Extended scalar sectors and dark matter at e^+e^- colliders

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Extended scalar sector

- Standard Model: simple scalar sector

$$V(\phi) = m^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

ϕ – complex $SU(2)$ doublet

- many signals of BSM physics
  - gravity
  - dark matter
  - dark energy
  - ν oscillations
  - muon $g-2$ (?)
  - W boson mass (?)
  - ...

- is the scalar sector really so simple?
  - SM + real singlet(s)
  - 2HDM
  - 2HDM + singlet (N2HDM)
  - SM + triplet(s)
  - ...

for a short review and references see Steggemann '20

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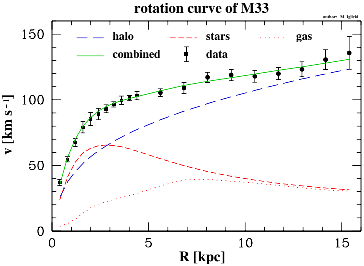
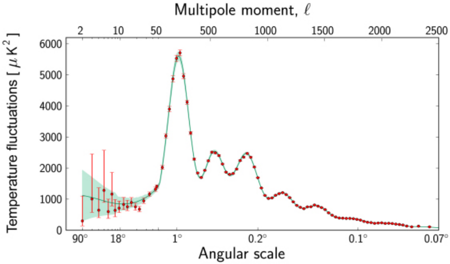
Evidence for dark matter

- rotation curves
- CMB fluctuations ⇒ ~~MOND~~
- gravitational lensing
- colliding clusters
- ...



source of the data: astro-ph/9909252

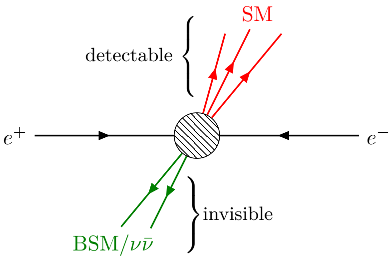


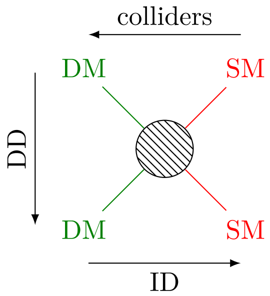
source: http://sci.esa.int/planck

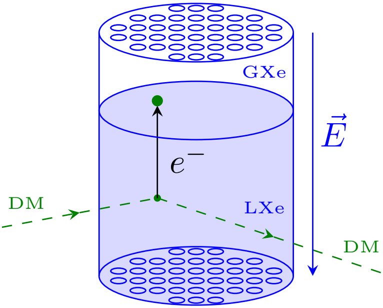

source: https://apod.nasa.gov/apod

DM believed to be beyond the Standard Model

- indirect detection: gamma rays
- direct detection: XENON etc.
- colliders: missing energy analysis

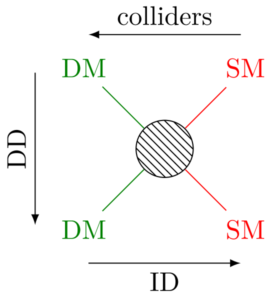


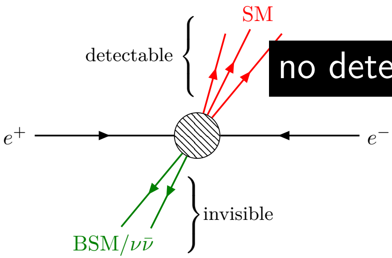




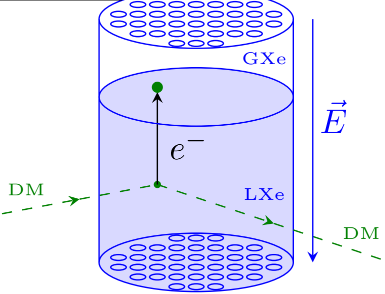
How to see something invisible?

- indirect detection: gamma rays
- direct detection: XENON etc.
- colliders: missing energy analysis





no detection so far :-(

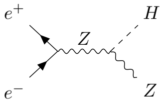


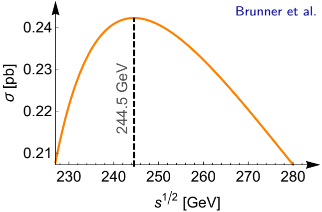
Future e^+e^- colliders

- signal to background ratio much better than at hadron colliders
- initial state precisely known (no parton distribution functions involved)
- circular colliders preferable at small E, linacs at high E ($\gtrsim 500$ GeV)
- near-future plans:
  - ILC (Japan?)
  - CEPC (China)
  - FCC-ee (Europe)
  - CLIC (Europe)

$\sqrt{s} = 240$–250 GeV (ZH-production peak)

ILC Int. Dev. Team '22
CEPC Phys. Study Group '22
Agapov et al. '22
Brunner et al. '22





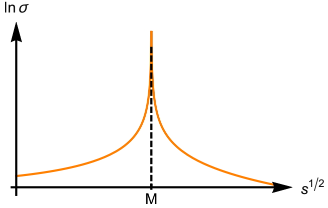
- other proposals
  - C^3 („Cool Copper Collider")
  - CERC („Circular Energy Recovery Collider")
  - ReLiC („Recycling Linear e^+e^- Collider")
  - ...

Bai et al. '21
Litvinenko et al. '22
Litvinenko et al. '22

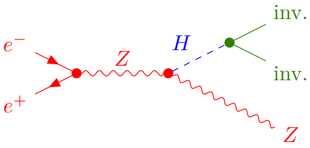
Collider search for new physics

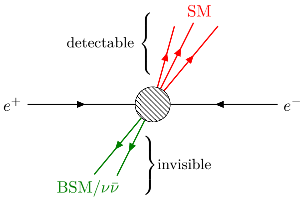
- resonance enhancements

$$\sigma \sim \frac{1}{(s-M^2)^2+M^2\Gamma^2} \longrightarrow$$



- invisible decays of H
  (missing-energy analysis)





- 3 simple (but not simplified) DM models of different spins: 0, 1, 1/2
- consider a mechanism of DM production at e^+e^- colliders
- take current experimental constraints into account
- check...
  - ...what range of parameters is still allowed?
  - ...how many DM-production events can we expect at future colliders?
  - ...whether it is possible to disentangle models of different spins?

during calculations, collider parameters of ILC are used

Theoretical models

The main properties of the models:

- simple, but renormalizable and QFT-consistent
- common parameter space
- DM connected to SM by the Higgs portal: $\kappa|H|^2|S|^2$
- mixing in the scalar sector

real-part fluctuations of a new singlet S and the Higgs doublet H

$$\Big\downarrow \times\ \mathcal{R} \equiv \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

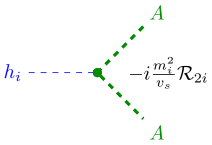
two Higgs-like mass eigenstates: h_1 $(m_1 = 125$ GeV) and h_2 $(m_2 = ?)$

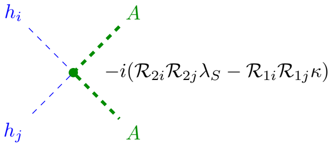
The models:

- pseudo-Goldstone (scalar) DM model $(s = 0)$ $\rightarrow$ dark scalar A
- vector DM model $(s = 1)$ $\rightarrow$ dark gauge vector X_μ
- fermion DM model $(s = 1/2)$ $\rightarrow$ dark Majorana fermion ψ

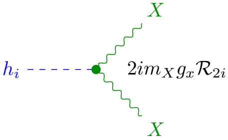
DM interactions

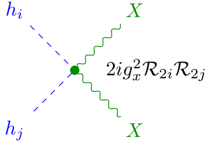
- pGDM



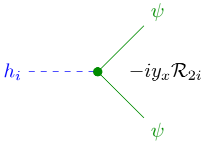


- VDM





- FDM



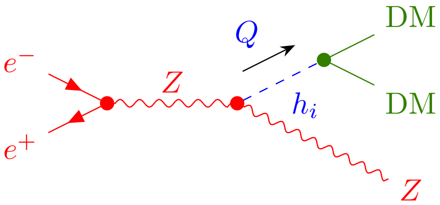
input parameters: $\underbrace{v,\ m_1}_{\text{assumed to be SM-like}},\ v_s,\ m_2,\ \sin\alpha,\ m_{\text{DM}} \longrightarrow \kappa \equiv \frac{(m_1^2 - m_2^2)\sin 2\alpha}{2vv_s}$

