Lattice QCD and Physics

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Need for Precision

- Beauty factories (Belle II, LHCb) producing B mesons (and other b-hadrons) at unprecedented rates.

- Similarly for charm factories (BES III, LHCb, Belle II).

- CEPC/FCC have potential to combine high rate and boost of LHC with cleanliness of asymmetric e^+e^- machines.

- Lingering puzzles in strangeness: will b anomalies linger too?

- Two-prong strategy:

  - determine parameters of the Standard Model;

  - study rare processes with well-predicted SM rate.

Outline

- Need for precision
- QCD and lattice gauge theory
- Some precise results from lattice QCD
  - Hadron masses and quark masses
  - Quark flavor
  - Muon g–2
- Enabling precision lattice QCD

Lattice QCD

QCD Lagrangian

- SU(3) gauge symmetry and $1 + n_f + 1$ parameters:

$$\mathcal{L}_{\mathrm{QCD}} = \frac{1}{g_0^2}\,\mathrm{tr}[F_{\mu\nu}F^{\mu\nu}] \qquad M_\Omega \text{ or similar, ;}$$

$$- \sum_f \bar{\psi}_f (\not{D} + m_f)\psi_f \qquad M_\pi,\ M_K,\ M_{J/\psi},\ M_\Upsilon,\ \ldots;$$

$$+ \frac{i\theta}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}[F_{\mu\nu}F_{\rho\sigma}] \qquad \theta = 0.$$

- Gauge coupling g_0 and quarks masses m_f are not directly measurable:
  - quarks bound into hadrons: use meson masses to fix m_f;
  - "dimensional transmutation"; use another mass to eliminate g_0.

QCD Lagrangian

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$$\mathcal{L}_{\rm QCD} = \frac{1}{g_0^2} \operatorname{tr}[F_{\mu\nu}F^{\mu\nu}] \qquad M_\Omega \text{ or similar, ;}$$

$$- \sum_f \bar{\psi}_f (\not{D} + m_f)\psi_f \qquad M_\pi,\ M_K,\ M_{J/\psi},\ M_\Upsilon,\ \ldots;$$

Humankind's most perfect theory—Wilczek

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QCD Functional Integral

- Everything from integrals:

$$\langle \mathscr{O}(U,\psi,\bar{\psi})\rangle = \frac{1}{Z}\int \mathscr{D}U\,\mathscr{D}\psi\,\mathscr{D}\bar{\psi}\,\mathscr{O}(U,\psi,\bar{\psi})e^{-S}$$

gluons

(anti)quarks

imaginary time:
$t = x^0 = -ix_4$

- Infinite spacetime lattice makes set of integration variables countable; finite lattice makes them finite.

- Markov chain Monte Carlo with important sampling:

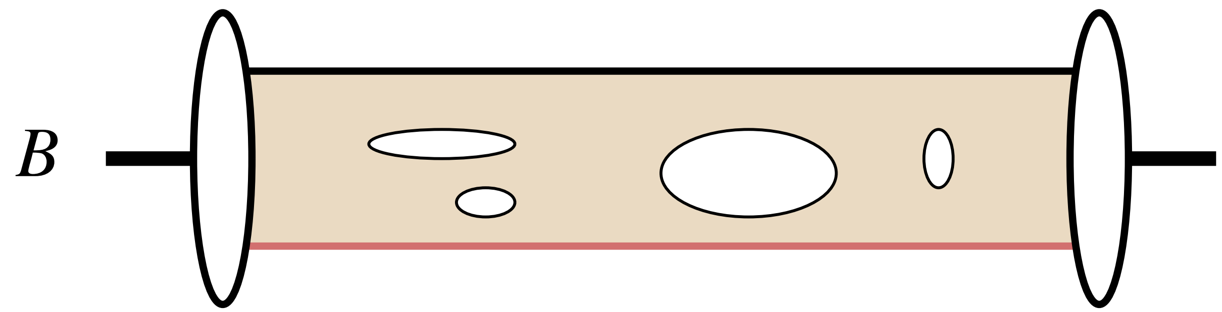
$$\langle \mathscr{O}(U,G)\rangle = \frac{1}{C}\sum_{c=0}^{C-1}\mathscr{O}(U,G)$$

Hybrid Monte Carlo (HMC)
Duane, Kennedy, Pendleton, Roweth
Phys. Lett. B 195 (1987) 216

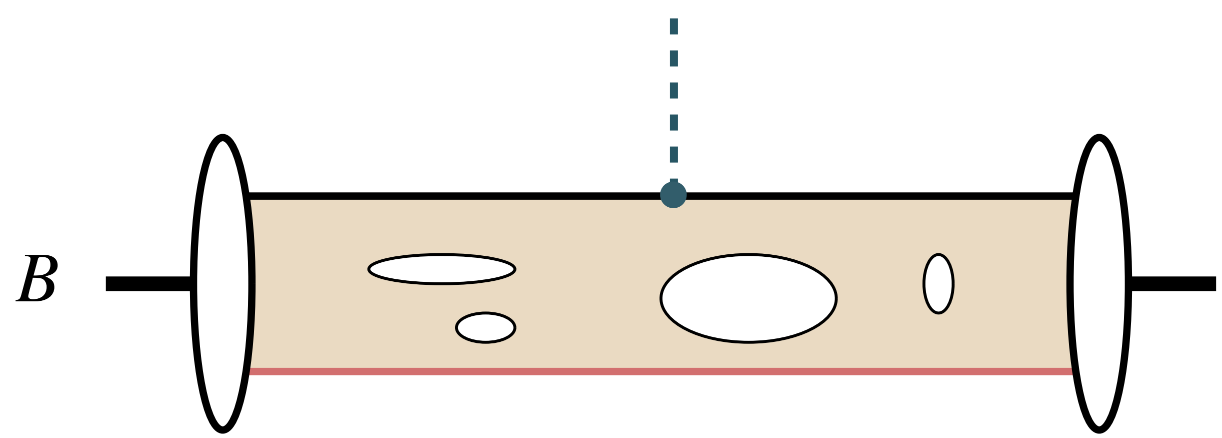
QCD Correlation Functions

- Everything (almost) from correlation functions:

- Masses, annihilation matrix elements:



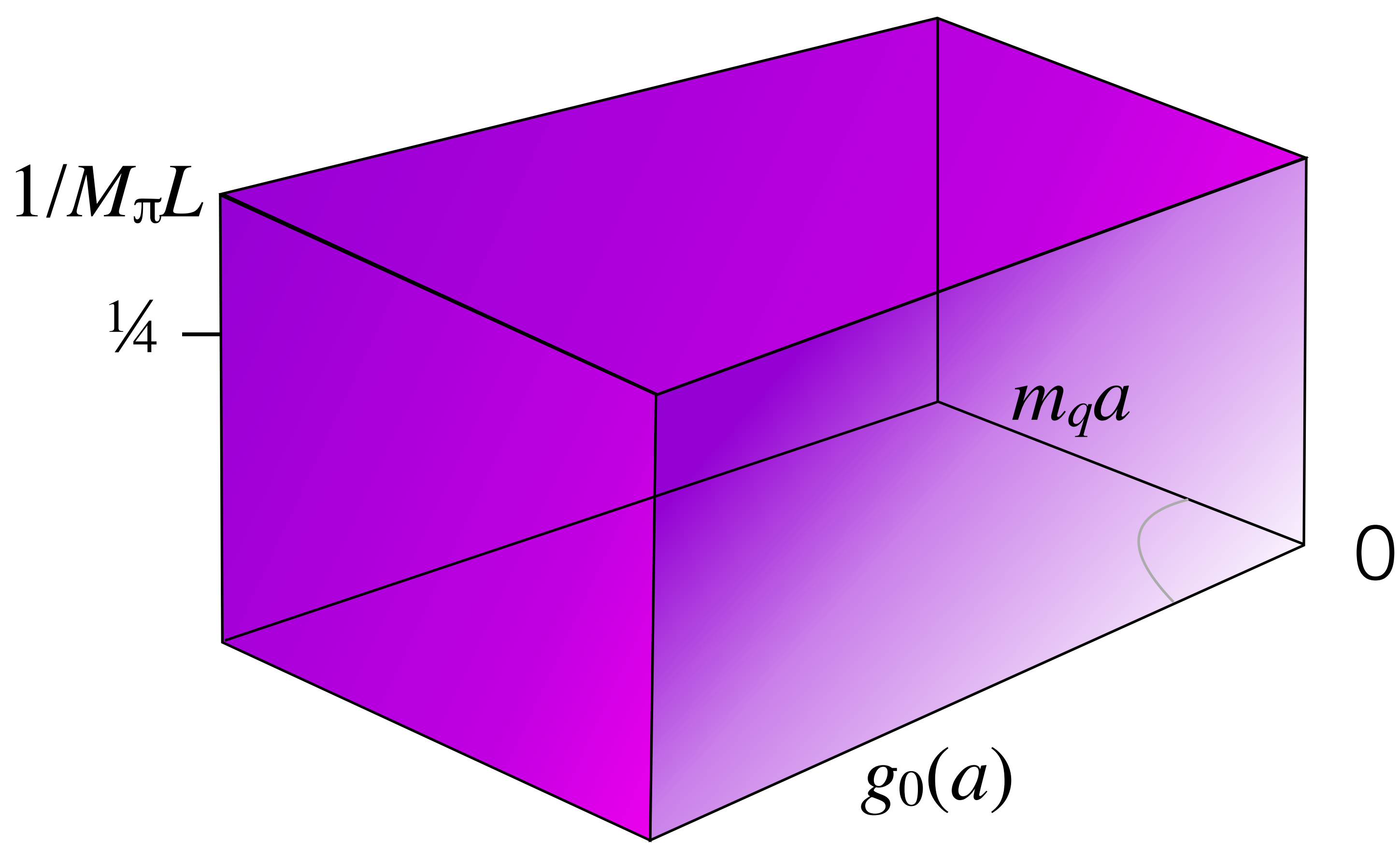
- Decay and mixing matrix elements:



- Valence & sea quarks: different parts of code, so masses can differ.

Lattice QCD Data

- Computer generates data in a slab of a 10-dimensional parameter space:



Sea	Valence
	m_u
$m'_u = m'_d$	m_d
m'_s	m_s
m'_c	m_c
	m_b

$m'_u \neq m'_d \Rightarrow$ 11d slab

- Combine data with effective field theories (EFT) [*e.g.*, hep-lat/0205021].

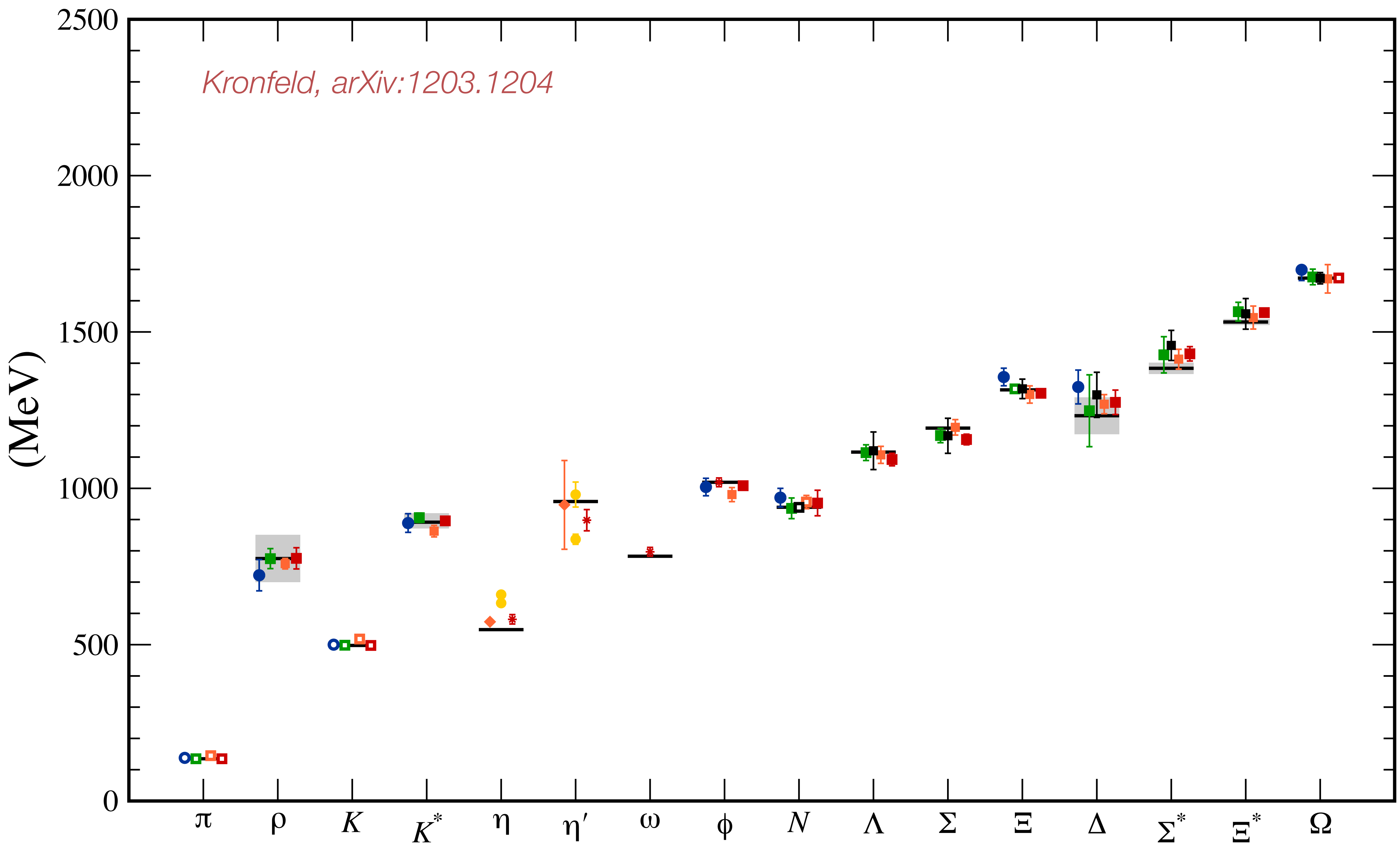
red EFT

- Effective field theories: Symanzik ($a \to 0$), chiral perturbation theory (m_q', $m_q \to m_u$, m_d), heavy quark theory ($m_Q \to m_b$, m_c).

Masses

π…Ω: BMW, MILC, PACS-CS, QCDSF; ETM (2+1+1);
η-η′: RBC, UKQCD, Hadron Spectrum (ω).

