New analyses of event shapes and the determination of α_s in e^+e^- annihilation

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International Workshop on the High Energy CEPC

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Based on arXiv:2112.06212; 1908.00060; 1902.01984, in collaboration with Stanley J. Brodsky, Xing-Gang Wu, Jian-Ming Shen, and Leonardo Di Giustino

Outline

一．Introduction

二．Principle of Maximum Conformality (PMC)

三．Event shape observables and a novel method for the determination of α_s at LEP

四．Event shape observables at CEPC

一. Introduction

A pQCD calculable quantity ρ can be expanded in perturbative series

$$\rho(\mu_R) = r_0 \alpha_s(\mu_R)[1 + \sum_{k=1}^{\infty} r_k(\frac{Q}{\mu_R}) \frac{\alpha_s^k(\mu_R)}{\pi^k}]$$

$$g_0 = Z_g \mu^{\varepsilon/2} g \quad (\varepsilon\text{=4-d})$$

$$\frac{\partial \rho(\mu_R)}{\partial \mu_R} \equiv 0$$

◆ infinite order, no scheme- and scale-dependence

◆ fixed-order, the prediction, scheme- and scale-dependence

一. Introduction

$$\frac{\partial \rho_n}{\partial \mu_R} \neq 0; \quad n - \textit{finite order}$$

Conventional Method

- Guessing a renormalization scale Q "typical momentum transfer", or to eliminate large logs or to improve convergence
- Varying the scale in a certain range, e.g. [Q/2, 2Q] to discuss its uncertainty
- Convergence is usually problematic due to $n!\beta_i^n \alpha_s^n$

一. Introduction

How to solve the SCALE

Brodsky–Lepage–Mackenzie method (BLM)

Cited 1214

PHYSICAL REVIEW D VOLUME 28, NUMBER 1 1 JULY 1983

On the elimination of scale ambiguities in perturbative quantum chromodynamics

Stanley J. Brodsky
Institute for Advanced Study, Princeton, New Jersey 08540
*and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305**

G. Peter Lepage
Institute for Advanced Study, Princeton, New Jersey 08540
*and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853**

Paul B. Mackenzie
Fermilab, Batavia, Illinois 60510
(Received 23 November 1982)

Principle of Minimum Sensitivity (PMS)

Cited 1200

PHYSICAL REVIEW D VOLUME 23, NUMBER 12 15 JUNE 1981

Optimized perturbation theory

P. M. Stevenson
Physics Department, University of Wisconsin-Madison, Madison, Wisconsin 53706
(Received 21 July 1980; revised manuscript received 17 February 1981)

RG-improved effective coupling method (FAC)

Cited 553

Volume 95B, number 1 PHYSICS LETTERS 8 September 1980

RENORMALIZATION GROUP IMPROVED PERTURBATIVE QCD

G. GRUNBERG [1]
Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853, USA

一. Introduction

ELSEVIER
Contents lists available at SciVerse ScienceDirect
Progress in Particle and Nuclear Physics
journal homepage: www.elsevier.com/locate/ppnp

Review

The renormalization scale-setting problem in QCD

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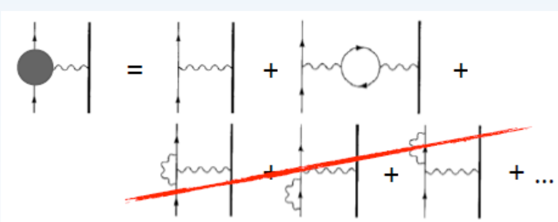
[b] SLAC National Accelerator Laboratory, Stanford University, CA 94039, USA
[c] CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230, Denmark

CrossMark

BLM/FAC/PMS

In the case of QED, the renormalization scale can be set unambiguously by using the Gell-Mann-Low method, which automatically sums all vacuum polarization contributions to the photon propagators to all orders.



BLM=> nf-term
BLM method reduces in the Abelian limit to the Gell-Mann-Low method

Quantum Electrodynamics at Small Distances
M. Gell-Mann and F. E. Low
Phys. Rev. **95**, 1300 – Published 1 September 1954

二. principle of maximum conformality

The PMC method extends the BLM scale-setting method to all orders

PHYSICAL REVIEW D 85, 034038 (2012)

Scale setting using the extended renormalization group and the principle of maximum conformality: The QCD coupling constant at four loops

Stanley J. Brodsky[1,*] and Xing-Gang Wu[1,2,†]

[1]*SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, California 94025, USA*
[2]*Department of Physics, Chongqing University, Chongqing 401331, China*
(Received 30 November 2011; published 22 February 2012)

PRL 109, 042002 (2012) PHYSICAL REVIEW LETTERS week ending 27 JULY 2012

Eliminating the Renormalization Scale Ambiguity for Top-Pair Production Using the Principle of Maximum Conformality

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[2]*Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China*
(Received 29 March 2012; published 23 July 2012)

PRL 110, 192001 (2013) PHYSICAL REVIEW LETTERS week ending 10 MAY 2013

Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

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CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230 Odense, Denmark and SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA

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(Received 13 January 2013; published 10 May 2013)

二. principle of maximum conformality

The scale dependence of the strong coupling constant is controlled by the renormalization group equation (RGE) via the β function:

$$\beta^{\mathcal{R}} = \mu_r^2 \frac{\partial}{\partial \mu_r^2} \left(\frac{\alpha_s^{\mathcal{R}}(\mu_r)}{4\pi} \right) = -\sum_{i=0}^{\infty} \beta_i^{\mathcal{R}} \left(\frac{\alpha_s^{\mathcal{R}}(\mu_r)}{4\pi} \right)^{i+2}$$

If one can find a proper way to sum up all known-type of *βi*-terms into the coupling constant, then one can determine the effective coupling for a specific process definitely at each perturbative order, and thus, the renormalization scale dependence can be greatly suppressed or even be eliminated.

二. principle of maximum conformality

Scale Setting Using the Extended Renormalization Group and the Principle of Maximum Conformality: the QCD Coupling Constant at Four Loops.
Phys.Rev. D85 (2012) 034038.

Eliminating the Renormalization Scale Ambiguity for Top-Pair Production Using the Principle of Maximum Conformality
Phys.Rev.Lett. 109 (2012) 042002.

Self-Consistency Requirements of the Renormalization Group for Setting the Renormalization Scale
Phys.Rev. D86 (2012) 054018.

Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD
Phys.Rev.Lett. 110 (2013) 192001.

The Renormalization Scale-Setting Problem in QCD
Prog.Part.Nucl.Phys. 72 (2013) 44-98.

Reanalysis of the BFKL Pomeron at the next-to-leading logarithmic accuracy
JHEP 1310 (2013) 117

Systematic Scale-Setting to All Orders: The Principle of Maximum Conformality and Commensurate Scale Relations
Phys.Rev. D89 (2014) 014027.

Renormalization Group Invariance and Optimal QCD Renormalization Scale-Setting
Rept.Prog.Phys. 78 (2015) 126201.

General Properties on Applying the Principle of Minimum Sensitivity to High-order Perturbative QCD Predictions
Phys.Rev. D91 (2015) , 034006.

Setting the renormalization scale in perturbative QCD: Comparisons of the principle of maximum conformality with the sequential extended Brodsky-Lepage-Mackenzie approach.
Phys.Rev. D91 (2015), 094028.

Degeneracy Relations in QCD and the Equivalence of Two Systematic All-Orders Methods for Setting the Renormalization Scale
Phys.Lett. B748 (2015) 13-18.

The Generalized Scheme-Independent Crewther Relation in QCD
Phys.Lett. B770 (2017) 494-499

Novel All-Orders Single-Scale Approach to QCD Renormalization Scale-Setting
Phys.Rev. D95 (2017) , 094006.

Renormalization scheme dependence of high-order perturbative QCD predictions
Phys.Rev. D97 (2018), 036024.

Novel demonstration of the renormalization group invariance of the fixed-order predictions using the principle of maximum conformality and the C -scheme coupling
Phys.Rev. D97 (2018), 094030.

The QCD Renormalization Group Equation and the Elimination of Fixed-Order Scheme-and-Scale Ambiguities Using the Principle of Maximum Conformality
Prog.Part.Nucl.Phys. 108 (2019) 103706

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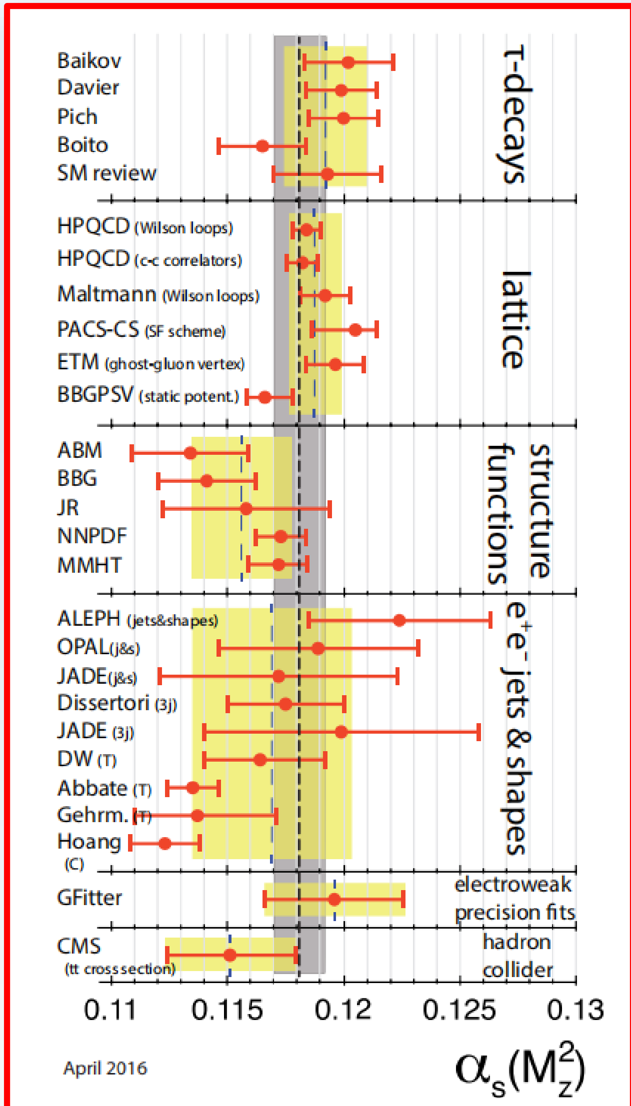
三. Event shapes and extracting α_s at LEP



