Complete two-loop electroweak corrections to $e^+e^- \to H\,Z$

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Based on work done with:
Xiang Chen, Chuan-Qi He, Zhao Li, Xiao Liu, Yan-Qing Ma

2209.14953

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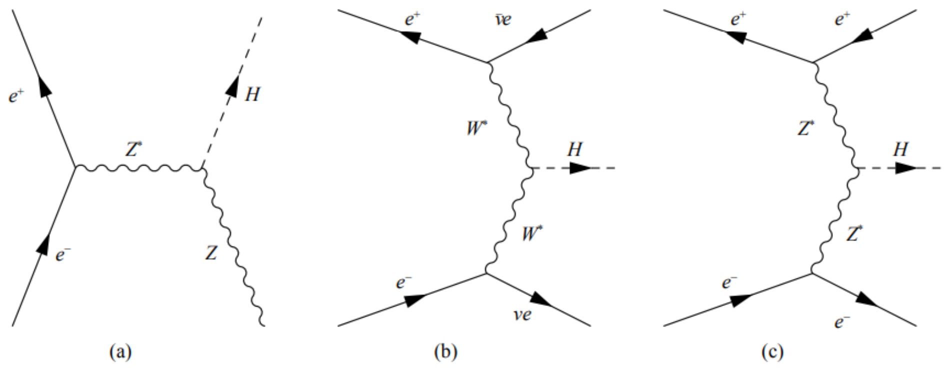
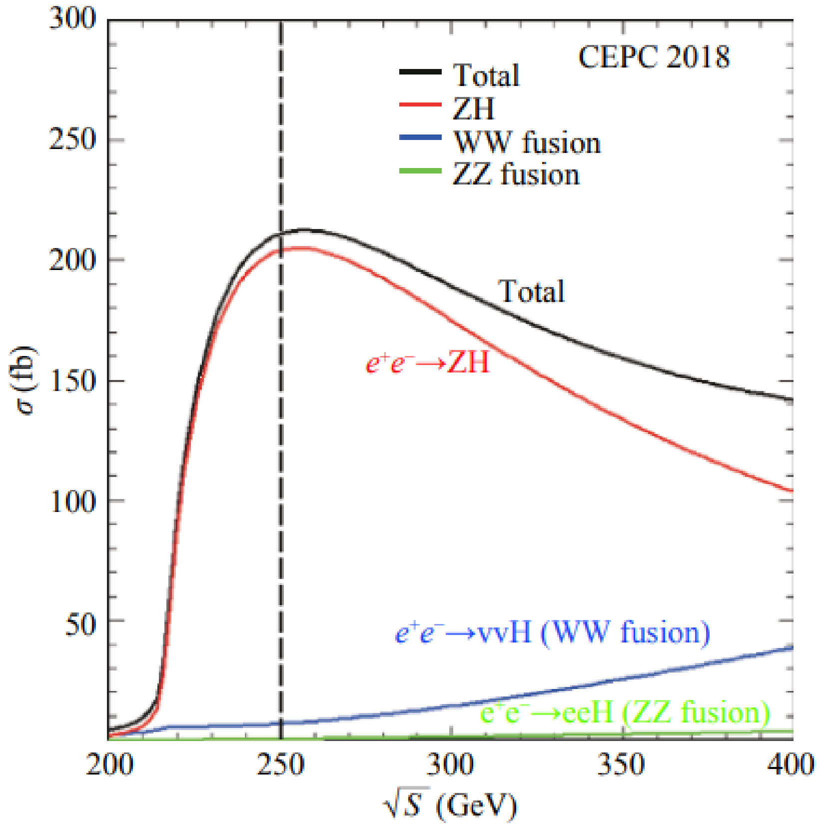
Higgs production at e^+e^- colliders

- Precise measurement of properties of Higgs
  - SM & BSM
- Higgs-Strahlungs(HZ) is the dominant contribution

at a typical center-of-mass energy 250GeV



Higgs-Strahlungs WW fusion ZZ fusion



[F. An et al: 1810.09037]

- σ_{HZ}~0.51% [CEPC Study Group: 1811.10545]

Theoretic efforts for Higgsstrahlung

- LO [J. Ellis, M. K. Gaillard, D. V. Nanopoulos, Nucl. Phys. B(1976)]
- NLO EW correction [J. Fleischer and F. Jegerlehner: Nucl. Phys. B(1983), B. A. Kniehl. Z. Phys. C(1992), A. Denner et al. Z. Phys. C(1992)]
- EW-QCD mixed correction

