

# Spin Resonance Free Booster For Future 100 km-scale Circular $e^+e^-$ Colliders



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## Introduction

**Obtaining high energy polarized beams through circular accelerators is a challenge due to depolarization resonances. Overcoming these resonances may need extra insertion elements or complicated operation.** For the problem we have studied the spin depolarization resonance structure of a booster lattice for the 100 km-scale Circular Electron Positron Collider (CEPC). The 100 km-scale booster lattice has a periodicity of 8 and each arc region contains hundreds of FODO cells, and features a very high periodicity in terms of the spin resonance structure. The first very strong intrinsic and imperfection resonances are beyond 120 GeV, within which the spin resonances are much weaker. Spin tracking simulations show that polarization can be mostly maintained in the fast ramping to 45.6 GeV and 80 GeV beam energies. Acceleration up to 120 GeV could suffer more depolarization and potential mitigation is discussed. **This study opens the way for injection of highly polarized beams into the collider rings, to enable resonant depolarization measurements as well as longitudinally polarized colliding beam experiments.**

## Background

**Depolarization Effect** According to theory of spin motion in circular accelerators, the spin will precess around  $\vec{n}$  which is determined by the lattice structure. The rate of spin precession around  $\vec{n}$  is described by the spin tune  $\nu_s$ . On the closed orbit,  $\nu_0 \approx G\gamma$  changes during acceleration, where  $G$  is the anomalous magnetic moment,  $G = 0.00115965$  for electron and positron,  $\gamma$  is the Lorentz factor. When crossing a spin resonance, which means  $\nu_s$  goes from below resonance location  $K$  to above  $K$ , depolarization occurs. For isolated resonance, this effect can be described by the Froissart-Stora formula:

$$\frac{P_f}{P_i} = 2e^{-\pi|\epsilon_K|^2/2\alpha} - 1 \quad (1)$$

where  $\epsilon_K$  is the resonance strength at  $G\gamma = K$ ,  $P_i$  and  $P_f$  are polarization before and after crossing through the resonance respectively. The crossing speed  $\alpha$  can be describe as:

$$\alpha = \frac{dG\gamma}{d\theta} \quad (2)$$

According to Eq 3, while  $\epsilon_K/\sqrt{\alpha} < 0.06$ , the polarization loss will be less than 1%, and  $\epsilon_K/\sqrt{\alpha} > 2$  can achieve a total spin flip. Otherwise depolarization will occur. It is showed in Figure 1 (a).

**Depolarization Resonances** There are 2 major types of resonances driven by different terms: the intrinsic spin resonances and the imperfection spin resonances. Both are due to coherent transverse spin kicks which occur primarily in the quadrupole fields of the lattice. Intrinsic spin resonances arise from vertical betatron oscillations, while imperfection spin resonances are due to vertical closed orbit distortions. For the case of a depolarizing intrinsic spin resonance this occurs whenever the spin tune  $G\gamma = N \pm Q_y$ . Here  $N$  is an arbitrary integer,  $Q_y$  is the vertical betatron tune. Assuming the accelerator is composed of  $P$  superperiods with  $M$  same FODO cells per superperiod, The strength of the intrinsic resonances can be written as:

$$\epsilon_K \approx \frac{G}{2\pi} \left( \frac{\gamma \epsilon_N}{\pi} \right)^{1/2} \{ E_P^+ [E_M^+ (g_F \sqrt{\beta_z(F)} - g_D \sqrt{\beta_z(D)}) e^{i(K \pm \nu_B)}] + X_{ms}^+ + E_P^- [E_M^- (g_F \sqrt{\beta_z(F)} - g_D \sqrt{\beta_z(D)}) e^{i(K \pm \nu_B)}] + X_{ms}^- \} \quad (3)$$

where  $\nu_B = MP\mu$  is the total accumulated phase advance in dipoles cells,  $2\pi\mu$  is the phase advance in each FODO cells.  $E_P^\pm$  and  $E_M^\pm$  are enhancement factors due to the  $P$  superperiods and the  $M$  FODO cells respectively:

$$E_P^\pm = e^{i\pi(P-1)\frac{K \pm \nu_B}{P}} \zeta_P \left( \frac{K \pm \nu_B}{P} \right) \quad (4)$$

$$E_M^\pm = e^{i\pi(M-1)\frac{K \pm \nu_B}{PM}} \zeta_P \left( \frac{K \pm \nu_B}{PM} \right) \quad (5)$$

