# Electroweak Phase Transition in 2HDM under Higgs, $Z$-pole, and $W$ precision measurements

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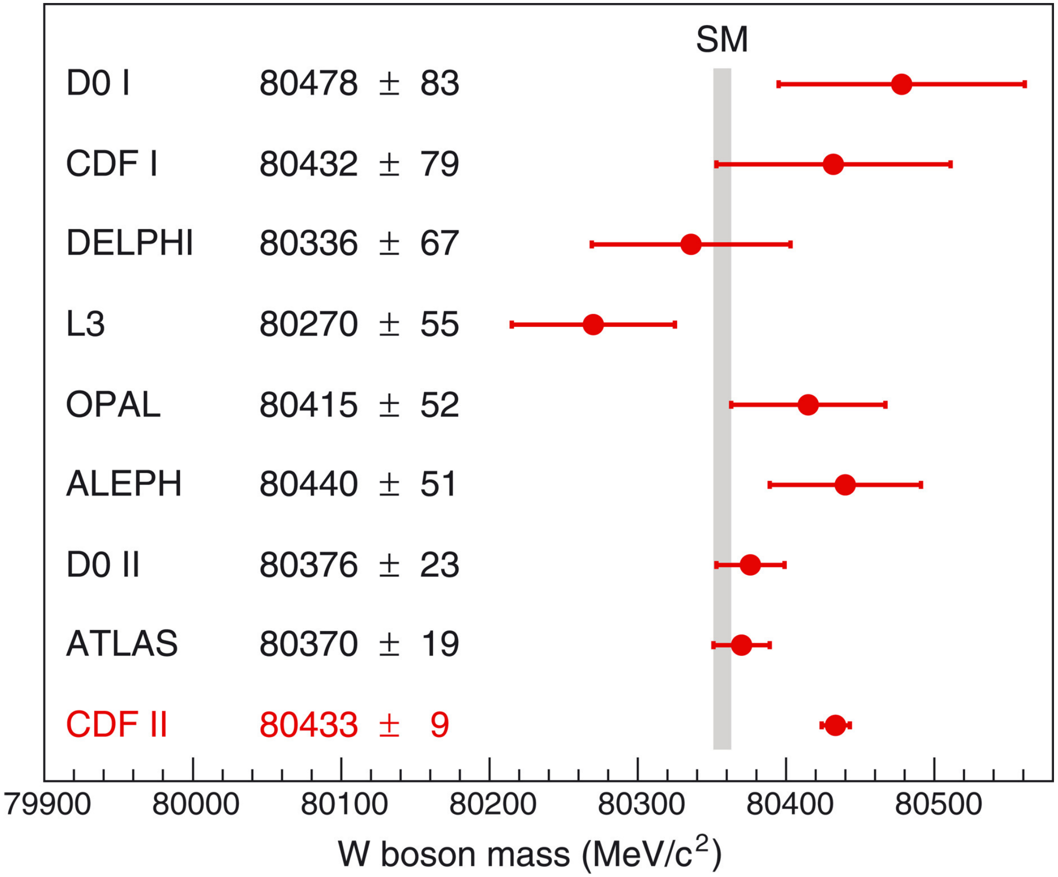
# CDF II $W$ Boson Mass

In April 2022, the CDF collaboration released a new measurement of the mass of the $W$ boson with the most precise to date, standing $7\sigma$ from the SM prediction



# Explanations?

1. An understanding of experimental assumptions and calibrations is important for theorists, and comparative studies are needed for other experimental measurements.

2. Accepting the CDF II result at face value, it must a clear hint of new physics.

$$\Delta m_W^2 = \frac{\alpha c_W^2 m_Z^2}{c_W^2 - s_W^2}\left[-\frac{\mathcal{S}}{2} + c_W^2 \mathcal{T} + \frac{c_W^2 - s_W^2}{4 s_W^2}\mathcal{U}\right] > 0$$

within the electroweak(EW) sector in terms of the oblique parameters $S$, $T$ and $U$

$\longrightarrow$ Indicates a sizable $T$, mass splitting in NP (SFOEWPT)

***consistent with EW, Higgs data, further?***

# Mini-review of 2HDM

We consider a CP conserving two-Higgs-doublet model scalar potential with a softly broken $Z_2$ symmetry to inhibit tree-level flavor changing neutral currents

## Two Higgs Doublet Model:

$$V^0(\Phi_1,\Phi_2) = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - m_{12}^2\left(\Phi_1^\dagger\Phi_2 + h.c.\right) + \frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2$$
$$+\lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger\Phi_2\right)^2 + h.c.\right]$$

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ (v_i + \phi_i^0 + iG_i)/\sqrt{2} \end{pmatrix}$$

$$v_u^2 + v_d^2 = v^2 = (246\mathrm{GeV})^2$$
$$\tan\beta = v_u/v_d$$

| | $\Phi_1$ | $\Phi_2$ |
|---|---|---|
| Type I | $u, d, \ell$ | |
| Type II | $u$ | $d, \ell$ |
| Lepton-specific | $u, d$ | $\ell$ |
| Flipped | $u, \ell$ | $d$ |

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}, \qquad \begin{aligned} A &= -G_1\sin\beta + G_2\cos\beta \\ H^\pm &= -\phi_1^\pm\sin\beta + \phi_2^\pm\cos\beta \end{aligned}$$

## Input Parameters

$$\boxed{m_{11}^2, m_{22}^2, m_{12}^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5} \rightarrow \boxed{v, \tan\beta, \cos(\beta-\alpha), m_{12}^2, m_h, m_H, m_A, m_{H^\pm}}$$

# Theoretical constraints on 2HDM

- **Vacuum stability**, potential bounded from below

$$\lambda_1 > 0\,,\;\; \lambda_2 > 0\,,\;\; \lambda_3 > -\sqrt{\lambda_1\lambda_2}\,,\;\; \lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1\lambda_2}$$

- **Perturbativity**, couplings are perturbative $|\lambda_i| \leq 4\pi$

- **Tree-level unitarity**, bounds on combinations of $\lambda_i$

$$\left|3(\lambda_1+\lambda_2) \pm \sqrt{9(\lambda_1-\lambda_2)^2 + 4(2\lambda_3+\lambda_4)^2}\right| < 16\pi\;,$$

$$\left|(\lambda_1+\lambda_2) \pm \sqrt{(\lambda_1-\lambda_2)^2 + 4\lambda_4^2}\right| < 16\pi\;,$$

$$\left|(\lambda_1+\lambda_2) \pm \sqrt{(\lambda_1-\lambda_2)^2 + 4\lambda_5^2}\right| < 16\pi\;,$$

$$|\lambda_3 + 2\lambda_4 \pm 3\lambda_5| < 8\pi\;,\;\; |\lambda_3 \pm \lambda_4| < 8\pi\;,\;\; |\lambda_3 \pm \lambda_5| < 8\pi$$

# Theoretical constraints on 2HDM

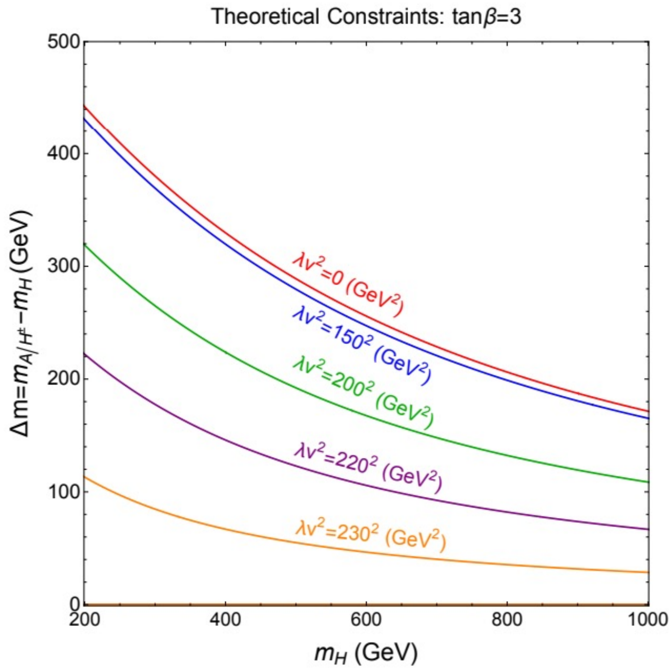
## How to understand these constraints?