α_s is a free parameter in QCD.

$$\alpha_s(M_Z^2) = 0.1181 \pm 0.0011 \ ,$$

0.9%

[Particle Data Group], Phys. Rev. D98, 030001 (2018)

三. Event shapes and extracting α_s at LEP

The classic event shapes: the thrust (T), the heavy jet mass (M_H^2/s), the wide and total jet broadenings B_W and B_T, the C-parameter (C)

$$T = \max_{\vec{n}} \left(\frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|} \right)$$

$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2\theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2},$$

$$\rho \equiv M_H^2/s = \max(M_1^2/s, M_2^2/s)$$

$$M_i^2/s = \frac{1}{E_{\rm vis}^2} \left(\sum_{k \in H_i} p_k \right)^2$$

$$B_W = \max(B_1, B_2)\,, \qquad B_T = B_1 + B_2\,.$$

$$B_i = \frac{\sum_{k \in H_i} |\vec{p}_k \times \vec{n}_T|}{2 \sum_k |\vec{p}_k|}\,.$$

Currently, the main obstacle for achieving a precise determination of $a_s(M_Z)$ is not the lack of precise experimental data, especially at Z^0 peak, but the ambiguity of theoretical predictions.

Rep. Prog. Phys. 69, 1771 (2006).

三. Event shapes and extracting α_s at LEP

The method for extracting $a_s(M_Z)$ in e^+e^- collider:

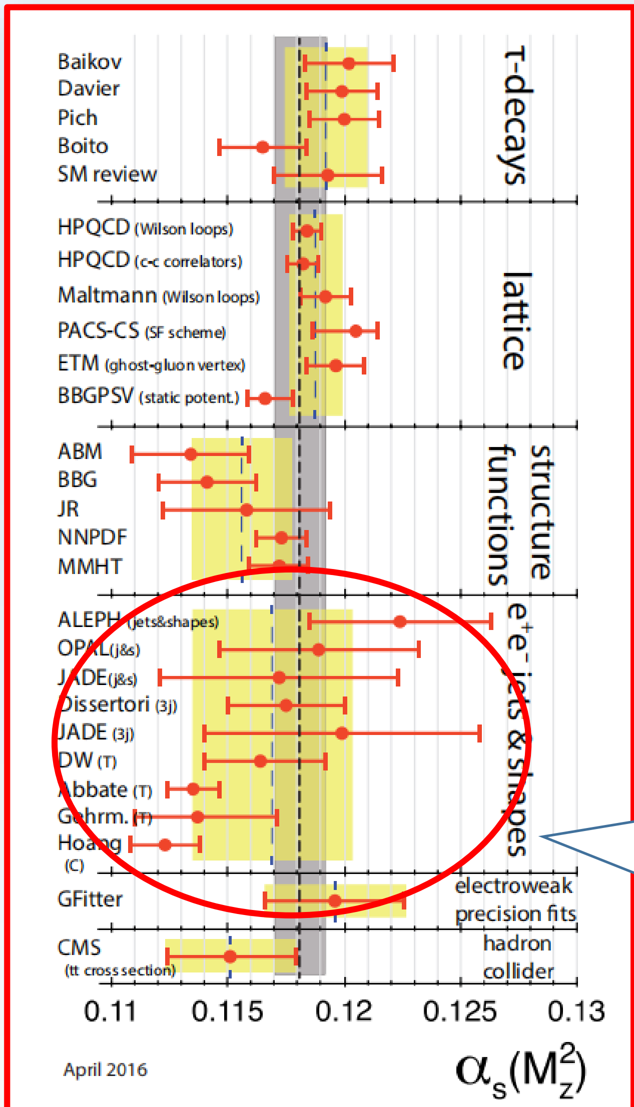
- predictions matched Monte Carlo models to correct for hadronization effects
- based on analytic calculations of non-perturbative and hadronization effects, using methods like power corrections, factorization of soft-collinear effective field theory, dispersive models and low scale QCD effective couplings

We note that there is criticism on both classes of α_s extractions described above: those based on corrections of non-perturbative hadronization effects using QCD-inspired Monte Carlo generators (since the parton level of a Monte Carlo simulation is not defined in a manner equivalent to that of a fixed-order calculation), as well as studies based on non-perturbative analytic calculations, as their systematics have not yet been fully verified. In particular, quoting rather small overall experimental, hadronization and theoretical uncertainties of only 2, 5 and 9 per-mille, respectively [425,427], seems unrealistic and has neither been met nor supported by other authors or groups.

?

[Particle Data Group],
Phys. Rev. D98, 030001 (2018)

三. Event shapes and extracting *αs* at LEP



419. G. Dissertori *et al.*, JHEP **0908**, 036 (2009).
420. G. Abbiendi *et al.*, Eur. Phys. J. **C71**, 1733 (2011).
421. S. Bethke *et al.*, [JADE Collab.], Eur. Phys. J. **C64**, 351 (2009).
422. G. Dissertori *et al.*, Phys. Rev. Lett. **104**, 072002 (2010).
423. J. Schieck *et al.*, Eur. Phys. J. **C73**, 2332 (2013).
424. R.A. Davison and B.R. Webber, Eur. Phys. J. **C59**, 13 (2009).
425. R. Abbate *et al.*, Phys. Rev. **D83**, 074021 (2011).
426. T. Gehrmann *et al.*, Eur. Phys. J. **C73**, 2265 (2013).
427. A.H. Hoang *et al.*, Phys. Rev. **D91**, 094018 (2015).
428. R. Frederix *et al.*, JHEP **1011**, 050 (2010).

- The $a_s(M_Z)$ are plagued by significant scale uncertainty
- Some extracted $a_s(M_Z)$ are deviated from the world average
- non-self-consistent

See Paolo Nason's talk

三. Event shapes and extracting α_s at LEP

The differential distribution for a event shape:

$$\frac{1}{\sigma_h}\frac{d\sigma}{d\tau} = \bar{A}(\tau)\, a_s(Q) + \bar{B}(\tau)\, a_s^2(Q) + \mathcal{O}(a_s^3).$$

$Q = \sqrt{s}$ using conventional method

$$\frac{1}{\sigma_h}\frac{d\sigma}{d\tau} = \bar{A}(\tau) a_s(\mu_r^{\rm pmc}) + \bar{B}(\tau,\mu_r)_{\rm con}\, a_s^2(\mu_r^{\rm pmc}) + \mathcal{O}(a_s^3)$$

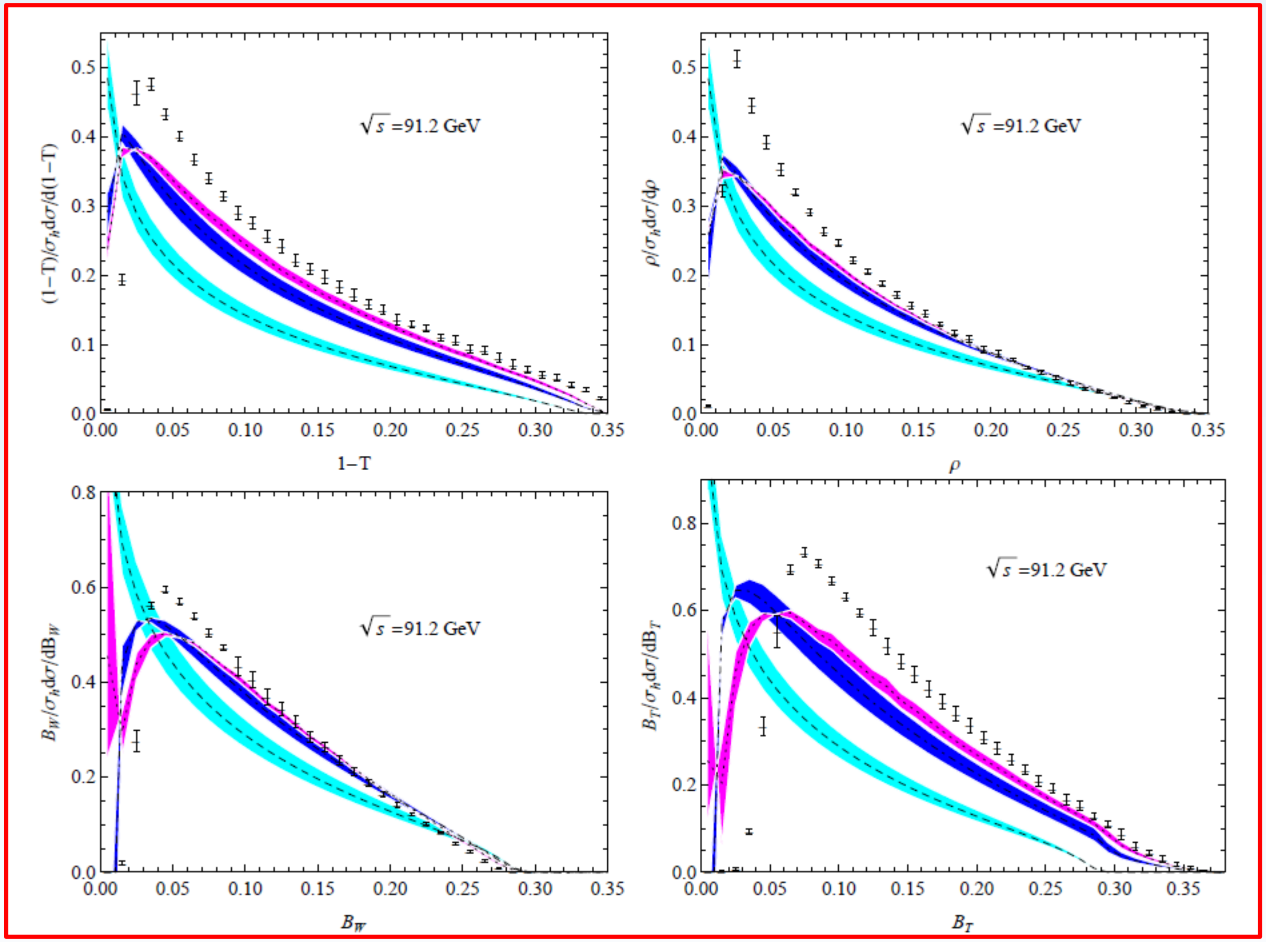
$$\bar{B}(\tau,\mu_r)_{\rm con} = \frac{11C_A}{4T_R}\bar{B}(\tau,\mu_r)_{n_f} + \bar{B}(\tau,\mu_r)_{\rm in},$$

