Shape Variables and Power Corrections

Paolo Nason,

INFN, sez. di Milano Bicocca and MPI, Munich

CEPC, 26/10/2022

UNIVERSITÀ DEGLI STUDI DI MILANO BICOCCA

INFN Istituto Nazionale di Fisica Nucleare

ACCADEMIA NAZIONALE DEI LINCEI

Alexander von Humboldt Stiftung/Foundation

Shape variables and QCD

Shape variables in e^+e^- annihilation are the simplest contest where we can study perturbative QCD.

For example, thrust:

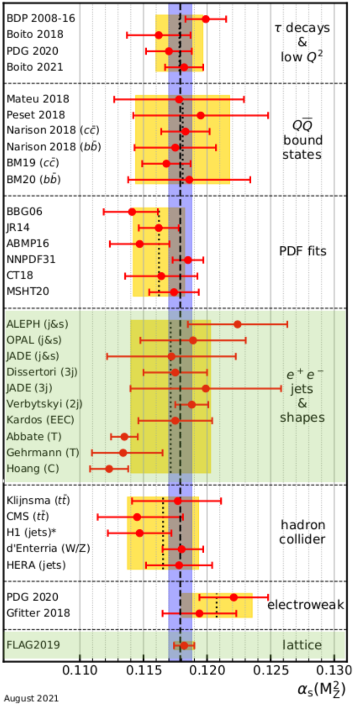
$$T = \max_{\vec{t}} \frac{\sum |\vec{p}_i \cdot \vec{t}|}{\sum |\vec{p}_i|}$$

equals 1 for two narrow back-to-back jets, and $2/3 < T < 1$ for three narrow jet.
Thus in the region $2/3 < T < 1$ the thrust distribution is proportional to α_s, and can be used for its determination.

On the other hand, the thrust distribution is sensitive to non-perturbative hadronization effects.

For example, the emission of a soft hadron with momentum 500 MeV, perpendicular to the thrust direction, affects the thrust by an amount $0.5/91 \approx 0.005$ on the Z peak. This shift in T can affect the thrust distribution by an amount of the order of 5%.

In practice non-perturbative corrections can reach the 10% level, and can affect at the same level the extracted value of α_S.



α_s determinations (PDG)

Determination of α_s from the first seven rows of the jets & shapes cathegory (highlighted in green) use Monte Carlo model to correct for non-perturbative effects.

The following three lines (Abbate, Gehrmann, Hoang) are based upon analytic modeling of non-perturbative effects.

Status

- The use of Monte Carlo modeling for hadronization corrections is not totally satisfying, since it lacks a sound theoretical basis.
- Analytic models seem to favour a too low value of α_s as compared to the world average and to the precise lattice determination.
- No bridge between MC and analytic models
- It is disturbing that we do not fully understand the role of non-perturbative effects at least in the simplest context where they can be studied.
- Understanding non-perturbative effects can have important consequences also for precision physics at hadron colliders, where linear power corrections can play an important role.

There are two broad classes of analytic methods:

- those based upon the so called "Dispersive approach", Based upon work of Dokshitzer, Webber, Marchesini, Salam and others. It is based upon the computation of the emission of a very soft gluon, with an associated non-perturbative coupling. The reference to Gehrmann in the previous slide refers to this method.
- Those based upon factorization, that separates the QCD calculation into a perturbative and non-perturbative contribution (a so called Shape Function), based upon work of Collins, Soper, Korchemsky, Sterman, and followed by a vast literature (Hoang, Stuart, Thaler, Mateu, Bauer, Schwartz and many others) using SCET. The references to Abbate and Hoang refer to this method.

These methods have however a common feature: the non-perturbative correction is computed in the two-jet limit, and then it is extrapolated to the three-jet region, where the measurement is performed.

Recent progress

There have been recently new findings regarding the structure of linear power corrections in collider observables:

- In ref. Eur.Phys.J.C 81 (2021), (Luisoni, Monni, Salam) it was shown that linear power corrections to the C parameter in the 3-jet symmetric limit are about 1/2 of those in the two jet limit.
- In ref. JHEP 01 (2022) 093, (Caola, Ferrario-Ravasio, Limatola, Melnikov, P.N.) it was demonstrated that linear power corrections are absent in sufficiently inclusive observables, in a variety of processes, in a model theory (large n_f QCD) that shares some properties with the full theory. These findings confirmed previous results obtained at the numerical level JHEP 06 (2021) 018, (Ferrario-Ravasio,Limatola,P.N.).
- The same findings opened the possibility to compute linear power corrections to shape variables in the 3-jet configuration arXiv:2204.02247, (Caola,Ferrario-Ravasio,Limatola,Melnikov, Ozcelik,P.N.)