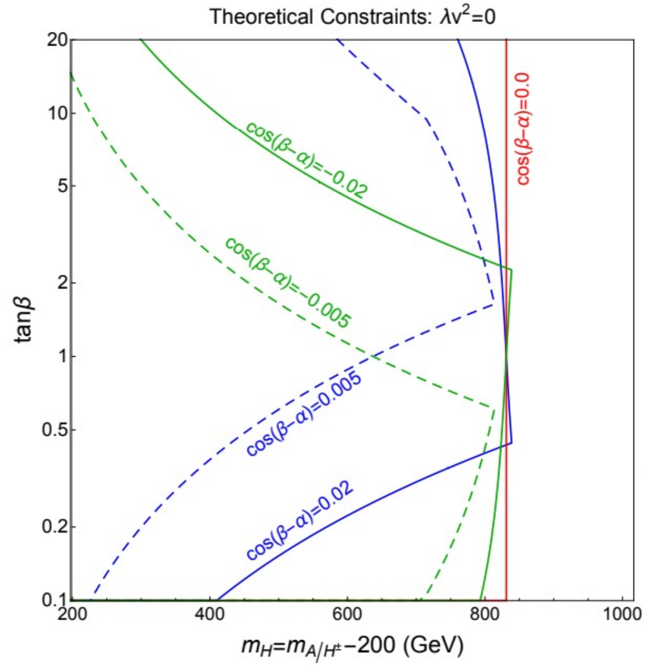
*Use relations between quartic couplings and the physical masses*

$$v^2\lambda_1 = m_h^2 - \frac{t_\beta\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{c_\beta^2} + \left(m_h^2 - m_H^2\right)\left[c_{\beta-\alpha}^2\left(t_\beta^2 - 1\right) - 2t_\beta s_{\beta-\alpha}c_{\beta-\alpha}\right]$$

$$\vdots$$

$$v^2\lambda_5 = m_H^2 - m_A^2 - \frac{m_{12}^2 - m_H^2 s_\beta c_\beta}{s_\beta c_\beta}$$




with

$$\lambda v^2 \equiv m_H^2 - m_{12}^2/(s_\beta c_\beta)$$

# Theoretical constraints on 2HDM

## How to understand these constraints?

*Use relations between quartic couplings and the physical masses*

$$v^2\lambda_1 = m_h^2 - \frac{t_\beta\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right)}{c_\beta^2} + \left(m_h^2 - m_H^2\right)\left[c_{\beta-\alpha}^2\left(t_\beta^2 - 1\right) - 2t_\beta s_{\beta-\alpha}c_{\beta-\alpha}\right]$$

$\vdots$

$$v^2\lambda_5 = m_H^2 - m_A^2 - \frac{m_{12}^2 - m_H^2 s_\beta c_\beta}{s_\beta c_\beta}$$

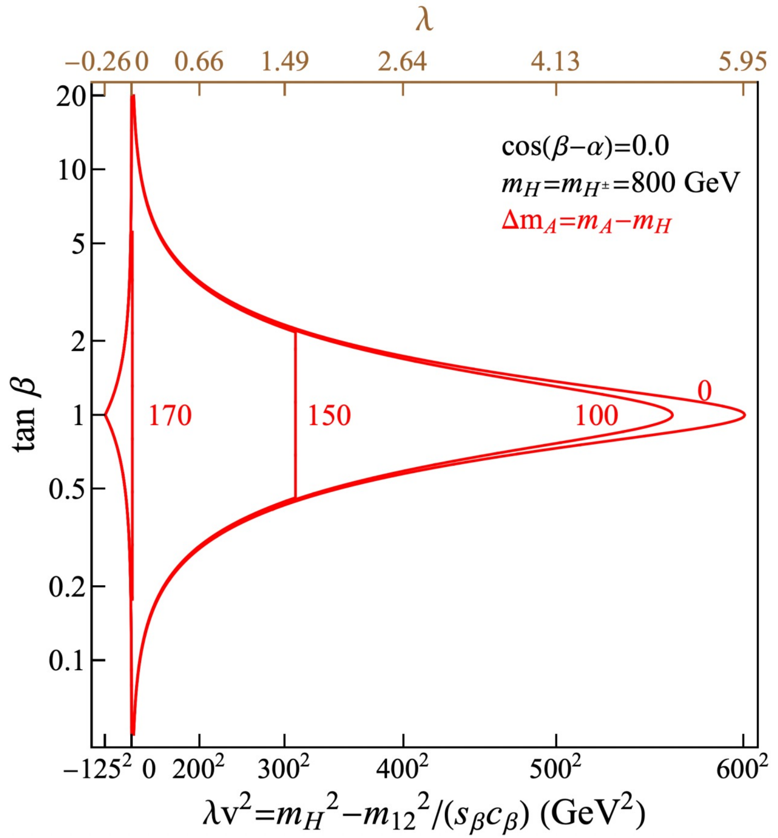
## Theoretical Constraints :

$$\lambda v^2 < 4\pi v^2,$$

$$\max\{\tan\beta, \cot\beta\} \lesssim \sqrt{(8\pi v^2)/(3\lambda v^2)},$$

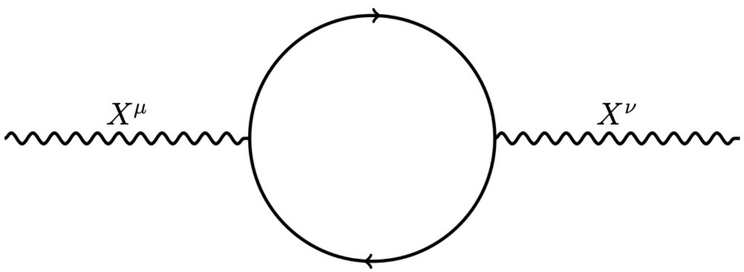
$$m_A^2 - m_H^2 \lesssim \mathcal{O}\left(4\pi v^2 - \lambda v^2\right),$$

$$m_{H^\pm}^2 - m_H^2 \lesssim \mathcal{O}\left(4\pi v^2 - \lambda v^2\right).$$



# Z-pole constraints on 2HDM

In the framework of the global EW fit ($Z$-pole observables), the $W$ boson mass can be predicted. Within the EW sector, the NP in the BSM can be parameterized in terms of the oblique parameters $S$, $T$ and $U$.




