D_s^* weak decays and the experiment potential

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arXiv:2203.06797, in collaboration with Y.H Ju, Q. Qin and F.S Yu

October 25, 2022

Overview

Motivation-desirability

- $\alpha_s : \alpha : G_F \sim \mathcal{O}(1) : \mathcal{O}(\frac{1}{137}) : \mathcal{O}(10^{-5})$
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- very hard to measure weak decay from strong and e.m interactions
- so the total widths of heavy-light vector mesons are still in lack
  - ⋆ $\Gamma_{D^{*+}} = 84.3 \pm 1.8\,\mathrm{keV} \quad \to D^0\pi^+, D^+\pi^0, D^+\gamma$
  - ⋆ $\Gamma_{D^{*0}} < 2.1\,\mathrm{MeV}$ $\quad \Gamma_{D_s^{*+}} < 1.9\,\mathrm{MeV}$ [PDG 2022]
    $\to D^0\pi^0, D^0\gamma \quad \to D_s^+\gamma, D_s^+\pi^0, D_s^+e^+e^-$
  - ⋆ $\Gamma_{B^*}, \Gamma_{B_s^*}$ no measurement

- but they are very important
  properties, structures, $g_{D_s^* D_s \gamma}$, non-perturbative approaches [Li 2020]

LCSRs, partial NLO

	$g_{D^{*+}D^+\gamma}$ (GeV^{-1})	$g_{D^{*0}D^0\gamma}$ (GeV^{-1})	$g_{D_s^{*+}D_s^+\gamma}$ (GeV^{-1})
this work	$-0.15^{+0.11}_{-0.10}$	$1.48^{+0.29}_{-0.27}$	$-0.079^{+0.086}_{-0.078}$
HHχPT [24]	-0.27 ± 0.05	2.19 ± 0.11	0.041 ± 0.056
HQET+VMD [35]	$-0.29^{+0.19}_{-0.11}$	$1.60^{+0.35}_{-0.45}$	$-0.19^{+0.19}_{-0.08}$
HQET+CQM [71]	$-0.38^{+0.05}_{-0.06}$	1.91 ± 0.09	–
Lattice QCD [32]	-0.2 ± 0.3	2.0 ± 0.6	–
LCSR [21]	-0.50 ± 0.12	1.52 ± 0.25	–
QCDSR [20]	$-0.19^{+0.03}_{-0.02}$	0.62 ± 0.03	-0.20 ± 0.03
RQM [72]	-0.44 ± 0.06	2.15 ± 0.11	-0.19 ± 0.03
experiment [16–18]	-0.47 ± 0.06	1.77 ± 0.03	–

- LCSRs, complete NLO corrections, $g_{D_s^* D_s \gamma} = 0.60^{+0.19}_{-0.18}$, [Pullin 2021]
- very sensitive to different contributions (radiative corrections, power corrections), a benchmark to probe the involved dynamics

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- but they are very important
  properties, structures, $g_{D_s^* D_s \gamma}$, non-perturbative approaches, et.al.,
- impressive lattice QCD evaluation

$$\Gamma_{D_s^{*+}} = 0.070(28)\,\mathrm{keV}$$ [HPQCD 2013]

the longest-lived charged vector meson

- encourage us to study the exclusive D_s^* weak decay

Motivation-significances

† leptonic decays, helicity enhanced $D_s^* \to l\nu$, decay constant

$$\Gamma_{D_s^* \to l\nu} = \frac{G_F^2}{12\pi} |V_{cs}|^2 f_{D_s^*}^2 m_{D_s^*}^3 \left(1 - \frac{m_l^2}{m_{D_{s*}^2}}\right)\left(1 + \frac{m_l^2}{m_{D_{s*}^2}}\right) = 2.44 \times 10^{-12}\,\mathrm{GeV}\,.$$

† semileptonic decays, $D_s^* \to \phi l\nu$, $|V_{cs}|$ and helicity form factors

- the least precisely determinations of CKM unitarity

$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1.026 \pm 0.022\,, \quad |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.025 \pm 0.022$$

$$|V_{cs}| = 0.987 \pm 0.011\,,$$

D_s^* weak decay are highly anticipated to reduce the uncertainty.