The results of Luisoni, Monni, Salam are based upon the so called "dispersive approach", where one assumes that the strong coupling at low energy can be given by an effective coupling

The results of Caola et al. and Ferrario-Ravasio et al. are obtained from the study of IR renormalons.

The two approaches are deeply related.

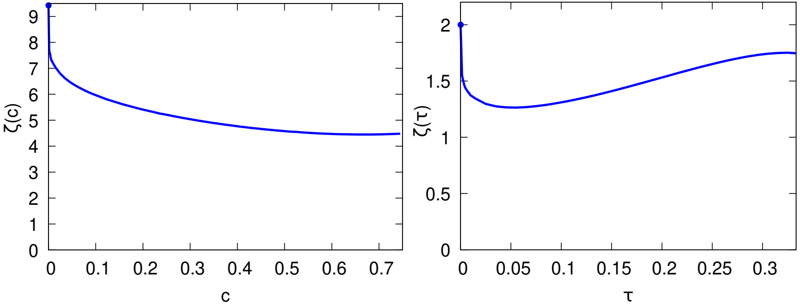
Non perturbative corrections are seen to arise from the variation of the shape variable when emitting a very soft gluon (gluer).

- The calculability of the non-perturbative correction in the two-parton case is based upon the fact that all shape variables have a well defined value for two-partons final states
(from a state with the emission of a gluer, there is a unique state without emission)
- The calculability of the non-perturbative correction to the C parameter in the three-jet symmetric limit (Luisoni et al.) is based upon the fact that the C parameter acquires a well-defined value near the 3-partons symmetric limit, up to quadratic effects in the deviation from the 3-partons symmetric configuration. (from a state with the emission of a gluer, there are several close states without emission, but C is not very sensitive to that)
- The calculability of the non-perturbative correction in the generic case is based upon the findings of Caola et al., that in suitable recoil schemes recoil effects cannot generate linear power corrections
(in general, differences in the state without emission don't generate linear corrections.)

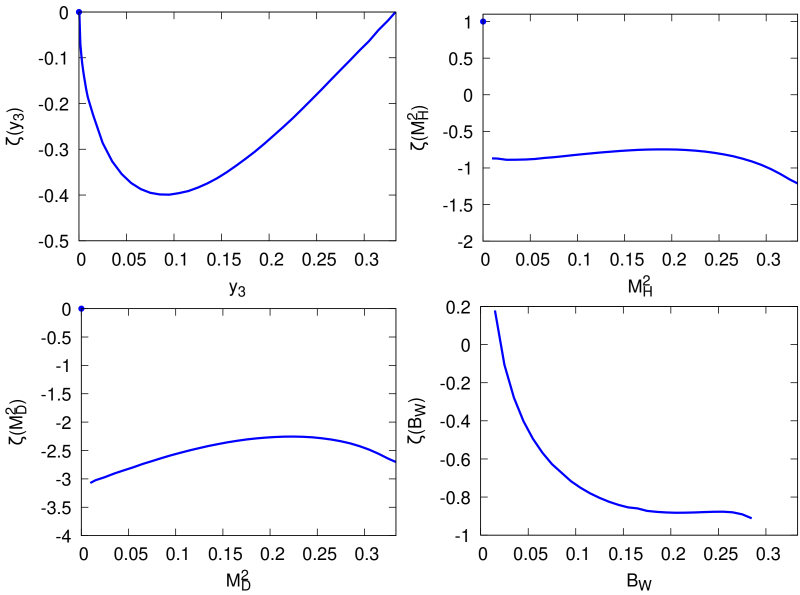
Non-perturbative corrections can be parametrized by a shift in the perturbative cumulant distribution:

$$\Sigma(s + H_{\mathrm{NP}}\zeta(s)) - \Sigma(s) \approx \frac{\mathrm{d}\sigma}{\mathrm{d}s} H_{\mathrm{NP}}\zeta(s), \qquad \Sigma(s) = \int \mathrm{d}\sigma(\Phi)\theta(s - s(\Phi))$$

and $H_{\mathrm{NP}} \approx \Lambda/Q$ is a non-perturbative parameter that is fully calculable in the large n_{f} approximation but must be fitted to data in real QCD.



