Resistive-wall impedance of an elliptical multilayer beam pipe and its induced incoherent tune shift

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Impedance of the elliptical multilayer chamber

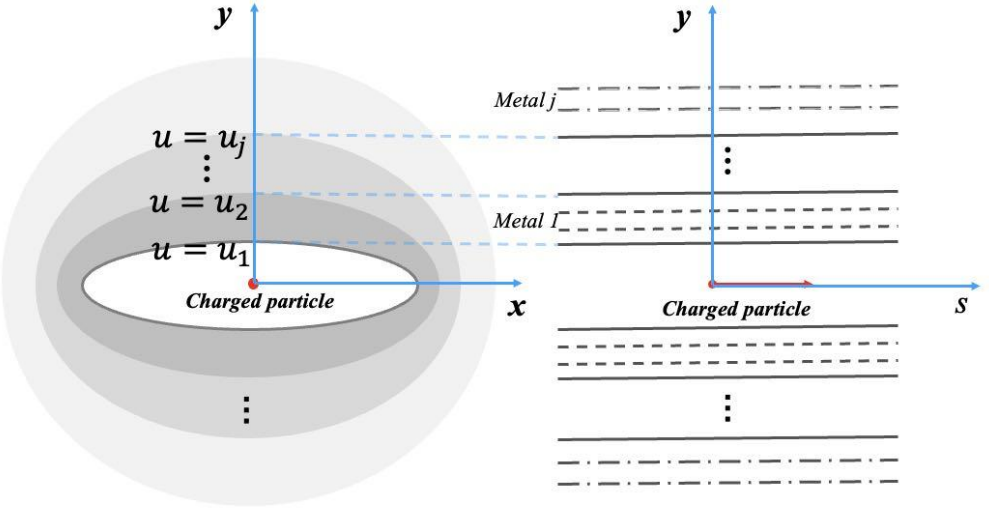


Fig.1 Cross section and side view of the elliptical beam pipe.

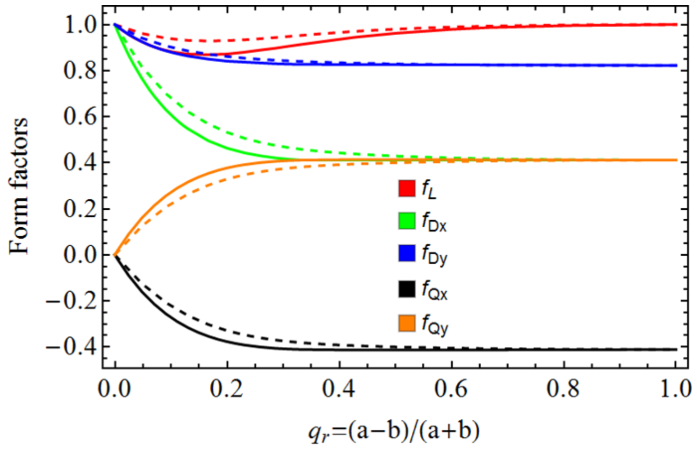


Fig.2 Form factors

Solid lines: Based on A. Lutman's method, directly solving the Maxwell's equations in the elliptic coordinate system and the impedances were determined by field matching at the interface between the vacuum and the metal walls.

Dashed lines: Yokoya form factor, based on K.Yokoya's method that derived impedance formulae by introducing the surface impedance.

The confocal elliptical beam pipe (as shown in Fig.1) is proposed to study the elliptical beam pipe with nonuniform thickness.