$\zeta$  is enhancement function :

$$\zeta_N(x) = \frac{\sin N\pi x}{\sin \pi x} \quad (6)$$

The factor  $E_P^\pm$  enhances the resonance strength  $P$  times at  $K = mP \pm \nu_B$ . Similarly, the factor  $E_M^\pm$  enhances the resonance strength  $M$  times at  $K = mPM \pm \nu_B$ . In a high energy accelerator  $M$  is normally much larger than  $P$ , so  $E_M^\pm$  will be the most important factor in the equation. Since  $mPM \pm \nu_B$  may not coincide with the resonance condition, important resonances occur at those  $K = nP \pm \nu_B$  such that they are closest to  $mPM \pm \nu_B$ . For the imperfection resonances driven by vertical closed orbit distortions. Since the orbit distortions repeat themselves every turn, the imperfection resonances occurs whenever the spin tune  $G\gamma = integer$ . Similarly to the analysis above, the strongest imperfection resonances will located at  $K = nPM \pm \nu_B$ . Its strength is also highly rely on the effect of closed orbit correction.

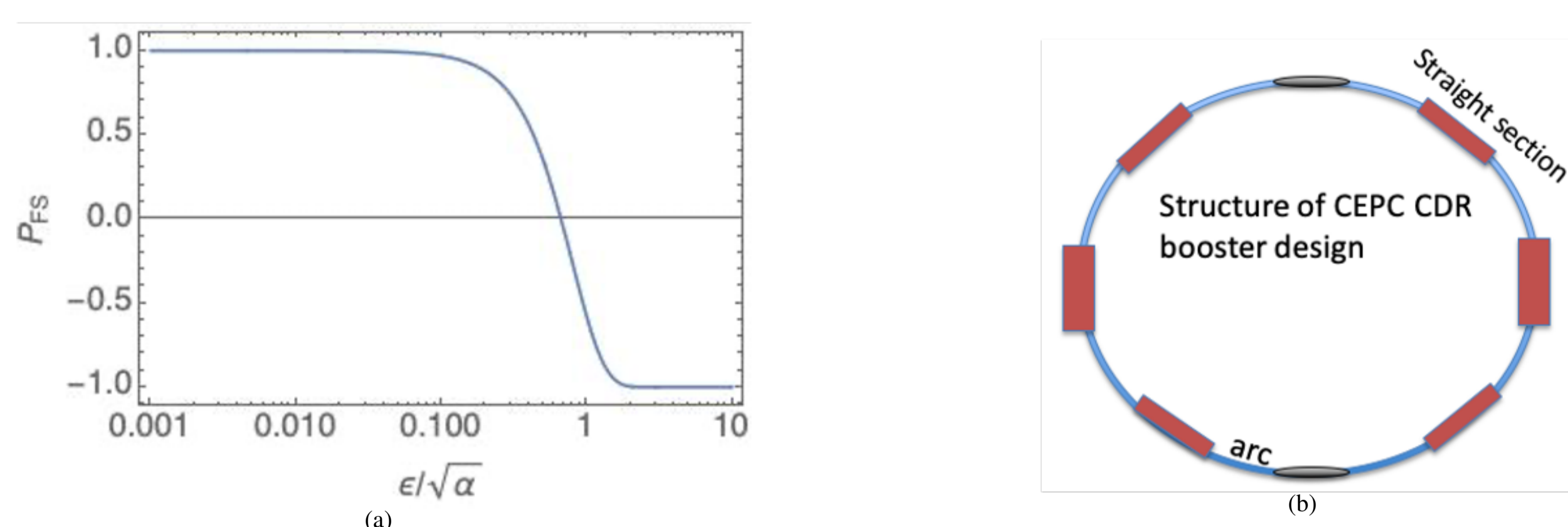


Figure 1: (a) is the curve of Froissart-Stora formula, which describes the spin behavior after crossing a isolated resonance; (b) is a simple schematic diagram of the CDR booster structure.

