Synchrotron radiation background

陈姗红

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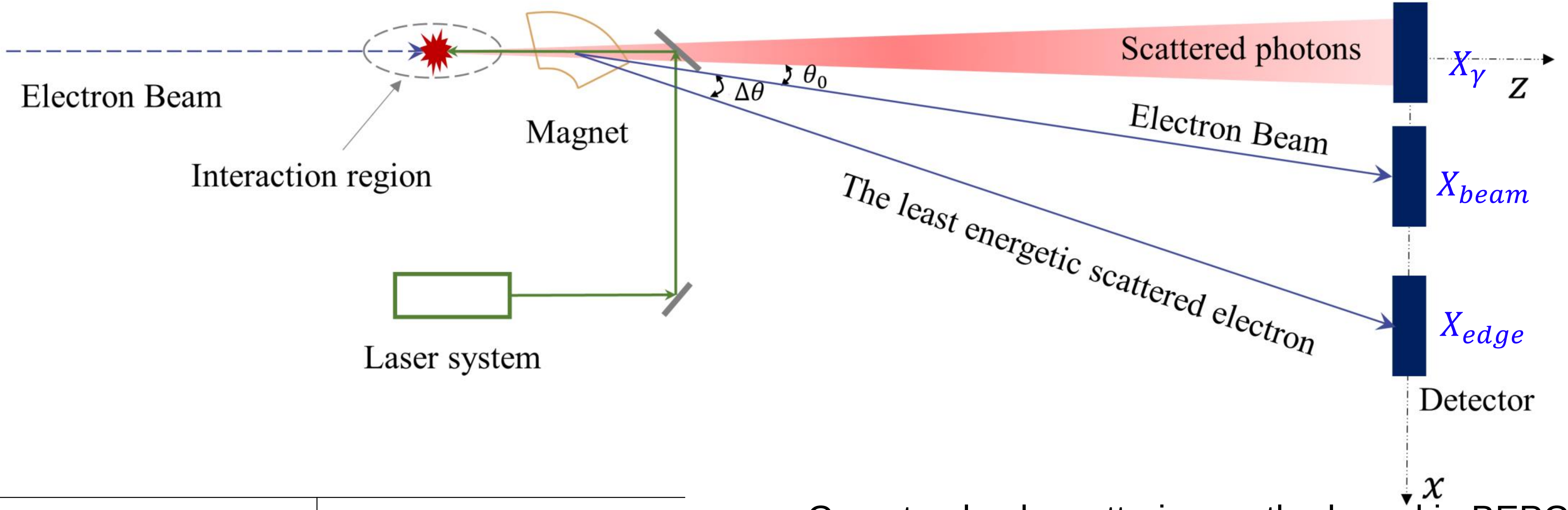
Outline

- Calibration of beam energy
- Properties of scattered photons
- Synchrotron radiation

Method of energy calibration

➢ Method: Compton back-scattering combining a bending magnet



Electron beam		**Nd:YAG Laser system**	
Energy (GeV)	120	λ(nm)	532
N_e	15×10^{10}	Energy(J)	0.1
Collision angle α		~ 2.35 mrad	
Compton scattering cross section		202 mb	

- Compton back-scattering method used in BEPC by measuring the energy of scattered photons with accuracy is 2×10^5.

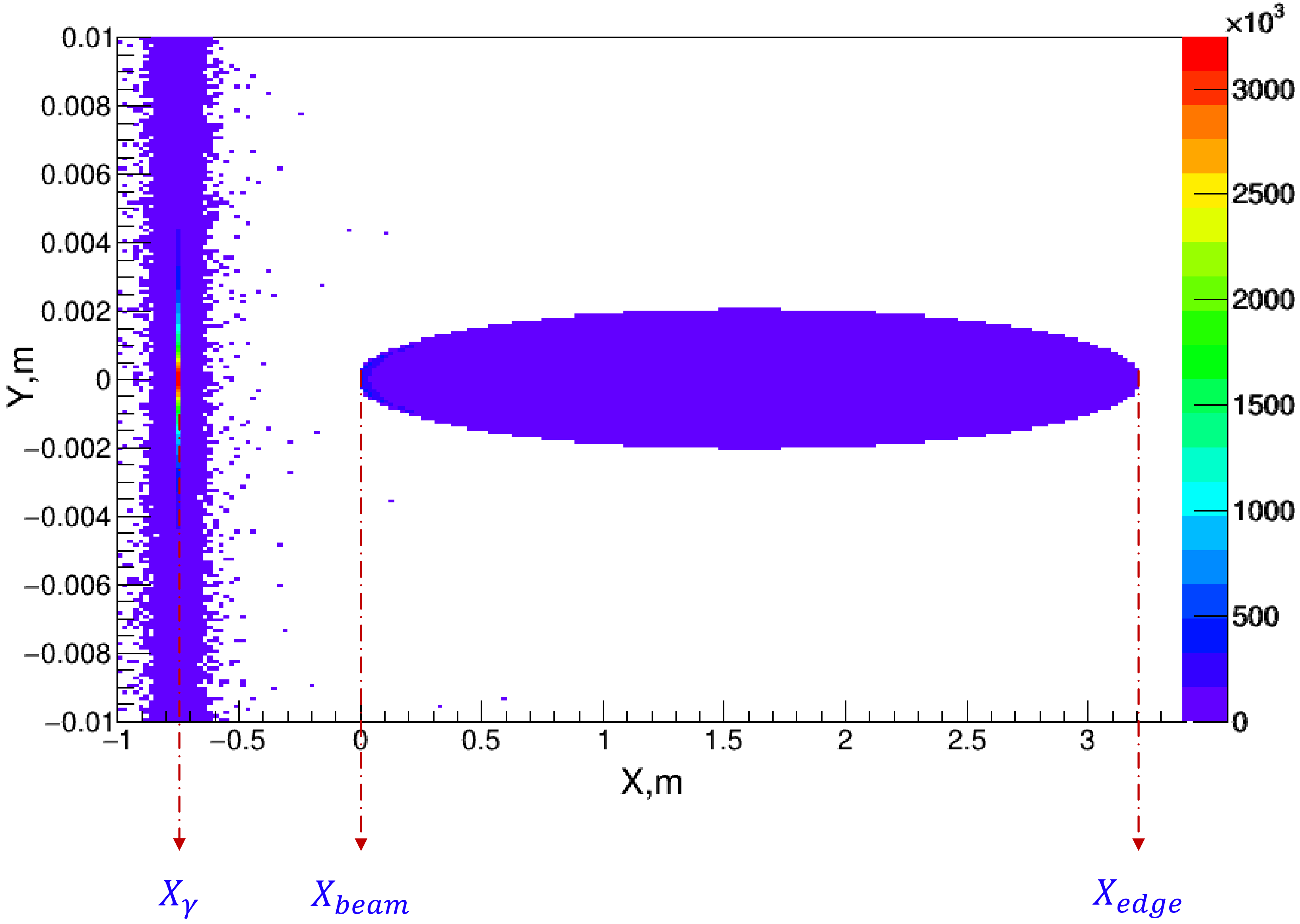
https://doi.org/10.1016/j.nima.2011.08.050

