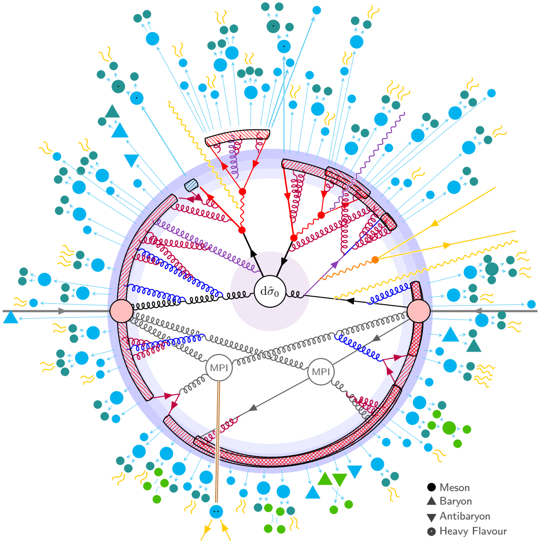
Department of Theoretical Physics

Parton showers for high precision e^+e^- collisions

Alexander Karlberg

The 2022 international workshop on the high energy Circular Electron-Positron Collider

Anatomy of a collision



- Hard Interaction
- Resonance Decays
- MECs, Matching & Merging
- FSR
- ISR*
- QED
- Weak Showers
- Hard Onium
- Multiparton Interactions
- Beam Remnants*
- Strings
- Ministrings / Clusters
- Colour Reconnections
- String Interactions
- Bose-Einstein & Fermi-Dirac
- Primary Hadrons
- Secondary Hadrons
- Hadronic Reinteractions

(: incoming lines are crossed)*

from Pythia 8.3 manual

The ubiquitous Parton Shower

Pythia 8

An introduction to PYTHIA 8.2 #1

Torbjörn Sjöstrand (Lund U., Dept. Theor. Phys.), Stefan Ask (Cambridge U.), Jesper R. Christiansen (Lund U., Dept. Theor. Phys.), Richard Corke (Lund U., Dept. Theor. Phys.), Nishita Desai (U. Heidelberg, ITP) et al. (Oct 11, 2014)

Published in: *Comput.Phys.Commun.* 191 (2015) 159-177 • e-Print: 1410.3012 [hep-ph]

pdf links DOI cite 4,050 citations

Herwig 7

Herwig++ Physics and Manual #1

M. Bahr (Karlsruhe U., ITP), S. Gieseke (Karlsruhe U., ITP), M.A. Gigg (Durham U., IPPP), D. Grellscheid (Durham U., IPPP), K. Hamilton (Louvain U.) et al. (Mar, 2008)

Published in: *Eur.Phys.J.C* 58 (2008) 639-707 • e-Print: 0803.0883 [hep-ph]

pdf links DOI cite 2,644 citations

Sherpa

Event generation with SHERPA 1.1 #1

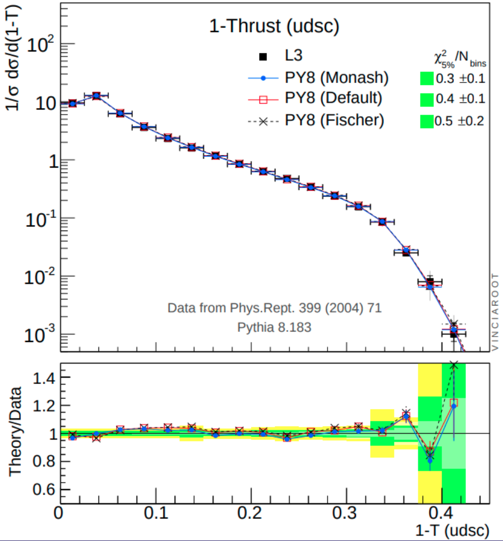
T. Gleisberg (SLAC), Stefan. Hoeche (Zurich U.), F. Krauss (Durham U., IPPP), M. Schonherr (Dresden, Tech. U.), S. Schumann (Edinburgh U.) et al. (Nov, 2008)

Published in: *JHEP* 02 (2009) 007 • e-Print: 0811.4622 [hep-ph]

pdf links DOI cite 3,386 citations

Parton Showers enter one way or another in almost 95% of all ATLAS and CMS analyses. Collider physics would not be the same without them.

