Two-loop Electroweak corrections with fermion loops for $e^+e^- \to ZH$

A. Freitas, Q. Song: arXiv:2209.07612

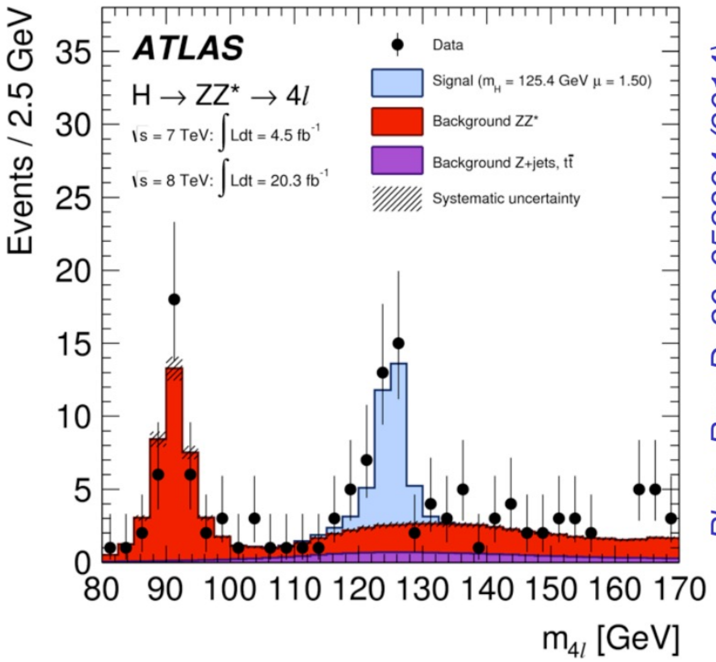
Qian Song
In collaboration with Ayres Freitas
University of Pittsburgh
CEPC Workshop, 24 Oct, 2022

Content

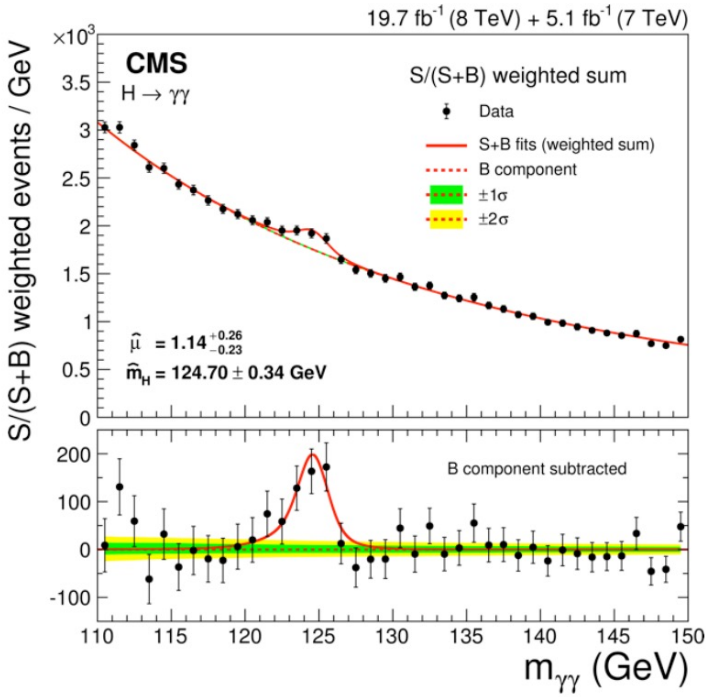
- Motivation
- Evaluation method
- Result
- Summary

Higgs discovery and property measurement

- Discovery of SM-like Higgs boson(2012,LHC) offers opportunity to investigate Higgs
- Higgs property measurements
  - agrees with the result SM predicted
  - BSM expects deviations at percent level
  - One promising way probing new physics: precision measurements of the properties of H
  - LHC is difficult to reach very high precision due to complicated background



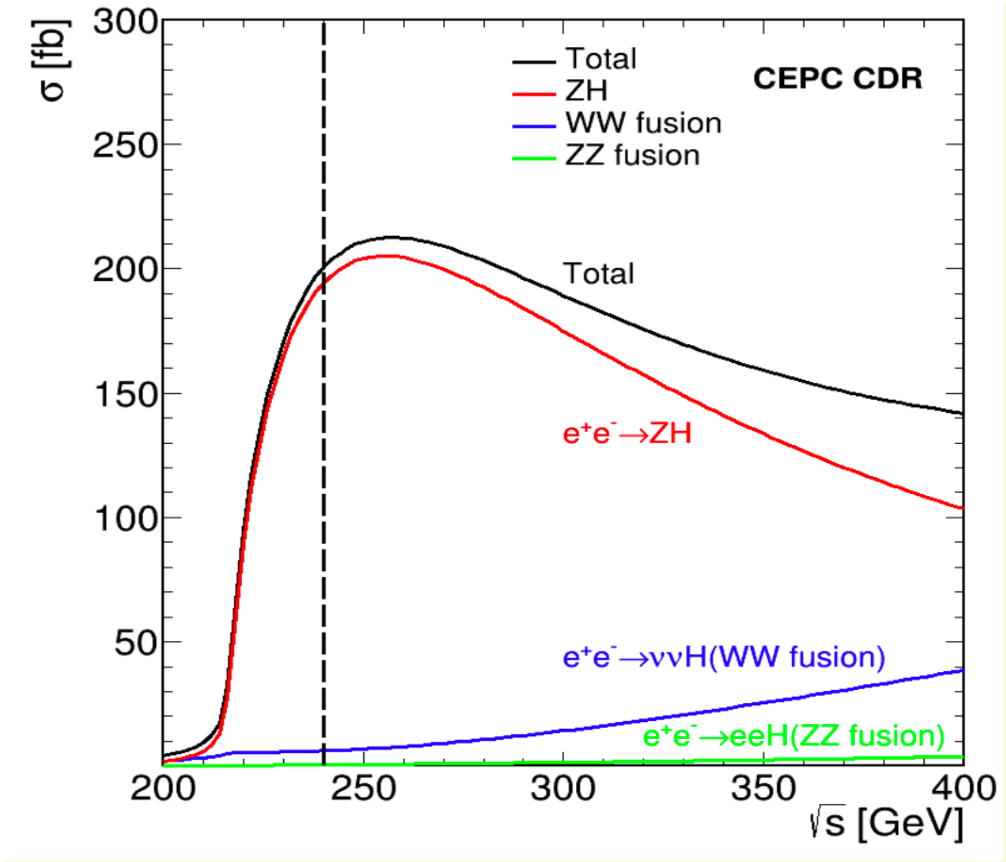
Phys. Rev. D. 90, 052004 (2014)

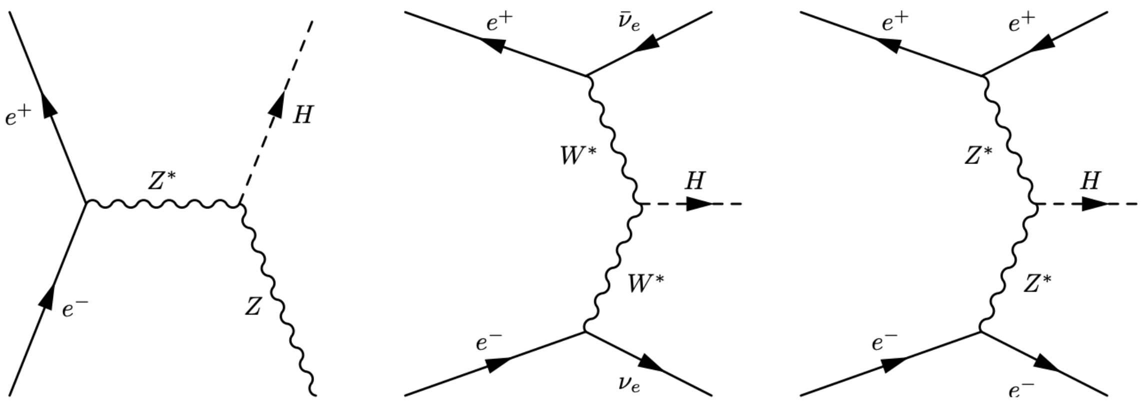


Eur. Phys. J. C 74 (2014) 3076

Higgs discovery and property measurement

- Higgs factories: such as CEPC,FCC-ee,ILC, e+e- collider ($\sqrt{s}$=240-250GeV)