$\alpha(m_z)$ scheme

$\sqrt{s}$ (GeV)	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NNLO}$ (fb)	$\sigma^{\rm exp.}_{\rm NNLO}$ (fb)
240	252.0	228.6	231.5	231.5
250	252.0	227.9	230.8	230.8
300	190.0	170.7	172.9	172.9
350	135.6	122.5	124.2	124.0
500	60.12	54.03	54.42	54.81

Y. Gong, Z. Li, X. Xu, L. L. Yang and X. Zhao: 1609.03955

renormalization scheme uncertainties

$\sqrt{s}$	schemes	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NNLO}$ (fb)
240	$\alpha(0)$	223.14 ± 0.47	229.78 ± 0.77	$232.21^{+0.75+0.10}_{-0.75-0.21}$
	$\alpha(M_Z)$	252.03 ± 0.60	$228.36^{+0.82}_{-0.81}$	$231.28^{+0.80+0.12}_{-0.79-0.25}$
	G_μ	239.64 ± 0.06	$232.46^{+0.07}_{-0.07}$	$233.29^{+0.07+0.03}_{-0.06-0.07}$
250	$\alpha(0)$	223.12 ± 0.47	229.20 ± 0.77	$231.63^{+0.75+0.12}_{-0.75-0.21}$
	$\alpha(M_Z)$	252.01 ± 0.60	$227.67^{+0.82}_{-0.81}$	$230.58^{+0.80+0.14}_{-0.79-0.25}$
	G_μ	239.62 ± 0.06	231.82 ± 0.07	$232.65^{+0.07+0.04}_{-0.07-0.07}$

Q.-F. Sun, F. Feng, Y. Jia and W.-L. Sang: 1609.03995

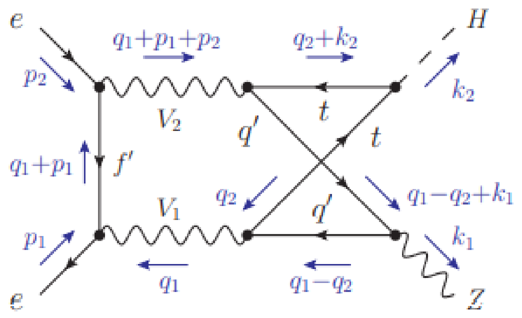
~1% corrections, significantly larger than the expected experimental accuracy (0.51%) !

Two-loop EW correction is indispensable!

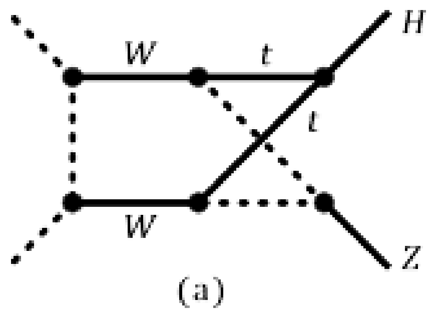
Two-loop EW calculation is challenging

➢Many mass scales

- It seems hopeless to compute Feynman integrals analytically
- Progress in numerical way
  - A class of box diagram



Q. Song and A. Freitas: 2101.00308



X. Liu and Y.-Q. Ma: 2107.01864

➢Many Feynman diagrams

- 25377 Feynman diagrams

[Z. Li, Y. Wang and Q. F. Wu 2012.12513]

- Require a systematic approach

Equipped with many state-of-art techniques,
the complete two-loop EW calculation is available now!

Toward a complete two-loop EW corrections

- A general procedure

Differential Equation

generate integrand → Integral reduction → Evaluate master integral

$$A = \sum_{i=1}^{\mathcal{O}(10^4)} f_i \times I_i$$

$$I_i = \sum_{j=1}^{\mathcal{O}(10^2)} c_{ij} M_j$$

$$I_i(p_i \cdot p_j, m_i^2) = \int \dots \int \frac{d^d k_1 \dots d^d k_h}{E_1^{a_1} \dots E_N^{a_N}} \quad (E_i = q_i^2 - m_i^2 + i0^+)$$

Feynman diagram

Integrate-By-Part(IBP)