$\Downarrow$

DM purely gravitational

if $m_1 = m_2$

Considered process



$$Q^2 = s - 2E_Z\sqrt{s} + m_Z^2 \equiv m_{\text{rec}}^2$$

$$\text{DM} = A,\ X_\mu,\ \psi$$

note: in principle $\Gamma_{1,2}$ model-dependent

the differential cross section:

$$\frac{d\sigma}{dm_{\text{rec}}^2} \sim \sigma_{\text{SM}}(m_{\text{rec}}^2) \cdot \sin^2\alpha\cos^2\alpha \cdot \left| \frac{1}{m_{\text{rec}}^2 - m_1^2 + i m_1\Gamma_1} - \frac{1}{m_{\text{rec}}^2 - m_2^2 + i m_2\Gamma_2} \right|^2$$

$$\times \sqrt{1 - 4\frac{m_{\text{DM}}^2}{m_{\text{rec}}^2}} \cdot \begin{cases} 1 & \text{(pGDM)} \\ 1 - 4\frac{m_{\text{DM}}^2}{m_{\text{rec}}^2} + 12\left(\frac{m_{\text{DM}}^2}{m_{\text{rec}}^2}\right)^2 & \text{(VDM)} \\ 2\frac{m_{\text{DM}}^2}{m_{\text{rec}}^2}\left(1 - 4\frac{m_{\text{DM}}^2}{m_{\text{rec}}^2}\right) & \text{(FDM)} \end{cases}$$

Considered process: shape of the differential cross section

$$\frac{d\sigma}{dm_{\text{rec}}^2} \sim \sigma_{\text{SM}}(m_{\text{rec}}^2)\cdot \sin^2\alpha\cos^2\alpha \cdot \left|\frac{1}{m_{\text{rec}}^2 - m_1^2 + i m_1\Gamma_1} - \frac{1}{m_{\text{rec}}^2 - m_2^2 + i m_2\Gamma_2}\right|^2$$

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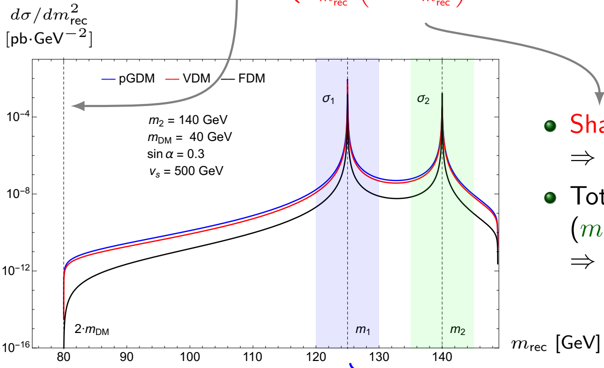
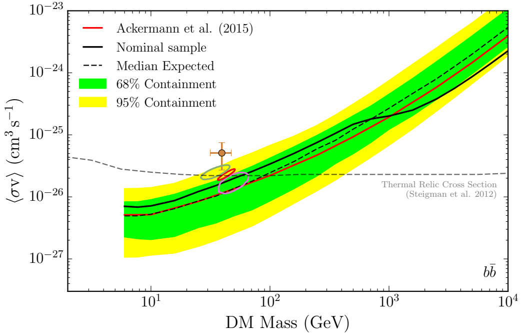


- Shape fitting
  $\Rightarrow$ spin hard to check
- Total σ as a function of $\sqrt{s}$
  ($m_{\text{rec}} = 2m_{\text{DM}}$ threshold)
  $\Rightarrow$ mass of DM possible to determine

after integration: $\sigma \approx \underset{\text{if } 2\,m_{\text{DM}}<m_1<\sqrt{s}-m_Z}{\sigma_1} + \underset{\text{if } 2\,m_{\text{DM}}<m_2<\sqrt{s}-m_Z}{\sigma_2}$

$\sigma_1 \equiv \sigma_{\text{SM}}(m_1)\cdot\cos^2\alpha\cdot \text{BR}(h_1 \to \text{DM})$ $\quad$ $\sigma_2 \equiv \sigma_{\text{SM}}(m_2)\cdot\sin^2\alpha\cdot\text{BR}(h_2 \to \text{DM})$

1. perturbativity (h_i-DM): $g_x, y_x < 4\pi \;\Rightarrow\; v_s > m_{\text{DM}}/4\pi$
2. $h^2\,\Omega_0^{\text{DM}} = 0.12 \pm 0.0012 \;\Rightarrow$ constraint on $\langle\sigma v\rangle_{\text{ann}} \;\Rightarrow$ constraint on κ^2 — Planck Col. '18
3. ID experiments $\Rightarrow$ limit on $\langle\sigma v\rangle_{\text{ann}} \Rightarrow$ limit on m_{DM} — Fermi-LAT & DES Cols. '16



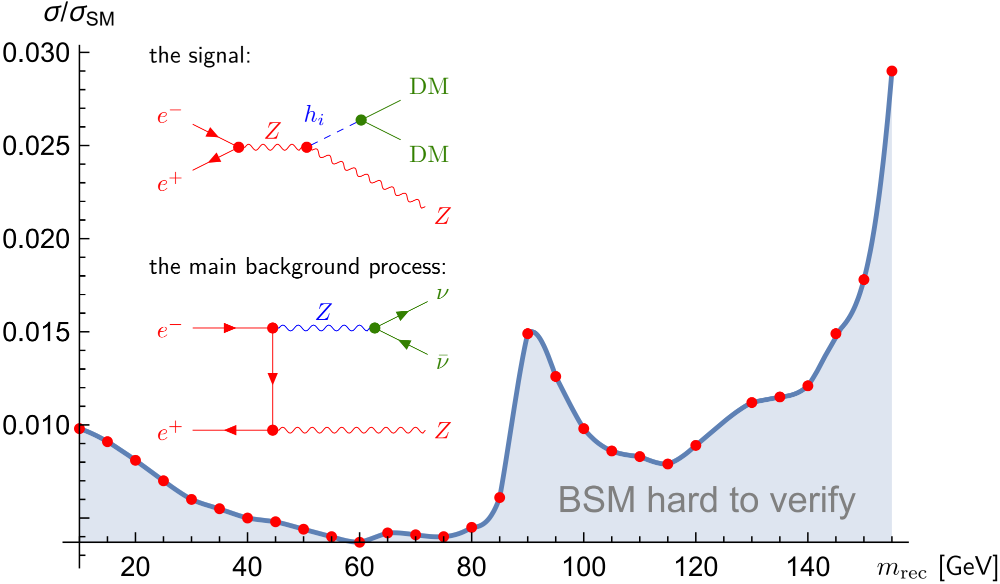
4. DD experiments and ν telescopes $\Rightarrow$ limit on $\sigma^{\text{DD}} \;\Rightarrow$ limit on κ^2 — XENON Col. '18, IceCube Col. '16
5. LHC $\Rightarrow$ $|\sin\alpha| \lesssim 0.3$ — Robens, Stefaniak '16
6. LHC $\Rightarrow$ BR$(h_1 \to \text{DM}) < 19\%$ — CMS Col. '18

Results: methodology

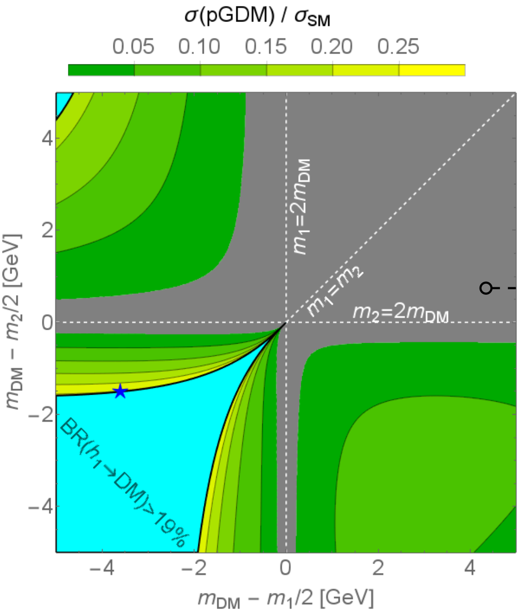
- constraints:
  - $|\sin\alpha| \lesssim 0.3$
  - relic density (Ω_0^{DM}) constraint on κ^2
  - $v_s < \frac{m_{\mathrm{DM}}}{4\pi}$
  - direct detection (DD) limit on κ^2
  - $\mathrm{BR}(h_1 \to \mathrm{DM}) < 19\%$