- heavy quark symmetry (HQS) has been examined in $\bar{B} \to D^*(D)l\bar{\nu}$, can also be tested in $D_s^*(D_s) \to \phi l^+\nu$
- lepton flavour university (LFU) in vector charm sector

† hadronic decays, $D_s^* \to \phi\rho, \phi\pi$, factorisation theorem or topological analysis

† inclusive decays, $D_s^* \to X_s l\nu$, HQET and reliability of power expansion

$D_s^* \to \phi$ helicity form factors

- heavy-to-light form factors (FFs) play the key role in weak decays
- both pert. and nonpert. physics enter into the game
- the measurement would reveal the inner structures of hadrons

- QCD-based approaches to calculate FFs, LCSRs, PQCD, LQCD, et al.
- implement of LCSRs in charm sector, $D \to \pi, K, \eta^{(\prime)}, \phi$ et.al
  [Khodjamirian 2000, Ball 2006, Offen 2013, Du 2003, Wu 2006]

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- $D_s^* \to \phi$ FFs in this work
  - † first LCSRs prediction of $V \to V'$ type FFs
  - † helicity decomposition with seven FFs, saying $\mathbf{00}, \mathbf{0}\pm, \pm\mathbf{0}, \pm\mp$
  - † LCSRs prediction is reliable in large recoiled region $[0, 0.4]\,\mathrm{GeV}^2$
  - † parameterisations to the full kinematical region $[0, 1.2]\,\mathrm{GeV}^2$

- experiment potential of D_s^* weak decays

$D_s^* \to \phi$ helicity form factors

- start with the correlation function

$$F_{\mu a}(p_1, q) = i \int d^4x\, e^{iq\cdot x} \langle \phi(p_2, \epsilon_2^*) \big| T\{J_\mu^W(x), J_a^V(0)\} \big| 0 \rangle \,,$$

- heavy-light weak current $J_\mu^W = \bar{s}\gamma_\mu(1-\gamma_5)c$ and vector current $J_a^V = \bar{c}\gamma_a s$
- modify the correlation function by multiplying $\bar{\epsilon}^\mu$ to obtain the helicity correlator

$$\bar{\epsilon}^\mu F_{\mu a}(p_1, q) = \sum_{i,j=0,\pm} \epsilon^*_{1a,i'} F_{ij}(q^2, p_1^2) \,, \quad i' = i + j$$

$i, j, i' = i + j$ denote the polarizations of the J_μ^W, ϕ meson and the J_a^V, respectively.

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$i, j, i' = i + j$ denote the polarizations of the J_μ^W, ϕ meson and the J_a^V, respectively.

- twofold ways to consider the correlation function
  - † at quark-gluon level by OPE, $\sim \sum_i H_i(u,\mu) \otimes \phi_i(u)$
  - † at hadron level, sum over intermediate states
  - † QCD asymptotic behaviour, quark-hadron duality to equal, s_0
  - † to improve the accuracy of duality, Borel transformation, M^2

- OPE is valid for the large energies of the final state meson $E_\phi \gg \Lambda_{QCD}$,

  $0 \leqslant |q^2| \leqslant m_{D_s^*}^2 - 2m_{D_s^*} E_\phi \equiv q^2_{\rm LCSR,max}, \Leftarrow q \cdot x \sim 0, x^2 \sim 0$
- $|q^2| \in [0, q^2_{\rm LCSR,max}] \sim m_c^2 - 2m_c\chi$ with a typical hadron scale $\chi \sim 500\,\mathrm{MeV}$, the lower part of $0 < |q^2| < (m_{D_s^*}^2 - m_\phi)^2 \equiv q_0^2 \approx 1.2\,\mathrm{GeV}^2$

† $|q^2| \to \mathcal{O}(m_c^2)$, the virtuality of c-quark decreases to a soft scale, OPE fails

† $|q^2|, |(p_2+q)^2| \ll m_c^2$, the intermediate c-quark filed has large virtuality,

LO,

$$S(x,0) = -i\langle 0|T\{c(x), \bar{c}(0)\}|0\rangle = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} \frac{\not{p} + m_c}{p^2 - m_c^2} \tag{3}$$