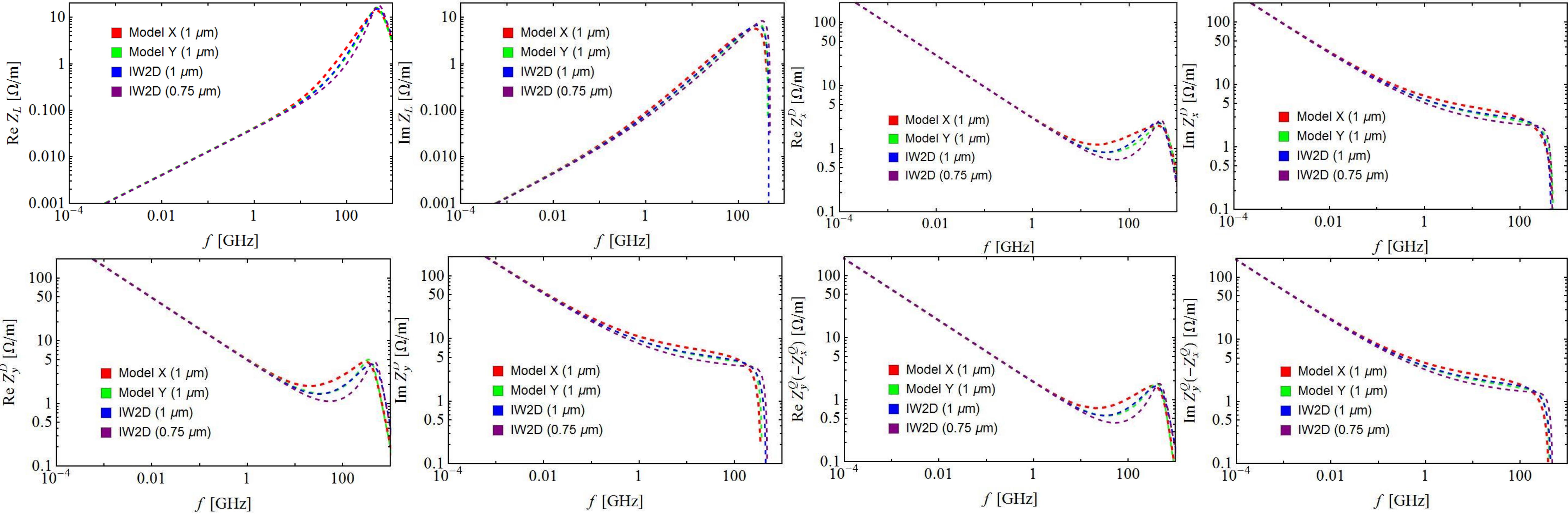
1. The analytical formulae allow us to evaluate the impedances with good convergency from the circular corss section to the two parallel plates taking any value of ellipticity in a large range of frequency.
2. The from factor are calculated and compared with Yokoya form factor, as shown in Fig.2

----At lower qr, small discrepancies for both longitudinal and transverse form factors are observed, which can be explained by the fact that the concept of surface impedance has been used in the K. Yokoya's method, instead of solving the Maxwell's equations directly as discussed in this report.

[1] **Y.T. Wang**, N. Wang, Q. Qin, G. Xu and S. Yue, Resistive-wall impedance of an elliptical beam pipe with multilayer, Nuclear Inst. and Methods in Physics Research, A, 1037, 166928 (2022).

3. To mitigate the impedance of the elliptical beam pipe with NEG coating, the influence of the nonuniform NEG coating on impedance is investigated.

Parameter: Copper beam pipe treated with NEG coating. Model X with horizontal thickness of 1 µm in red lines; Model Y with vertical thickness of 1 µm in green lines. IW2D with thickness of 1 µm and 0.75 µm in blue and purple lines.



From the results we can conclude that the impedances are more sensitive to the vertical thickness of the NEG coating, other than the horizontal thickness. Therefore, in order to reach low impedance and high reliability at the same time concerning the NEG coating chambers, we can potentially increase the thickness of coating in the horizontal plane while keep the thin layer in the vertical plane.