  $\Big\}$ taken into account numerically
- free parameters: m_2, m_{DM}, $\sin\alpha$, $v_s \to \kappa^2$
- $\kappa^2 = \kappa^2(\Omega_0^{\mathrm{DM}}) \Rightarrow$ 3 parameters left
- cross section maximized with respect to $\sin\alpha$
  $\Rightarrow \sin\alpha \to \sin\alpha_{\max} = 0.3$, free parameters: m_2, m_{DM}

The 95% CL sensitivity limit for ILC



Results: maximal cross section for pGDM



parameters of ⋆

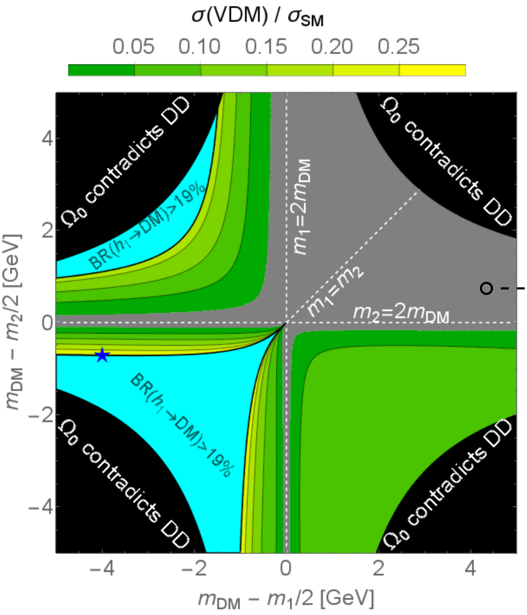
$m_2 = 120.8$ GeV
$m_{DM} = 58.9$ GeV
$\sin\alpha = 0.30$
$v_s = 646$ GeV

$\Gamma_1 = 7.4 \cdot 10^{-3}$ GeV
$\Gamma_2 = 9.8 \cdot 10^{-3}$ GeV
BR($h_1 \to$ DM) $= 19\%$
BR($h_2 \to$ DM) $= 95\%$

$\boxed{\sigma = 62 \text{ fb} \to 1.24 \cdot 10^5 \text{ events}}$

below the 95% CL
limit for the ILC
1903.01629

Results: maximal cross section for VDM



parameters of ⋆

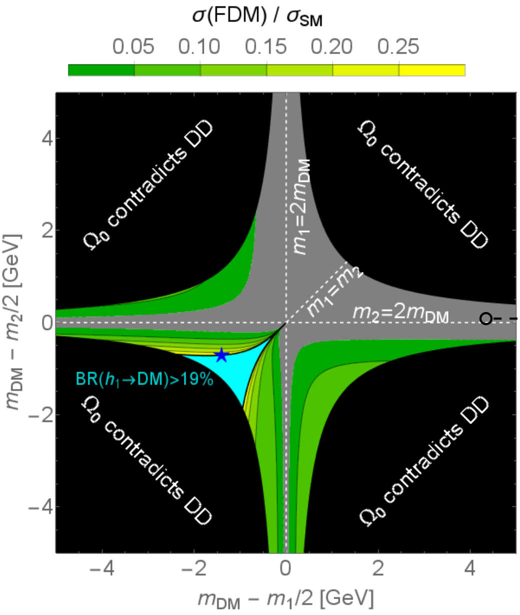
$m_2 = 118.4$ GeV
$m_{\text{DM}} = 58.5$ GeV
$\sin\alpha = 0.30$
$v_s = 561$ GeV

$\Gamma_1 = 7.4 \cdot 10^{-3}$ GeV
$\Gamma_2 = 6.4 \cdot 10^{-3}$ GeV
$\text{BR}(h_1 \to \text{DM}) = 18\%$
$\text{BR}(h_2 \to \text{DM}) = 92\%$

$\boxed{\sigma = 61 \text{ fb} \to 1.22 \cdot 10^5 \text{ events}}$

below the 95% CL limit for the ILC
1903.01629

Results: maximal cross section for FDM



parameters of ⋆

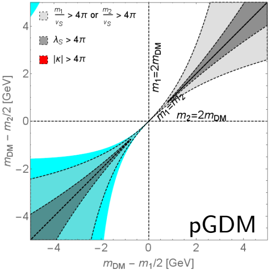
$m_2 = 123.6$ GeV
$m_{\text{DM}} = 61.1$ GeV
$\sin\alpha = 0.30$
$v_s = 76$ GeV

$\Gamma_1 = 7.4 \cdot 10^{-3}$ GeV
$\Gamma_2 = 5.9 \cdot 10^{-3}$ GeV
$\text{BR}(h_1 \to \text{DM}) = 18\%$
$\text{BR}(h_2 \to \text{DM}) = 91\%$

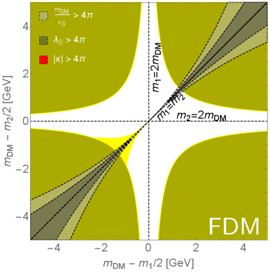
$\boxed{\sigma = 59 \text{ fb} \to 1.18 \cdot 10^5 \text{ events}}$

below the 95% CL limit for the ILC
1903.01629

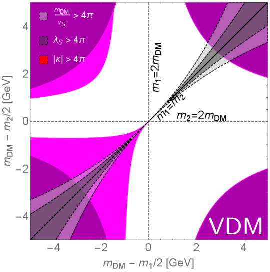
White: pGDM allowed,
Cyan: pGDM forbidden ($BR_{h_1 \to DM} > 19\%$)



White: FDM allowed, Yellow: FDM forbidden
(light: $BR_{h_1 \to DM} > 19\%$, dark: σ_{DD} too large)

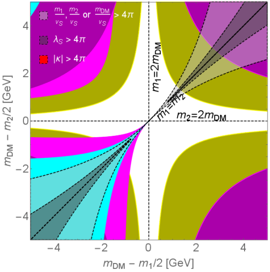


White: VDM allowed, Magenta: VDM forbidden
(light: $BR_{h_1 \to DM} > 19\%$, dark: σ_{DD} too large)



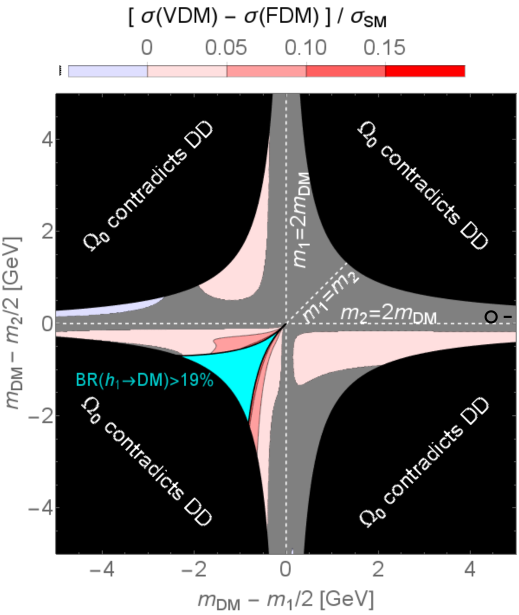
all plots combined:

Yellow: FDM forbidden, pGDM and VDM allowed
Magenta: FDM and VDM forbidden, pGDM allowed
Cyan: all forbidden, White: all allowed