NLO, $\mathcal{O}(\alpha_s)$ correction with gluon interactions $\cdots$

- only ϕ meson is on shell, $p_2^2 = m_\phi^2$, dispersion intergral in $(p_2+q)^2$

$$\bar{\epsilon}^\mu F_{\mu a}(q, p_1) = \epsilon^*_{1a,i'} \sum_{i,j} F_{ij}^{\rm OPE}(q^2, (p_2+q)^2) \tag{4}$$

$$F_{ij}^{\rm OPE}(q^2,(p_2+q)^2) = \sum \int_0^1 du T^{(n)}(u, q^2, p_1^2)\phi^{(n)}(u)$$

$$= \frac{1}{\pi}\int_0^1 du \sum_n \frac{{\rm Im}F_{n,ij}^{\rm OPE}(q^2,u)}{[-u(p_2+q)^2 - \bar{u}q^2 + u\bar{u}m_\phi^2 + m_c^2]^n}\,. \tag{4}$$

- the hadron dispersion relation in $p_1^2 > 0$

$$F_{ij}(q^2, p_1^2) = \frac{1}{\pi} \int_{m_c^2}^{\infty} ds \, \frac{\mathrm{Im}\, F_{ij}(q^2, s)}{s - p_1^2} = \frac{\rho_{ij}^0}{m_{D_s^*}^2 - p_1^2} + \int_{s_0}^{\infty} ds \, \frac{\rho_{ij}'^h(q^2, s)}{s - p_1^2} \tag{5}$$

$$\epsilon_{1a,i'}^* \rho_{ij}^0(q^2) = \bar{\epsilon}_i^\mu \langle \phi(p_2, \epsilon_2^*) | J_\mu^W(x) | D_s^*(\epsilon_1, p_1) \rangle \langle D_s^*(\epsilon_1^*, p_1) | J_a^V(0) | 0 \rangle \tag{6}$$

- matrix elements and the helicity form factors

$$\bar{\epsilon}_i^\mu \langle \phi(p_2, \epsilon_2^*) | \bar{s} J_{\mu,j}^W c | D_s^*(\epsilon_1, p_1) \rangle \equiv H_{ij}(q^2) \,, \quad \langle D_s^{*+}(p_1, \epsilon_1^*) | \bar{s} \gamma_a c | 0 \rangle = \epsilon_{1a}^* m_{D_s^*} f_{D_s^*} \,,$$

- isolate the ground state contribution

$$F_{ij}(q^2, p_1^2) = \frac{m_{D_s^*} f_{D_s^*} H_{ij}(q^2)}{m_{D_s^*}^2 - p_1^2} + \int_{s_0}^{\infty} ds \, \frac{\rho_{ij}'^h(q^2, s)}{s - p_1^2}. \tag{7}$$

- the same correlator in OPE calculation Eq.(4) and hadron interpolation Eq.(7)
- QCD property, like $F_\pi(q^2)$ and $G_\pi(s)$ have the similar asymptotic behaviour
- semi-local duality $s \equiv s(q^2, u) = \bar{u} m_\phi^2 + (m_c^2 - \bar{u} q^2)/u$

$$\frac{1}{\pi} \int_{s_0}^{\infty} ds \, \frac{u^2(s)}{[u^2(s) m_\phi^2 - q^2 + m_c^2]} \sum_n \frac{\mathrm{Im} F_{n,ij}^{\mathrm{OPE}}(q^2, s)}{u^n(s)[s - (p_2 + q)^2]^n} \Big|_{q^2, (p_2+q)^2 < 0} = \int_{s_0}^{\infty} ds \, \frac{\rho_{ij}^{\prime h}(q^2, s)}{s - p_1^2}$$

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† Borel trans. to suppress the pollution introduced by duality

$$\hat{B} \left[\int_{u_0}^{1} du \frac{F(u)}{\triangle} \right] = \int_{u_0}^{1} du \frac{F(u)}{u} e^{-s(u)/M^2}, \cdots$$

† $\mu_f^2 = m_{D_s^*}^2 - m_c^2 = 1.66^2 \, \mathrm{GeV}^2$, $\mu_f^2 < M^2 \sim \mathcal{O}(u m_{D_s^*}^2 + \bar{u} Q^2 - u \bar{u} m_\phi^2) < s_0$, $s_0 \approx (m_{D_s^*} + \chi)^2$

