Dressing jets with flavour in an infrared safe way

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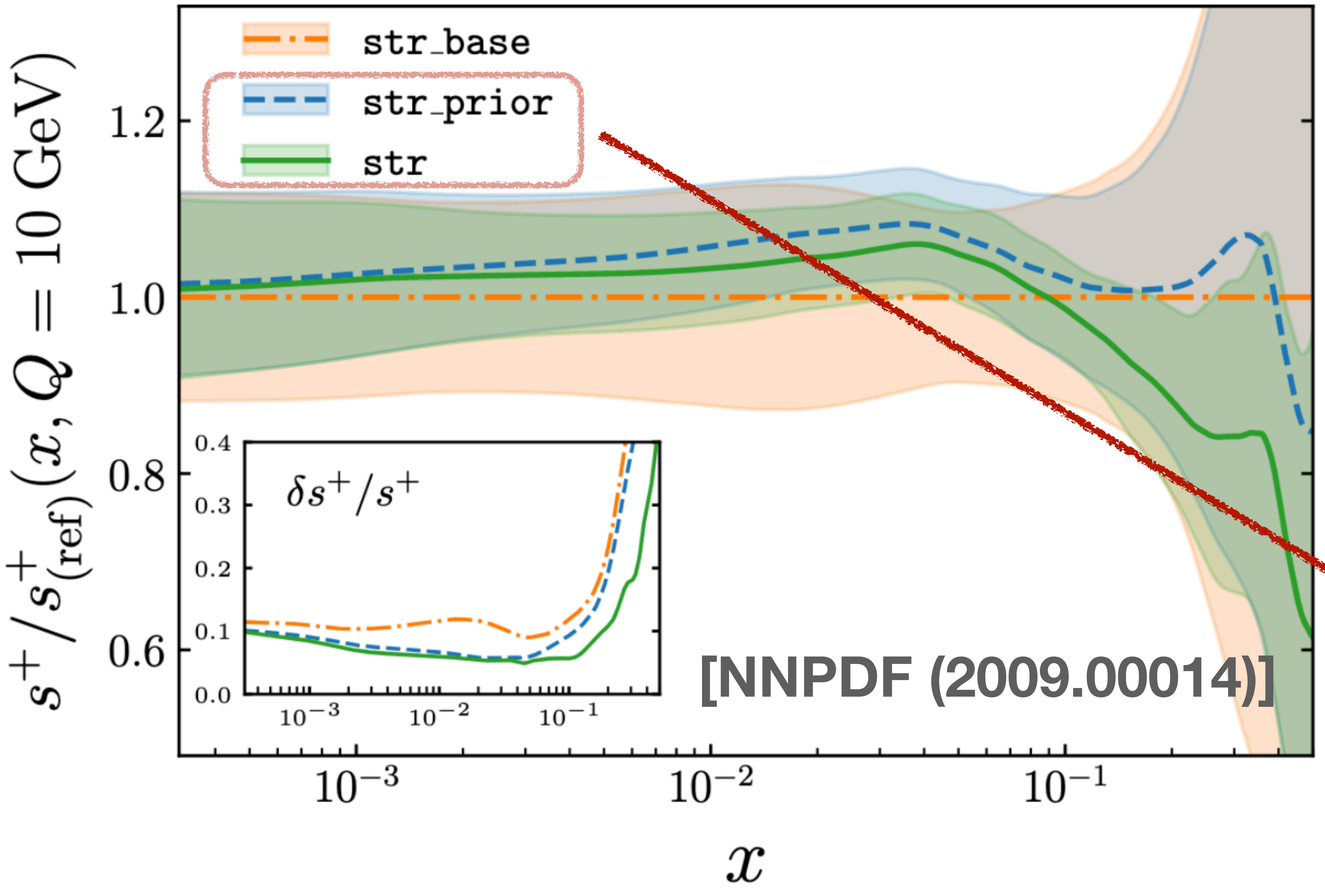
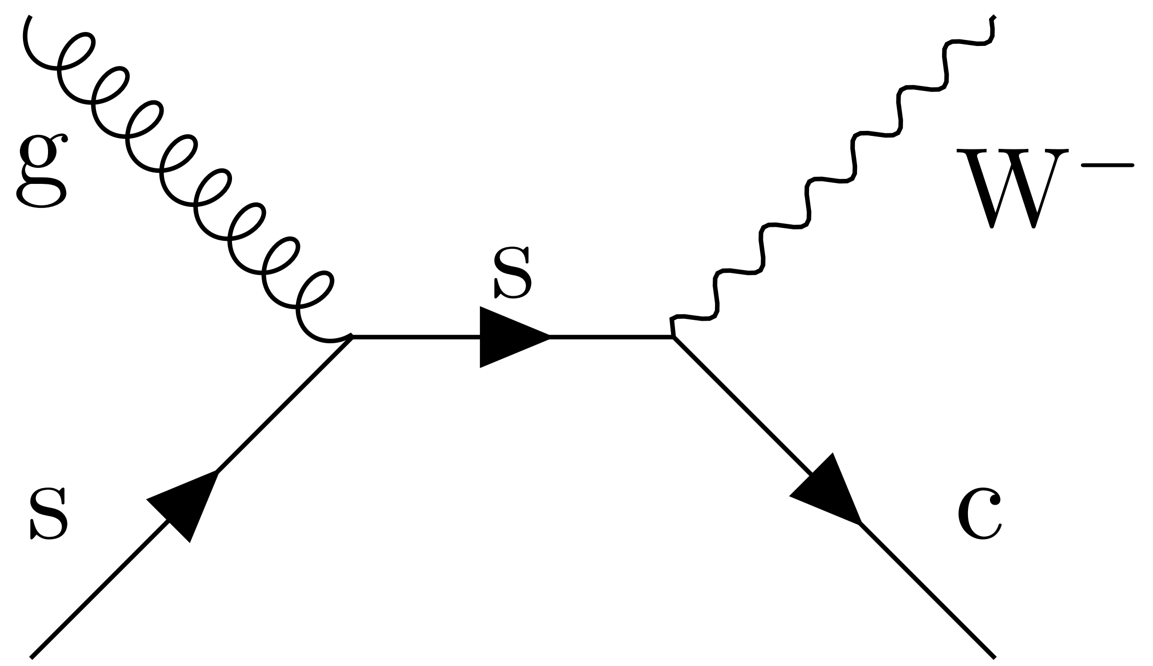
Why flavoured jets?



An example: W+c-jet
unique probe into the strange PDF

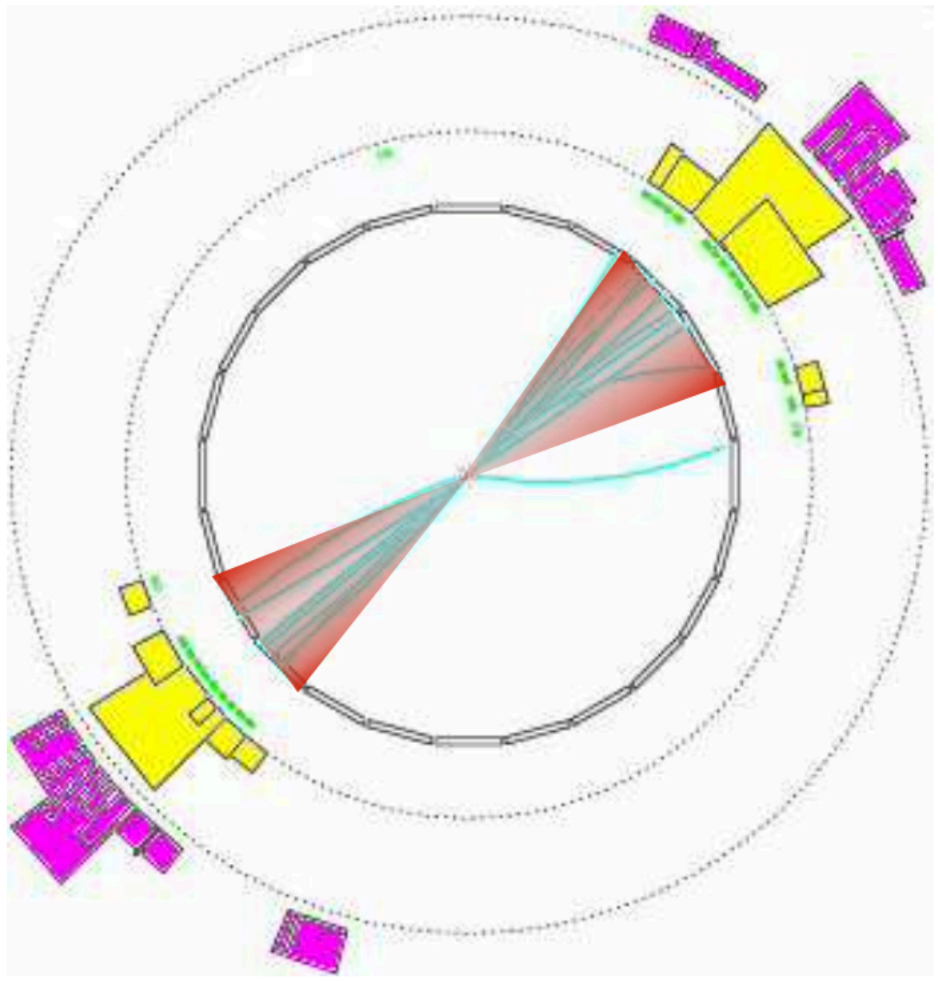


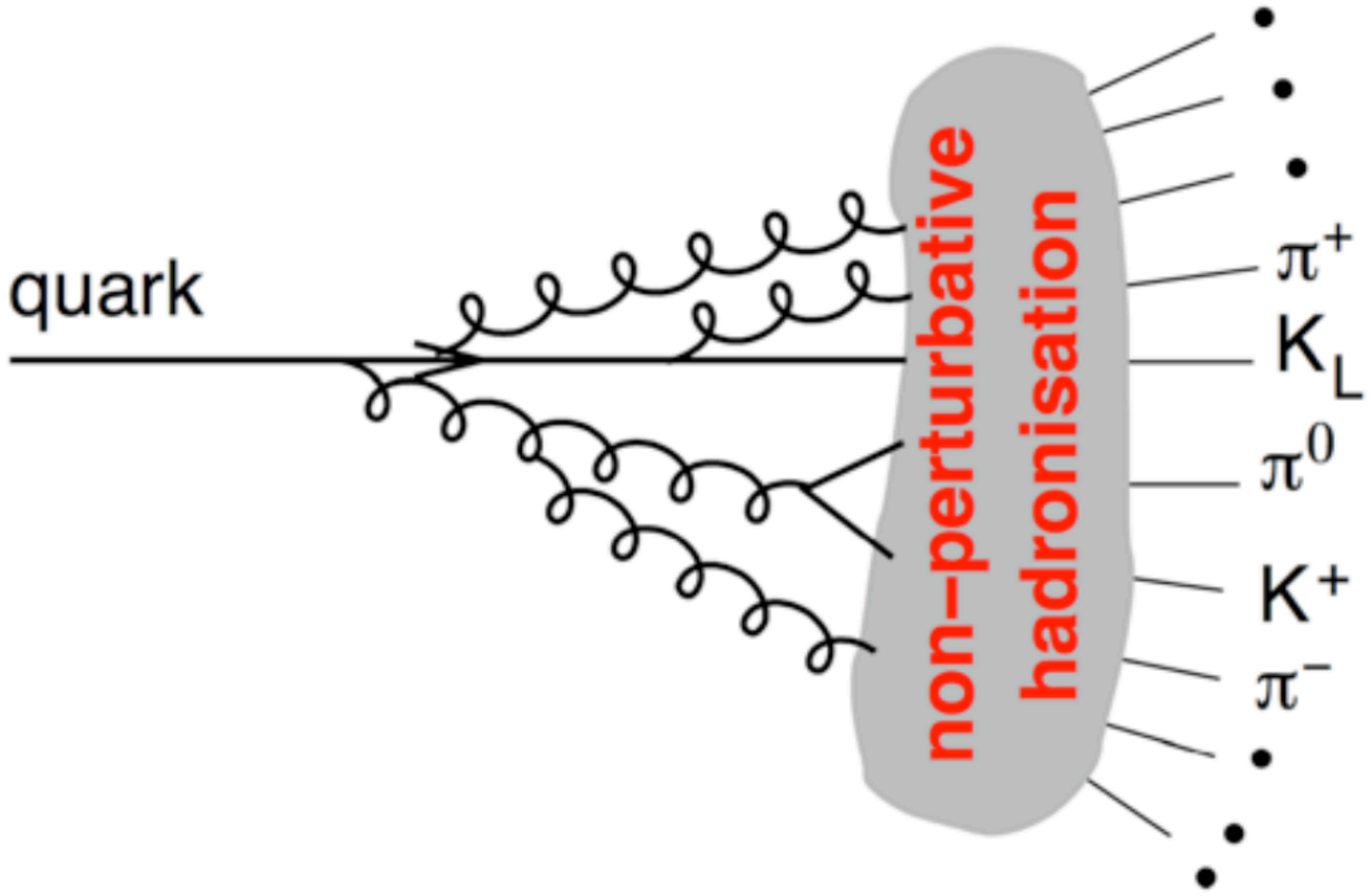
contain [ATLAS (1402.6263)] and [CMS (1310.1138)] 7 TeV data

