A New Method of Extracting Strong Coupling from Energy Correlators

Zhen Xu

with Wen Chen, Jun Gao, Yibei Li, Xiaoyuan Zhang, HuaXing Zhu,
in preparation

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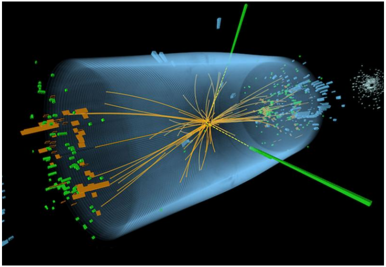
Outline

- Introduction to energy correlators and applications to α_s-determination
- Higher point energy correlators and projected energy correlators
- Resummation of projected 3-point energy correlator in collinear limit
- α_s-determination from ratios of energy correlators and jet substructure

Precision Collider Physics and Event Shapes



Event shape variables for testing QCD and for understanding its dynamics:

Thrust :

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

Energy-energy correlator :

$$\mathrm{EEC}(\chi) = \frac{\sum_{i,j} E_i E_j \delta(\cos\chi - \cos\chi_{ij})}{\left(\sum_i E_i\right)^2}$$

Other important examples include jet-broadening, spherocity and C-parameter, etc.

Introduction to Energy-Energy Correlator

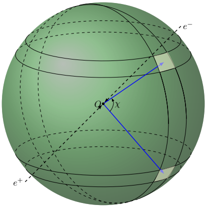
The energy-energy correlator in terms of $z = (1 - \cos\theta)/2$,
[Basham, Brown, Ellis, Love, PRL. 41, 1585 (1978)]

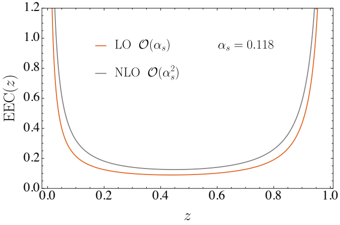
$$\mathrm{EEC}(z) \equiv \frac{1}{\sigma_0}\frac{d\sigma}{dz} = \sum_{i,j}\int d\sigma\, \frac{E_i E_j}{Q^2}\,\delta\left(z - \frac{1-\cos\theta_{ij}}{2}\right)$$

Born-level gives only collinear contribution $\delta(z)$ and back-to-back contribution $\delta(1-z)$, with the long-known one-loop correction

$$\frac{1}{\sigma_0}\frac{d\sigma^{e^+e^-}(z)}{dz} = \frac{\alpha_s(\mu)}{4\pi}C_F\left\{\frac{13}{24}\delta(z) + (-2\zeta_2 - 4)\delta(1-z) + \frac{3}{2}\frac{1}{[z]_+} - 2\left[\frac{\ln(1-z)}{1-z}\right]_+ - \frac{3}{[1-z]_+} + \frac{1}{2z^5}\left[-9z^4 - 6z^3 - 42z^2 + 36z + 4(-z^4 - z^3 + 3z^2 - 15z + 9)\ln(1-z)\right]\right\}$$

First event shape variable calculated analytically at NLO. See also Reynaldi's talk.
[L. Dixon, M.X. Luo, V. Shtabovenko, T.Z. Yang, H.X. Zhu, 1801.03219]





Resummation of Energy-Energy Correlator in Collinear Limit

The energy-energy correlator in terms of $z = (1 - \cos\theta)/2$,

$$\text{EEC}(z) \equiv \frac{1}{\sigma_0}\frac{d\sigma}{dz} = \sum_{i,j}\int d\sigma\, \frac{E_i E_j}{Q^2}\,\delta\left(z - \frac{1-\cos\theta_{ij}}{2}\right)$$

In the collinear limit, interesting for jet-substructure, dominated by large logarithms $\ln z$.