† compromise between the overwhelming ground state and the convergent OPE,

$$\frac{d}{d(1/M^2)} \ln H_{ij}(q^2) = 0 \, .$$

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- $s_0 = 6.8 \pm 1.0 \,\mathrm{GeV}^2$, $M^2 = 4.50 \pm 1.0 \,\mathrm{GeV}^2$ and $q^2_{\mathrm{LCSR,max}} = 0.4 \,\mathrm{GeV}^2$
- the sum rule with leading power approximation

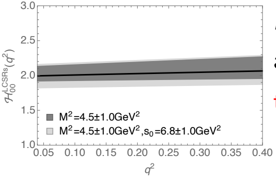
$$\frac{1}{\pi} \int_{u_0}^{1} du \, \frac{\mathrm{Im} F_{1,ij}^{\mathrm{OPE}}(q^2 < 0, u)}{u} \, e^{-s(u)/M^2} = m_{D_s^*} f_{D_s^*} \, H_{ij}(q^2 > 0) \, e^{-m_{D_s^*}^2/M^2} \tag{11}$$

- $\bar{m}_c(m_c) = 1.30\,\mathrm{GeV}$, $m_{D_s^*} = 2.112\,\mathrm{GeV}$ and $f_{D_s^*} = 0.274\,\mathrm{GeV}$

modified FFs $\mathcal{H}_{ij} \equiv q^2 H_{ij}$

LCSRs paras

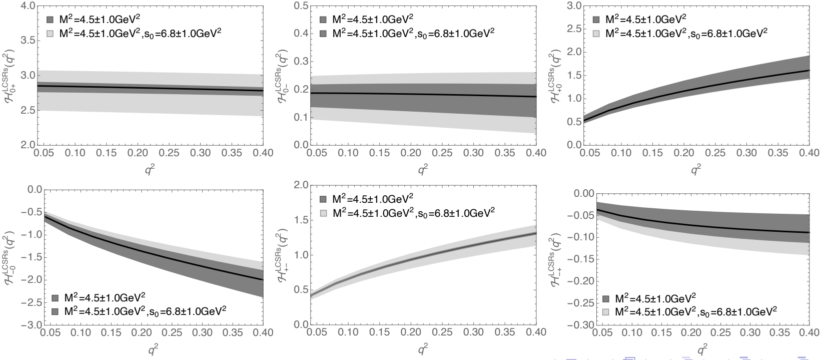
10%-20% uncertainty



$\bar{m}_c(m_c) = 1.30 \pm 0.10\,\mathrm{GeV}$

another 10%-20% uncertainty

the missing radiative corrections



- The (modified) helicity form factors at the full recoiled point $q^2 = 0$ are

$$
\begin{aligned}
&\mathcal{H}_{00}(0) = 1.99^{+0.15+0.32}_{-0.17-0.30}\,, \\
&\mathcal{H}_{0+}(0) = 2.86^{+0.21+0.48}_{-0.35-0.48}\,, \quad \mathcal{H}_{0-}(0) = 0.19^{+0.05+0.09}_{-0.09-0.07}\,, \\
&H_{+0}(0) = 2.67^{+0.47+0.31}_{-0.26-0.29}\,, \quad H_{-0}(0) = -2.92^{-0.53-0.35}_{+0.33+0.32}\,, \\
&H_{+-}(0) = 2.11^{+0.17+0.40}_{-0.27-0.28}\,, \quad H_{-+}(0) = -0.19^{-0.06-0.01}_{+0.11+0.03}\,,
\end{aligned}
$$

- the center values of several orthogonal Lorentz form factors as

$$
\begin{aligned}
&\mathcal{V}_1(0) - \mathcal{V}_2(0) = -1.86\,, \quad \mathcal{V}_5(0) = 2.46\,, \\
&\mathcal{A}_1(0) + \mathcal{A}_2(0) = -1.63\,, \quad \mathcal{V}_6(0) = -0.26\,.
\end{aligned}
$$

- compared with the result obtained from light-front quark model [Chang 2019]
- in fact they show a good consistence after considering the different definitions