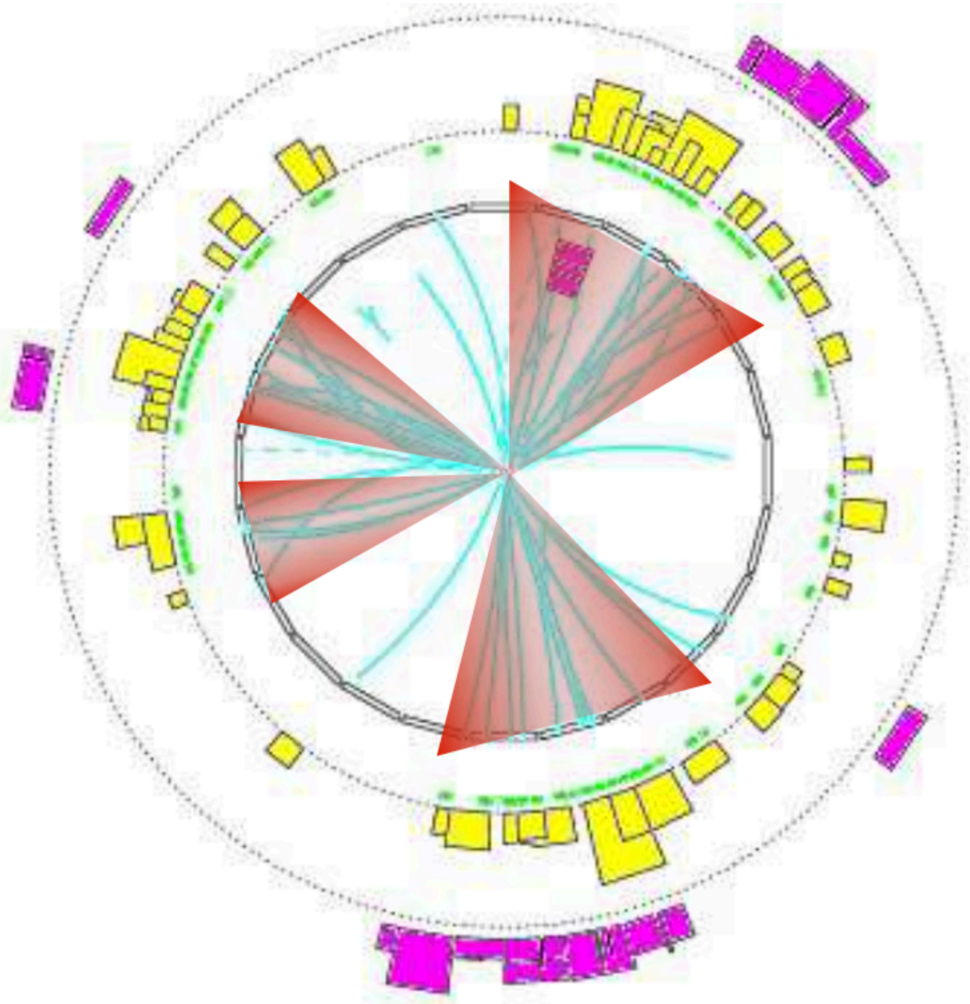
… but **flavoured jets appear everywhere**:
top physics, Higgs physics, new physics searches, …

What are jets?

Naive definition: **collimated bunch of hadrons** flying roughly in the same direction







2 clear jets

3 jets?
or 4 jets?

Proper definition: a collection of hadrons defined by means of a **jet algorithm**

"Jet [definitions] are legal contracts between theorists and experimentalists"
MJ Tannenbaum

IRC safety

An observable is **infrared and collinear safe** if, in the limit of a **collinear splitting**, or the **emission of an infinitely soft** particle, the observable remains **unchanged:**

$$O(X; p_1, \ldots, p_n, p_{n+1} \to 0) \to O(X; p_1, \ldots, p_n)$$
$$O(X; p_1, \ldots, p_n \parallel p_{n+1}) \to O(X; p_1, \ldots, p_n + p_{n+1})$$

This property ensures cancellation of **real** and **virtual** divergences in higher order calculations

If we wish to be able to calculate a jet rate in perturbative QCD the jet algorithm that we use must be IRC safe:
soft emissions and collinear splittings must not change the hard jets

slide stolen from Matteo Cacciari

Popular jet algorithms

e^+e^- colliders

$$d_{ij} = 2\min(E_i^2, E_j^2)\,(1 - \cos\theta_{ij})$$

k_t or Durham algorithm

pp colliders

$$d_{ij} = \min(p_{T,i}^{2p}, p_{T,j}^{2p})\frac{\Delta y^2 + \Delta\phi^2}{R^2}\,, \quad d_{iB} = p_{T,i}^{2p}$$

$p = 1$: k_t algorithm

$p = 0$: Cambridge/Aachen algorithm

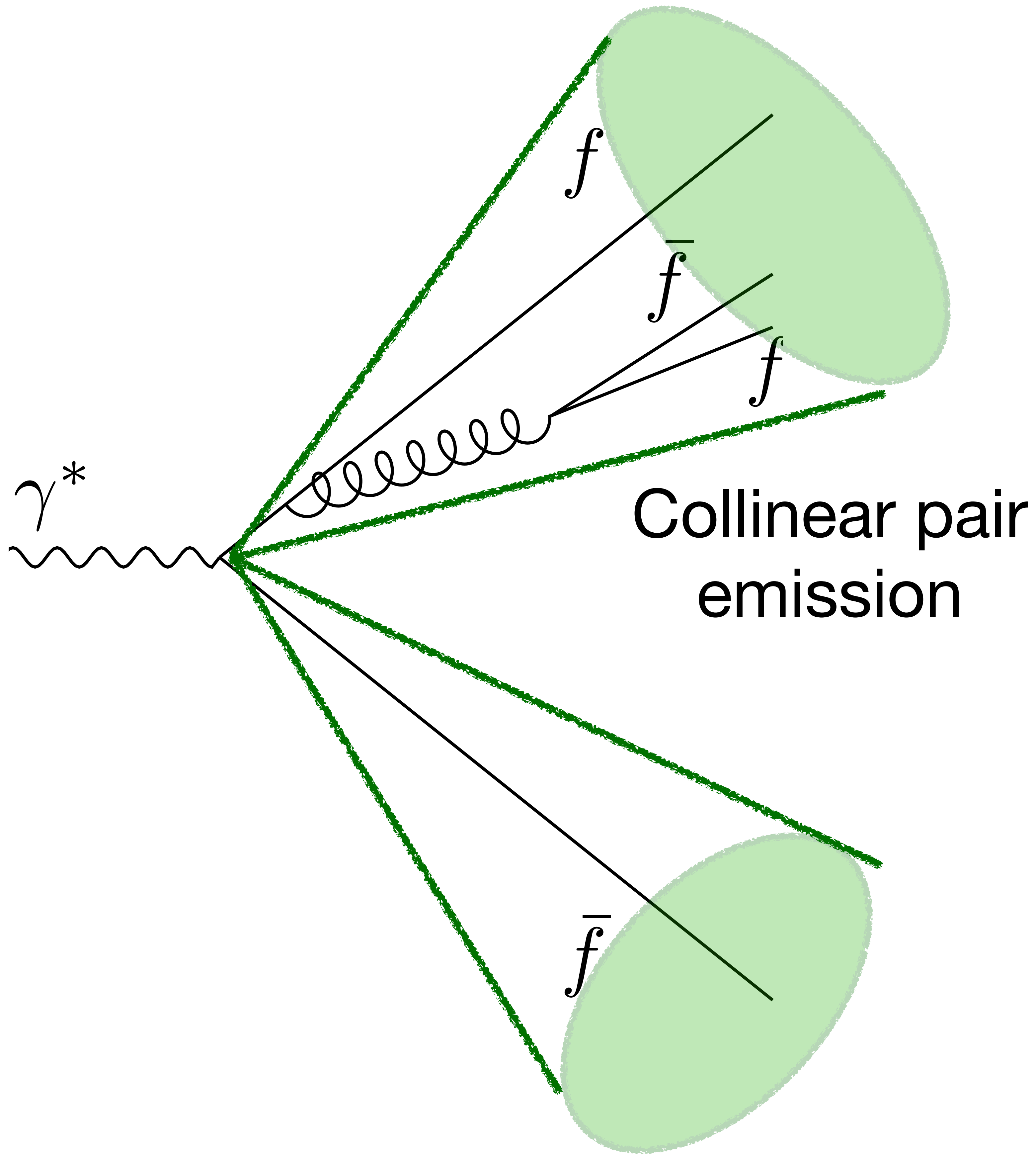
$p = -1$: anti-k_t algorithm

All IRC safe

IRC safe flavour definition of jets?

$e^+e^- \rightarrow$ jets with the k_t algorithm (flavour agnostic)
Issues at $\mathcal{O}(\alpha_s^2)$, when on top of $e^+e^- \rightarrow f\bar{f}$ we add a $f\bar{f}$ pair coming from gluon splitting.

When the $f\bar{f}$ pair from gluon is soft or collinear, the jet algorithm must return two flavoured jets.



Collinear safe
(as any IRC safe flavour-agnostic algorithm)

IRC safe flavour definition of jets?

$e^+e^- \to$ jets with the k_t algorithm (flavour agnostic)
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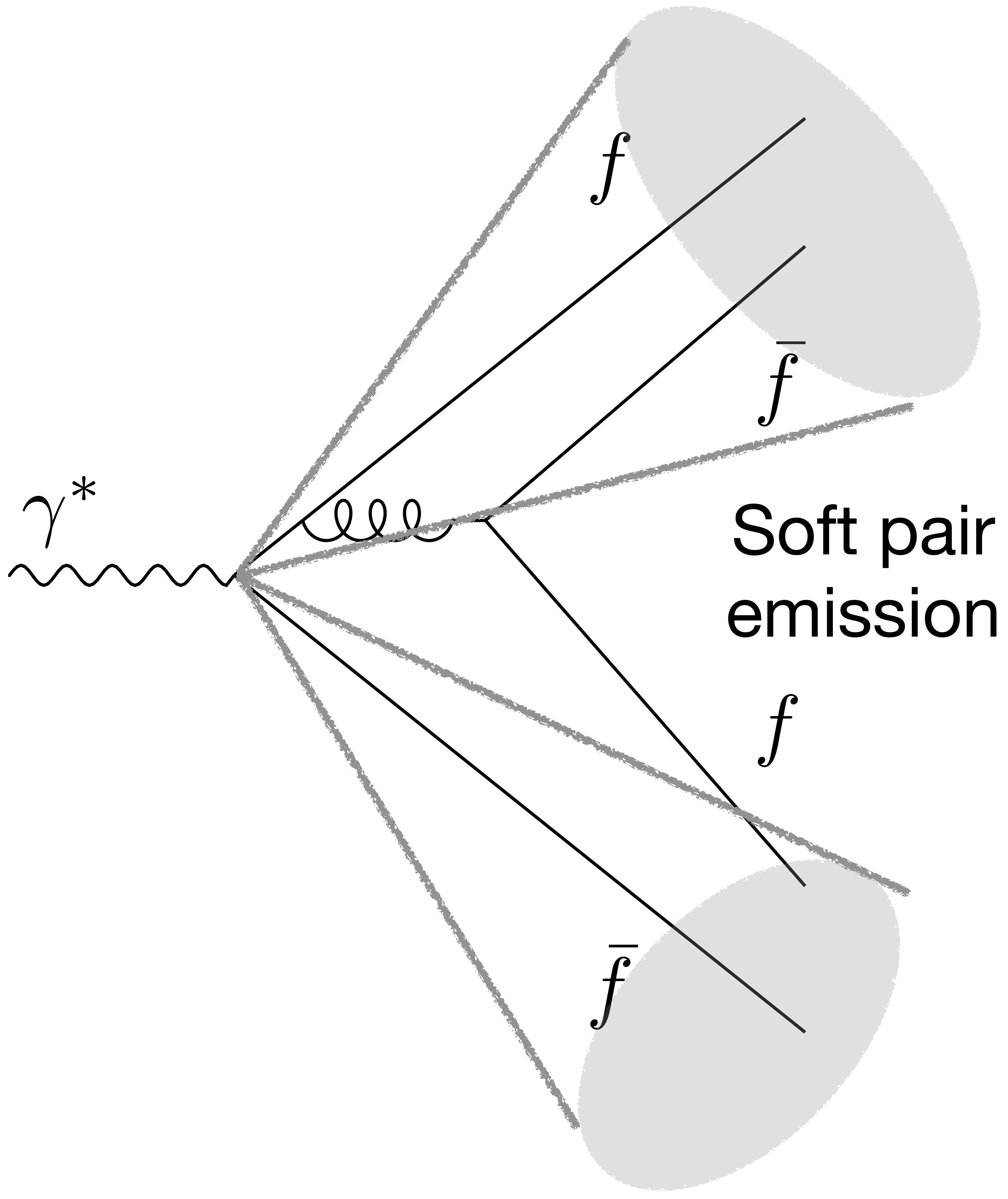