$$\begin{aligned}\frac{1}{\sigma_0}\frac{d\sigma^{e^+e^-}(z)}{dz} &= \frac{2a_s}{z} + \frac{a_s^2}{z}\left(-\frac{173}{15}\ln z + \cdots\right)\\ &\quad + \frac{a_s^3}{z}\left[\frac{20317}{450}\ln^2 z + \ln z\left(\frac{3704}{81}\zeta_3 - \frac{343252}{1215}\zeta_2 - \frac{686702711}{1093500}\right) + \cdots\right]\\ &= \frac{2a_s}{z} + \frac{a_s^2}{z}(-11.5333\ln z + 81.4809) + \frac{a_s^3}{z}\left(45.1489\ln^2 z - 1037.73\ln z + 2871.36\right)\end{aligned}$$

The logarithmic series to all-orders,

$$\frac{1}{\sigma_0}\frac{d\sigma}{dz} = \sum_{L=1}^{\infty}\sum_{j=-1}^{L-1} a_s^L c_{L,j}\,\mathcal{L}^j(z) + \ldots \qquad \text{where} \quad \mathcal{L}^j(z) = \left[\ln^j z/z\right]_+$$

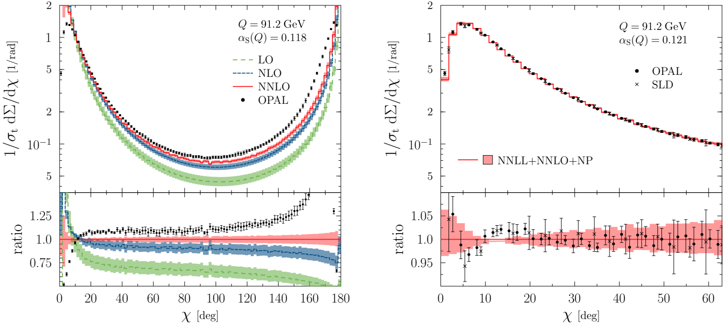
$$= \underbrace{\sum_{L=1}^{\infty} a_s^L\, c_L \left[\ln^{L-1} z/z\right]_+}_{\text{LL}} + \underbrace{\sum_{L=2}^{\infty} a_s^L\, d_L \left[\ln^{L-2} z/z\right]_+}_{\text{NLL}} + \underbrace{\sum_{L=3}^{\infty} a_s^L f_L \left[\ln^{L-3} z/z\right]_+}_{\text{NNLL}} + \cdots$$

All-order resummation achieved through to NNLL via RG evolution.

[L. Dixon, I. Moult, H.X. Zhu, arXiv:1905.01310]

Strong Coupling Determination from Energy Correlators

· EEC at e^+e^- colliders, in back-to-back limit ($\chi \equiv \pi - \theta_{ij}$ here)



[Z. Tulipant, A. Kardos, G. Somogyi, arXiv:1708.04093]

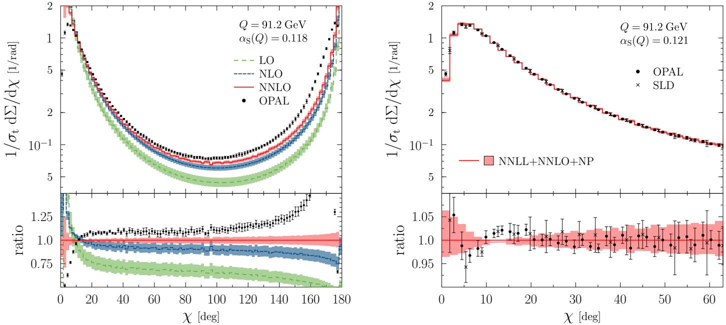
- Resummation is required in both collinear and back-to-back limits.
- Non-perturbative corrections $\sim \Lambda_{\rm QCD}$ are sizable.

  non-perturbative correction in the back-to-back limit parametrized with

$$S_{\rm NP} = e^{-\frac{1}{2}a_1 b^2}(1 - 2a_2 b)$$

Strong Coupling Determination from Energy Correlators

- EEC at e^+e^- colliders, in back-to-back limit ($\chi \equiv \pi - \theta_{ij}$ here)



[Z. Tulipant, A. Kardos, G. Somogyi, arXiv:1708.04093]