$D_s^* \to \phi$ helicity form factors small recoiling

- form factors with small recoiling, $q^2 \in [q^2_{\mathrm{LCSR,max}}, q_0^2] \sim [0.4, 1.2]\,\mathrm{GeV}^2$
- consider two parameterisations $q_0^2 \equiv (m^2_{D_s^*} - m_\phi)^2$

† reproduce the LCSRs predictions in the lower interval $[0, q^2_{\mathrm{LCSR,max}}]$

† provide an extrapolation in $[q^2_{\mathrm{LCSR,max}}, q_0^2]$ with the expected analytical properties

- BCL model: z-series expansion [Bourrely 2008]

$$F^{(i)}(q^2 > 0) = \frac{a_{F^{(i)}}(q^2)}{1 - q^2/m^2_{D1}} \left\{1 + b_{F^{(i)}}\,[z(q^2) - z(0)]\right\}, \quad z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \tag{12}$$

$$\sqrt{q^2} H_{ij}(q^2) = \frac{1 + b_{ij}\,[z(q^2) - z(0)]}{\left(1 - q^2/m^2_{D1}\right)} \mathcal{K}_{ij}(q^2) \left[\frac{a^1_{ij}\lambda^{3/2}}{m_{D_s^*} m_\phi (m^2_{D_s^*} - m^2_\phi)} + \frac{a^2_{ij}\lambda}{(m^2_{D_s^*} - m^2_\phi)} + a^3_{ij}\lambda^{1/2}\right] \text{Kinematics}$$

$$+\kappa_{ij} q_0 \left[m_{D_s^*} \frac{1 + b_1^{\mathrm{ED}} z'(q^2)}{1 - q^2/m^2_{D1}} + m_\phi \frac{1 + b_2^{\mathrm{ED}} z'(q^2)}{1 - q^2/m^2_{D1}}\right] . \text{Endpoint relations} \tag{13}$$

- Wigner-Eckart theorem: the helicity information at endpoint is only governed by the Clebsch-Gordan coefficients [Hiller 2014, Grattrex 2016, Hiller 2021]

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† helicity independent dynamics is absorbed in to the matrix element M_{111}

$$H_{\lambda_q \bar{\lambda}_\phi}(q_0^2) = C_{\lambda_{cs} \lambda_q \bar{\lambda}_\phi} M_{111} \,. \quad (14)$$

$$H_{00}(q_0^2) \propto C_{000}^{111} = 0 \,, \quad H_{01}(q_0^2) : H_{0\bar{1}}(q_0^2) \propto C_{101}^{111} : C_{\bar{1}0\bar{1}}^{111} = -\frac{1}{2} : \frac{1}{2} \,,$$

$$H_{10}(q_0^2) : H_{\bar{1}0}(q_0^2) \propto C_{110}^{111} : C_{\bar{1}\bar{1}0}^{111} = \frac{1}{2} : -\frac{1}{2} \,, \quad H_{\bar{1}1}(q_0^2) : H_{1\bar{1}}(q_0^2) \propto C_{011}^{111} : C_{01\bar{1}}^{111} = -\frac{1}{2} : \frac{1}{2} \,, \quad (15)$$