Soft **unsafe**
due to polluting large-angle soft pair

The flavour-k_t algorithm

[Banfi, Salam, Zanderighi (hep-ph/0601139)]

1. Introduce a distance measure $d_{ij}^{(F)}$ between every pair of partons i, j:

$$d_{ij}^{(F,\alpha)} = (\Delta\eta_{ij}^2 + \Delta\phi_{ij}^2) \times \begin{cases} \max(k_{ti}, k_{tj})^\alpha \min(k_{ti}, k_{tj})^{2-\alpha}\,, & \text{softer of } i,j \text{ is flavoured,} \\ \min(k_{ti}^2, k_{tj}^2)\,, & \text{softer of } i,j \text{ is flavourless,} \end{cases} \tag{17}$$

as well as distances to the two beams,

$$d_{iB}^{(F,\alpha)} = \begin{cases} \max(k_{ti}, k_{tB}(\eta_i))^\alpha \min(k_{ti}, k_{tB}(\eta_i))^{2-\alpha}\,, & i \text{ is flavoured,} \\ \min(k_{ti}^2, k_{tB}^2(\eta_i))\,, & i \text{ is flavourless,} \end{cases} \tag{18}$$

and an analogous definition of $d_{i\bar{B}}^{(F,\alpha)}$ involving $k_{t\bar{B}}(\eta_i)$ instead of $k_{tB}(\eta_i)$ (both defined as in eqs. (15) and (16)).[9] As in section 2 we have introduced a class of measures, parametrised by $0 < \alpha \leq 2$.

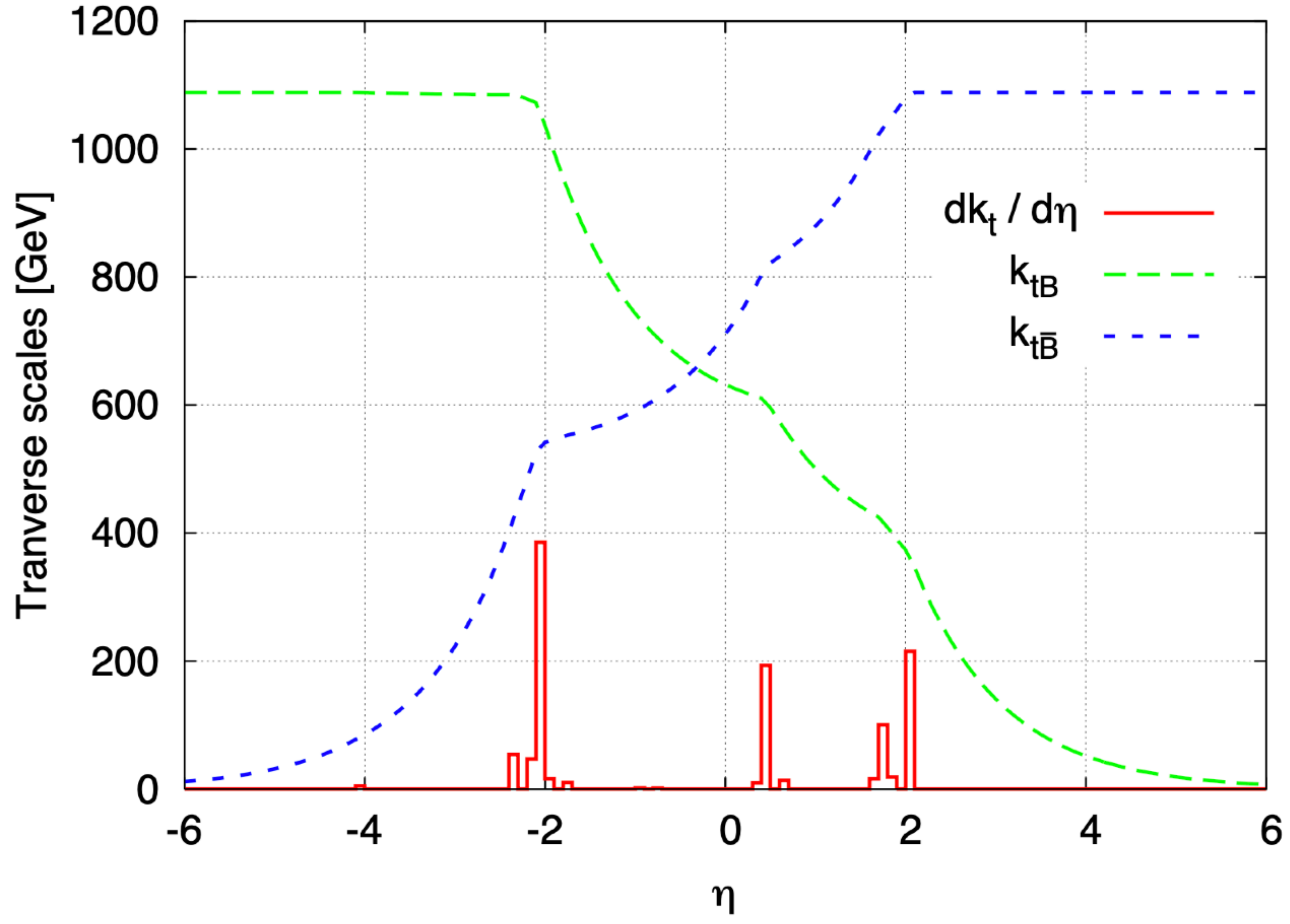
2. Identify the smallest of the distance measures. If it is a $d_{ij}^{(F,\alpha)}$, recombine i and j; if it is a $d_{iB}^{(F,\alpha)}$ ($d_{i\bar{B}}^{(F,\alpha)}$) declare i to be part of beam B ($\bar{B}$) and eliminate i; in the case where the $d_{iB}^{(F,\alpha)}$ and $d_{i\bar{B}}^{(F,\alpha)}$ are equal (which will occur if i is a gluon), recombine with the beam that has the smaller $k_{tB}(\eta_i)$, $k_{t\bar{B}}(\eta_i)$.

3. Repeat the procedure until all the distances are larger than some d_{cut}, or, alternatively, until one reaches a predetermined number of jets.[10,11]

Modified beam distance:

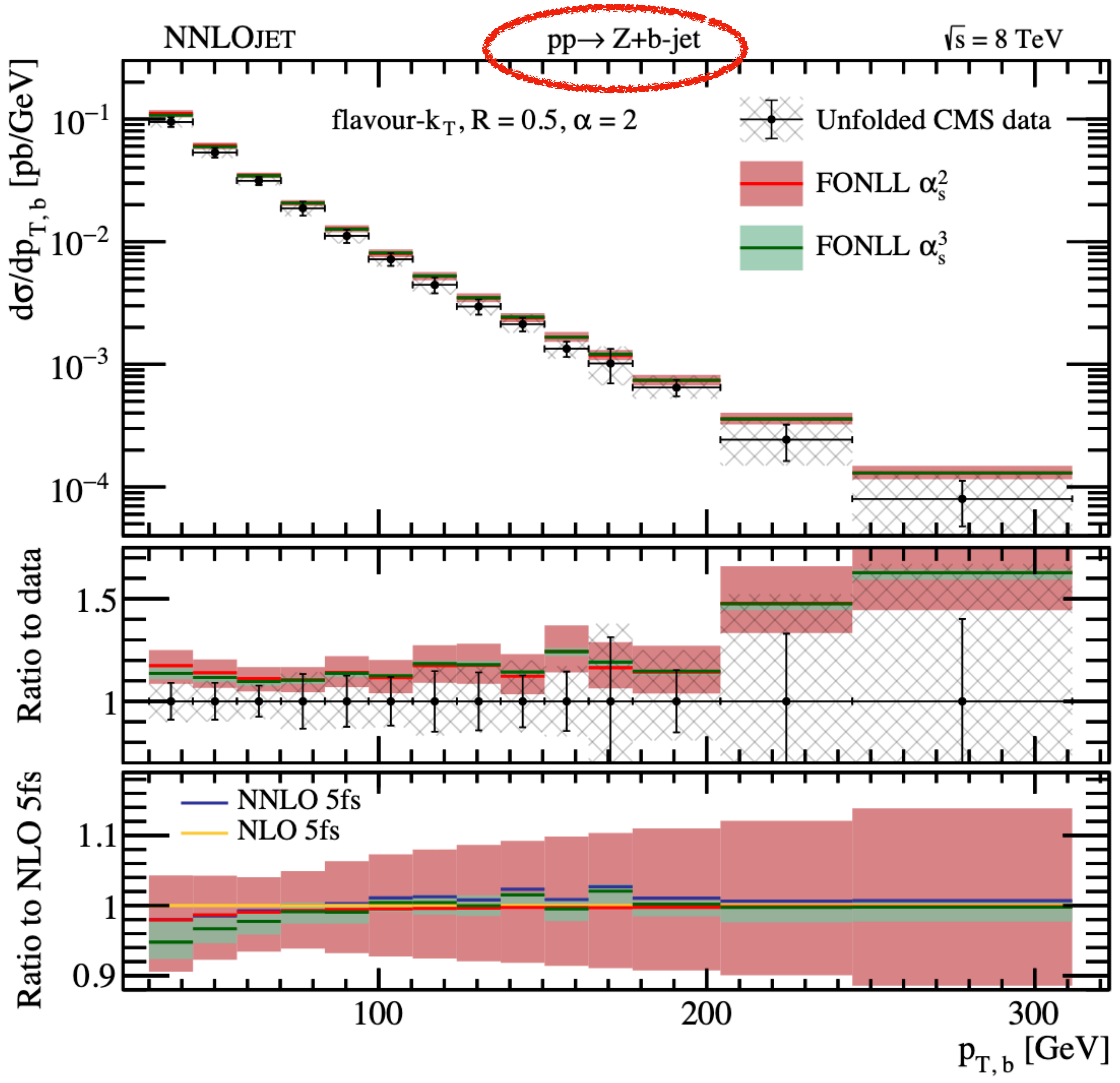
$$k_{tB}(\eta) = \sum_i k_{ti}\left(\Theta(\eta_i - \eta) + \Theta(\eta - \eta_i)e^{\eta_i - \eta}\right)$$

$$k_{t\bar{B}}(\eta) = \sum_i k_{ti}\left(\Theta(\eta - \eta_i) + \Theta(\eta_i - \eta)e^{\eta - \eta_i}\right)$$