Result from performing three-parameter fits with data,

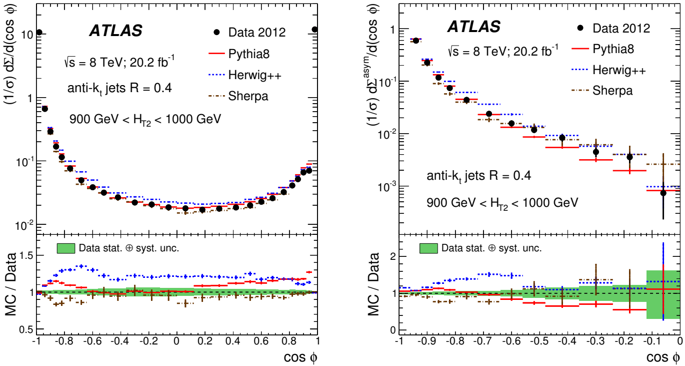
$$\alpha_s(M_Z) = 0.121^{+0.001}_{-0.003}, \quad a_1 = 2.47^{+0.48}_{-2.38}\ \mathrm{GeV}^2, \quad a_2 = 0.31^{+0.27}_{-0.05}\ \mathrm{GeV}$$

- Accurate determination of $\alpha_s(M_Z)$ at the percent level.
- Both the perturbative accuracy, fixed-order and resummed, and the knowledge of non-perturbative corrections are important.

Strong Coupling Determination from Energy Correlators

- Transverse Energy-Energy Correlator [A.Ali, E.Pietarinen, W.J.Stirling, PLB, 447 (1984)]

$$\mathrm{TEEC} = \sum_{a,b} \int d\sigma_{pp\to a+b+X} \frac{2E_{T,a}E_{T,b}}{|\sum_i E_{T,i}|^2} \delta(\cos\phi_{ab} - \cos\phi),$$

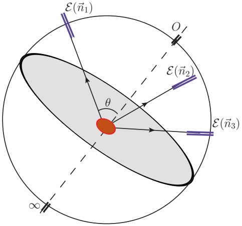


[ATLAS Collaboration, arXiv:1707.02562]

$$\alpha_s(M_Z) = 0.1162 \pm 0.0011\ (\mathrm{exp.})\ {}^{+0.0076}_{-0.0061}\ (\mathrm{scale}) \pm 0.0018\ (\mathrm{PDF}) \pm 0.0003\ (\mathrm{NP})$$

$$\alpha_s(M_Z) = 0.1196 \pm 0.0013\ (\mathrm{exp.})\ {}^{+0.0061}_{-0.0013}\ (\mathrm{scale}) \pm 0.0017\ (\mathrm{PDF}) \pm 0.0004\ (\mathrm{NP})$$

Generalization to Higher Point Energy Correlators



Three-point energy correlator (EEEC)

$$\frac{1}{\sigma_{\text{tot}}}\frac{d^3\sigma}{dx_1 dx_2 dx_3} = \sum_{ijk}\int d\sigma \frac{E_i E_j E_k}{Q^3} \times \delta\left(x_1 - \frac{1-\cos\theta_{jk}}{2}\right)\delta\left(x_2 - \frac{1-\cos\theta_{ik}}{2}\right)\delta\left(x_3 - \frac{1-\cos\theta_{ij}}{2}\right)$$

captures non-trivial shape dependence of the events or jets.

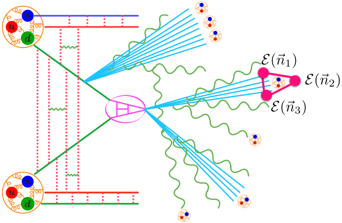
- defined and calculated in the collinear limit in [H. Chen, M.X. Luo, I. Moult, T.Z. Yang, X.Y. Zhang, H.X. Zhu, arXiv:1912.11050];
- full analytic calculation at LO in $\mathcal{N} = 4$ SYM [K. Yan, X.Y. Zhang, 2203.04349] , and in QCD [T.Z. Yang, X.Y. Zhang, 2208.01051].

Show interesting mathematical structure and benefit the understanding of trijet events as well as jet-substructure.