$= i\Pi_{XX}(p^2)g^{\mu\nu}$

$$S \equiv \frac{4c_W^2 s_W^2}{\alpha}\left[\Pi'_{ZZ}(0) - \frac{c_W^2 - s_W^2}{c_W s_W}\Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0)\right],$$

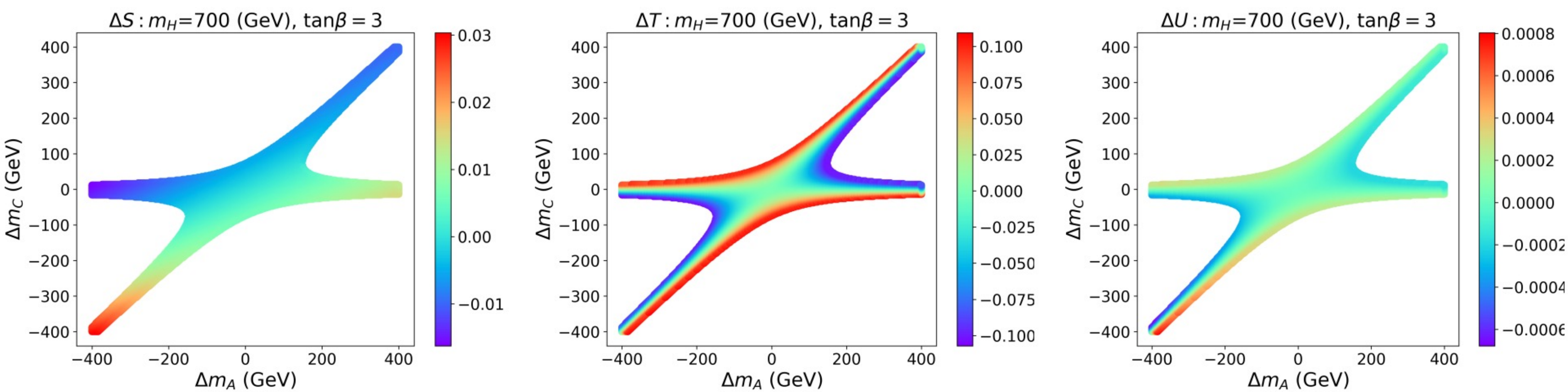
$$T \equiv \frac{1}{\alpha}\left[\frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2}\right],$$

$$U \equiv \frac{4s_W^2}{\alpha}\left[\Pi'_{WW}(0) - \frac{c_W}{s_W}\Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0)\right] - S,$$

# Z-pole constraints on 2HDM

$Z$-pole physics at LEP impose strong constraints on oblique parameters (on the 2HDM)

| | Current | | | | CEPC | | | | FCC-ee | | | | ILC | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\sigma$ | correlation | | | $\sigma$ | correlation | | | $\sigma$ | correlation | | | $\sigma$ | correlation | | |
| | | $S$ | $T$ | $U$ | $(10^{-2})$ | $S$ | $T$ | $U$ | $(10^{-2})$ | $S$ | $T$ | $U$ | $(10^{-2})$ | $S$ | $T$ | $U$ |
| $S$ | $0.04 \pm 0.11$ | 1 | 0.92 | $-0.68$ | 1.82 | 1 | 0.9963 | $-0.9745$ | 0.370 | 1 | 0.9898 | $-0.8394$ | 2.57 | 1 | 0.9947 | $-0.9431$ |
| $T$ | $0.09 \pm 0.14$ | – | 1 | $-0.87$ | 2.56 | – | 1 | $-0.9844$ | 0.514 | – | 1 | $-0.8636$ | 3.59 | – | 1 | $-0.9569$ |
| $U$ | $-0.02 \pm 0.11$ | – | – | 1 | 1.83 | – | – | 1 | 0.416 | – | – | 1 | 2.64 | – | – | 1 |

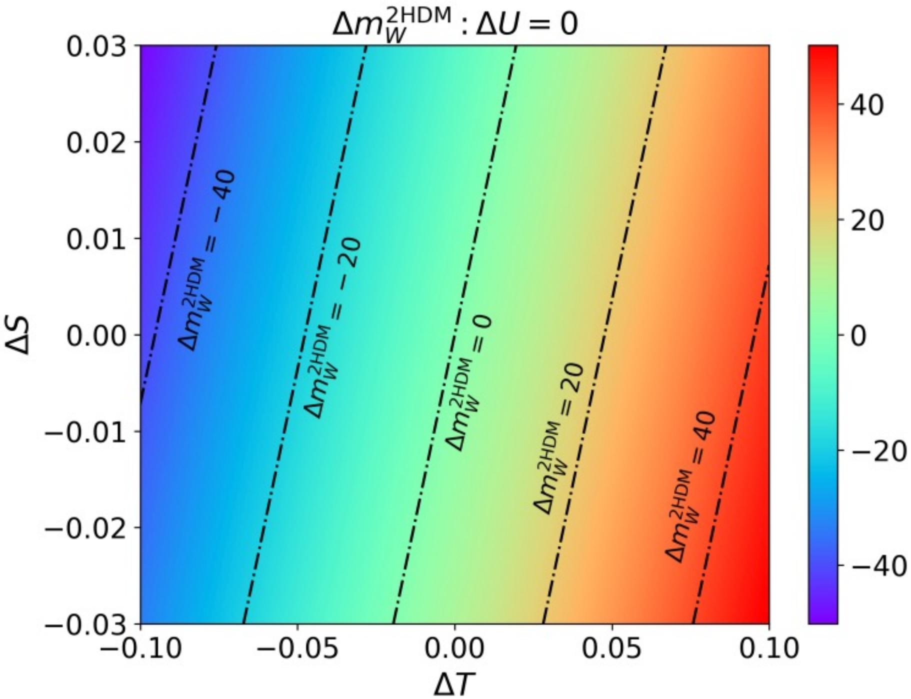


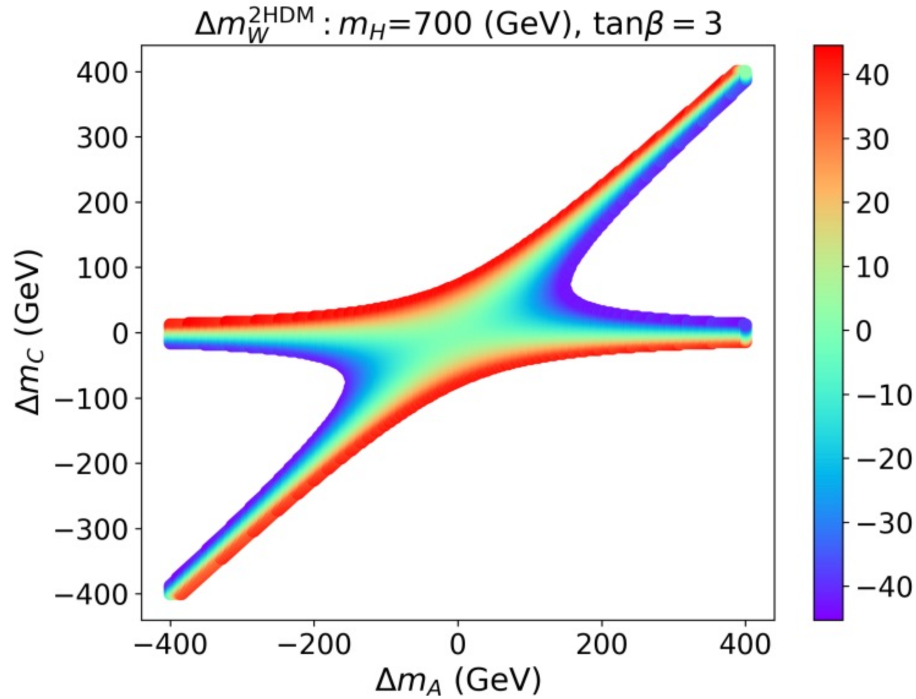
# Hierarchical 2HDM to Solve $M_W$ Anomaly