$$\mu_r^{\rm pmc} = \mu_r \exp\left[\frac{3\bar{B}(\tau,\mu_r)_{n_f}}{4T_R\bar{A}(\tau)} + \mathcal{O}(a_s)\right].$$

三. Event shapes and extracting α_s at LEP

Conventional results at 91.2 GeV

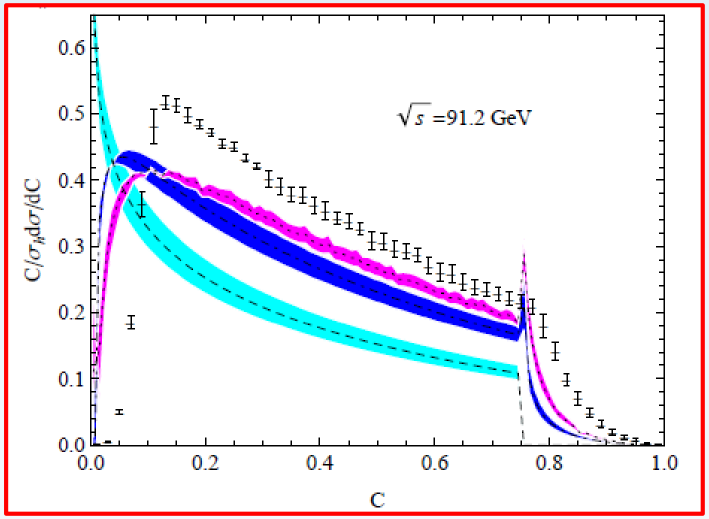
Central values are Q = 91.2 GeV , the errors are [Q/2, 2Q].



Phys. Rev. Lett. 99, 132002
JHEP 0712, 094
Phys.Rev. Lett. 101, 162001
JHEP 0906, 041

三. Event shapes and extracting *αs* at LEP

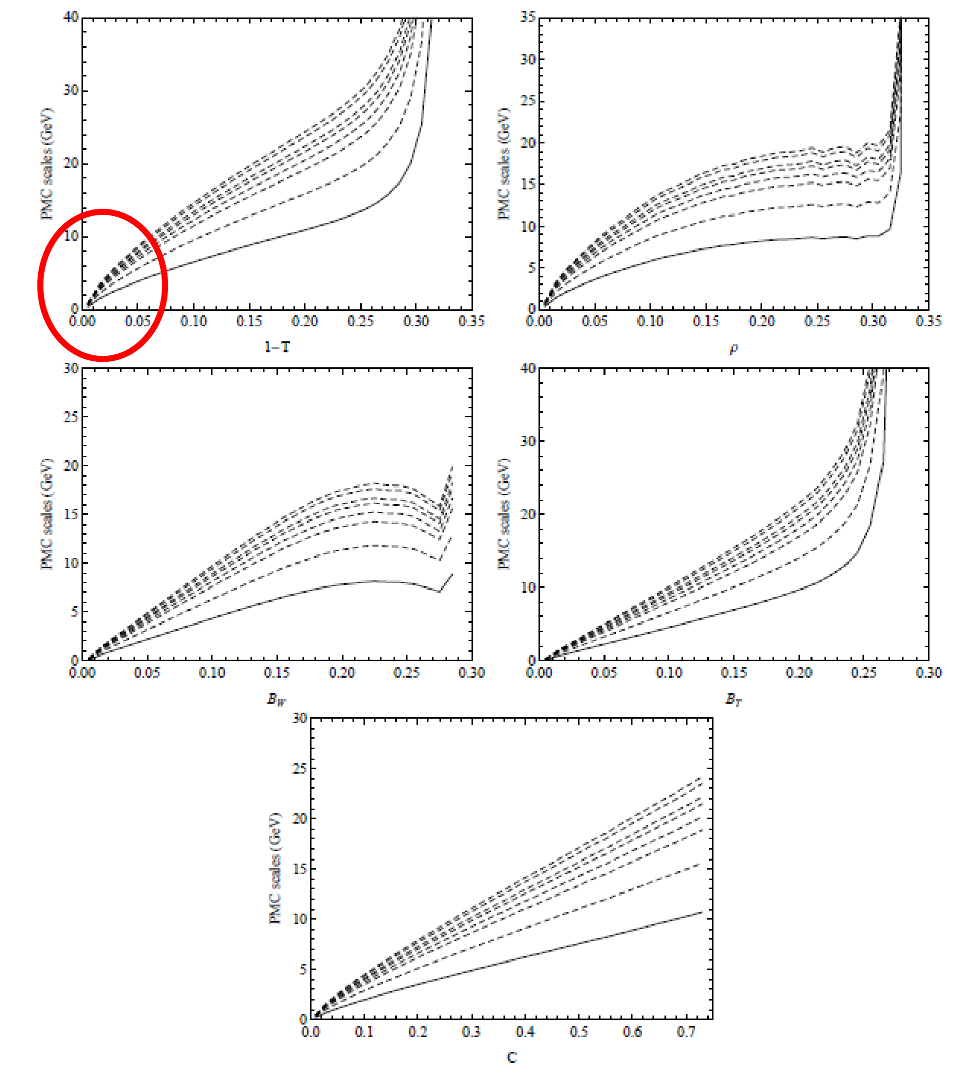
Event shapes using the conventional method:



- The NLO and NNLO are large and the pQCD series shows a slow convergence.
- Estimating the unknown higher order QCD by varying the scale [1/2Q, 2Q] is unreliable.
- The predictions are plagued by scale uncertainty, and even up to NNLO, the predictions do not match the data.
- The extracted coupling constants are deviated from the world average, and are also plagued by scale uncertainty.

三. Event shapes and extracting α_s at LEP

PMC scales:



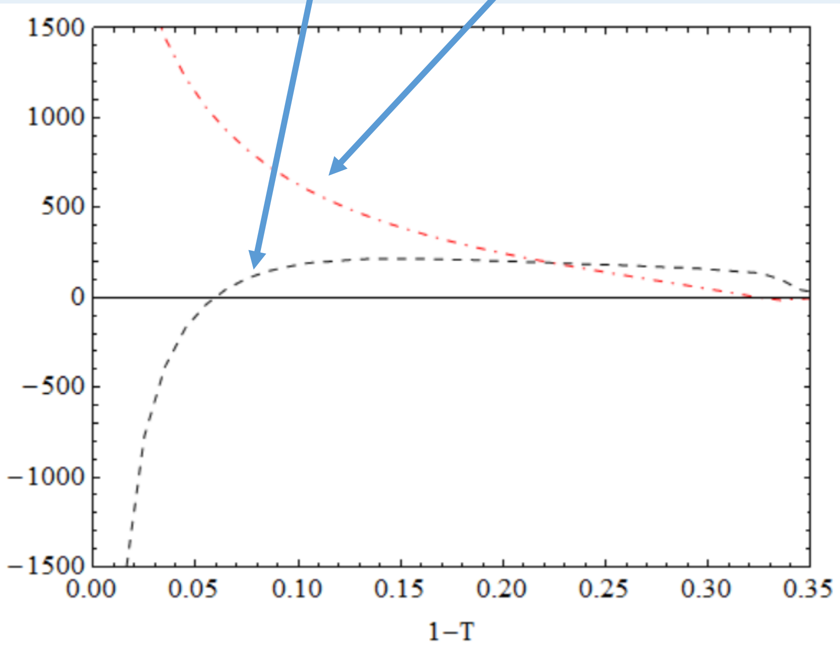
- Remarkably, the PMC scales change dynamically with event shapes;
- The quarks and gluons have soft virtuality near the two-jet region. The PMC scales are very soft in this region, while in the regions away from the two-jet region, the PMC scales are increased, as expected;
- The PMC scales are small in the wide kinematic regions compared to the conventional method \sqrt{s};
- The PMC scales increase with the center-of-mass energy;
- yields the correct physical behavior, and similar behavior are obtained in the SCET theory and other literatures (ZPA 339, 189; EPJC 74, 2896).