The ubiquitous Parton Shower



Collider scene currently dominated by LHC, but the physics program relies on legacy e^+e^- colliders like LEP, for tuning of event generators.

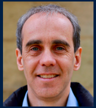
There is excellent agreement, after tuning, of for instance the Pythia8 generator and data taken at L3.

The high precision which future e^+e^- colliders will likely reach, has to be met with equally high theoretical precision.

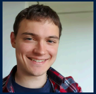
[Plot from Skands, et al. (1404.5630)]

The PanScales collaboration

Oxford


Gavin Salam


Melissa van Beekveld


Rok Medves


Frederic Dreyer


Ludo Scyboz


Jack Helliwell

CERN


Mrinal Dasgupta


Gregory Soyez


Pier Monni


AKB


Alba Soto-Ontoso


Silvia Ferrario Ravasio

UCL


Keith Hamilton

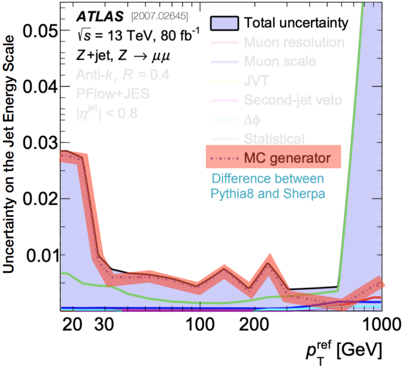

Rob Verheyen

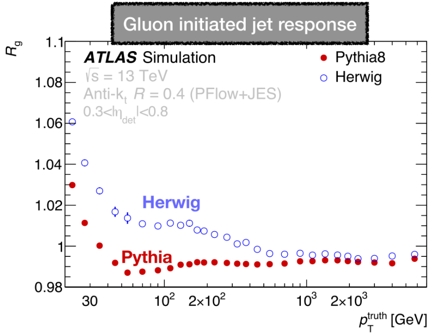
Manchester


Basem El-Menoufi

Differences matter!

Jet energy calibration uncertainties feed in to all jet analyses at the LHC





Differences amongst MC generators is the dominant uncertainty

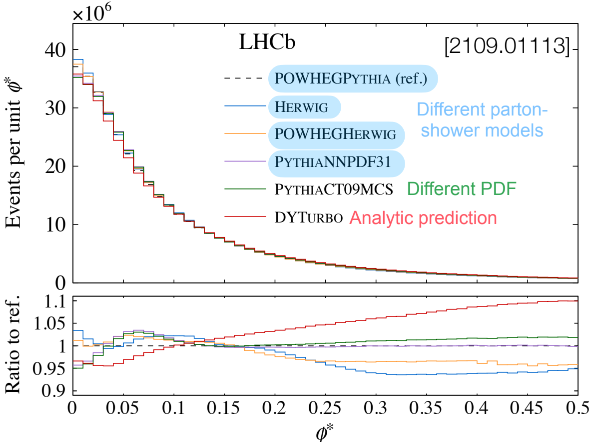
But differences matter…

Consider measurement of W boson mass

Measurements of p_T^Z in $Z/\gamma^* \to l^+l^-$ decays used to validate the MC predictions for p_T^W

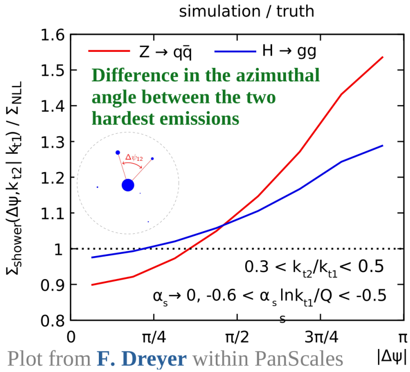
The envelope of shifts in m_W originating from differences in these shower predictions is the dominant theory uncertainty (11 MeV)

$$m_W = 80354 \pm 23_{\text{stat}} \pm 10_{\text{exp}} \pm 17_{\text{theory}} \pm 9_{\text{PDF}} \text{ MeV}$$