- The technique is "non-destructive": $\sim 10^6$ Compton scattered particles in one collision.

Spatial distribution of scattered particles

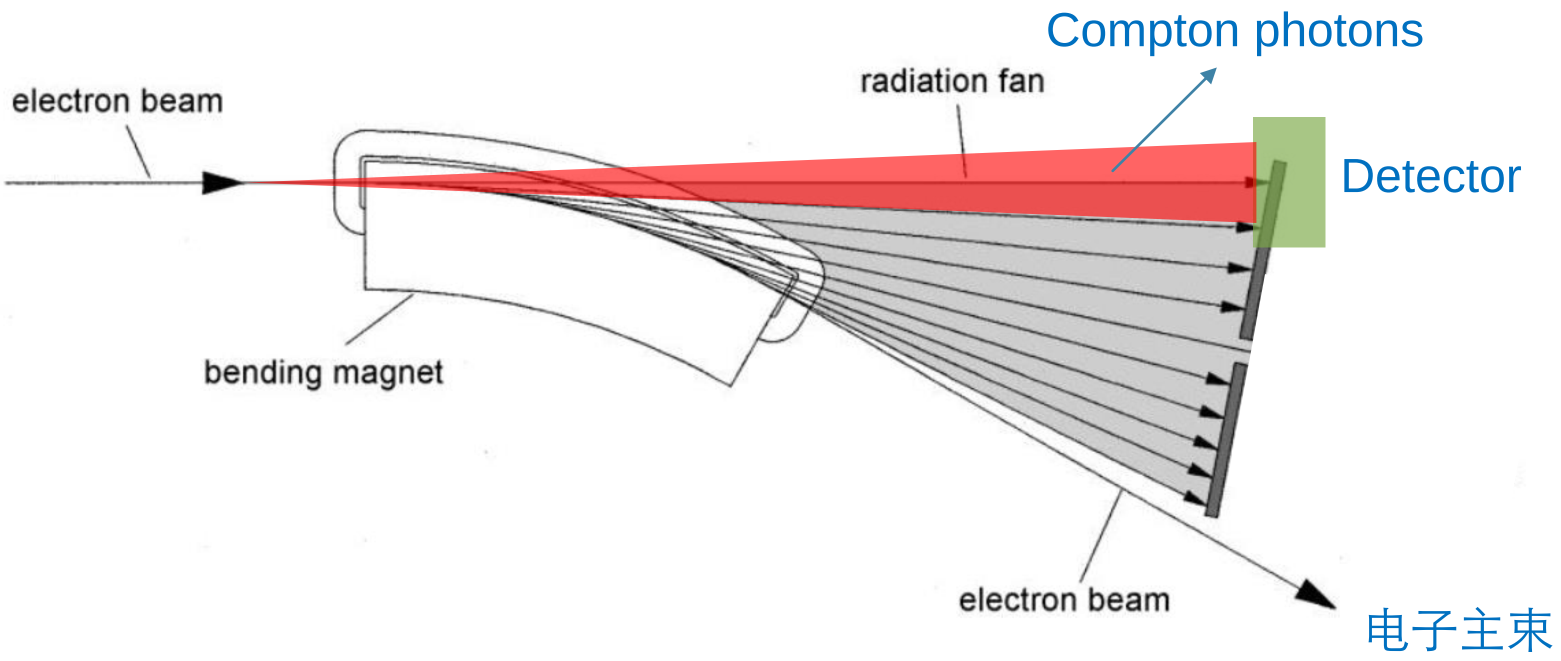
- Beam energy can be calibrated by:
  - Position of the main electron beam particles(X_{beam}).
  - Position of scattered photons(X_{γ}).
  - Position of the scattered electrons with the least energy(X_{edge}).

$$E_{beam} = \frac{(m_e c^2)^2}{4w_0} \frac{X_{edge} - X_{beam}}{X_{beam} - X_{\gamma}}$$



arXiv:1803.09595, 2018.