Differential equation

- Another approach * [A. Freitas and Qian Song, 2209.07612]

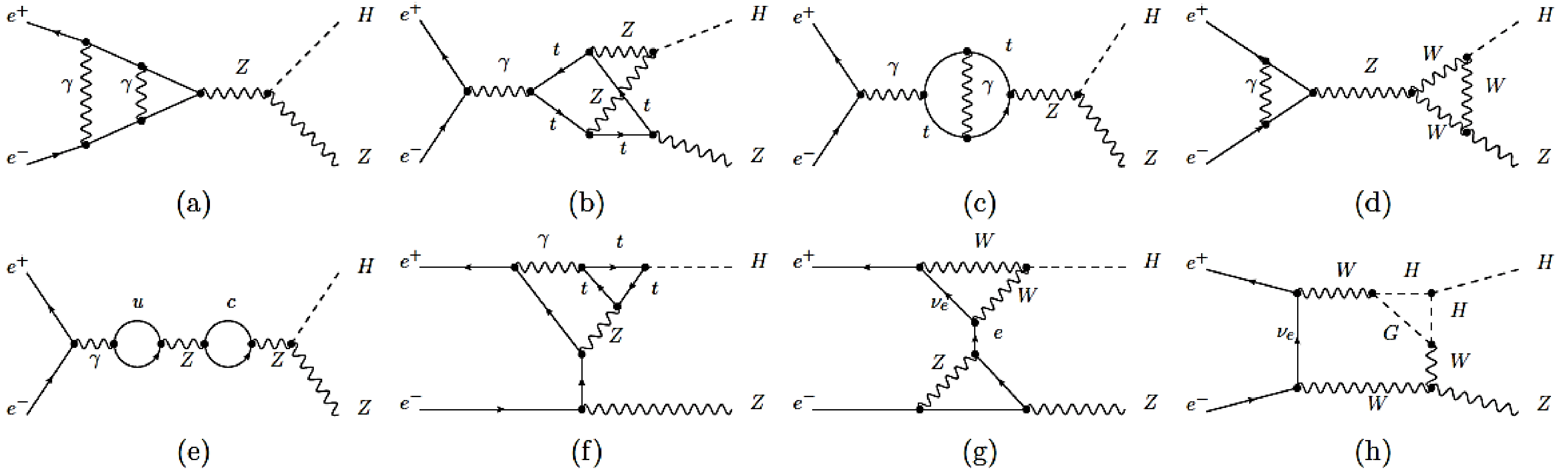
Talk by Qian Song

- No integral reduction
- Dispersion relations + Feynman parametrizations

Step 1: Generate integrand

Feynman Amplitude

- 25377 Feynman diagrams (QGRAF and FeynArts) [P. Nogueira: J. Comput. Phys(1993) P. Nogueira: Comput. Phys. Commun(2021)]
- 372 integral families [T. Hahn, Comput. Phys. Commun(2001)]
- Naïve γ_5 scheme M. S. Chanowitz, M. Furman, I. Hinchliffe. Nucl. Phys. B(1979)
  - Keep the anticommutation relation
  - KKS scheme: take average of reading points J. G. Korner, D. Kreimer, K. Schilcher. Z. Phys. C(1992)



Step 2: Integral reduction

status of IBP reduction

Chetyrkin and Tkachov, NPB(1981)

- IBP is taking center stage in precision calculation
  - Laporta algorithm [Laporta, Int.J.Mod.Phys.A(2000)]
- Finite fields and functional reconstruction

[Von Manteuffle and R. M. Schabinger, Phys. Lett. B 2015]
Kira, Usovitsch et al, arXiv:2008.06494
FiniteFlow, Peraro, JHEP (2019)
Fire 6, Smirnov, Comput.Phys.Commun. (2020)
CARAVEL, Abreu, et al, Comput. Phys. Commun. (2021)

- New ideas
  - Less sample
    - Better basis (UT / quasi finite integral / ϵ factorized …) [J. M. Henn, Phys. Rev. Lett(2013), J. Usovitsch arXiv: 2002.08173, A. V. Smirnov and V. A. Smirnov arXiv: 2002.08042, E. Panzer, 2015, von Manteuffel, Panzer, Schabinger, JHEP(2015)]
    - Denominator guessing[S. Abreu et al, Phys. Rev. Lett (2019), M. Heller, von Manteuffel. Comput. Phys. Commun(2022)]
  - Fast sampling
    - Syzygy [Gluza, Kajda and Kosower. Phys. Rev. D(2011), Kasper J. Larsen and Yang Zhang. Phys. Rev.D (2016)]
    - Block-triangular form [Xin Guan, Xiao Liu and Yan-Qing Ma, *Chin.Phys.C* (2020)]