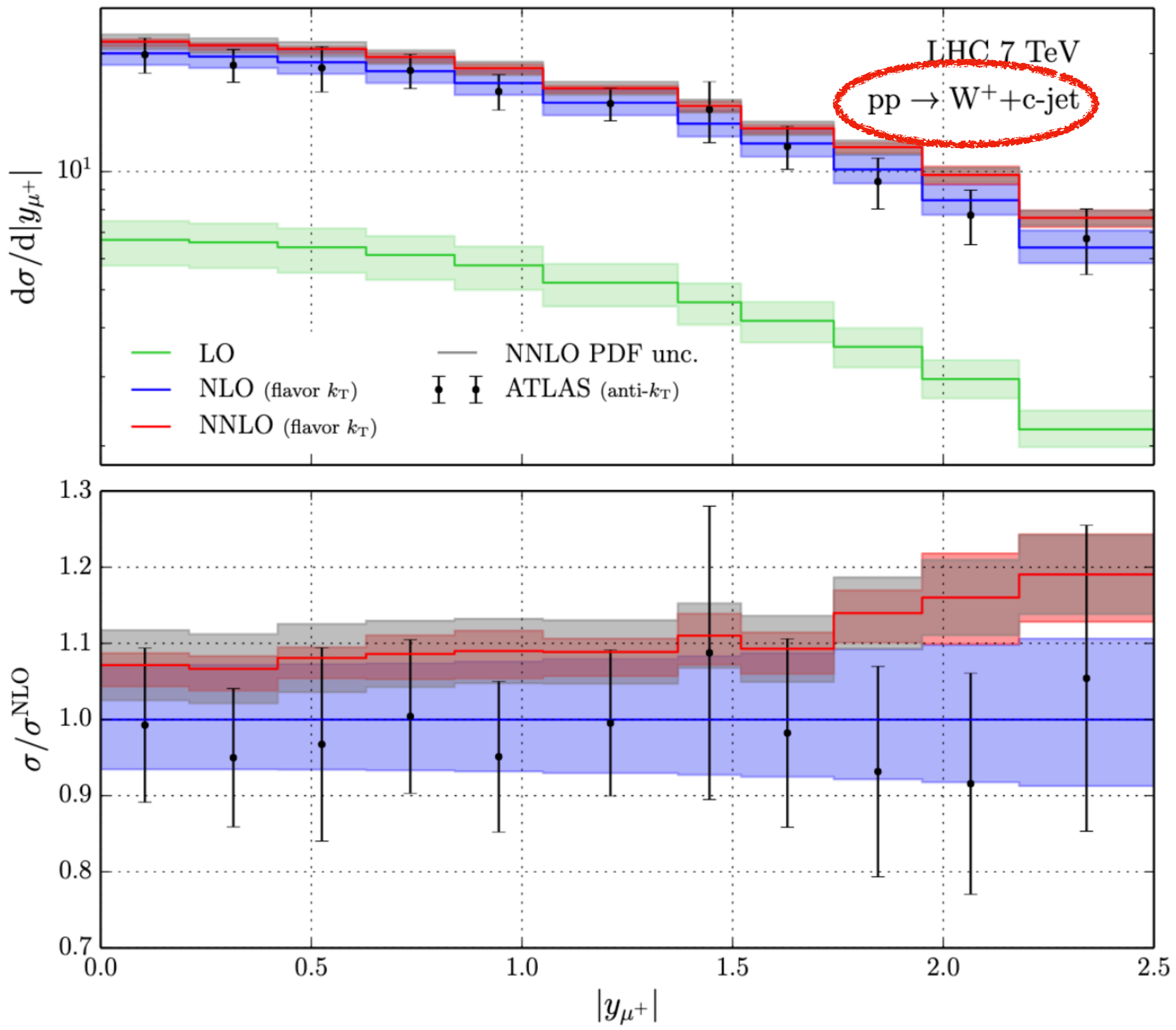


IRC flavour safe to all orders,
but different kinematics
(because new distance)

NNLO predictions with flavour-k_t



[Gauld et al. (2005.03016)]



[Czakon et al. (2011.01011)]

Comparison with experimental data **not straightforward**

Recent proposals

- based on Soft Drop grooming techniques
  [Caletti, Larkoski, Marzani, Reichelt (2205.01109)]
- through the alignment of flavoured particles along the Winner-Take-All axis
  [Caletti, Larkoski, Marzani, Reichelt (2205.01117)]
- through a modification of anti-k_t clustering distance
  [Czakon, Mitov, Poncelet (2205.11879)]
- with successive iterations of flavour-k_t and anti-k_t
  [Caletti, Fedkevych, Marzani, Reichelt (2108.10024)]
- using jet angularities and primary Lund jet plane as discriminants
  [Fedkevych, Khosa, Marzani, Sforza (2202.05082)]

However, none of the above reproduces the same jets as anti-k_t,
can be applied to a generic process with one or more jets
and it is IRC safe to all orders.

The flavour dressing algorithm

[Gauld, Huss, GS (2208.11138)]

Flavour assignment *factorised* from jet reconstruction:
we assign flavour to flavour-agnostic jets in an IRC safe way

Inputs:

- *Flavour agnostic jets* $\{j_k\}$
- *Flavoured clusters* $\{\hat{f}_i\}$
- *Association criterion*
- *Accumulation criterion*

The flavour dressing algorithm: inputs

- *Flavour agnostic jets* $\{j_k\}$*:*
  set of jets obtained with an IRC safe jet algorithm (e.g. gen-k_t family), possibly after a fiducial selection.

- *Flavoured clusters* $\{\hat{f}_i\}$

- *Association criterion*

- *Accumulation criterion*

The flavour dressing algorithm: inputs

- *Flavour agnostic jets* $\{j_k\}$

- *Flavoured clusters* $\{\hat{f}_i\}$*:*
  built out of quarks (e.g. c, b) or stable heavy-flavour hadrons (e.g. D, B), by dressing them with radiation close in angle (see below)
  “Naked” flavoured objects are collinear unsafe

- *Association criterion*

- *Accumulation criterion*

The flavour dressing algorithm: inputs

- *Flavour agnostic jets* $\{j_k\}$
- *Flavoured clusters* $\{\hat{f}_i\}$
- *Association criterion:* whether f_i is "associated" to j_k
  At parton-level simply if f_i is a constituent of j_k
  Other options: $\Delta R(f_i, j_k) < R_{\text{tag}}$, ghost association, …
  Flavour assignment based only on the association is not IRC safe
- *Accumulation criterion*

The flavour dressing algorithm: inputs

- *Flavour agnostic jets* $\{j_k\}$
- *Flavoured clusters* $\{\hat{f}_i\}$
- *Association criterion*
- *Accumulation criterion:* how to “sum” flavours
  - sum flavoured if unequal number of f and $\bar{\mathrm{f}}$ (need charge information)
  - sum flavoured if odd number of f or $\bar{\mathrm{f}}$ (if no charge information)

Definition of flavoured cluster $\hat{f}_i$

1. Initialise a set with all the flavourless objects p_i (particles used as input to jets) and all the flavoured objects f_i (bare flavours), avoiding double counting if necessary.

2. Find the pair with the smallest angular distance ΔR_{ab}:
   — flavourless p_a, p_b: combine p_a and p_b into a flavourless p_{ab};
   — flavoured f_a, f_b: remove both from the set;
   — flavoured f_a, unflavoured p_b: remove p_b from the set and check a Soft Drop criterion

$$\frac{\min(p_{t,a}, p_{t,b})}{(p_{t,a} + p_{t,b})} > z_{\text{cut}} \left(\frac{\Delta R_{ab}}{\delta R} \right)^{\beta}$$

   to **recombine collinear while preserving soft.** [default: $\delta R = 0.1$, $z_{\text{cut}} = 0.1$, $\beta = 2$]
   If satisfied, combine f_a and p_b into a flavoured f_{ab}.

3. Iterate while there are at least two objects in the set until $\Delta R_{ab} > \delta R$.
   The momentum of $\hat{f}_i$ is given by the accumulated momentum into f_i.

The flavour dressing algorithm

1. Define tag_k = flavoured clusters assigned to jet j_k (initialised as empty for all jets) and populate set of distances:
   - $d(\hat{f}_i, \hat{f}_j)$ between flavoured clusters;
   - $d(\hat{f}_i, \hat{j}_k)$ if flavoured cluster $\hat{f}_i$ *associated* to jet j_k
   - $d_B(\hat{f}_i)$ if $\hat{f}_i$ not associated to any jet.

   Distances (including beam) inherited from the flavour-k_t algorithm:

$$d(a,b) = \Delta R_{ab}^2 \max\left(p_{T,a}^{\alpha}, p_{T,b}^{\alpha}\right) \min\left(p_{T,a}^{2-\alpha}, p_{T,b}^{2-\alpha}\right)$$

$$d_{B\pm}(f) = \max\left(p_{t,f}^{\alpha}, p_{t,B_\pm}^{\alpha}(y_f)\right)\ \min\left(p_{t,f}^{2-\alpha}, p_{t,B_\pm}^{2-\alpha}(y_f)\right)$$

The flavour dressing algorithm

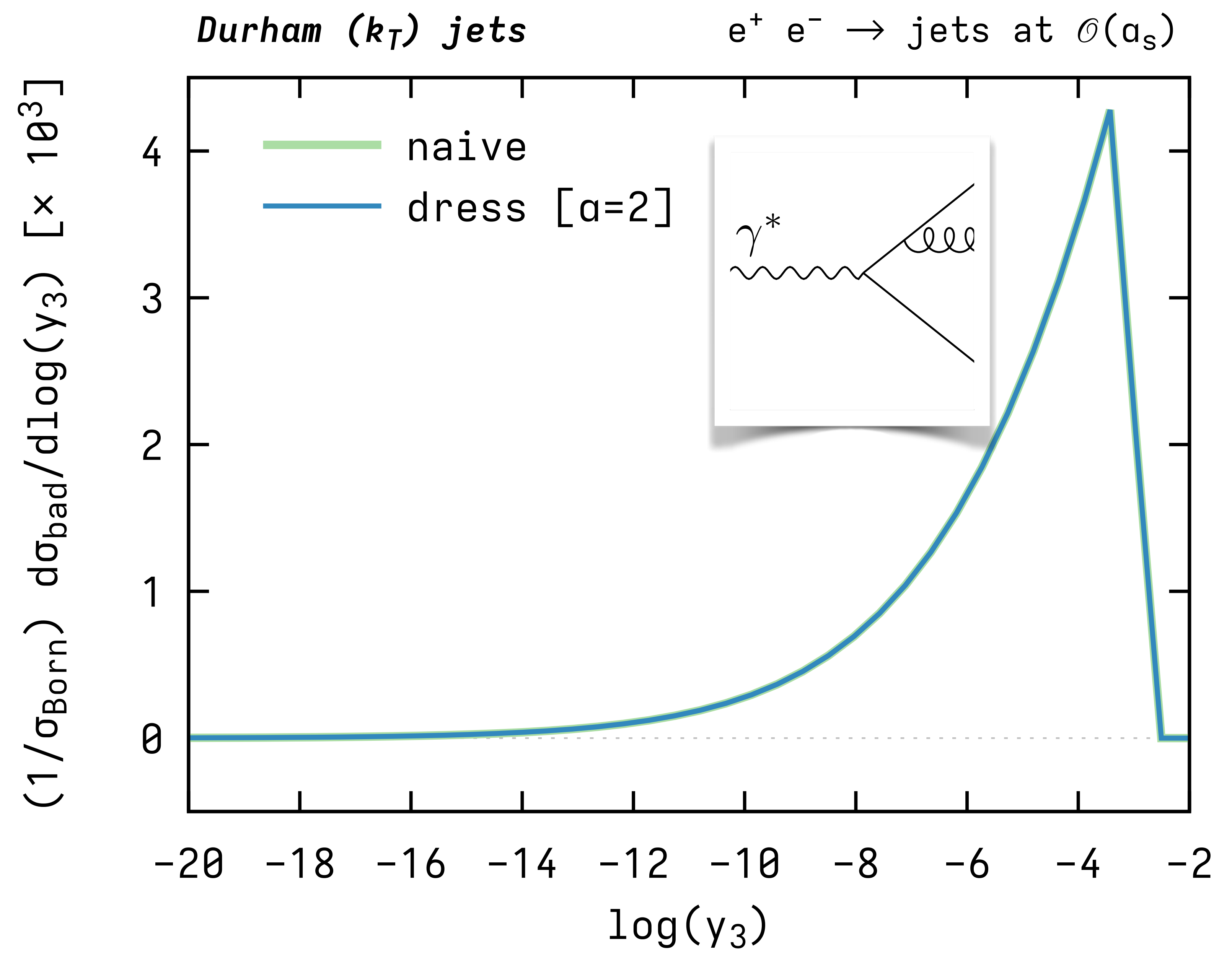
2. While the set of distances is not empty, select the smallest distance:
   $\rightarrow d(\hat{f}_i, \hat{f}_j)$:
   the two flavours "annihilate", hence remove distances that involve $\hat{f}_i$ or $\hat{f}_j$;
   $\rightarrow d(\hat{f}_i, \hat{j}_k)$:
   update $\mathrm{tag}_k = \mathrm{tag}_k \cup \{f_i\}$, then remove distances that involve $\hat{f}_i$.
   $\rightarrow d_B(\hat{f}_i)$:
   discard flavour $\hat{f}_i$ and remove all entries that involve $\hat{f}_i$.

3. Assign flavour to jet j_k according to tag_k and *accumulation* criterion.

IRC safety test in $e^+e^- \to$ jets

Vanishing mis-identification of flavours in the fully unresolved regime = IRC safety

only soft and/or collinear radiation



Any gen-k_t algo is safe!