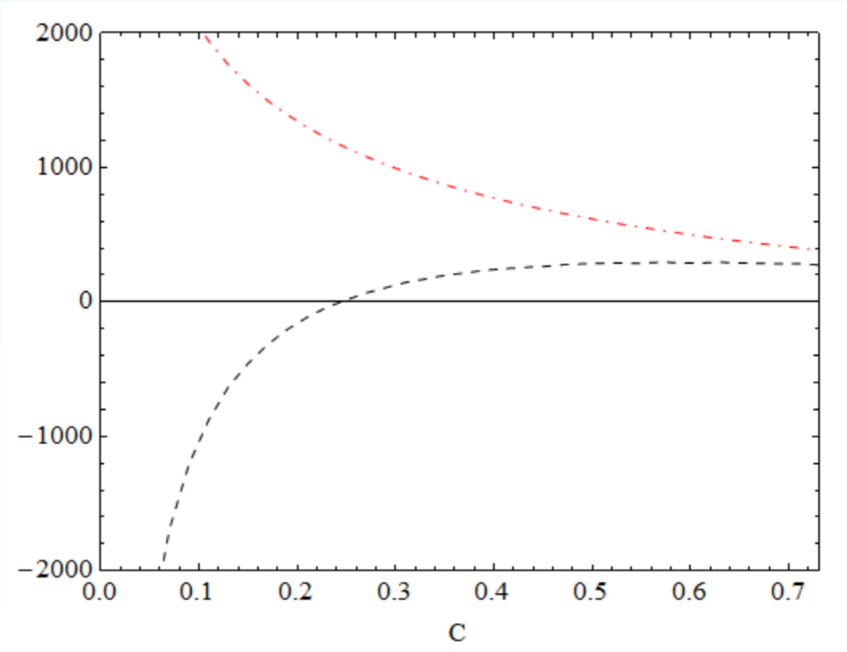
三. Event shapes and extracting αs at LEP

Perturbative coefficients:

$$\bar{B}(y,\mu_r) = \bar{B}(y,\mu_r)_{\rm con} + \bar{B}(y,\mu_r)_{\beta_0}\cdot\beta_0,$$

In addition to the PMC scales, the behavior of the PMC conformal coefficients is very different from that of the conventional scale-setting method.

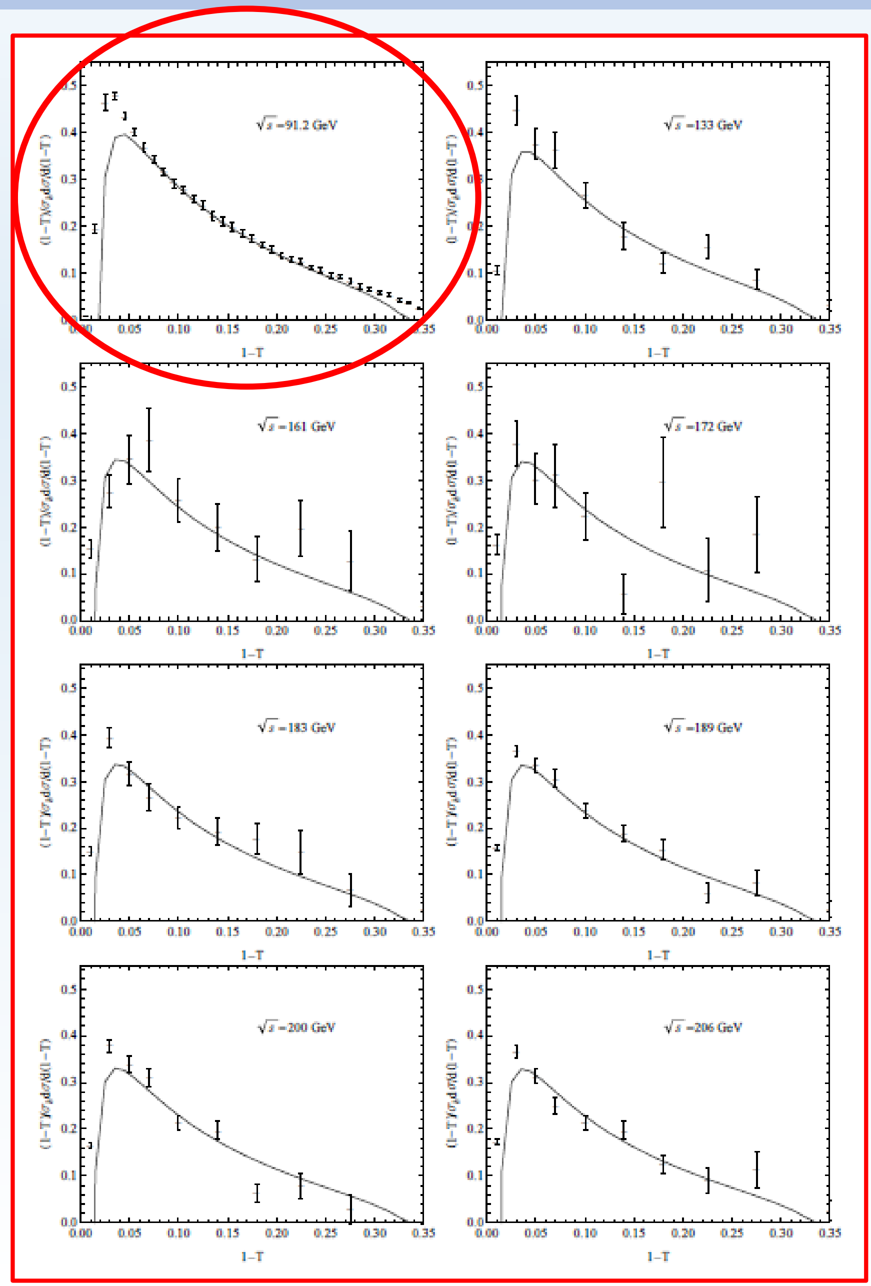




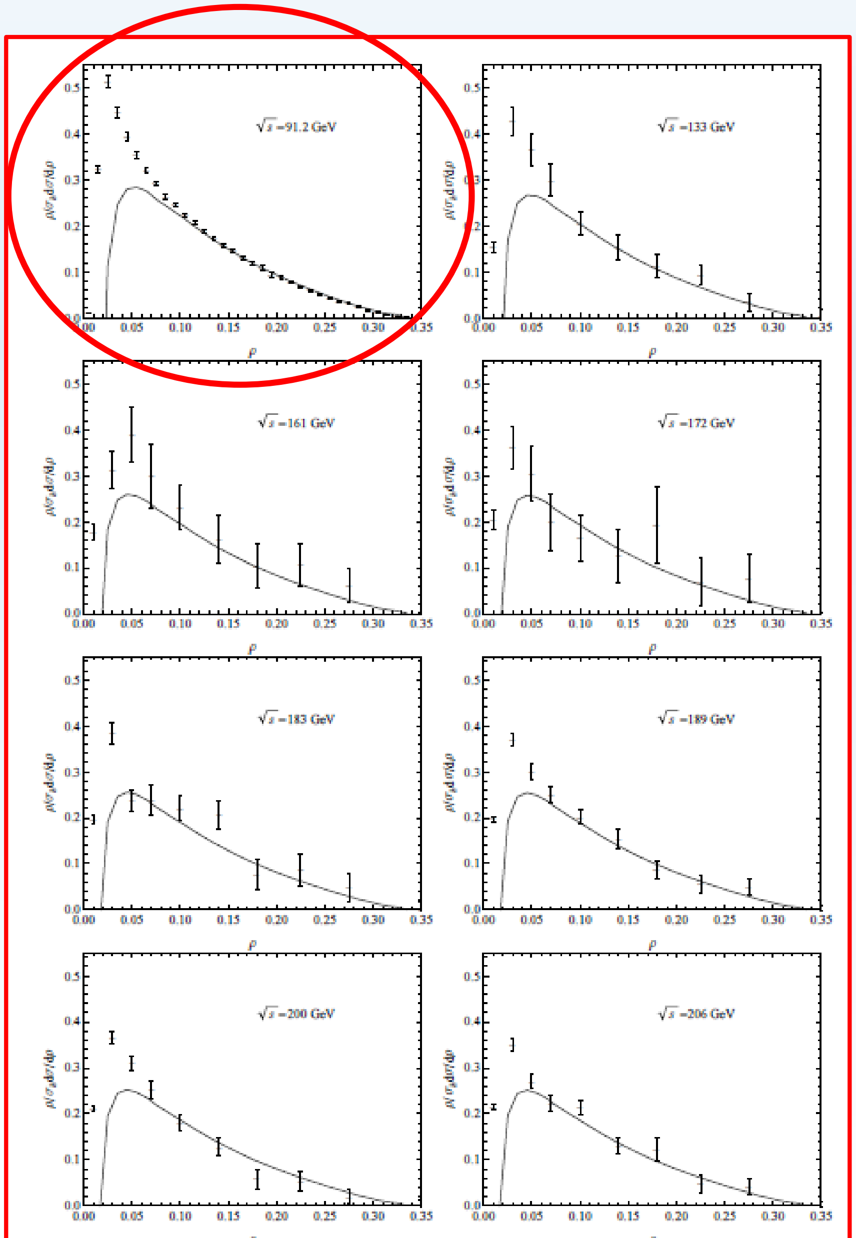
三. Event shapes and extracting α_s at LEP