Hadron Spectrum

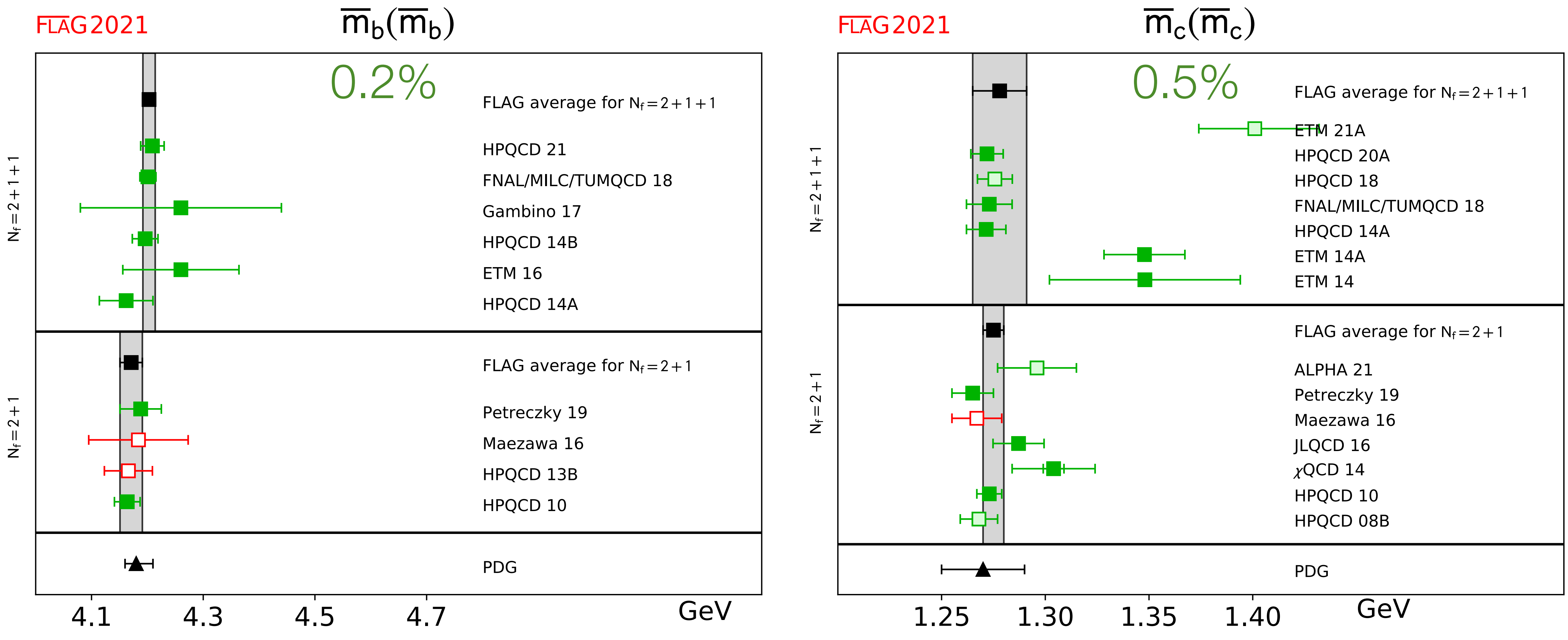


Hadron Spectroscopy

- Not much interest in normal hadron masses these days:
  - even M_n–M_p studied vs. m_d–m_u & α a while ago [arXiv:1406.4088].
- Lots of interest in exotic hadrons (*XYZ*, tetraquarks, pentaquarks): lattice QCD ↔ structure [USQCD-WP, SnowWP].
- Most precisely calculable masses are pseudoscalar meson masses:
  - adjust bare quark masses until n_f of them agree with experiment—π, $K^0 \pm K^+$, $D_{(s)}$ (η_c), $B_{(s)}$ (η_b);
  - convert them to quark masses in renormalization schemes used in continuum QCD ($\overline{\text{MS}}$, RGI).

Quark Masses: bottom, charm

- Flavor Lattice Averaging Group (FLAG), arXiv:2111.09849.



- Green passes all quality criteria; solid enters average (hatched superseded).

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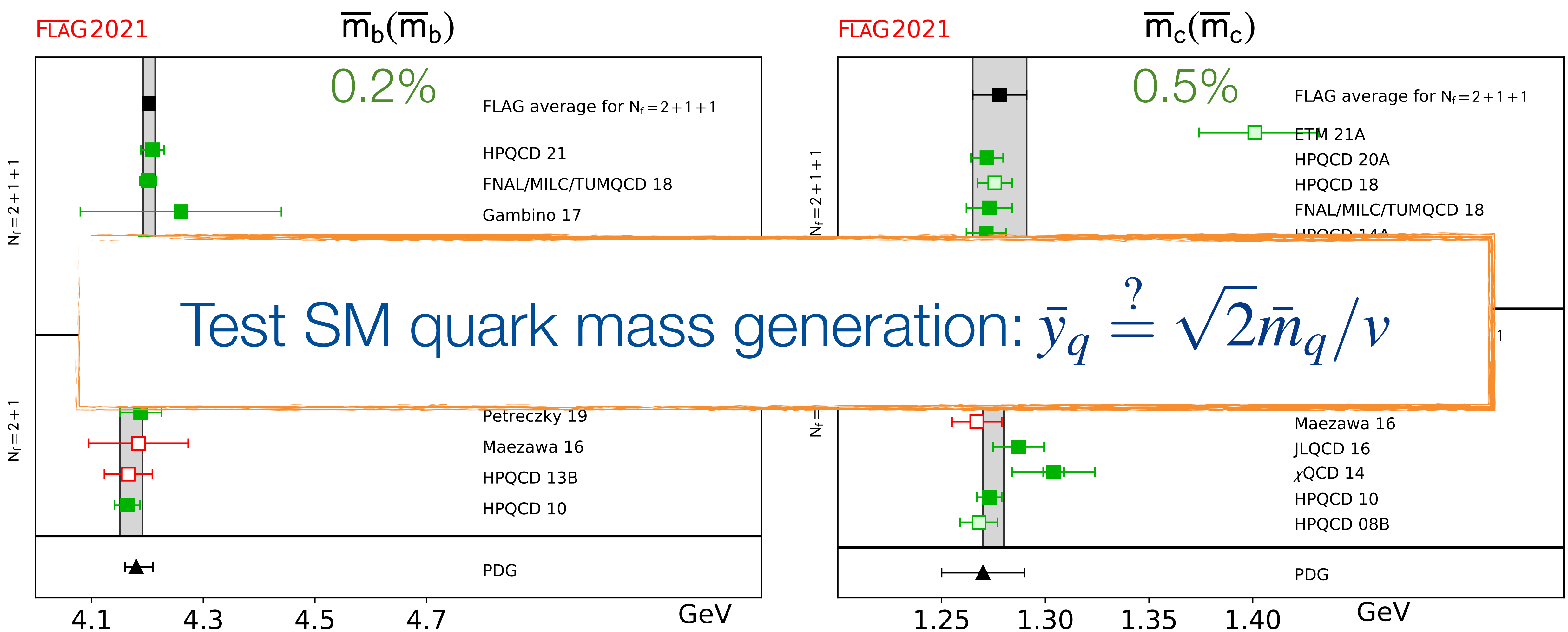
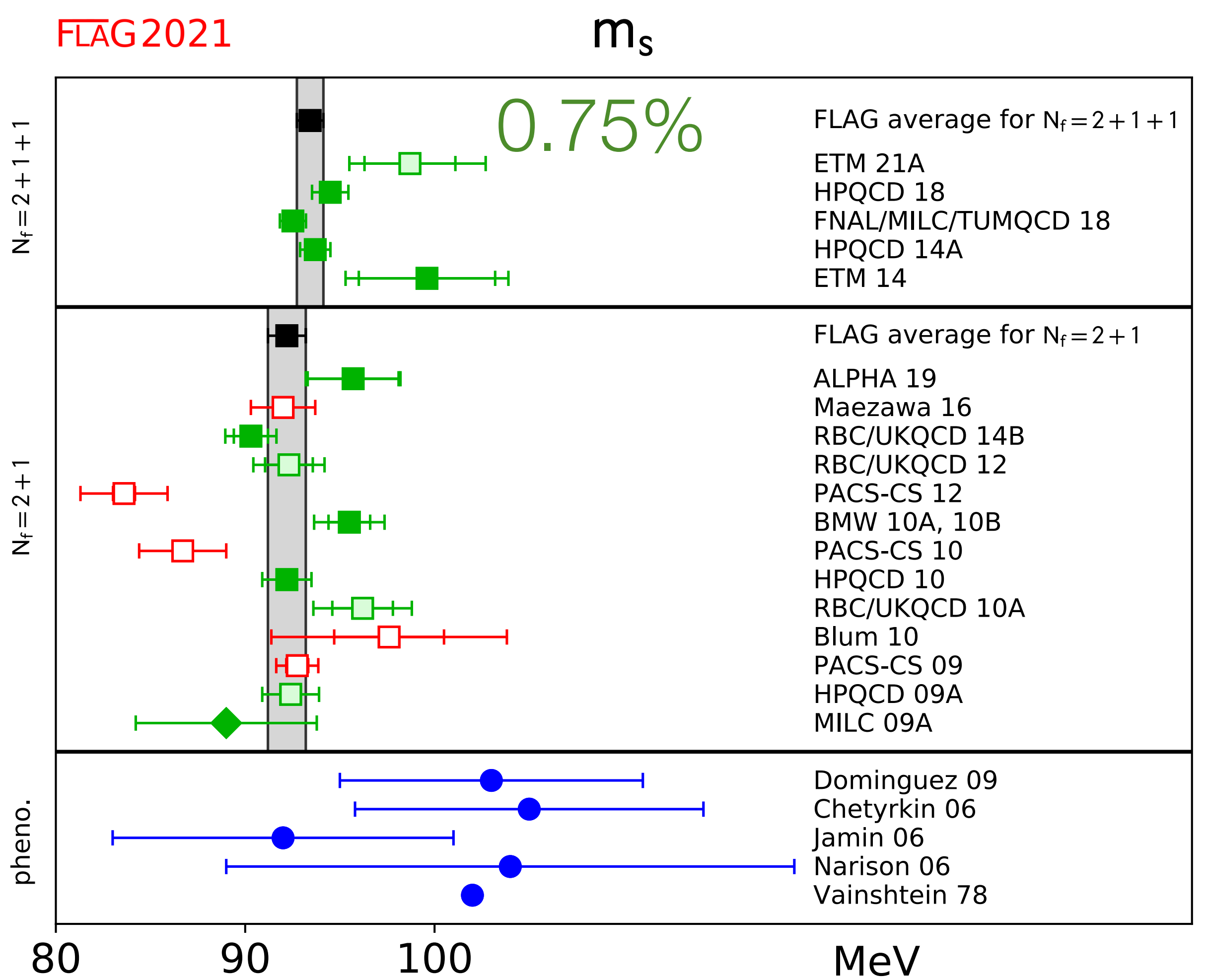
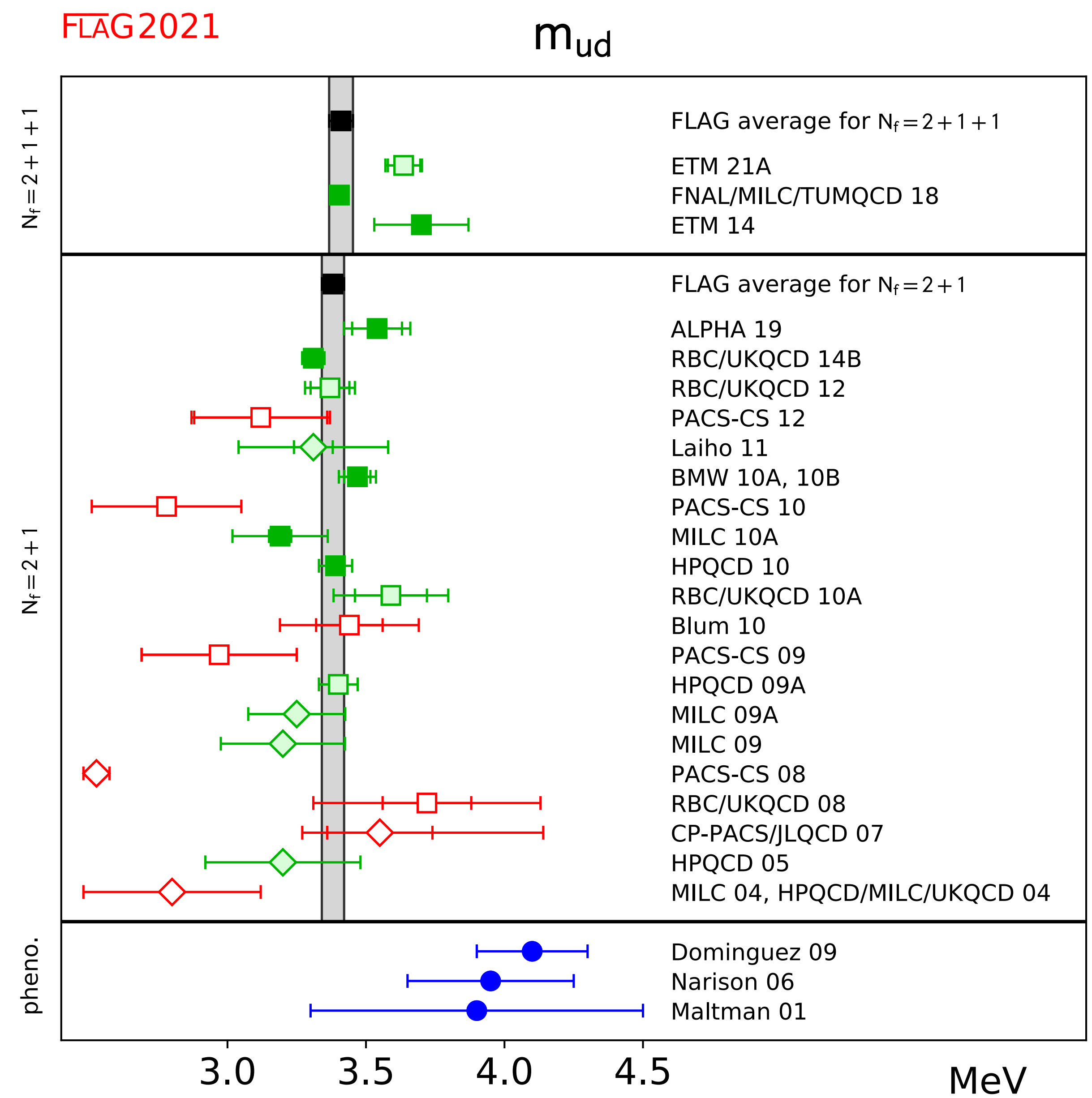


Test SM quark mass generation: $\bar{y}_q \overset{?}{=} \sqrt{2}\bar{m}_q/v$

- Green passes all quality criteria; solid enters average (hatched superseded).