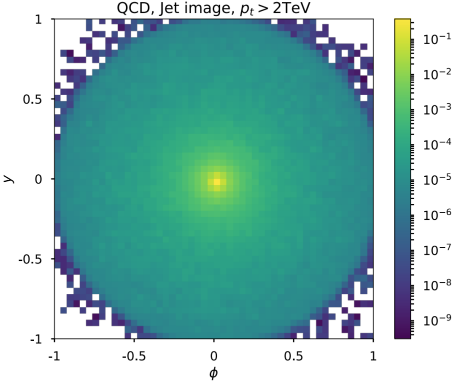


$$\phi^* = \frac{\tan((\pi - \Delta\phi)/2)}{\cosh(\Delta\eta/2)} \sim \frac{p_T^Z}{m_{ll}}$$ [1009.1580]

Machine learning and jet sub-structure



Plot from **F. Dreyer** within PanScales



[1511.05190]

Machine learning might learn un-physical "features" from MC → can significantly impact the potential of new physics searches.

A Parton Shower in a nutshell

In one line: A Parton Shower is an iterative stochastic algorithm that takes n particles and maps them to to $n+1$ particles.

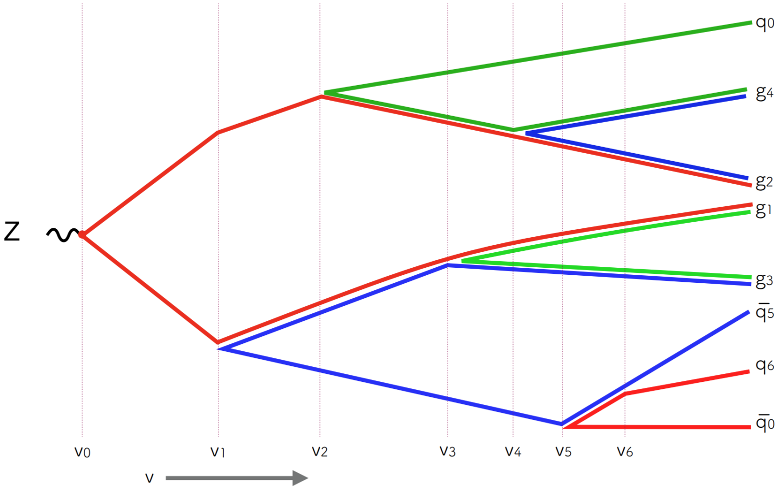
In order to do so one needs:

- A kinematic ordering variable, v, so that every phase space point is only reached once (and a cut-off $v_{cut} \sim \Lambda_{\mathrm{QCD}}$)
  - $\rightarrow$ Standard dipole showers take $v \sim k_T$ but many sensible choices exists
- A recoil map $\{p_n\} \rightarrow \{p_{n+1}\}$ to ensure momentum conservation and on-shellness of final-state particles
  - $\rightarrow$ Typically either local (only splitting dipole takes recoil) or global (all partons take recoil)
- An evolution equation governing the probability for a splitting $\widetilde{\widetilde{ij}} \rightarrow ijk$ to take place

$$\mathrm{d}\mathcal{P}_{\widetilde{\widetilde{ij}} \rightarrow ijk} \sim \frac{\alpha_s}{\pi} \mathrm{d} \ln v \, \mathrm{d}\bar{\eta} \, \frac{\mathrm{d}\phi}{2\pi} \left[g(\bar{\eta}) z_i P_{ik}(z_i) + g(-\bar{\eta}) z_j P_{jk}(z_j) \right] \tag{1}$$

! Constrained by QCD

A Parton Shower in a nutshell



courtesy G. Salam

Accuracy of Parton Showers

How do you even define the accuracy of an algorithm as described above?