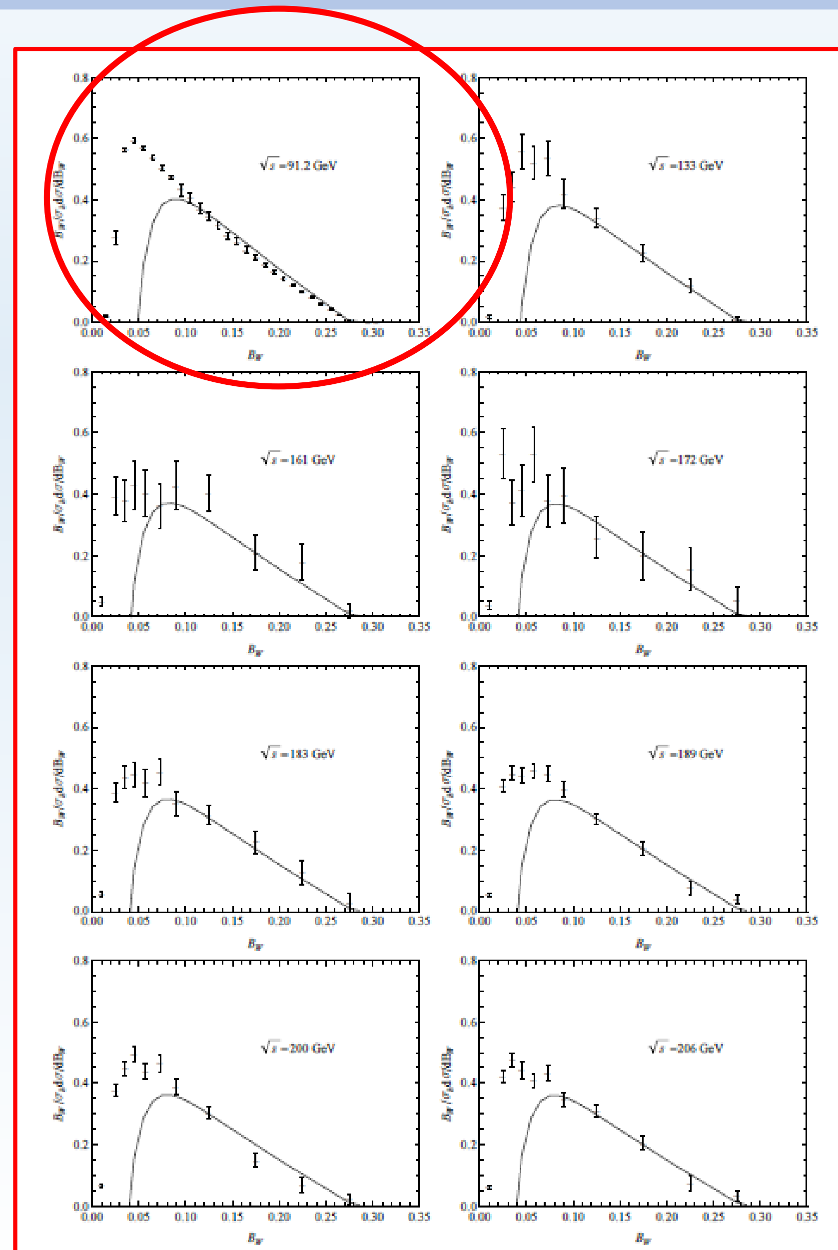
- The resulting PMC predictions are increased in wide kinematic regions compared to the conventional predictions.

- Since the PMC scales are independent of the choice of renormalization scale and the conformal coefficients are also renormalization scale independent, the PMC predictions eliminate the renormalization scale uncertainty.

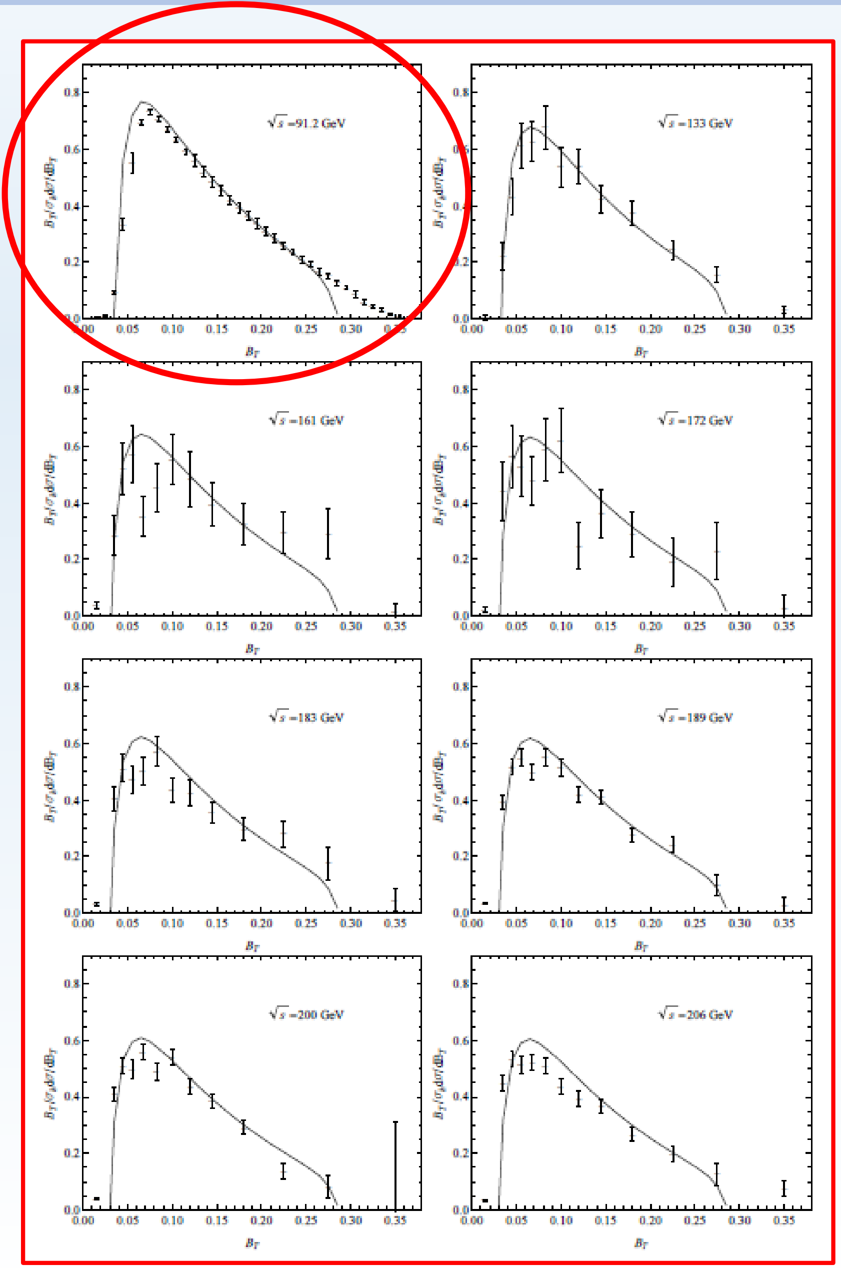


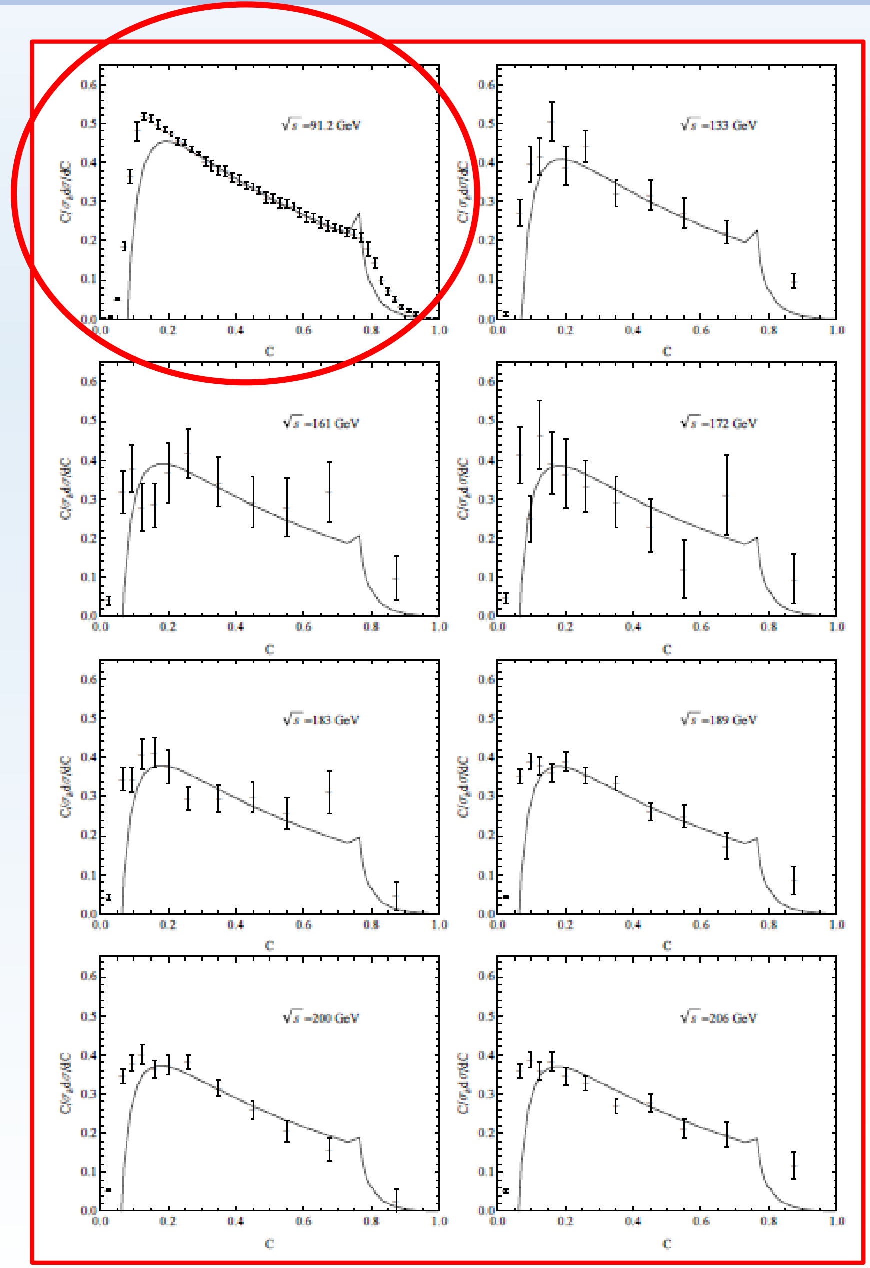
三. Event shapes and extracting αs at LEP