$\Rightarrow$ there exist regions where

- only pGDM and VDM are allowed
- only pGDM is allowed
- nothing is allowed



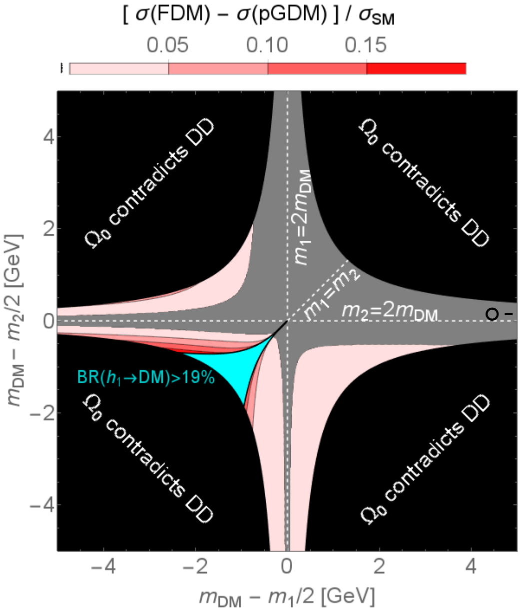
below the 95% CL limit for the ILC

1903.01629

$\Downarrow$

models are indistinguishable

Results: pseudo-Goldstone vs fermion



below the 95% CL limit for the ILC

1903.01629

$\Downarrow$

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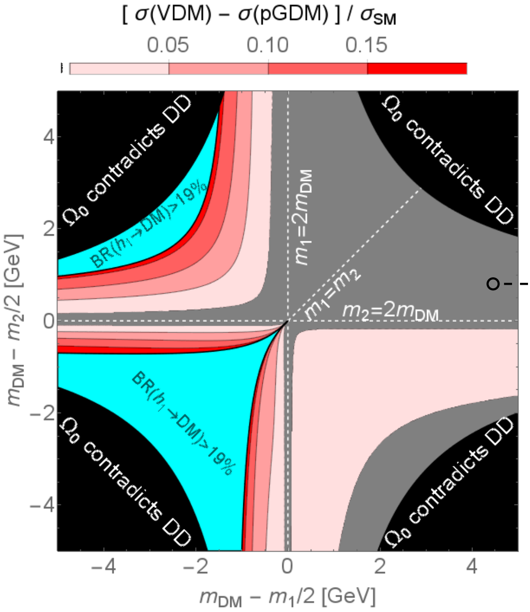
Results: pseudo-Goldstone vs vector



below the 95% CL limit for the ILC
1903.01629
⇓
models are indistinguishable

Summary

——— in general ———

- Standard Model of particle physics certainly not complete
- e^+e^- colliders especially useful in search for scalar-mediated BSM physics

——— our work ———

We introduce 3 simple models:

- pseudo-Goldstone dark matter model ($s = 0$)
- vector dark matter model ($s = 1$)
- fermion dark matter model ($s = 1/2$)

which are QFT-consistent and share common parameter space

Comparing models of various spins we conclude that...

- ...maximal cross section is similar in all 3 cases ($\sim 1.2 \cdot 10^5$ events / 20 y)
- ...signal-to-background ratio can be $\sim 10\%$
- ...allowed range of parameters is largest for pGDM
- ...there are regions where cases of different spins could be disentangled

But...

- ...if parameters far from optimal, DM of any spin hard to be detected
- ...the collider is necessary

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Thank you!

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But...

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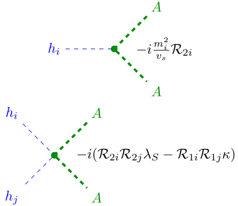
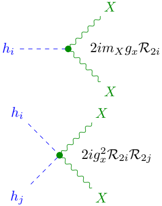
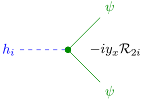
- ...the collider is necessary

a lot of

BACKUP SLIDES

Theoretical models – comparison

	PSEUDO-GOLDSTONE DM MODEL	VECTOR DM MODEL	FERMION DM MODEL
symmetry group	$\mathcal{G} = \mathcal{G}_{\text{SM}} \times \mathbb{Z}_2 \times U(1)_X$ global, softly broken	$\mathcal{G} = \mathcal{G}_{\text{SM}} \times U(1)_X$ local	$\mathcal{G} = \mathcal{G}_{\text{SM}} \times \mathbb{Z}_4$
new states ($\mathcal{G}_{\text{SM}}$-even)	complex scalar S $\quad (q = (1,1))$	gauge vector X_μ complex scalar S $\quad (q = 1)$	LH fermion χ $\quad (q = 1)$ real scalar S $\quad (q = 2)$
Lagrangian	$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{DM}} - V(H,S)$		
scalar potential	$V(H,S) = -\mu_H^2 \lvert H\rvert^2 + \lambda_H \lvert H\rvert^4$ $-\mu_S^2 \lvert S\rvert^2 + \lambda_S \lvert S\rvert^4$ $+\kappa \lvert H\rvert^2 \lvert S\rvert^2 + \mu^2 (S^2 + S^{*2})$	$V(H,S) = -\mu_H^2 \lvert H\rvert^2 + \lambda_H \lvert H\rvert^4$ $-\mu_S^2 \lvert S\rvert^2 + \lambda_S \lvert S\rvert^4$ $+\kappa \lvert H\rvert^2 \lvert S\rvert^2$	$V(H,S) = -\mu_H^2 \lvert H\rvert^2 + \lambda_H \lvert H\rvert^4$ $-\frac{\mu_S^2}{2} S^2 + \frac{\lambda_S}{4} S^4$ $+\frac{\kappa}{2} \lvert H\rvert^2 S^2$
SSB	$H \to (\pi^+, v + h + i\pi^0)^T/\sqrt{2}$ $S \to (v_s + \phi + iA)/\sqrt{2}$	$H \to (\pi^+, v + h + i\pi^0)^T/\sqrt{2}$ $S \to (v_s + \phi + i\sigma)/\sqrt{2}$	$H \to (\pi^+, v + h + i\pi^0)^T/\sqrt{2}$ $S \to v_s + \phi$
Higgs sector mixing	$\begin{pmatrix} h \\ \phi \end{pmatrix} = \mathcal{R} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$ $\quad \mathrm{tg}\, 2\alpha = \frac{\kappa v v_s}{\lambda_H v^2 - \lambda_S v_s^2}$		
dark state	$A \equiv \sqrt{2}\, \mathcal{I}m\, S \quad (m_A^2 = -4\mu^2)$	$X_\mu \quad (m_X^2 = g_x^2 v_s^2)$	$\psi \equiv \chi + \chi^c \quad (m_\psi^2 = y_x^2 v_s^2)$
dark matter interactions			

input parameters: $\underbrace{v,\, m_1,}_{\text{assumed to be SM-like}}\ v_s,\ m_2,\ \sin\alpha,\ m_{\text{DM}} \longrightarrow \kappa \equiv \frac{(m_1^2 - m_2^2)\sin 2\alpha}{2 v v_s}$ $\Rightarrow$ DM purely gravitational if $m_1 = m_2$

Pseudo-Goldstone DM model

- Gauge group: $\mathcal{G} = \underbrace{SU(3)_c \times SU(2)_L \times U(1)_Y}_{\text{Standard Model gauge group}} \times \mathbb{Z}_2 \times \overbrace{U(1)_X}^{\text{global symmetry}}$
- $U(1)_X$ softly broken to $\mathbb{Z}_2'$
- Complex scalar S introduced, $\quad \mathbb{Z}_2 : S \to S^* \,, \quad \mathbb{Z}_2' : S \to -S$
- Lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + (\partial^\mu S)^* (\partial_\mu S) - V(H,S)$$

$$V(H,S) = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \mu_S^2 |S|^2 + \lambda_S |S|^4 + \underbrace{\kappa |H|^2 |S|^2}_{\text{Higgs portal coupling}} + \overbrace{\mu^2 (S^2 + S^{*2})}^{U(1)_X\text{-breaking term}}$$

$$V(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \underbrace{\kappa|H|^2|S|^2}_{\text{Higgs portal coupling}} + \overbrace{\mu^2(S^2 + S^{*2})}^{U(1)_X\text{-breaking term}}$$

- SSB $\quad H \to (\pi^+, v + h + i\pi^0)^T/\sqrt{2} \qquad S \to (v_s + \phi + iA)/\sqrt{2}$
- We introduce h_1, h_2