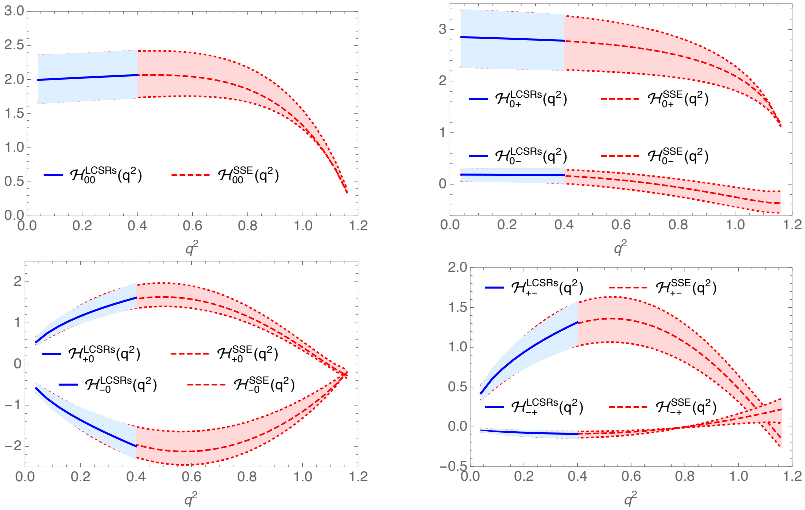
- kinematical factors

$$\mathcal{K}_{00}(q^2) = \frac{\lambda^{1/2}}{m_{D_s^*} m_\phi} \,, \quad \mathcal{K}_{ij \neq 00} = 1 \,, \quad \kappa_{00} = 0, \quad \kappa_{0\mp} = \kappa_{\pm 0} - \kappa_{\pm\mp} = \pm 1 \,,$$

$$z'(q^2) = z(q^2) - z(0) \,, \quad q_1^2 = q_0^2 - q^2 \,, \quad a_{0\pm}^1 = a_{\pm 0}^1 = a_{\pm\mp}^1 = 0 \,, \quad a_{0\pm}^2 = 0 \,. \quad (16)$$

- The end-point relations $\mathcal{H}_{\mathbf{00}}^{\rm SSE}(q_0^2) = 0$, $|\mathcal{H}_{ij\neq\mathbf{00}}^{\rm SSE}(q_0^2)| = 0.23^{+0.18}_{-0.23}$
- play an important role to set down the shapes of helicity form factors in the small recoiled regions where the LCSRs calculation is failed
- BK mode: two-pole parameterisation [Becirevic 1999], show almost the same result

$D_s^* \to \phi$ helicity form factors whole recoiled regions



Exclusive D_s^* weak decays

- dependence on the $q^2_{\rm LCSR,max}$ choice are checked by varying it in $[0.2, 0.4]\,{\rm GeV}^2$, $\mathcal{H}_{ij}(1.2\,{\rm GeV}^2)$ change no more than four percents.

- semileptonic decays $D_s^* \to \phi l \nu_l$

$$\frac{d\Gamma_{ij}(q^2)}{dq^2} = \frac{G_F^2|V_{cs}|^2}{192\pi^3 m_{D_s^*}^3}\lambda^{1/2}(m^2_{D_s^*}, m^2_\phi, q^2)\, q^2\, |H_{ij}(q^2)|^2 \,,$$

$$\Gamma_{D_s^* \to \phi l \nu_l} = \frac{1}{3}\int_0^{q_0^2} dq^2 \sum_{i,j=0,\pm} \frac{d\Gamma_{ij}(q^2)}{dq^2} = \left(3.28^{+0.82}_{-0.71}\right) \times 10^{-14}\,{\rm GeV} \,.$$

- hadronic decays (naive factorisation)

$$\mathcal{A}(D_s^{*+} \to \phi\pi^+) = (-i)\frac{G_F}{\sqrt{2}} V_{cs}\, a_1\, m_\pi f_\pi \sum_{i=0,\pm} H_{0j}(m_\pi^2) \,,$$

$$\mathcal{A}(D_s^{*+} \to \phi\rho^+) = \frac{G_F}{\sqrt{2}} V_{cs}\, a_1\, m_\rho f_\rho^{\parallel(\perp)} \sum_{i,j} H_{ij}(m_\rho^2) \,.$$

$a_1(\mu) = 0.999,\ f_\pi = 0.130\,{\rm GeV},\ f_\rho^\parallel = 0.210\,{\rm GeV}$

$$\Gamma_{D_s^{*+}\to\phi\pi^+} = \left(3.81^{+1.52}_{-1.33}\right) \times 10^{-14}\,{\rm GeV}\,, \qquad \Gamma_{D_s^{*+}\to\phi\rho^+} = \left(1.16^{+0.42}_{-0.39}\right) \times 10^{-13}\,{\rm GeV}\,.$$

- the result of $\phi\pi$ channel is marginally consistent with the PQCD [Yang 2022]