Quark Masses: strange, down/up





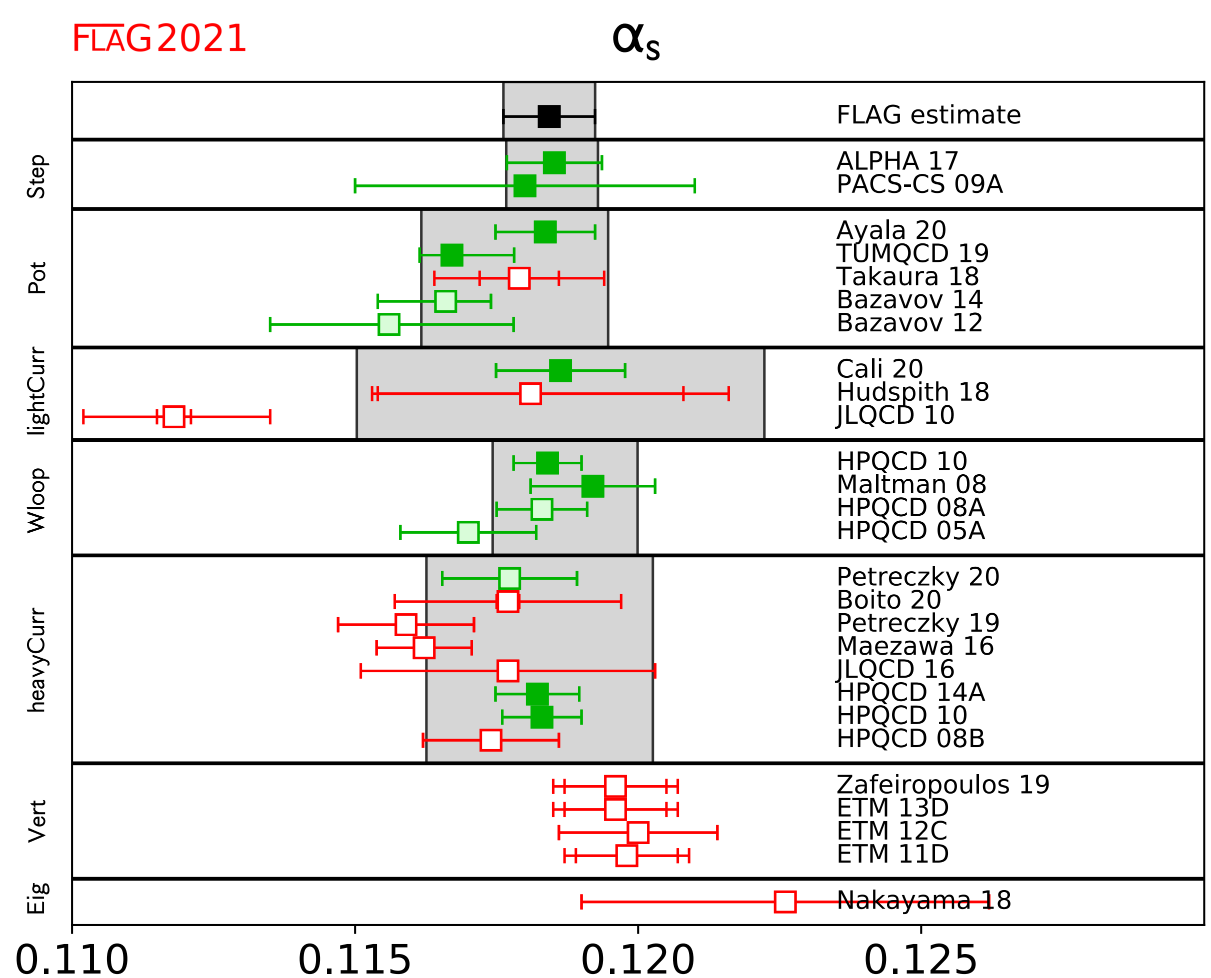
$$m_{u,\overline{\rm MS}}(2\ {\rm GeV}) = 2.130(41)\ {\rm MeV}$$

$$m_{d,\overline{\rm MS}}(2\ {\rm GeV}) = 4.675(56)\ {\rm MeV}$$

- arXiv:1802.04248, arXiv:1805.06225

Strong Coupling $\alpha_s(\mu)$

- Numerous methods:
  - different observables;
  - different probes;
  - different systematics.
- Consider “heavyCurr” ☛
  - same PT as in $e^+e^- \to Q\bar{Q}$ determination.
- SnowWP on α_s.



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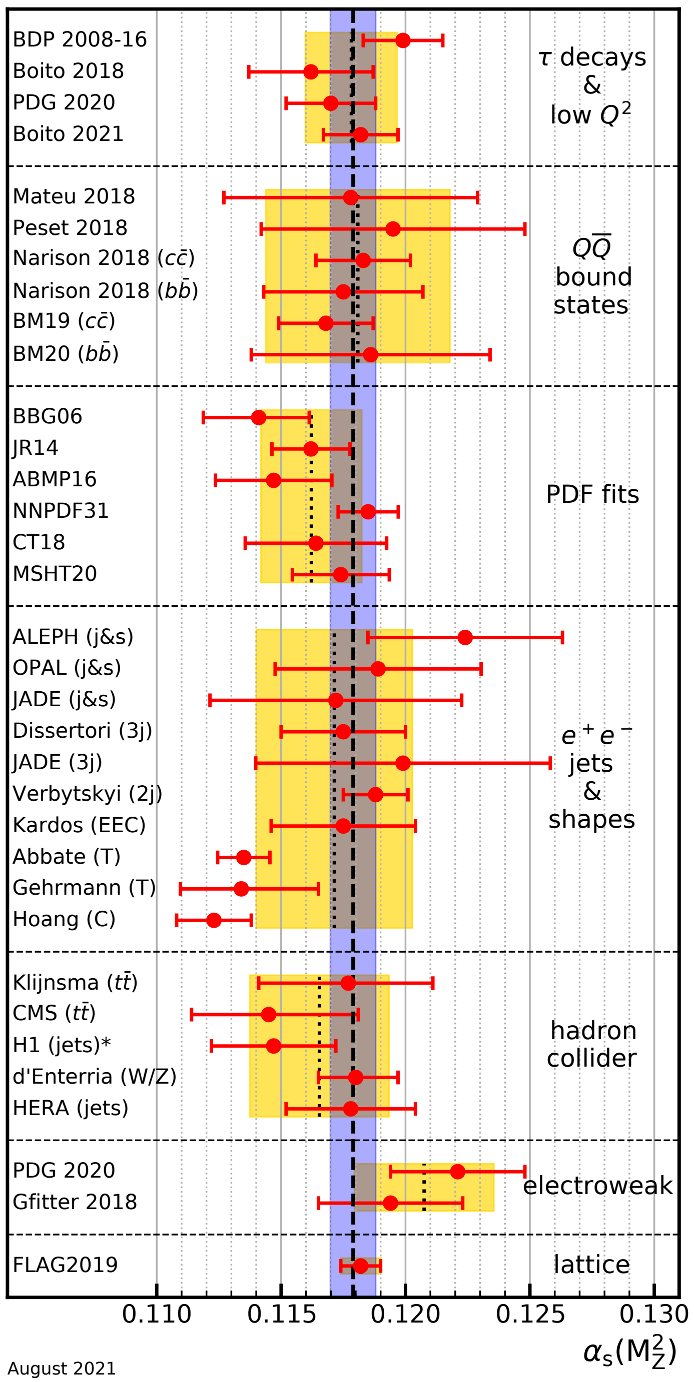

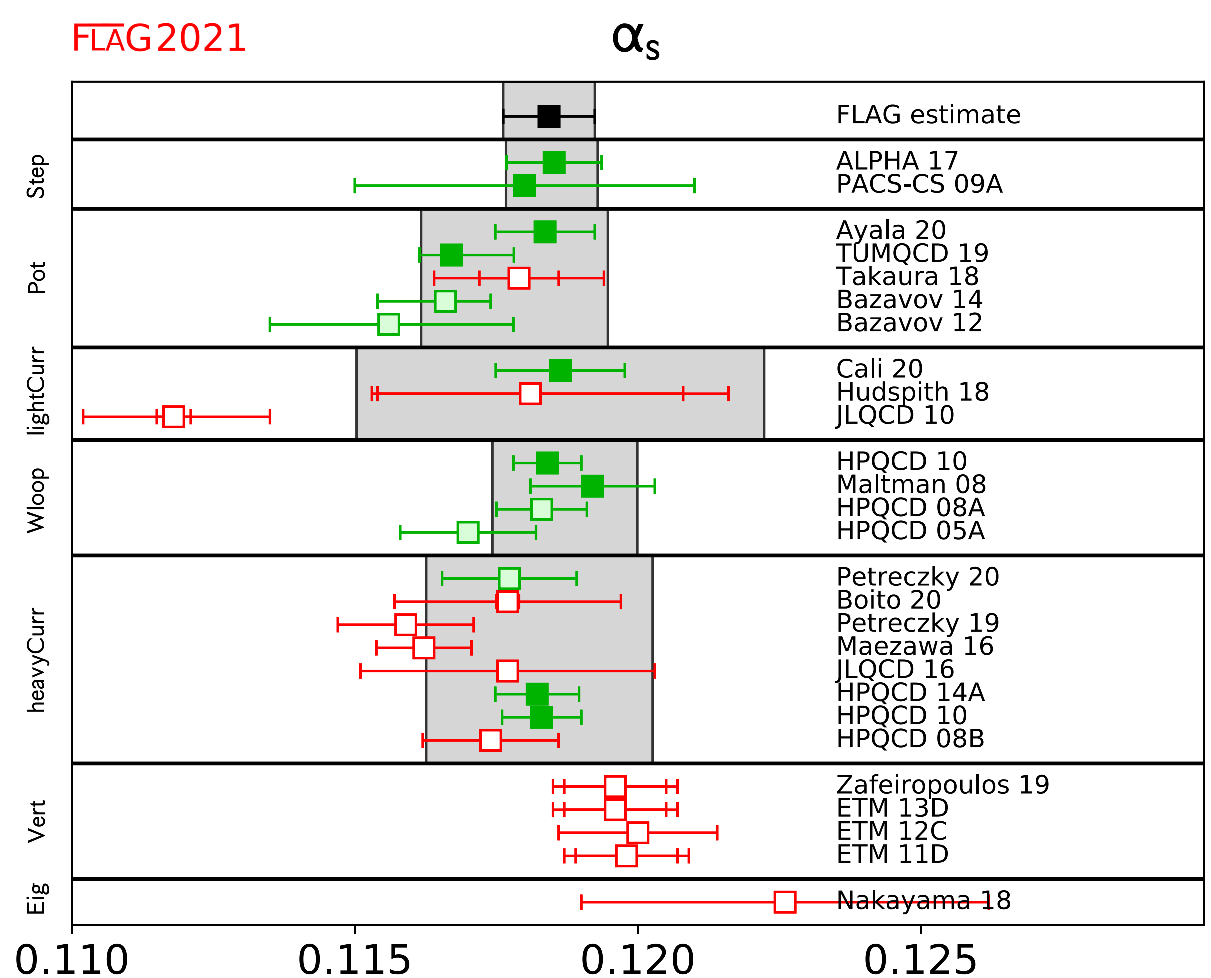

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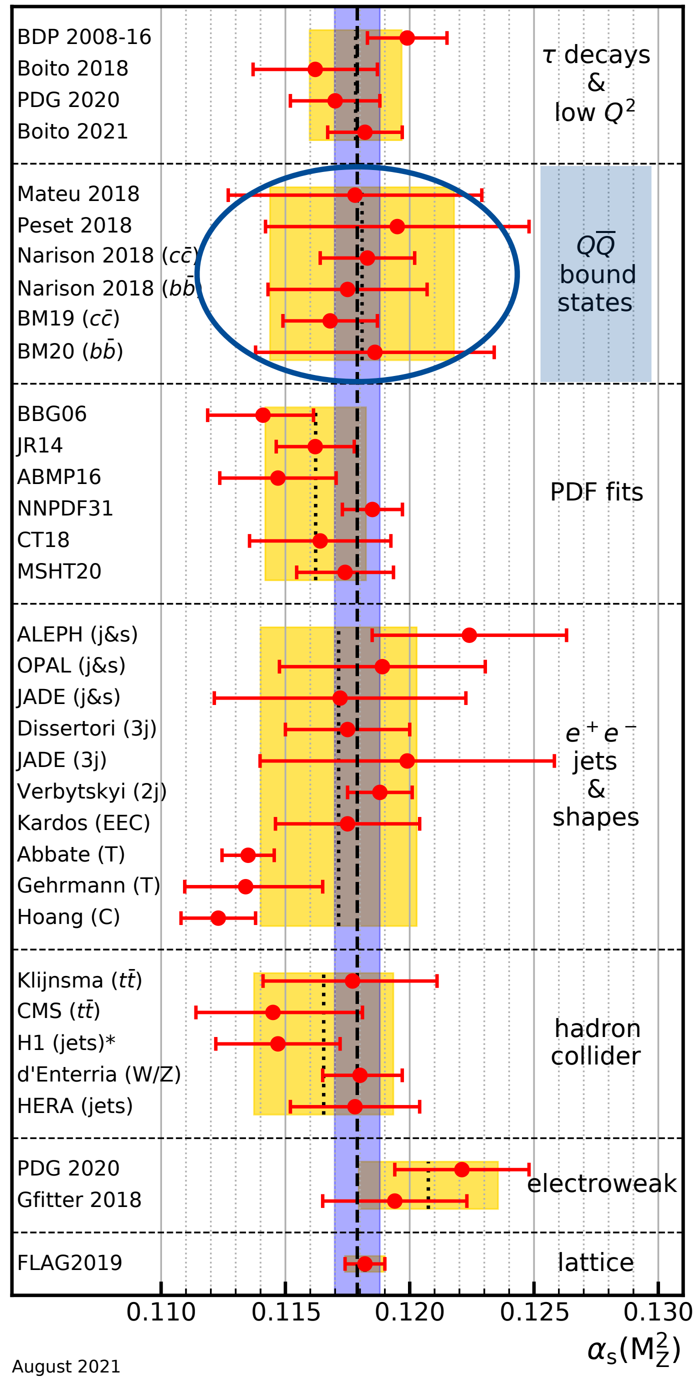