The dot in the plots represents the constant value that was used in earlier studies. Notice that the value of ζ at the symmetric point is about one/half of the value at $c = 0$, consistent with Luisoni etal.



In some cases the ζ function is negative!

Implications

- There is a clear indication that the non-perturbative correction in the two jet limit cannot be safely extrapolated in the region where α_s is fitted.
- There is a hint that the small values of α_s found in fits using analytic models may be due to this assumption
- It is likely that this is not the whole story, and more needs to be understood before these findings can be safely used.

Mass corrections

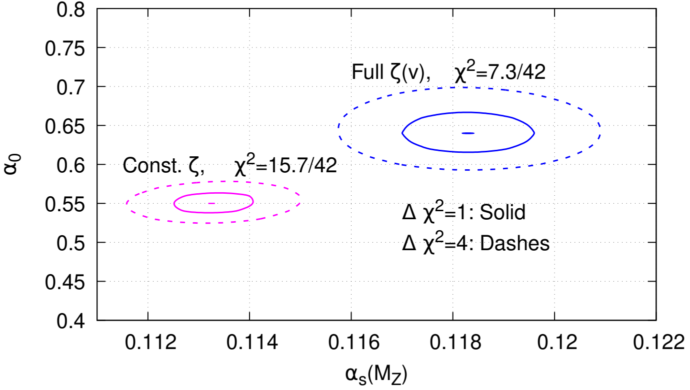
- Shape variables are defined for massless partons, and the analytic models refer to the "massless" definition.
- Final state hadrons are massive; so the definition of the shape variables must be extended to massive objects. This leads to ambiguities in the definition.
- This problem has been extensively studied in JHEP 05 (2001) 061, (Salam, Wicke). Three mass schemes where proposed:
  - the p scheme, where the energy of a particle is set equal to the modulus of the 3-momentum;
  - the E scheme, where the modulus of the momentum is set equal to the energy;
  - the D scheme ("Decay scheme"), where massive hadrons are decayed isotropically into a pair of fictitious massless particles before the shape variable is computed.

In the following I will illustrate preliminary results (Zanderighi,P.N., in preparation) obtained by fitting ALEPH data.
The non-perturbative shift for C and t are available from arXiv:2204.02247, Caola et al..
In Zanderighi,P.N. we also computed it for the y_3 in the Durham scheme, the Heavy jet mass M_h^2 and the heavy-light mass difference $M_h^2 - M_l^2$. and the broadening of the wide jet B_W.

- The perturbative theoretical errors were estimated with a 3-point scale variation $\mu_R/Q = 0.25, 0.5, 1$. An estimate of the error on the non-perturbative component was also included and added in quadrature.
- Diagonal terms of the covariant matrix were computed by summing in quadrature the systematic statistical and theoretical errors. The off-diagonal terms were computed as $E_{ij} = \min(\delta\sigma^2_{\mathrm{syst},i}, \delta\sigma^2_{\mathrm{syst},j})$ (the so called minimal-overlap model).
- We adopted the E scheme as our default treatment of hadron masses. We computed the associated bin migration matrix using Pythia8. Using Herwig7 we obtain compatible results with a slightly worse χ^2.

PRELIMINARY RESULTS

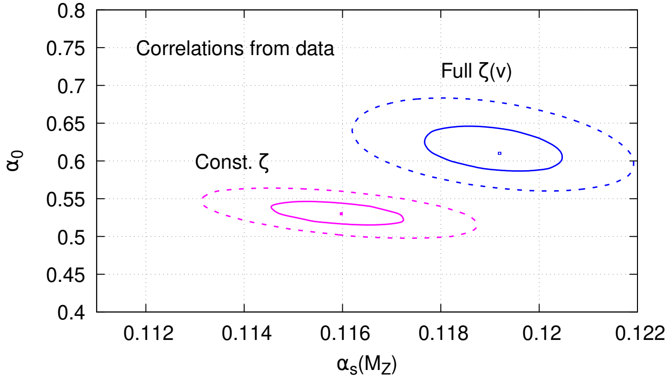
Simultaneous fit to C, t and y_3, both for our newly computed $\zeta(v)$, and, for comparison, with $\zeta(v) \to \zeta(0)$ (traditional method for the computation of power corrections).



