

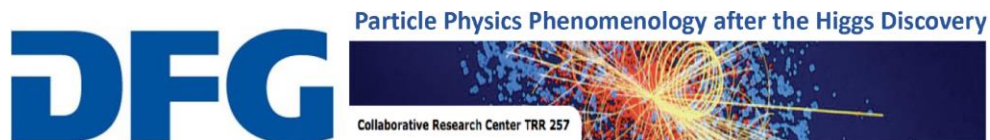
# Standard Model Predictions and LFU violation in $B$ decays

---

Nico Gubernari

International Workshop on the High Energy  
Circular Electron Positron Collider

Nanjing University and IHEP (ONLINE) – 25-Oct-2022



# Introduction

# Flavour changing currents

flavour changing charged currents (FCCC) occur at tree level (mediated by  $W^\pm$ ) in the SM

flavour changing neutral currents (FCNC) absent at tree level in the SM

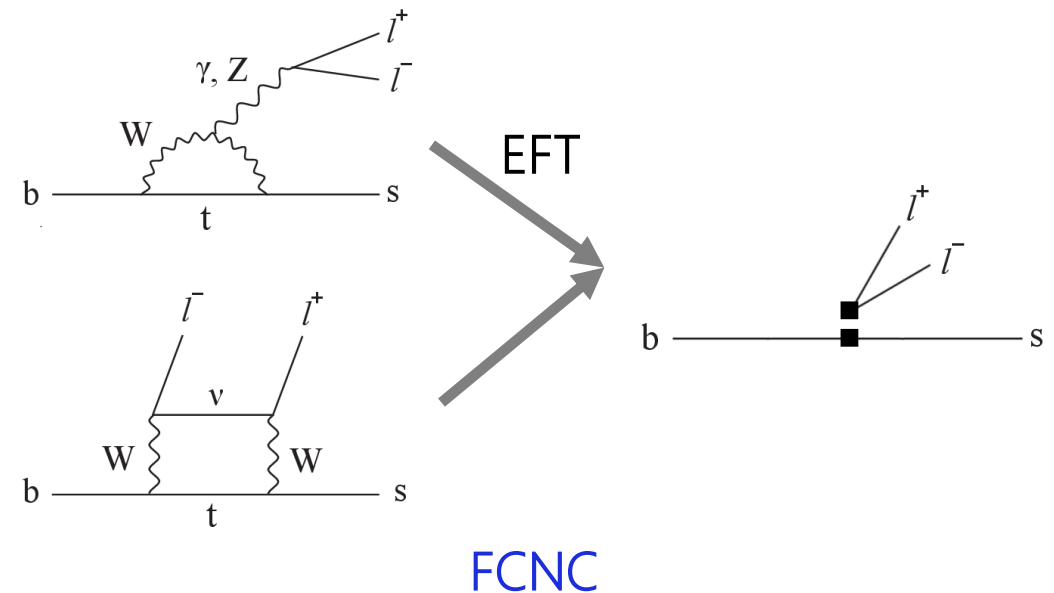
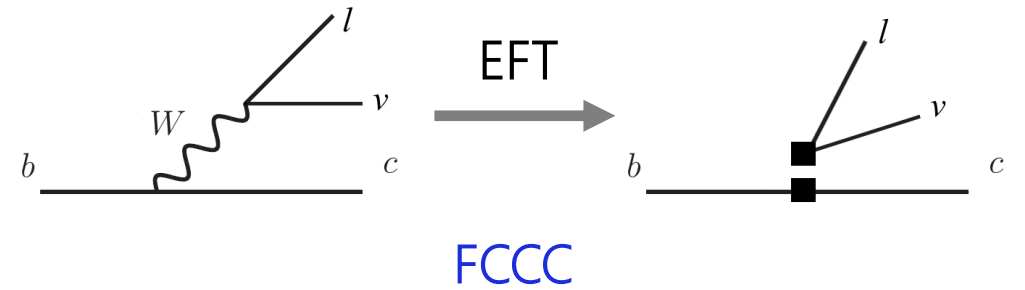
FCNC are loop, GIM and CKM suppressed in the SM

FCNC sensitive to new physics contributions probe the SM through indirect searches

integrate out DOF heavier than the  $b$



weak effective field theory



# Hadronic matrix elements

study  $B$ -meson decays to test the SM (neglect QED corrections)

$$\text{FCCC} \quad \langle D^{(*)} \ell \nu_\ell | \mathcal{O}_{eff} | B \rangle = \langle \ell \nu_\ell | \mathcal{O}_{lep} | 0 \rangle \langle D^{(*)} | \mathcal{O}_{had} | B \rangle$$

$$\text{FCNC} \quad \langle K^{(*)} \ell \ell | \mathcal{O}_{eff} | B \rangle = \langle \ell \ell | \mathcal{O}_{lep} | 0 \rangle \langle K^{(*)} | \mathcal{O}_{had} | B \rangle + \text{non-fact.}$$

leptonic matrix elements: perturbative objects, high accuracy

hadronic matrix elements: non-perturbative QCD effects, usually large uncertainties

decay amplitudes depend on:

- local hadronic matrix elements  
(form factors)  
 $\langle K^{(*)} | \mathcal{O}(0) | B \rangle$   
 $\langle D^{(*)} | \mathcal{O}(0) | B \rangle$
- nonlocal hadronic matrix elements  
(soft gluon contributions  
to the charm-loop)  
 $\langle K^{(*)} | \mathcal{O}(0, x) | B \rangle$