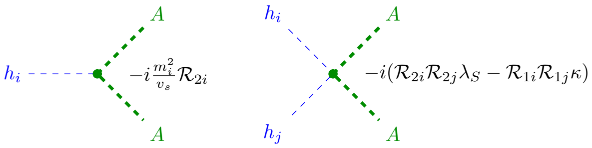
$$\mathrm{tg}\, 2\alpha = \kappa v v_s / (\lambda_H v^2 - \lambda_S v_s^2)$$

$$\begin{pmatrix} h \\ \phi \end{pmatrix} = \underbrace{\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}}_{\mathcal{R}} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

$$m_1^2 = \lambda_H v^2 (1 + \sec 2\alpha) + \lambda_S v_s^2 (1 - \sec 2\alpha)$$
$$m_2^2 = \lambda_H v^2 (1 - \sec 2\alpha) + \lambda_S v_s^2 (1 + \sec 2\alpha)$$

$$\begin{aligned} V(H,S) \longrightarrow\ & \frac{1}{2} m_1^2 h_1^2 + \frac{1}{2} m_2^2 h_2^2 - 2\mu^2 A^2 + \frac{m_1^2 \sin^2\alpha + m_2^2 \cos^2\alpha}{8 v_s^2} A^4 \\ & - \sum_{i=1,2} \frac{1}{2} \frac{m_i^2 \mathcal{R}_{2i}}{v_s} A^2 h_i + \sum_{i,j=1,2} \frac{1}{4} (\mathcal{R}_{2i}\mathcal{R}_{2j}\lambda_s - \mathcal{R}_{1i}\mathcal{R}_{1j}\kappa) A^2 h_i h_j \\ & + \text{const} + \mathcal{O}\left([h_1, h_2]^3\right) \end{aligned}$$

- $A \equiv \sqrt{2}\, \mathcal{I}m\,(S)$ is a massive dark particle, $m_A^2 = -4\mu^2$



Vector DM model

- Gauge group: $\mathcal{G} = \underbrace{SU(3)_c \times SU(2)_L \times U(1)_Y}_{\text{Standard Model gauge group}} \times \overbrace{U(1)_X}^{\text{gauge symmetry}}$
- $U(1)_X$ gauge vector X_μ and complex scalar S introduced
- Discrete $\mathbb{Z}_2$ symmetry: $X_\mu \to -X_\mu$, $S \to S^*$
  $\Rightarrow$ no kinetic mixing with SM fields
- Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{DM}} - V(H,S)$$

$$\mathcal{L}_{\text{DM}} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + (\mathcal{D}^\mu S)^*(\mathcal{D}_\mu S)$$

$$V(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \underbrace{\kappa|H|^2|S|^2}_{\text{Higgs portal coupling}}$$

$$\mathcal{L}_{\mathsf{DM}} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + (\mathcal{D}^\mu S)^*(\mathcal{D}_\mu S)$$

$$V(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \underbrace{\kappa|H|^2|S|^2}_{\text{Higgs portal coupling}}$$

- SSB $\quad H \to (\pi^+, v + h + i\pi^0)^T/\sqrt{2} \qquad S \to (v_s + \phi + i\sigma)/\sqrt{2}$
- We introduce h_1, h_2

$$\begin{pmatrix} h \\ \phi \end{pmatrix} = \underbrace{\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}}_{\mathcal{R}} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

$$\operatorname{tg} 2\alpha = \kappa v v_s/(\lambda_H v^2 - \lambda_S v_s^2)$$

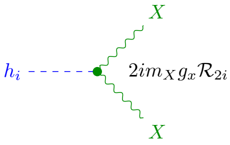
$$m_1^2 = \lambda_H v^2(1 + \sec 2\alpha) + \lambda_S v_s^2(1 - \sec 2\alpha)$$

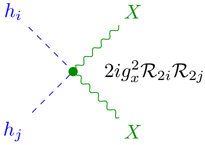
$$m_2^2 = \lambda_H v^2(1 - \sec 2\alpha) + \lambda_S v_s^2(1 + \sec 2\alpha)$$

$$\mathcal{L}_{\mathsf{DM}} \longrightarrow -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{1}{2}\partial^\mu\phi\,\partial_\mu\phi + \frac{g_x^2 v_s^2}{2}X^\mu X_\mu + \frac{g_x^2}{2}X^\mu X_\mu\phi^2 + g_x^2 v_s X^\mu X_\mu \phi$$

$$V(H,S) \longrightarrow \frac{1}{2}m_1^2 h_1^2 + \frac{1}{2}m_2^2 h_2^2 + \text{const} + \mathcal{O}\left([\text{field}]^3\right)$$

- X_μ is a massive dark particle, $m_X^2 = g_x^2 v_s^2$





Fermion DM model

- Gauge group: $\mathcal{G} = \underbrace{SU(3)_c \times SU(2)_L \times U(1)_Y}_{\text{Standard Model gauge group}} \times \mathbb{Z}_4$
- Left-handed fermion χ and real scalar S introduced,
  $\mathbb{Z}_4 : \chi \to i\chi\,,\ S \to -S$
- Lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \mathcal{L}_{\mathsf{DM}} - V(H,S)$$

$$\mathcal{L}_{\mathsf{DM}} = i\bar{\chi}\not{\partial}\chi + \frac{1}{2}\partial^\mu S\,\partial_\mu S - \frac{y_x}{2}(\bar{\chi}^c\chi + \bar{\chi}\chi^c)S$$

$$V(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \frac{\mu_S^2}{2}S^2 + \frac{\lambda_S}{4}S^4 + \underbrace{\frac{\kappa}{2}|H|^2S^2}_{\text{Higgs portal coupling}}$$

$$\mathcal{L}_{\text{DM}} = i\bar{\chi}\partial\!\!\!/\chi + \frac{1}{2}\partial^\mu S\,\partial_\mu S - \frac{y_x}{2}(\bar{\chi}^c\chi + \bar{\chi}\chi^c)S$$

$$V(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \frac{\mu_S^2}{2}S^2 + \frac{\lambda_S}{4}S^4 + \underbrace{\frac{\kappa}{2}|H|^2S^2}_{\text{Higgs portal coupling}}$$

- SSB $\quad H \to (\pi^+, v + h + i\pi^0)^T/\sqrt{2} \qquad S \to v_s + \phi$
- We introduce h_1, h_2

$$\operatorname{tg} 2\alpha = \kappa v v_s/(\lambda_H v^2 - \lambda_S v_s^2)$$

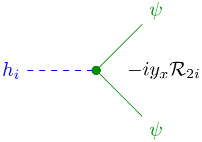
$$\begin{pmatrix} h \\ \phi \end{pmatrix} = \underbrace{\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}}_{\mathcal{R}} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

$$m_1^2 = \lambda_H v^2(1 + \sec 2\alpha) + \lambda_S v_s^2(1 - \sec 2\alpha)$$
$$m_2^2 = \lambda_H v^2(1 - \sec 2\alpha) + \lambda_S v_s^2(1 + \sec 2\alpha)$$

$$\mathcal{L}_{\text{DM}} \longrightarrow \frac{i}{2}\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}\partial^\mu\phi\,\partial_\mu\phi - \frac{y_x v_s}{2}\bar{\psi}\psi - \frac{y_x}{2}\bar{\psi}\psi\phi$$

$$V(H,S) \longrightarrow \frac{1}{2}m_1^2h_1^2 + \frac{1}{2}m_2^2h_2^2 + \text{const} + \mathcal{O}\left([\text{field}]^3\right)$$

- Majorana fermion $\psi \equiv \chi + \chi^c$ is a massive dark particle, $m_\psi^2 = y_x^2 v_s^2$



Stability conditions

$$V_{\text{pGDM}}(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|H|^2|S|^2 + \mu^2(S^2 + S^{*2})$$

$$V_{\text{VDM}}(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|H|^2|S|^2$$

$$V_{\text{FDM}}(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \frac{\mu_S^2}{2}S^2 + \frac{\lambda_S}{4}S^4 + \frac{\kappa}{2}|H|^2S^2$$

- asymptotic positivity of the scalar potential
  - $\lambda_S > 0\,,\ \ \lambda_H > 0$
  - $\kappa > -2\sqrt{\lambda_H\lambda_S}$ $\leftarrow$ always satisfied:

$$\kappa = \frac{(m_1^2 - m_2^2)\sin 2\alpha}{2vv_s} > -\sqrt{\left[\frac{(m_1^2 - m_2^2)\sin 2\alpha}{2vv_s}\right]^2 + \frac{m_1^2m_2^2}{v^2v_s^2}} = -2\sqrt{\lambda_H\lambda_S}$$

- minimum at $v, v_s > 0$ (μ^2 only for pGDM):
  - $\kappa^2 < 4\lambda_H h\lambda_S$
  - $\mu^2 < 0$
  - $2\lambda_S\mu_H^2 - \kappa(\mu_S^2 - 2\mu^2) > 0$
  - $2\lambda_H(\mu_S^2 - 2\mu^2) - \kappa\mu_H^2 > 0$