Synchrotron radiation background



Bending magnet	
B	0.5 T
length	3 m

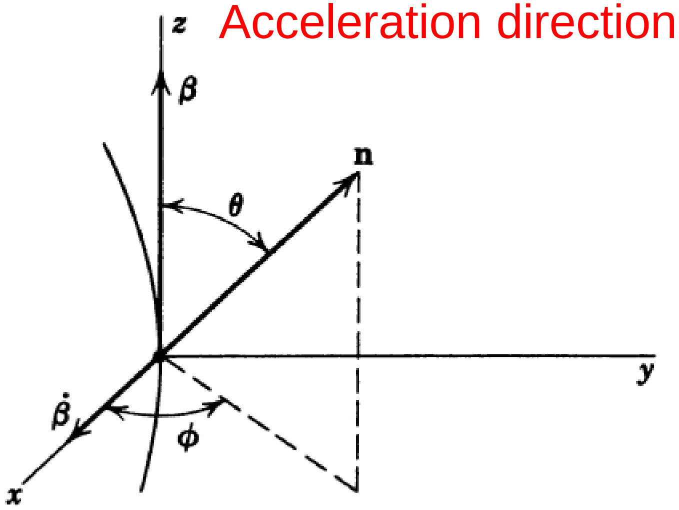
Theory

- For a horizontally accelerated electron, per solid angle

$$d\Omega = sin\vartheta d\vartheta d\varphi$$

ϑ is polar angle;
φ is azimuth angle

Acceleration direction



velocity direction

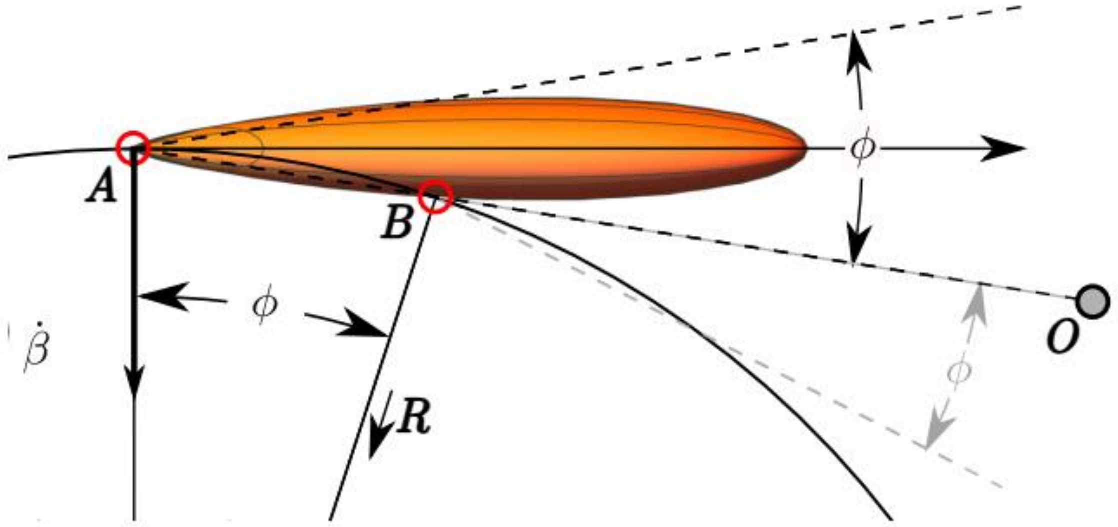


Figure 2.4: Synchrotron radiation cone of width $\phi = 1/\gamma$ sweeping across an observation point O while the photon source is moving from points A to B on the circular trajectory.

SR properties

- Bending radius

Bending radius of charged particle path in homogeneous field B at energy E is

$$\frac{1}{\rho}[\mathrm{m}] = \frac{eB}{\beta E} = 0.2998 \frac{B[\mathrm{T}]}{\beta E[\mathrm{GeV}]} \qquad (1)$$

Magnet parameter: B = 0.5 T, length = 3 m;
CEPC Higgs mode: $E_{beam} = 120\ GeV$

$$\frac{1}{\rho}[m] = 0.2998\frac{0.5}{120} = 0.0012$$

$$\rho = 800.5337m$$

1. Critical energy

发射的光子的特征能量 E_c 与特征波长 λ_c 是:

$$E_c = \hbar\frac{3}{2}\frac{c\gamma^3}{R} = 4.7948\ MeV$$

连续光谱的特征波长是:

$$\lambda_c = \frac{4\pi}{3}\rho\gamma^{-3} = 2.5876e\text{-}13\mathrm{m}$$

Theory

当一个带电粒子在一个弯转磁铁中作圆周运动时， 若在圆形轨道外一个固定点上观察，可以接受到电子轨道上的一小段圆弧上来的同步辐射，这一小段辐射脉冲可以表示同步辐射的频谱。

同步辐射的完全谱分布：

$$\frac{d^2P}{d\Omega d\omega}=\frac{P_\gamma\gamma}{\omega_c}F(\omega,\psi), \qquad (5.11)$$