# Interesting observables

define observables smartly to reduce the hadronic uncertainties

test the lepton flavour universality to test the SM

**lepton flavour universality** = the 3 lepton generations have the same couplings to the gauge bosons

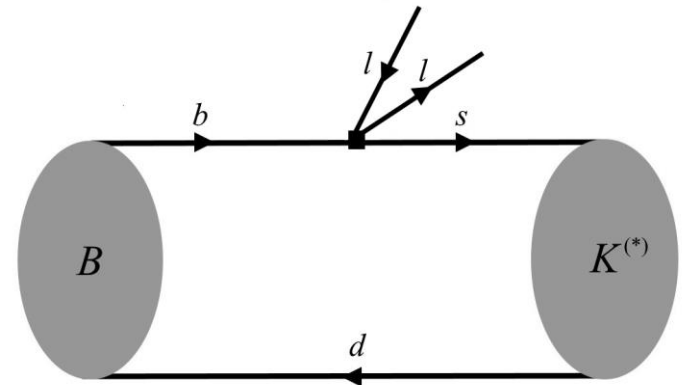
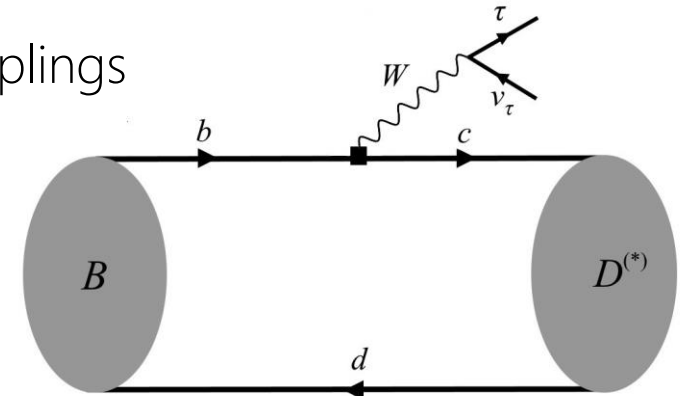
violations of lepton flavour universality  $\Rightarrow$  new physics

observables to test LFU

$$R_{D^{(*)}} = \frac{\Gamma(B \rightarrow D^{(*)} \nu \tau)}{\Gamma(B \rightarrow D^{(*)} \nu \mu)}$$

$$R_{K^{(*)}} = \frac{\Gamma(B \rightarrow K^{(*)} \mu^+ \mu^-)}{\Gamma(B \rightarrow K^{(*)} e^+ e^-)}$$

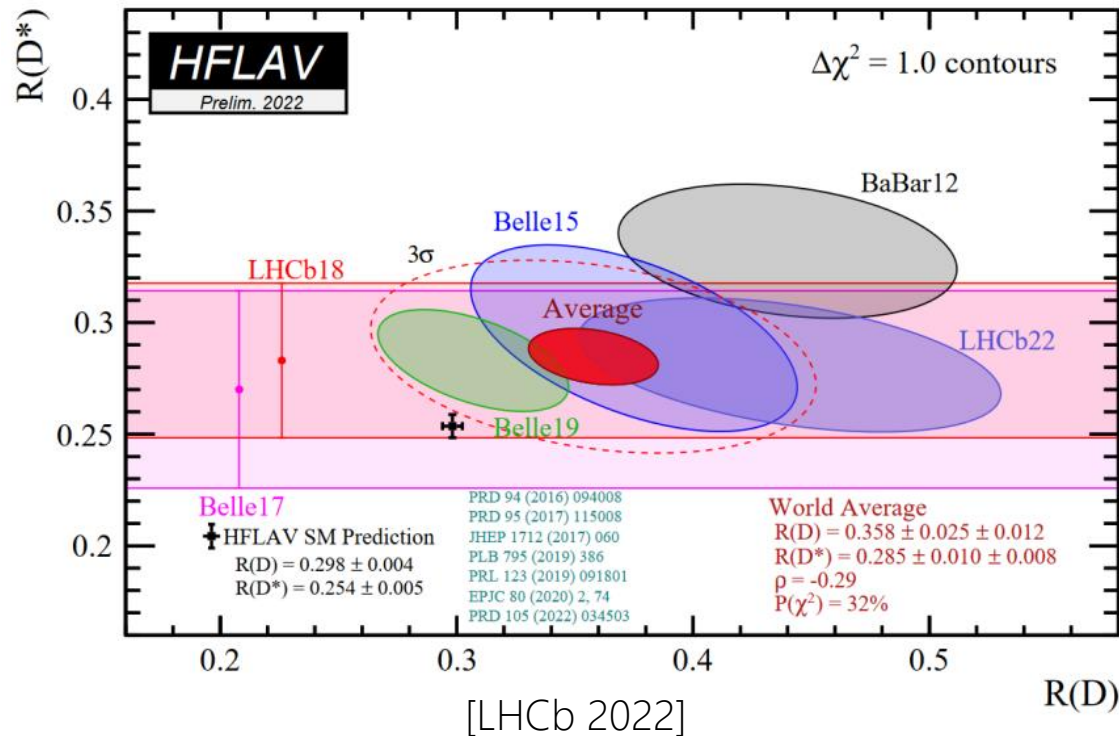
another test of the SM: angular observables in  $B \rightarrow K^* \ell \ell$  (e.g.  $P'_5$ )



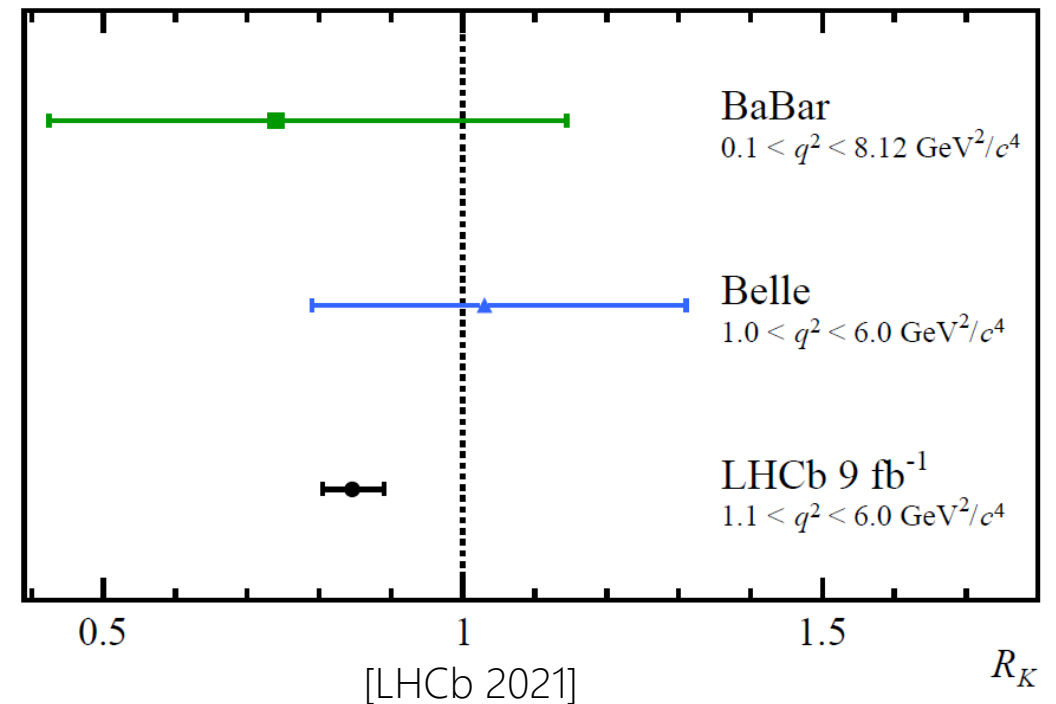
# B-anomalies

**B-anomalies** = tension between experimental measurements and theoretical predictions in B-meson decays involving different observables ( $R_{D^{(*)}}, R_{K^{(*)}}, P_5'$  ...) and experiments