Input parameters

$$V_{\mathrm{pGDM}}(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|H|^2|S|^2 + \mu^2(S^2 + S^{*2})$$

$$V_{\mathrm{VDM}}(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|H|^2|S|^2$$

$$V_{\mathrm{FDM}}(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \frac{\mu_S^2}{2}S^2 + \frac{\lambda_S}{4}S^4 + \frac{\kappa}{2}|H|^2S^2$$

- Input parameters: $\underbrace{v,\, m_1,}_{\text{assumed to be SM-like}}\ v_s,\ m_2,\ \sin\alpha,\ m_{\mathrm{DM}}$
- Other parameters of the models in terms of the input parameters:

$$\mu^2 = -\frac{1}{4}m_{\mathrm{DM}}^2\ (\mathrm{pGDM}) \qquad g_x = \frac{m_{\mathrm{DM}}}{v_s}\ (\mathrm{VDM}) \qquad y_x = \frac{m_{\mathrm{DM}}}{v_s}\ (\mathrm{FDM})$$

$$\lambda_H = \frac{m_1^2\cos^2\alpha + m_2^2\sin^2\alpha}{2v^2} \qquad \lambda_S = \frac{m_1^2\sin^2\alpha + m_2^2\cos^2\alpha}{2v_s^2}$$

$$\mu_H^2 = \frac{1}{2}m_1^2\cos^2\alpha + \frac{1}{2}m_2^2\sin^2\alpha + \frac{1}{4}\frac{v_s}{v}(m_1^2 - m_2^2)\sin 2\alpha$$

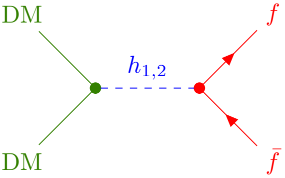
$$\mu_S^2 = \frac{1}{2}m_1^2\sin^2\alpha + \frac{1}{2}m_2^2\cos^2\alpha + \frac{1}{4}\frac{v}{v_s}(m_1^2 - m_2^2)\sin 2\alpha$$

$$\kappa = \frac{(m_1^2 - m_2^2)\sin 2\alpha}{2vv_s}$$

- $m_2 = m_1 \Rightarrow \kappa = 0 \Rightarrow$ no Higgs portal $\Rightarrow$ DM completely decoupled

Relic density constraint on indirect detection Steigman et al.

$$\langle\sigma v\rangle^{\text{ID}}_{f\bar f} = \frac{m_{\text{DM}} m_f^2}{\pi}\,\frac{\kappa^2\cdot(m^2_{\text{DM}}-m_f^2)^{3/2}}{D_1 D_2}\times \qquad \longleftarrow \quad D_i = (4m^2_{\text{DM}}-m_i)^2+m_i^2\Gamma_i^2$$

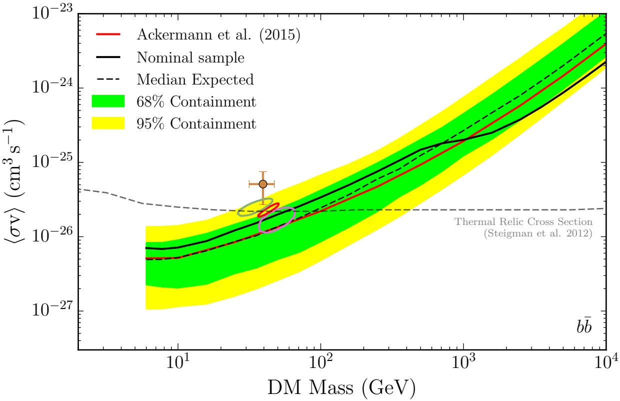


$$\times\begin{cases}12 & \text{pGDM}\\ 1 & \text{VDM}\\ \frac{9}{4}\left(\frac{m_{\text{DM}}}{T}\right)^{-1} & \text{FDM}\end{cases} + \text{[higher orders in } (m_{\text{DM}}/T)^{-1}]$$

m_f^2 factor $\Rightarrow$ $b\bar b$ contribution dominates.

- If $\langle\sigma v\rangle^{\text{ID}} = \sigma_0(m_{\text{DM}}/T)^{-n}$ then $h^2\Omega_0^{\text{DM}} \sim (n+1)(m_{\text{DM}}/T_f)^{n+1}/\sigma_0$
- Correct $h^2\Omega_0^{\text{DM}} \longleftrightarrow \langle\sigma v\rangle\Big|_{\text{now}} = (T_0/T_f)^n\cdot\underbrace{(n+1)\cdot 1.9\cdot 10^{-9}\ \text{GeV}^{-2}}_{\langle\sigma v\rangle|_{\text{freeze out}}}$
- $T_f \sim m_{\text{DM}}/25$, hence

$$\kappa^2 = 3.5\cdot 10^{-10}\ \text{GeV}^{-4}\,\frac{D_1 D_2}{m_{\text{DM}}(m^2_{\text{DM}}-m_b^2)^{3/2}}\cdot\begin{cases}1/12 & \text{pGDM}\\ 1 & \text{VDM}\\ \frac{8m_\psi}{9T_f}\approx 22 & \text{FDM}\end{cases}$$



value of $\langle \sigma v \rangle\big|_{\text{freeze out}}$ corresponding to correct Ω_0^{DM}

$$\langle \sigma v \rangle\big|_{\text{now}} = (T_0/T_f)^n \cdot \langle \sigma v \rangle\big|_{\text{freeze out}}$$

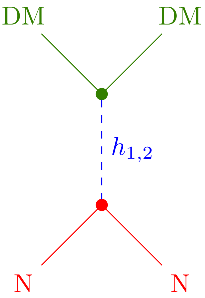
- pGDM, VDM: $n = 0 \;\Rightarrow\; m_{\text{DM}} \gtrsim 30$ GeV
- FDM: $n = 1 \;\Rightarrow\; \langle \sigma v \rangle\big|_{\text{now}}$ orders of magnitude lower than the limit

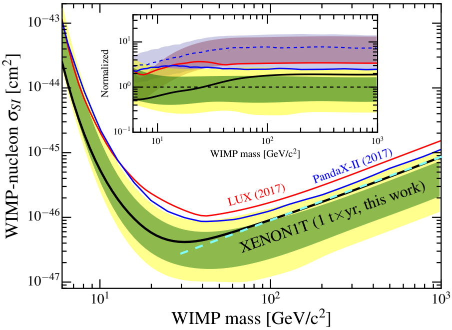
$$\sigma_{\mathsf{SD}} = 0 \qquad \text{(no axial couplings in our models)}$$

$$\sigma_{\mathsf{SI}} \sim \kappa^2 \cdot \frac{m_{\mathsf{DM}}^2}{m_2^4} \begin{cases} \left[\frac{\mathcal{A}}{64\pi^2 v v_s^2}\right]^2 & \text{(pGDM)} \\ 1 & \text{(VDM), (FDM)} \end{cases}$$

$\mathcal{A}$ – combination of 1-loop integrals

(the tree level vanishes for pGDM)





$$\frac{\sigma_{\mathsf{SI}}}{1\ \mathsf{cm}^2} \lesssim \frac{m_{\mathsf{DM}}}{1\ \mathsf{GeV}} \cdot 10^{-48.05}$$

$$\Downarrow$$

$$\boxed{\kappa^2 < \frac{m_2^4}{m_{\mathsf{DM}}} \cdot 2.5 \cdot 10^{-11}\ \mathsf{GeV}^{-3} \times \times \begin{cases} \left[\frac{\mathcal{A}}{64\pi^2 v v_s^2}\right]^{-2} & \text{(pGDM)} \\ 1 & \text{(VDM), (FDM)} \end{cases}}$$

$\sigma_{\text{SD}} = 0$ (no axial couplings in our models)

$$\sigma_{\text{SI}} \sim \kappa^2 \cdot \frac{m_{\text{DM}}^2}{m_2^4} \begin{cases} \left[\frac{\mathcal{A}}{64\pi^2 v v_s^2}\right]^2 & \text{(pGDM)} \\ 1 & \text{(VDM), (FDM)} \end{cases}$$