Higgs discovery and property measurement

- e+e- colliders can measure H properties with very high precision due to large statistics, high luminosity, clean environment
- For example: expected precision on $\sigma(e^+e^- \to ZH)$
  - CEPC:0.5%. Y.Fang et al `18
- Reconstruct Higgs with Recoil Mass Method

$$\kappa_Z^2 = \frac{\sigma(e^+e^- \to ZH)^{exp}}{\sigma(e^+e^- \to ZH)^{theory}} \qquad \kappa_Z = \frac{g_{\mathrm{HZZ}}}{g_{\mathrm{HZZ}}^{\mathrm{SM}}}$$

- Theoretical precision on $\sigma(e^+e^- \to ZH)$ should be comparable with experiment

Theoretical corrections

- Perturbative expansion with respect to EW coupling(α) and QCD coupling(α_s)

$$\sigma(e^+e^- \to ZH) \propto |MM^*| = |(M^{\text{tree}} + M^{\alpha} + M^{\alpha_s\alpha} + \ldots)(M^{\text{tree}} + M^{\alpha} + M^{\alpha_s\alpha} + \ldots)^*|$$

$$= \underbrace{|M^{\text{tree}} M^{\text{tree}*}|}_{\text{LO}} + \underbrace{2Re|M^{\text{tree}} M^{\alpha *}|}_{\text{NLO}} + \underbrace{2Re|M^{\text{tree}} M^{\alpha_s\alpha *}|}_{\text{NNLO(EW+QCD)}}$$

$$+ \underbrace{|M^{\alpha} M^{\alpha *}| + 2Re|M^{\text{tree}} M^{\alpha^2 *}|}_{\text{NNLO(EW+EW)}} + \underbrace{2Re|M^{\alpha} M^{\alpha_s\alpha}|}_{\text{NNNLO(EW +EW+QCD)}} + \ldots$$

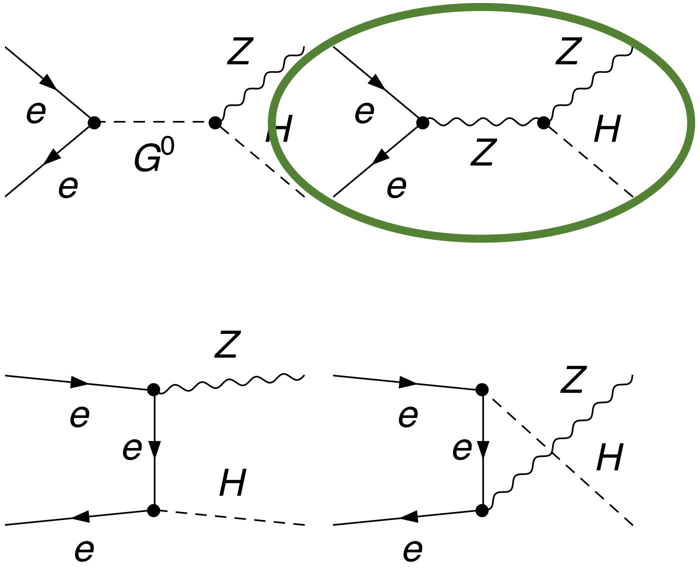
Theoretical corrections

- Perturbative expansion with respect to EW coupling(α) and QCD coupling(α_s)

$$\sigma(e^+e^- \to ZH) \propto |MM^*| = |(M^{\text{tree}} + M^{\alpha} + M^{\alpha_s\alpha} + \dots)(M^{\text{tree}} + M^{\alpha} + M^{\alpha_s\alpha} + \dots)^*|$$

$$= \underbrace{|M^{\text{tree}}M^{\text{tree}*}|}_{\text{LO}} + \underbrace{2Re|M^{\text{tree}}M^{\alpha *}|}_{\text{NLO}} + \underbrace{2Re|M^{\text{tree}}M^{\alpha_s\alpha *}|}_{\text{NNLO(EW+QCD)}}$$

$$+ \underbrace{|M^{\alpha}M^{\alpha *}| + 2Re|M^{\text{tree}}M^{\alpha^2 *}|}_{\text{NNLO(EW+EW)}} + \underbrace{2Re|M^{\alpha}M^{\alpha_s\alpha}|}_{\text{NNNLO(EW +EW+QCD)}} + \dots$$



- LO : only consider s channel(ignore electron mass)

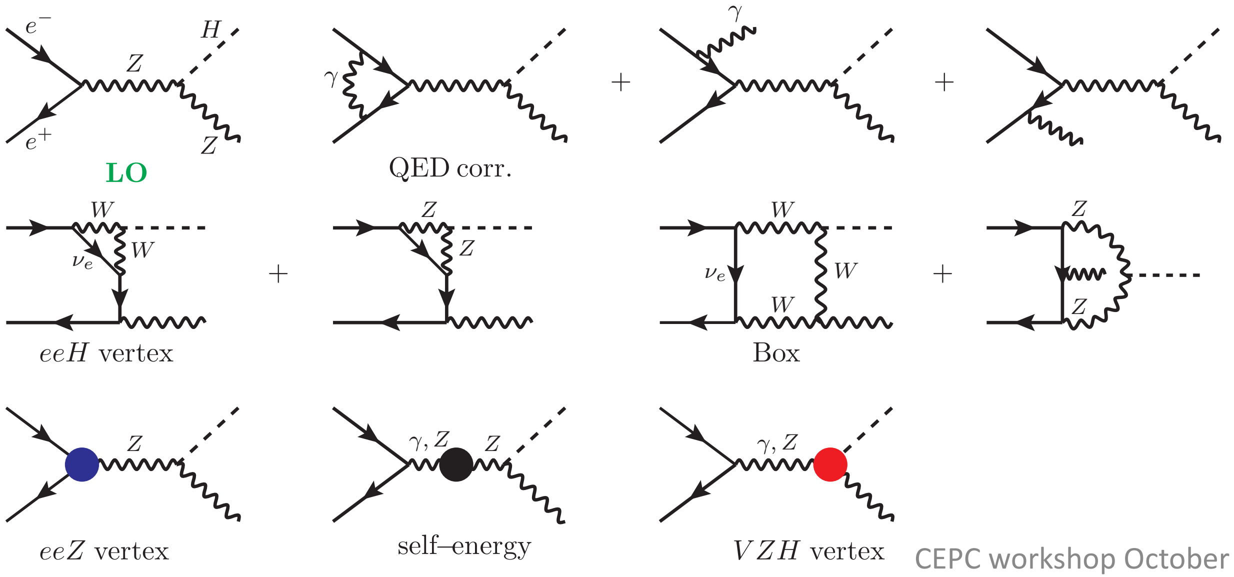
Theoretical corrections

- Perturbative expansion with respect to EW coupling(α) and QCD coupling(α_s)

$$\sigma(e^+e^- \to ZH) \propto |MM^*| = |(M^{\text{tree}} + M^{\alpha} + M^{\alpha_s\alpha} + \ldots)(M^{\text{tree}} + M^{\alpha} + M^{\alpha_s\alpha} + \ldots)^*|$$

$$= \underbrace{|M^{\text{tree}} M^{\text{tree}*}|}_{\text{LO}} + \underbrace{2Re|M^{\text{tree}} M^{\alpha*}|}_{\text{NLO}} + \underbrace{2Re|M^{\text{tree}} M^{\alpha_s\alpha*}|}_{\text{NNLO(EW+QCD)}}$$

$$+ \underbrace{|M^{\alpha} M^{\alpha*}| + 2Re|M^{\text{tree}} M^{\alpha^2*}|}_{\text{NNLO(EW+EW)}} + \underbrace{2Re|M^{\alpha} M^{\alpha_s\alpha}|}_{\text{NNNLO(EW +EW+QCD)}} + \ldots$$