Experimental potential

- with the lattice evaluation of $\Gamma_{D_s^*} = (0.70 \pm 0.28) \times 10^{-8}\,\mathrm{GeV}$ [HPQCD 2013]

$$\mathcal{B}(D_s^* \to l\nu) = (3.49 \pm 1.40) \times 10^{-5}\,, \quad \mathcal{B}(D_s^* \to \phi l\nu) = (0.47\,^{+0.12}_{-0.10} \pm 0.19) \times 10^{-6}\,,$$
$$\mathcal{B}(D_s^{*+} \to \phi\pi^+) = (0.54\,^{+0.22}_{-0.19} \pm 0.22) \times 10^{-6}\,, \quad \mathcal{B}(D_s^{*+} \to \phi\rho^+) = (1.65\,^{+0.61}_{-0.56} \pm 0.66) \times 10^{-6}\,.$$

- Belle II   clear background

† 2022, $400\,\mathrm{fb}^{-1}$, reconstruct 2×10^5 data samples of $D_s^*(D_s)$ from $\phi\pi$ channel

† phase 3 running (2024-2026), $10\,\mathrm{ab}^{-1}$, $\mathcal{O}(1 \times 10^7)$ data sample of $D_s^*(D_s)$

† the number of D_s^* production is $\mathcal{O}(10^9)$ with $\mathcal{B}(D_s \to \phi\pi) = (4.5 \pm 0.4)\,\%$

† excellent potential to study the D_s^* weak decays, $50\,\mathrm{ab}^{-1}$ is hottest expected

- LHCb   excellent particle identification to distinguish K, π and μ

† the channel $D_s^* \to \phi(KK)\pi$ with the D_s^* producing by $B_s \to D_s^* \mu\nu$

- BESIII   low background

† directly produced from e^+e^- collision at the $D_s D_s^*$ threshold

† have collected 3.07×10^6 D_s^* mesons with the $3.2\,\mathrm{fb}^{-1}$ data at $4.178\,\mathrm{GeV}$

† provides the good chance for the leptonic decay $D_s^* \to l\nu$, Statistical error

Experimental potential

CEPC CDR

Particle	Tera-Z	Belle II	LHCb
b hadrons			
B^+	6×10^{10}	3×10^{10} (50 ab^{-1} on $\Upsilon(4S)$)	3×10^{13}
B^0	6×10^{10}	3×10^{10} (50 ab^{-1} on $\Upsilon(4S)$)	3×10^{13}
B_s	2×10^{10}	3×10^{8} (5 ab^{-1} on $\Upsilon(5S)$)	8×10^{12}
b baryons	1×10^{10}		1×10^{13}
Λ_b	1×10^{10}		1×10^{13}
c hadrons			
D^0	2×10^{11}		
D^+	6×10^{10}		
D_s^+	3×10^{10}		
Λ_c^+	2×10^{10}		
τ^+	3×10^{10}	5×10^{10} (50 ab^{-1} on $\Upsilon(4S)$)	

BESIII $\mathcal{O}(10^6) D_s^+/D_s^{*+}$ production
Belle II $\mathcal{O}(10^9) D_s^+/D_s^{*+}$ production

Table 2.4: Collection of expected number of particles produced at a tera-Z factory from 10^{12} Z-boson decays. We have used the hadronization fractions (neglecting p_T dependencies) from Refs. [431, 432] (see also Ref. [433]). For the decays relevant to this study we also show the corresponding number of particles produced by the full 50 ab^{-1} on $\Upsilon(4S)$ and 5 ab^{-1} on $\Upsilon(5S)$ runs at Belle II [430], as well as the numbers of b hadrons at LHCb with 50 fb^{-1} (using the number of $b\bar{b}$ pairs within the LHCb detector acceptance from [435] and the hadronization fractions from [431]).

- CEPC low background

† without doubt, could do much more/precise study on D_s^* weak decays.

Conclusion

- we study the D_s^* weak decay
- we discuss the experiment potentials and CEPC is highly anticipated

† first direct measurement of weak decays of vector meson

† shine light on the study of $\Gamma_{D_s^*}$ and $g_{D_s^* D_s \gamma} \cdots$

† new playground to examine SM, like the $|V_{cs}|$ and the CKM unitarity

† check the heavy quark spin symmetry

The End, Thanks.