The $W$ boson mass can be expressed in terms of $STU$ as

$$m_W^{\rm 2HDM} = m_W^{\rm SM}\left[1+\frac{\alpha {c_W}^2}{2\left({c_W}^2-{s_W}^2\right)}T(1+\delta\rho^{\rm 2HDM})+\frac{\alpha}{8{s_W}^2}U-\frac{\alpha}{4\left({c_W}^2-{s_W}^2\right)}S\right]$$

where $\delta\rho^{\rm 2HDM}\sim\frac{|\lambda_{hhh}^{\rm 2HDM}|^2}{12\pi^2 m_h^2}$ charactrizes higher order 2HDM effects from enhanced Higgs boson self-interactions. Given the current constraint on $\kappa_{hhh}=\lambda_{hhh}^{\rm 2HDM}/\lambda_{hhh}^{\rm SM}$ is between (−1.0, 6.6), $\delta\rho^{\rm 2HDM}$ is only $\mathcal{O}(0.01)$, which can be neglected.



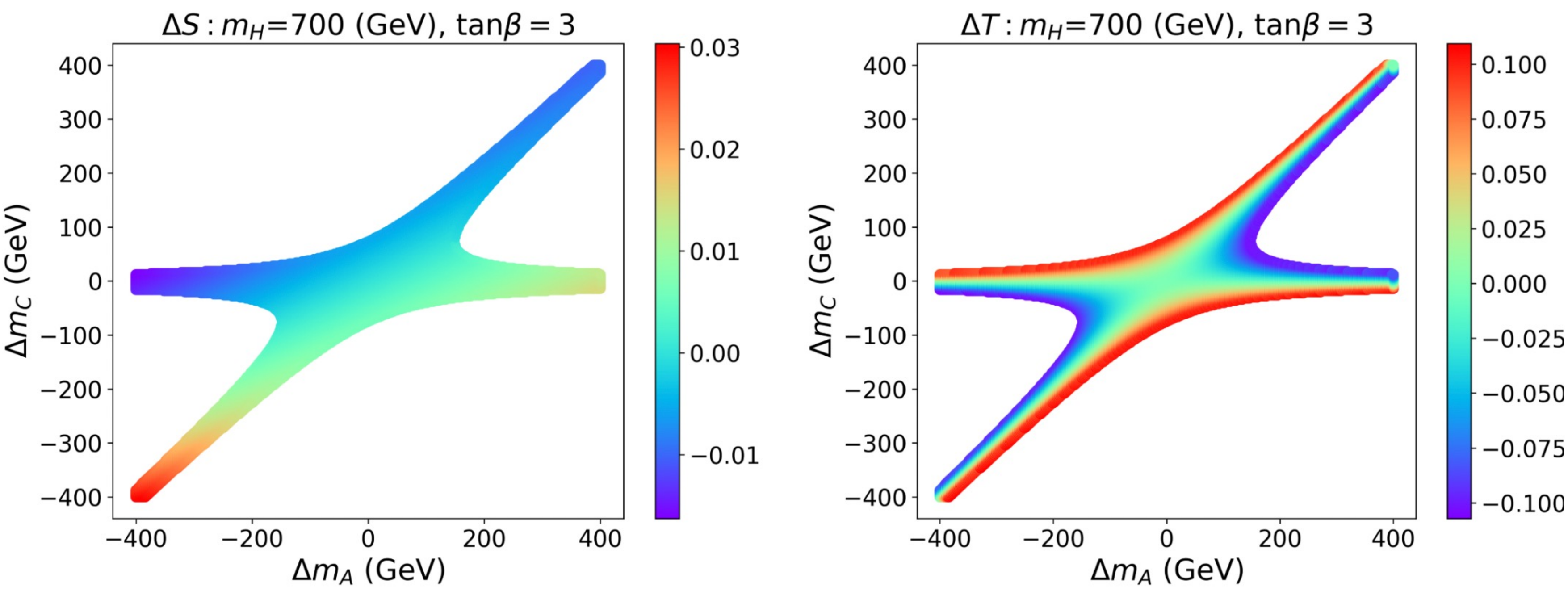
# Hierarchical 2HDM to Solve $M_W$ Anomaly

## How to understand?

*Notice that the mass shift and T plot have same features, U is neglectable*

$$T \xRightarrow{alignment} \frac{1}{16\pi s_W^2 c_W^2 m_Z^2}\left[F\left(m_{H^\pm}^2, m_A^2\right) + F\left(m_{H^\pm}^2, m_H^2\right) - F\left(m_A^2, m_H^2\right)\right]$$

$$\approx \frac{1}{12\pi s_W^2 c_W^2 m_Z^2}\Delta m_C(\Delta m_C - \Delta m_A) + \mathcal{O}(\Delta m^3)$$



# What Else Does Hierarchical Spectrum Buy us?

Let's examine the electroweak phase transition in 2HDM

## Thermal corrected effective potential

$$V(\phi_1,\phi_2,T) = V^0(\phi_1,\phi_2) + V^{\rm CW}(\phi_1,\phi_2) + V^{\rm CT}(\phi_1,\phi_2) + V^{\rm T}(\phi_1,\phi_2,T)$$

The strength of the electroweak phase transition is quantified by $\xi_c = v_c/T_c$, where $T_c$ is critical temperature of the phase transition, and $v_c$ is the VEV of the broken minimum at this temperature. To have a strong enough phase transition to account for baryogenesis, $\xi_c \gtrsim 1$ (we relax this to 0.9 to accout for the theoretical uncertainty).

## In high temperature limit

$$V(\phi_h,T) \approx (DT^2-\mu^2)\phi_h^2 - ET\phi_h^3 + \frac{\tilde\lambda}{4}\phi_h^4 \longrightarrow \xi_c \equiv \frac{v_c}{T_c} \approx \frac{2E}{\tilde\lambda}$$

in 2HDM $E = \frac{1}{12\pi}\left[6\frac{m_W^3}{v^3} + 3\frac{m_Z^3}{v^3} + \frac{m_h^3}{v^3}\right] + \boxed{E_{(H/A/H^\pm)}}$ contributions from the non-SM Higgs bosons

$$-\frac{1}{12\pi}T(m_\alpha^2(\phi_h))^{3/2} = -\frac{1}{12\pi}T(M^2+\lambda_\alpha\phi_h^2)^{3/2}$$

$$\approx \begin{cases} -\frac{T}{12\pi}\lambda_\alpha^{3/2}\phi_h^3, & M^2 \ll \lambda_\alpha\phi_h^2 \\ -\frac{T}{12\pi}M^3\left(1+\frac{3}{2}\frac{\lambda_\alpha\phi_h^2}{M^2}\right), & M^2 \gg \lambda_\alpha\phi_h^2 \end{cases} \longrightarrow E_{(\alpha)} \approx \begin{cases} \frac{1}{12\pi}\lambda_\alpha^{3/2} = \frac{1}{12\pi}\frac{m_\alpha^3}{v^3}, & M^2 \ll \lambda_\alpha\phi_h^2 \\ 0, & M^2 \gg \lambda_\alpha\phi_h^2 \end{cases}$$