3.2 $\sigma$  tension



3.1 $\sigma$  tension



Standard Model predictions

# Methods to compute hadronic matrix elements

non-perturbative techniques are needed  
to compute hadronic matrix elements



## Lattice QCD (LQCD)

numerical evaluation of correlators in a  
finite and discrete space-time

local matrix elements (usually at high  $q^2$ )

nonlocal matrix elements still  
work in progress

[see Kronfeld talk]

## Light-cone sum rules (LCSRs)

based on unitarity, analyticity, and  
quark-hadron duality approximation

need universal non-perturbative inputs

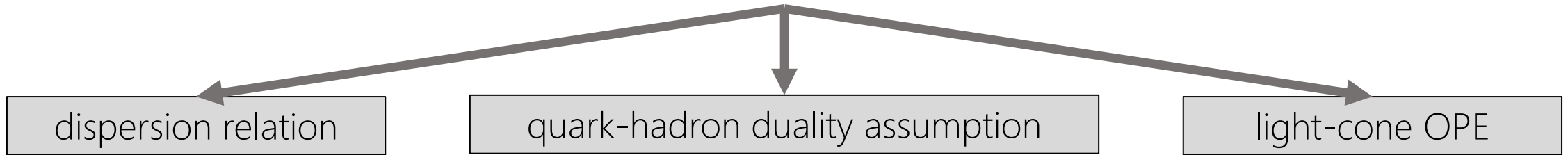
applicable for both local and nonlocal  
matrix elements (at low  $q^2$ )



# Light-cone sum rules in a nutshell

light-cone sum rules (LCSRs) are a method to calculate hadronic matrix elements

method based on:



## Pros

- easy to adapt to different transitions
- compute hadronic matrix elements not accessible yet with lattice QCD
- effective at small  $q^2$   
(complementary to lattice QCD)

## Cons

- quark-hadron duality assumption
- non-perturbative inputs  
(distribution amplitudes)
- large uncertainties

# Definition of the form factors

form factors (FFs) parametrize exclusive hadronic matrix elements

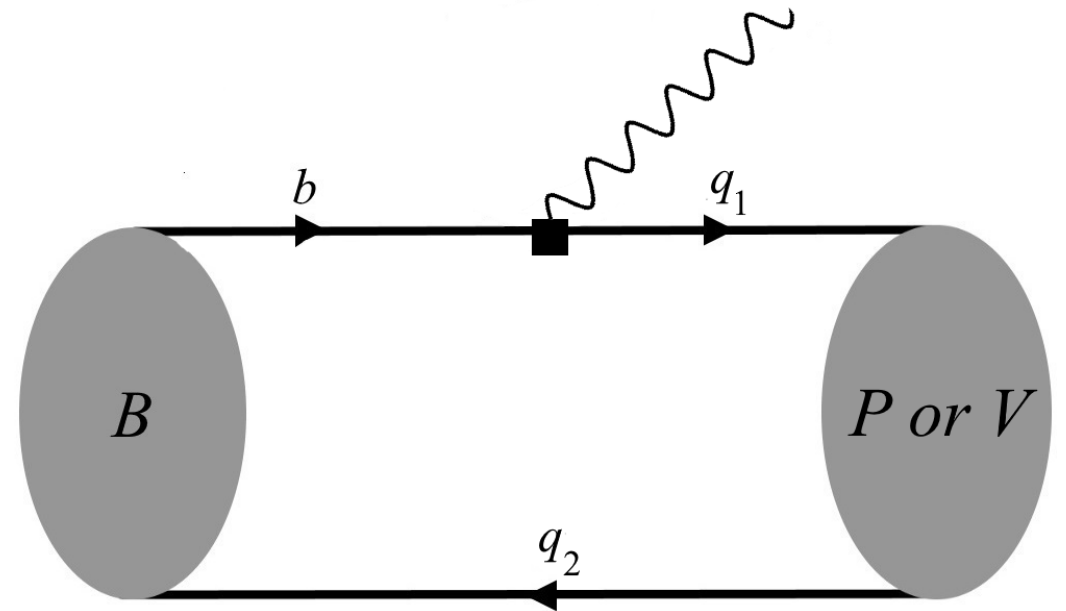
$$\langle P(k) | \bar{q}_1 \gamma_\mu b | B(q+k) \rangle = 2 k_\mu f_+(q^2) + q_\mu (f_+(q^2) + f_-(q^2))$$

$$\langle P(k) | \bar{q}_1 \sigma_{\mu\nu} q^\nu b | B(q+k) \rangle = \frac{i f_T(q^2)}{m_B + m_P} (q^2 (2k + q)_\mu - (m_B^2 - m_P^2) q_\mu)$$

decomposition follows from Lorentz invariance

FFs are functions of the momentum transferred  $q^2$   
( $q^2$  is the dilepton mass squared)

3 independent  $B$  to pseudoscalar meson ( $P$ ) FFs  
7 independent  $B$  to vector meson ( $V$ ) FFs



# State of the art

	Transition	LQCD	LCSR
$b \rightarrow s$	$B \rightarrow K$	whole $q^2$ range	$q^2 < 12 \text{ GeV}^2$
	$B \rightarrow K^*$	high $q^2$	$q^2 < 6 \text{ GeV}^2$
	$B_s \rightarrow \phi$	high $q^2$	$q^2 < 6 \text{ GeV}^2$
$b \rightarrow c$	$B \rightarrow D$	high $q^2$	$q^2 < 0 \text{ GeV}^2$
	$B \rightarrow D^*$	high $q^2$	$q^2 < 0 \text{ GeV}^2$
	$B_s \rightarrow D_s$	whole $q^2$ range	$q^2 < 0 \text{ GeV}^2$
	$B_s \rightarrow D_s^*$	whole $q^2$ range	$q^2 < 0 \text{ GeV}^2$

# Parametrization for $\mathcal{F}_\lambda$

obtain FFs in the whole semileptonic region by

1. **combining** LQCD calculations at high  $q^2$  and LCSR calculations at low  $q^2$  (w/ or w/o using HQET)
2. **extrapolate** LQCD calculations to the low  $q^2$  region

FFs are analytic functions of  $q^2$  (branch cut for  $q^2 > t_+ = (M_B + M_P)^2$ )