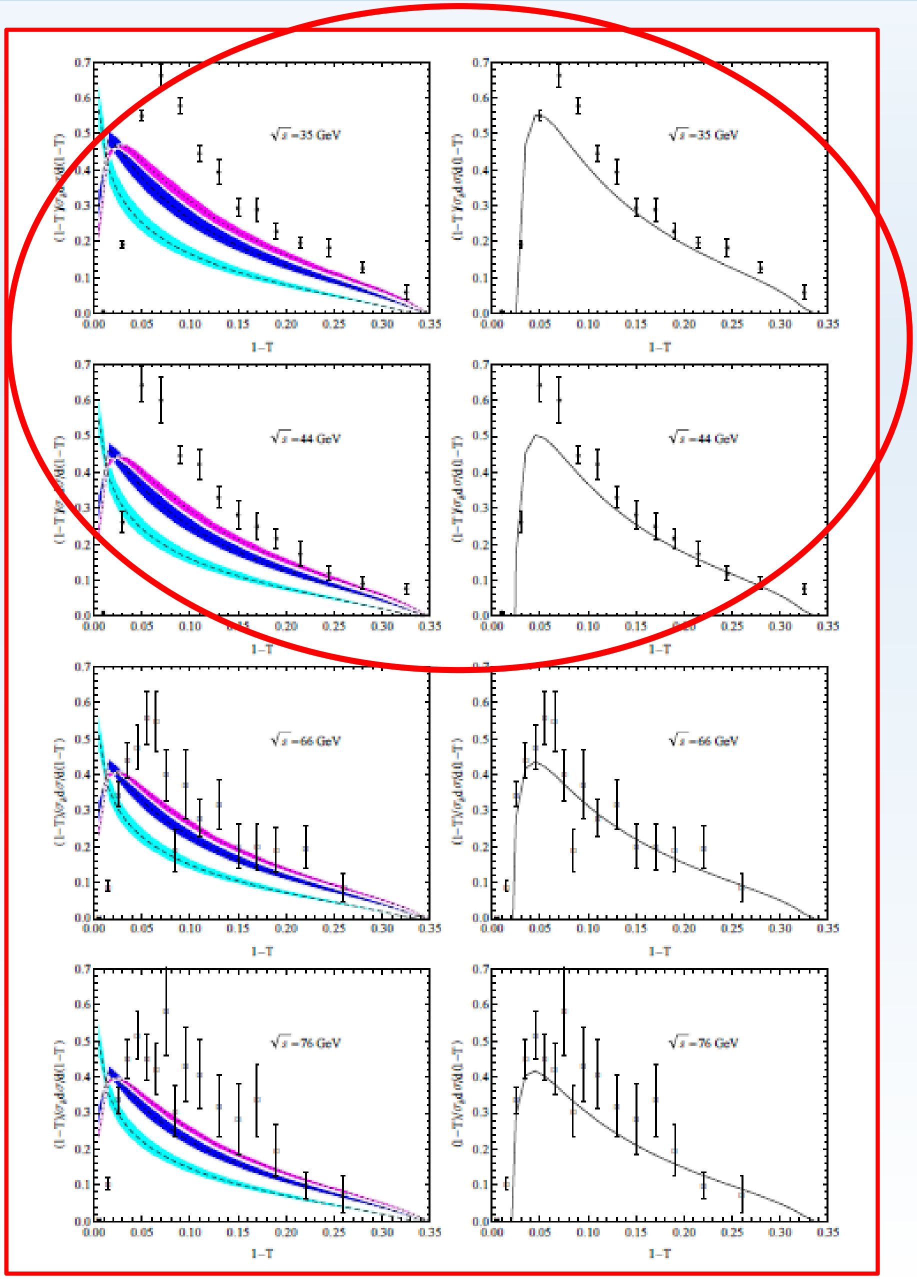
三. Event shapes and extracting α_s at LEP



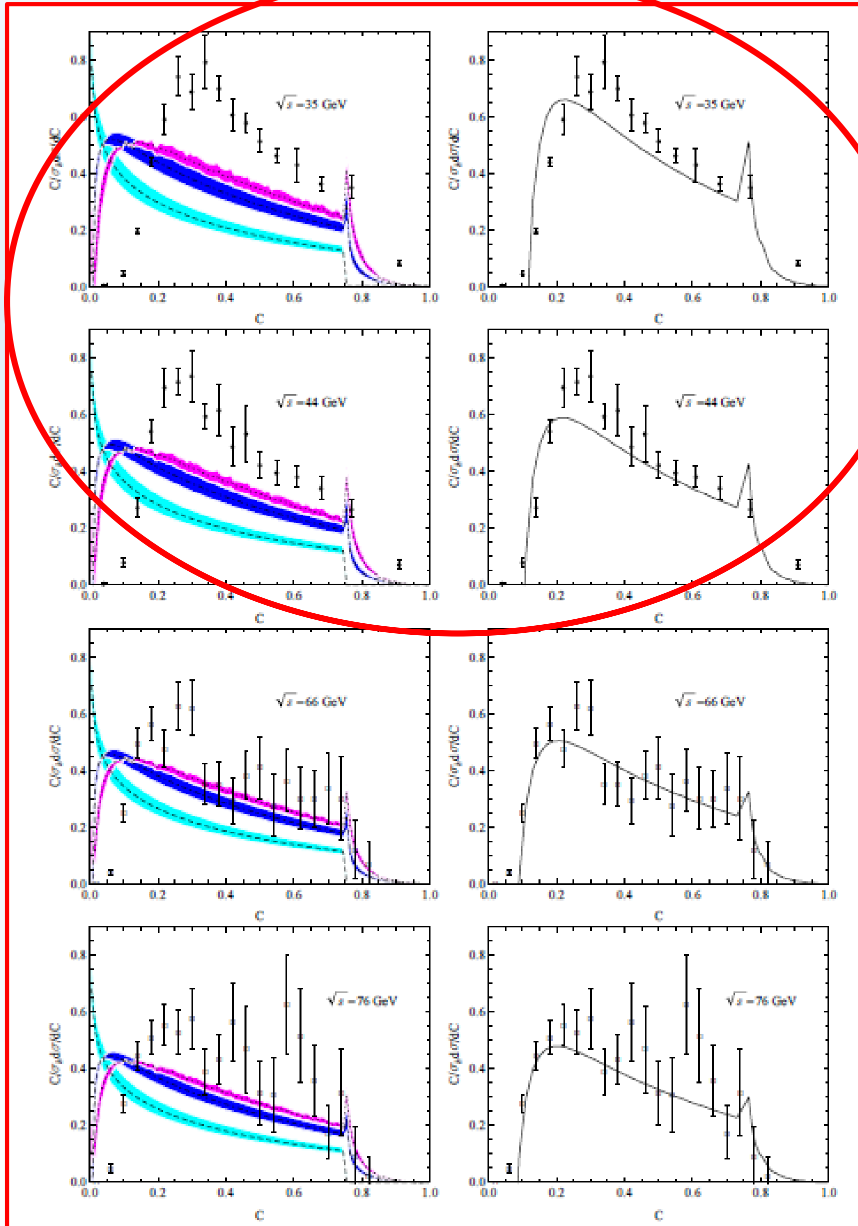


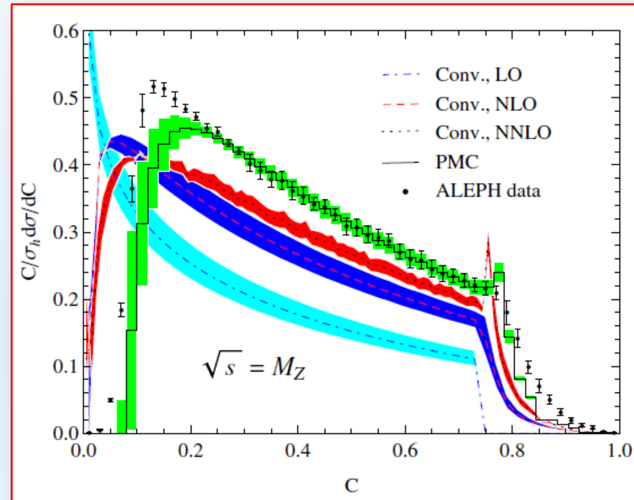
三. Event shapes and extracting αs at LEP

Event shape distributions below $Z0$ peak



三. Event shapes and extracting α_s at LEP

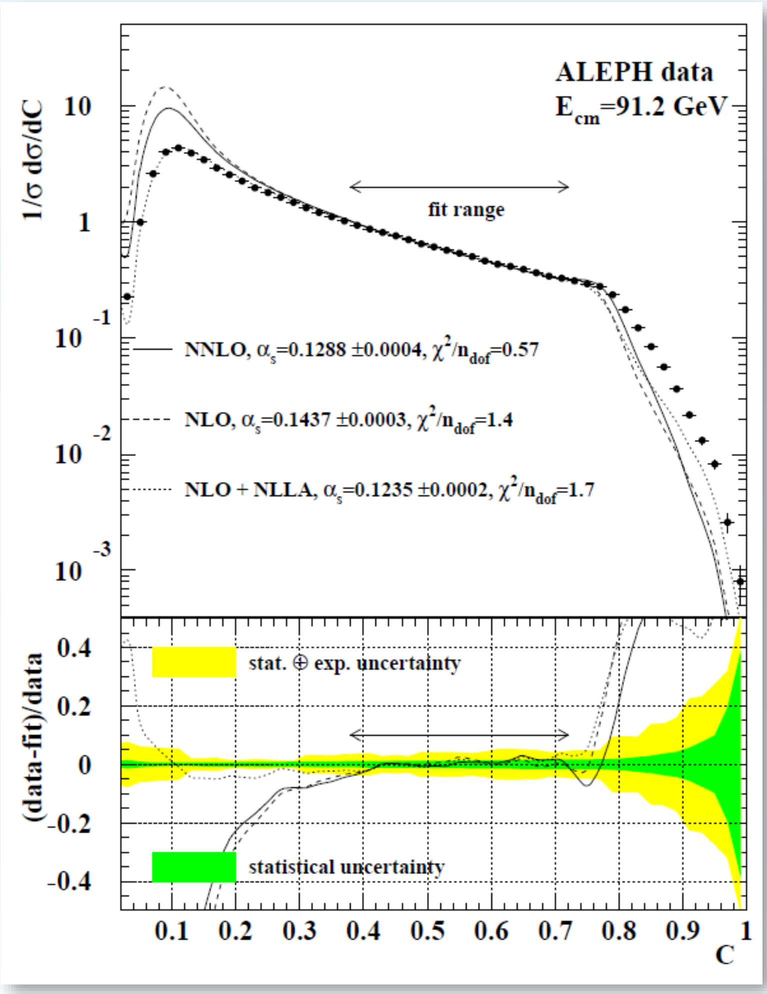




- The PMC predictions are greatly increased in wide kinematic regions, which leads PMC results to be closer to the experimental data.