Block-triangular form reduction

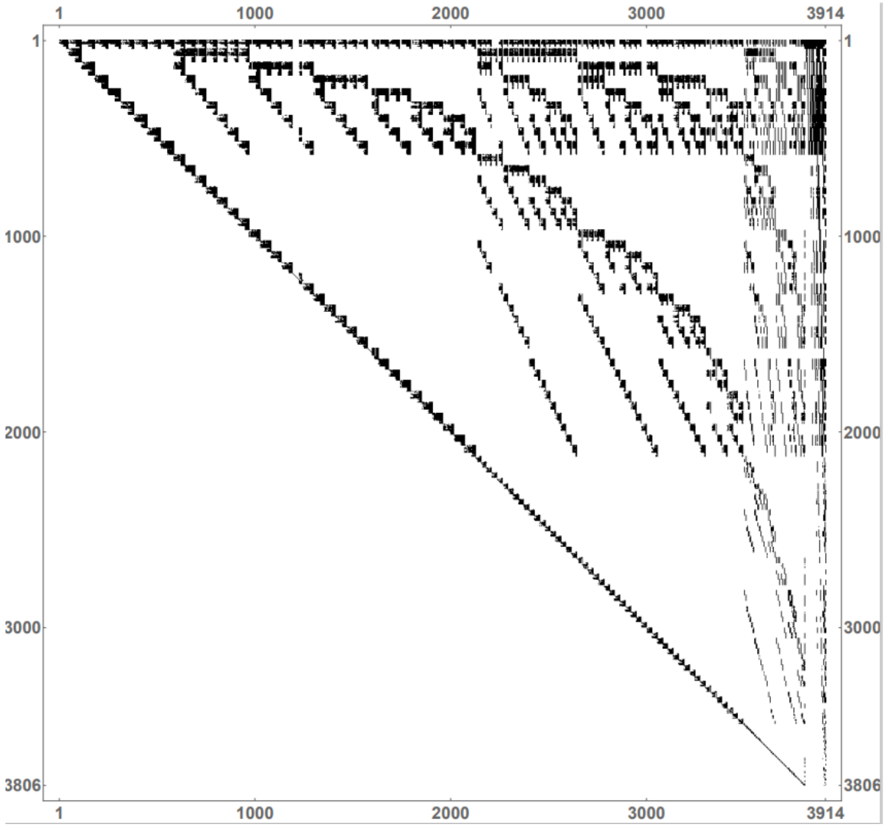
Xin Guan, Xiao Liu and Yan-Qing Ma, *Chin.Phys.C* (2020)

➢ Improved linear system

- Search algorithm
- as many equations as target integrals
- Nice block-triangular structure, ideal for numerical sampling

➢ Program development

- Blade: a package for block-triangular form improved Feynman integrals decomposition.
- Applied to this work and our recent another work[Xiang Chen, Xin Guan, Chuan-Qi He, Xiao Liu, and Yan-Qing Ma. 2209.14259]. (Talk by Xiang Chen)
- Reduces the computational time by several times to 2 orders, depending on families.
- To release the package in the near future



Matrix plot of block-triangular relations for double-pentagon topology

Step 3: Evaluate master integrals

Numerical differential equation

- ➢Widely used recently
  - systematic

R. N. Lee, V. A. Smironov JHEP (2018)
R. Bonciani, G. Degrassi, P. P. Giardino, R. Grober, Comput.Phys.Commun (2019)
H. Frellesvig, M. Hidding, L. Maestri, F. Moriello, G. Salvatori, JHEP (2020)
Long Cheng, Czakon and Niggetiedt, JHEP(2021)
Fael, Lange, Schonwald and Steinhauser, Phys.Rev.Lett(2022)
Martijin Hidding, Comput.Phys.Commun (2021)
Xiao Liu and Yan-Qing Ma, arXiv: 2201.11669

