})^2$。

Table: $q^2 = (3, 5, 7)GeV^2$ 处$B \to D_2^*$ 跃迁过程$Z_k(q^2)$ 的数值结果。

$B \to D_2^*$	$q^2 = 3GeV^2$	$q^2 = 5GeV^2$	$q^2 = 7GeV^2$
$Z_1(q^2)$	-1.28×10^{-2}	-2.18×10^{-2}	-3.11×10^{-2}
$Z_2(q^2)$	-2.92×10^{-4}	-3.00×10^{-4}	-1.40×10^{-4}

Table: $B \to D_2^*$ 过程形状因子b_1 和b_2 的数值。

$B \to D_2^*$	V	A_0	A_1	A_2	T_1	T_2	T_3
b_1	$-6.59(-6.20)$	$-6.64(-6.20)$	$-3.19(-3.00)$	$-6.44(-6.10)$	$-6.61(-6.30)$	$-2.47(-2.40)$	$-5.85(-5.91)$
b_2	5.94	6.51	2.44	5.93	4.87	1.17	0.50



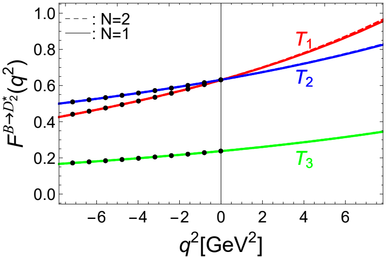


Figure: $B \to D_2^*$ 过程在$N = 1$ 和$N = 2$ 截断方案下对q^2 的依赖图。

$\mathcal{F}$	$F(0)$	b_1	$\mathcal{F}$	$F(0)$	b_1	$\mathcal{F}$	$F(0)$	b_1
$V^{D\to a_2}$	$0.96^{+0.18}_{-0.16}$	$-4.60^{+3.60}_{-2.70}$	$V^{D\to K_2^*}$	$1.00^{+0.15}_{-0.16}$	$-4.60^{+2.84}_{-2.32}$	$V^{D_s\to K_2^*}$	$0.98^{+0.22}_{-0.19}$	$-4.80^{+4.00}_{-3.10}$
$A_0^{D\to a_2}$	$0.62^{+0.07}_{-0.07}$	$-3.70^{+3.90}_{-2.90}$	$A_0^{D\to K_2^*}$	$0.68^{+0.06}_{-0.08}$	$-3.70^{+3.10}_{-2.48}$	$A_0^{D_s\to K_2^*}$	$0.58^{+0.08}_{-0.08}$	$-4.20^{+4.50}_{-3.40}$
$A_1^{D\to a_2}$	$0.63^{+0.11}_{-0.10}$	$-2.00^{+2.60}_{-1.80}$	$A_1^{D\to K_2^*}$	$0.71^{+0.09}_{-0.10}$	$-1.90^{+1.98}_{-1.63}$	$A_1^{D_s\to K_2^*}$	$0.61^{+0.12}_{-0.10}$	$-2.60^{+3.00}_{-2.30}$
$A_2^{D\to a_2}$	$0.45^{+0.10}_{-0.08}$	$-3.70^{+2.70}_{-1.90}$	$A_2^{D\to K_2^*}$	$0.58^{+0.11}_{-0.10}$	$-4.90^{+2.58}_{-1.97}$	$A_2^{D_s\to K_2^*}$	$0.51^{+0.14}_{-0.10}$	$-5.10^{+3.40}_{-2.50}$