其中Ω是 solid angle，ω_c为临界频率，$F(\omega,\psi)$为谱的角分布函数，用 $F_\sigma(\omega,\psi)$和 $F_\pi(\omega,\psi)$代表水平和垂直偏振分量：

$$P_\gamma=\frac{2}{3}\frac{e^2c}{4\pi\varepsilon_0}\frac{\beta^4\gamma^4}{\rho^2} \qquad \omega_c=\frac{3c\gamma^3}{2\rho}, \qquad (5.12)$$

$$F(\omega,\psi)=F_\sigma(\omega,\psi)+F_\pi(\omega,\psi), \qquad (5.13)$$

$$F(\omega,\psi)=\left(\frac{3}{2\pi}\right)^3\left(\frac{\omega}{2\omega_c}\right)^2(1+\gamma^2\psi^2)^2\left[K_{2/3}^2(\xi)+\frac{\gamma^2\psi^2}{1+\gamma^2\psi^2}K_{1/3}^2(\xi)\right], \qquad (5.14)$$

$$F_\sigma(\omega,\psi)=\left(\frac{3}{2\pi}\right)^3\left(\frac{\omega}{2\omega_c}\right)^2(1+\gamma^2\psi^2)^2K_{2/3}^2(\xi)$$

$$F_\pi(\omega,\psi)=\left(\frac{3}{2\pi}\right)^3\left(\frac{\omega}{2\omega_c}\right)^2(1+\gamma^2\psi^2)\gamma^2\psi^2K_{1/3}^2(\xi), \qquad (5.15)$$

这里，K 为分数阶修正 Bessel 函数，ξ则为

$$\xi=\frac{\omega}{2\omega_c}(1+\gamma^2\psi^2)^{3/2}。 \qquad (5.16)$$

Theory

$$\frac{d^2P}{d\Omega d\omega}=\frac{P_\gamma \gamma}{\omega_c}F(\omega,\psi)\,,$$

将谱的角功率分布对Ω进行积分，得到同步辐射的谱分布：

$$\frac{dP}{d\omega}=\frac{P_\gamma}{\omega_c}S\left(\frac{\omega}{\omega_c}\right), \quad (5.17)$$

其中

$$S\left(\frac{\omega}{\omega_c}\right)=S_\sigma\left(\frac{\omega}{\omega_c}\right)+S_\pi\left(\frac{\omega}{\omega_c}\right)=\frac{9\sqrt{3}\omega}{8\pi\omega_c}\int_{\omega/\omega_c}^{\infty}K_{5/3}(\zeta)d\zeta, \quad (5.18)$$

$$S_\sigma\left(\frac{\omega}{\omega_c}\right)=\frac{9\sqrt{3}\omega}{16\pi\omega_c}\left[\int_{\omega/\omega_c}^{\infty}K_{5/3}(\zeta)d\zeta+K_{2/3}\left(\frac{\omega}{\omega_c}\right)\right]$$

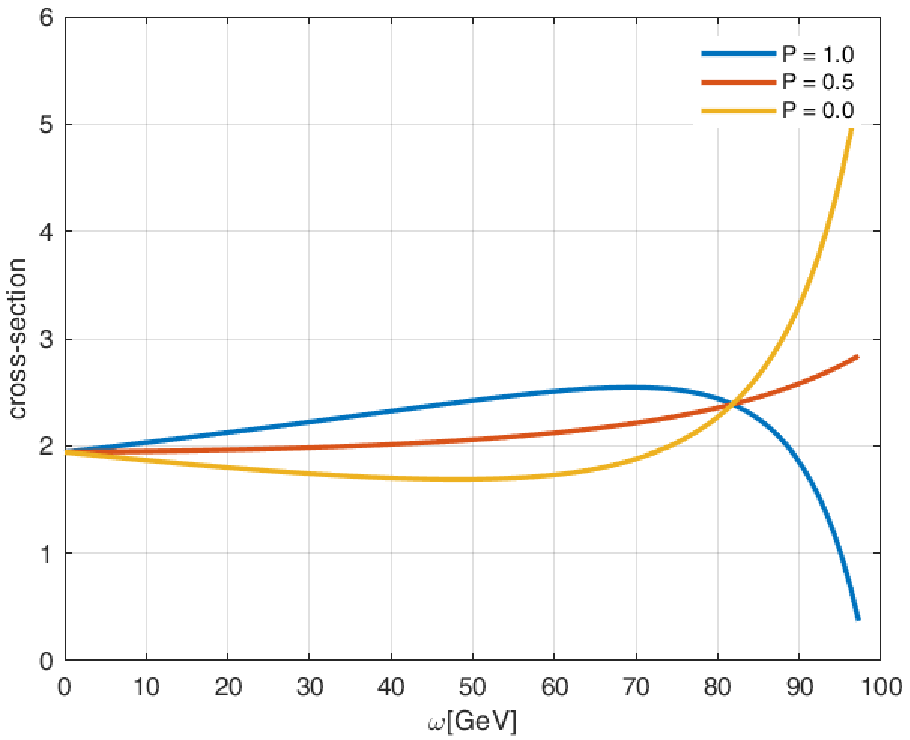
$$S_\pi\left(\frac{\omega}{\omega_c}\right)=\frac{9\sqrt{3}\omega}{16\pi\omega_c}\left[\int_{\omega/\omega_c}^{\infty}K_{5/3}(\zeta)d\zeta-K_{2/3}\left(\frac{\omega}{\omega_c}\right)\right]。 \quad (5.19)$$