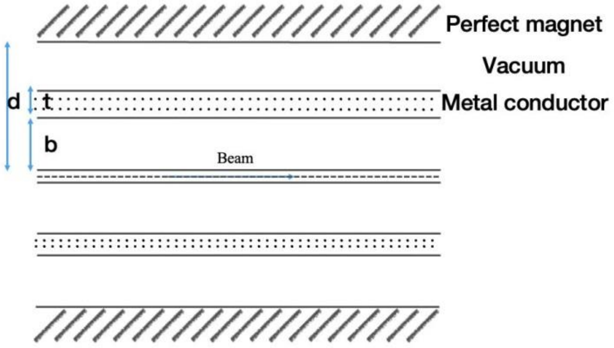
2. Tune shift due to quadrupolar impedance

● Background

1. Quadrupolar tune shift is widely observed. Two theoretical models developed by A.Chao and Y.Shobuda to reproduce the measurements.

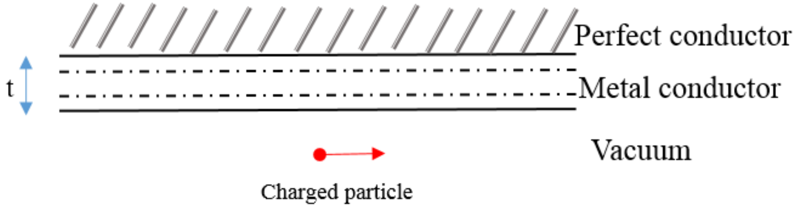
2. A.Chao's model

$$\frac{dv_{x,y}}{dI} = \pm \frac{\pi r}{48 v_{x,y}} \frac{Z_0}{E/e} \left(\frac{R}{b}\right)^2 \frac{L}{C}$$



3. Y.Shobuda's model

$$\frac{dv_{x,y}}{dI} = \mp D_{2xy} \frac{L < \beta_{x,y} > }{4\pi} \frac{Z_0}{E/e} \frac{t}{\pi b^2}$$



4. Comprehensive studies on SOLEIL shows, in the long range regime dealing with multibunch beams, the measurement result lies somewhere between the two models [3].

[1] A.Chao, S.Heifets, and B.Zotter, Tune shifts of bunch trains due to resistive vacuum chambers without circular symmetry, Phys. Rev. Accel. Beams 8, 042801 (2005)
[2] Y. Shobuda and K.Yokoya, Resistive wall impedance and tune shift for a chamber with a finite thickness, Phys. Rev. E, 66, 056501 (2002)
[3] P. Brunelle, R. Nagaoka, R. Sreedharan, Measurement and analysis of the impact of transverse incoherent wakefields in a light source storage ring, Phys. Rev. Accel. Beams 19 (4) (2016) 044401.

Tune shift due to quadrupolar impedance

Incoherent tune shift due to the quadrupolar impedance is derived from beam oscillation equations

$$\ddot{y}_n(t)+\omega_\beta^2 y_n(t) = -\frac{Nr_0c}{\gamma T_0}\sum_{m=0}^{M-1}\sum_{k=0}^{\infty}[y_m(t-(k+\frac{m-n}{M})T_0)W_1^d(-(k+\frac{m-n}{M})C)$$
$$+y_n(t)W_1^q(-(k+\frac{m-n}{M})C)],\quad (n=0,\ 1,\ ...,\ M-1),$$

$$\Omega-\omega_\beta = -i\frac{Nr_0c}{2\gamma\omega_\beta T_0^2}\sum_{\mu=0}^{M-1}\sum_{p=-\infty}^{\infty}[Z_1^d((\mu+pM)\omega_0+\omega_\beta)+Z_1^q(pM\omega_0)]$$

DC value of the quadrupolar impedance dominates the tune shift.

Incoherent tune slope: $$\frac{d\Delta\nu_\beta}{dI} = \frac{C}{8\pi^2(E/e)\cdot\nu_\beta}\sum_{p=-\infty}^{\infty}\mathrm{Im}Z_1^q(pM\omega_0)$$

With the impedance model, both the conductivity and permeability of different layers with finite thickness can be considered.