$T_1^{D\to a_2}$	$0.61^{+0.08}_{-0.08}$	$-4.06^{+3.16}_{-2.90}$	$T_1^{D\to K_2^*}$	$0.68^{+0.07}_{-0.09}$	$-3.63^{+2.27}_{-2.08}$	$T_1^{D_s\to K_2^*}$	$0.58^{+0.09}_{-0.09}$	$-4.58^{+3.40}_{-3.42}$
$T_2^{D\to a_2}$	$0.61^{+0.08}_{-0.08}$	$4.78^{+2.06}_{-1.38}$	$T_2^{D\to K_2^*}$	$0.68^{+0.08}_{-0.09}$	$4.64^{+1.66}_{-1.29}$	$T_2^{D_s\to K_2^*}$	$0.58^{+0.09}_{-0.09}$	$4.23^{+1.87}_{-1.23}$
$T_3^{D\to a_2}$	$0.22^{+0.03}_{-0.06}$	$-4.85^{+1.81}_{-1.55}$	$T_3^{D\to K_2^*}$	$0.17^{+0.07}_{-0.10}$	$-2.85^{+1.35}_{-1.58}$	$T_3^{D_s\to K_2^*}$	$0.15^{+0.05}_{-0.08}$	$-4.21^{+1.97}_{-2.42}$
$V^{D_s\to f_2'}$	$1.19^{+0.19}_{-0.20}$	$-4.40^{+3.00}_{-2.50}$	$V^{\eta_c(1S)\to D_2^*}$	$2.87^{+1.47}_{-0.93}$	$-19.4^{+10.8}_{-10.4}$	$V^{\eta_c(1S)\to D_{s2}^*}$	$3.67^{+1.53}_{-1.12}$	$-14.6^{+9.59}_{-8.34}$
$A_0^{D_s\to f_2'}$	$0.72^{+0.07}_{-0.08}$	$-3.70^{+3.42}_{-2.80}$	$A_0^{\eta_c(1S)\to D_2^*}$	$0.96^{+0.22}_{-0.20}$	$-18.5^{+11.4}_{-10.8}$	$A_0^{\eta_c(1S)\to D_{s2}^*}$	$1.28^{+0.25}_{-0.24}$	$-13.3^{+9.59}_{-8.85}$
$A_1^{D_s\to f_2'}$	$0.77^{+0.10}_{-0.11}$	$-2.10^{+2.21}_{-1.91}$	$A_1^{\eta_c(1S)\to D_2^*}$	$1.10^{+0.34}_{-0.27}$	$-15.8^{+9.23}_{-8.67}$	$A_1^{\eta_c(1S)\to D_{s2}^*}$	$1.47^{+0.35}_{-0.32}$	$-10.8^{+7.86}_{-7.32}$
$A_2^{D_s\to f_2'}$	$0.67^{+0.15}_{-0.13}$	$-4.90^{+2.99}_{-2.36}$	$A_2^{\eta_c(1S)\to D_2^*}$	$0.98^{+0.55}_{-0.30}$	$-14.2^{+13.0}_{-10.1}$	$A_2^{\eta_c(1S)\to D_{s2}^*}$	$1.13^{+0.62}_{-0.35}$	$-8.39^{+13.4}_{-10.9}$
$T_1^{D_s\to f_2'}$	$0.73^{+0.08}_{-0.10}$	$-3.64^{+2.59}_{-2.31}$	$T_1^{\eta_c(1S)\to D_2^*}$	$1.04^{+0.27}_{-0.23}$	$-22.1^{+10.9}_{-10.4}$	$T_1^{\eta_c(1S)\to D_{s2}^*}$	$1.38^{+0.30}_{-0.28}$	$-16.9^{+9.07}_{-8.56}$