The central value is at $\alpha_s(M_Z) = 0.1182$, $\alpha_0 = 0.64$. The "traditional" method leads to smaller values of α_s.

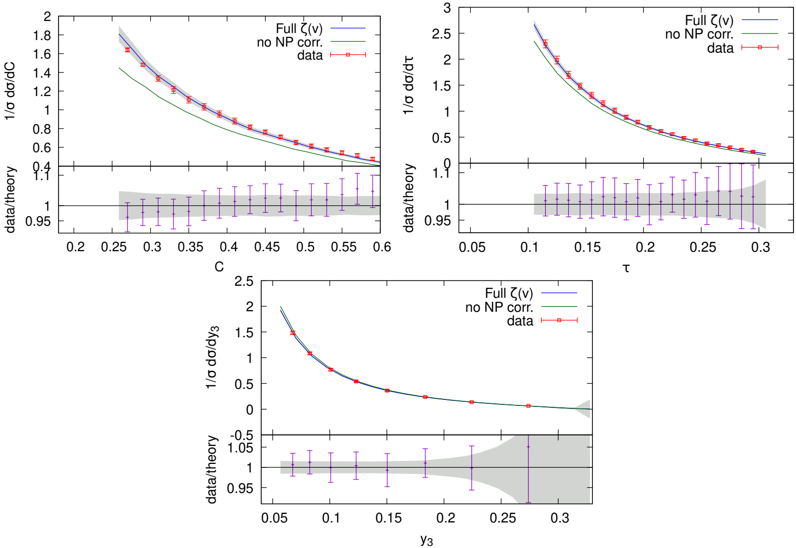
PRELIMINARY RESULTS

Different correlation computation, using correlation data not publicly available (thanks to Hasko Stenzel).

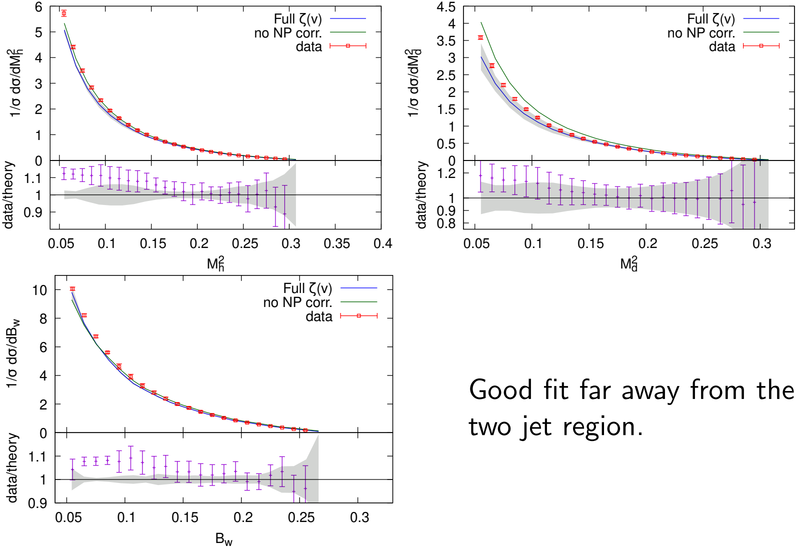


The central value is at $\alpha_s(M_Z) = 0.1192$, $\alpha_0 = 0.61$. The "traditional" method leads to smaller values of α_s, but higher than the previous case.

Quality of the fit for C, τ and y_3, using the new calculation of the non-perturbative effect (i.e. the full $\zeta(v)$ dependence.)

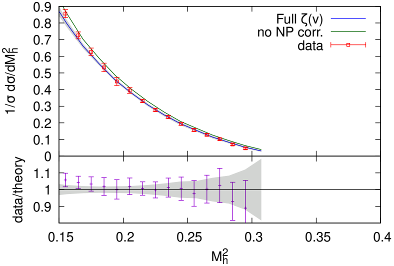


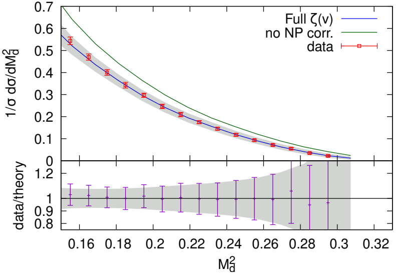
Prediction for M_H^2, M_D^2 and B_W using the values of α_s and α_0 obtained by fitting C, τ and y_3.