[1] **Y.T. Wang**, N. Wang, H.S. Xu and G. Xu, Tune shift due to the quadrupolar resistive-wall impedance of an elliptical beam pipe, Nuclear Inst. and Methods A, 166414, 2022

Application in CEPC Z operation

With frequency decreases, the skin depth increases and the background material outside the beam pipe should be considered.

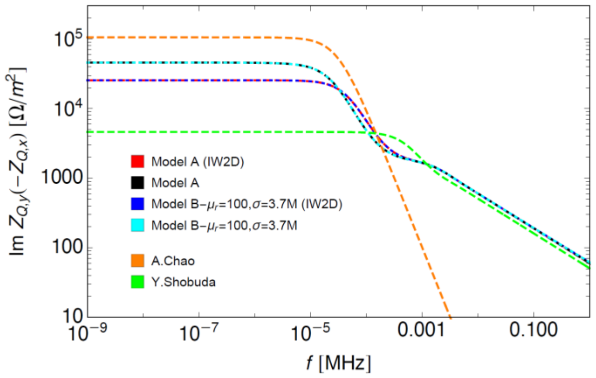
Two impedance models:(the product of the form factors and the impedance for the circular chamber[1])

Model A: Vacuum outside the chamber

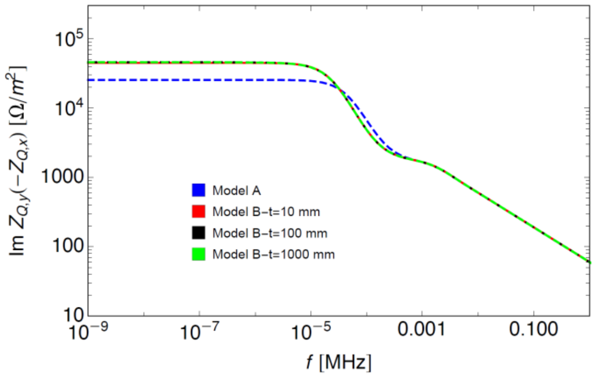
3mm Cu + Vacuum (Infinite)

Model B: Magnet outside the chamber

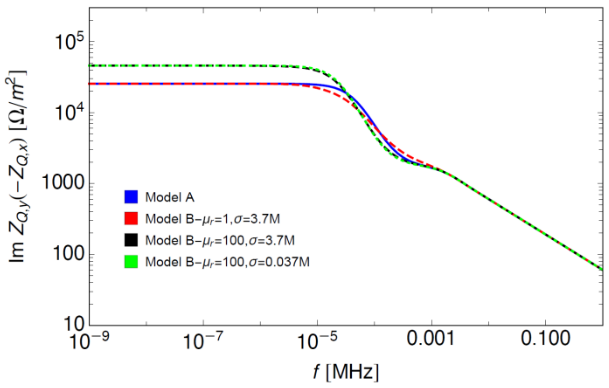
3mm Cu + 100mm Magnet (μ=100) + Vacuum (Infinite)



Comparison between different impedance models



Dependence on the thickness of the magnets



Dependence on the permeability and conductivity of the magnets

Parameters	Values
Size	28×37.8 mm
Conductivity of Copper	58 MS/m
Thickness of chamber	3 mm
Conductivity of Magnet	3.7 MS/m
Relative Permeability	100
Thickness of Magnet	100 mm

Model	Tune slope (x/y) (A^{-1})	Form factor f_y^Q $\left(-f_x^Q\right)$
A.Chao	0.65/–0.64	0.41
Y.Shobuda	0.06/–0.06	0.28
Model A	0.16/–0.15	0.33
Model B	0.28/–0.28	0.33
Model A (20%)+Model B (80%)	0.26/–0.25	0.33

- Consider that 80% of the collider is surrounded by magnets, the tune shifts are 0.26/-0.25 (A^{-1})
- Main difference among different methods can be explained by the impedance model and the form factors used.

[1] N. Wang and Q. Qin, Resistive-wall impedance of two-layer tube, Phys. Rev. ST Accel. Beams 10 (2007), 111003.