$T_2^{D_s\to f_2'}$	$0.73^{+0.08}_{-0.09}$	$6.30^{+1.89}_{-1.85}$	$T_2^{\eta_c(1S)\to D_2^*}$	$1.04^{+0.27}_{-0.23}$	$28.4^{+7.53}_{-13.8}$	$T_2^{\eta_c(1S)\to D_{s2}^*}$	$1.38^{+0.30}_{-0.28}$	$88.1^{+26.1}_{-21.7}$
$T_3^{D_s\to f_2'}$	$0.15^{+0.07}_{-0.12}$	$-2.27^{+1.09}_{-1.83}$	$T_3^{\eta_c(1S)\to D_2^*}$	$0.16^{+0.18}_{-0.23}$	$-41.1^{+8.96}_{-10.6}$	$T_3^{\eta_c(1S)\to D_{s2}^*}$	$0.35^{+0.28}_{-0.35}$	$-30.0^{+6.30}_{-2.50}$
$V^{B_c\to B_2^*}$	$13.4^{+5.30}_{-4.11}$	$-87.6^{+26.7}_{-23.5}$	$V^{B_c\to B_{s2}^*}$	$14.8^{+5.59}_{-4.12}$	$-74.7^{+20.2}_{-14.1}$			
$A_0^{B_c\to B_2^*}$	$2.01^{+0.33}_{-0.37}$	$-19.3^{+3.46}_{-6.39}$	$A_0^{B_c\to B_{s2}^*}$	$2.45^{+0.11}_{-0.14}$	$-20.5^{+5.34}_{-4.03}$			
$A_1^{B_c\to B_2^*}$	$2.42^{+0.60}_{-0.55}$	$-35.4^{+1.13}_{-3.74}$	$A_1^{B_c\to B_{s2}^*}$	$2.94^{+0.10}_{-0.30}$	$-35.4^{+2.79}_{-3.63}$			
$A_2^{B_c\to B_2^*}$	$2.51^{+2.42}_{-1.15}$	$-31.6^{+12.0}_{-14.1}$	$A_2^{B_c\to B_{s2}^*}$	$3.45^{+3.15}_{-1.60}$	$-29.4^{+16.5}_{-7.91}$			
$T_1^{B_c\to B_2^*}$	$2.24^{+0.43}_{-0.44}$	$-77.1^{+10.1}_{-11.7}$	$T_1^{B_c\to B_{s2}^*}$	$2.69^{+0.43}_{-0.49}$	$-74.5^{+4.74}_{-3.43}$			
$T_2^{B_c\to B_2^*}$	$2.25^{+0.45}_{-0.45}$	$89.2^{+22.9}_{-20.2}$	$T_2^{B_c\to B_{s2}^*}$	$2.69^{+0.10}_{-0.18}$	$117^{+23.5}_{-26.4}$			
$T_3^{B_c\to B_2^*}$	$-0.06^{+0.90}_{-1.43}$	$-60.1^{+13.0}_{-24.8}$	$T_3^{B_c\to B_{s2}^*}$	$-0.52^{+1.29}_{-1.82}$	$-61.1^{+21.3}_{-26.1}$			