- NLO: unpolarized beam: 5-10%

  J.Fleischer, F.Jegerlehner \`83

  B.A.Kniehl \`92

  A.Denner, J.Kublbeck, R.Mertig, M.Bohm \`92

  polarized beam: 10-20%

  S.Bondarenko, Y.Dydyshka, L.Kalinovskaya, L.Rumyantsev, R.Sadykov and V.Yermolchyk \`18

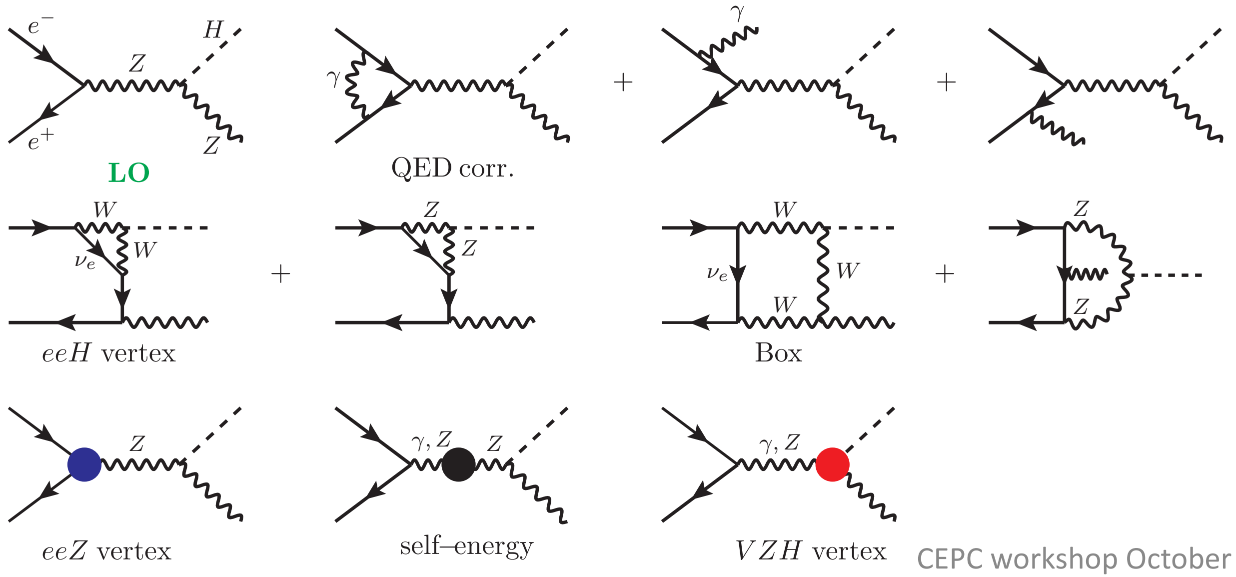
Theoretical corrections

- Perturbative expansion with respect to EW coupling(α) and QCD coupling(α_s)

$$\sigma(e^+e^- \to ZH) \propto |MM^*| = |(M^{\text{tree}} + M^{\alpha} + M^{\alpha_s\alpha} + \ldots)(M^{\text{tree}} + M^{\alpha} + M^{\alpha_s\alpha} + \ldots)^*|$$

$$= \underbrace{|M^{\text{tree}} M^{\text{tree}*}|}_{\text{LO}} + \underbrace{2Re|M^{\text{tree}} M^{\alpha*}|}_{\text{NLO}} + \underbrace{2Re|M^{\text{tree}} M^{\alpha_s\alpha*}|}_{\text{NNLO(EW+QCD)}}$$

$$+ \underbrace{|M^{\alpha} M^{\alpha*}| + 2Re|M^{\text{tree}} M^{\alpha^2*}|}_{\text{NNLO(EW+EW)}} + \underbrace{2Re|M^{\alpha} M^{\alpha_s\alpha}|}_{\text{NNNLO(EW +EW+QCD)}} + \ldots$$



- NNLO(EW+QCD):

  0.4-1.3% $(\alpha(0), \alpha(M_z), G_\mu)$

  Q. F. Sun, F. Feng, Y. Jia and W. L. Sang `16

  1.3% $(\overline{MS}, \alpha(M_z))$

  Y. Gong, Z. Li, X. Xu, L. L. Yang and X. Zhao `17

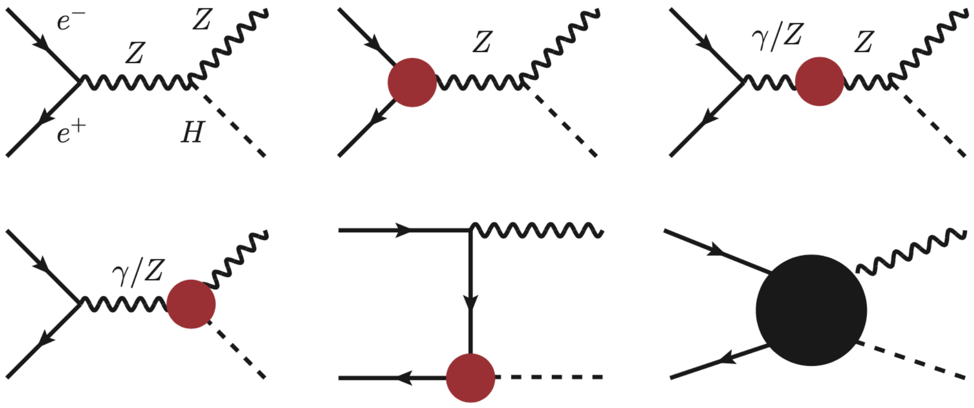
Theoretical corrections

- Perturbative expansion with respect to EW coupling(α) and QCD coupling(α_s)

$$\sigma(e^+e^- \to ZH) \propto |MM^*| = |(M^{\text{tree}} + M^{\alpha} + M^{\alpha_s\alpha} + \ldots)(M^{\text{tree}} + M^{\alpha} + M^{\alpha_s\alpha} + \ldots)^*|$$

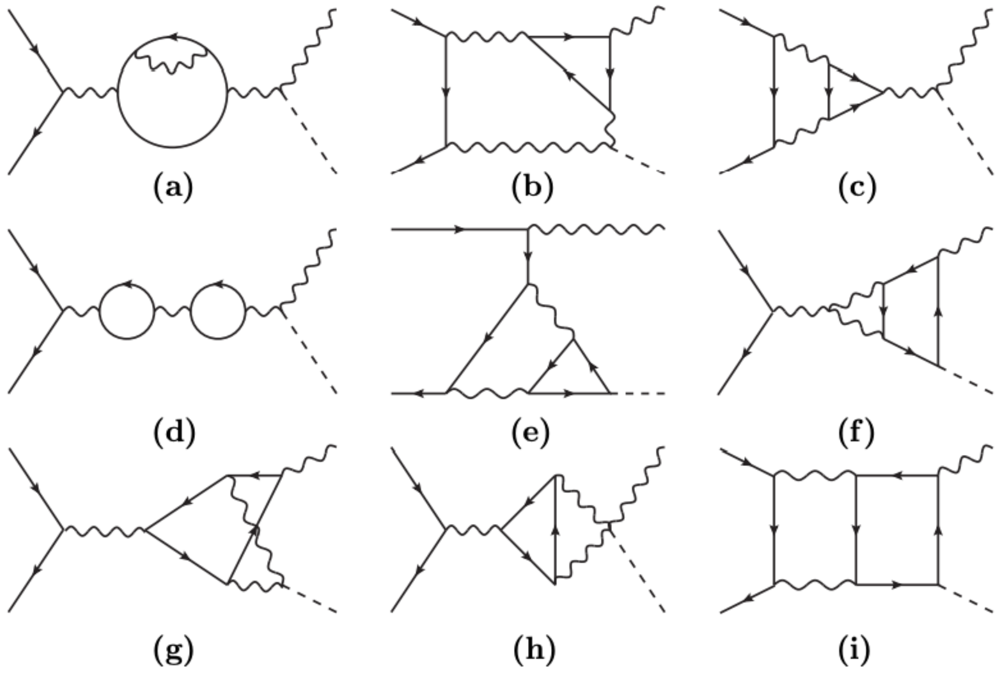
$$= \underbrace{|M^{\text{tree}} M^{\text{tree}*}|}_{\text{LO}} + \underbrace{2Re|M^{\text{tree}} M^{\alpha*}|}_{\text{NLO}} + \underbrace{2Re|M^{\text{tree}} M^{\alpha_s\alpha*}|}_{\text{NNLO(EW+QCD)}}$$