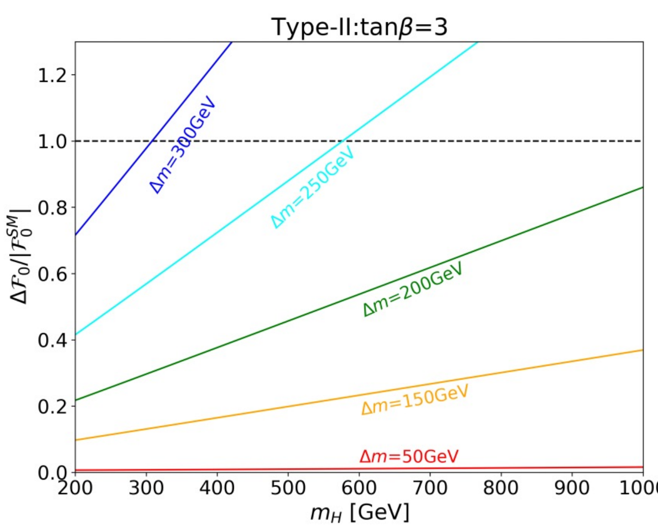
# What Else Does Hierarchical Spectrum Buy Us?

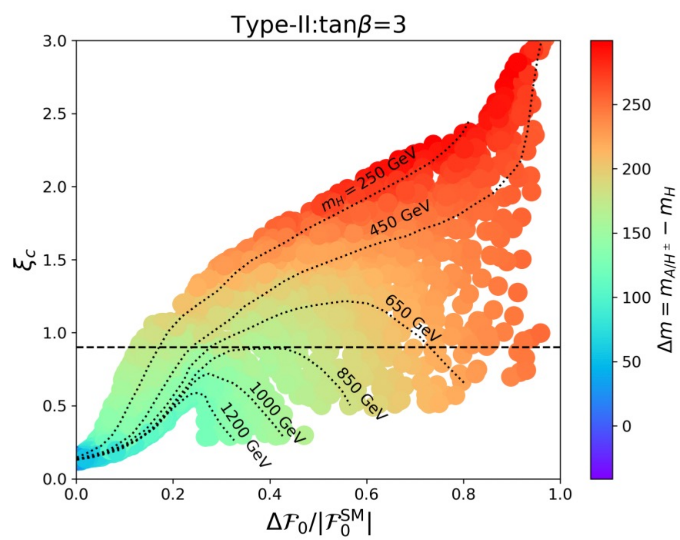
High temperature limit analysis indicates SFOEWPT prefers a non-degenerate mass spectrum in 2HDM.

The previous studies showed that the amount by which the VEV is “uplifted” with respect to the SM case, $\Delta\mathcal{F}_0/|\mathcal{F}_{\mathrm{SM}}| \equiv (\mathcal{F}_0 - \mathcal{F}_{\mathrm{SM}})/|\mathcal{F}_{\mathrm{SM}}|$, constitutes a good indicator of the increase in the strength of the EWPT.

$$\Delta\mathcal{F}_0 = \frac{1}{64\pi^2}\left[\boxed{\left(m_h^2 - 2M^2\right)^2\left(\frac{3}{2} + \frac{1}{2}\log\left[\frac{4m_A m_H m_{H^\pm}^2}{\left(m_h^2 - 2M^2\right)^2}\right]\right)} + \frac{1}{2}\left(m_A^4 + m_H^4 + 2m_{H^\pm}^4\right) + \left(m_h^2 - 2M^2\right)\left(m_A^2 + m_H^2 + 2m_{H^\pm}^2\right)\right]$$

$$\sim \frac{\left(m_h^2 - 2M^2\right)^2}{2M}\left(3M + M\ln\frac{4M^4}{(m_h^2 - 2M^2)^2} + \Delta m_A + 2\Delta m_C\right)$$