## Results

**CEPC CDR Booster Lattice** CEPC Booster will be built in the same tunnel as the collider ring, and it will be a ring with circumference about 100km. In the conceptual design report(CDR) released in 2018, standard FODO cells have been chosen for the booster lattice. The phase advance of each cell is 90/90 degrees in the horizontal and vertical planes, the total length of each FODO structure is 101 m. The whole ring is consist of 8 arc regions and straight sections connecting them. 97 FODO structures make up an octant. At the two ends of each octant, there are dispersion suppressors. Between the octant there are straight sections connecting them. The structure is showed in Figure 1 (b). Count the dispersion suppressors on both sides of each octant, each superperiod is composed of  $M=99$  periodic structures. The parameters of the CDR ring is showed in Table 1. In order to accelerate beams to 120 GeV, the correspond  $G\gamma$  is about 272, a  $\nu_B$  less than then  $G\gamma$  means that at least 1 strong intrinsic resonance may be crossed during the ramping and it may cause much polarization loss. Meanwhile, during accelerating to 45.6 GeV or 80 GeV, no strong resonances will be encountered thus we can get high enegygy polarized beams without extra efforts. Our inferences are confirmed in simulations, results are showed below Figure 2-4.

Table 1: Parameters of CEPC CDR Booster	
P: number of periodicities	8
M: number of unit cells in each arc region (per period)	99
$\nu_y$ : total betatron phase advance/( $2\pi$ )	261.2
$\nu_B$ : total betatron phase advance in arc regions/( $2\pi$ )	198