Generalization to Higher Point Energy Correlators

Projected N-point energy correlator (ENC) [H. Chen, I. Moult, X.Y. Zhang, H.X. Zhu, 2004.11381]

$$\frac{d\sigma^{[N]}}{dx_L} = \sum_n \sum_{1\le i_1,\cdots i_N\le n} \int d\sigma \frac{\prod_{a=1}^N E_{i_a}}{Q^N} \delta(x_L - \max\{x_{i_1,i_2}, x_{i_1,i_3}, \cdots x_{i_{N-1},i_N}\})$$



- Nice representation with energy flow operators.

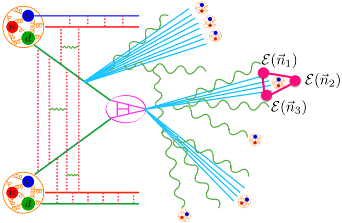
$$\frac{d\sigma^{[N]}}{dx_L} = \int d\Omega_{\vec{n}_1} \int d\Omega_{\vec{n}_2} \delta\Big(x_L - \frac{1-\vec{n}_1\cdot\vec{n}_2}{2}\Big) \prod_{k=3}^{N} \int d\Omega_{\vec{n}_k}$$

$$\times\, \Theta(\{\vec{n}\}) \int d^4x\, \frac{e^{iq\cdot x}}{Q^N} \langle 0|\mathcal{O}^\dagger(x)\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\ldots\mathcal{E}(\vec{n}_N)\mathcal{O}(0)|0\rangle$$

Generalization to Higher Point Energy Correlators

Projected N-point energy correlator (ENC) [H. Chen, I. Moult, X.Y. Zhang, H.X. Zhu, 2004.11381]

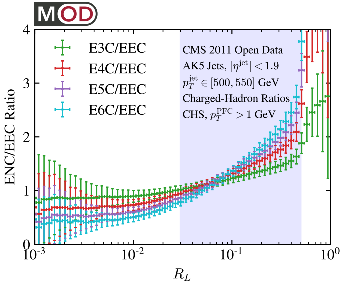
$$\frac{d\sigma^{[N]}}{dx_L} = \sum_n \sum_{1\le i_1,\cdots i_N\le n} \int d\sigma \frac{\prod_{a=1}^N E_{i_a}}{Q^N} \delta(x_L - \max\{x_{i_1,i_2}, x_{i_1,i_3}, \cdots x_{i_{N-1},i_N}\})$$



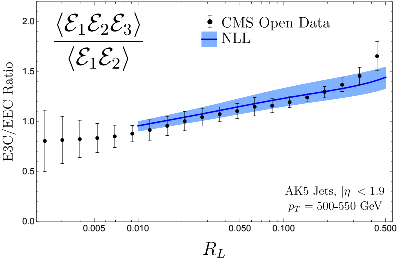
- Nice representation with energy flow operators.
- Ratio between ENC and EEC more robust to non-perturbative effects.
- Enjoy simple factorization property, facilitating the resummation.

Generalization to Higher Point Energy Correlators

Projected N-point energy correlator measured within jets
[H. Chen, I. Moult, X.Y. Zhang, H.X. Zhu, 2004.11381]



[P.T. Komiske, I. Moult, J. Thaler, H.X. Zhu, 2201.07800]



[K. Lee, B. Mecaj, I. Moult, 2205.03414]

- Distinguished for understanding the jet-substructure.
- Connects the formal theoretic developments to experimental observations.

Factorization Theorem and Resummation

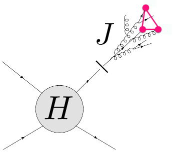
Factorization of projected N-point correlators in the collinear limit $x_L \to 0$,
[H. Chen, I. Moult, X.Y. Zhang, H.X. Zhu, 2004.11381]

$$\Sigma^{[N]}\left(x_L, \ln\frac{Q^2}{\mu^2}, \mu\right) = \int_0^{x_L} dx'_L \, \frac{d\sigma^{[N]}}{dz}\left(x'_L, \ln\frac{Q^2}{\mu^2}, \mu\right) \overset{x_L \to 0}{=} \int_0^1 dx \, x^N \vec{J}^{[N]}\left(\ln\frac{x_L x^2 Q^2}{\mu^2}, \mu\right) \cdot \vec{H}\left(x, \ln\frac{Q^2}{\mu^2}, \mu\right)$$