When applying perturbation theory to total cross sections, it is easy to talk about the accuracy (LO, NLO, NNLO, ...)

$$\sigma = \sum_n c_n \alpha_S^n \tag{2}$$

Similarly for logarithmically enhanced observables we may talk about their logarithmical accuracy (LL, NLL, NNLL, ...)

$$\sigma(\mathcal{O} < e^L) = \sigma_{tot} \exp\left[\frac{1}{\alpha_S} g_1(\alpha_S L) + g_2(\alpha_S L) + \alpha_S g_3(\alpha_S L) + \cdots\right] \tag{3}$$

when $\alpha_S << 1, \quad \alpha_S L \sim -1.$

But both of these equations are *observable* dependent.

Accuracy of Parton Showers

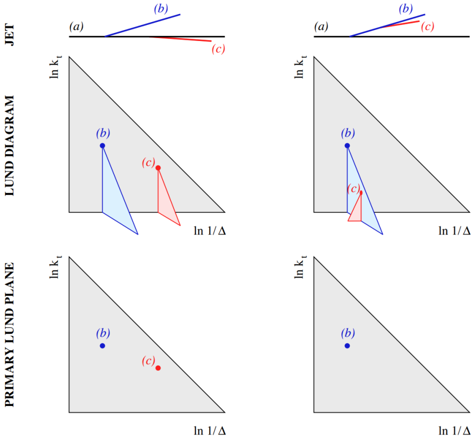
At colliders we can ask arbitrary questions about an event. The same is true for parton showers (+ hadronisation), e.g.

- Number (multiplicity) of particles in event (or jet)
- Energy in detector slice
- Angular distributions inside jets
- Even if we don't ask, machine learning might...

We therefore need to establish how to determine the logarithmic accuracy with which a parton shower can make predictions.

To do so we need to introduce the *Lund Plane* (B. Andersson et al (1989) & F. Dreyer et al. [1807.04758])

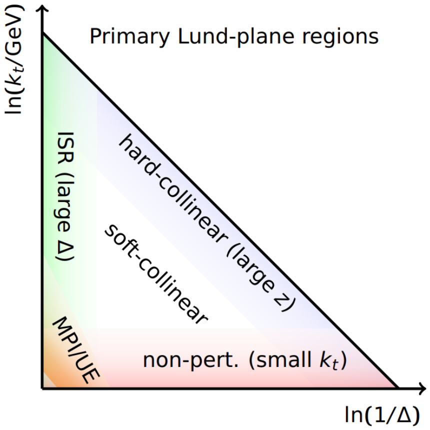
The Lund Plane



[1807.04758]

- Cluster the event with the Cambridge/Aachen algorithm, producing an angular ordered clustering sequence.
- Decluster the last clustering and record the transverse momentum and the opening angle of the declustering (plus other kinematics).
- Iterate along the hardest branch after each declustering to produce the *primary* Lund Plane.
- Following the softer branch produces the secondary, tertiary, etc Lund Plane.
- One can impose cuts easily on the declusterings (e.g. that they satisfy $z > z_{\mathrm{cut}}$)

Logarithms in the Lund Plane



[1807.04758]

- The emission probability in the Lund Plane is then
$$d\rho \sim \alpha_S \, d\ln k_T \, d\ln\theta$$
- Hence emissions that are well-separated in *both* directions are associated with *double logarithms* of the form $\alpha_S^n L^{2n}$
- Emissions separated along one direction are associated with *single logarithms* of the form $\alpha_S^n L^n$
- Emissions that are close in the Lund Plane are associated with a factor α_S^n
- We are now ready to state the PanScales NLL criteria for Parton Showers

NLL accurate Parton Showers

Fixed Order Matrix Element Condition

- Shower must reproduce fixed order n-particle matrix elements when emissions are well-separated in the Lund Plane, ie when the cross section is logarithmically enhanced.
- Supplement this with unitarity, 2-loop running and correct cusp anomalous dimension

Resummation Condition

- Shower must reproduce known NLL analytical resummations
- Global event shapes
- Multiplicity
- Non-global observables (slice observables), technically at LL

[1805.09327] & [2002.11114]

NLL accurate Parton Showers

Fixed Order Matrix Element Condition

- Fairly straightforward. Generate n emissions with your shower and compare to either factorised matrix elements (numerically very stable) or a full matrix element in some kinematic limit.
- Be careful to cover the collinear/soft phase space.