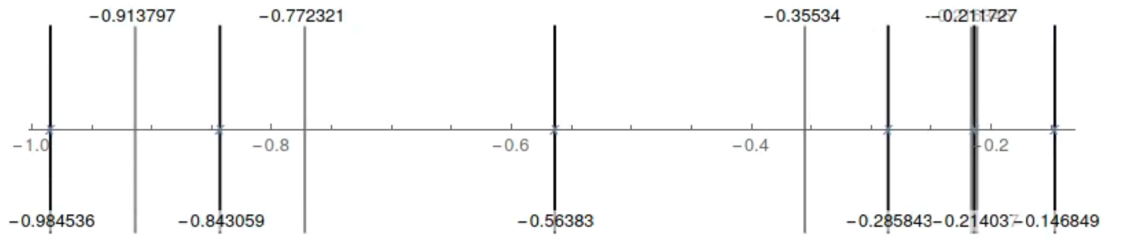
- ➢A general procedure
  - Construct differential equation and obtain boundary condition(non-trivial)

$$\frac{d\, I_i(\epsilon,x)}{d\,x} = \sum_j A_{ij}\, I_j(\epsilon,x)$$

  - Generalized series expansion around singularities

$$I(\epsilon,x) = \sum_{\mu,k,n} c_{\mu,k,n}(\epsilon)\, x^{\mu(\epsilon)} \log^k x\;\; x^n$$

  - Taylor expansions around regular points to cover the whole physical region(for e.g phase space integration).



Technique details

➢Construct differential equation

- Employ symmetries among families to get a minimal set of master integrals (7675 in total)
- Mandelstam variables: $s = (k_1 + k_2)^2,\ t = (k_1 - k_3)^2$
- DE w.r.t the massless parameter: t/m_t^2 (for any fixed s)

➢Boundary condition

[Xiao Liu and Yan-Qing Ma, arXiv: 2201.11669]

- we use AMFlow to compute master integrals at $\frac{t}{m_t^2} = -1/2$ with high precision

Talk by Xiao Liu

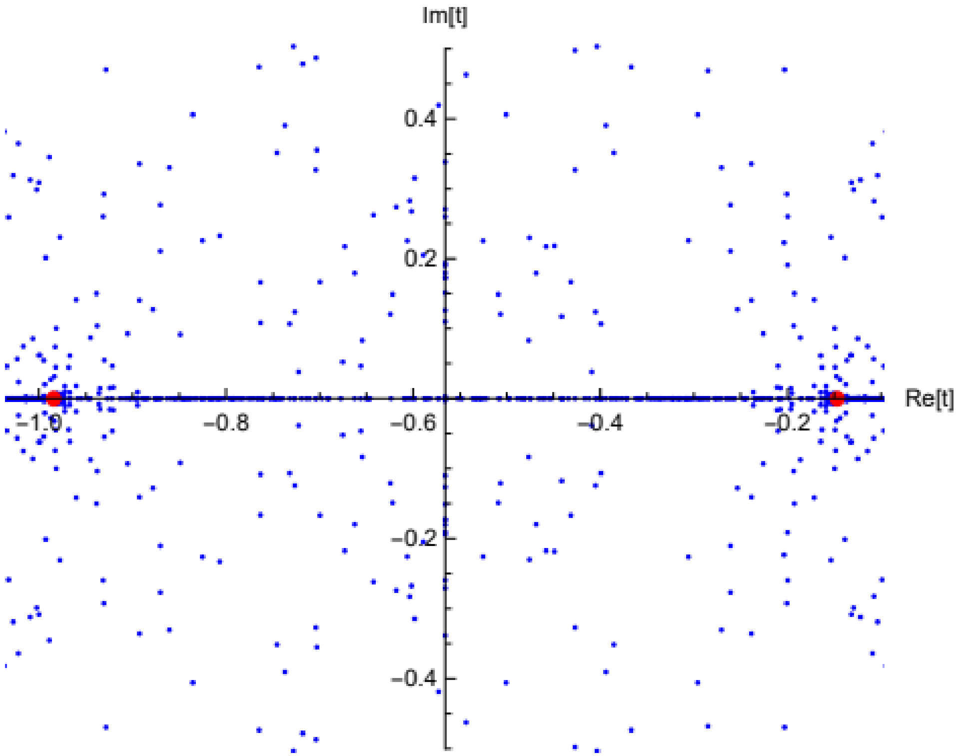
- We implement the interface between Blade and AMFlow to speed up calculation