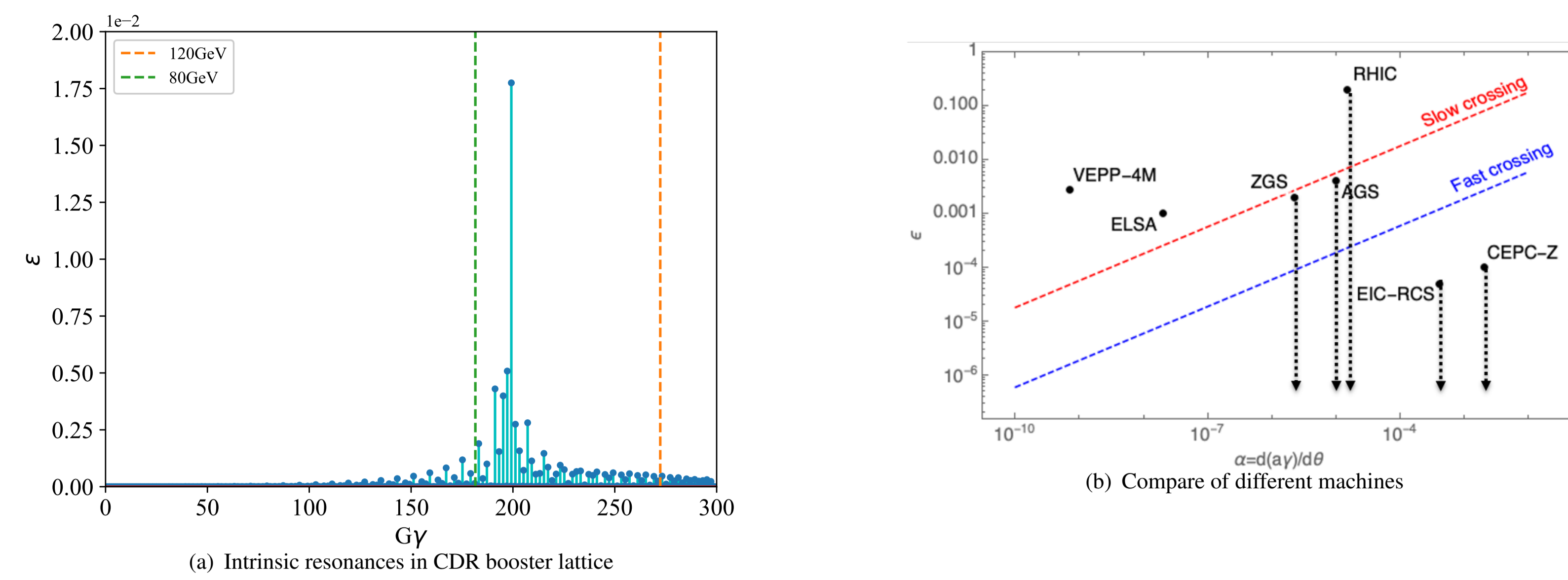


Figure 2: (a) shows the structure of intrinsic resonances, as expected, a strong peak at  $G\gamma \approx 198$  occurs. Accelerating to 45 or 80 GeV are not affected by the peak. (b) Due to the high crossing speed and lower resonance strength ( $W$  and  $Z$  Mode), the total ramping process is within 'fast crossing' region, thus the polarization loss is negligible.