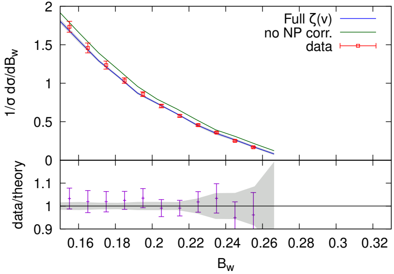


Good fit far away from the two jet region.

Prediction for M_H^2, M_D^2 and B_W using the values of α_s and α_0 obtained by fitting C, τ and y_3.

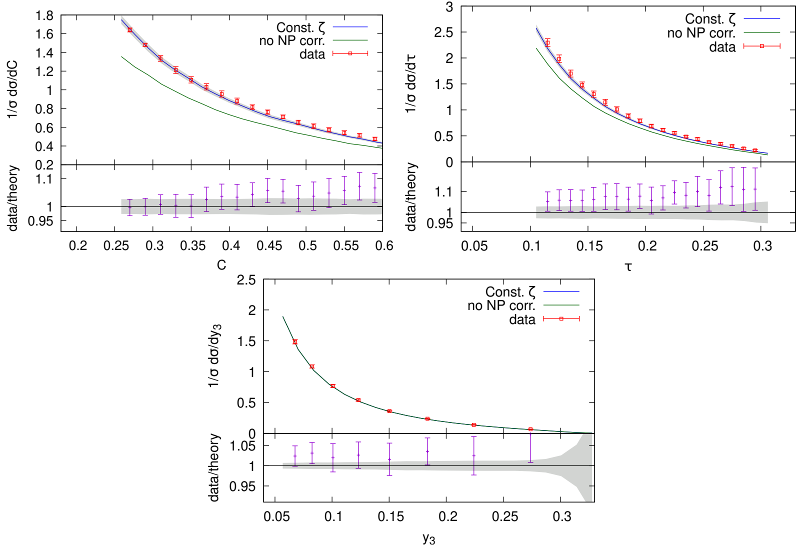




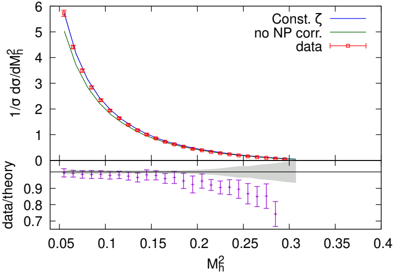


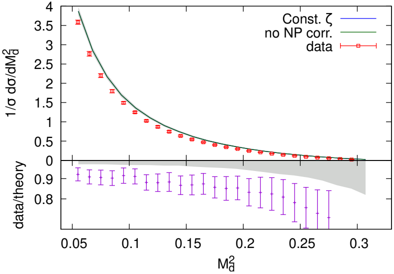
Good fit far away from the two jet region.

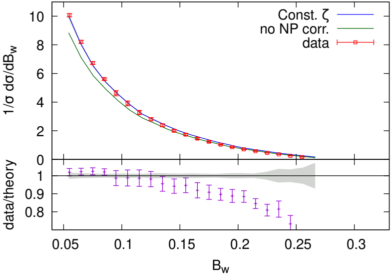
Quality of the fit for C, τ and y_3, obtained setting $\zeta(v) = \zeta(0)$.



Prediction for M_H^2, M_D^2 and B_W using the fitted values of α_s and α_0 obtained by fitting C, τ and y_3.

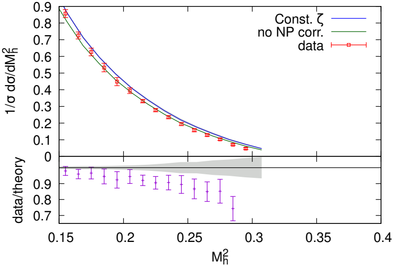


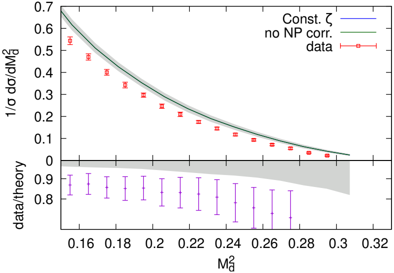


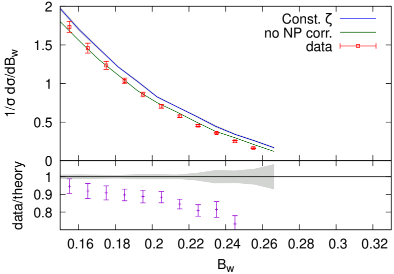


No Good fit in any region.