$\mathcal{F}$	$F(0)$	b_1	$\mathcal{F}$	$F(0)$	b_1	$\mathcal{F}$	$F(0)$	b_1
$V^{B\to a_2}$	$0.24^{+0.04}_{-0.04}$	$-5.30^{+1.10}_{-0.98}$	$V^{B\to K_2^*}$	$0.28^{+0.05}_{-0.05}$	$-5.50^{+1.10}_{-0.99}$	$V^{B\to D_2^*}$	$0.75^{+0.13}_{-0.16}$	$-6.20^{+0.52}_{-0.50}$
$A_0^{B\to a_2}$	$0.21^{+0.03}_{-0.03}$	$-5.40^{+0.74}_{-0.61}$	$A_0^{B\to K_2^*}$	$0.24^{+0.04}_{-0.04}$	$-5.60^{+0.74}_{-0.59}$	$A_0^{B\to D_2^*}$	$0.64^{+0.11}_{-0.11}$	$-6.20^{+0.53}_{-0.51}$
$A_1^{B\to a_2}$	$0.19^{+0.04}_{-0.03}$	$-2.30^{+0.52}_{-0.37}$	$A_1^{B\to K_2^*}$	$0.22^{+0.07}_{-0.01}$	$-2.50^{+0.51}_{-0.34}$	$A_1^{B\to D_2^*}$	$0.63^{+0.11}_{-0.12}$	$-3.00^{+0.51}_{-0.48}$
$A_2^{B\to a_2}$	$0.17^{+0.03}_{-0.02}$	$-4.80^{+0.83}_{-0.90}$	$A_2^{B\to K_2^*}$	$0.20^{+0.04}_{-0.03}$	$-5.20^{+1.00}_{-0.92}$	$A_2^{B\to D_2^*}$	$0.58^{+0.12}_{-0.12}$	$-6.10^{+0.52}_{-0.53}$
$T_1^{B\to a_2}$	$0.19^{+0.03}_{-0.03}$	$-5.40^{+0.75}_{-0.61}$	$T_1^{B\to K_2^*}$	$0.23^{+0.04}_{-0.04}$	$-5.60^{+0.75}_{-0.59}$	$T_1^{B\to D_2^*}$	$0.63^{+0.11}_{-0.12}$	$-6.30^{+0.52}_{-0.51}$
$T_2^{B\to a_2}$	$0.19^{+0.03}_{-0.03}$	$-2.00^{+0.42}_{-0.30}$	$T_2^{B\to K_2^*}$	$0.23^{+0.04}_{-0.04}$	$-2.20^{+0.41}_{-0.26}$	$T_2^{B\to D_2^*}$	$0.63^{+0.11}_{-0.12}$	$-2.40^{+0.20}_{-0.33}$
$T_3^{B\to a_2}$	$0.12^{+0.01}_{-0.05}$	$-4.90^{+1.35}_{-0.92}$	$T_3^{B\to K_2^*}$	$0.13^{+0.03}_{-0.01}$	$-5.20^{+3.10}_{-0.62}$	$T_3^{B\to D_2^*}$	$0.24^{+0.01}_{-0.06}$	$-5.91^{+0.47}_{-0.83}$
$V^{B_s\to K_2^*}$	$0.23^{+0.04}_{-0.05}$	$-6.50^{+0.42}_{-0.28}$	$V^{B_s\to f_2'}$	$0.32^{+0.05}_{-0.06}$	$-6.30^{+0.48}_{-0.37}$	$V^{B_s\to D_{s2}^*}$	$0.95^{+0.18}_{-0.17}$	$-6.50^{+0.66}_{-0.66}$
$A_0^{B_s\to K_2^*}$	$0.18^{+0.04}_{-0.04}$	$-6.80^{+0.42}_{-0.29}$	$A_0^{B_s\to f_2'}$	$0.26^{+0.05}_{-0.05}$	$-6.50^{+0.47}_{-0.37}$	$A_0^{B_s\to D_{s2}^*}$	$0.76^{+0.13}_{-0.13}$	$-6.60^{+0.67}_{-0.67}$