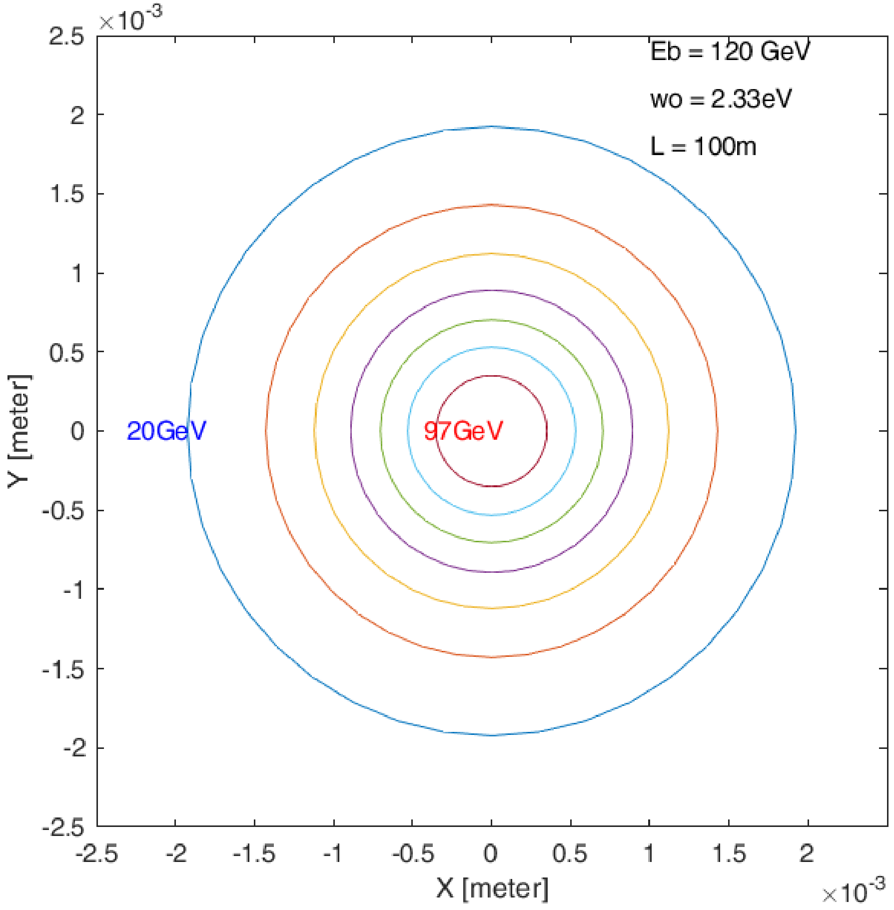
将谱的角功率分布对频率积分,得到:

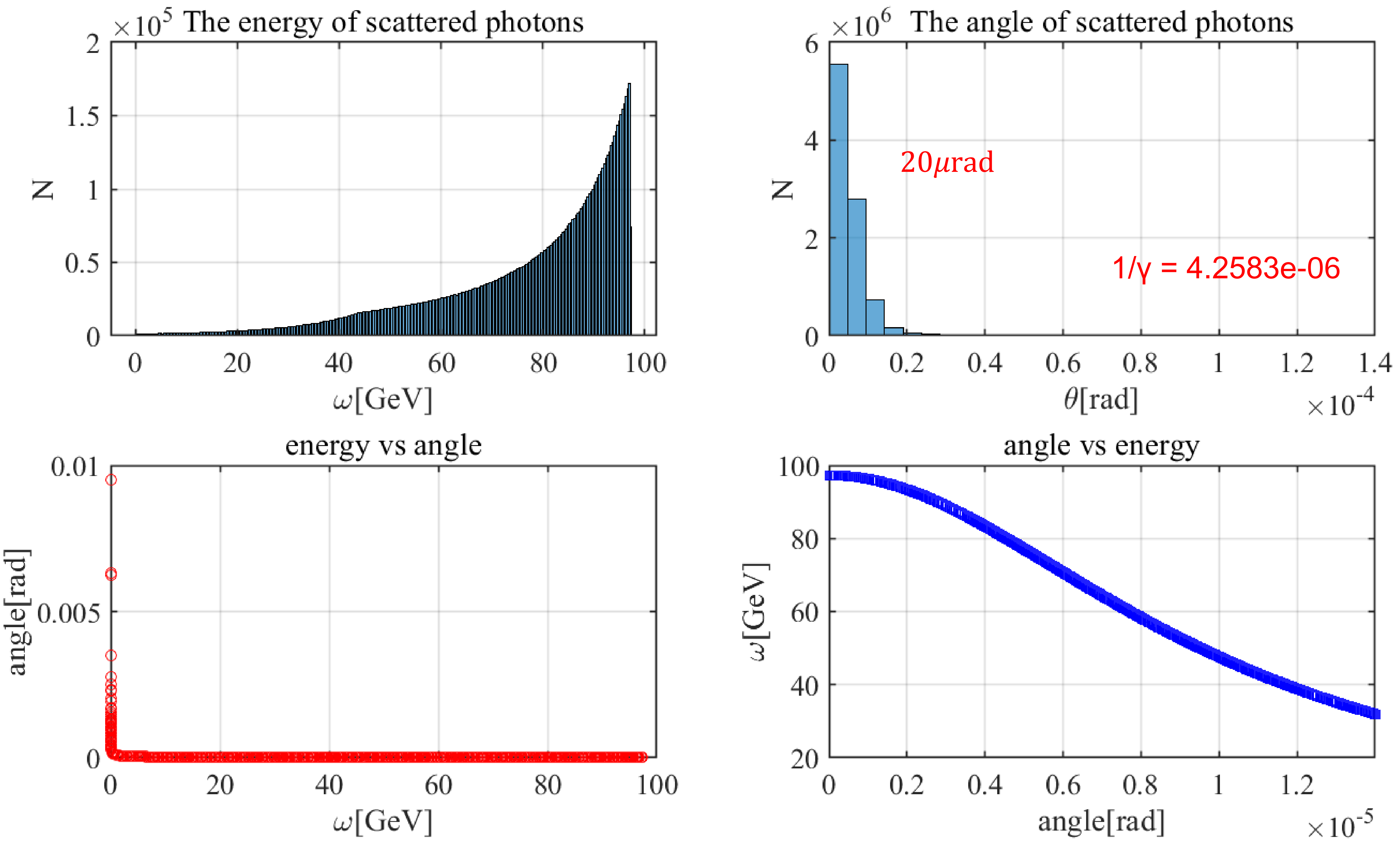
$$\frac{dP}{d\Omega}=\frac{21}{32}\frac{P_\gamma}{2\pi}\frac{\gamma}{(1+\gamma^2\psi^2)^{5/2}}\left[1+\frac{5}{7}\frac{\gamma^2\psi^2}{1+\gamma^2\psi^2}\right]。$$