Renormalization Group Equations for the Hard and Jet functions

$$\frac{d\vec{H}\left(x, \ln\frac{Q^2}{\mu^2}, \mu\right)}{d\ln\mu^2} = -\int_x^1 \frac{dy}{y} \widehat{P}_T(y) \cdot \vec{H}\left(\frac{x}{y}, \ln\frac{Q^2}{\mu^2}\right)$$

$$\frac{d\vec{J}^{[N]}\left(\ln\frac{x_L Q^2}{\mu^2}, \mu\right)}{d\ln\mu^2} = \int_0^1 dy \, y^N \vec{J}^{[N]}\left(\ln\frac{x_L y^2 Q^2}{\mu^2}\right) \cdot \widehat{P}_T(y)$$



Resummation achieved by evolving the Jet function and Hard function to a common scale.

Towards NNLL for ENC in Collinear Limit:

- Jet function and hard function through to 2-loop needed as boundary conditions.
- Time-like splitting function and β-function to 3-loop.

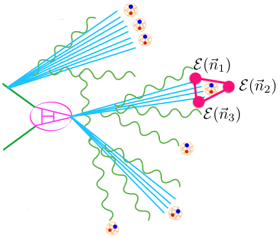
Calculation of the Two-loop Jet Function for E3C

The gluon jet function for EEEC, then project to the largest angle,

$$J_g(x_1, x_2, x_3, Q, \mu^2) = \int \frac{dl^+}{2\pi} \frac{1}{2(N_C^2-1)} \mathrm{Tr} \int d^4x e^{il\cdot x} \langle 0 | \mathcal{B}^{a,\mu}_{n,\perp}(x) \boxed{\widehat{\mathcal{M}}_{\mathrm{EEEC}}} \delta(Q + \bar{n}\cdot\mathcal{P}) \delta^2(\mathcal{P}_\perp) \mathcal{B}^{a,\mu}_{n,\perp}(0) | 0 \rangle$$

$$\widehat{\mathcal{M}}_{\mathrm{EEEC}} \to \sum_{i,j,k} \frac{E_i E_j E_k}{Q^3} \delta\left(x_1 - \frac{\theta_{ij}^2}{4}\right) \delta\left(x_2 - \frac{\theta_{jk}^2}{4}\right) \delta\left(x_3 - \frac{\theta_{ki}^2}{4}\right)$$

Similarly for quark jet function with collinear quark field $\chi_n \equiv W_n^\dagger \xi_n$.



- Contact contributions (i, j, k not all non-identical) effectively depend on only one angle.

$$\delta\left(x - \frac{1-\cos\theta_{ij}}{2}\right) = \frac{(p_i \cdot p_j)}{x} \delta\left[2x(p_i\cdot Q)(p_j\cdot Q) - p_i\cdot p_j\right]$$

$$= \frac{1}{2\pi i} \frac{(p_i\cdot p_j)}{x} \left\{ \frac{1}{\left[2x(p_i\cdot Q)(p_j\cdot Q) - p_i\cdot p_j\right] - i0} - \frac{1}{\left[2x(p_i\cdot Q)(p_j\cdot Q) - p_i\cdot p_j\right] + i0} \right\}$$

IBP reduction to linear combination of same master integrals as in analytic EEC calculation at $\mathcal{O}(\alpha_s^2)$. [L. Dixon, M.X. Luo, V. Shtabovenko, T.Z. Yang, H.X. Zhu, 1801.03219]

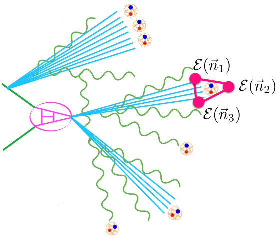
Calculation of the Two-loop Jet Function for E3C