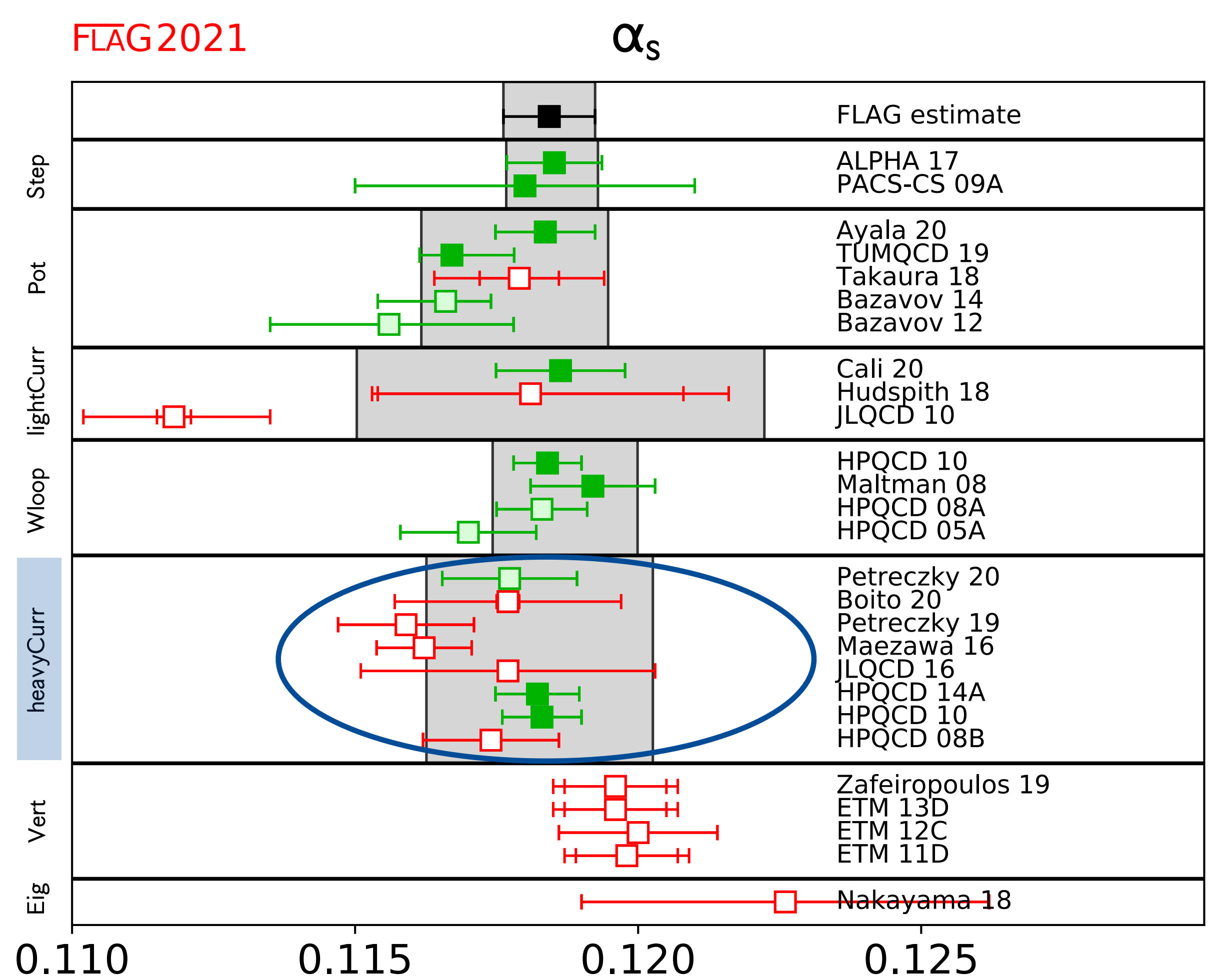

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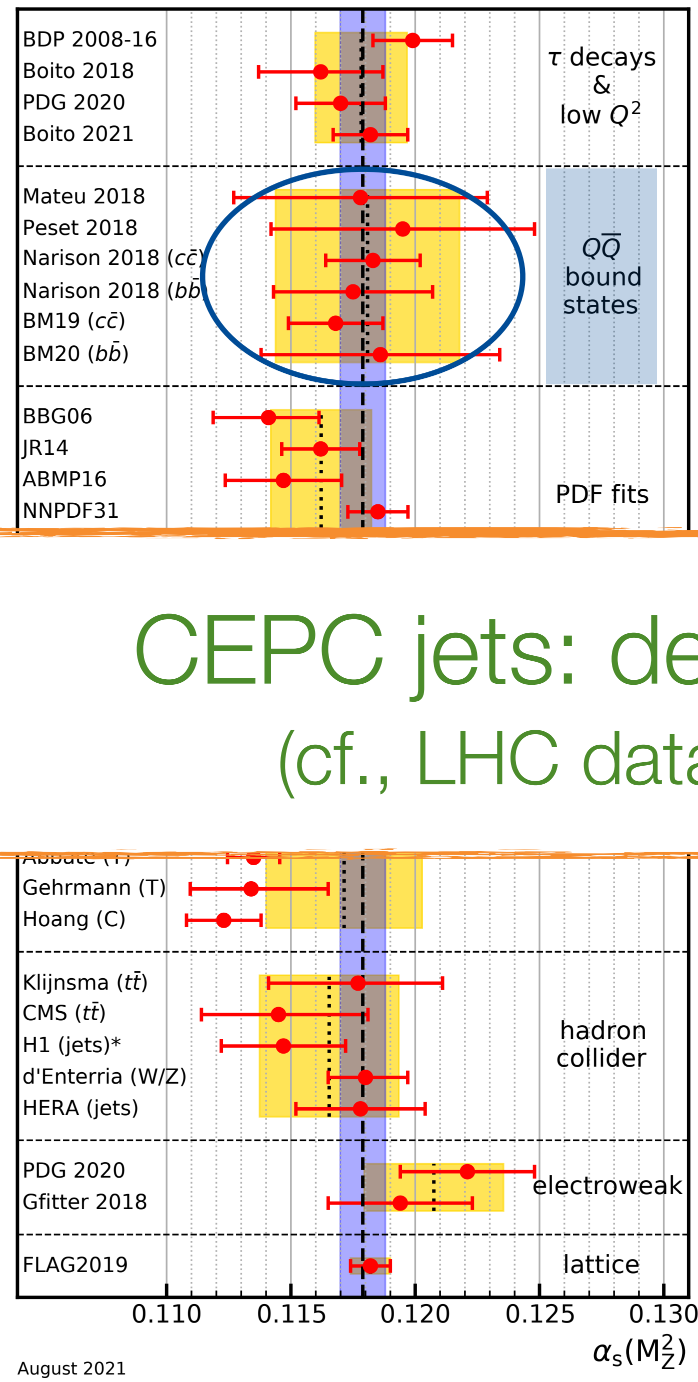
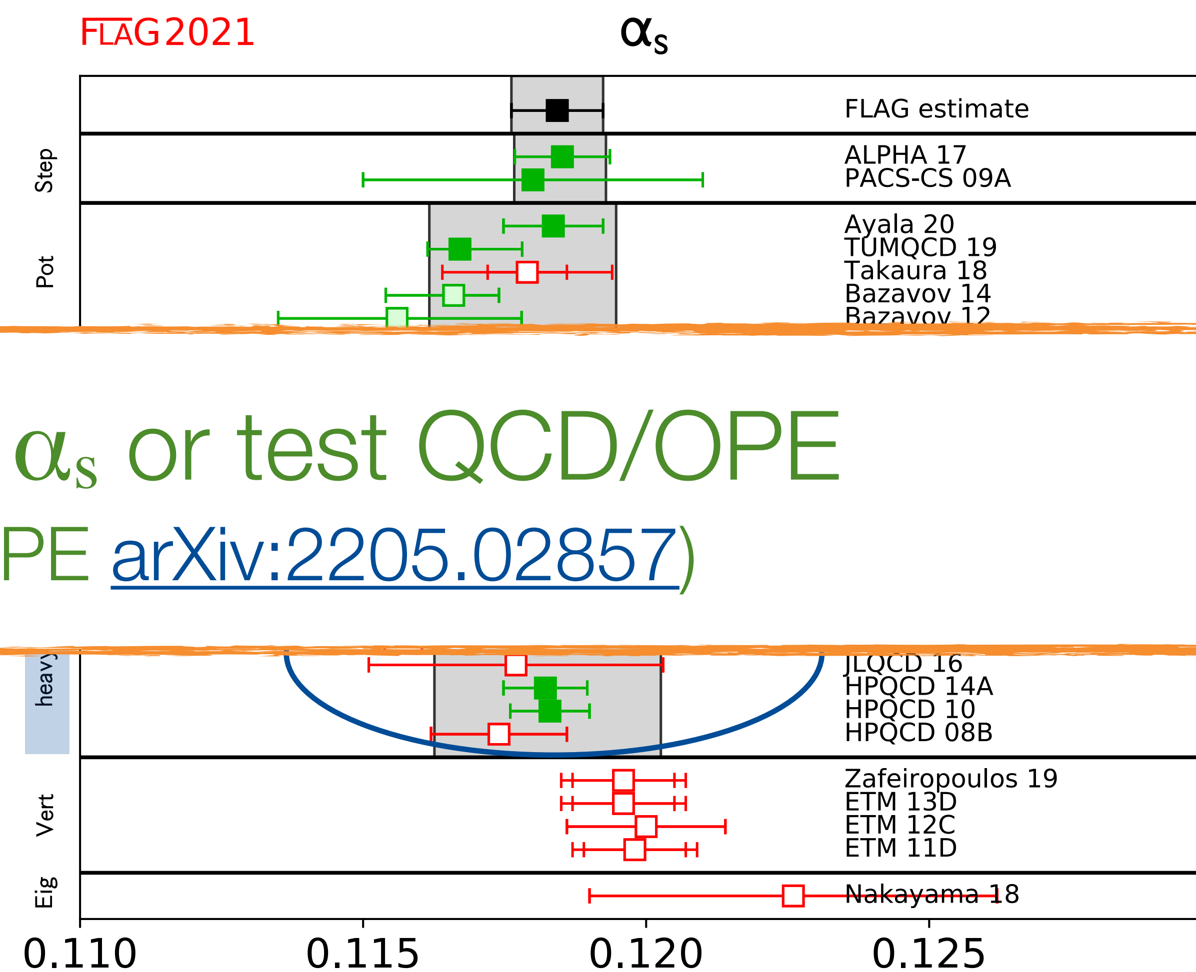


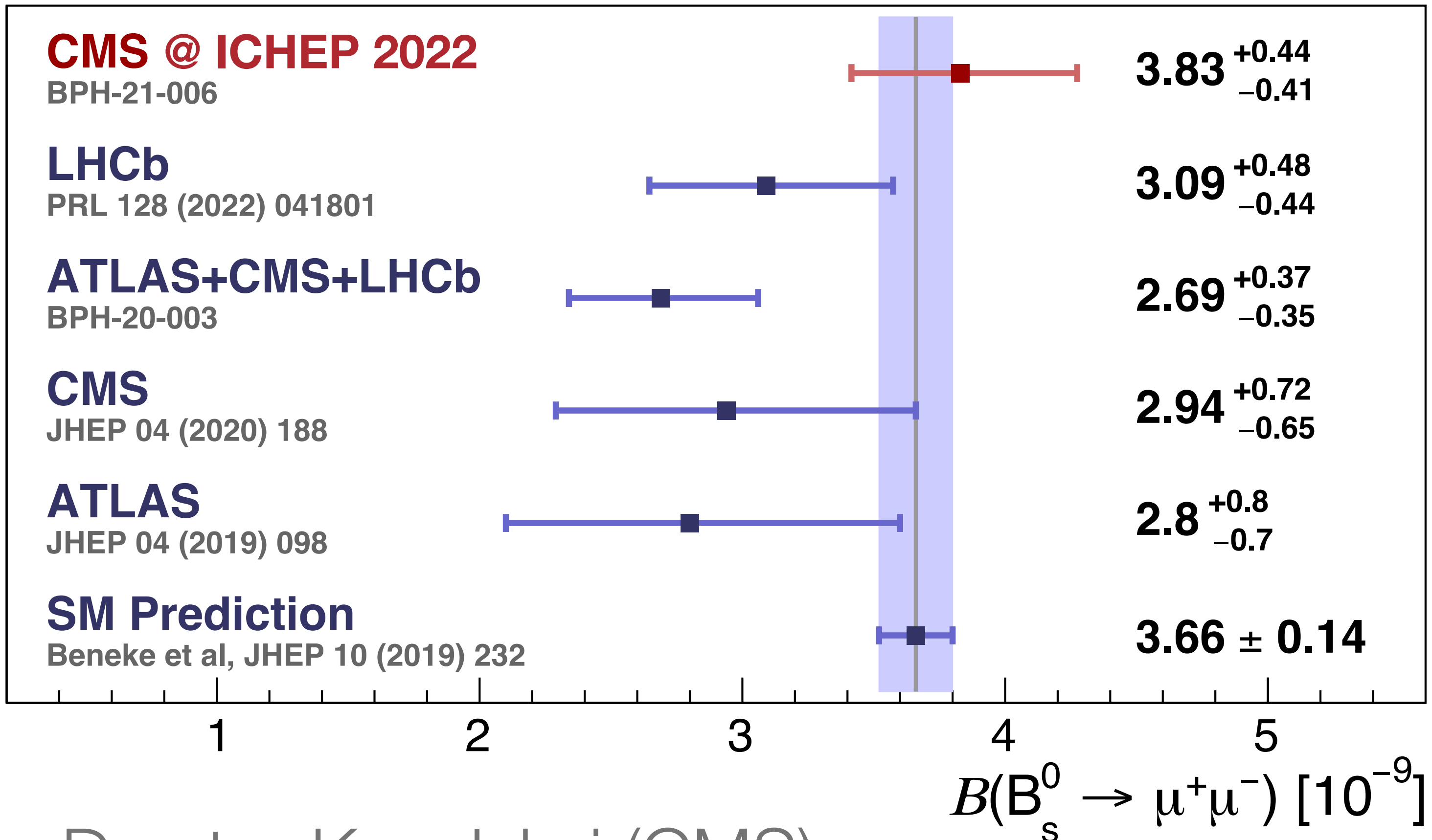


CEPC jets: determine α_s or test QCD/OPE

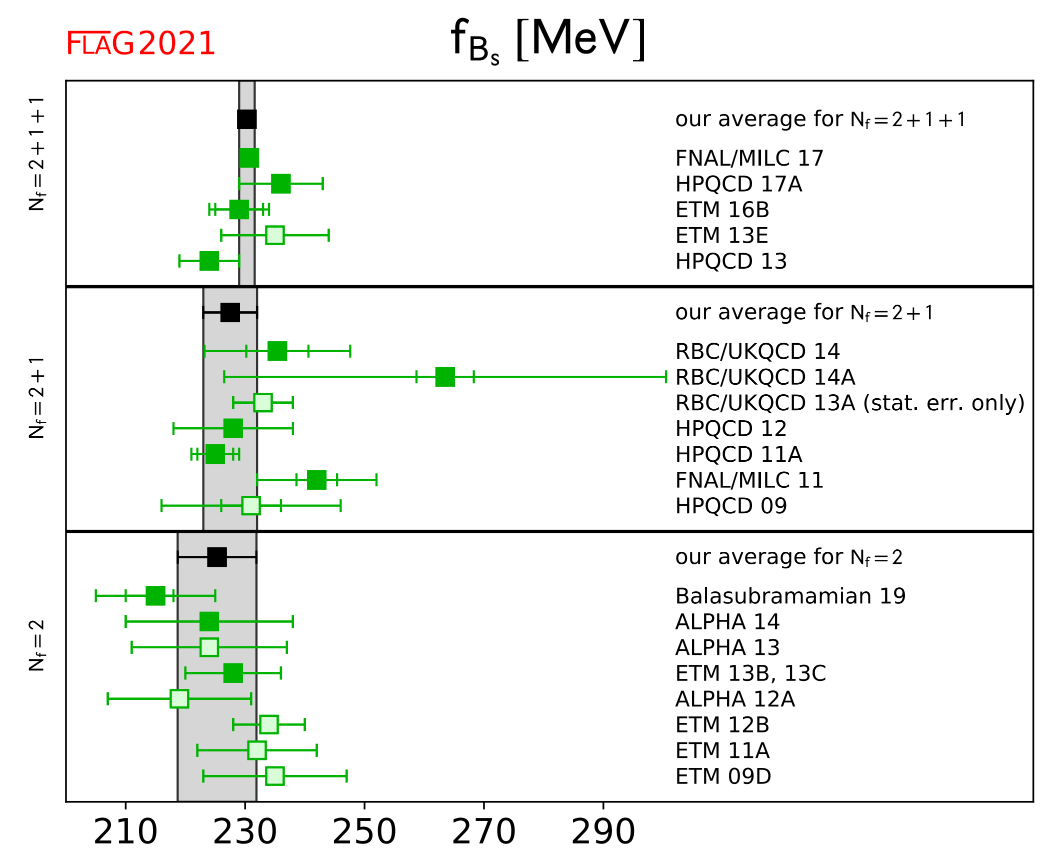
(cf., LHC data testing OPE arXiv:2205.02857)

Matrix Elements

Leptonic Decays



- CKM: $B^+ \to \tau^+\nu$.
- BSM: $B_s^0 \to \mu^+\mu^-$.
- ~0.7% uncertainty.