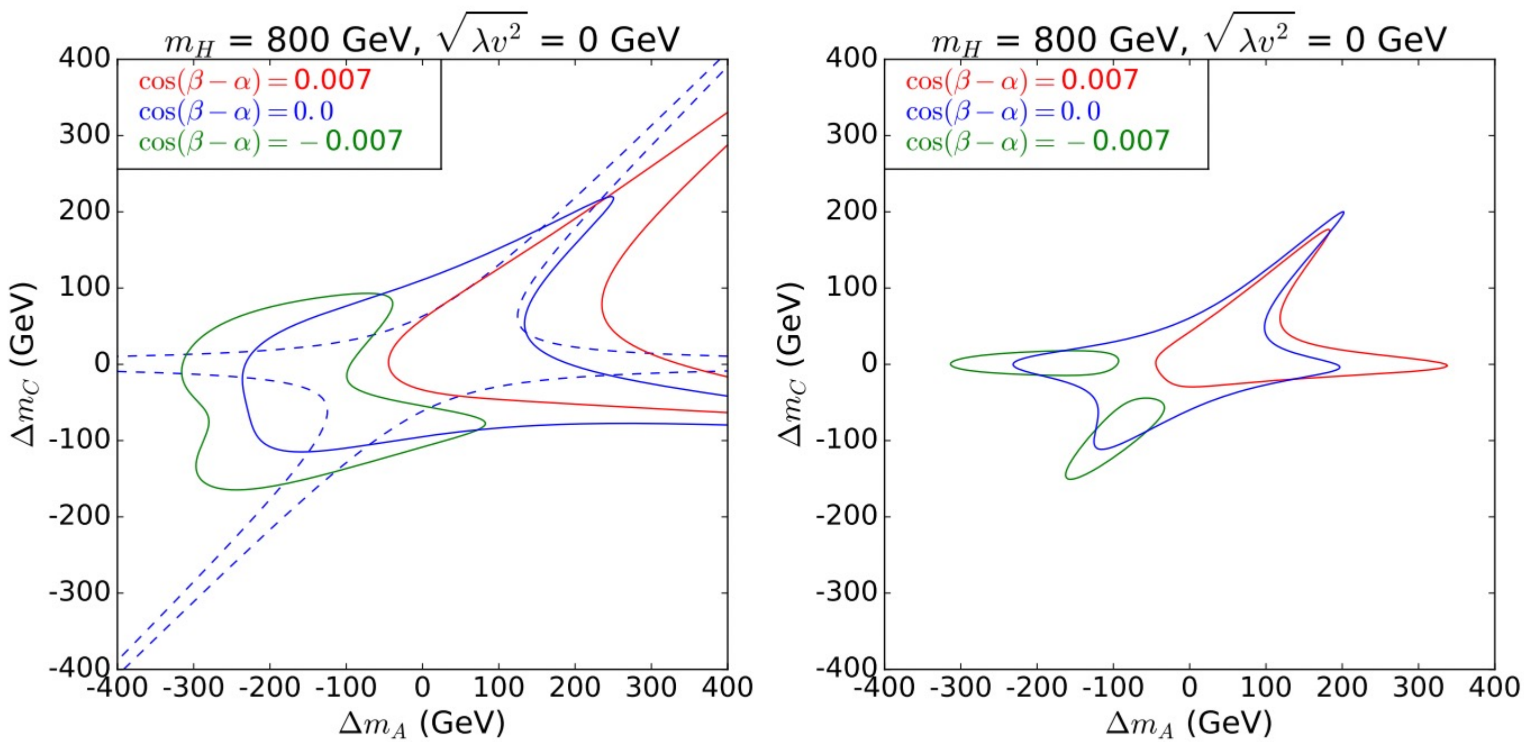



# Other constraints on 2HDM

- Direct searches on non-SM Higgses at LEP and LHC
- Flavor physics constraints

  Constraints on the masses of non-SM Higgses
- Higgs precision measurements

  Complementary to $Z$-pole observables and also constrain the mass splitting

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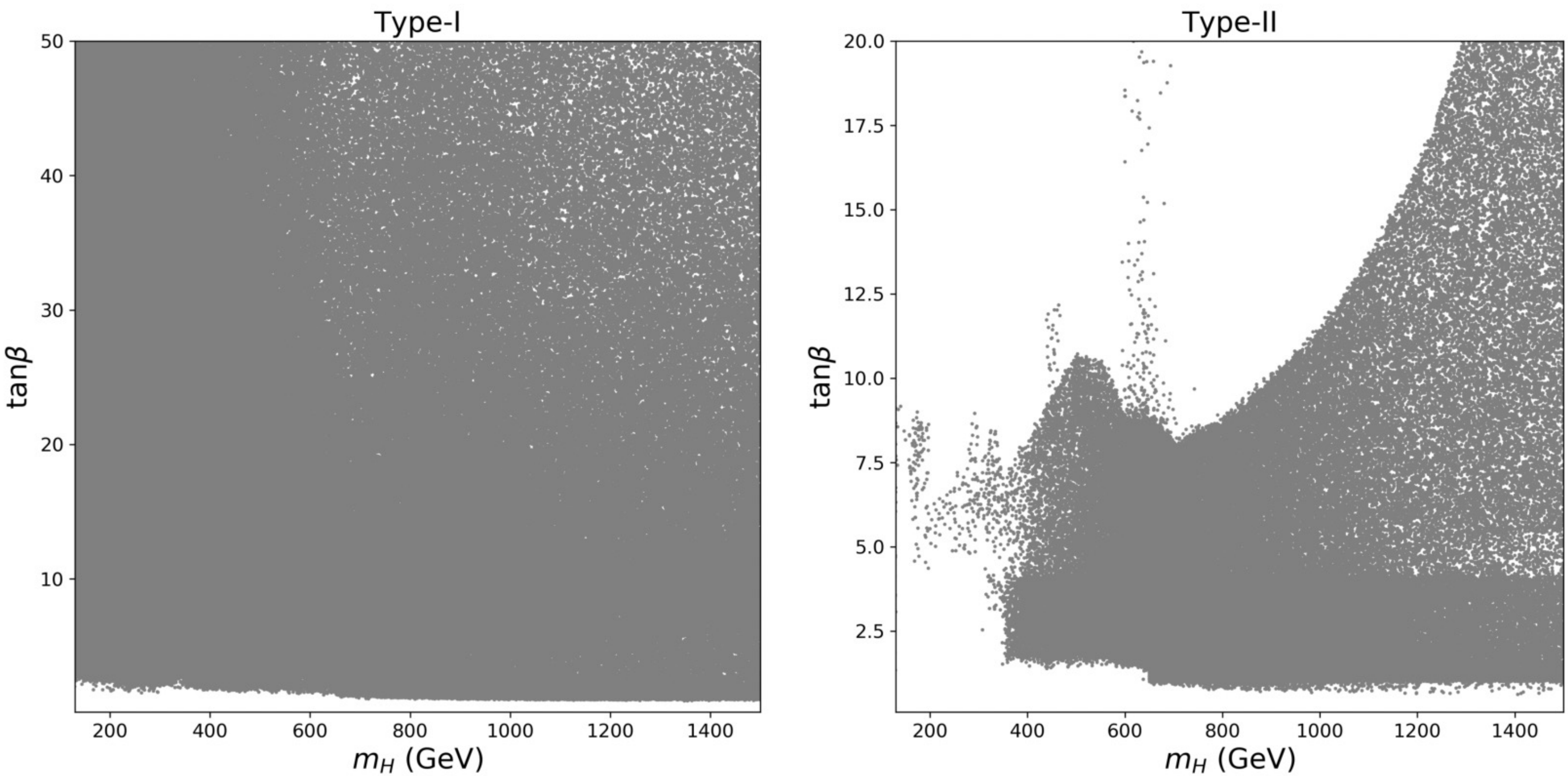
# Other constraints on 2HDM

Current constraints from LHC Run-II, including Higgs physics



# Our Study

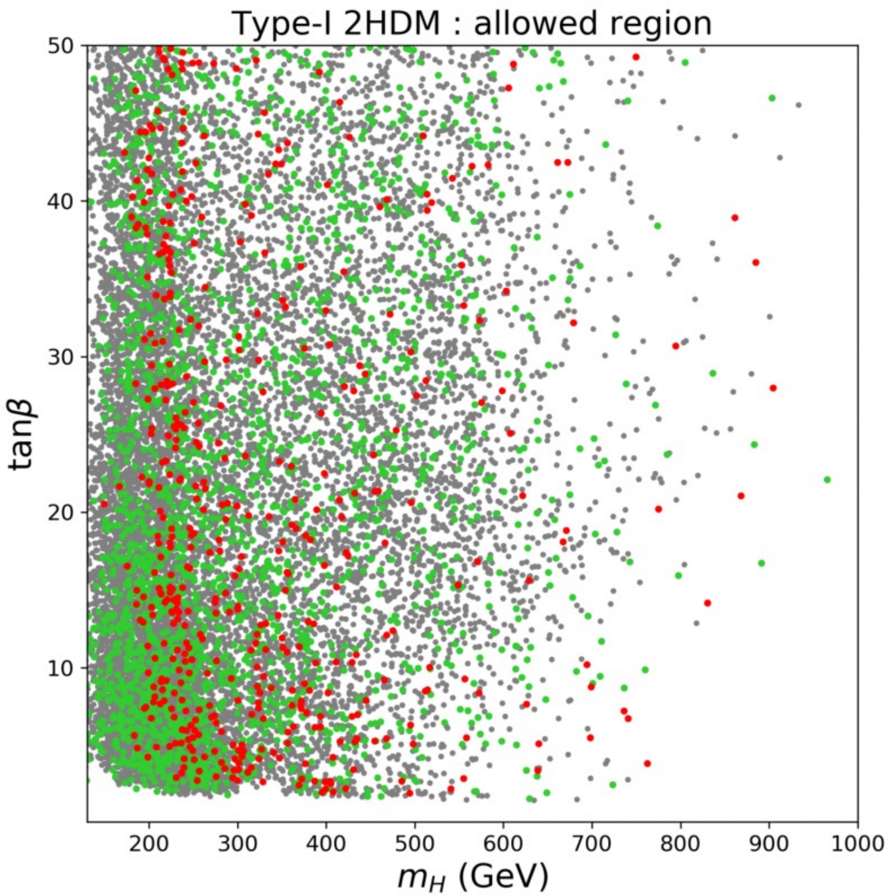
We perform a 6 parameters random scan for both Type-I and –II