Resummation Condition

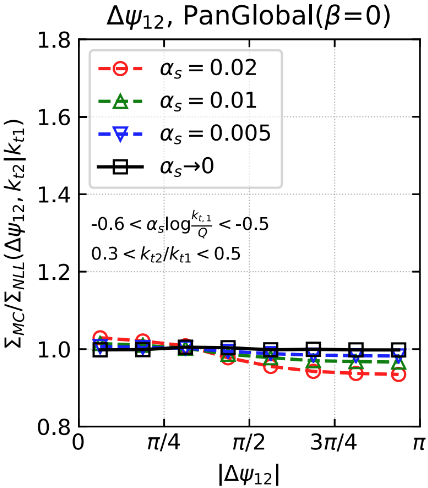
- This in general is trickier for 2 reasons:
- Requires the existence of NLL analytical results.
- Can't just compare

$$\frac{\Sigma^{\mathrm{PS}}(\alpha_S L)}{\Sigma^{\mathrm{NLL}}(\alpha_S L)} = \frac{\Sigma^{\mathrm{PS}}(\alpha_S L)}{\sigma_{tot}\exp\left[\frac{1}{\alpha_S} g_1(\alpha_S L) + g_2(\alpha_S L)\right]}$$

as the shower in general induces spurious higher order terms.

- How do we disentangle spurious "NNLL" terms from genuine NLL violations?

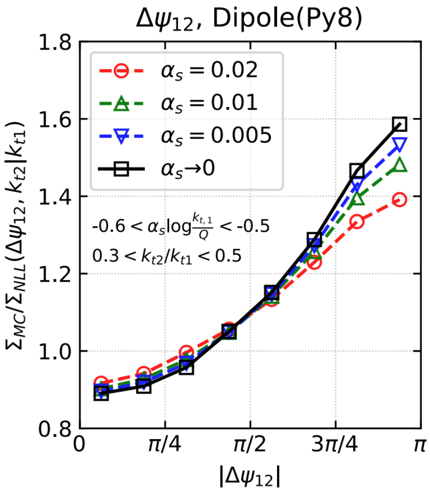
NLL tests



[2002.11114]

- Run the full shower with a specific (finite) value of $\alpha_S = \alpha_S(Q)$ and measure your favourite observable (that you can resum to NLL)
- Take the ratio to NLL and see that it is not flat.
- To see if there is an NLL mistake reduce α_S while keeping $\alpha_S L$ fixed, ie include more collinear and soft emissions.
- Genuine NLL effects are $(\alpha_S L)^n$ and are therefore unchanged. NNLL on the other hand goes as $\alpha_S(\alpha_S L)^n$ and should therefore vanish.
- Go as small in α_S as possible and extract $\alpha_S \to 0$.
- Now is it flat?