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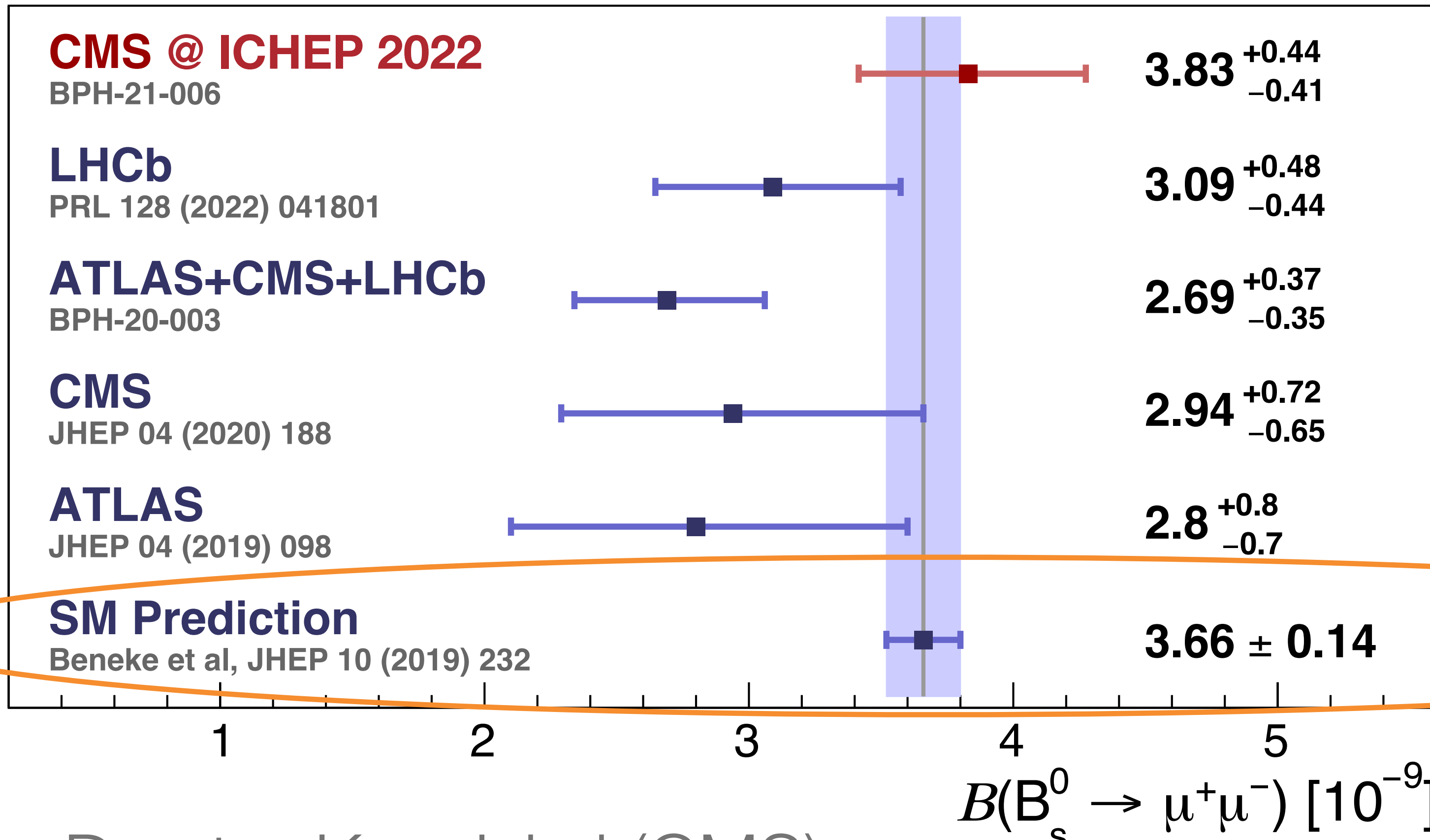

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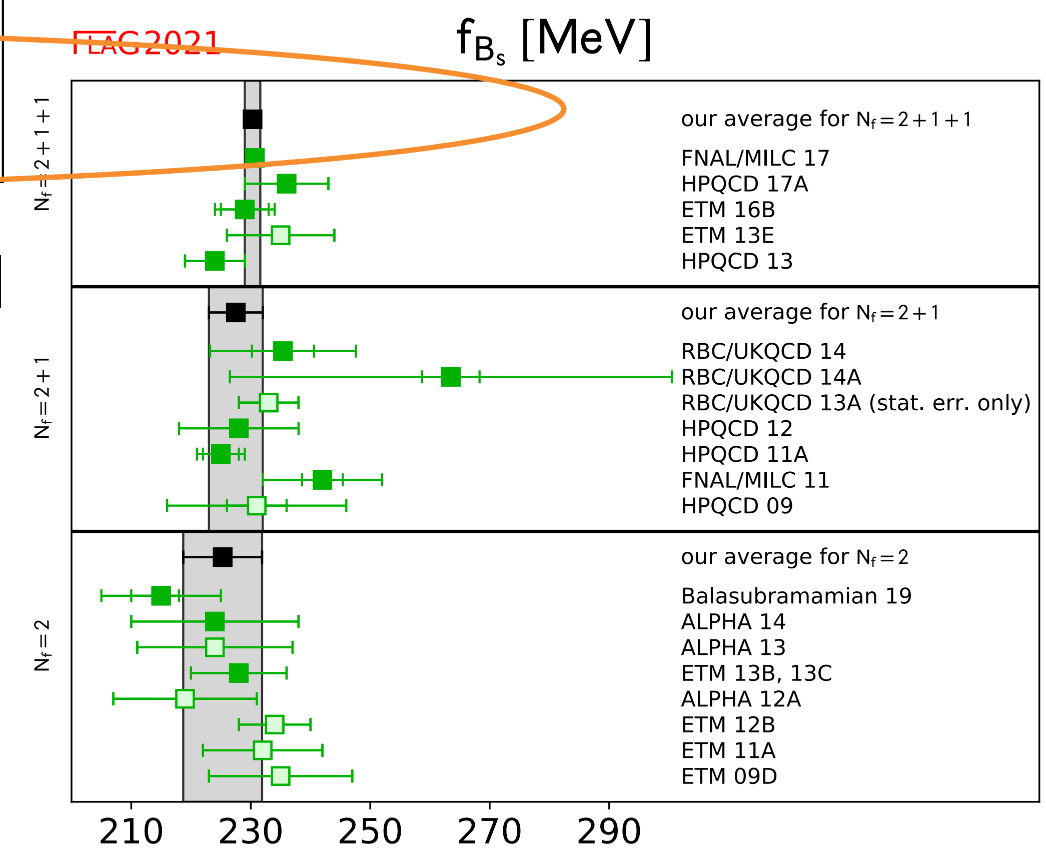

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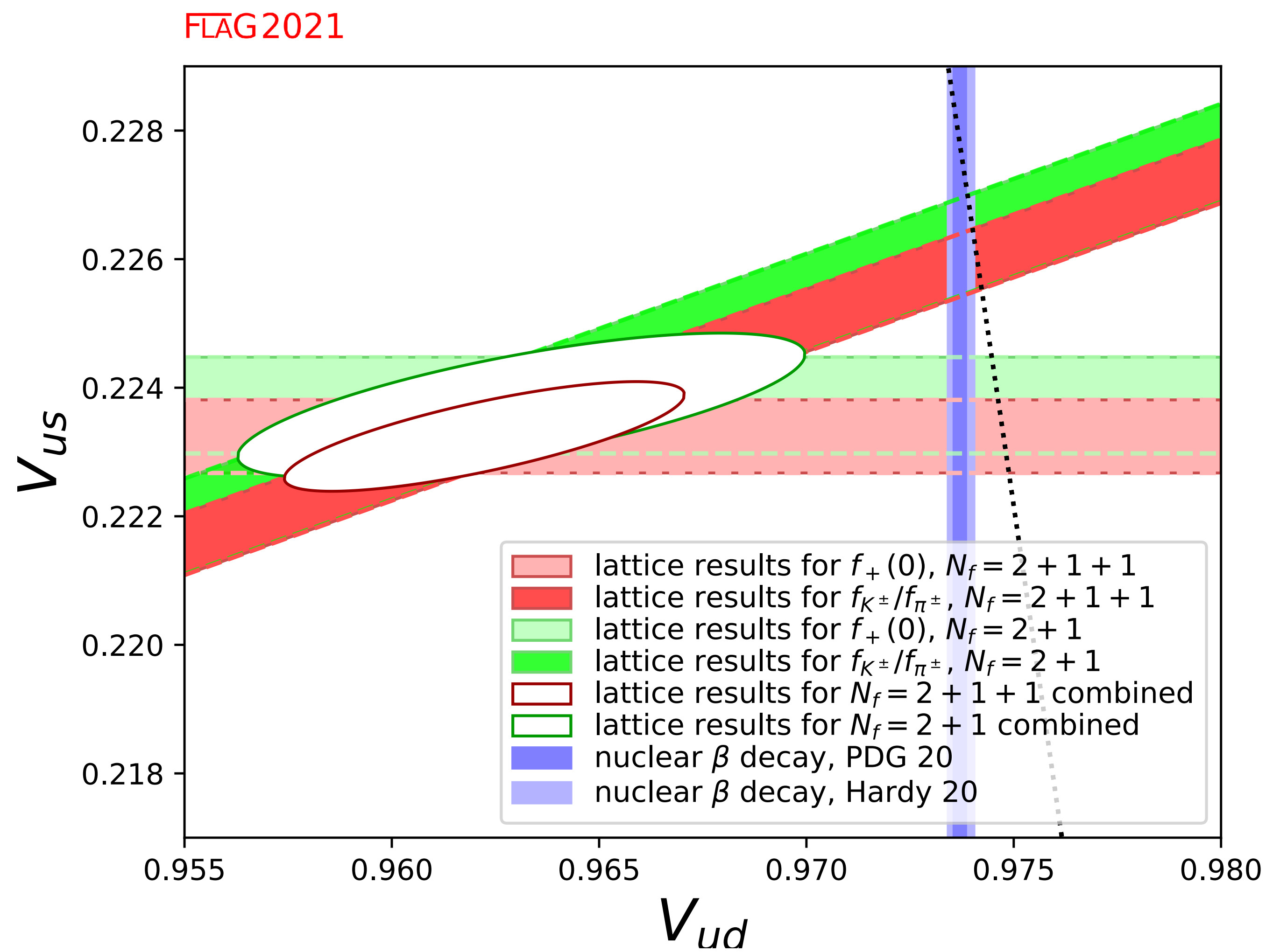
Cabibbo Angle Anomaly

- Leptonic decays π_{l2}/K_{l2} with f_K/f_π yield $|V_{ud}|/|V_{us}|$.

- Semileptonic decay K_{l3} with $f_+(0)$ yields $|V_{us}|$.

- Superallowed nuclear decay with nuclear theory yields $|V_{ud}|$.

- Unitarity crisis (3.2σ):



$$1 - |V_{ud}|^2_{0^+} - |V_{us}|^2_{K_{l3}} = 0.0021(2)(2)(2)(2)(5)_{\text{NS}}$$

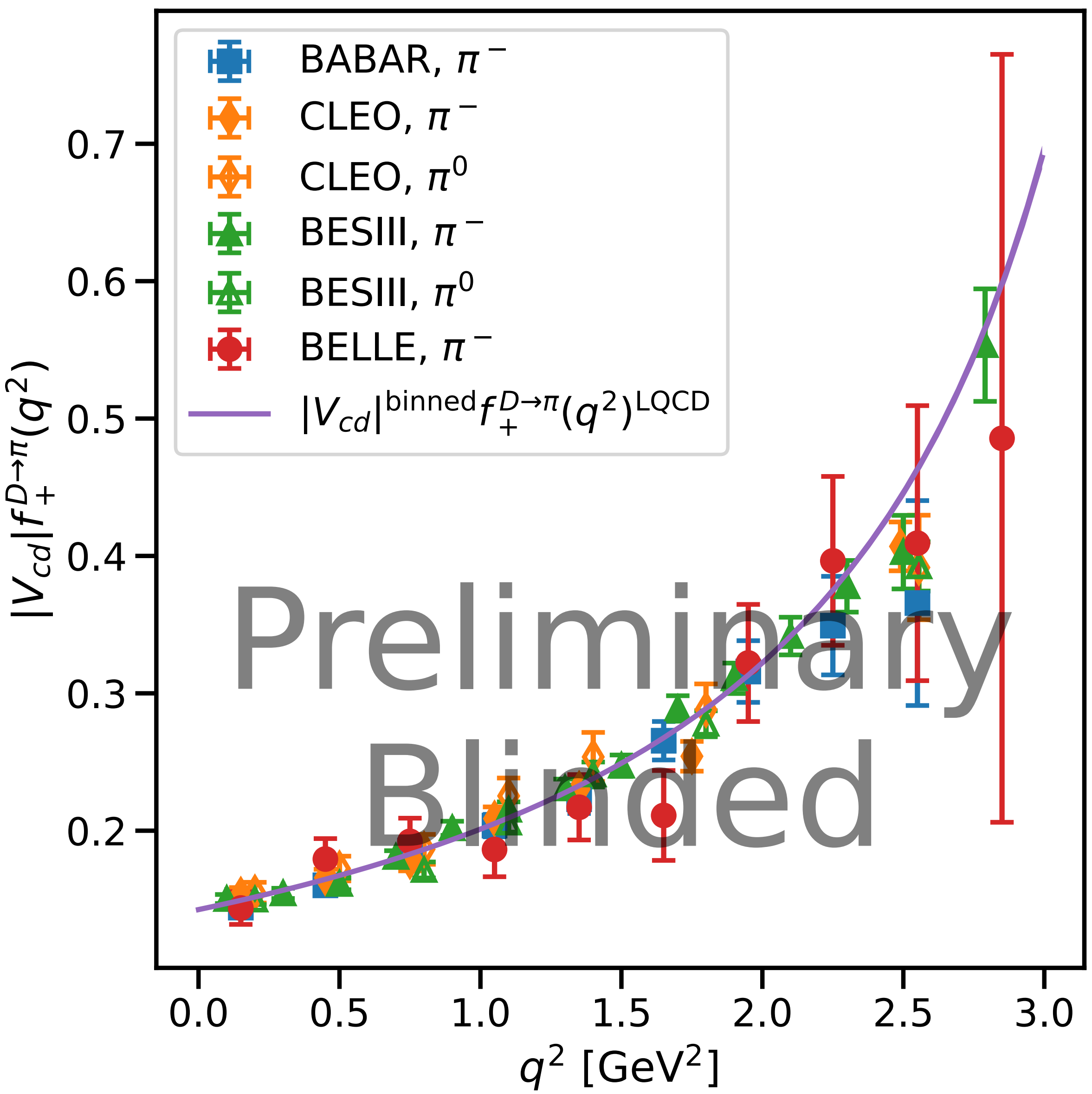
[arXiv:2107.14708]

nuclear structure

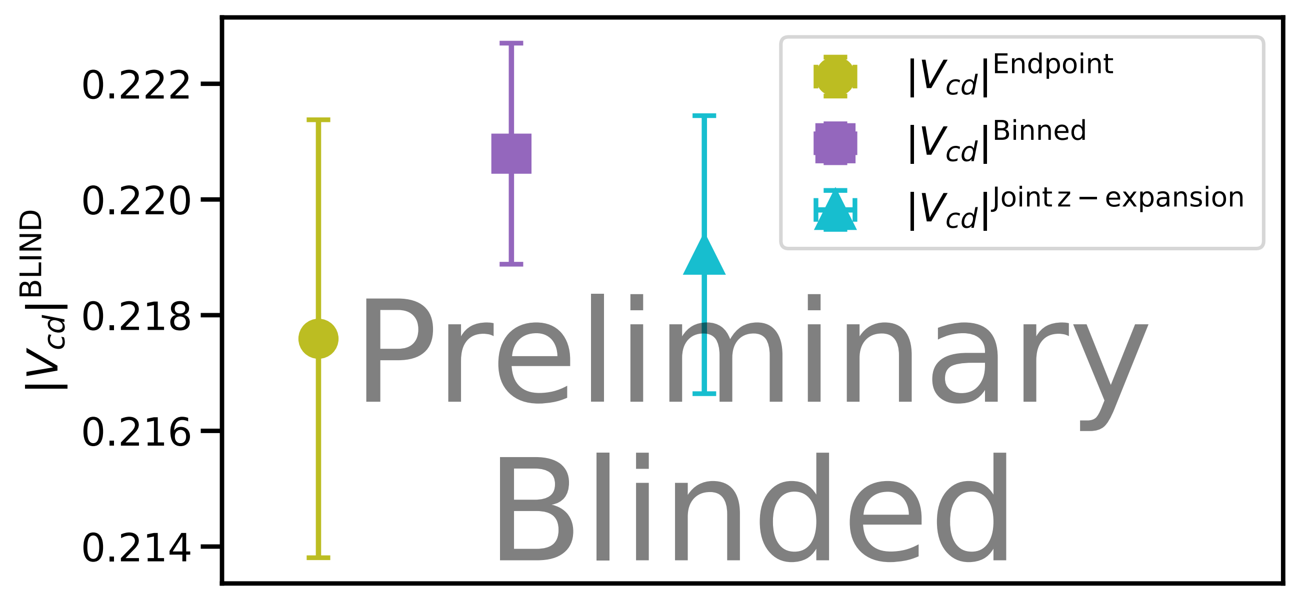
- Improve NS or (some day) just use π & K.