$A_1^{B_s\to K_2^*}$	$0.17^{+0.03}_{-0.04}$	$-4.00^{+0.08}_{-0.04}$	$A_1^{B_s\to f_2'}$	$0.24^{+0.05}_{-0.05}$	$-3.70^{+0.19}_{-0.10}$	$A_1^{B_s\to D_{s2}^*}$	$0.74^{+0.15}_{-0.14}$	$-3.40^{+0.67}_{-0.67}$
$A_2^{B_s\to K_2^*}$	$0.16^{+0.03}_{-0.03}$	$-6.10^{+0.31}_{-0.18}$	$A_2^{B_s\to f_2'}$	$0.22^{+0.04}_{-0.04}$	$-6.00^{+0.40}_{-0.31}$	$A_2^{B_s\to D_{s2}^*}$	$0.66^{+0.14}_{-0.13}$	$-6.00^{+0.64}_{-0.65}$
$T_1^{B_s\to K_2^*}$	$0.17^{+0.04}_{-0.03}$	$-6.80^{+0.42}_{-0.28}$	$T_1^{B_s\to f_2'}$	$0.25^{+0.05}_{-0.04}$	$-6.50^{+0.47}_{-0.37}$	$T_1^{B_s\to D_{s2}^*}$	$0.75^{+0.14}_{-0.14}$	$-6.70^{+0.67}_{-0.67}$
$T_2^{B_s\to K_2^*}$	$0.17^{+0.04}_{-0.03}$	$-3.70^{+0.02}_{-0.12}$	$T_2^{B_s\to f_2'}$	$0.25^{+0.05}_{-0.04}$	$-3.40^{+0.09}_{-0.03}$	$T_2^{B_s\to D_{s2}^*}$	$0.75^{+0.14}_{-0.14}$	$-2.30^{+0.12}_{-0.38}$
$T_3^{B_s\to K_2^*}$	$0.10^{+0.02}_{-0.02}$	$-6.30^{+0.30}_{-0.18}$	$T_3^{B_s\to f_2'}$	$0.14^{+0.01}_{-0.05}$	$-6.10^{+0.57}_{-0.61}$	$T_3^{B_s\to D_{s2}^*}$	$0.29^{+0.32}_{-0.16}$	$-6.00^{+2.35}_{-5.80}$
$V^{B_c\to D_2^*}$	$0.35^{+0.09}_{-0.10}$	$-14.0^{+1.10}_{-0.99}$	$V^{B_c\to D_{s2}^*}$	$0.65^{+0.19}_{-0.13}$	$-13.0^{+2.40}_{-2.60}$	$V^{B_c\to \chi_{c2}(1P)}$	$1.54^{+0.51}_{-0.38}$	$-13.0^{+2.90}_{-2.80}$
$A_0^{B_c\to D_2^*}$	$0.19^{+0.06}_{-0.06}$	$-15.0^{+0.97}_{-0.91}$	$A_0^{B_c\to D_{s2}^*}$	$0.37^{+0.08}_{-0.09}$	$-13.0^{+0.83}_{-0.86}$	$A_0^{B_c\to \chi_{c2}(1P)}$	$0.96^{+0.25}_{-0.20}$	$-13.0^{+3.00}_{-2.90}$
$A_1^{B_c\to D_2^*}$	$0.19^{+0.06}_{-0.06}$	$-12.0^{+1.90}_{-1.70}$	$A_1^{B_c\to D_{s2}^*}$	$0.36^{+0.08}_{-0.09}$	$-10.0^{+1.40}_{-1.40}$	$A_1^{B_c\to \chi_{c2}(1P)}$	$0.97^{+0.28}_{-0.22}$	$-9.80^{+3.20}_{-3.10}$
$A_2^{B_c\to D_2^*}$	$0.18^{+0.06}_{-0.06}$	$-13.0^{+1.40}_{-1.30}$	$A_2^{B_c\to D_{s2}^*}$	$0.31^{+0.08}_{-0.08}$	$-11.0^{+1.20}_{-1.20}$	$A_2^{B_c\to \chi_{c2}(1P)}$	$0.87^{+0.32}_{-0.22}$	$-12.0^{+3.00}_{-3.00}$