➢Solve differential equation

- Frobenius method and generalized series expansion[S. Pozzorini and E. Remiddi, Comput. Phys. Commun(2006), F. Moriello, JHEP(2020), H. Frellesvig, M. Hidding, L. Maestri, F. Moreillo et al, JHEP(2020) M. Hidding:Comput. Phys. Commun(2021)]
- Care should be paid to treat singularities

Treatment of singularities

- $\mathcal{O}(10^3)$ singularities
  - Cumbersome and inefficient to do series expansion at each singularity.
- Necessary to classify singularities
  - Physical: e.g. threshold, predicted by unitarity cuts Analytic continuation is needed.
  - Non-physical: integrals are regular in physical sheet, singular on other Riemann sheets, e.g. pseudo-threshold,
  - Spurious: integrals are always regular



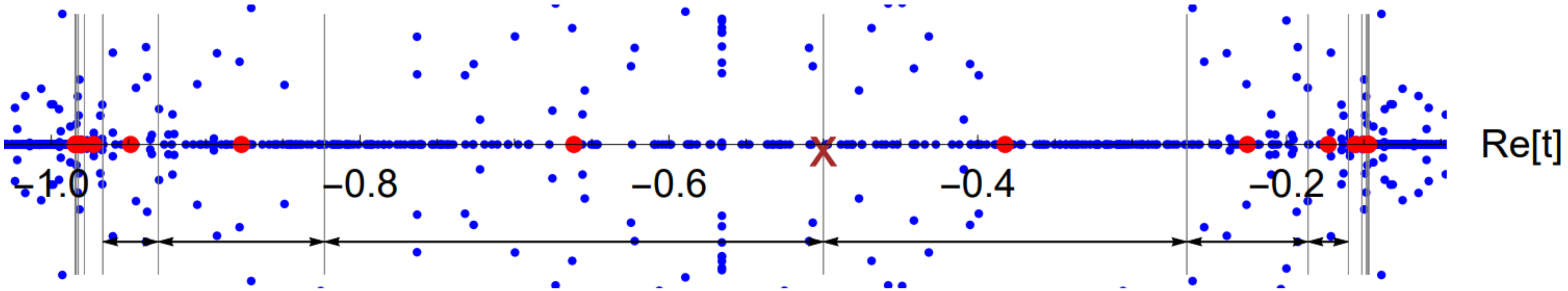
Singularities distribution

Trait of expansion

- A limited range of convergence for series expansion.
- For regular point: it’s convergent radius is only limited by the nearest physical singularity.
- For physical singularity: it’s convergent radius is not limited by spurious singularity.
- Neighboring non-physical/spurious singularities would affect numerical stability during intermediate computation, which could be resolved by keeping higher number of (inexact)digits.

Based on these observations, it is straightforward to design line segment.

Strategy to design line segment



Demonstration of the line segment of the master integrals in the physical region. “x” point: boundary condition at $t/m_t^2 = -1/2$. “ • ”: expansion point. “↔ “: the maximum distance that the expansion could estimate.

- Learn physical singularities
- Add additional regular expansion points to move close to the nearest physical singularity
- Line segment *:

{[-0.984536,-0.984536,-0.984472], [-0.984472,-0.984467,-0.984462], ⋯ , [-0.823024,-0.661512,-0.5], [-0.5,-0.382283,-0.264566],⋯, [-0.146893,-0.146849,-0.146849]}

* We show numeric value of segments, while the real expansion is carried out at exact number points.

A very economical way to cover the entire region!