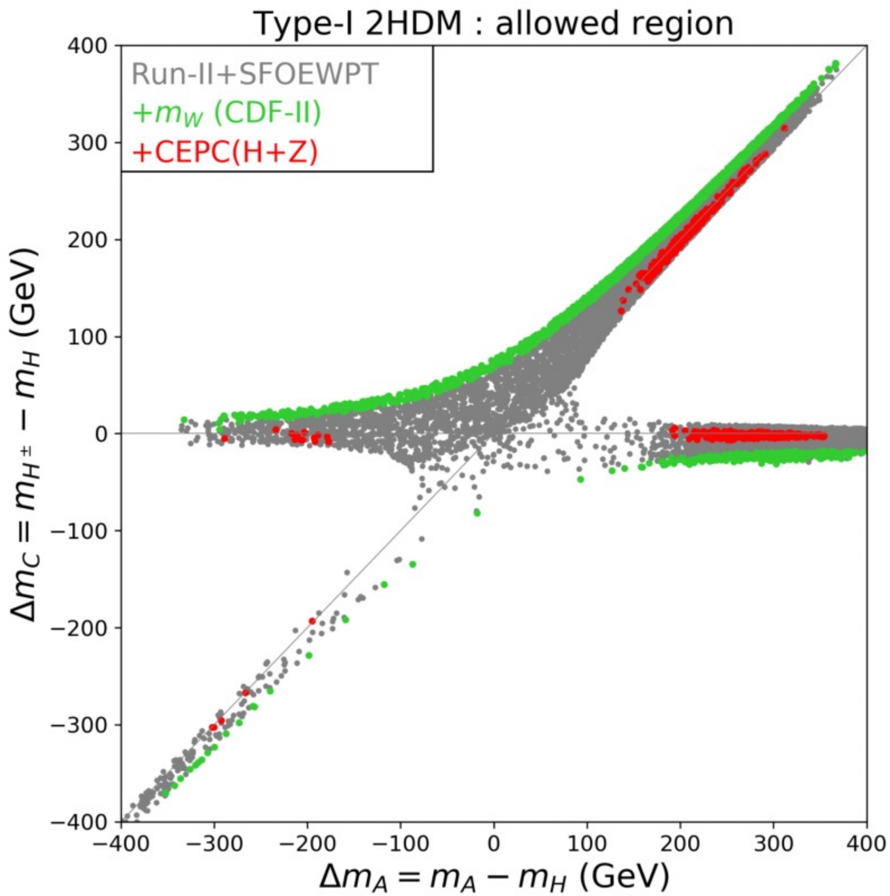
$$\tan\beta \in (0.2, 50), |\cos(\beta-\alpha)| < 0.5\,, \quad m_{A/H^\pm} \in (10, 1500)\ \text{GeV}\,,$$
$$m_{12}^2 \in (0, 1500^2)\ \text{GeV}^2, \qquad m_H \in (130, 1500)\ \text{GeV}.$$

with in total 1 billion points.

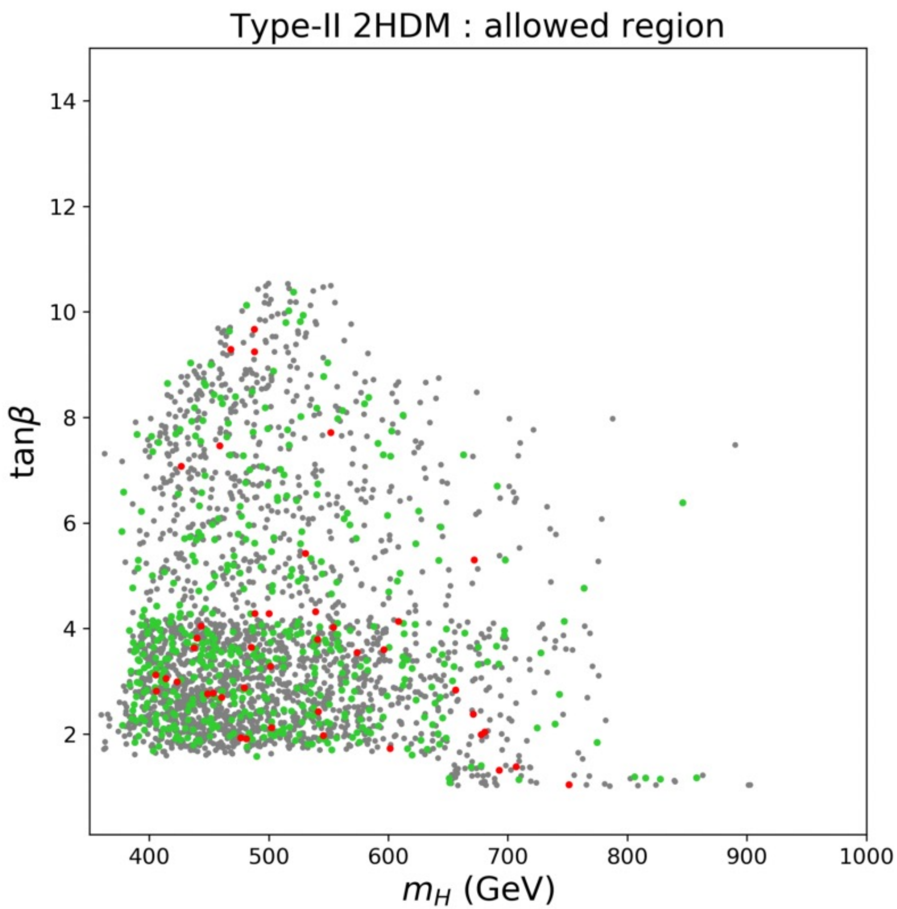
After considering various theoretical and experimental constraints, 1 million points are left; and further requiring a SFOEWPT, a few thousands of points allowed for Type-I (much less for Type-II) are shown, whose statistic is sufficient.