Coming Soon: $|V_{cd}|$ and $|V_{cs}|$

- Semileptonic decays: great way to determine CKM: check if shapes agree & the fit relative normalization. Plots by Will I. Jay.

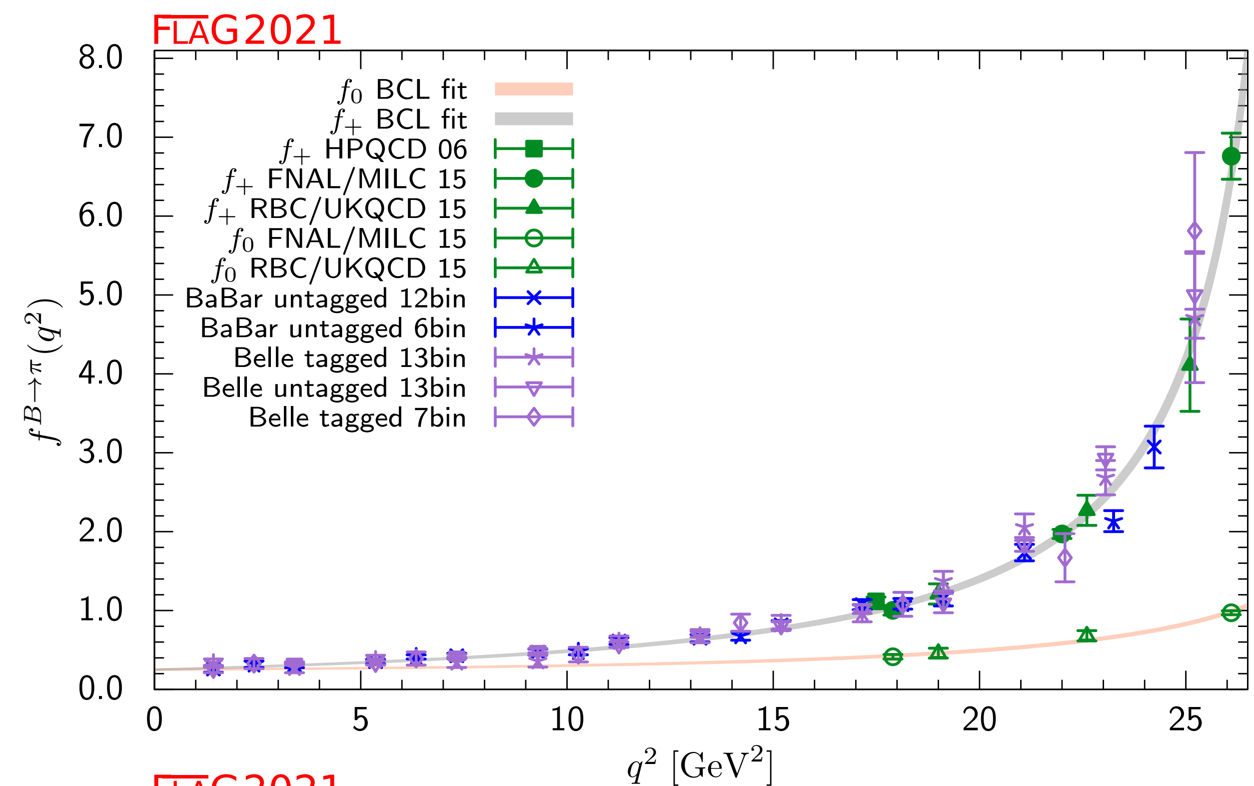


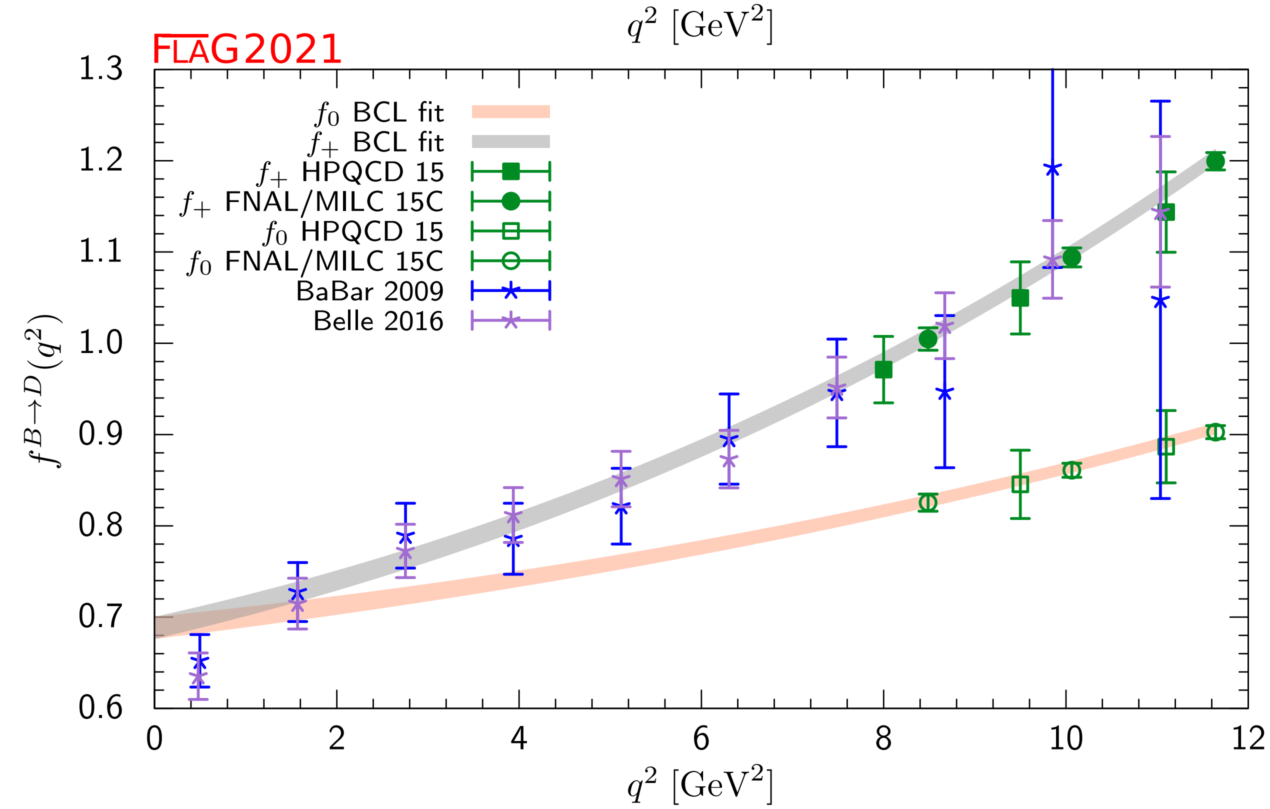
- $|V_{cd}|$ & $|V_{cs}|$ to ~1%.
- Same approach for $|V_{cb}|$ & $|V_{ub}|$: (<)1% for $|V_{ub}|$ ($|V_{cb}|$) in ~2 years.



Semileptonic B-Meson Decays

- $B \to \pi l \nu = |V_{ub}|$; $B \to D l \nu = |V_{cb}|$.
- Same approach as previous slide.
- Three collaborations; two sets of ensembles.
- Existing results are, by now, several years old:
    - 2+1+1-flavor results in a couple years [cf. arXiv:2111.05184].
- (click on plots ☞ full set of references)