$$+ \underbrace{|M^{\alpha} M^{\alpha*}| + 2Re|M^{\text{tree}} M^{\alpha^2*}|}_{\text{NNLO(EW+EW)}} + \underbrace{2Re|M^{\alpha} M^{\alpha_s\alpha}|}_{\text{NNNLO(EW +EW+QCD)}} + \ldots$$



Theoretical corrections

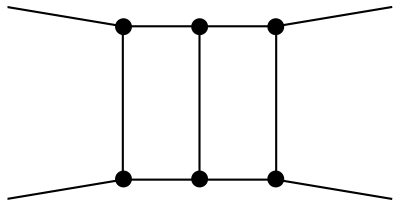
- NNLO(EW+EW): complete result in progress
- Complete result
  - with fermion loops $N_f > 0$
  - without fermion loops $N_f = 0$
- Correction for $N_f > 0$ dominate due to:
  - large top-quark Yukawa coupling
  - flavor number enhancement
- Correction for $N_f > 0$: 0.7% ($\alpha(M_z)$) (this talk)



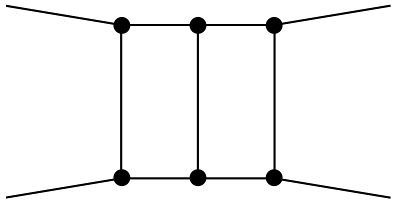
Examples of two-loop EW Feynman diagrams with at least one closed fermion loop

Evaluation method

- Analytical result:
  - optimal, convenient for phenomenology analysis
  - is expressed with harmonic polylogarithmic functions, etc
- NNLO correction:
  - simple diagrams have analytical expressions
    - massless double-box diagram V.A.Smirnov `99
    - double-box diagram with 4 massive and 3 massless lines V.A.Smirnov `00
  - generally difficult for analytical result: require more knowledge about special functions



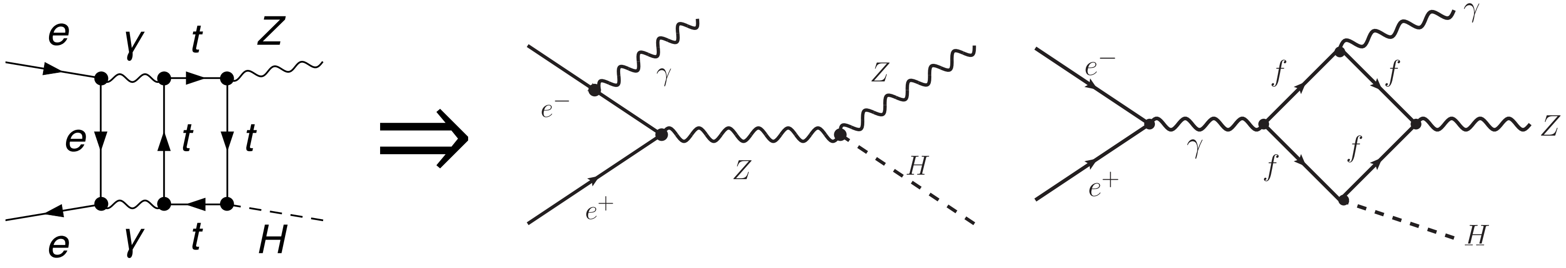
Evaluation method



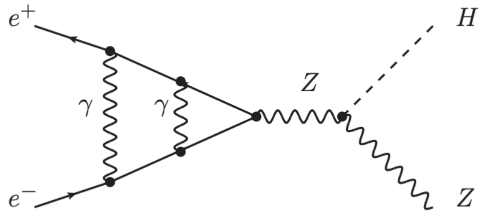
- Numerical calculation:
  - Feynman parametrization F. Yuasa, E. de Doncker, N. Hamaguchi, T. Ishikawa, K. Kato, Y. Kurihara, J. Fujimoto and Y. Shimizu `12
    - applied to calculate double-box diagrams
    - takes few days integrand converges slowly
  - Differential equation X. Chen, X. Guan, C-Q. He, Z.Li, X. Liu `22
    - applied to calculate complete NNLO(EW+EW) result
    - more details → See Xin Guan's talk
  - √ Feynman parametrization and dispersion relation A. Freitas, Q. Song 22`
    - applied to calculate complete NNLO(EW+EW) result
    - finished NNLO(EW+EW) diagrams with fermion loops
    - not automatic, case-by-case UV divergence treatment
    - numerically stable, with good precision(4-digit precision)
    - evaluation is fast, takes few minutes

Preliminary

- dimensional regularization to regulate UV divergence, trace calculation in D dim
  - Trace not involving γ_5 is evaluated in D dimension(NDR) because of UV div
  - Trace involving γ_5 is treated in 4 dimension because that part is UV finite
    $\Rightarrow$ zero by summing over all fermions, SM free of anomaly
- On-Shell renormalization: cancel all UV divergences
- IR divergence:
  - spurious IR divergence: regulate with a small photon mass
  - physical IR divergence: cancel with real photon emission, not include in our calculation



Example diagram with spurious IR div



Example diagram with physical IR div

Calculation process

- Generate Feynman diagrams with FeynArts T.Hahn `00
- Calculate unpolarized squared amplitude with FeynCalc V. Shtabovenko, R. Mertig and F. Orellana `01
- Use dispersion relation and Feynman parameterization to simplify squared amplitude
  - Using private code, Squared amplitude is expressed as ≤ 3-fold numerical integral

$$|MM^*| = \int dx \int d\sigma \Delta B_{\mu\nu}(\sigma, m_1^2, m_2^2) \times (c_1 A_0 + c_2 B_0 + c_3 C_0 + c_4 D_0 + c_{ij} D_{ij} + \ldots)$$

analytically known analytically known, implemented from looptools packag

  - UV finite diagram: straight forward
  - UV divergent diagram: subtraction terms
- Squared amplitude is evaluated numerically in C++ with LoopTools package, Gauss-Kronrod quadrature in Boost package T.Hahn `98 https://www.boost.org/

UV divergent diagram

- Subtraction terms to deal with UV divergence:
  - subtract few simple terms(I_{subtra}) to make it UV finite
  - I_{subtra} must be simple enough to be integrated analytically
  - add I_{subtra} back analytically

$$I_{\mathrm{div}} = \underbrace{I_{\mathrm{div}} - I_{\mathrm{subtra}}}_{\text{UV finite, Numerical}} + \underbrace{I_{\mathrm{subtra}}}_{\text{Analytical}}$$

Integrated in C++ Derive analytical result in MMA

- 3 types subtraction terms $\leftrightarrow$ 1 global divergence + 2 local divergences