$T_1^{B_c\to D_2^*}$	$0.19^{+0.06}_{-0.06}$	$-15.0^{+0.98}_{-0.91}$	$T_1^{B_c\to D_{s2}^*}$	$0.37^{+0.08}_{-0.08}$	$-13.0^{+0.84}_{-0.86}$	$T_1^{B_c\to \chi_{2c}(1P)}$	$0.96^{+0.26}_{-0.21}$	$-13.0^{+3.00}_{-2.90}$
$T_2^{B_c\to D_2^*}$	$0.19^{+0.06}_{-0.06}$	$-12.8^{+1.90}_{-1.50}$	$T_2^{B_c\to D_{s2}^*}$	$0.37^{+0.08}_{-0.08}$	$-9.30^{+1.90}_{-1.60}$	$T_2^{B_c\to \chi_{2c}(1P)}$	$0.96^{+0.26}_{-0.20}$	$-7.70^{+2.50}_{-2.80}$
$T_3^{B_c\to D_2^*}$	$0.08^{+0.03}_{-0.01}$	$-15.5^{+6.00}_{-4.75}$	$T_3^{B_c\to D_{s2}^*}$	$0.15^{+0.03}_{-0.04}$	$-13.0^{+1.00}_{-1.00}$	$T_3^{B_c\to \chi_{2c}(1P)}$	$0.29^{+0.03}_{-0.07}$	$-13.0^{+1.70}_{-3.10}$
$V^{\eta_b(1S)\to B_2^*}$	$0.61^{+0.22}_{-0.19}$	$-57.4^{+3.37}_{-3.43}$	$V^{\eta_b(1S)\to B_{s2}^*}$	$0.90^{+0.27}_{-0.24}$	$-56.0^{+4.06}_{-4.05}$			
$A_0^{\eta_b(1S)\to B_2^*}$	$0.19^{+0.07}_{-0.06}$	$-58.5^{+1.18}_{-1.27}$	$A_0^{\eta_b(1S)\to B_{s2}^*}$	$0.29^{+0.09}_{-0.08}$	$-57.2^{+1.27}_{-1.22}$			
$A_1^{\eta_b(1S)\to B_2^*}$	$0.21^{+0.08}_{-0.07}$	$-53.0^{+1.81}_{-1.80}$	$A_1^{\eta_b(1S)\to B_{s2}^*}$	$0.32^{+0.11}_{-0.09}$	$-51.5^{+1.26}_{-1.22}$			
$A_2^{\eta_b(1S)\to B_2^*}$	$0.23^{+0.10}_{-0.08}$	$-52.8^{+0.73}_{-0.84}$	$A_2^{\eta_b(1S)\to B_{s2}^*}$	$0.35^{+0.14}_{-0.11}$	$-51.4^{+0.56}_{-0.69}$			
$T_1^{\eta_b(1S)\to B_2^*}$	$0.20^{+0.06}_{-0.07}$	$-58.7^{+0.95}_{-1.06}$	$T_1^{\eta_b(1S)\to B_{s2}^*}$	$0.30^{+0.10}_{-0.08}$	$-57.4^{+0.26}_{-0.21}$			
$T_2^{\eta_b(1S)\to B_2^*}$	$0.20^{+0.06}_{-0.07}$	$-54.1^{+1.78}_{-1.75}$	$T_2^{\eta_b(1S)\to B_{s2}^*}$	$0.30^{+0.10}_{-0.08}$	$-52.2^{+0.52}_{-0.44}$			
$T_3^{\eta_b(1S)\to B_2^*}$	$0.02^{+0.01}_{-0.02}$	$-73.6^{+25.9}_{-17.8}$	$T_3^{\eta_b(1S)\to B_{s2}^*}$	$0.03^{+0.02}_{-0.03}$	$-71.4^{+22.9}_{-11.9}$			