The gluon jet function for EEEC, then project to the largest angle,

$$J_g(x_1, x_2, x_3, Q, \mu^2) = \int \frac{dl^+}{2\pi} \frac{1}{2(N_C^2-1)} \mathrm{Tr} \int d^4x e^{il\cdot x} \langle 0 | \mathcal{B}^{a,\mu}_{n,\perp}(x) \boxed{\widehat{\mathcal{M}}_{\mathrm{EEEC}}} \delta(Q + \bar{n}\cdot\mathcal{P})\delta^2(\mathcal{P}_\perp)\mathcal{B}^{a,\mu}_{n,\perp}(0)|0\rangle$$

$$\widehat{\mathcal{M}}_{\mathrm{EEEC}} = \sum_{i,j,k} \frac{E_iE_jE_k}{Q^3} \delta\left(x_1 - \frac{\theta_{ij}^2}{4}\right)\delta\left(x_2 - \frac{\theta_{jk}^2}{4}\right)\delta\left(x_3 - \frac{\theta_{ki}^2}{4}\right)$$

Similarly for quark jet function with collinear quark field $\chi_n \equiv W_n^\dagger \xi_n$.



- Non-identical contributions (i, j, k all non-identical) contain Θ-functions to keep the largest angle.

$$\int_S d\mathrm{Re}(z) d\mathrm{Im}(z) J_{\widehat{ijk}}$$
$$= A \cdot \int_S d\mathrm{Re}(z) d\mathrm{Im}(z) \int \mathrm{d}\Phi_c^{(3)} \frac{4g^4}{s_{123}^2} \sum_{i,j,k} P_{ijk} \widehat{\mathcal{M}}_{\mathrm{EEEC}}$$

IBP reduction in the parametric representation. [W. Chen, arXiv:1902.10387; 1912.08606; 2007.00507]

Performed also direct integration with the proper parametrization.

Calculation of the Two-loop Jet Function for E3C

Combining the contact contributions and the non-identical contributions,

$$\frac{dJ_q^{\text{2-loop}}}{dx_L} \overset{\mu=Q}{=} \left(\frac{\alpha_s}{4\pi}\right)^2 \Big\{ \delta(x_L) \times j_q^{(2)} \;\; \rightarrow \text{jet function constant}$$

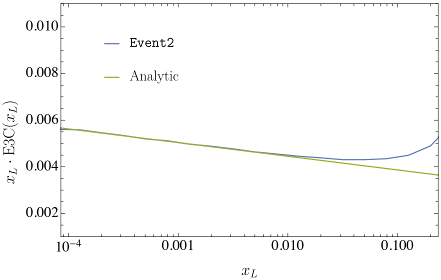
$$+ \underbrace{\frac{1}{x_L}\Big[C_F T_F n_f\Big(0.695 \ln x_L - 4.144\Big) + C_F^2\Big(2.944 \ln x_L - 1.050\Big) + C_F C_A\Big(-2.448 \ln x_L + 11.13\Big)\Big]}_{\rightarrow \text{ leading power distribution}}\Big\}$$

$$\frac{dJ_g^{\text{2-loop}}}{dx_L} \overset{\mu=Q}{=} \left(\frac{\alpha_s}{4\pi}\right)^2 \Big\{ \delta(x_L) \times j_g^{(2)} \;\; \rightarrow \text{jet function constant}$$

$$+ \frac{1}{x_L}\Big[C_F T_F n_f\Big(-0.4125 \ln x_L + 0.0875\Big) + C_A T_F n_f\Big(-0.4125 \ln x_L - 7.064\Big)$$

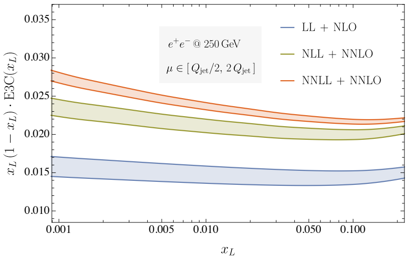
$$+ C_A^2\Big(0.28 \ln x_L + 20.69\Big) + T_F^2 n_f^2\Big(0.2 \ln x_L - 1.092\Big)\Big]\Big\}$$

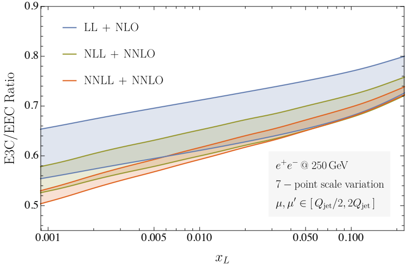
checking the leading power distribution with `Event2`.