Calculation process

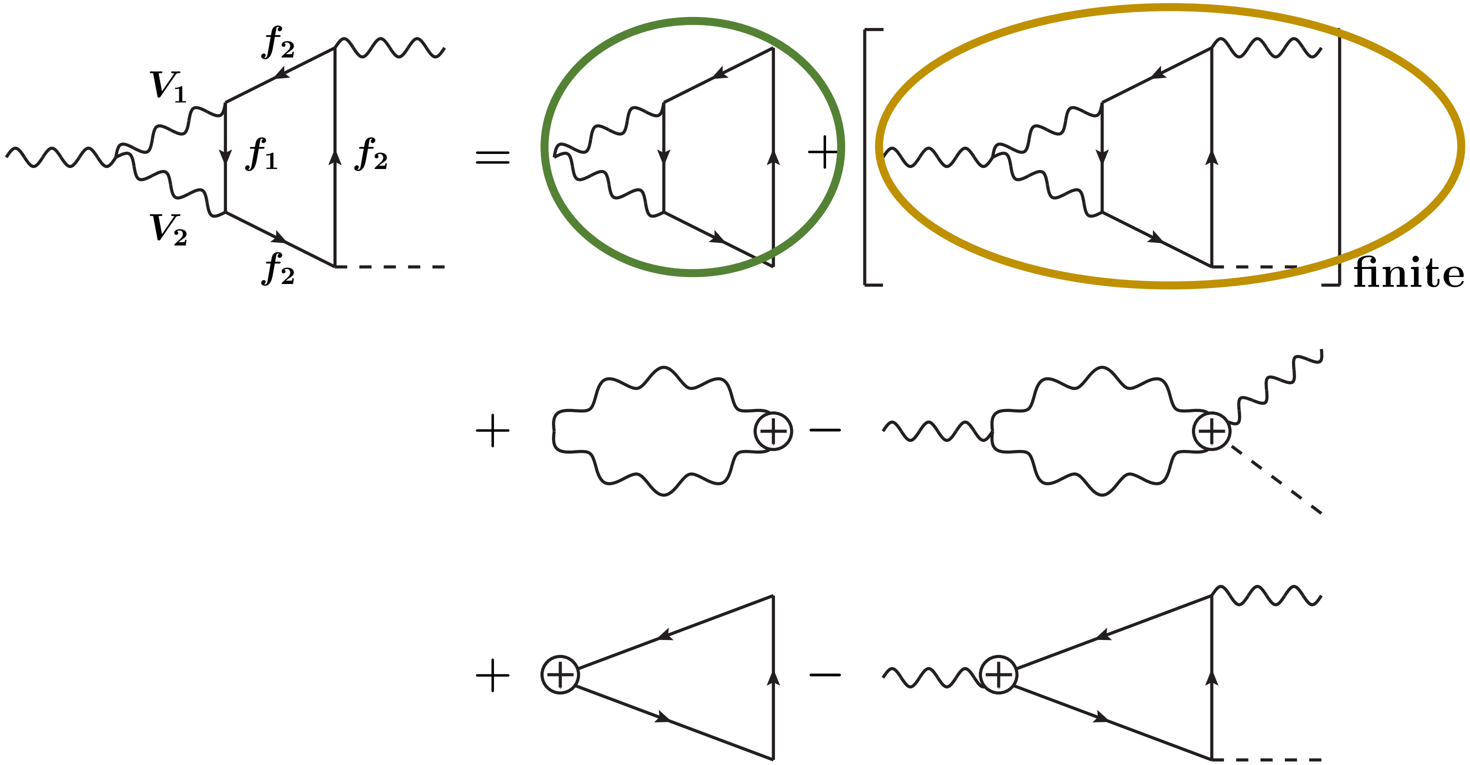
- Generate Feynman diagrams with FeynArts T.Hahn `00
- Calculate unpolarized squared amplitude with FeynCalc V. Shtabovenko, R. Mertig and F. Orellana `01
- Use dispersion relation and Feynman parameterization to simplify squared amplitude
  - Using private code, Squared amplitude is expressed as ≤ 3-fold numerical integral

$$|MM^*| = \int dx \int d\sigma \, \underbrace{\Delta B_{\mu\nu}(\sigma, m_1^2, m_2^2)}_{\text{analytically known}} \times \underbrace{(c_1 A_0 + c_2 B_0 + c_3 C_0 + c_4 D_0 + c_{ij} D_{ij} + ...)}_{\text{analytically known, implemented from looptools packag}}$$

  - UV finite diagram: straight forward
  - UV divergent diagram: subtraction terms
- Squared amplitude is evaluated numerically in C++ with LoopTools package, Gauss-Kronrod quadrature in Boost package T.Hahn `98 https://www.boost.org/

UV divergent diagram: VZH vertex(2 local + 1 global divergence)

$$I_{\text{div}} = \underbrace{I_{\text{div}} - I_{\text{subtra}}}_{\text{UV finite, Numerical}} + \underbrace{I_{\text{subtra}}}_{\text{Analytical}}$$



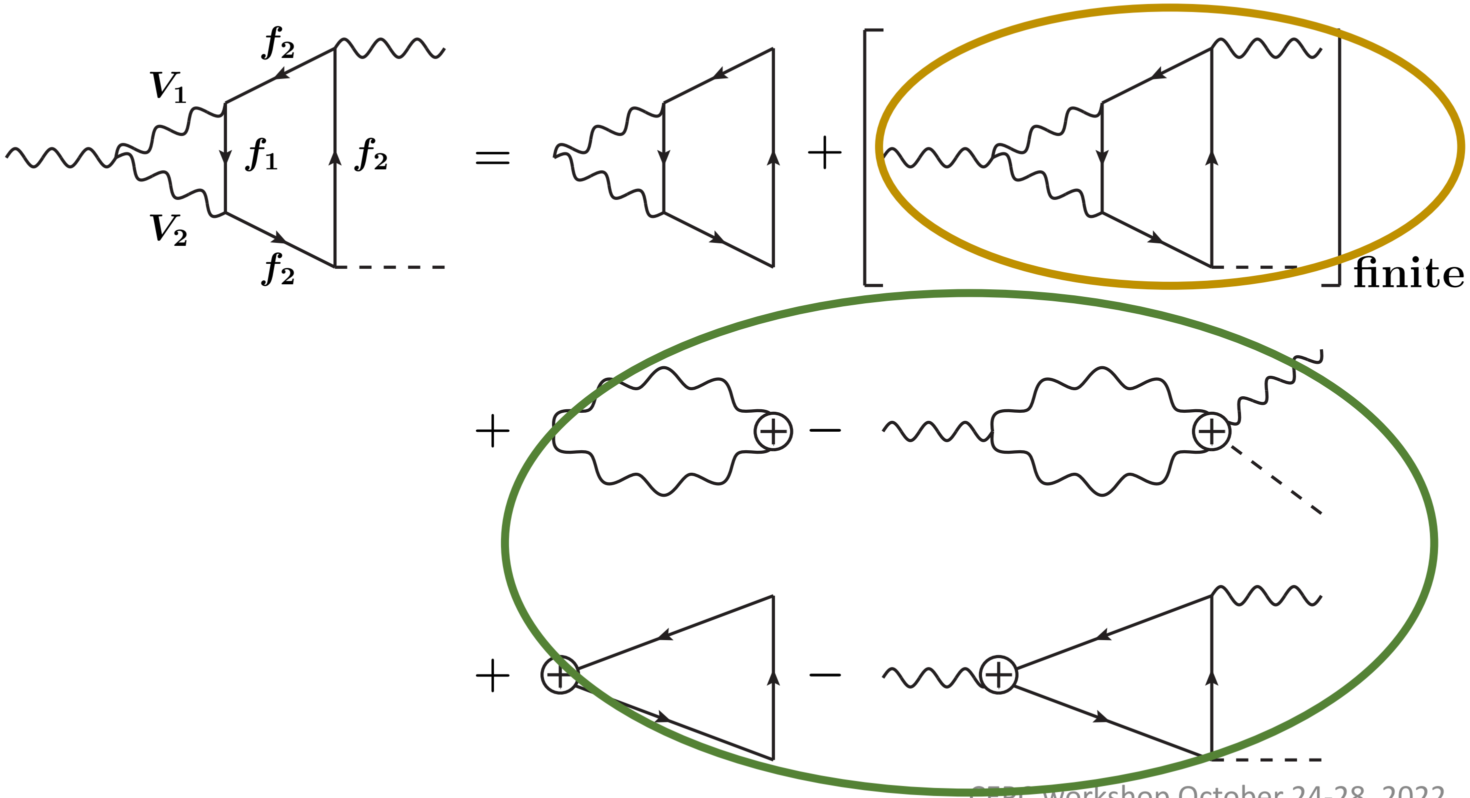
- Vacuum diagram(no legs) with same intermediate particles
- cancel global divergence
- analytical expression
  - tensor decomposition: reduce tensor integral to scalar integral
  - reduce scalar integral to master one using FIRE