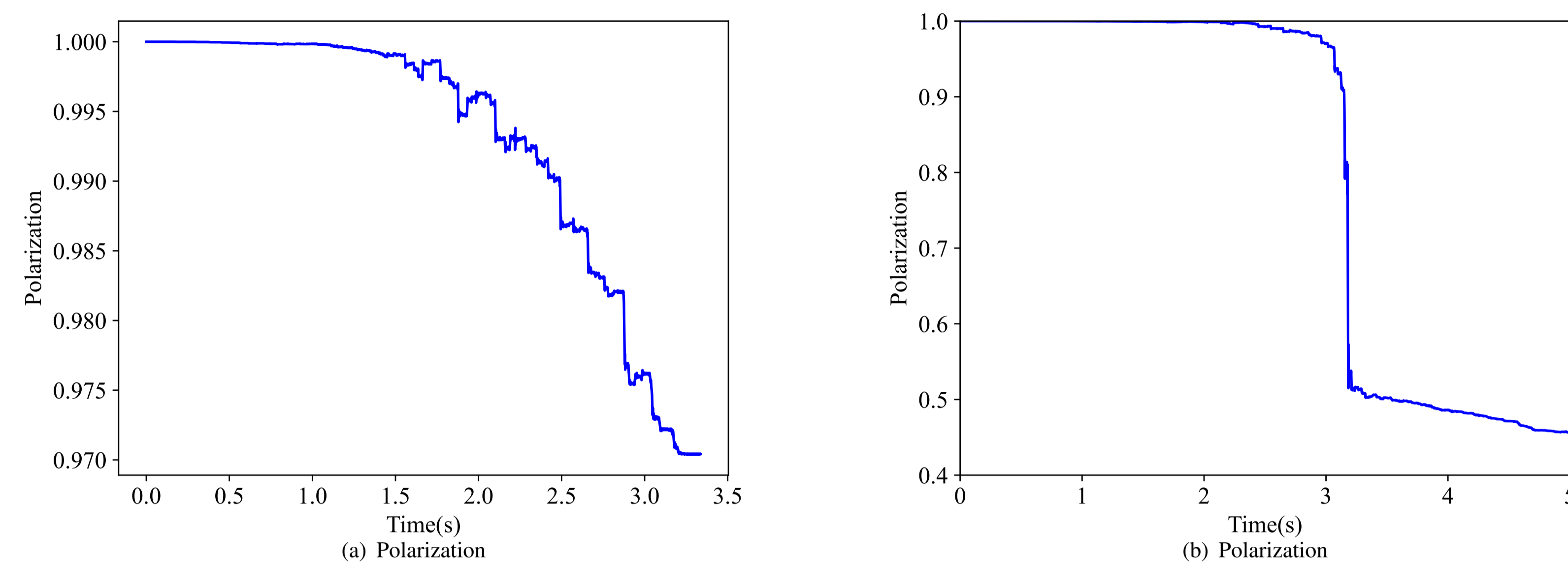


Figure 3: (a) The polarization during acceleration to 80 GeV, the polarization loss is little (b) The polarization during acceleration to 120 GeV, it's clearly to see the main depolarization occurs at the peak of the intrinsic resonance.

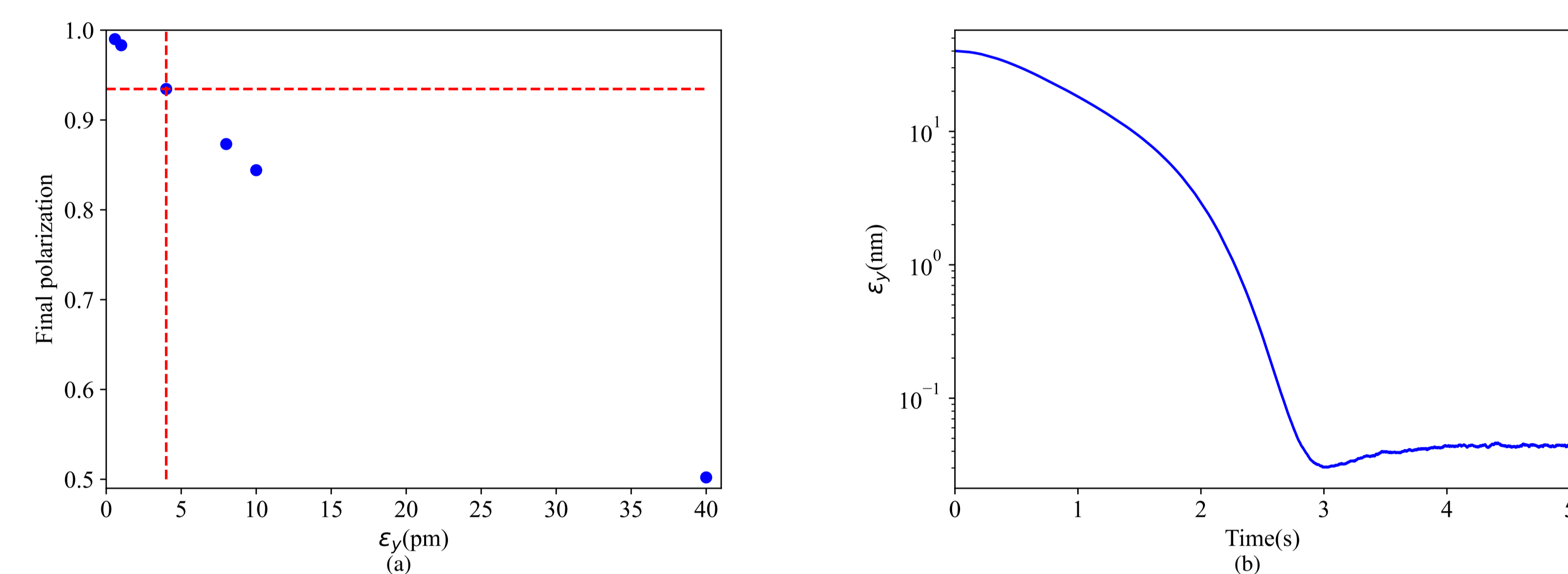


Figure 4: (a) The intrinsic resonance are proportional to the emittance, When crossing the main intrinsic resonance the emittance will be equilibrium emittance, so reduce equilibrium emittance can reduce the polarization loss. As shown, 90% polarization need about 4 pm emittance, this is an achievable goal through further optimization. (b) The beam is injected with a relatively high emittance (40 nm), the emittance damps to equilibrium emittance during the acceleration, in CDR lattice, equilibrium emittance is about 40 pm.