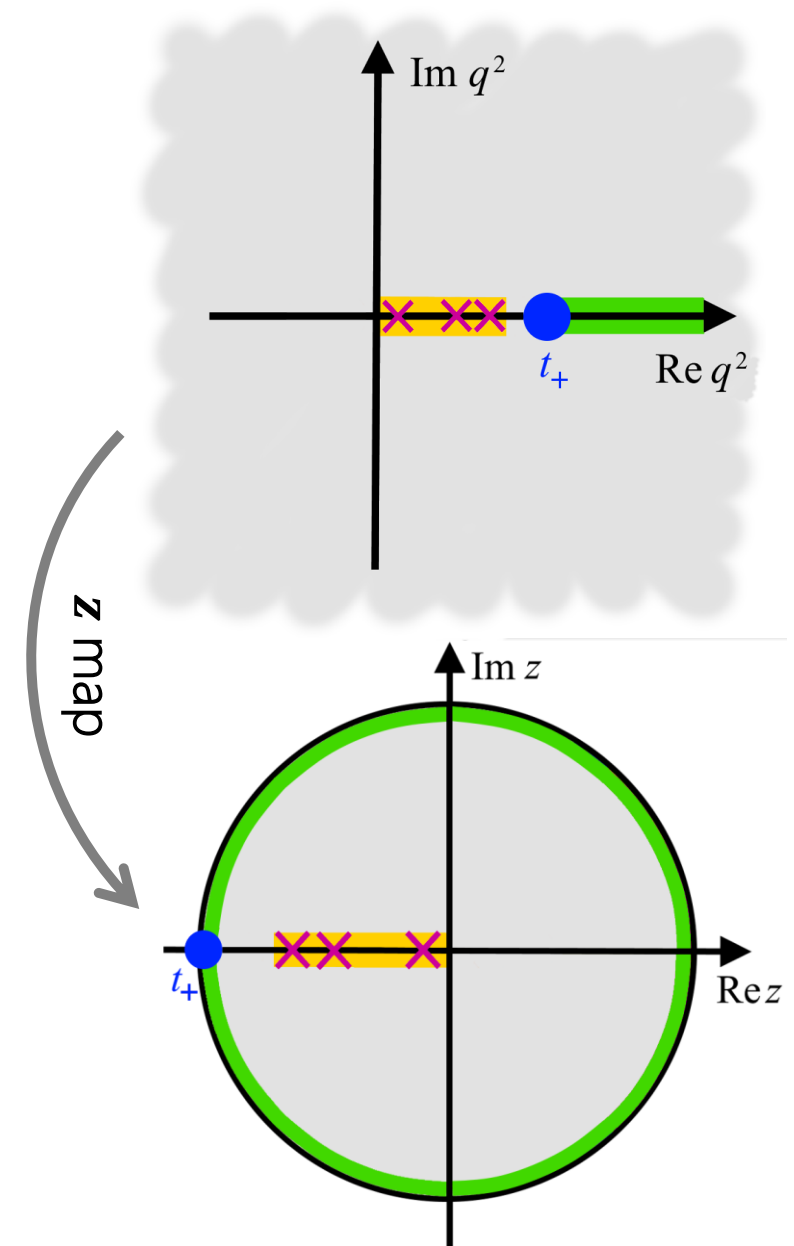
fit results to a  **$z$  parametrization** (standard approach) e.g. BGL:

[Boyd/Grinstein/Lebed 1997]