NLL tests



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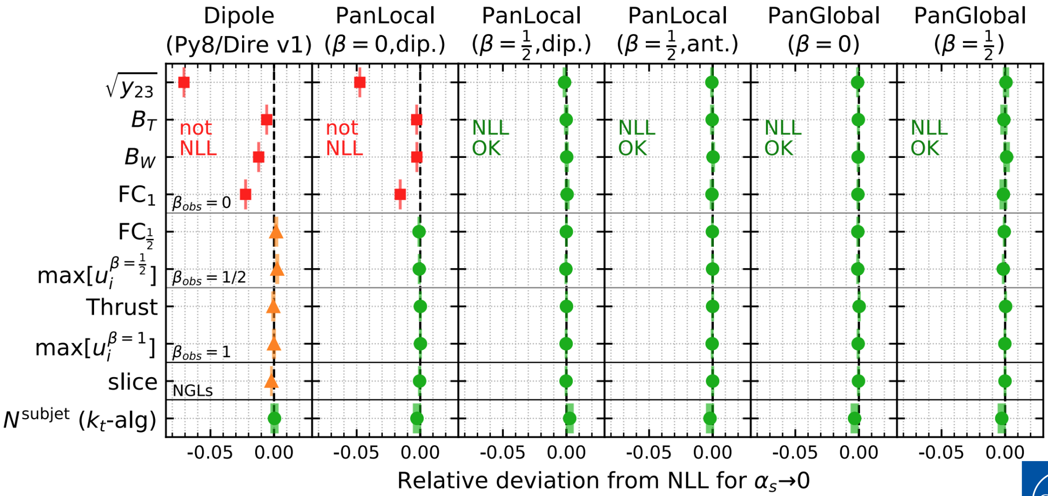
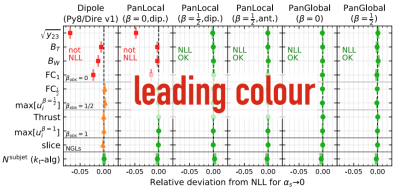
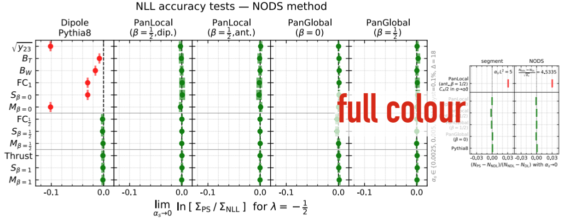
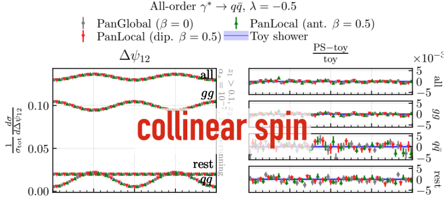
NLL tests summary

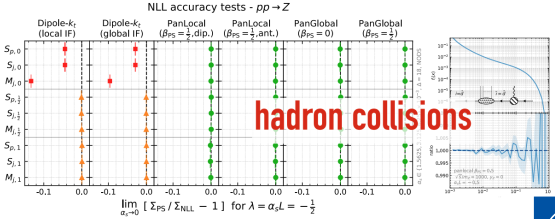


NLL tests summary

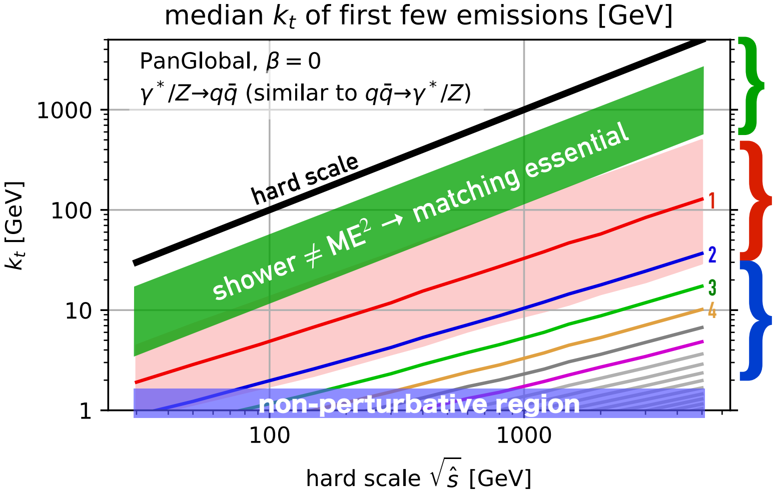








Where is shower accuracy useful / necessary?