**More discussion on lattice design** In the CDR design, during ramping to 120GeV, crossing a strong resonance at  $\nu_B$  is inevitable since  $\nu_B < G\gamma(120GeV)$ . So we try to use a lattice with more betatron phase advance for better polarization performance. Specifically, we choose a lattice with  $\nu_y = 353.28$  and  $\nu_B = 281$ , and we have carefully studied the polarization performance of the lattice, Results are showed below.

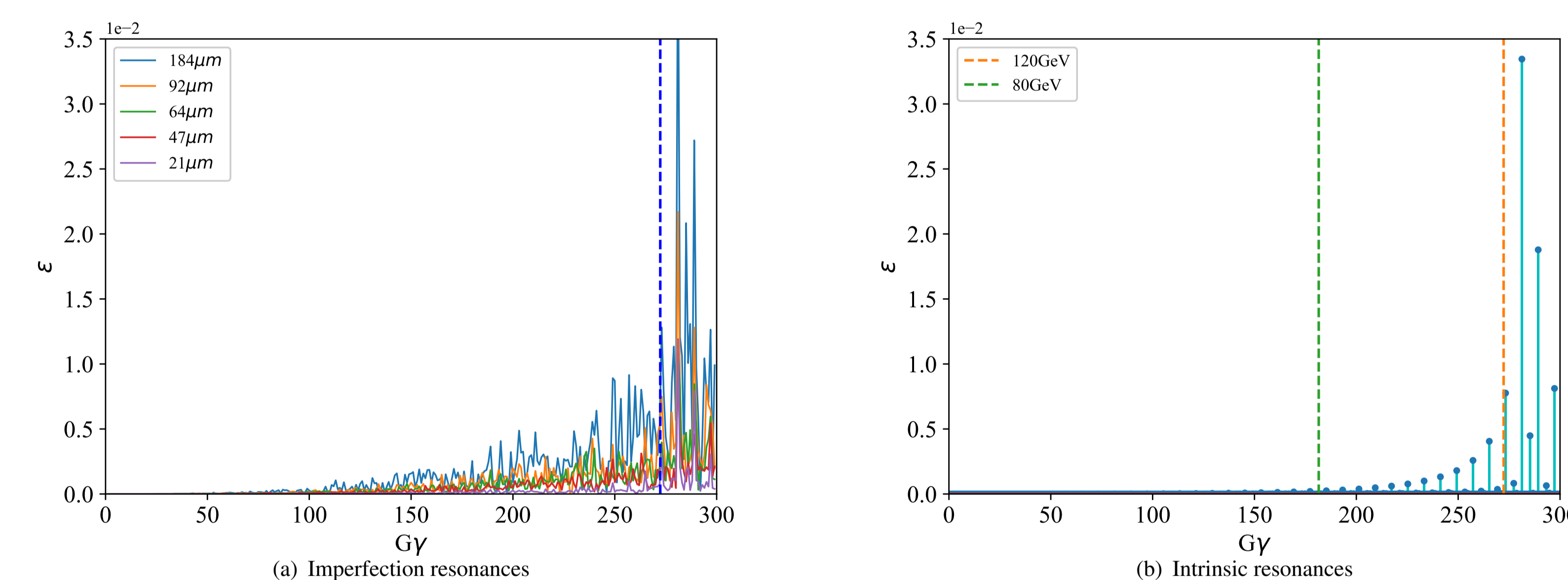


Figure 5: Two major resonances in new lattice. There are peaks at both intrinsic and imperfection resonances near  $G\gamma \approx 281$ . But accelerating to 120 GeV need not to crossing them, this makes the polarization performance much better than the CDR lattice.