    A.V.Smirnov and F.S.Chukharev `19
  - master integral can be evaluated analytically with TVID

    B.Stefan, F.Ayres and W.Daniel `19

UV divergent diagram: VZH vertex(2 local + 1 global divergence)

$$I_{\text{div}} = \underbrace{I_{\text{div}} - I_{\text{subtra}}}_{\text{UV finite, Numerical}} + \underbrace{I_{\text{subtra}}}_{\text{Analytical}}$$



- Product of one loop functions
- analytically known: product of one-loop functions
- cancel local divergences from two subloops

Numerical result

input parameters:

$m_W = 80.352$ GeV

$m_Z = 91.1535$ GeV

$m_H = 125.1$ GeV

$m_t = 172.76$ GeV

$m_{f\neq t} = 0$

$\alpha^{-1} = 137.036$

$\Delta\alpha = 0.059$

$\sqrt{s} = 240$ GeV

$\theta = \pi/2$

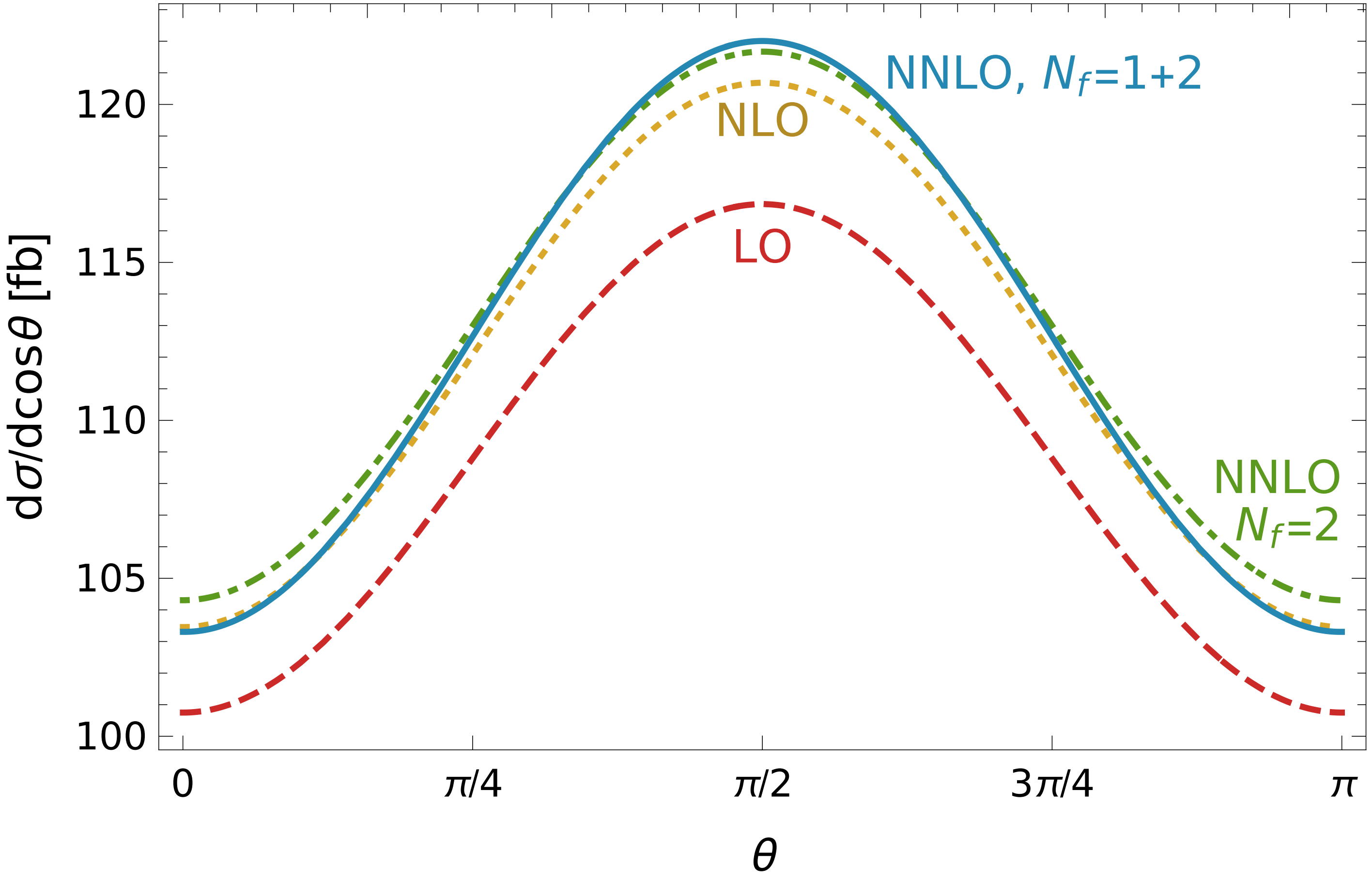
	(fb)	Contribution	(fb)
σ^{LO}	222.958		
σ^{NLO}	229.893	N_f: number of fermion loops	
		$\mathcal{O}(\alpha_{N_f=1})$	21.130
		$\mathcal{O}(\alpha_{N_f=0})$	−14.195
σ^{NNLO}	231.546	N_f: number of fermion loops	
		$\mathcal{O}(\alpha^2_{N_f=2})$	1.881
		$\mathcal{O}(\alpha^2_{N_f=1})$	−0.226

- $\mathcal{O}(\alpha) = 3\%$ bosonic and fermionic contribution partially cancel
- $\mathcal{O}(\alpha^2) = 0.7\%$ comparable with experimental precision
- $\mathcal{O}(\alpha^2_{N_f=2}) \gg \mathcal{O}(\alpha^2_{N_f=1})$ large top mass and flavor number enhancement; accidental numerical cancellation;

Numerical result

Contribution for N_f=1:

- difference between solid bule line and dashed green curves;
- larger than 0 for $|\cos\theta| > 0.59$
- smaller than 0 for $|\cos\theta| < 0.59$