IRC safety test in $e^+e^- \to$ jets

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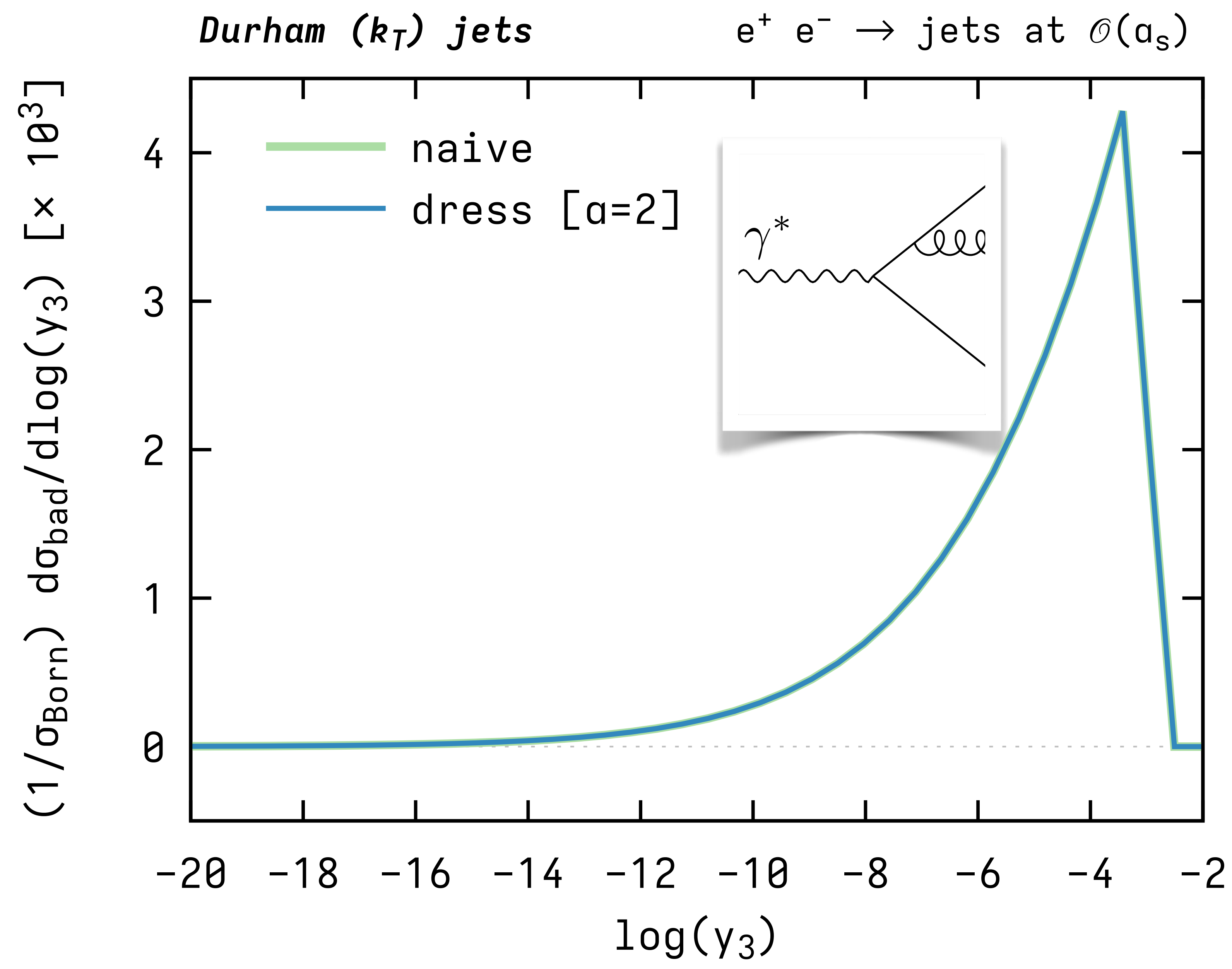
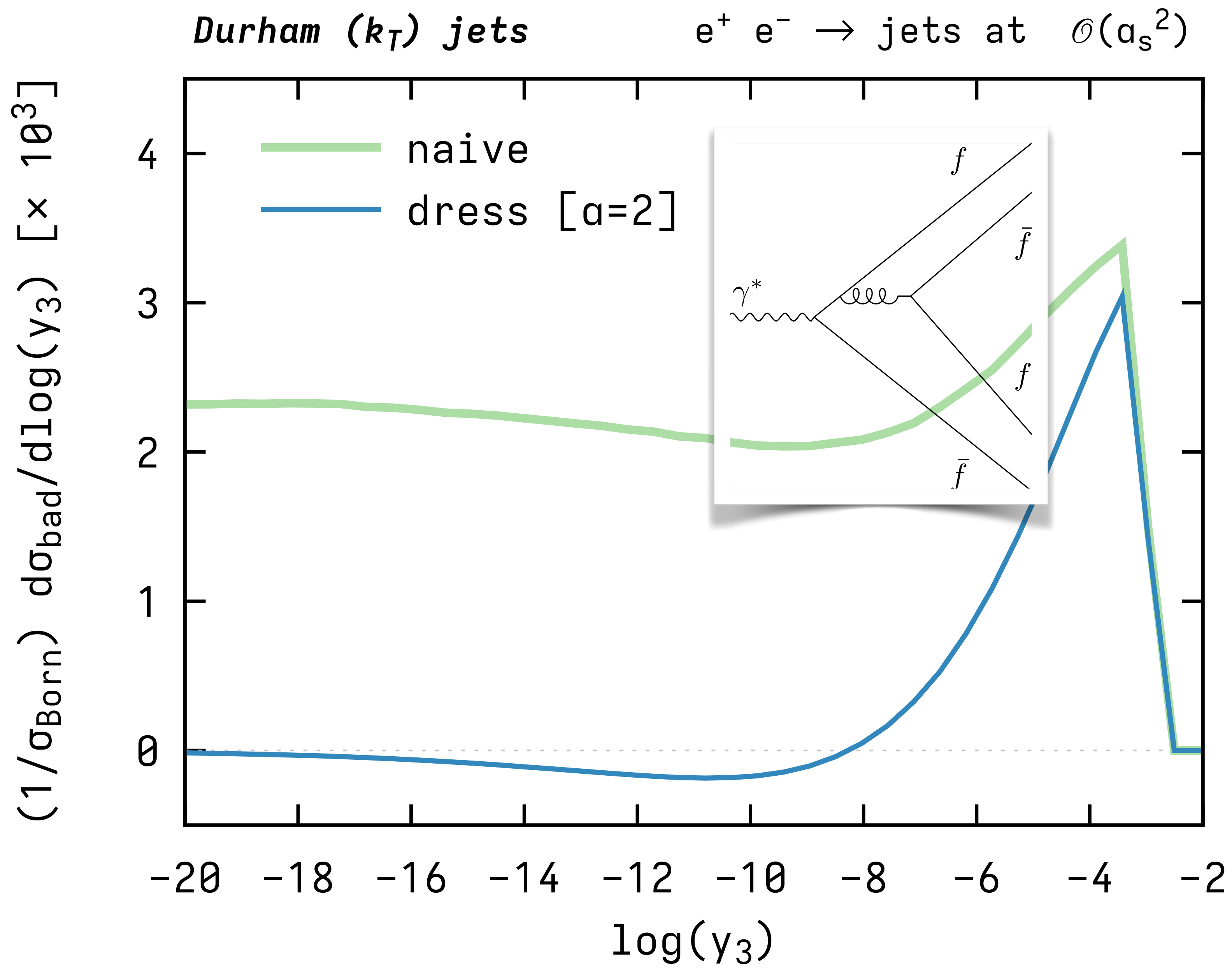




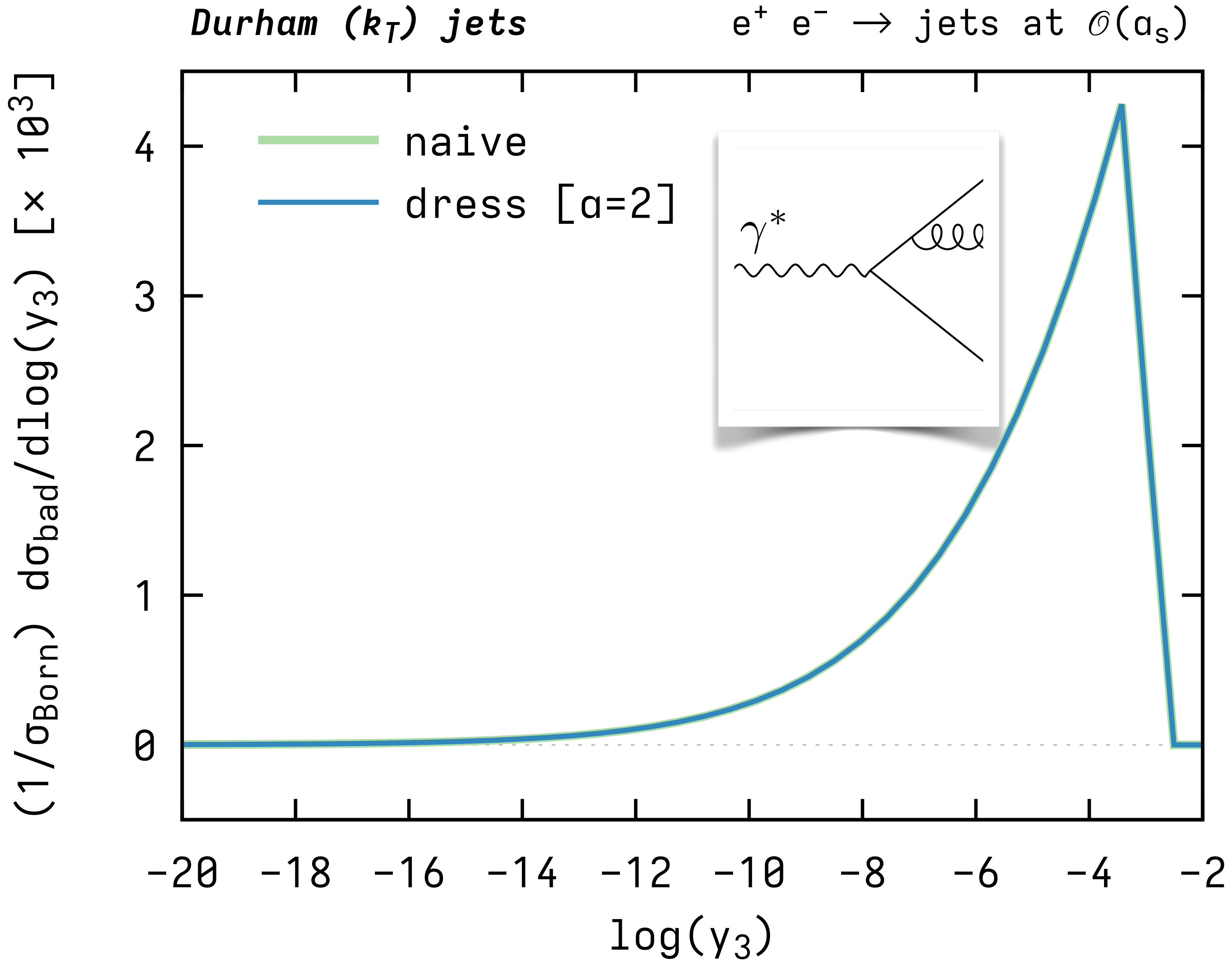
Any gen-k_t algo is safe!