散射光子分布计算

Properties of scattered photons







×10^5 The energy of scattered photons
N
ω[GeV]
×10^6 The angle of scattered photons
20μrad
1/γ = 4.2583e-06
N
θ[rad]
×10^-4
energy vs angle
angle[rad]
ω[GeV]
angle vs energy
ω[GeV]
angle[rad]
×10^-5

Properties of scattered photons

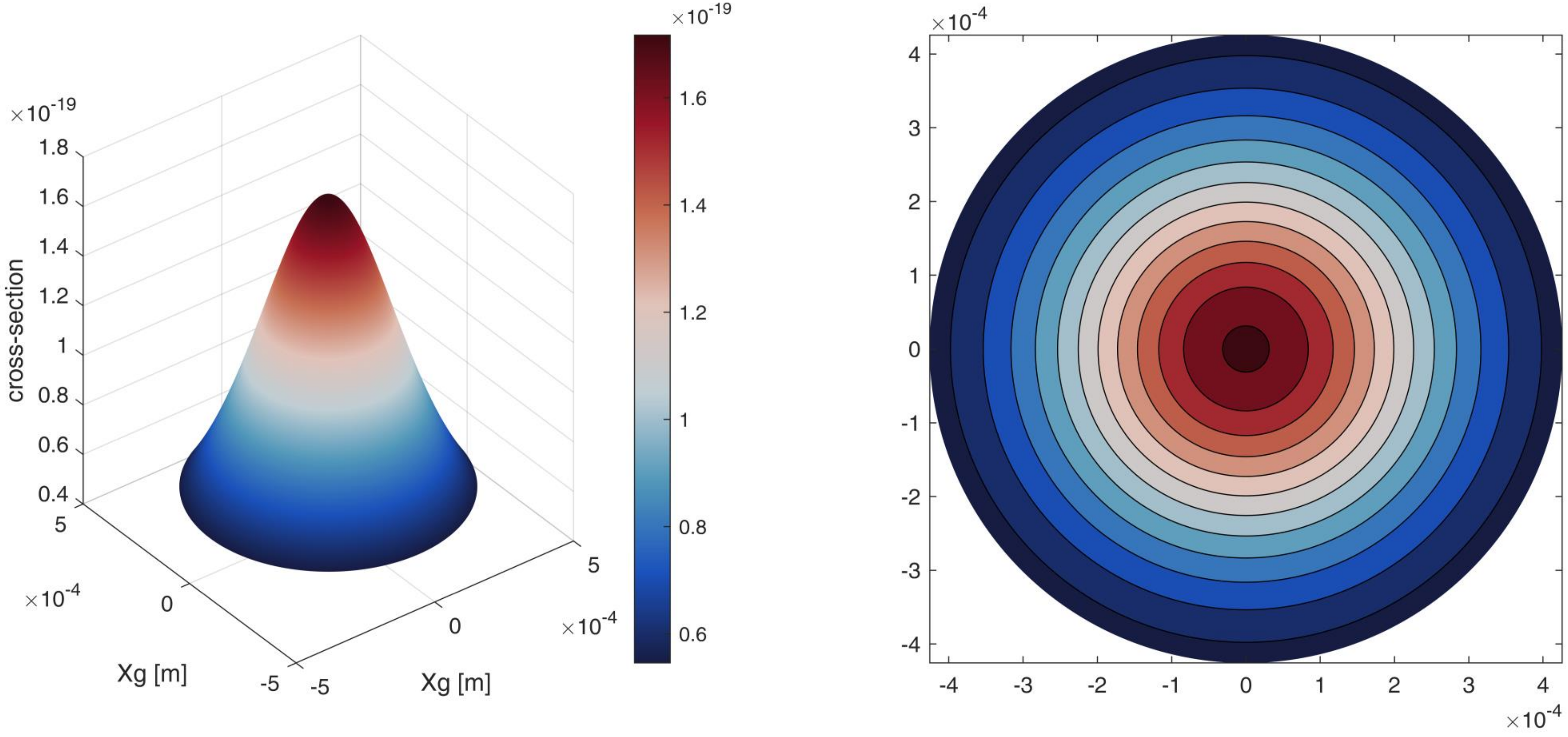


Figure. The computed spatial photons of Compton gamma-ray photons by a head-on a circularly polarized 532 nm laser with an 120 GeV electron beam.