E3C Distribution and its Ratio to EEC at NNLL+NNLO

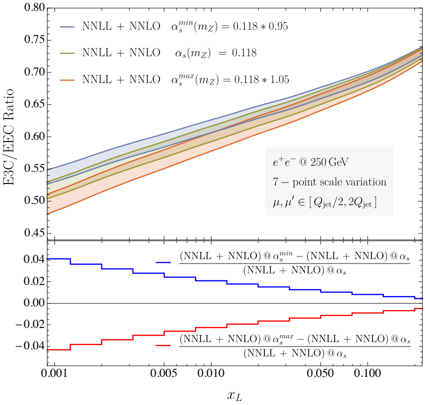
Preliminary results





- Evident convergence from LL to NNLL, with decreasing scale uncertainty.
- Uncertainty bands for E3C distribution are not overlapping.
  - Higher order perturbative contributions sizable.
- The E3C/EEC has better convergence and is advantageous in α_s-determination.

E3C/EEC Ratio and Strong Coupling Constant



- Varying the value of $\alpha_s(m_Z)$ changes the E3C/EEC ratio at the same level.
- Measurements can also be performed with jet-substructure.

So far, parton level only.
Non-perturbative QCD corrections can be sizable in the collinear limit $x_L \to 0$.

Non-Perturbative Corrections to the ENC

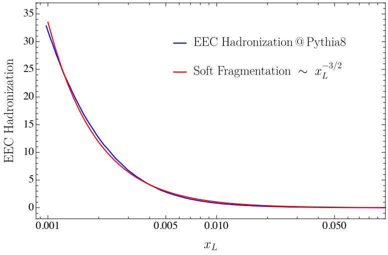
In the collinear limit, the ENC is sensible to the low-energy non-perturbative QCD effects.

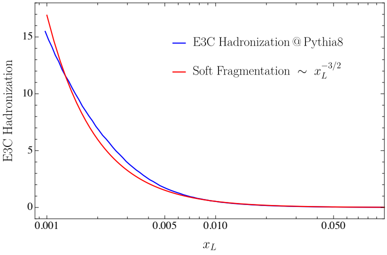
$$\frac{d\sigma^{\mathrm{PT}}}{dx_L} \sim \frac{\alpha_s(\mu)}{x_L} \cdot \left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)^0 \cdot \left[\sum_{n=1}^{\infty} a_s^n\, c_n \, [\ln^n x_L\ ] + \cdots\right] \qquad \text{(Perturbative)}$$

$$\frac{d\sigma^{\mathrm{NP}-soft}}{dx_L} \sim \frac{\alpha_s(\mu)}{x_L^{3/2}} \cdot \left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)^1 + \cdots \qquad \text{(Dispersive analysis at LL)}$$

$$\frac{d\sigma^{\mathrm{NP}-collinear}}{dx_L} \sim \frac{\alpha_s^2(\mu)}{x_L^2} \cdot \left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)^2 + \cdots \qquad \text{(Renormalon analysis at NLL)}$$

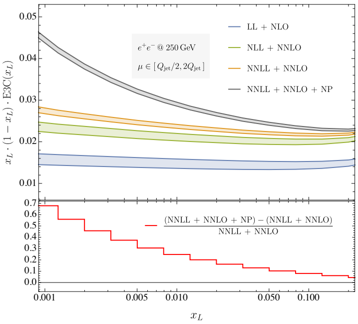
Soft contribution gives leading non-perturbative power correction linear in Λ_{QCD}.