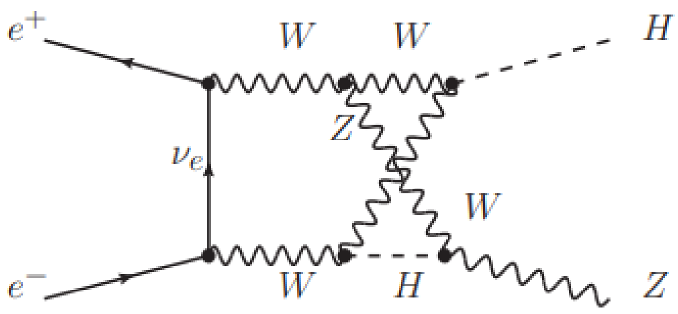
A look at expansion

$$
\begin{aligned}
&I_{7662}(t)_{[-0.5,-0.382,-0.264]} = \\
&(0.2905261664 - 16.4865783263I) - (0.359534597908 + 0.927758324598I)\left(\frac{32025}{83773} + t\right) \\
&- (0.564246296565 + 0.712877830925I)\left(\frac{32025}{83773} + t\right)^2 - (0.150810825876 + 0.125923285480I)\left(\frac{32025}{83773} + t\right)^3 + \cdots \\
&+ \epsilon \left\{(10.7287530015 - 65.4953069777I) - (1.80036205321 + 5.12759811617I)\left(\frac{32025}{83773} + t\right)\right. \\
&\left. - (3.23111689383 + 4.21419323769I)\left(\frac{32025}{83773} + t\right)^2 - (1.00115179359 8 + 0.833557992156I)\left(\frac{32025}{83773} + t\right)^3 + \cdots\right\} \\
&+ \epsilon^2 \left\{(31.4875482644 - 137.8079193836I) - (5.0724571740 + 14.4734637605I)\left(\frac{32025}{83773} + t\right)\right. \\
&\left. - (9.8770844526 + 12.6735856110I)\left(\frac{32025}{83773} + t\right)^2 - (3.45057089286 + 2.78762538182I)\left(\frac{32025}{83773} + t\right)^3 + \cdots\right\} \\
&+ \cdots
\end{aligned}
$$

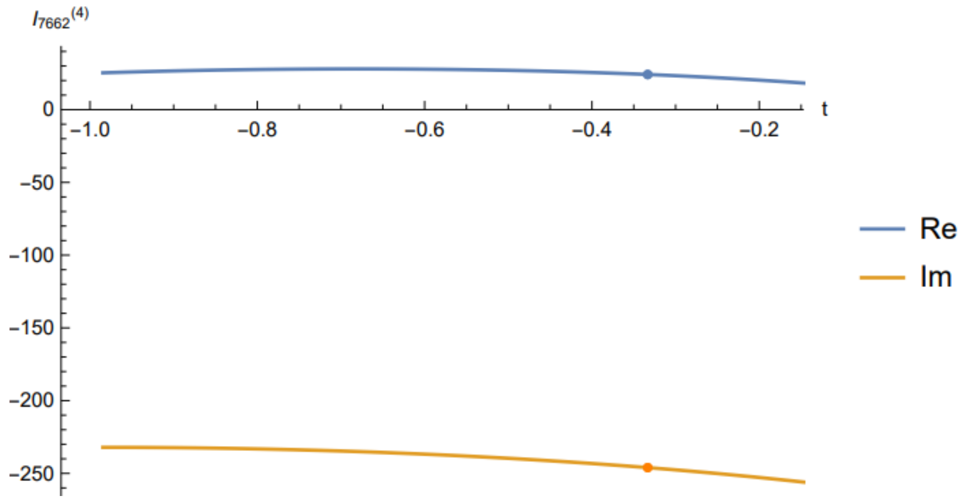
* We show truncated coefficients here while we keep thousands of digits in full computation

Numerical results

➢ Master integrals



e.g. one of the most complicated family



Plots of the corner integral, for $m_H^2 = \frac{12}{23},\ m_Z^2 = \frac{23}{83}, m_W^2 = \frac{14}{65}, s = \frac{83}{43}, m_t = 1$ as a function of t, at order ϵ^4, obtained by series expanding along the line segment. The solid point represents values computed with AMFlow at $t = -\frac{1}{3}$, which provides highly nontrivial self-consistency check of results.

➢ Bare squared amplitude

- Piecewise function of t : high precision, efficient evaluations
- Remains both UV and IR divergence

Renormalization and Infrared subtraction

- Renormalization
- Infrared subtraction
  - Real-emission: soft and final-state collinear divergence
  - Collinear factorization

We are still working on these issues

Summary and outlook

- We calculate the complete two loop electroweak corrections to H+Z production at the future Higgs factory for the first time.
- Block-triangular form is a very efficient reduction method.
- AMFlow and numerical differential equation is a systematic and efficient method to evaluate Feynman integrals.
- The methods employed in this work allow us to calculate higher order corrections to many other important processes at e^+e^- colliders.

Thanks!