Compton photons

- Samples

$$X_\gamma = L_1 \theta_\gamma cos\varphi$$

$$Y_\gamma = L_1 \theta_\gamma sin\varphi$$

- Fit function

$$d\sigma_0 = \frac{r_e^2}{\kappa^2(1+u)^3}\left(\kappa(1+(1+u)^2) - 4\frac{u}{\kappa}(1+u)(\kappa-u)\left[1-\xi_\perp \cos(2(\varphi-\varphi_\perp))\right]\right) du\, d\varphi,$$

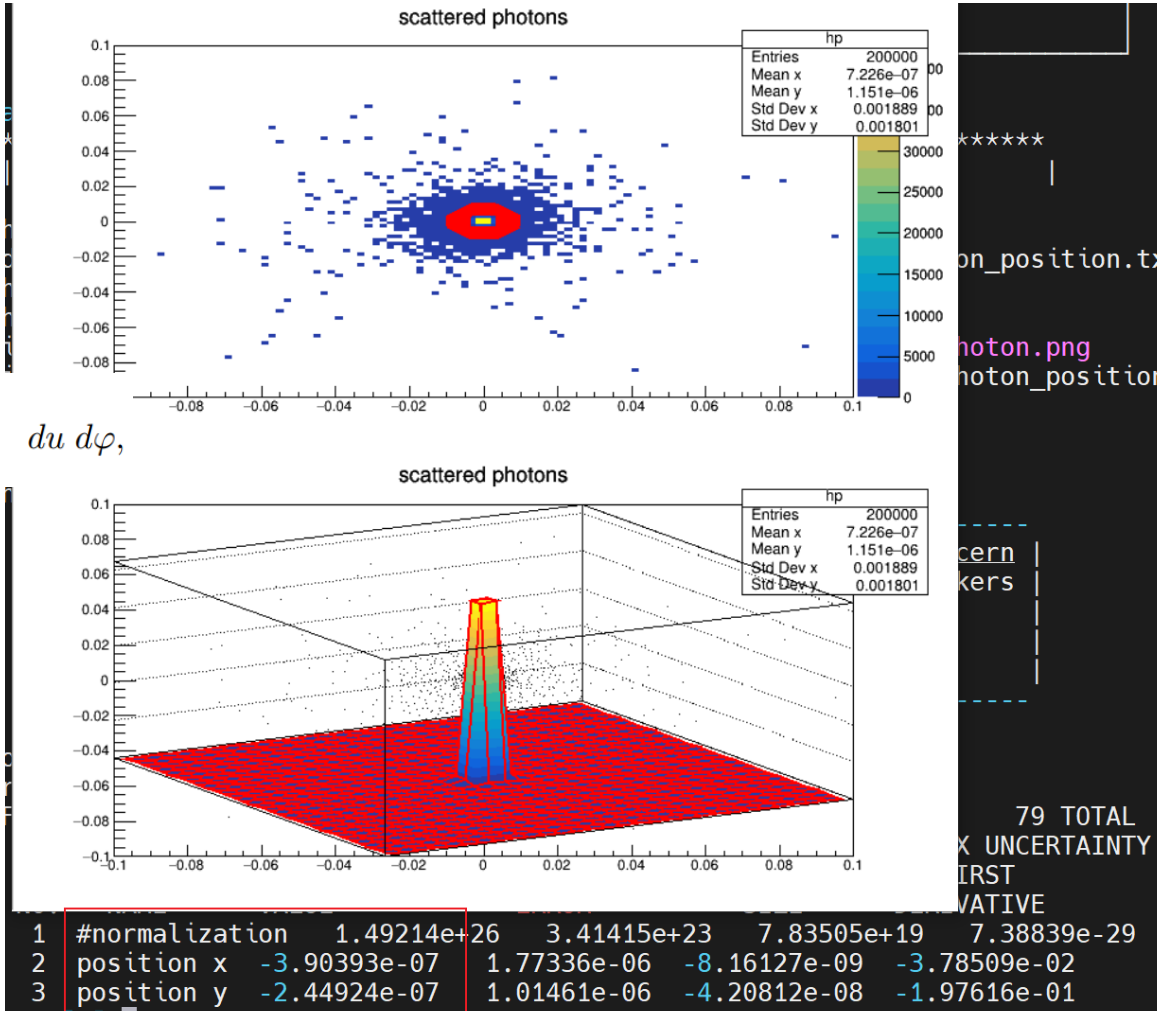
$$\frac{d\sigma_0}{dX_\gamma dY_\gamma} = Jacobian \cdot \frac{d\sigma_0}{dud\varphi}$$

$$X_\gamma = X - X_0$$

$$Y_\gamma = Y - Y_0$$

拟合参数

- Fit results



同步辐射计算

Synchrotron radiation background

更实用的是弯铁中每单位转角中的光子数，即光通量为

$$\frac{dN_{ph}}{d\psi}=\frac{1}{\hbar\omega}\frac{d^2P}{d\omega d\psi}$$

$$=\frac{1}{\hbar\omega}\frac{1}{2\pi}\frac{dP}{d\omega}\Delta\omega$$

$$=\frac{P_r}{2\pi\hbar\omega_c}\cdot\frac{\Delta\omega}{\omega}\cdot S(\omega/\omega_c) \qquad (\text{Ⅱ}.73)$$