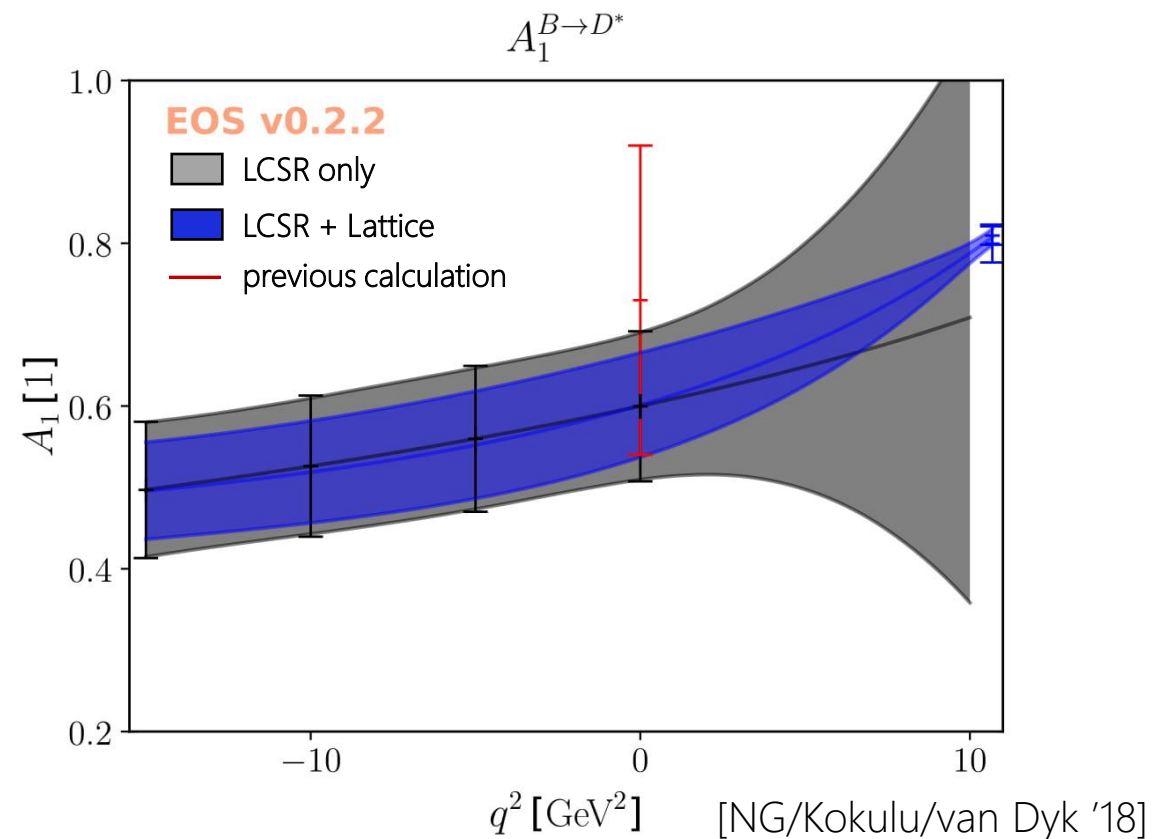
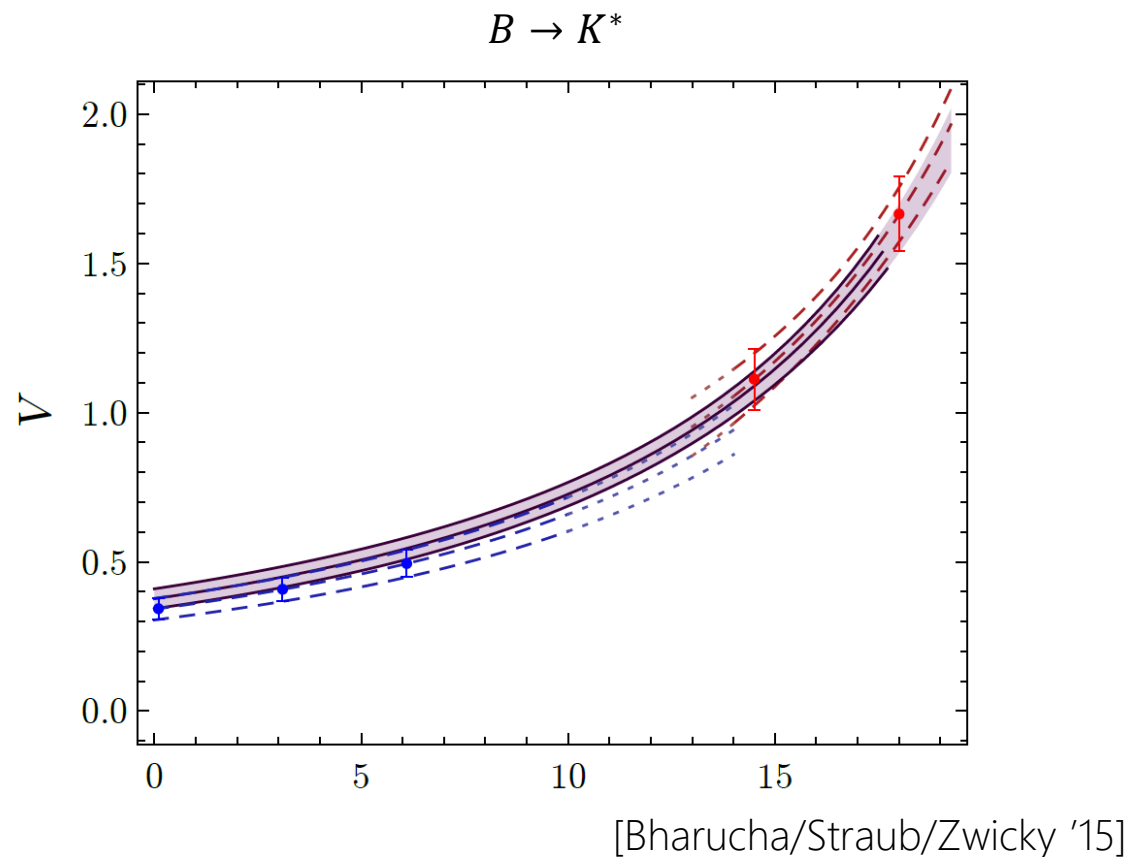
$$\text{FFs} = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{\infty} \alpha_k z^k$$

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+}}{\sqrt{t_+ - q^2} + \sqrt{t_+}}$$

$\mathcal{P}(z)$  Blaschke factor,  $\phi(z)$  outer function (known functions)



# Combine lattice QCD and LCSRs for local FFs



obtain the FF values to the whole spectrum (no additional assumptions required)

good agreement between lattice and LCSR's calculations

# Unitarity bounds

unitarity (or dispersive) bounds are model independent constraints on the FFs

$$\sum_{k=0}^{\infty} |a_k|^2 < 1$$

control the truncation error

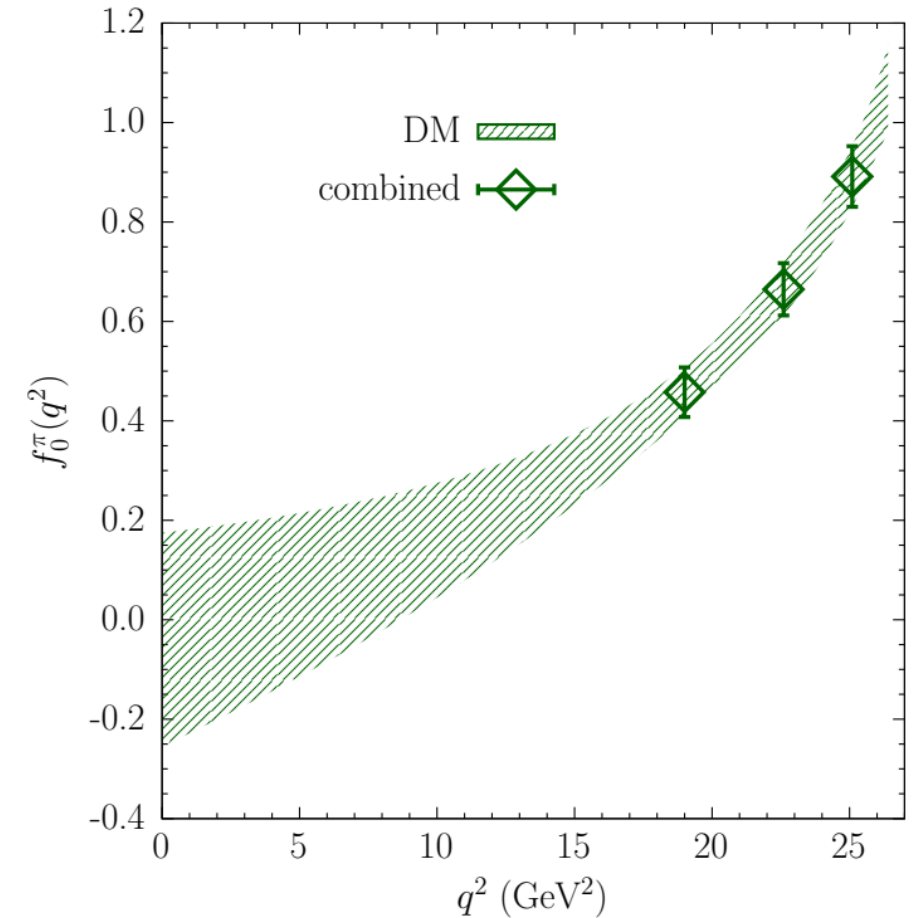
alternative way to implement these constraints  
→ **dispersive matrix (DM) method**