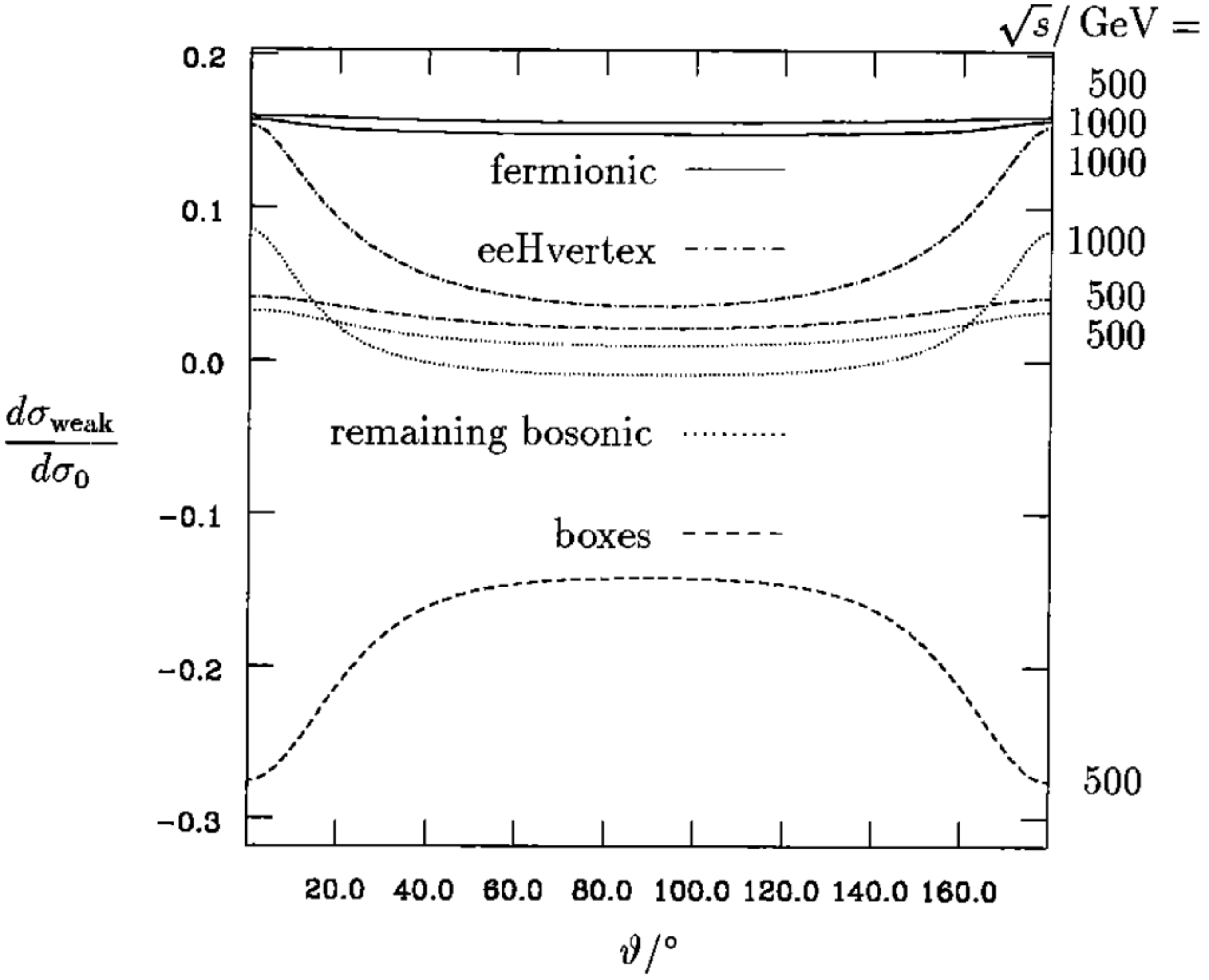


Summary

- Two-loop Electroweak correction must be included because of expected high precision at future e^+e^- colliders
- Diagrams with closed fermionic loops are dominated, UV divergent in general
- Using Feynman parametrization and dispersion relation, squared amplitude can be simplified to 3/2/1-fold integral
- Numerical integration takes few minutes with 4-digit precision
- Subtraction terms to deal with UV divergence, canceled with CT
- NNLO(EW+EW) for $N_f = 2$ is much greater than $N_f = 1$
- For future: polarized cross section, real correction, unstable Z&H decay, bosonic correction estimation, renormalization scheme dependence…

Thank you!

Numerical result at NLO



Polarized cross section at NLO

P_{e^-}	P_{e^+}	σ^{hard}, fb	σ^{Born}, fb	$\sigma^{\text{1-loop}}$, fb	δ, %
0	0	82.0(1)	225.59(1)	206.91(1)	-8.28(1)
-0.8	0	47.6(1)	266.05(1)	223.52(2)	-15.99(1)
-0.8	-0.6	46.3(1)	127.42(1)	111.76(2)	-12.29(1)
-0.8	0.6	147.1(1)	404.69(1)	335.28(1)	-17.15(1)

Table 1: Hard, Born and 1-loop cross sections in fb of the process $e^+e^- \to ZH$ and relative correction δ in percents for energy 250 GeV and various polarizations of initial particles produced by SANC.

Missing term estimation

- A simple method is based on the assumption that the perturbation series follows roughly a geometric progression, such as

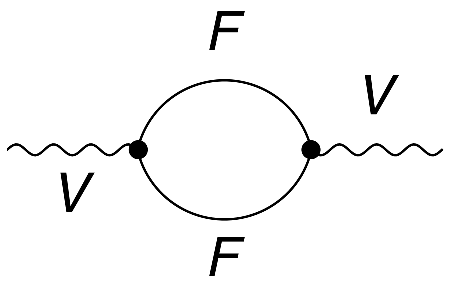
$$\mathcal{O}(\alpha^2\alpha_s) = \frac{\mathcal{O}(\alpha^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s)$$

- One is called the "Traditional Blue Band Method". It is based on the fact that the results by using different method, different renormalization scheme, differ from each other.
- A different approach is that for each type of unknown corrections the relevant enhancement factors are kept and remaining dimensionless loop integral is set to be 1.

Box diagram with self-energy fermionic subloop

- Self-energy loop is UV divergent.

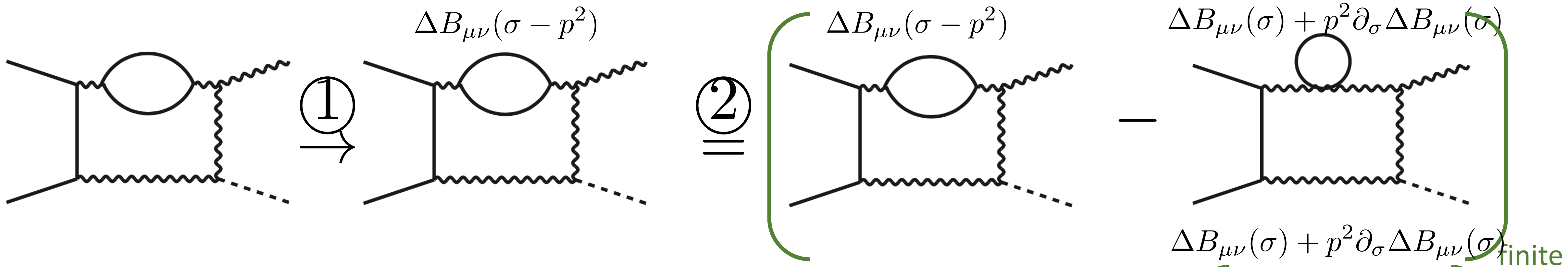
$$\int d^4q \frac{q^\mu q^\nu}{(q^2-m_1^2)((q+p)^2-m_2^2)} \overset{q\to\infty}{\approx} \int dq\, q = \infty$$

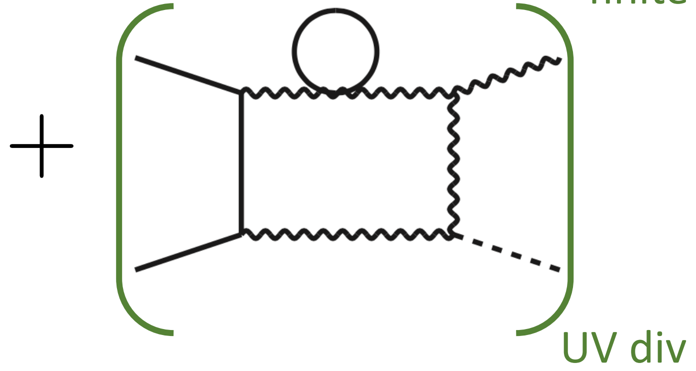


- With dispersion relation, the UV performance at large q becomes UV divergence at large σ

$$\int d^4q \frac{q^\mu q^\nu}{(q^2-m_1^2)((q+p)^2-m_2^2)}$$
$$= \int_{\sigma_0}^{\infty} d\sigma \frac{\Delta B^{\mu\nu}(\sigma, m_1^2, m_2^2)}{\sigma - p^2} \overset{\sigma\to\infty}{\approx} \int d\sigma = \infty$$