Prediction for M_H^2, M_D^2 and B_W using the fitted values of α_s and α_0 obtained by fitting C, τ and y_3.







No Good fit in any region.

- The heavy jet mass, mass difference and y broadening are well fitted far enough away from the two jet region with the newly computed ζ functions.
- On the other hand, it seems impossible to fit them using a constant, two-jet limit ζ.

Variation	α_s	α_0	χ^2	$\chi^2/N_{\rm deg}$
Default setup	**0.1182**	**0.64**	**7.3**	**0.17**
Renormalization scale $Q/4$	0.1202	0.60	9.1	0.21
Renormalization scale Q	0.1184	0.68	8.7	0.20
NP scheme (B)	0.1198	0.77	7.0	0.16
NP scheme (C)	0.1206	0.80	5.4	0.12
NP scheme (D)	0.1194	0.66	5.8	0.13
P-scheme	0.1158	0.62	10.7	0.24
D-scheme	0.1198	0.79	5.7	0.13
no scheme	0.1176	0.58	9.2	0.21
No heavy to light correction	0.1186	0.67	6.8	0.16
`Herwig` 6	0.1180	0.59	15.9	0.36
`Herwig` 7	0.1180	0.60	12.0	0.27
Ranges (2)	0.1174	0.62	12.7	0.23
Ranges (3)	0.1188	0.69	2.7	0.08
Replica method (around average)	0.1192	0.61	7.0	0.16
Replica method (around default)	0.1192	0.61	7.0	0.16
y_3 clustered	0.1174	0.66	8.2	0.19

We have considered several variations of the methods. They lead substantially to the same picture, with a spread in the value of α_s of the order of 2%. Hadron mass effects are particularly important ...

Opportunity for future e^+e^- colliders: CEPC

I estimate the following number of hadronic events:

E_{CM}	σ (nb)	Int. Lum. (ab^{-1})	Num. had. events
91.2	33.1	100	3.31×10^{12}
160	0.026	6	1.5×10^8
240	0.009	20	1.8×10^8
360	0.0039	1	3.9×10^6

Comparing to what was collected on the Z peak at LEP1 (16×10^6 events), we may infer that statistics should not be a limiting factor.

Opportunity for future e^+e^- colliders: FCC-ee

Phase	Run duration (years)	Center-of-mass Energies (GeV)	Integrated Luminosity (ab^{-1})
FCC-ee-Z	4	88-95	150
FCC-ee-W	2	158-162	12
FCC-ee-H	3	240	5
FCC-ee-tt	5	345-365	1.5

I estimate the following Z/γ^* hadronic cross sections:

E_{CM}	σ (nb)	Num. had. events
91.2	33.1	5.0×10^{12}
160	0.026	0.31×10^{9}
240	0.009	0.45×10^{8}
350	0.0039	0.58×10^{7}

comparable number of events.

Opportunity for future e^+e^- colliders

- We can expect hard to quantify systematics for running off the Z peak.
- We can expect a negligible statistical error at the higher energies.

Opportunity for future e^+e^- colliders

- Would an N^{3}LO calculation useful?
  An N^{3}LO calculation would be by itself of great value. For example, to see if the factorial growth associated to renormalons becomes visible.
- Power corrections are large on the Z peak (of order 10%). Perturbative uncertainties are of order 2%. At the highest energies NP corrections should be roughly a factor of 4 smaller, leading to further constraining power on the fits.
- Mass effects are partially uderstood in the two-jet limit. The situation in the three-jet case is quite unclear. More work is needed along that line.

Opportunity for future e^+e^- colliders

- Something new has being recently understood on power corrections. The theoretical consequences of these findings have yet to be fully explored. Hopefully these finding will bring in better agreement between power correction estimates obtained with shower Monte Carlo and those obtained from analytic methods generators.
- The impact of N^{3}LO calculations for shape variables will strongly depend upon the development of our understanding of NP effects.
- This understanding can also be tested now, by using preserved LEP data.
- Depending upon these developments, the availability of high statistics data at higher energies may allow for a high precision determination of α_s at high energy, to be contrasted with the low energy determination of the Lattice approach.