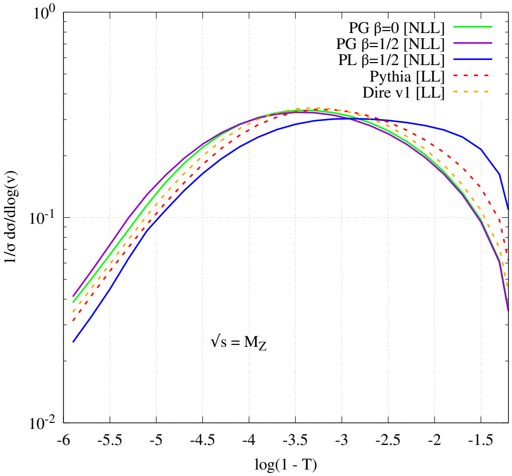
Full matrix-element needed for $k_t \gtrsim 0.1\sqrt{\hat{s}}$?

k_t of first emission (median and 68% interval)

median k_t of 2nd, 3rd, etc. emissions

the shower will be attempting to get all of these "right", together with the virtual corrections

Comparing parton showers



The PanScales showers all have the same formal NLL accuracy.

There can be a significant spread between predictions of the various showers, here shown for Thrust, indicating that spurious NNLL terms are large.

The showers in particular show a large spread than the two not-NLL showers, Pythia and Dire, do.

Are NLL showers less accurate than LL showers??

Comparing parton showers

NO! If we include scale variations this becomes very clear. For showers that have been established to be NLL accurate, for an emission carrying away a momentum fraction z, the emission strength is taken proportional to[1]

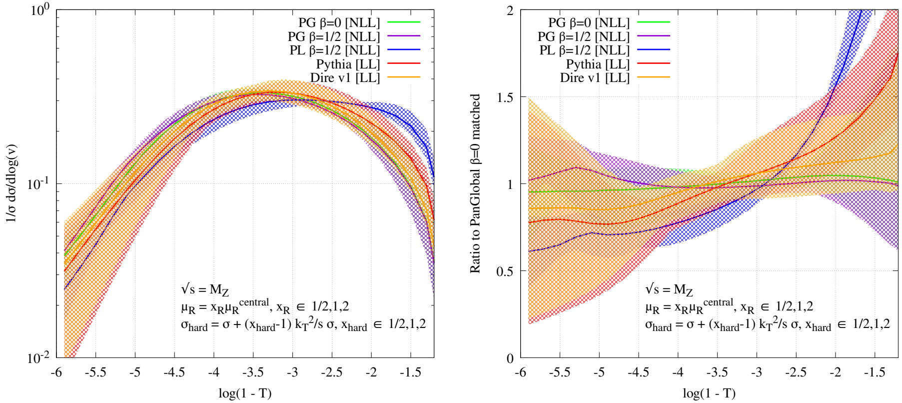
$$\alpha_S(\mu_R)\left(1+\frac{K\alpha_S(\mu_R)}{2\pi}+\frac{2(1-z)\beta_0\alpha_S(\mu_R)}{2\pi}\ln(x_R)\right), \quad \mu_R = x_R\mu_R^{\text{central}}. \tag{4}$$

The factor $1-z$ ensures that we only apply the scale compensation in the soft limit, and not the hard where the shower does include all the necessary ingredients. For showers that are not LL we include the term proportional to K (CMW scheme) but omit the $1-z$ term.
In order to assess missing terms in the hard matching region we take the emission strengt proportional to (unless matching that emission)

$$1+(x_{\text{hard}}-1)\frac{k_T^2}{s} \tag{5}$$

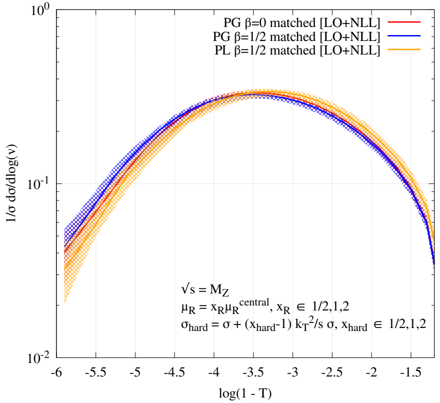
[1] Inspired by Mrenna & Skands [1605.08352]