Box diagram with self-energy fermionic subloop





1. Feynman parametrization and dispersion relation
   divergence of fermionic loop $\rightarrow \Delta B_{\mu\nu}(\sigma - p^2)$at large σ

2. $\lim_{\sigma\to\infty} \Delta B_{\mu\nu}(\sigma - p^2) - \Delta B_{\mu\nu}(\sigma) - p^2\partial_\sigma \Delta B_{\mu\nu}(\sigma) \to \text{finite}$

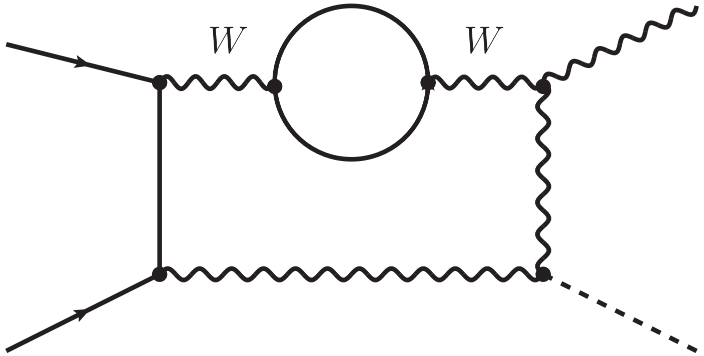
3. $\Delta B_{\mu\nu}(\sigma) + p^2\partial_\sigma \Delta B_{\mu\nu}(\sigma)$ can be integrated analytically:

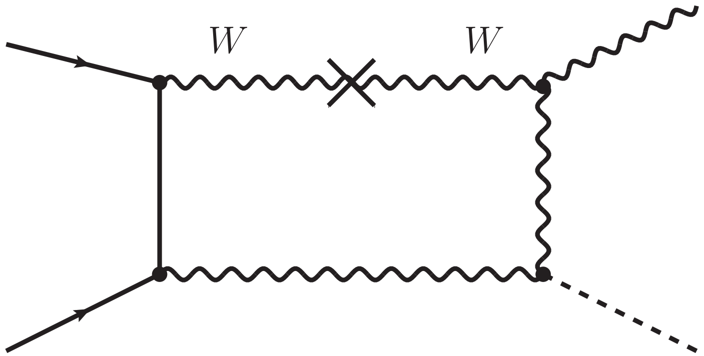
$$\int d\sigma \Delta B_{\mu\nu}(\sigma) + p^2\partial_\sigma \Delta B_{\mu\nu}(\sigma) = B_{\mu\nu}(0) + p^2 \frac{\partial B_{\mu\nu}(ps)}{\partial ps}|_{ps\to 0}$$

   bosonic subloop is expressed using scalar PaVe functions

4. Divergent part cancels with CT diagram

Box diagram with self-energy fermionic subloop





div: (6.449289*10^-7 + 3.513202*10^-15*I)/ϵ
finite: -6.847403*10^-6 – 3.854979*10^-14*I

div: (-6.449289*10^-7 -7.518501*10^-15*I)/ϵ
finite: 6.963528*10^-6 + 8.438749*10^-14*I

Sum of loop and CT is UV finite

Polarized amplitude

- Polarized squared amplitude can be easily computed by multiplying projection operator with fermion spinor
  $M_{LR} = M_{RL} = 0$

$$u(p_1) \to u_L(p_1) = \frac{1-\gamma^5}{2} u(p_1)$$
$$\to u_R(p_1) = \frac{1+\gamma^5}{2} u(p_1)$$

- Derive relation between polarized squared amplitude and unpolarized square amplitude
  $|M^0(M^2_{LL,RR})^*| = C_{LL,RR} \times |M^0(M^2)^*|$

Number of gauge bosons connected with ee beam $\Leftarrow$

gauge bosons V_i	unpolarized square amplitude	$e^-_L e^+_L$	$e^-_R e^+_R$
$V_1 = \gamma$	$\lvert M_0^Z M_2^\gamma \rvert = \frac{1}{4}(\lvert M^Z_{0,L} M^\gamma_{2,L}\rvert + \lvert M^Z_{0,R} M^\gamma_{2,R}\rvert)$	$\frac{g_L \times 4 \times \lvert M_0^Z M_2^{\gamma\gamma}\rvert}{g_L + g_R}$	$\frac{g_R \times 4 \times \lvert M_0^Z M_2^{\gamma\gamma}\rvert}{g_L + g_R}$
$V_1 = Z$	$\lvert M_0^Z M_2^Z \rvert = \frac{1}{4}(\lvert M^Z_{0,L} M^Z_{2,L}\rvert + \lvert M^Z_{0,R} M^Z_{2,R}\rvert)$	$\frac{g_L^Z \times 4 \times \lvert M_0^Z M_2^Z\rvert}{g_L^Z + g_R^Z}$	$\frac{g_R^Z \times 4 \times \lvert M_0^Z M_2^Z\rvert}{g_L^Z + g_R^Z}$
$V_1 = \gamma, V_2 = \gamma$	$\lvert M_0^Z M_2^{\gamma\gamma} \rvert = \frac{1}{4}(\lvert M^Z_{0,L} M^{\gamma\gamma}_{2,L}\rvert + \lvert M^Z_{0,R} M^{\gamma\gamma}_{2,R}\rvert)$	$\frac{g_L^Z \times 4 \times \lvert M_0^Z M_2^{\gamma\gamma}\rvert}{g_L^Z + g_R^Z}$	$\frac{g_R^Z \times 4 \times \lvert M_0^Z M_2^{\gamma\gamma}\rvert}{g_L^Z + g_R^Z}$
$V_1 = \gamma, V_2 = Z$	$\lvert M_0^Z M_2^{\gamma Z} \rvert = \frac{1}{4}(\lvert M^Z_{0,L} M^{\gamma Z}_{2,L}\rvert + \lvert M^Z_{0,R} M^{\gamma Z}_{2,R}\rvert)$	$\frac{(g_L^Z)^2 \times 4 \times \lvert M_0^Z M_2^{\gamma Z}\rvert}{(g_L^Z)^2 + (g_R^Z)^2}$	$\frac{(g_R^Z)^2 \times 4 \times \lvert M_0^Z M_2^{\gamma Z}\rvert}{(g_L^Z)^2 + (g_R^Z)^2}$
$V_1 = Z, V_2 = Z$	$\lvert M_0^Z M_2^{ZZ} \rvert = \frac{1}{4}(\lvert M^Z_{0,L} M^{ZZ}_{2,L}\rvert + \lvert M^Z_{0,R} M^{ZZ}_{2,R}\rvert)$	$\frac{(g_L^Z)^3 \times 4 \times \lvert M_0^Z M_2^{ZZ}\rvert}{(g_L^Z)^3 + (g_R^Z)^3}$	$\frac{(g_R^Z)^3 \times 4 \times \lvert M_0^Z M_2^{ZZ}\rvert}{(g_L^Z)^3 + (g_R^Z)^3}$
$V_1 = W, V_2 = W$	$\lvert M_0^Z M_2^{WW} \rvert = \frac{1}{4}(\lvert M^Z_{0,L} M^{WW}_{2,L}\rvert$	$4 \times \lvert M_0^Z M_2^{WW}\rvert$	0