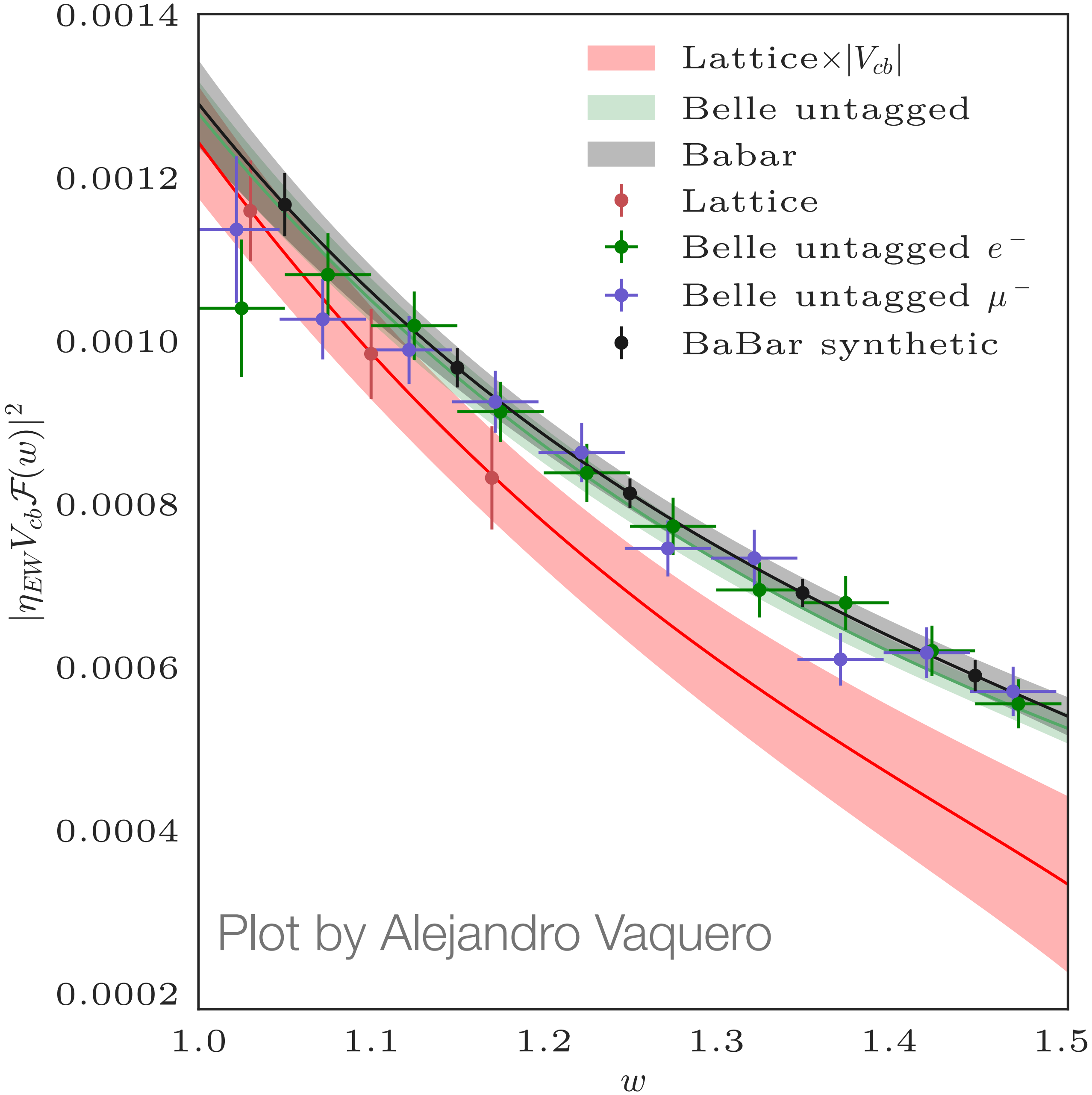


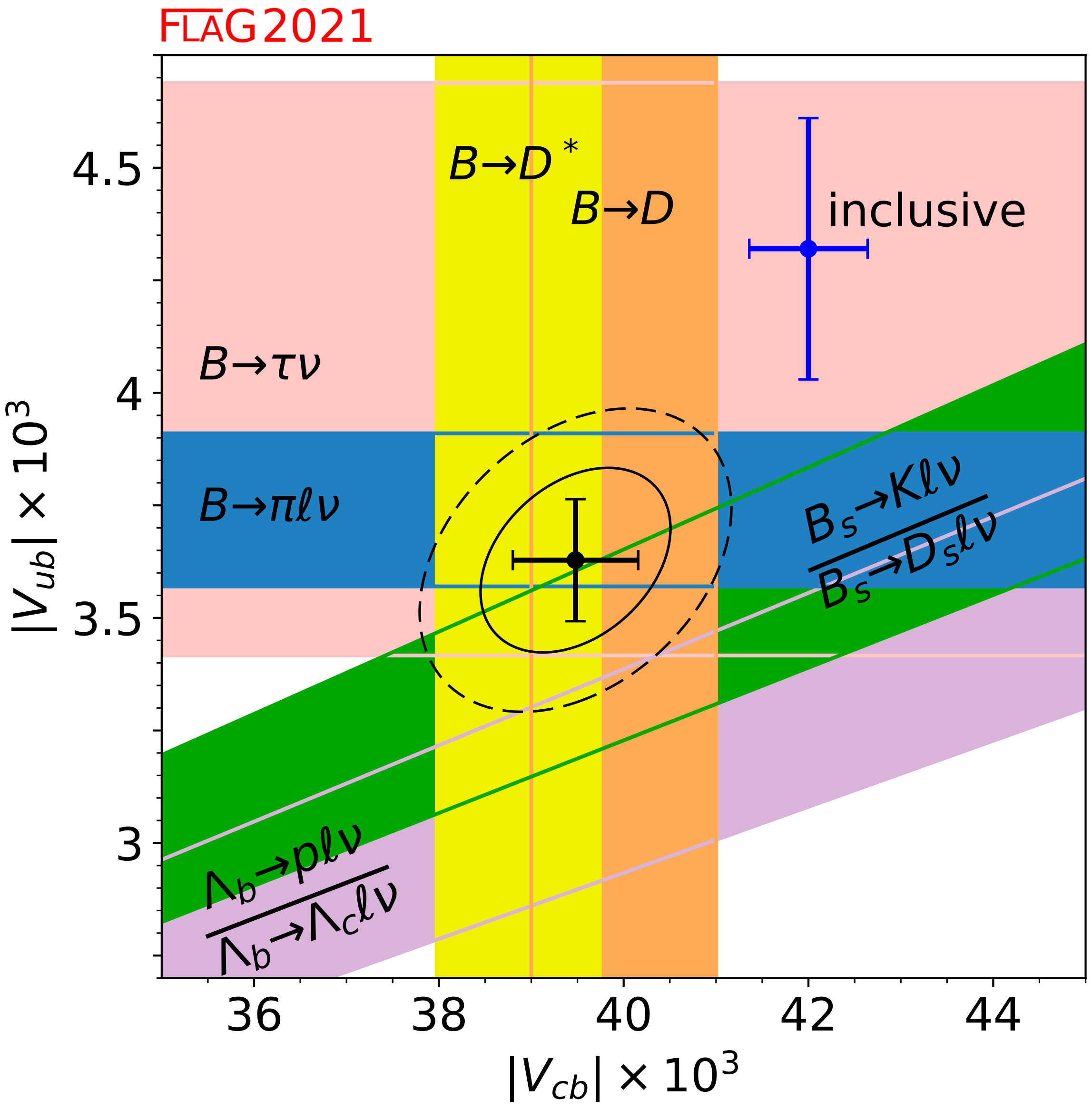


Semileptonic B-Meson Decays II

arXiv:2105.14019

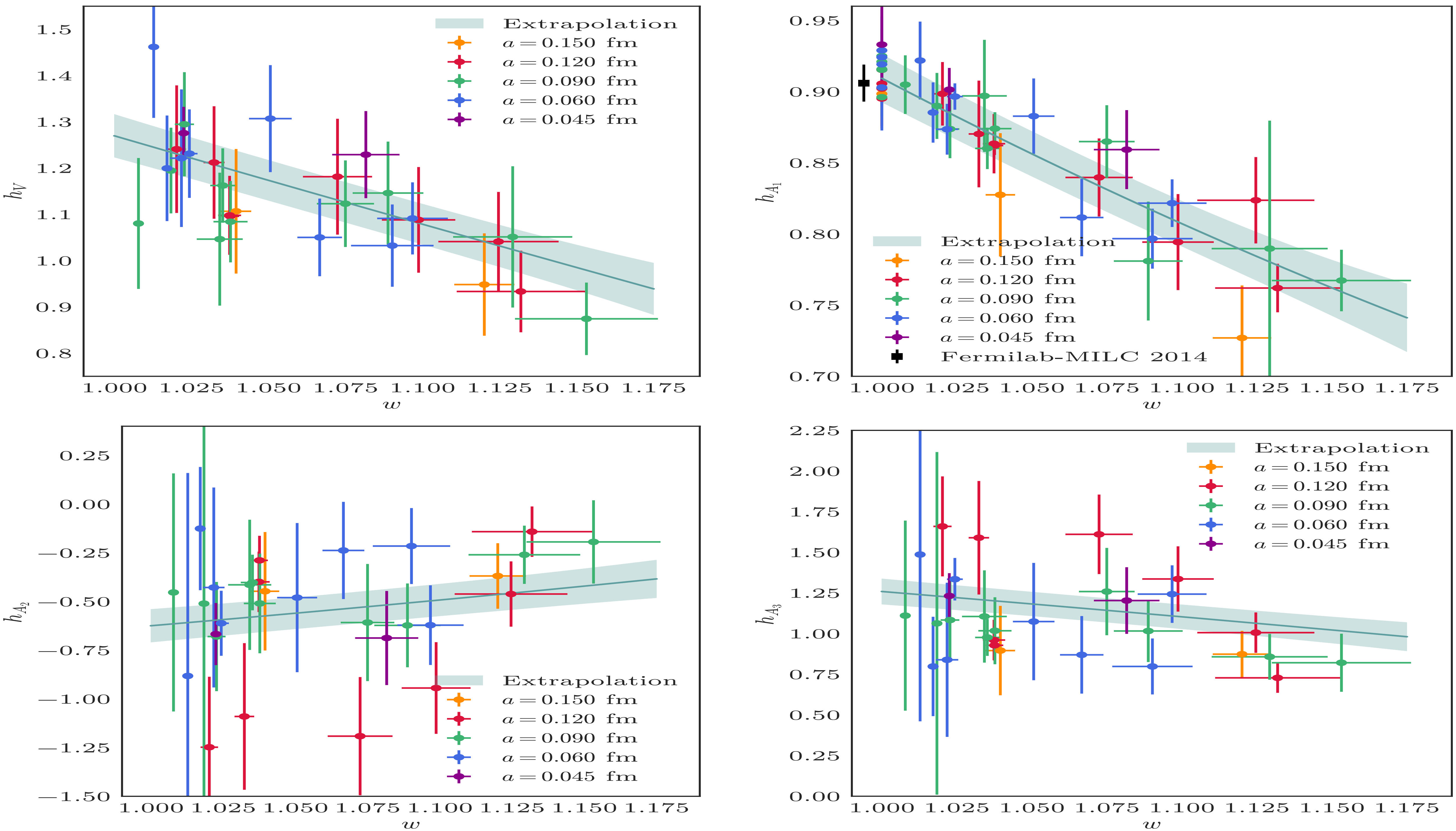
- Result: $10^3|V_{cb}| = 38.40(68_{\text{th}})(0.34_{\text{expt}})(0.18_{\text{EM}})$, still in tension with inclusive.





Form Factors

arXiv:2105.14019



An Error Budget

arXiv:2105.14019

Source	$h_V\,(\%)$	$h_{A_1}\,(\%)$	$h_{A_2}\,(\%)$	$h_{A_3}\,(\%)$
Chiral-continuum fit error	4.2	2.0	17.4	6.9
(Statistics)	(3.7)	(1.2)	(16.9)	(6.3)
(Chiral-continuum extr.)	(0.8)	(0.9)	(1.7)	(0.5)
(LQ and HQ discretization)	(2.6)	(1.3)	(9.7)	(4.4)
(HQ mistuning)	(0.0)	(0.0)	(1.7)	(0.0)
(Matching $O(am_c\alpha_s)$)	(0.3)	(0.2)	(1.7)	(0.5)
LQ mistuning	0.0	0.0	0.1	0.0
Matching $O(\alpha_s^2)$	0.7	0.3	0.5	0.3
Scale setting	0.0	0.0	0.3	0.1
Isospin effects	0.1	0.1	0.4	0.2
Finite volume	—	—	—	—
Total error	4.3	2.0	17.4	6.9

Flavor Anomalies

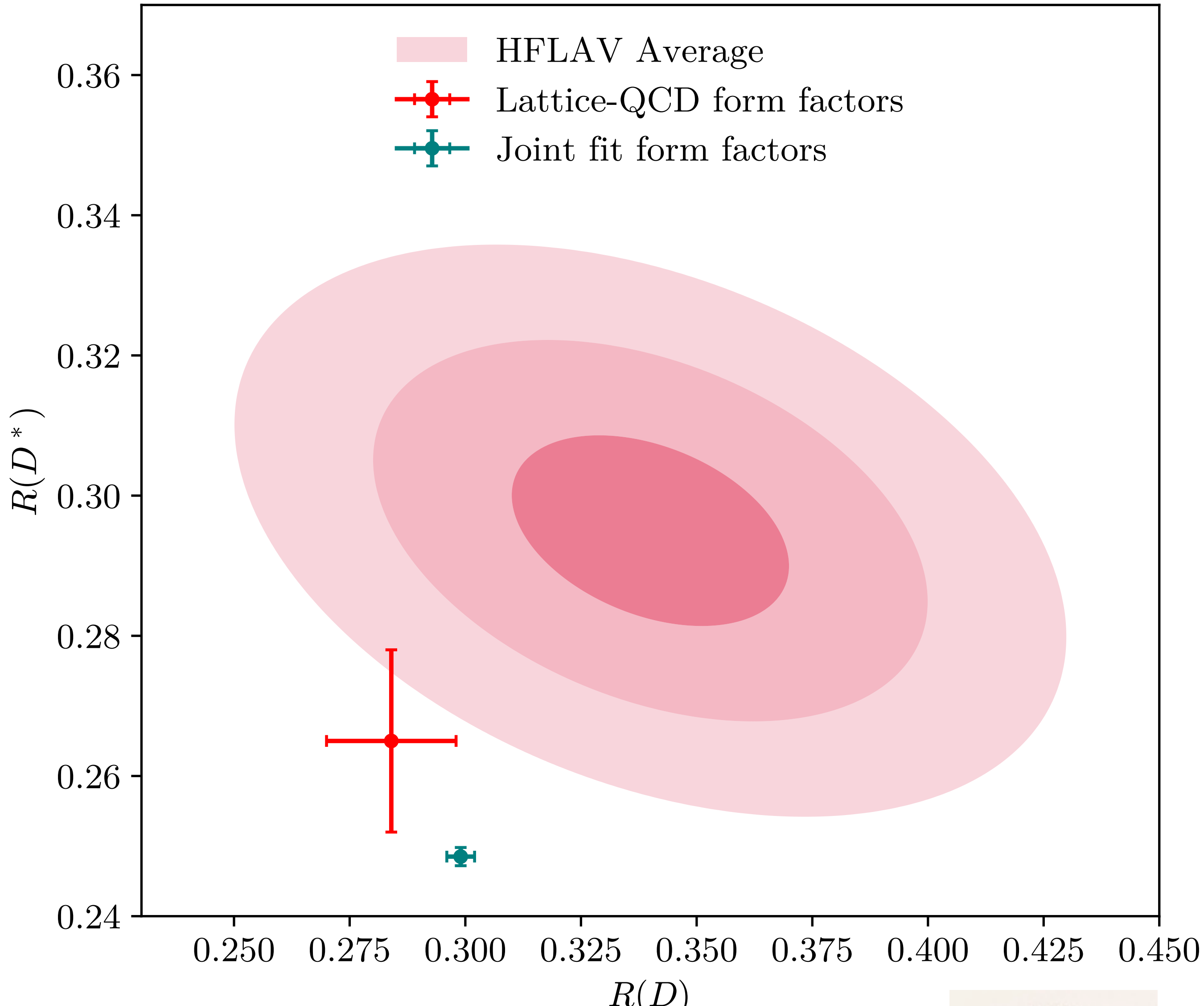
arXiv:2105.14019

- Lepton-flavor universality violation:

$$R(D^{(*)}) = \frac{\mathscr{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathscr{B}(B \to D^{(*)}\ell\nu_\ell)}$$

$$\ell = e, \mu$$

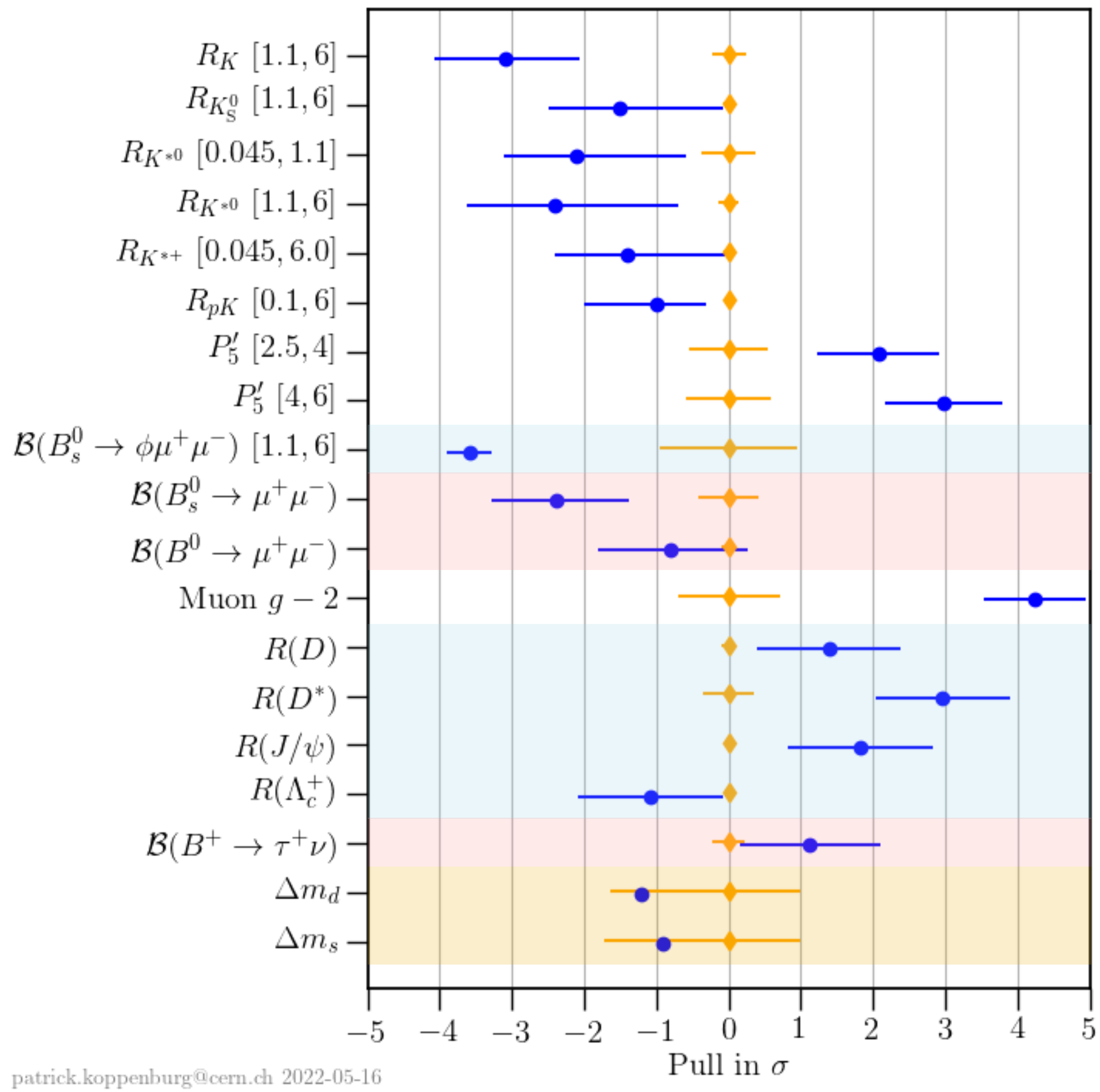
- Lattice-QCD only form factors +: errors comparable to data.
- Form factors from $|V_{cb}|$ fit +: errors negligible—
  - ~4σ discrepancy.



- Plot by Alejandro Vaquero.

Flavor Anomalies—Summary

- Many rely on QCD input:
  - decay constants;
  - form factors;
  - four-quark operators.
- (Angular observables and LFUV profit from, but don't rely on, form factors.)
- Plot by Patrick Koppenburg (LHCb).



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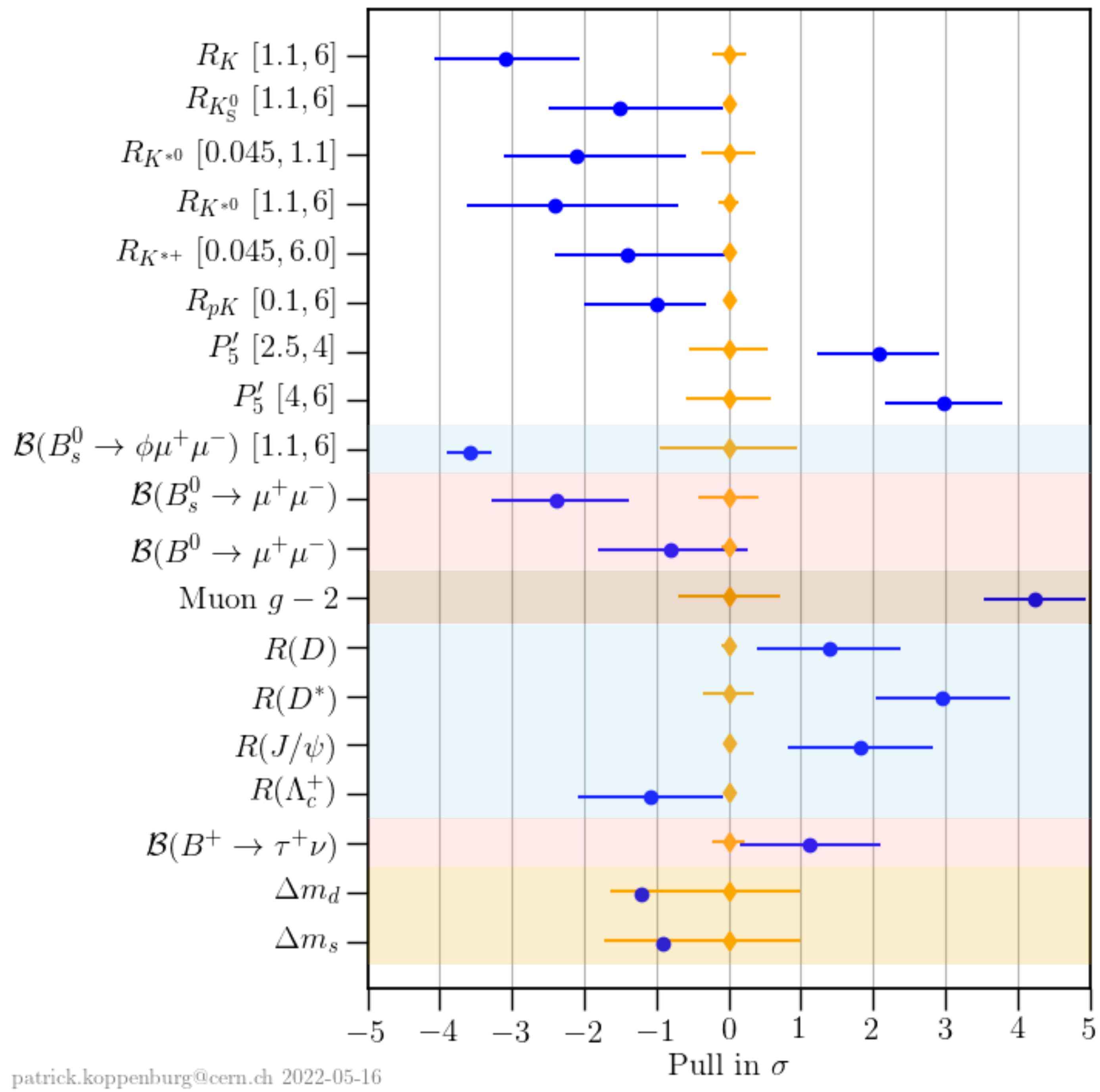
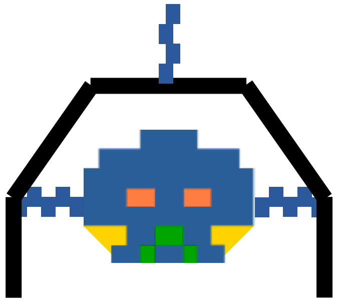