[Bourenly et al. 1981] [Lellouch 1995]

define a matrix with a positive determinant to obtain a bounds on the FFs

“parametrization-free” approach

BGL and DM are equivalent and both model independent



[Martinelli/Simula/Vittorio 2022]

# More on the $B_{(s)} \rightarrow D_{(s)}^{(*)}$ FFs

use heavy-quark limit ( $m_{b,c} \rightarrow \infty$ ) to relate  $B_{(s)} \rightarrow D_{(s)}$  FFs to  $B_{(s)} \rightarrow D_{(s)}^*$  FFs

expand  $B_{(s)} \rightarrow D_{(s)}^{(*)}$  FFs in the heavy-quark limit

$$FF^{B \rightarrow D^{(*)}}(q^2) = c_0 \xi(q^2) + c_1 \frac{\alpha_s}{\pi} C_i(q^2) + c_2 \frac{1}{m_b} L_i(q^2) + c_3 \frac{1}{m_c} L_i(q^2) + c_4 \frac{1}{m_c^2} l_i(q^2)$$

$$FF^{B_s \rightarrow D_s^{(*)}}(q^2) = c_0 \xi^s(q^2) + c_1 \frac{\alpha_s}{\pi} C_i(q^2) + c_2 \frac{1}{m_b} L_i^s(q^2) + c_3 \frac{1}{m_c} L_i^s(q^2) + c_4 \frac{1}{m_c^2} l_i(q^2)$$

include  $\Lambda_{\text{QCD}}/m_c^2$  corrections [Bordone/Jung/van Dyk 2019]

all  $B \rightarrow D^{(*)}$  and  $B_s \rightarrow D_s^{(*)}$  FFs parametrized in terms of 14 Isgur-Wise (IW) functions

$\Lambda_{\text{QCD}}/m_c^2 \sim 5\%$  are essential given the current (theory and experimental) precision

alternative approach Bernlochner et al. 2022

→ model the  $\Lambda_{\text{QCD}}/m_c^2$  corrections to reduce the number of parameters

# More on the $B_{(s)} \rightarrow D_{(s)}^{(*)}$ FFs

constrain IW functions with

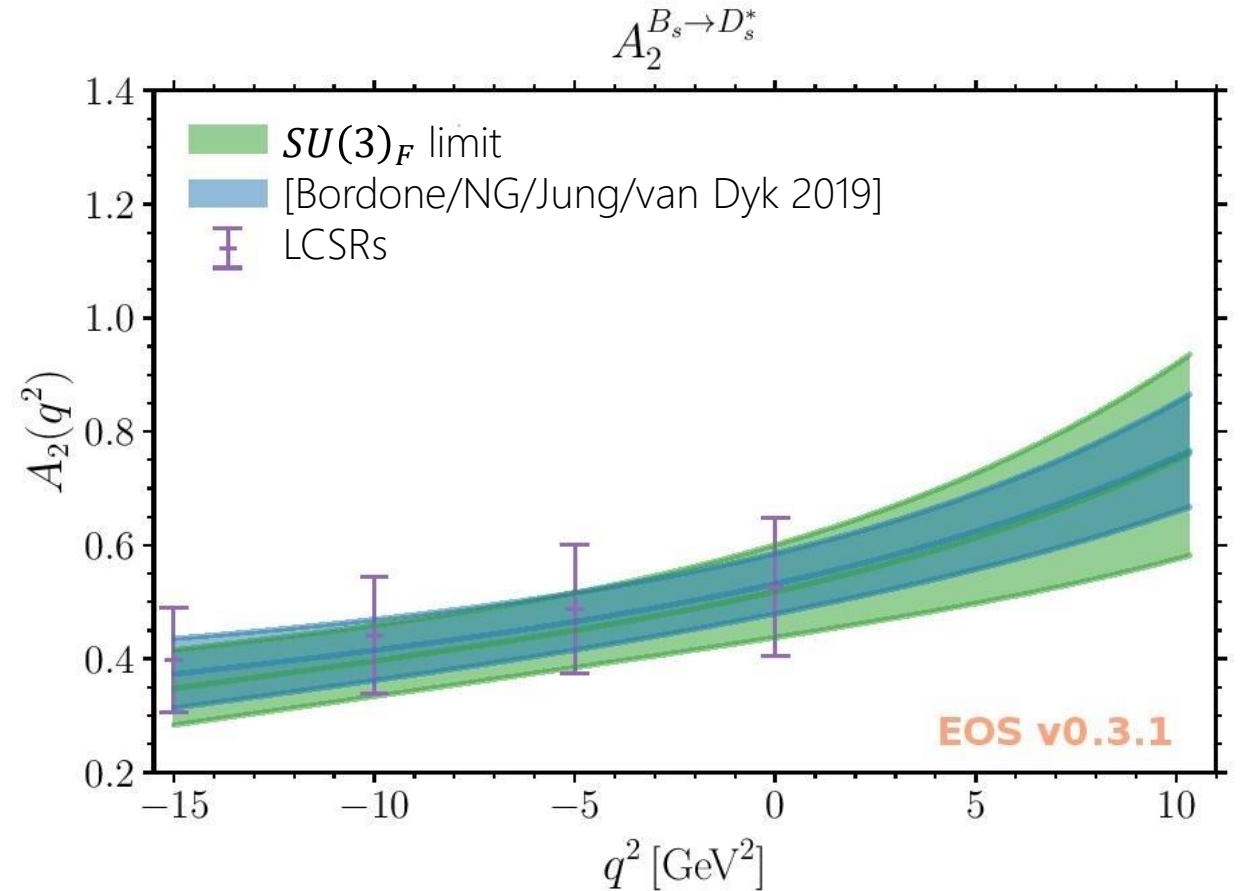
- lattice QCD (where available)
- light-cone sum rules for the FFs
- SVZ sum rules for Isgur-Wise functions
- w/ and w/o exp data
- unitarity bounds

results for all  $B \rightarrow D^{(*)}$  FFs and  $B_s \rightarrow D_s^{(*)}$  FFs  
in the whole physical phase space