- There are some deviations near the two-jet and multijet regions, since there are large logarithms that spoil the perturbative regime of the QCD. The resummation of large logarithms is thus required for the PMC results especially near the two-jet regions.

三. Event shapes and extracting α_s at LEP



JHEP 0802 (2008) 040

$Q = \sqrt{s} = M_Z$

Conv.

- ✓ One value α_s at scale M_Z is extracted ($\alpha_s(M_Z)$).
- ✓ the fit range of T (C) distribution is narrow.
- ✓ the fit range is arbitrary, different fit range leads to different α_s.

三. Event shapes and extracting α_s at LEP

$Q = \sqrt{s} = M_Z$

4 < Q < 16 GeV

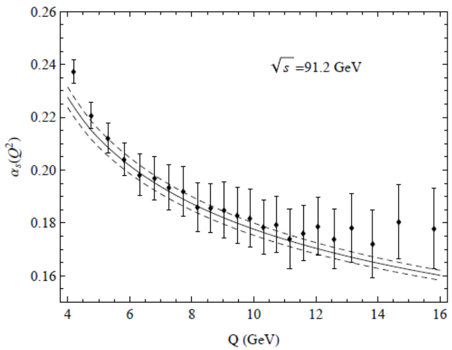


Figure 12. The extracted running coupling $\alpha_s(Q^2)$
paring the PMC predictions with the ALEPH data [
GeV. As a comparison, the solid line is the world av
and two dashed lines represent its uncertainty.

6 < Q < 9 GeV

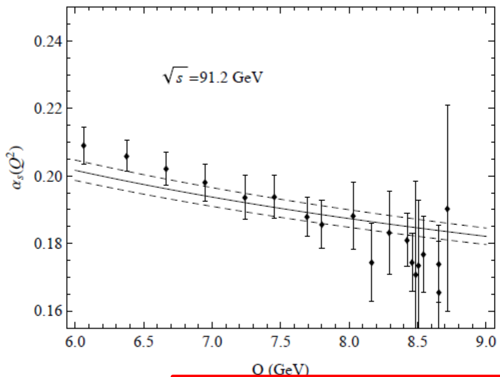


Figure 13. Similar to Fig. (12), but α_s

4 < Q < 7 GeV

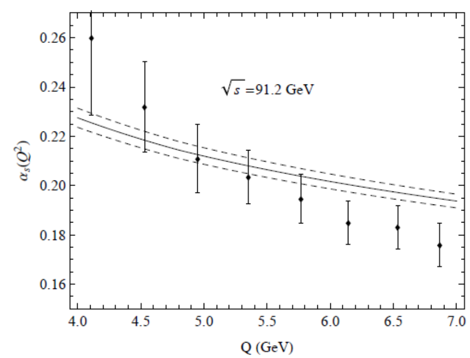


Figure 14. Similar to Fig. (12), but $\alpha_s(Q^2)$ extracted from the wide jet broadening (B_W) distribution.

三. Event shapes and extracting αs at LEP

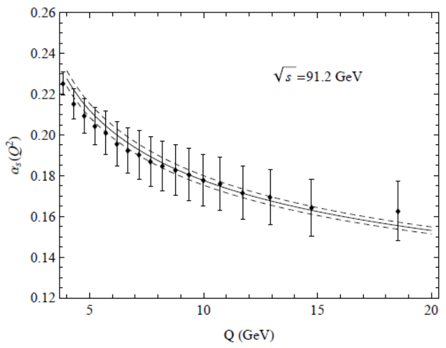


Figure 15. Similar to Fig. (12), but $\alpha_s(Q^2)$ extracted from the total jet broadening (B_T) distribution.

4 < Q < 19 GeV

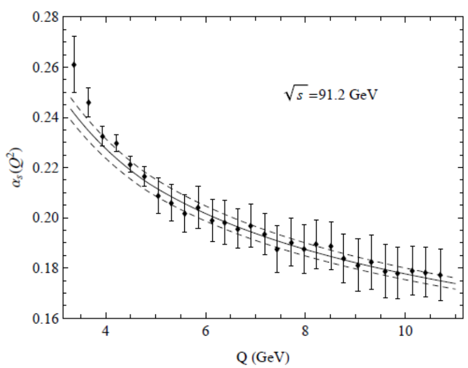


Figure 16. Similar to Fig. (12), but $\alpha_s(Q^2)$ extracted from the C-parameter (C) distribution.

3 < Q < 11 GeV

- ✓ The extracted α_s are in agreement with the world average in wide range of Q.
- ✓ The extracted α_s are not plagued by scale uncertainty.
- ✓ Since PMC scale varies with event shapes, we can extract the strong coupling at a wide scale range using the experimental data at single center-of mass-energy.

In QED, the running of the QED coupling at a wide scale range can be determined from events at a single energy

e.g., (OPAL Collaboration), EPJC 45, 1 (2006)

三. Event shapes and extracting α_s at LEP

the mean value of event shapes,

$$\langle y \rangle = \int_0^{y_0} \frac{y}{\sigma_h} \frac{d\sigma}{dy} dy,$$

- ✓ it involves an integration over the full phase space.
- ✓ it provides an important complement to the differential distributions and to determinate α_s

$$\mu_r^{\rm pmc}|_{\langle 1-T\rangle} = 0.0695\sqrt{s}, \text{ and } \mu_r^{\rm pmc}|_{\langle C\rangle} = 0.0656\sqrt{s},$$

- ✓ PMC scales of differential distribution are also very small.
- ✓ the average of the PMC scale for differential distribution is close to the scale of mean value. self-consistent.

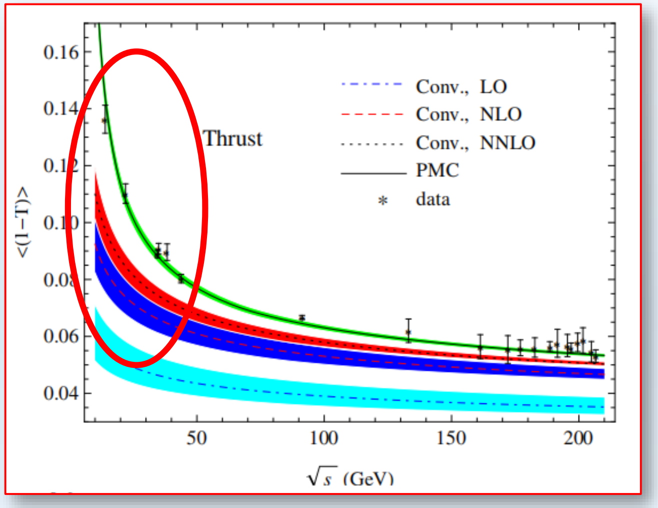
$\mu_r^{\rm pmc} \ll \sqrt{s}$ is also suggested by

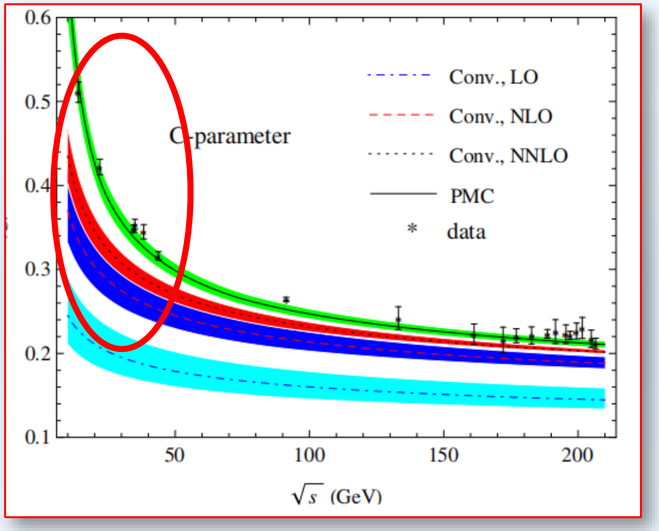
Studies of QCD at e^+e^- centre-of-mass energies between 91 and 209 GeV

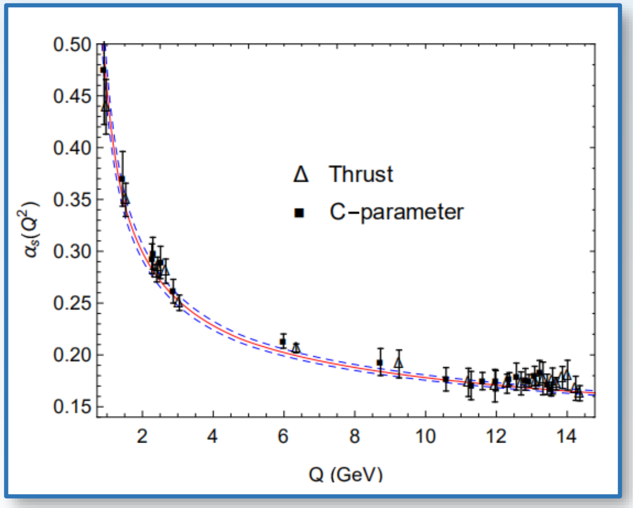
The ALEPH Collaboration

Eur. Phys. J. C 35, 457 – 486 (2004)

三. Event shapes and extracting *αs* at LEP







三. Event shapes and extracting αs at LEP

$$\alpha_s(M_Z^2) = 0.1185 \pm 0.0011(\text{Exp.}) \pm 0.0005(\text{Theo.}) = 0.1185 \pm 0.0012, \quad (3)$$

T

$$\alpha_s(M_Z^2) = 0.1193^{+0.0009}_{-0.0010}(\text{Exp.})^{+0.0019}_{-0.0016}(\text{Theo.}) = 0.1193^{+0.0021}_{-0.0019}, \quad (4)$$

C

Cited by LHeC and FCC group and PDG

[Particle Data Group], Prog. Theor. Exp. Phys. 2020 (2020), 083C01.