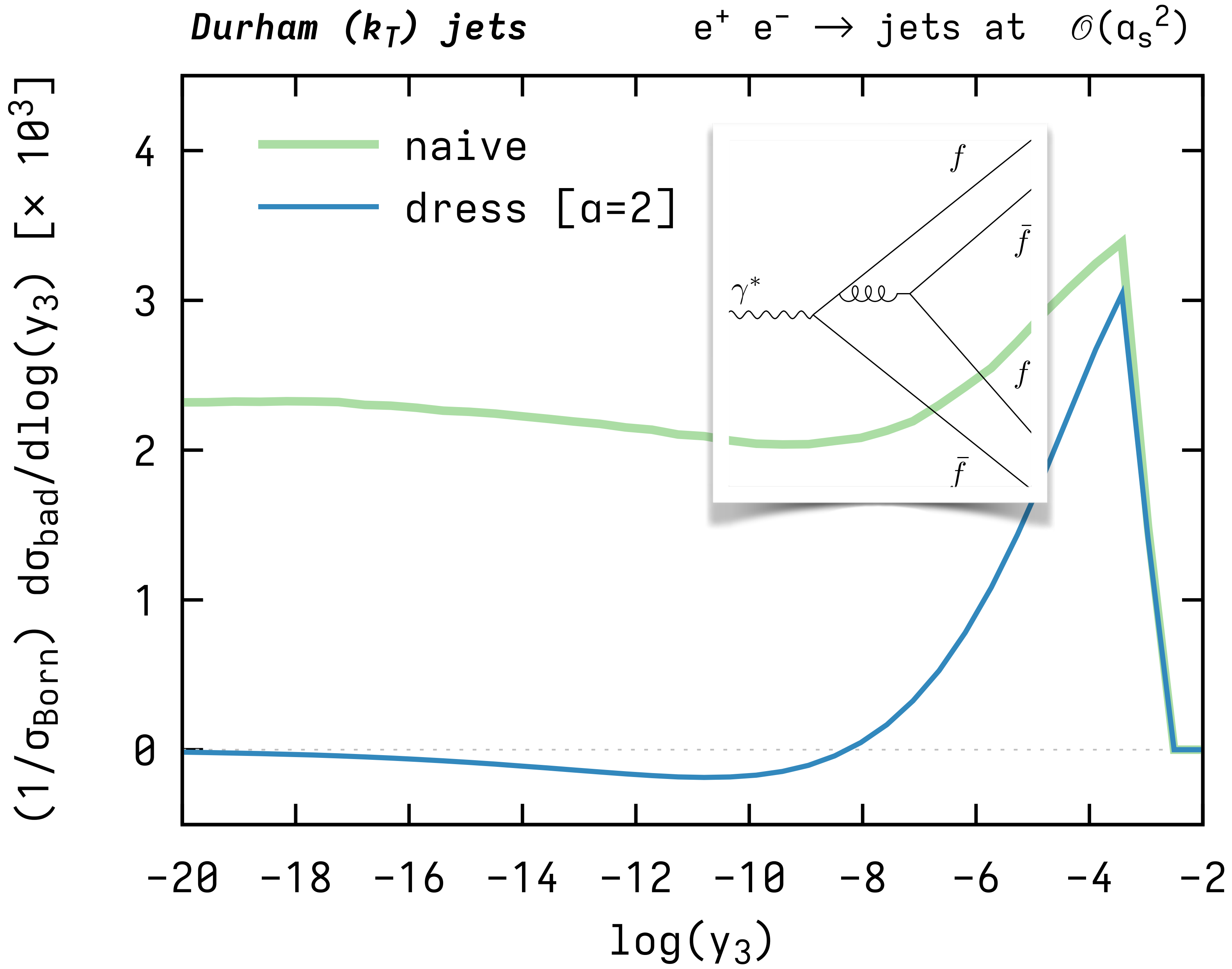
Naive dressing unsafe,
flavour dressing safe!

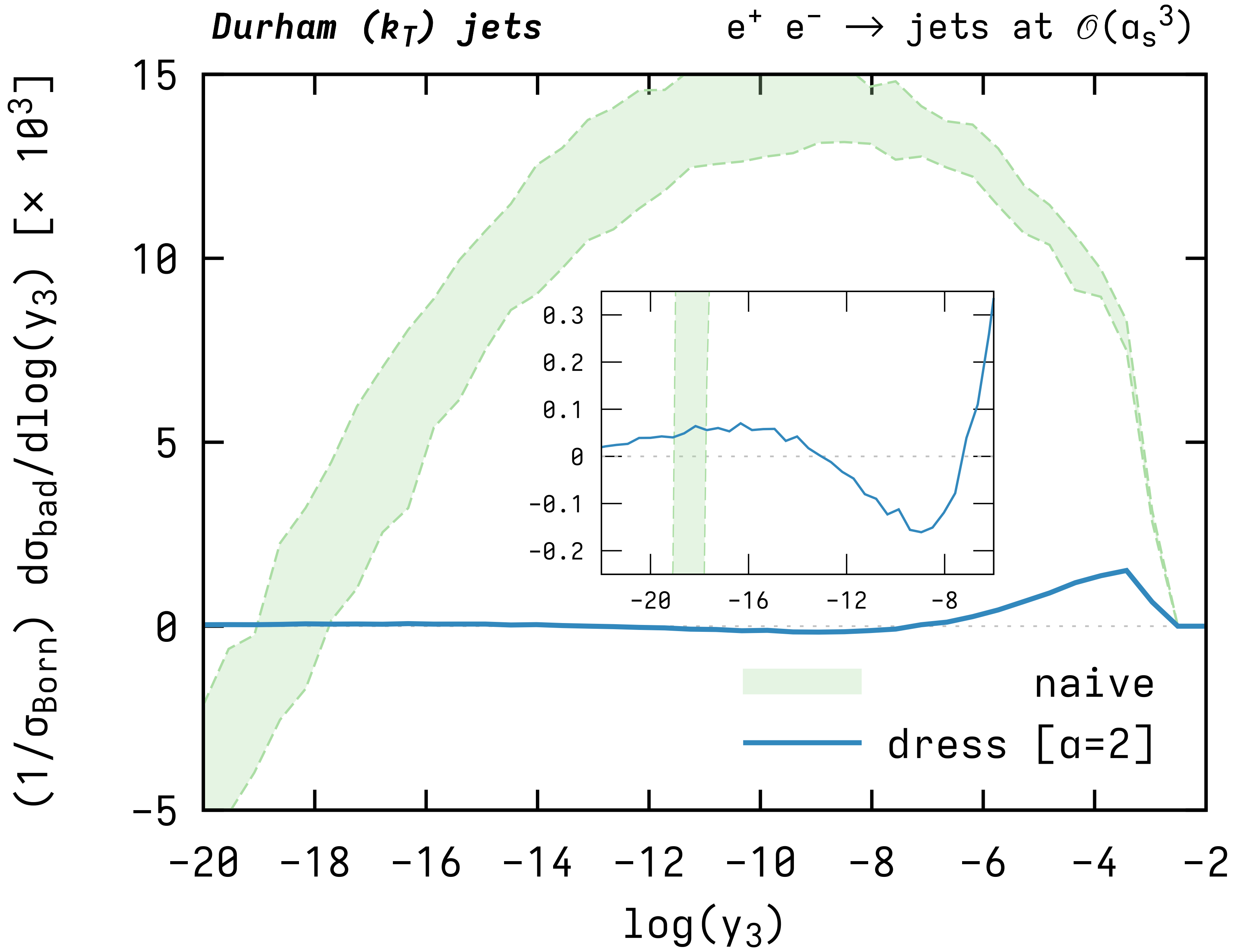
IRC safety test in $e^+e^- \to$ jets

Vanishing mis-identification of flavours in the fully unresolved regime = IRC safety

only soft and/or collinear radiation





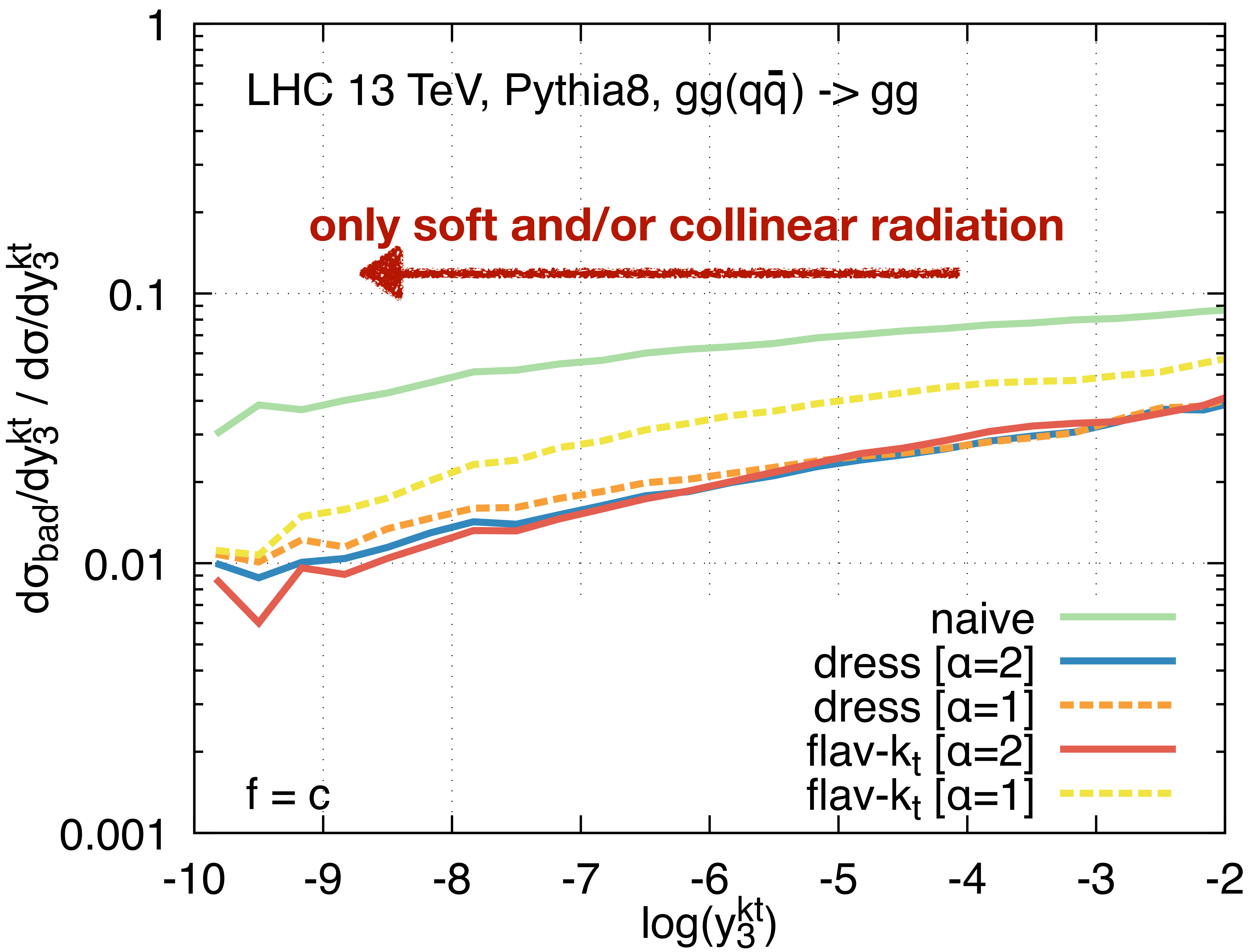


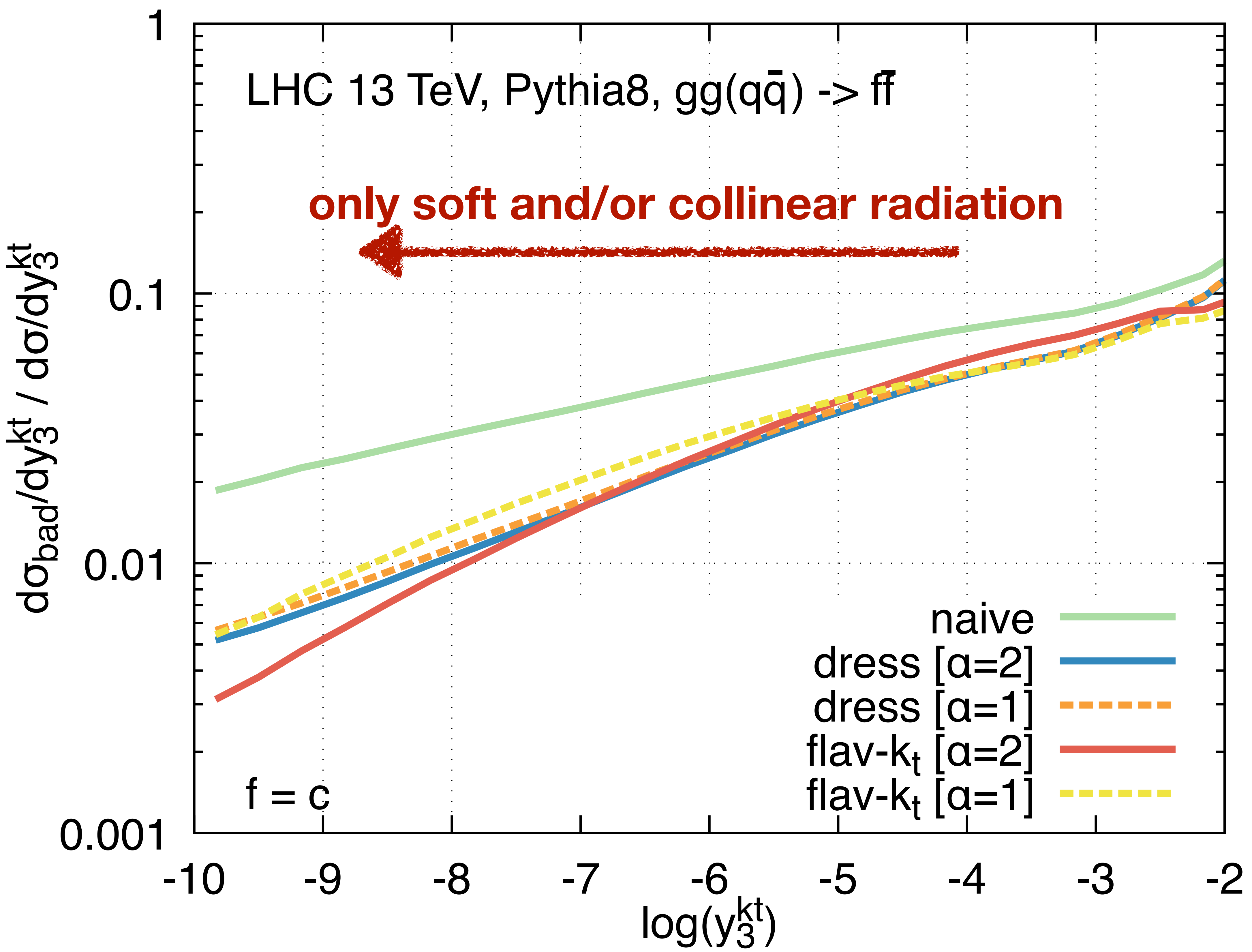
Any gen-k_t algo is safe!