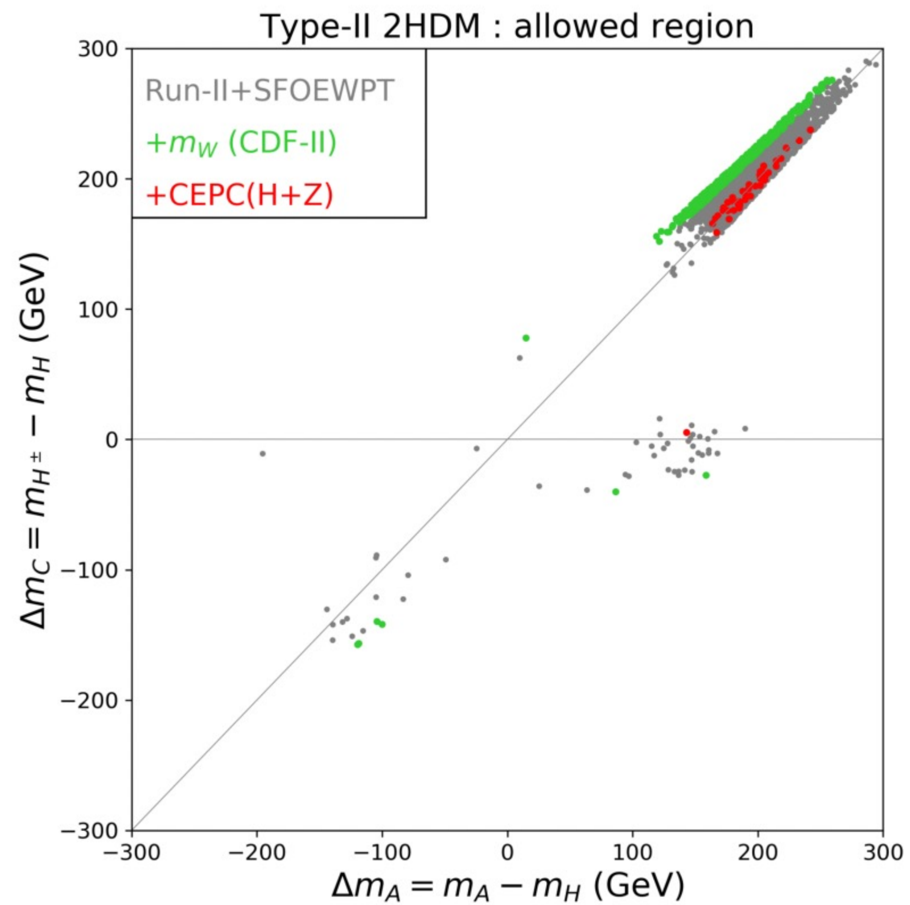
# Results for Type-I



# Results for Type-II



# Summary of Results

## Type-I:

$$m_H \in (125, 950)\ \mathrm{GeV}\,, \Delta m_{A/C} \in (-400, 400)\ \mathrm{GeV}, \tan\beta \in (1, 50)$$

2 regions

$\Delta m_C > 0\ \&\ \Delta m_C > \Delta m_A$

$\Delta m_C < 0\ \&\ \Delta m_C < \Delta m_A$

$\Delta m_A = 0$ is possilbe

## Type-II:

$$m_H \in (125, 900)\ \mathrm{GeV}\,, \Delta m_{A/C} \in (-200, 300)\ \mathrm{GeV}, \tan\beta \in (1, 10)$$

2 regions

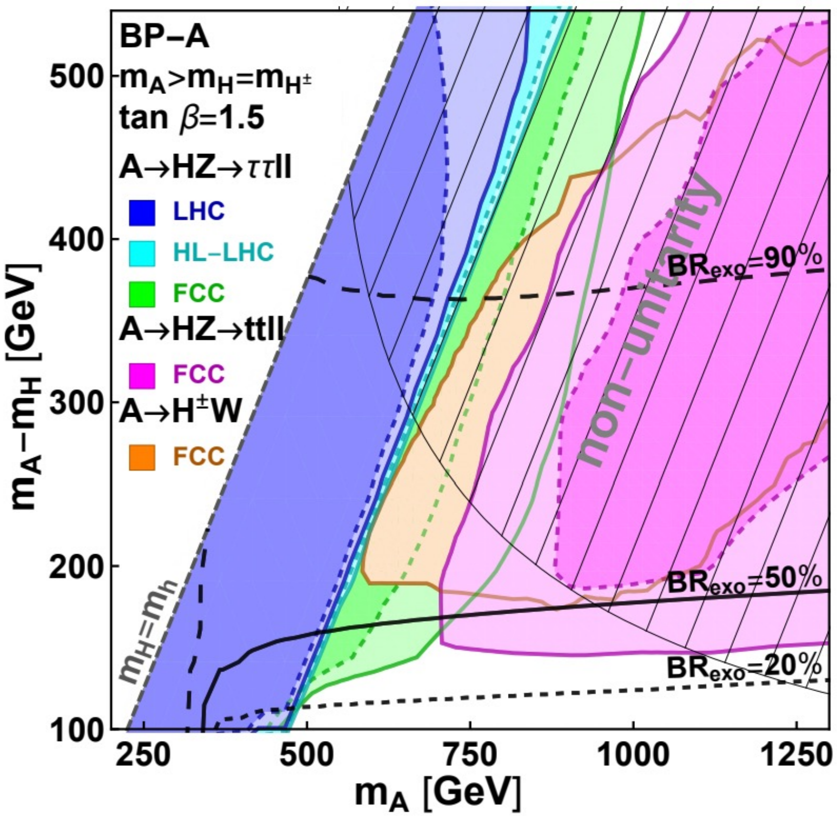
$\Delta m_C > 0\ \&\ \Delta m_C > \Delta m_A$

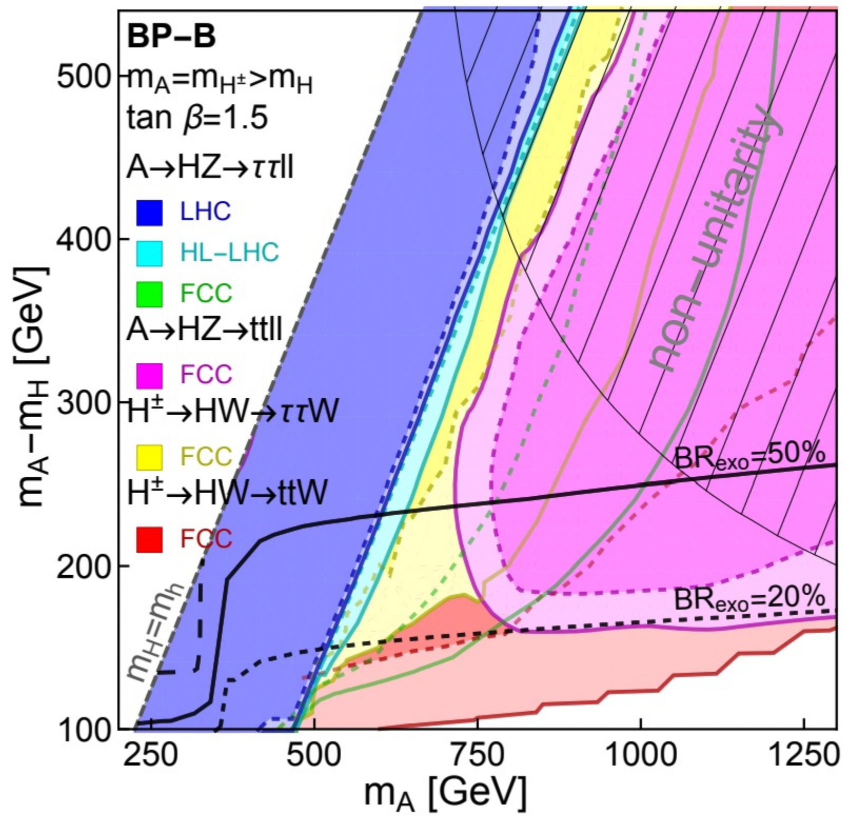
$\Delta m_C < 0\ \&\ \Delta m_C < \Delta m_A$

$\Delta m_{A/C} \neq 0$

# Direct Searches via Exotic Higgs Decays

The mass splitting gives extra decay channels for the non-SM Higgses which might suppress the traditional search channels via di-boson or di-fermion final state. Also notice that $g_{HVV} \propto \cos(\beta - \alpha) \sim 0$ in alignment limit.

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F. Kling, H Li, S Li, A. Pyarelal, **HS**, S. Su, and W. Su

# conclusion

1. A hierarchical mass spectrum in 2HDM can explain CDF II $m_W$
2. SFOEWPT also favors a non-SM Higgs mass splitting, and gives a mass limit ($m_H \lesssim 1$ TeV)
3. Higgs precision measurements are sensitive to the mass splitting
4. Future precision measurements at CEPC, ILC, or FCC-ee can test hierarchical 2HDM, no deviation to SM observed at Higgs physics may lead to tension with CDF II $m_W$
5. Hadron colliders can explore almost all parameter space of hierarchical 2HDM via exotic Higgs decay channel

# Thanks!