The Large Hadron-Electron Collider at the HL-LHC
LHeC Collaboration and FCC-he Study Group (P. Agostini (Santiago
CERN-ACC-Note-2020-0002, JLAB-ACP-20-3180
e-Print: **arXiv:2007.14491 [hep-ex]** | PDF

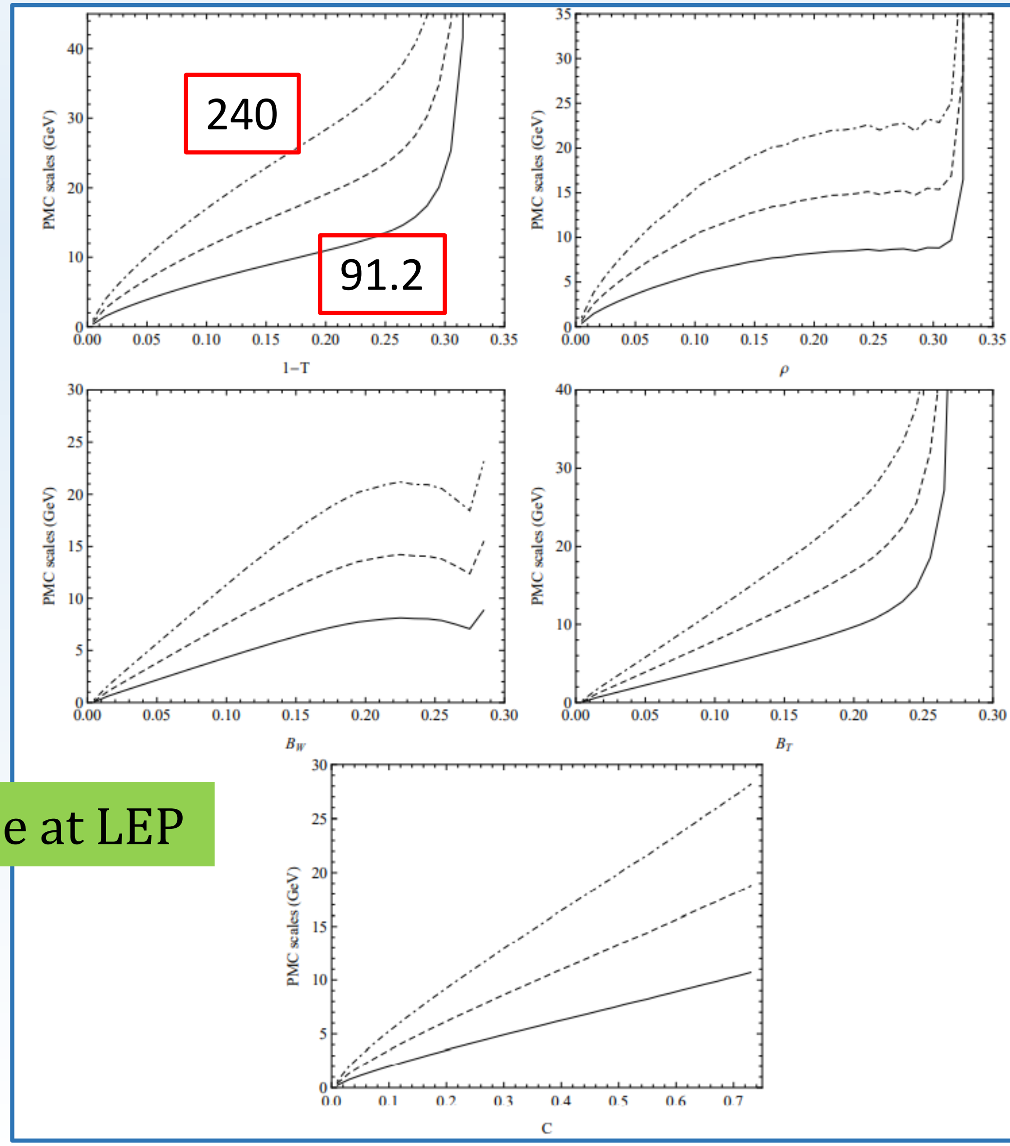
PDG

mean value for other event shapes, EEC,

四. Event shapes at CEPC

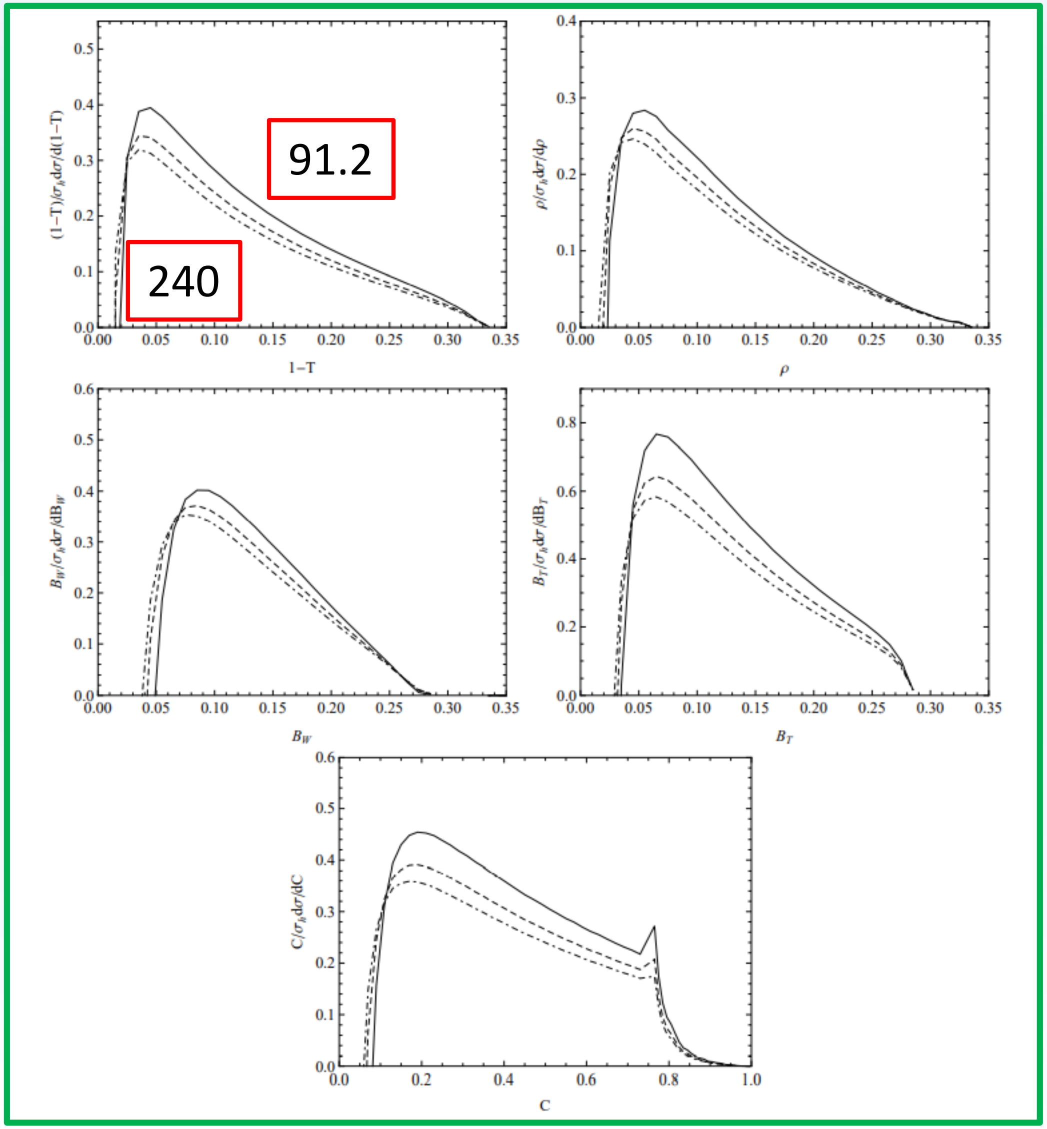
We calculate the classical event shapes at the CEPC at 91.2, 160 and 240 GeV.

PMC scales for event shape observables at CEPC



Similar to the case of the PMC scale at LEP

四. Event shape observables at CEPC



Our precise and scale-independent predictions for event shape observables, and a novel way to verify the running of αs(Q^2) call for the precise measurements at CEPC.

Summary

$$\beta^{\mathcal{R}} = \mu_r^2 \frac{\partial}{\partial \mu_r^2}\left(\frac{\alpha_s^{\mathcal{R}}(\mu_r)}{4\pi}\right) = -\sum_{i=0}^{\infty} \beta_i^{\mathcal{R}} \left(\frac{\alpha_s^{\mathcal{R}}(\mu_r)}{4\pi}\right)^{i+2}$$

- PMC method reduces in the Abelian limit to the Gell-Mann-Low method
- To eliminate the renormalization scheme-and-scale ambiguities
- There is no renormalon divergence in the pQCD series
- The more convergent perturbative series is in general achieved

thanks