using these FFs results calculate

BRs, angular observables, and LFU ratios  $R(D_{(s)}^{(*)})$

extract  $|V_{cb}| = (40.2 \pm 1.0)10^{-3}$



$$R(D) = 0.2981 \pm 0.0029, \quad R(D_s) = 0.2971 \pm 0.0034,$$

$$R(D^*) = 0.2504 \pm 0.0026, \quad R(D_s^*) = 0.2472 \pm 0.0077.$$



# More on the $b \rightarrow s$ transitions

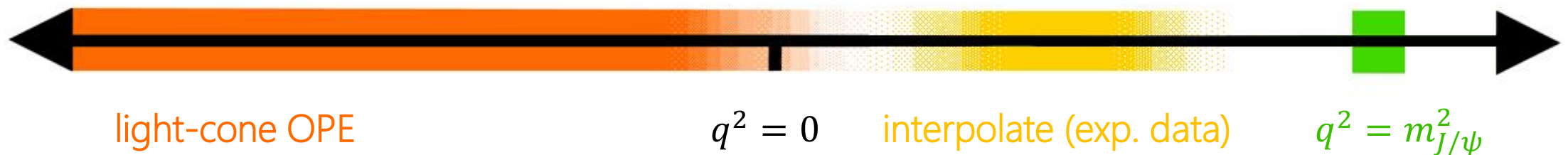
rare decays amplitude written in term of (local) FFs and non-local FFs

$$\mathcal{A}(B \rightarrow K^{(*)} \ell \ell) = \mathcal{N} \left[ (C_9 L_V^\mu + C_{10} L_A^\mu) \mathcal{F}_\mu - \frac{L_V^\mu}{q^2} (C_7 \mathcal{F}_{T,\mu} + \mathcal{H}_\mu) \right]$$

(local) FFs  $\mathcal{F}_\mu$  already discussed in the previous slides

calculate non-local FFs  $\mathcal{H}_\mu$  (charm-loop effects):

1. calculated  $\mathcal{H}_\mu$  using an Operator Product Expansion (OPE) at negative  $q^2$  [Khodjamirian et al. 2010]
2. extract  $\mathcal{H}_\mu$  at  $q^2 = m_{J/\psi}^2$  from  $B \rightarrow K^{(*)} J/\psi$  and  $B_s \rightarrow \phi J/\psi$  measurements
3. interpolate these results using a parametrization to obtain  $\mathcal{H}_\mu$  in the low  $q^2$  ( $0 < q^2 < 8 \text{ GeV}^2$ ) region



# Non-local form factors predictions

propose a new parametrization for  $\mathcal{H}_\lambda$  [NG/van Dyk/Virto 2020]

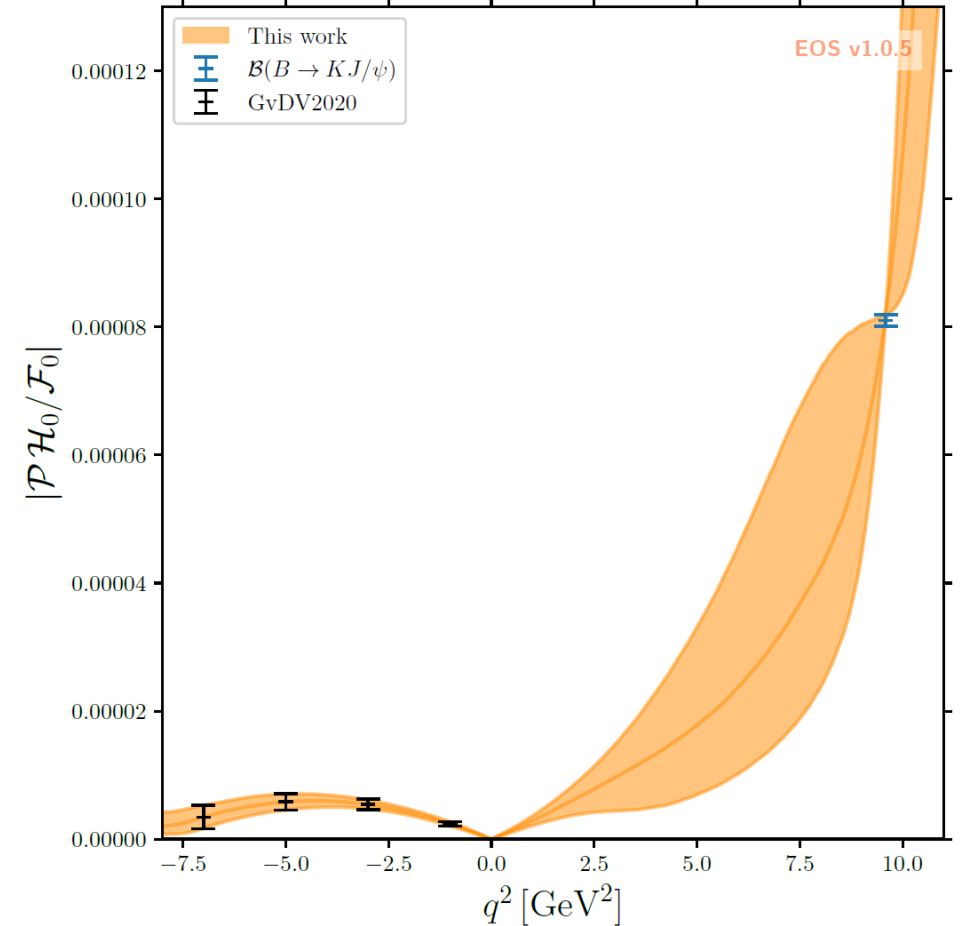
$$\mathcal{H}_\lambda \cong \sum_{n=0}^{\infty} \beta_n p_n(z)$$