上式中的 ψ 为水平偏转角，绕储存环一圈为 2π，$\hbar=\frac{h}{2\pi}$ 为简约化的普朗克常数。把 $P_r=\frac{2}{3}r_e m_0 c^2\frac{c\gamma^4}{\rho^2}$ 和 $\hbar\omega_c=\frac{3}{2}\hbar c\frac{\gamma^3}{\rho}$ 代入式(Ⅱ.73)，则

$$\frac{dN_{ph}}{d\psi}=\frac{4\alpha}{9}\gamma f_{rev}\frac{\Delta\omega}{\omega}S(\omega/\omega_c) \qquad (\text{Ⅱ}.74)$$

式(Ⅱ.74)中的 $\alpha=\frac{e^2}{\hbar c}$ 为精细结构常数，在实用单位制中 $\alpha=7.29735\times10^{-3}$，$f_{rev}=\frac{c}{2\pi\rho}$ 是电子的回旋频率。式(Ⅱ.74)表示一个电子在 $d\psi$ 角度内产生的光子数，如果有 N_e 个电子在储存环中回旋，它产生的光子通量为式(Ⅱ.74)的 N_e 倍，那末一个电流为 I($I=f_{rev}eN_e$)的循环束流，产生的光子数则为[48]

$$\frac{dN_{ph}}{d\psi}=\frac{4\alpha}{9}\frac{E}{m_0c^2}\frac{I}{e}\frac{\Delta\omega}{\omega}S(\omega/\omega_c)$$

$$=C_\psi IE\frac{\Delta\omega}{\omega}S(\omega/\omega_c) \qquad (\text{Ⅱ}.75)$$

这就是光通量公式(7.13)，它是单位时间、单位水平偏转角内辐射的光子数。此处 I 为储存环中的循环束流，单位为安培；E 为电子的能量，单位是 GeV；$\frac{\Delta\omega}{\omega}$ 是带宽，通常取 0.1%；$S(\omega/\omega_c)$ 是光谱函数，数值见表 7.1；C_ψ 是一个常数

$$C_\psi=\frac{4\alpha}{9m_0c^2e}=3.96139\times10^{16}\frac{\text{光子数}}{\text{秒}\cdot\text{毫弧度}\cdot\text{安培}\cdot\text{GeV}}$$

Nph = 3.3416e+13 [光子数/(秒·毫弧度·安培·GeV)]

Synchrotron radiation background

回旋频率： $f_{eq} = 3000$

束团数目: N = 242

单发束团一次的SR flux: Nph = 4.6028e+07 [光子数/(秒·毫弧度·安培·GeV)]

Detector acceptance angle = 2/γ = 0.0085 mrad

⬇

Nph = 391238

Synchrotron radiation flux

$$\frac{dN_{ph}}{d\psi} = \frac{4\alpha}{9} \frac{E}{m_0c^2} \frac{I}{e} \frac{\Delta\omega}{\omega} S(\omega/\omega_c)$$

$$= C_\psi IE \frac{\Delta\omega}{\omega} S(\omega/\omega_c)$$

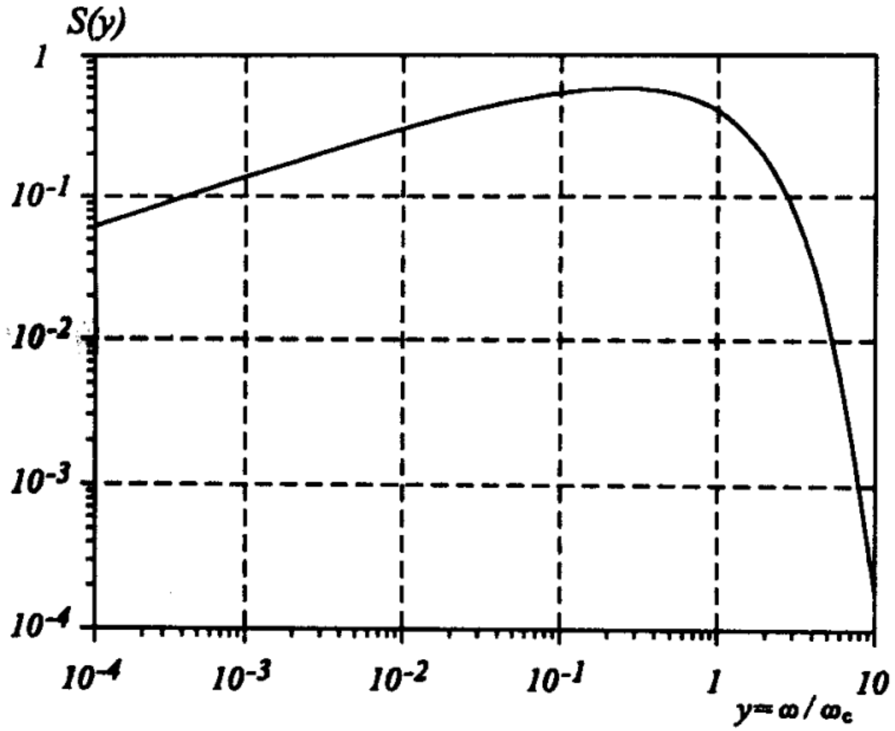


图 7.1 同步辐射光谱的谱适函数 $S(y)$ 曲线

Back-up