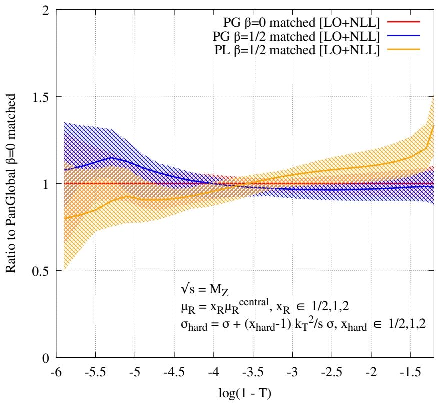
Scale variations in LL and NLL showers



Shower variations reduced significantly in the NLL showers. Showers also almost fully inside LL shower variations. Large discrepancies in hard region expected - can be fixed by matching.

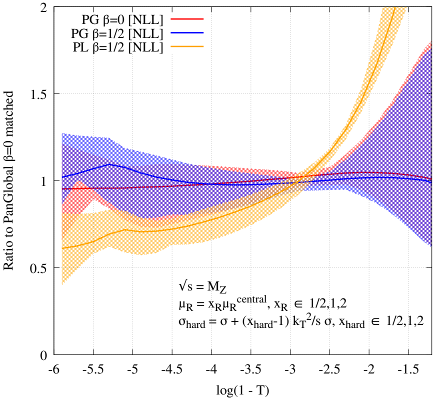
Scale variations in matched NLL showers

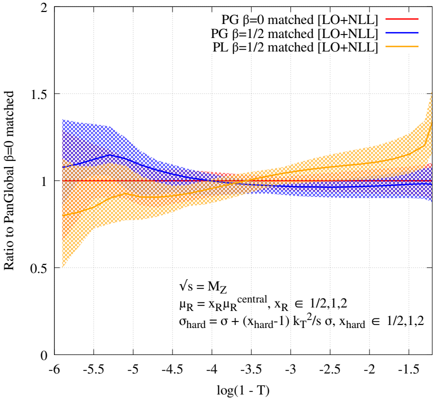




When matching LO matrix elements to the PanScales showers the agreement improves everywhere, not just in the hard region. Probably due to dominance of the first emission. Hard variations significantly decreased.

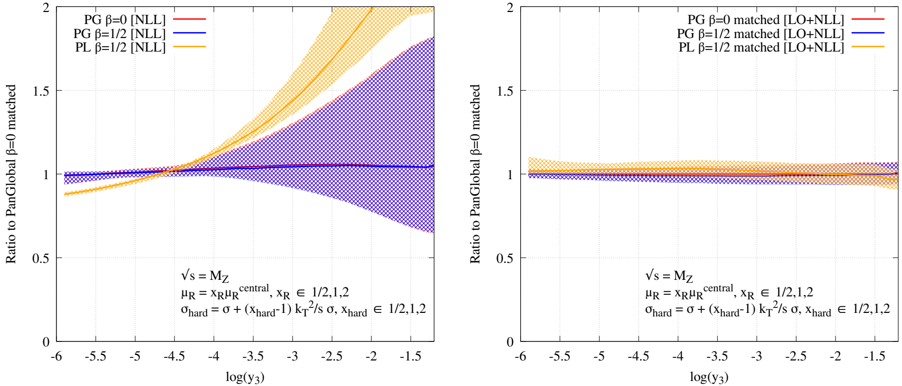
Scale variations with and without matching





When matching LO matrix elements to the PanScales showers the agreement improves everywhere, not just in the hard region. Probably due to dominance of the first emission.

Scale variations with and without matching for y_3



The agreement after matching is even better for Cambridge y_3.

Conclusions

- Parton showers with controlled logarithmic accuracy are emerging.[2]
- Such a program is mandatory for precision QCD studies, and by the time the CEPC starts running we can expect logaritmically accurate showers to be the standard
- With logaritmic control we can also assign meaningful uncertainties to shower predictions, thereby making them real predictions.
- Still many developments to come...

[2] See also recent work by Forshaw, Holguin, Plätzer (CVolver), Nagy, Soper (Deductor), Herren, Höche, Krauss, Reichelt, Schönherr (Alaric)