first bound for the non-local FFs

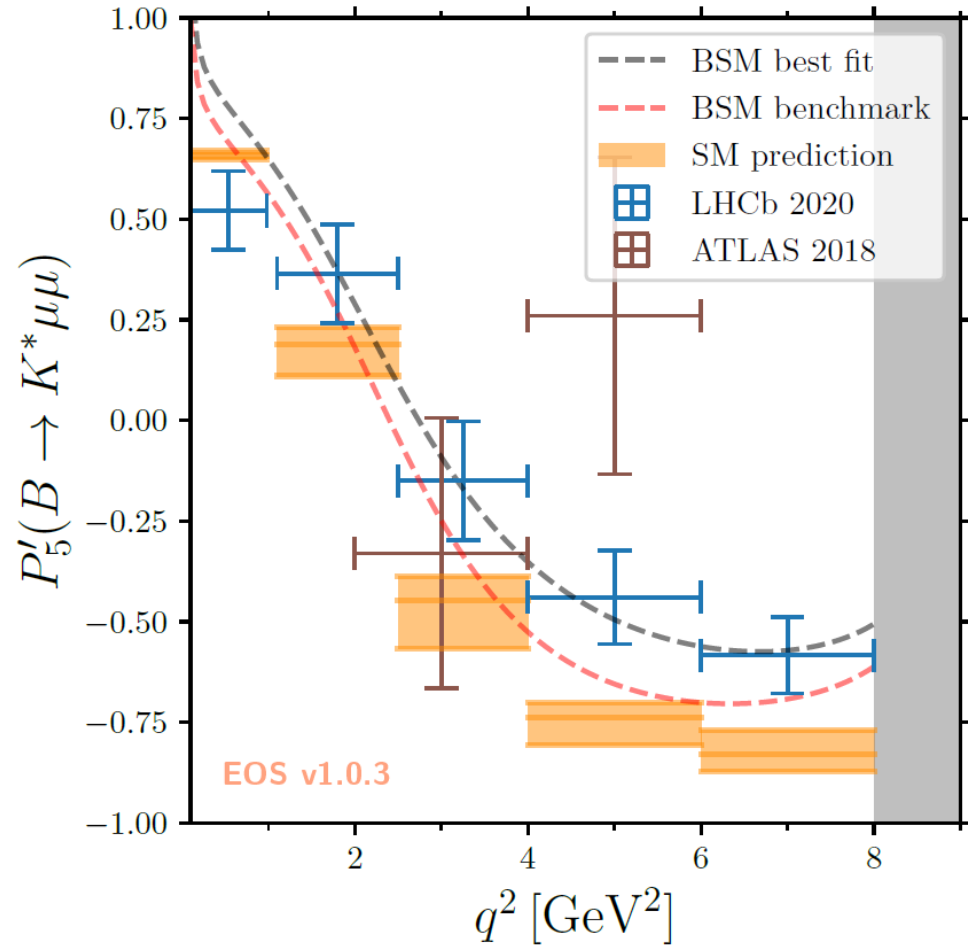
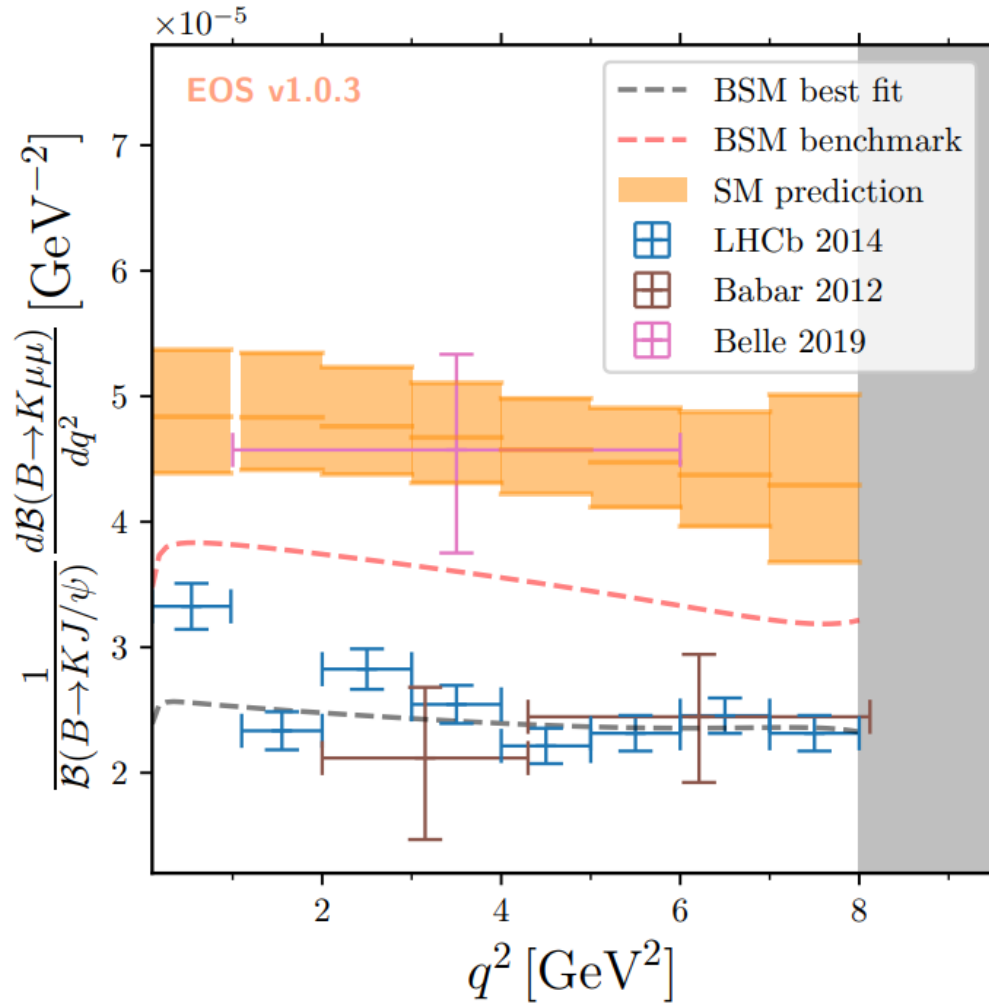
$$1 > \sum_{n=0}^{\infty} |\beta_n|^2$$

fit the  $z$  parametrization theoretical and exp. results  
(see previous slide)

use the results for results for local  $\mathcal{F}_\lambda$  and non-local  $\mathcal{H}_\lambda$  FFs  
to predict observables in  $B \rightarrow K^{(*)} \ell^+ \ell^-$  decays



# SM predictions for $B \rightarrow K^{(*)} \mu^+ \mu^-$



[NG/Reboud/van Dyk/Virto 2022]

confirm **sizable tension** between SM predictions and experimental results

Conclusions and outlook

# Conclusion and outlook

## $b \rightarrow c$ transitions:

- $B_{(s)} \rightarrow D_{(s)}^{(*)}$  FFs – lattice QCD (and LCSRs) calculations available
- use HQET and dispersive bounds for better precision
- non-local effects absent (neglect QED corrections)  $\rightarrow$  **very precise SM predictions**

## $b \rightarrow s$ transitions:

- $B \rightarrow K^{(*)}$  and  $B_s \rightarrow \phi$  FFs – lattice QCD (and LCSRs) calculations available
- non-local effects implies large uncertainties
- control these uncertainties (use dispersive bounds)  $\rightarrow$  systematically improve SM predictions

## Take-home message

SM predictions in semileptonic and rare  $B_{(s)}$  decays are (or will be) under control and can be improved

CEPC offers unique opportunities for flavour physics that we look forward to explore ( $B_s$  meson decay in a clean environment,  $Z \rightarrow X\gamma$  ... see A. Kwok and J. Kamenik talk)

Thank you!