Figure: $D\to(a_2,K_2^*)$，$D_s\to(K_2^*,f_2')$，$\eta_c(1S)\to(D_2^*,D_{s2}^*)$，$B_c\to(B_2^*,B_{s2}^*)$，$B\to(a_2,K_2^*,D_2^*)$，$B_s\to(K_2^*,f_2',D_{s2}^*)$，$B_c\to(D_2^*,D_{s2}^*,\chi_{c2}(1P))$，$\eta_b(1S)\to(B_2^*,B_{s2}^*)$ 形状因子的拟合结果。

Table: 本文在$q^2=0$计算的$B\to a_2$ 和$B\to K_2^*$ 形状因子的理论预测，以及采用QCD SR和传统CLF QM计算的结果。

	$B\to a_2$				$B\to K_2^*$			
	this work	LCSR	pQCD	CLF	this work	LCSR	pQCD	CLF
$V(0)$	$0.24^{+0.04}_{-0.04}$	$0.18^{+0.12}_{-0.07}$	$0.18^{+0.05}_{-0.04}$	0.28	$0.28^{+0.05}_{-0.05}$	$0.22^{+0.11}_{-0.08}$	$0.21^{+0.06}_{-0.05}$	0.29
$A_0(0)$	$0.21^{+0.03}_{-0.03}$	$0.30^{+0.06}_{-0.05}$	$0.18^{+0.06}_{-0.04}$	0.24	$0.24^{+0.04}_{-0.04}$	$0.30^{+0.06}_{-0.05}$	$0.18^{+0.05}_{-0.04}$	0.23
$A_1(0)$	$0.19^{+0.04}_{-0.03}$	$0.16^{+0.09}_{-0.05}$	$0.11^{+0.03}_{-0.03}$	0.21	$0.22^{+0.07}_{-0.01}$	$0.19^{+0.09}_{-0.07}$	$0.13^{+0.04}_{-0.03}$	0.22
$A_2(0)$	$0.17^{+0.03}_{-0.02}$	$0.07^{+0.08}_{-0.03}$	$0.06^{+0.02}_{-0.01}$	0.19	$0.20^{+0.04}_{-0.03}$	$0.11^{+0.05}_{-0.06}$	$0.08^{+0.03}_{-0.02}$	0.21
$T_1(0)$	$0.19^{+0.03}_{-0.03}$	$0.15^{+0.09}_{-0.05}$	$0.15^{+0.04}_{-0.03}$		$0.23^{+0.04}_{-0.04}$	$0.19^{+0.09}_{-0.06}$	$0.17^{+0.05}_{-0.04}$	0.28
$T_2(0)$	$0.19^{+0.03}_{-0.03}$	$0.15^{+0.09}_{-0.05}$	$0.15^{+0.04}_{-0.03}$		$0.23^{+0.04}_{-0.04}$	$0.19^{+0.09}_{-0.06}$	$0.17^{+0.05}_{-0.04}$	0.28
$T_3(0)$	$0.16^{+0.01}_{-0.05}$	$0.07^{+0.06}_{-0.03}$	$0.13^{+0.04}_{-0.03}$		$0.12^{+0.03}_{-0.01}$	$0.09^{+0.06}_{-0.04}$	$0.14^{+0.05}_{-0.03}$	-0.25

Table: $B \to \bar{D}_2^* \ell^+ \nu_\ell (\ell = e, \mu)$和$B \to \bar{D}_2^* \tau^+ \nu_\tau$ 分支比的数值结果。

	type-II	type-I	QCD SR	LCSR
$\mathcal{B}(B \to \bar{D}_2^* \ell^+ \nu_\ell)$	$(1.23^{+0.36}_{-0.39}) \times 10^{-2}$	$(0.63^{+0.39}_{-0.34}) \times 10^{-2}$	$(1.01^{+0.30}_{-0.30}) \times 10^{-3}$	$(3.80^{+0.74}_{-0.74}) \times 10^{-2}$
$\mathcal{B}(B \to \bar{D}_2^* \tau^+ \nu_\tau)$	$(0.49^{+0.15}_{-0.16}) \times 10^{-3}$	$(0.22^{+0.14}_{-0.13}) \times 10^{-3}$	$(0.16^{+0.06}_{-0.06}) \times 10^{-3}$	$(1.50^{+0.28}_{-0.28}) \times 10^{-3}$

$$\mathcal{B}(B^+ \to \bar{D}_2^{*0} \ell^+ \nu_\ell) = \begin{cases} (0.92^{+0.10+0.41}_{-0.10-0.35}) \times 10^{-2}\,, \\ (1.20^{+0.29+0.37}_{-0.29-0.47}) \times 10^{-2}\,, \end{cases}$$

$$R_{D_2^*} \equiv \frac{\Gamma(B \to \bar{D}_2^* \tau^+ \nu_\tau)}{\Gamma(B \to \bar{D}_2^* \ell^+ \nu_\ell)}$$

$$R_{D_2^*} = 0.040^{+0.002}_{-0.002} (0.035^{+0.004}_{-0.003})\,, \qquad \mathrm{type-II(type-I)}$$

总结与展望：

- CLF QM会出现协变性破坏和两种自洽性问题：一种是由非零的类光矢量ω依赖项引起的，这部分贡献同时破坏了洛伦兹协变性；另一种是由于采用不同处理强子矩阵元方法导致的。
- 通过修改协变框架和光前框架的对应关系（type-II方案），即可同时解决自洽性和协变性问题。
- 以上研究结果不仅能够改进CLF计算框架，而且可以为介子唯象学研究提供必要的非微扰输入。

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