$\mathcal{A}$ – combination of 1-loop integrals

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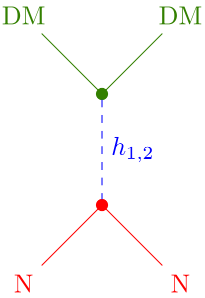
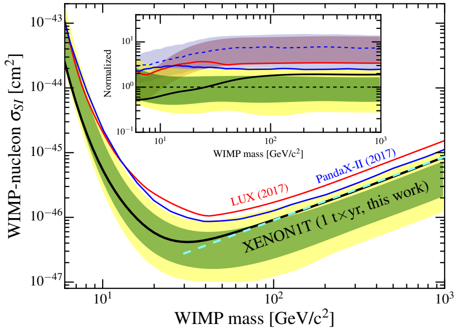




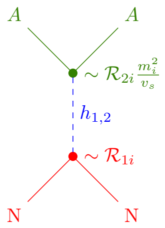
$$\frac{\sigma_{\text{SI}}}{1\ \text{cm}^2} \lesssim \frac{m_{\text{DM}}}{1\ \text{GeV}} \cdot 10^{-48.05}$$

$$\Downarrow$$

$$\boxed{\kappa^2 < \frac{m_2^4}{m_{\text{DM}}} \cdot 2.5 \cdot 10^{-11}\ \text{GeV}^{-3} \times \times \begin{cases} \cancel{\left[\frac{\mathcal{A}}{64\pi^2 v v_s^2}\right]^{-2} \quad \text{(pGDM)}} \\ 1 \quad \text{(VDM), (FDM)} \end{cases}}$$

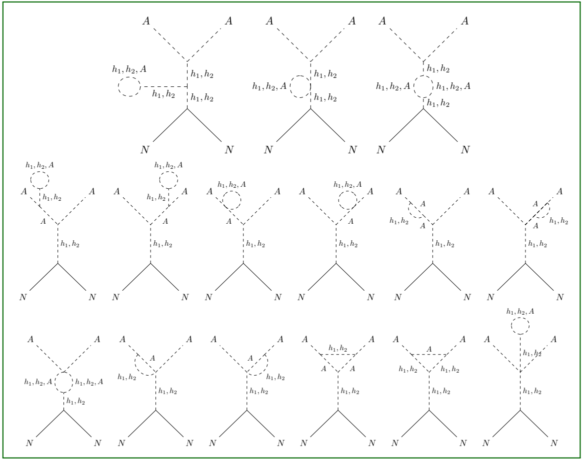
huge!

Direct detection – the pseudo-Goldstone case Azevedo et al.



the tree level:

$$\sigma \sim \frac{(\cos\alpha\sin\alpha)^2}{v_s^2}\left|\frac{m_1^2}{Q^2-m_1^2}-\frac{m_2^2}{Q^2-m_2^2}\right|^2 \xrightarrow{Q^2\to 0} 0$$



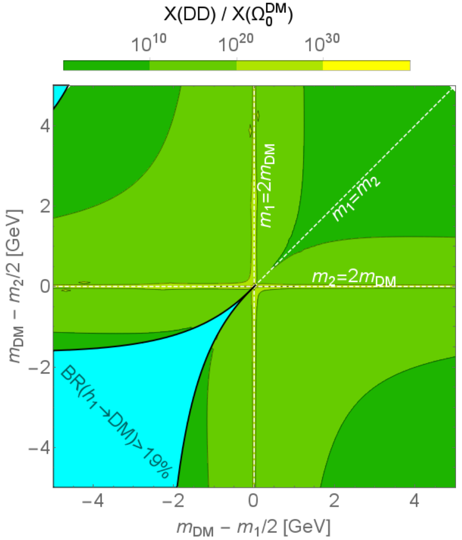
the 1-loop level:

$$\sigma \sim \mathcal{A}^2 \longleftarrow$$

DD vs Ω_0^{DM} limit for pGDM

huge!

$$\kappa^2(\mathrm{DD}) \equiv \frac{m_2^4}{m_{\mathrm{DM}}} \cdot 2.5 \cdot 10^{-11}\ \mathrm{GeV}^{-3} \begin{cases} \left[\frac{\mathcal{A}}{64\pi^2 v v_s^2}\right]^{-2} & \text{(pGDM)} \\ 1 & \text{(VDM), (FDM)} \end{cases}$$



IceCube and DD limits Wikström, Edsjö

- DM annihilation rate in the Sun: Γ_A
- Capture rate: C_C
- Change of DM's amount N in the Sun

$$\dot{N} = C_C - C_A N^2$$

where $C_A \equiv 2 \cdot \Gamma_A / N^2 = \text{const}$

- Solution

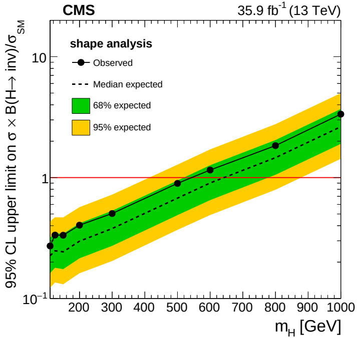
$$N(t) = \sqrt{C_C/C_A} \cdot \tanh(t\sqrt{C_C C_A})$$

- For large t

$$N(t) \to \sqrt{C_C/C_A}$$

$$\Rightarrow \quad \boxed{\Gamma_A = \tfrac{1}{2} C_C}$$

- C_C expressible in terms of DM-nucleon cross section
- Γ_A calculable from muon flux measurements



$$\sigma \approx \sigma_1 \cdot \mathbb{1}_{2\,m_{\rm DM}<m_1<\sqrt{s}-m_Z} + \sigma_2 \cdot \mathbb{1}_{2\,m_{\rm DM}<m_2<\sqrt{s}-m_Z}$$

$$\sigma_1 \equiv \sigma_{\rm SM}(m_1)\cdot\cos^2\alpha\cdot{\rm BR}(h_1 \to {\rm DM})$$

$$\sigma_2 \equiv \sigma_{\rm SM}(m_2)\cdot\sin^2\alpha\cdot{\rm BR}(h_2 \to {\rm DM})$$

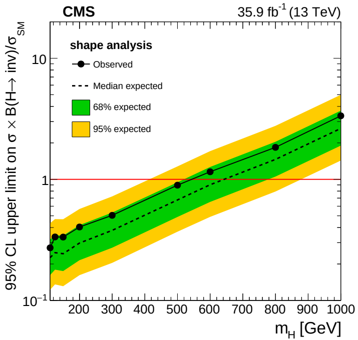
Conditions:

$$(1) \qquad \sigma_1 < 0.19\,\sigma_{\rm SM}(m_1)$$

$$(2) \qquad \log\left[\frac{\sigma_2}{\sigma_{\rm SM}(m_2)}\right] < 0.0011\cdot\frac{m_2}{1\ {\rm GeV}} - 0.63 \qquad (\sqrt{s} = 13\ {\rm TeV})$$

For $\sin\alpha < 0.3$ condition (2) always satisfied:

$$\left.\frac{\sigma_2}{\sigma_{\rm SM}(m_2)}\right|_{\sqrt{s}=13\ {\rm TeV}} = \sin^2\alpha\cdot{\rm BR}(h_2 \to {\rm DM}) < 10^{-1} < 10^{\,0.0011\cdot\frac{m_2}{1\ {\rm GeV}}-0.63}$$



$$\sigma \approx \sigma_1 \cdot \mathbb{1}_{2\,m_{\mathsf{DM}}<m_1<\sqrt{s}-m_Z} + \sigma_2 \cdot \mathbb{1}_{2\,m_{\mathsf{DM}}<m_2<\sqrt{s}-m_Z}$$

$$\sigma_1 \equiv \sigma_{\mathsf{SM}}(m_1) \cdot \cos^2\alpha \cdot \mathsf{BR}(h_1 \to \mathsf{DM})$$

$$\sigma_2 \equiv \sigma_{\mathsf{SM}}(m_2) \cdot \sin^2\alpha \cdot \mathsf{BR}(h_2 \to \mathsf{DM})$$

Conditions:

(1) $$\boxed{\sigma_1 < 0.19\,\sigma_{\mathsf{SM}}(m_1)}$$

(2) $$\log\left[\frac{\sigma_2}{\sigma_{\mathsf{SM}}(m_2)}\right] < 0.0011 \cdot \frac{m_2}{1\ \mathsf{GeV}} - 0.63 \qquad (\sqrt{s} = 13\ \mathsf{TeV})$$