- $D_s^* \to \phi$ form factors

$$
\begin{aligned}
&\langle \phi(p_2, \epsilon_2^*)| \bar{s}\gamma_\mu(1-\gamma_5)c | D_s^*(\epsilon_1, p_1)\rangle \\
= \quad & (\epsilon_1 \cdot \epsilon_2^*)\Big[p_{1\mu}\mathcal{V}_1(q^2) - p_{2\mu}\mathcal{V}_2(q^2)\Big] + \frac{(\epsilon_1 \cdot q)(\epsilon_2^* \cdot q)}{m_{D_s^*}^2 - m_\phi^2}\Big[p_{1\mu}\mathcal{V}_3(q^2) + p_{2\mu}\mathcal{V}_4(q^2)\Big] \\
- \quad & (\epsilon_1 \cdot q)\epsilon_{2\mu}^*\mathcal{V}_5(q^2) + (\epsilon_2 \cdot q)\epsilon_{1\mu}^*\mathcal{V}_6(q^2) - i\varepsilon_{\mu\nu\rho\sigma}\epsilon_1^\rho\epsilon_2^{*\sigma}\Big[p_1^\nu\mathcal{A}_1(q^2) + p_2^\nu\mathcal{A}_2(q^2)\Big] \\
+ \quad & i\varepsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma \frac{1}{m_{D_s^*}^2 - m_\phi^2}\Big[\epsilon_1^\nu(\epsilon_2^* \cdot q)\mathcal{A}_3(q^2) - \epsilon_2^\nu(\epsilon_1^* \cdot q)\mathcal{A}_4(q^2)\Big]
\end{aligned}
$$

- relations between helicity FFs and orthogonal Lorentz FFs

$$
\begin{aligned}
H_{\mathbf{00}}(q^2>0) &= \frac{(m_{D_s^*}^2 + m_\phi^2 - q^2)\lambda^{1/2}\Big[-\mathcal{V}_1(q^2) + \mathcal{V}_2(q^2)\Big]}{4\sqrt{q^2}m_\phi m_{D_s^*}} + \frac{\lambda^{3/2}\Big[\mathcal{V}_3(q^2) + \mathcal{V}_4(q^2)\Big]}{8\sqrt{q^2}m_\phi m_{D_s^*}(m_{D_s^*}^2 - m_\phi^2)} \\
&- \frac{\lambda^{1/2}\Big[(m_{D_s^*}^2 - m_\phi^2 - q^2)\mathcal{V}_5(q^2) - (m_{D_s^*}^2 - m_\phi^2 + q^2)\mathcal{V}_6(q^2)\Big]}{4\sqrt{q^2}m_\phi m_{D_s^*}}, \\
H_{\mathbf{0}\pm}(q^2>0) &= -\frac{\lambda^{1/2}\Big[\mathcal{V}_1(q^2) - \mathcal{V}_2(q^2)\Big]}{2\sqrt{q^2}} \mp \frac{\Big[(m_{D_s^*}^2 - m_\phi^2 + q^2)\mathcal{A}_1(q^2) + (m_{D_s^*}^2 - m_\phi^2 - q^2)\mathcal{A}_2(q^2)\Big]}{2\sqrt{q^2}}, \\
H_{\pm\mathbf{0}}(q^2>0) &= \frac{\lambda^{1/2}\mathcal{V}_6(q^2)}{2m_\phi} \pm \frac{\lambda\mathcal{A}_3(q^2)}{4m_\phi(m_{D_s^*}^2 - m_\phi^2)} \pm \left[\frac{\left(m_{D_s^*}^2 + m_\phi^2 - q^2\right)\mathcal{A}_1(q^2)}{2m_\phi} + m_\phi\mathcal{A}_2(q^2)\right], \\
H_{\mp\pm}(q^2>0) &= \frac{\lambda^{1/2}\mathcal{V}_5(q^2)}{2m_{D_s^*}} \mp \frac{\lambda\mathcal{A}_4(q^2)}{4m_{D_s^*}(m_{D_s^*}^2 - m_\phi^2)} \mp \left[m_{D_s^*}\mathcal{A}_1(q^2) + \frac{\left(m_{D_s^*}^2 + m_\phi^2 - q^2\right)\mathcal{A}_2(q^2)}{2m_{D_s^*}}\right].
\end{aligned}
$$