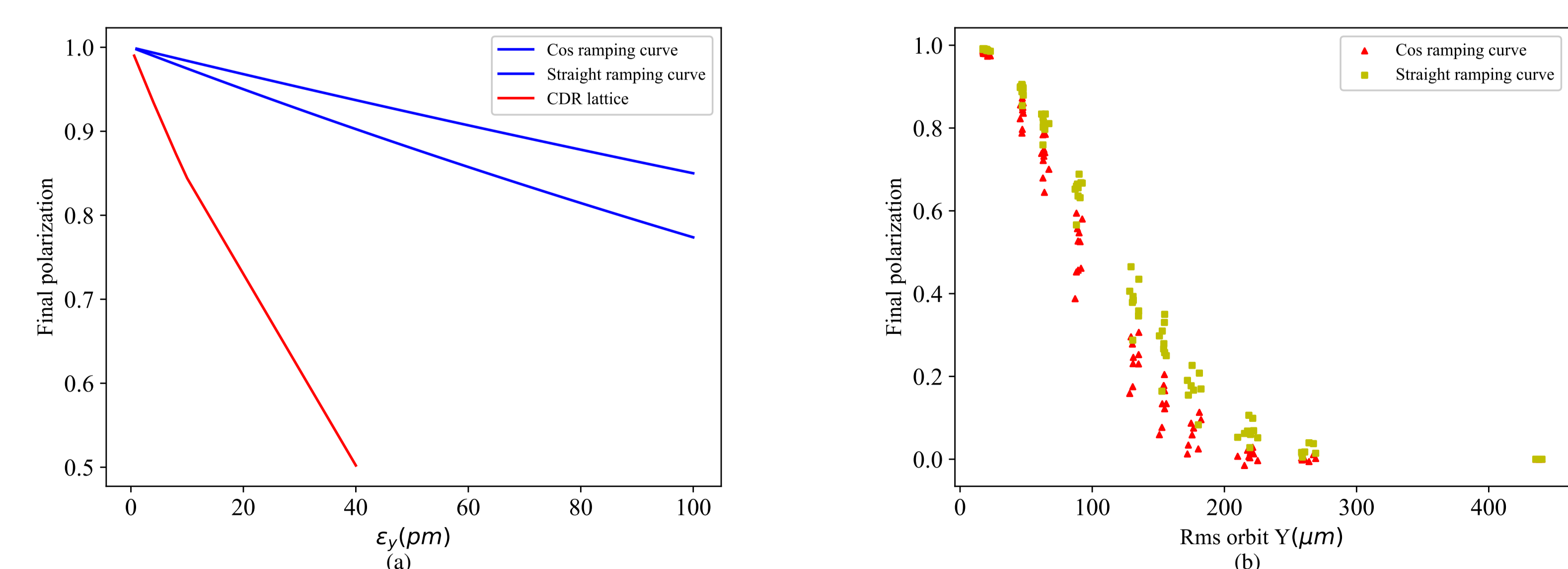


Figure 6: Influence of intrinsic resonances and imperfection resonances in the new lattice. In the new lattice, the polarization performance in same emittance is much better. The strength of imperfection resonances is sensitive to the closed-orbit. According to calculation, 80% polarization requires rms orbit to below 60  $\mu m$ .

**Other** In addition to the effect of the structure of the ring on polarization discussed above, There are some other factors need to be considered:

- Ramping curve: The ramping speed will smoothly transition to 0 at the end of the ramping process, this means at the end of the acceleration the crossing speed can be small, so the polarization loss become larger.
- Synchrotron radiation: Since the acceleration is a quick process, the effect of SR can be ignored.
- Dynamic error: Simulation results shows that 0.5% dynamic error has no effect on our conclusions.
- Tapering: Preliminary results show it has a greater impact to the imperfection resonances at the peak, so the results of CDR lattice may need further research.