Naive dressing unsafe, flavour dressing safe!

Naive dressing unsafer, flavour dressing still safe!

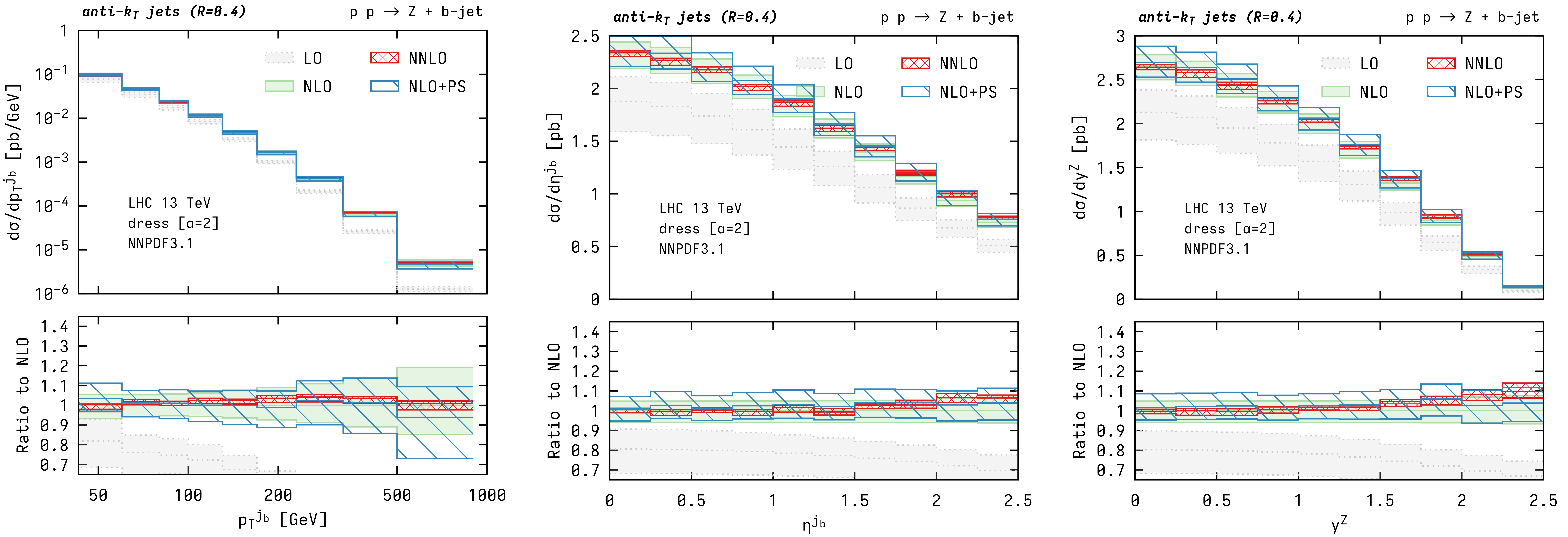
IRC sensitivity in $2 \to 2$ QCD events in pp





Flavour dressing approaches zero faster than a naive flavour tagging as $y_3^{k_t} \to 0$

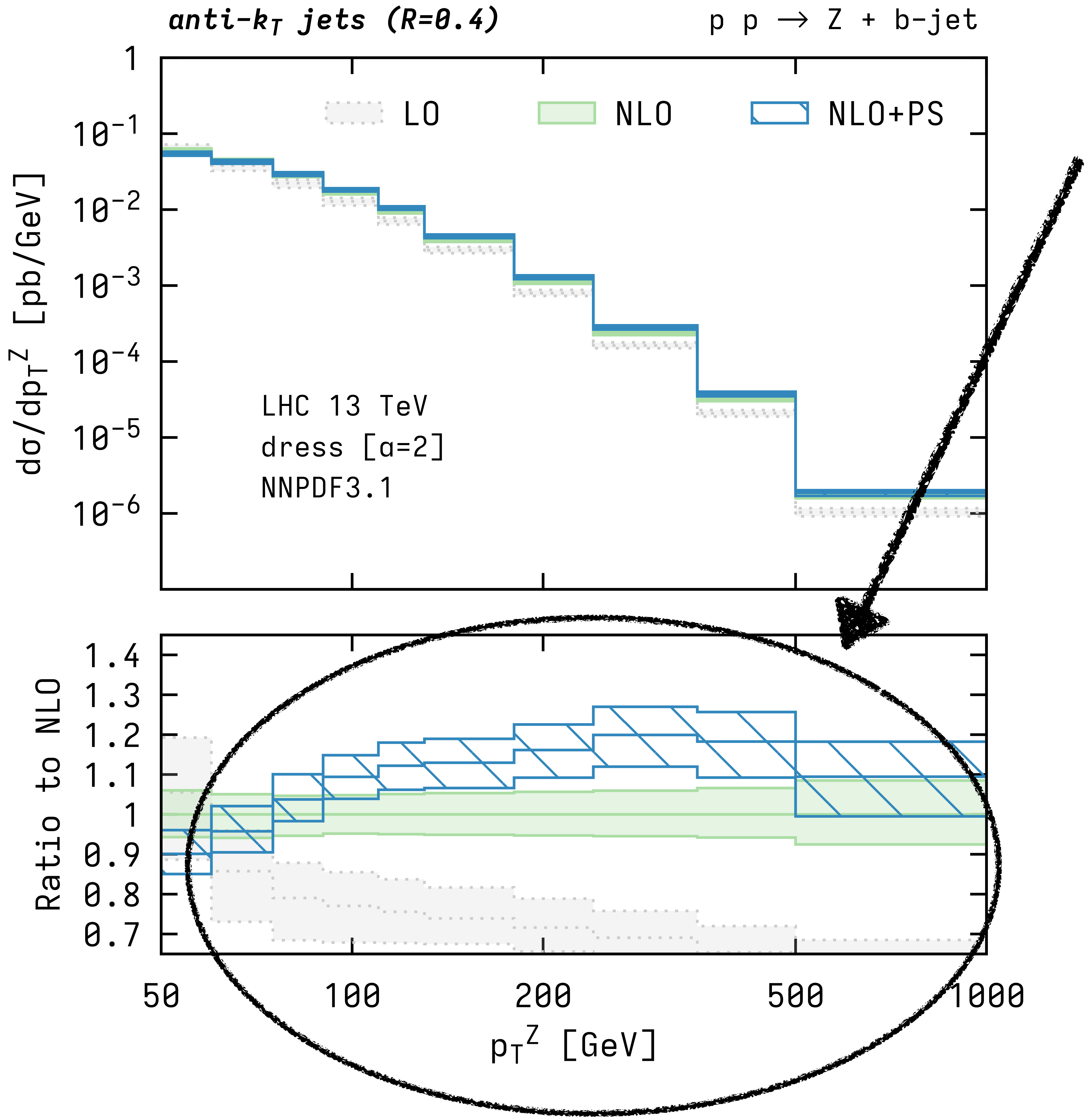
Test in a realist scenario: $Z + b$-jet



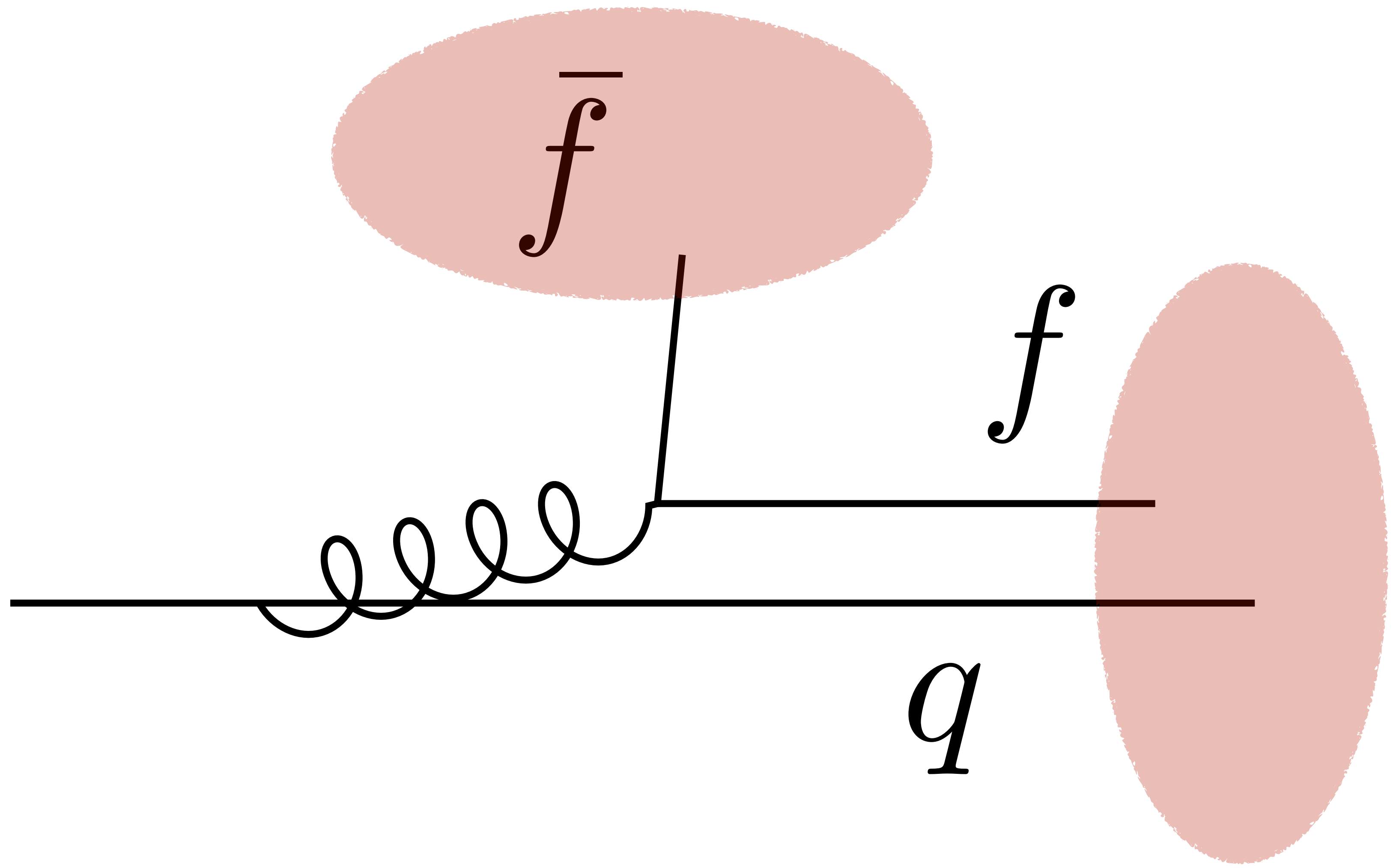
Remarkable agreement between (N)NLO and NLO+PS