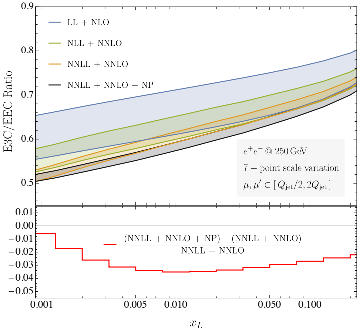




Extract the non-perturbative parameter from Pythia data to include $O(\Lambda_{\mathrm{QCD}})$ correction.

Including the Non-Perturbative Corrections





- In the collinear limit, hadronization correction to E3C is enhanced compared with the perturbative contribution.
- Hadronization correction to E3C/EEC ratio is in fact largely reduced.
  - Extraction of the strong coupling more reliable and accurate.

Summary

- Energy correlators are powerful event-shape variables in precision collider physics.
- E3C and its ratio with EEC obtained at NNLL+NNLO accuracy.
- Non-perturbative corrections to ENC are sizable, and are largely cancelled in ratios.
- E3C/EEC ratio in collinear limit sensitive to the strong coupling and provides a new method for its extraction from measurements within jets.

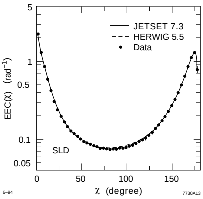
Outlook

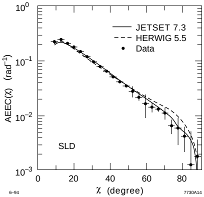
- Higher point ($N > 3$) energy correlators at NNLL
  - two-loop jet function constants
- Resummation for LHC processes
  - more complicated hard function
- Pinning down more accurately the non-perturbative corrections

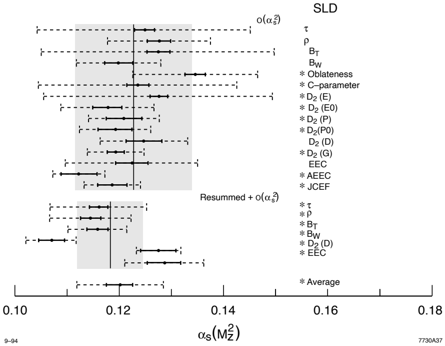
Extra Slides

Strong Coupling Determination from Energy Correlators

- e^+e^- colliders [SLD Collaboration, arXiv:hep-ex/9501003]







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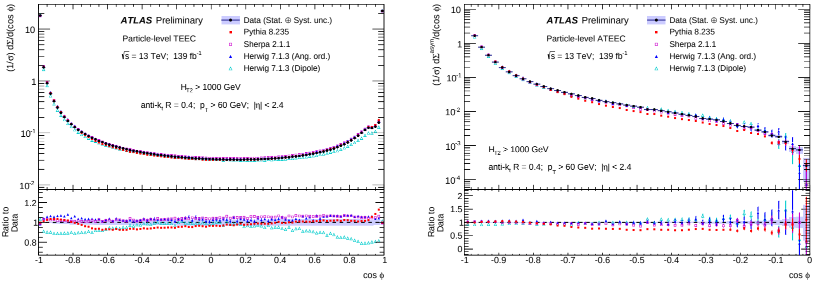
- Transverse Energy-Energy Correlator [A.Ali, E.Pietarinen, W.J.Stirling, PLB, 447 (1984)]

$$\text{TEEC} = \sum_{a,b} \int d\sigma_{pp\to a+b+X} \frac{2E_{T,a}E_{T,b}}{|\sum_i E_{T,i}|^2} \delta(\cos\phi_{ab} - \cos\phi)\,,$$



[ATLAS Collaboration, ATLAS-CONF-2020-025]

$$\alpha_s(M_Z) = 0.1196 \pm 0.0001(\text{stat.}) \pm 0.0004(\text{syst.})^{+0.0071}_{-0.0104}(\text{scale}) \pm 0.0011(\text{PDF}) \pm 0.0002(\text{NP})$$

$$\alpha_s(M_Z) = 0.1195 \pm 0.0002(\text{stat.}) \pm 0.0006(\text{syst.})^{+0.0084}_{-0.0106}(\text{scale}) \pm 0.0009(\text{PDF}) \pm 0.0003(\text{NP})$$