For $\sin\alpha < 0.3$ condition (2) always satisfied:

$$\left.\frac{\sigma_2}{\sigma_{\mathsf{SM}}(m_2)}\right|_{\sqrt{s}=13\ \mathsf{TeV}} = \sin^2\alpha \cdot \mathsf{BR}(h_2 \to \mathsf{DM}) < 10^{-1} < 10^{\,0.0011\cdot\frac{m_2}{1\ \mathsf{GeV}} - 0.63}$$

Considered process: degenerated case

$$\frac{d\sigma}{dm^2_{\text{rec}}} = \frac{d\sigma}{dQ^2} = \frac{\sigma_{\text{SM}}(Q^2)v^2}{32\pi^2} \frac{Q^4 \cdot \mathcal{X}}{\left[(Q^2-m_1^2)^2+(m_1\Gamma_1)^2\right]\left[(Q^2-m_2^2)^2+(m_2\Gamma_2)^2\right]} \times$$

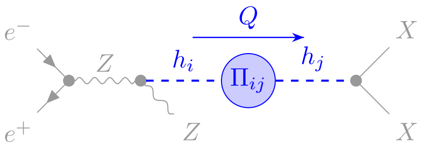
$$\times \sqrt{1-4\frac{m^2_{\text{DM}}}{Q^2}} \cdot \begin{cases} 1 & \text{(pGDM)} \\ 1-4\frac{m^2_{\text{DM}}}{Q^2}+12\left(\frac{m^2_{\text{DM}}}{Q^2}\right)^2 & \text{(VDM)} \\ 2\frac{m^2_{\text{DM}}}{Q^2}\left(1-4\frac{m^2_{\text{DM}}}{Q^2}\right) & \text{(FDM)} \end{cases}$$

$$\left.\begin{aligned} \Gamma_1 &\xrightarrow{m_2\to m_1} \cos^2\alpha\,\Gamma_{h\to\text{SM}} + \sin^2\alpha\,\Gamma_{h\to\text{DM}} \\ \Gamma_2 &\xrightarrow{m_2\to m_1} \sin^2\alpha\,\Gamma_{h\to\text{SM}} + \cos^2\alpha\,\Gamma_{h\to\text{DM}} \end{aligned}\right\}$$

$$\mathcal{X} \equiv (\cos\alpha\sin\alpha)^2 \frac{(m_1^2-m_2^2)^2+(m_1\Gamma_1-m_2\Gamma_2)^2}{v^2 v_s^2}\Bigg|_{m_1=m_2} \neq 0$$

but $\kappa = \frac{(m_1^2-m_2^2)\sin 2\alpha}{2vv_s} \xrightarrow{m_2\to m_1} 0 \quad !$

Considered process: degenerated case. The propagator



$$\Pi_{ij} \equiv \Pi_{ij}(Q^2) \equiv i \cdot \mathcal{I}m\,[\text{self energy}]$$
$$\Pi_{ii}(m_i^2) = i m_i \Gamma_i$$

the matrix element:

$$\begin{aligned}\mathcal{M} &= \mathcal{M}_{e^+e^-\to Zh_i}(Q^2)\cdot \Delta_{ij}(Q^2)\cdot \mathcal{M}_{h_j\to XX}(Q^2) = \\ &= \mathcal{M}_{e^+e^-\to Zh}(Q^2)\cdot \underbrace{\mathcal{R}_{1i}\cdot \Delta_{ij}(Q^2)\cdot \mathcal{R}_{2j}}_{\widehat{\Delta}(Q^2)}\cdot \mathcal{M}_{h\to XX}(Q^2)\ ,\end{aligned}$$

the propagator:

$$\begin{aligned}\widehat{\Delta}(Q^2) &= \mathcal{R}_{1i}\mathcal{R}_{2j}\cdot \frac{1}{\det D}\overbrace{\begin{bmatrix} Q^2 - m_2^2 + \Pi_{22} & -\Pi_{12} \\ -\Pi_{21} & s - m_1^2 + \Pi_{11}\end{bmatrix}}^{D}_{ij} = \\ &= \sin\alpha\cos\alpha\cdot\frac{(m_1^2 - m_2^2) - [(\Pi_{11} - \Pi_{22}) - (\operatorname{tg}\alpha\cdot\Pi_{12} - \operatorname{ctg}\alpha\cdot\Pi_{21})]}{(Q^2 - m_1^2 + \Pi_{11})(Q^2 - m_2^2 + \Pi_{22}) - \Pi_{12}\Pi_{21}}\end{aligned}$$

$$\widehat{\Delta}(Q^2) = \sin\alpha\cos\alpha \cdot \frac{(m_1^2 - m_2^2) - [(\Pi_{11} - \Pi_{22}) - (\operatorname{tg}\alpha \cdot \Pi_{12} - \operatorname{ctg}\alpha \cdot \Pi_{21})]}{(Q^2 - m_1^2 + \Pi_{11})(Q^2 - m_2^2 + \Pi_{22}) - \Pi_{12}\Pi_{21}}$$

- If $|m_1 - m_2| \gg \Gamma_1, \Gamma_2$, then $\longleftarrow$ in practice, only this case relevant

$$\begin{aligned}\widehat{\Delta}(Q^2) &\approx \sin\alpha\cos\alpha \cdot \frac{m_1^2 - m_2^2}{(Q^2 - m_1^2 + \Pi_{11})(Q^2 - m_2^2 + \Pi_{22})} \\ &\approx \sin\alpha\cos\alpha \cdot \left[\frac{1}{Q^2 - m_1^2 + i m_1 \Gamma_1} - \frac{1}{Q^2 - m_2^2 + i m_2 \Gamma_2}\right] \\ &\equiv \widehat{\Delta}^{(\mathsf{BW})}(Q^2)\end{aligned}$$

- If $m_1 \sim m_2$, an explicit calculation shows that

$$[(\Pi_{11} - \Pi_{22}) - (\operatorname{tg}\alpha \cdot \Pi_{12} - \operatorname{ctg}\alpha \cdot \Pi_{21})]\Big|_{m_1 = m_2} = 0$$

$$\Rightarrow \quad \widehat{\Delta}(Q^2)\Big|_{m_1 = m_2} = 0$$

The h_1's and h_2's decay widths

$$\Gamma_{h_i \to ab} \equiv \frac{\Pi^{ab}_{ii}(m_i^2)}{i m_i}$$

$$\Downarrow$$

- $\Gamma_{h_i \to \text{DM}} = \frac{\mathcal{R}_{2i}^2}{v_s^2} \frac{m_i^3}{32\pi} \sqrt{1 - \frac{4 m_{\text{DM}}^2}{m_i^2}} \begin{cases} 1 & \text{(pGDM)} \\ 1 - 4\frac{m_{\text{DM}}^2}{m_i^2} + 12\left(\frac{m_{\text{DM}}^2}{m_i^2}\right)^2 & \text{(VDM)} \\ 2\frac{m_{\text{DM}}^2}{m_i^2}\left(1 - 4\frac{m_{\text{DM}}^2}{m_i^2}\right) & \text{(FDM)} \end{cases}$
- $\Gamma_{h_i \to \text{SM}} = \mathcal{R}_{1i}^2 \cdot \gamma(m_i)$ (γ – decay width of SM Higgs particle of given mass)
- $\Gamma_{h_1 \to h_2 h_2} = \sin^2\alpha \, \cos^2\alpha \, (m_1^2 + 2m_2^2)^2 \left(\frac{\cos\alpha}{v_s} + \frac{\sin\alpha}{v}\right)^2 \frac{\sqrt{m_1^2 - 4m_2^2}}{32\pi m_1^2} \approx$
  $\approx \frac{\sin^2\alpha \, \cos^4\alpha}{v_s^2}(m_1^2 + 2m_2^2)^2 \frac{\sqrt{m_1^2 - 4m_2^2}}{32\pi m_1^2}$
- $\Gamma_{h_2 \to h_1 h_1} = \sin^2\alpha \, \cos^2\alpha \, (2m_1^2 + m_2^2)^2 \left(\frac{\sin\alpha}{v_s} - \frac{\cos\alpha}{v}\right)^2 \frac{\sqrt{m_2^2 - 4m_1^2}}{32\pi m_2^2} \approx$
  $\approx \frac{\sin^2\alpha \, \cos^4\alpha}{v^2}(2m_1^2 + m_2^2)^2 \frac{\sqrt{m_2^2 - 4m_1^2}}{32\pi m_2^2}$