→ for most distributions **largely insensitive to all-order corrections**

Test in a realist scenario: $Z + b$-jet

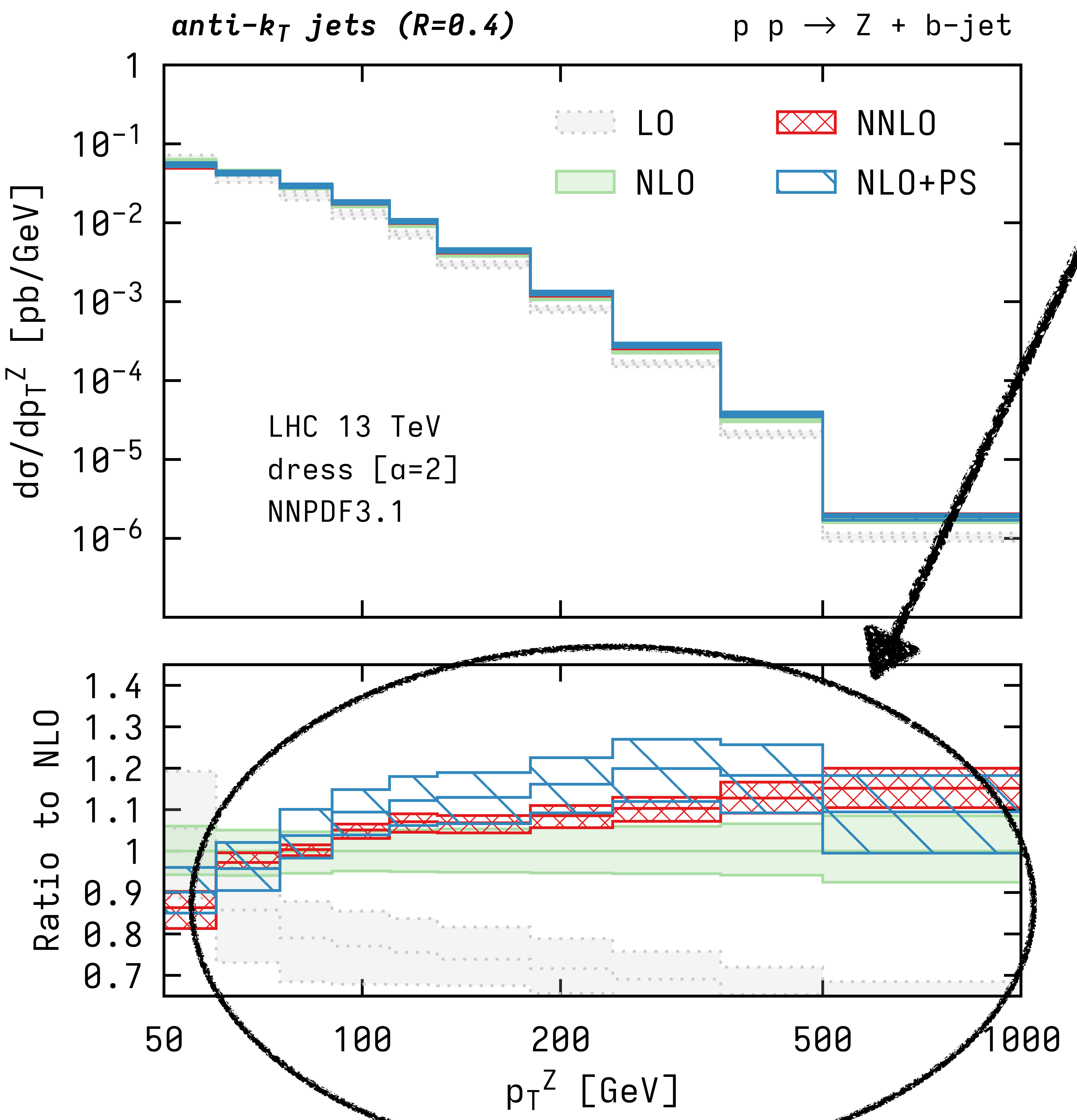


Some sensitivity observed in p_T^Z, likely due to:



Even if IRC finite, it leads to large migration of (unflavoured)-jet into the b-jet sample.

Test in a realist scenario: $Z + b$-jet



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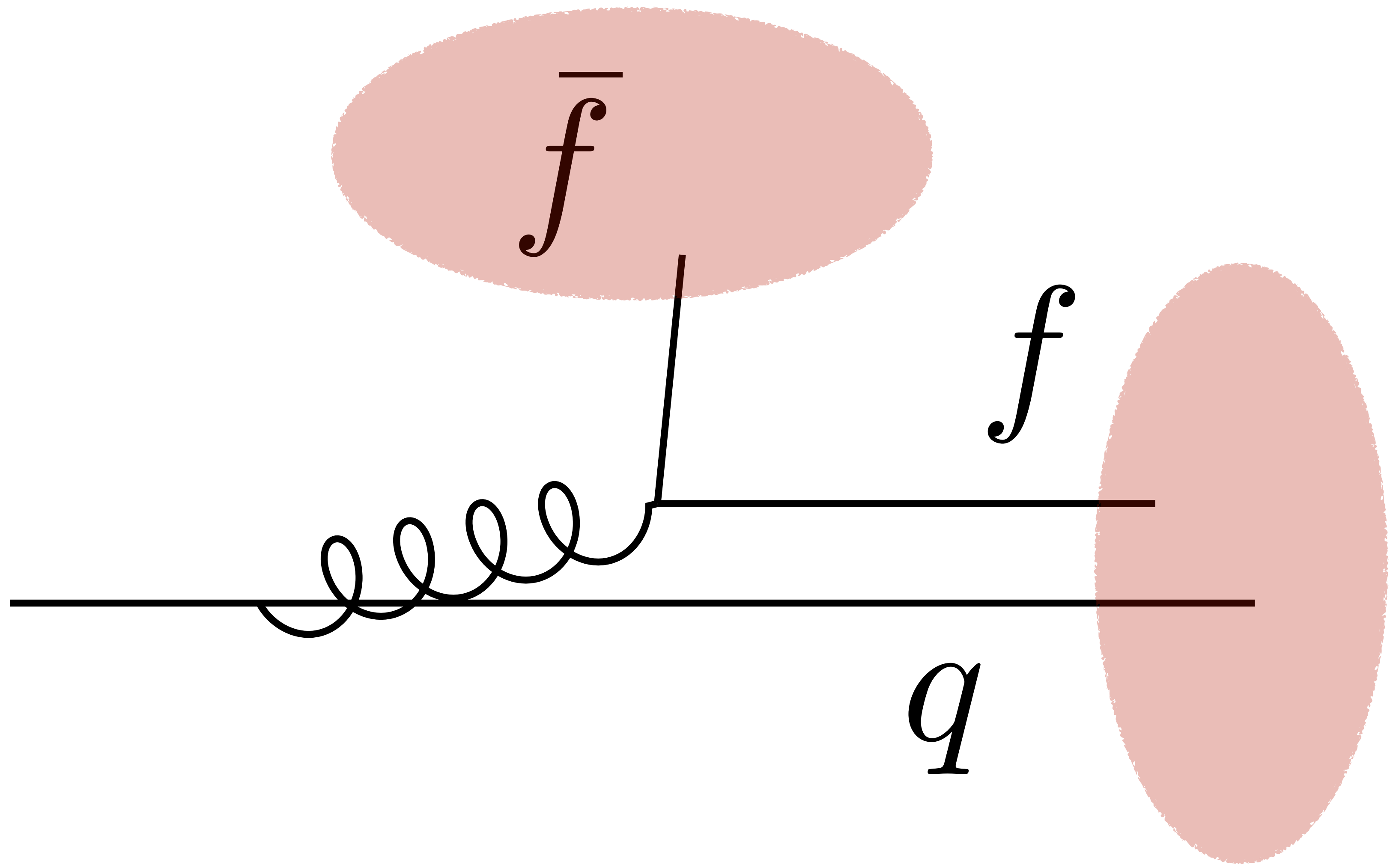


Effect **captured at NNLO**

Conclusions

With favour dressing, **flavour assignment *factorised* from the initial jet reconstruction**, hence it can be **combined with any IRC safe definition of a jet**

Thanks to an IRC safe flavour assignment to all orders in perturbation theory, we can compute massless fixed-order predictions, and in the case of massive calculations, we have a suppressed sensitivity on mass logarithms $\log(Q^2/m_f)$

Interesting to explore: experimental feasibility of the algorithm, how flavour dressing behaves for other processes and observables, and how it compares to the other approaches recently proposed.

BACKUP

Flavour anti-k_t algorithm

[Czakon, Mitov, Poncelet (2205.11879)]

Recent proposal: modify anti-k_t distance when flavoured particles involved

$$d_{ij} = R^2 \min(k_{T,i}^{-2}, k_{T,j}^{-2}) \cdot S_{ij}^{a}\,, \quad d_B = k_{T,i}^{-2}$$

where $S_{ij} \neq 1$ only when i and j are of opposite flavour

$$S_{ij}^{a} = 1 - \theta(1-\kappa)\cos\left(\frac{\pi}{2}\kappa\right)\,, \quad \kappa = \frac{1}{a}\frac{k_{T,i}^2 + k_{T,j}^2}{2\,k_{T,\max}^2}$$

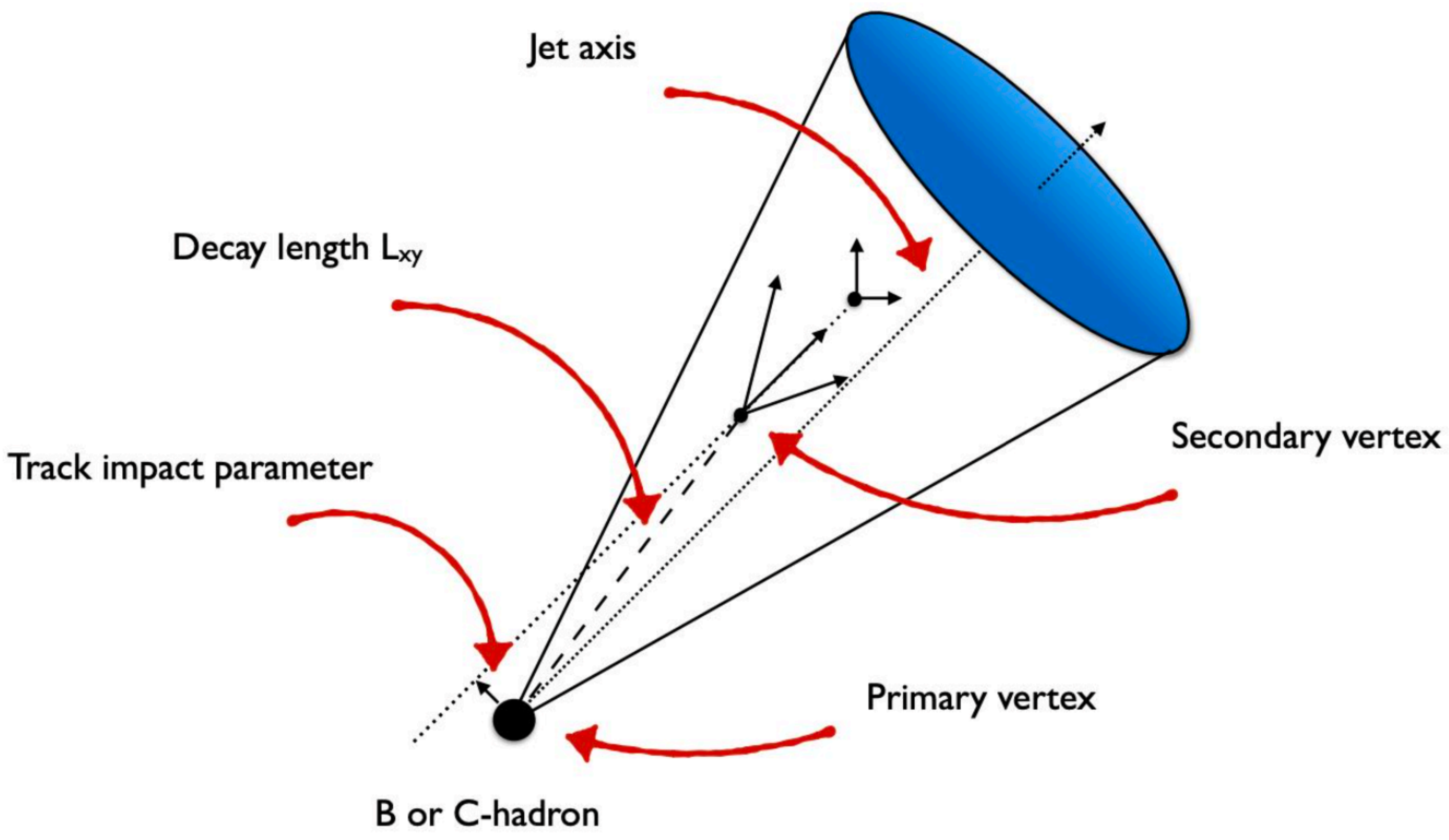
One recovers (IRC flavour unsafe) anti-k_t jets when $a \to 0$.

Quite significant dependence of the result on the parameter a.

Flavoured jets (experiments)

A jet defined as flavoured if it contains at least one heavy hadron within $\Delta R < R$ from the jet axis and with $p_T > p_{T,\mathrm{cut}}$ (*naive* tagging)

This is the "truth" labelling used in Monte Carlo samples, used to train a ML architecture ("High-level tagger") which adopts low-level variables as inputs



IRC flavour safety

The experimental definition is both **collinear and soft unsafe**

"A jet defined as flavoured if it contains
at least one heavy hadron
within $\Delta R < R$ from the jet axis
and with $p_T > p_{T,\mathrm{cut}}$"

Soft large angle $g \to b\bar{b}$
polluting different jets

$g \to b\bar{b}$ is always flavoured
even in the collinear limit
(an "even tag" removal is
enough to fix this)

$b \to bg$ collinear with the gluon
carrying most of the momentum
(would an identified particle, hence FF)