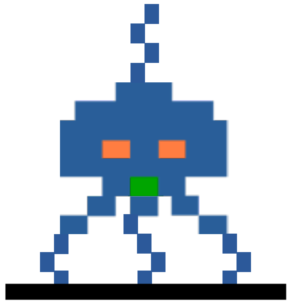
Muon $g-2$

Muon g–2: Hadronic Contributions

- Putative 4.2σ discrepancy between experiment and SM (see below).
- SM uncertainty mostly from two virtual hadronic processes:
  - hadronic vacuum polaration (HVP)—
    - 6845(40) = 6931(40) – 98.3(7) + 12.4(1);



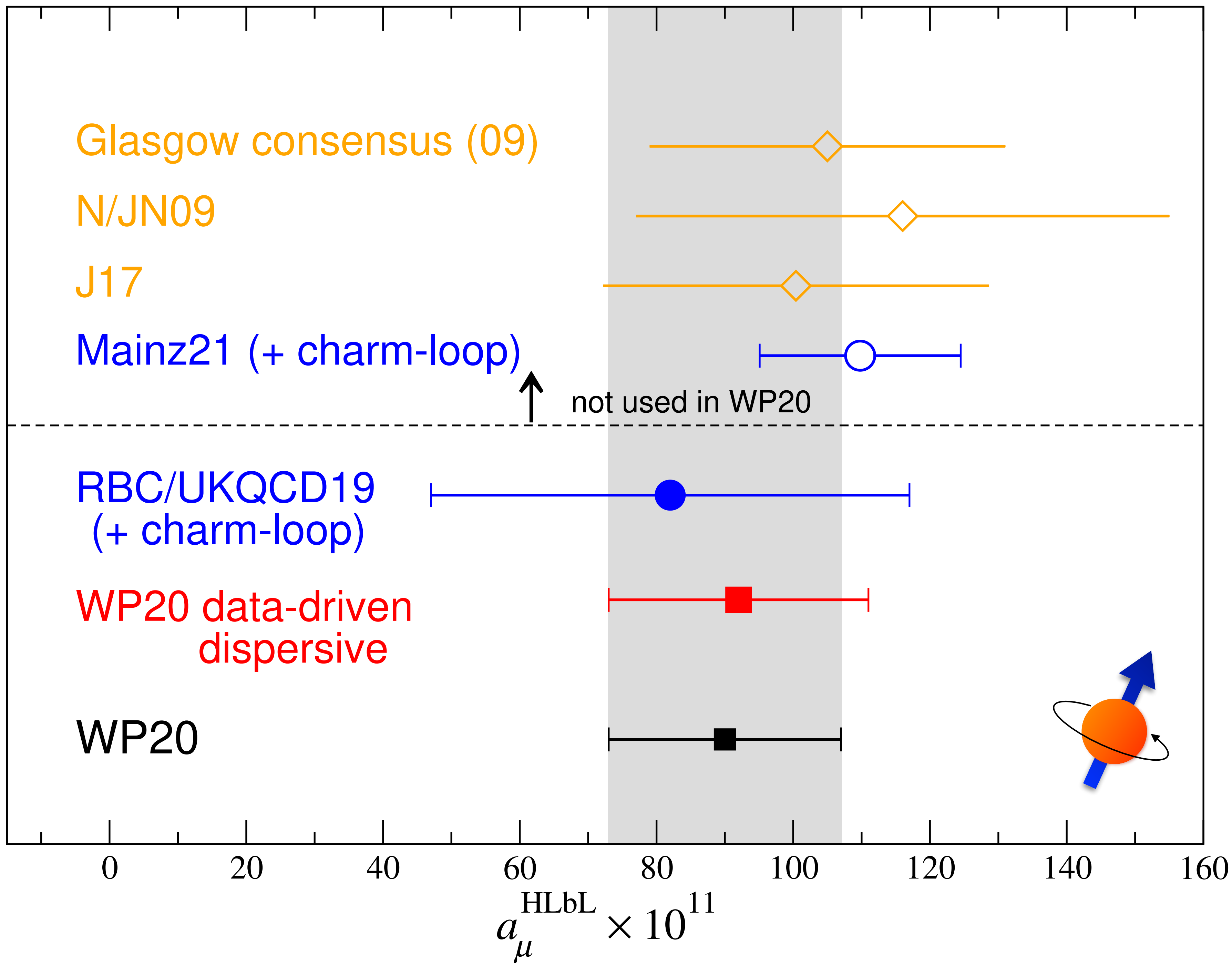
  - hadronic light-by-light scattering (HLbL)—
    - 92(18) = 90(17) + 2(1).



- Two methods: experimental data + dispersion theory OR lattice QCD:
  - Can we get from 6931(40) [actually 7075(55)] to 7000(13)?

Hadronic Light-by-Light

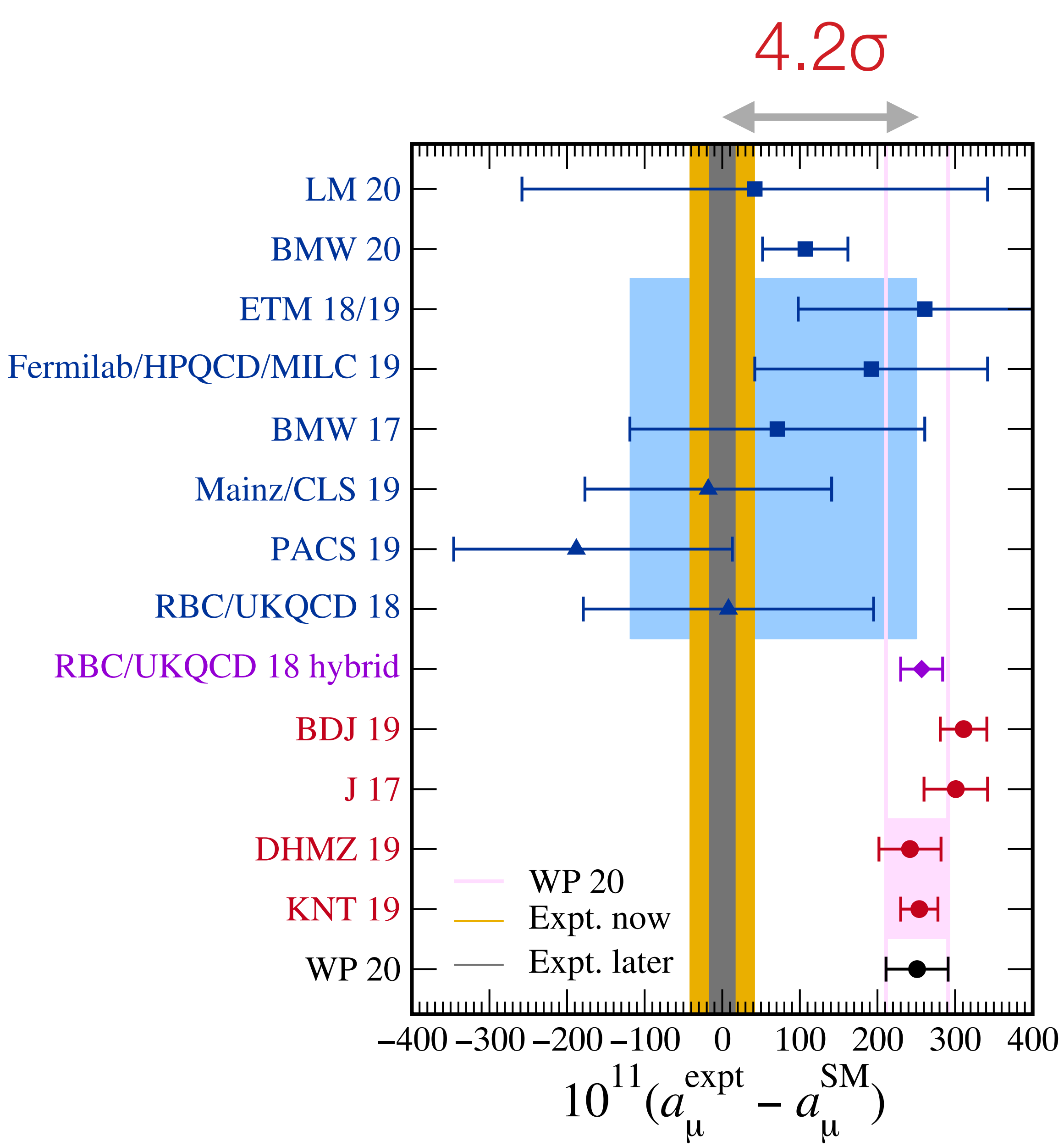
- Lattice QCD and dispersive method both support model estimates.
- Agreement is too good and error is too small for HLbL to explain the discrepancy.
- Plot from Mg–2TI SnowWP.



Hadronic Vacuum Polarization

- Discrepancy: HVP vs. BSM?!?

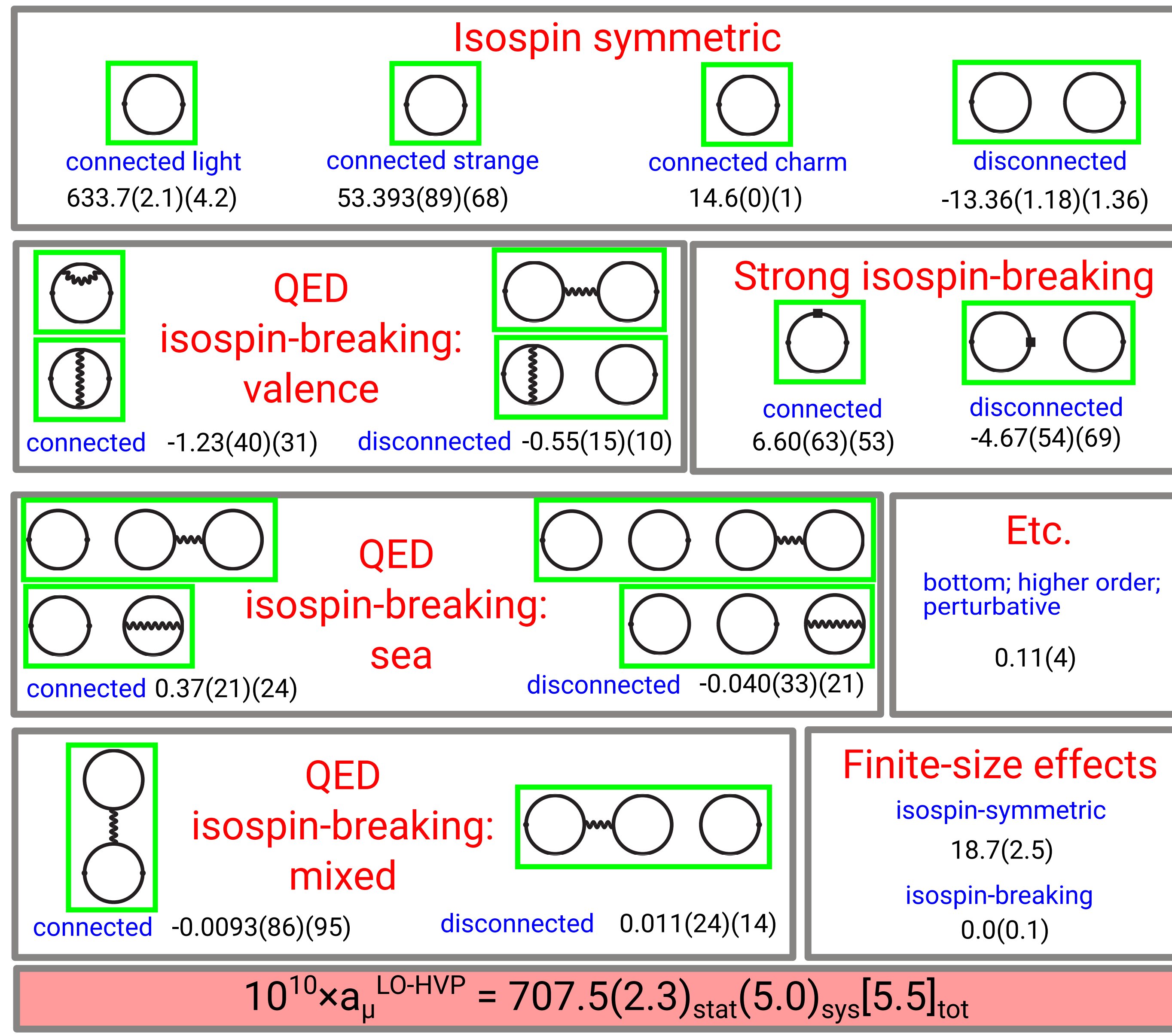
[Mg–2TI, SnowWP]



	$10^{11}[a_\mu = (g{-}2)_\mu/2]$	uncertainty now	uncertainty target
QED	116 584 718 .93(10)		
EW	153 .6(1.0)		
HVP LO+NLO+NNLO	6845 (40)		(13)
HLbL LO+NLO	95 (18)		(9)
SM total	116 591 810 (43)		(16)
BNL 821 ⊕ FNAL 989	116 592 061 (41)		(16)
Expt – SM	251 (59)		(22)

Many Contributions

- *u*, *d*, *s* connected dominates.
- Also need conversion from QCD units to muon units: m_μ/M_Ω.
- Need independent lattice-QCD calculations:
  - Fermilab Lattice+MILC+HPQCD; Aubin *et alia*;
  - χQCD; RBC+UKQCD;
  - Mainz/CLS; ETM;
  - BMW.



BMW arXiv:2002.12347

Road to Precision

- Goal is 0.2%, which will be $0.5\%/\sqrt{\text{few}}$.
- Road to precision will involve jumping through windows [arXiv:1801.07224]:

$$a_\mu = a_\mu^{\mathrm{SD}}(t_0,\Delta) + a_\mu^{\mathrm{W}}(t_0,t_1,\Delta) + a_\mu^{\mathrm{LD}}(t_1,\Delta)$$

- TI put forward intermediate window as a useful point of comparison.
- Also amenable to $R(e^+e^-)$ via Laplace transform.
- Very active: arXiv:2111.15329 (ETM), arXiv:2204.01280 (χQCD), arXiv:2206.06582 (Mainz/CLS), arXiv:2206.15084 (ETM), arXiv:2207.04765 (Fermilab/HPQCD/MILC).

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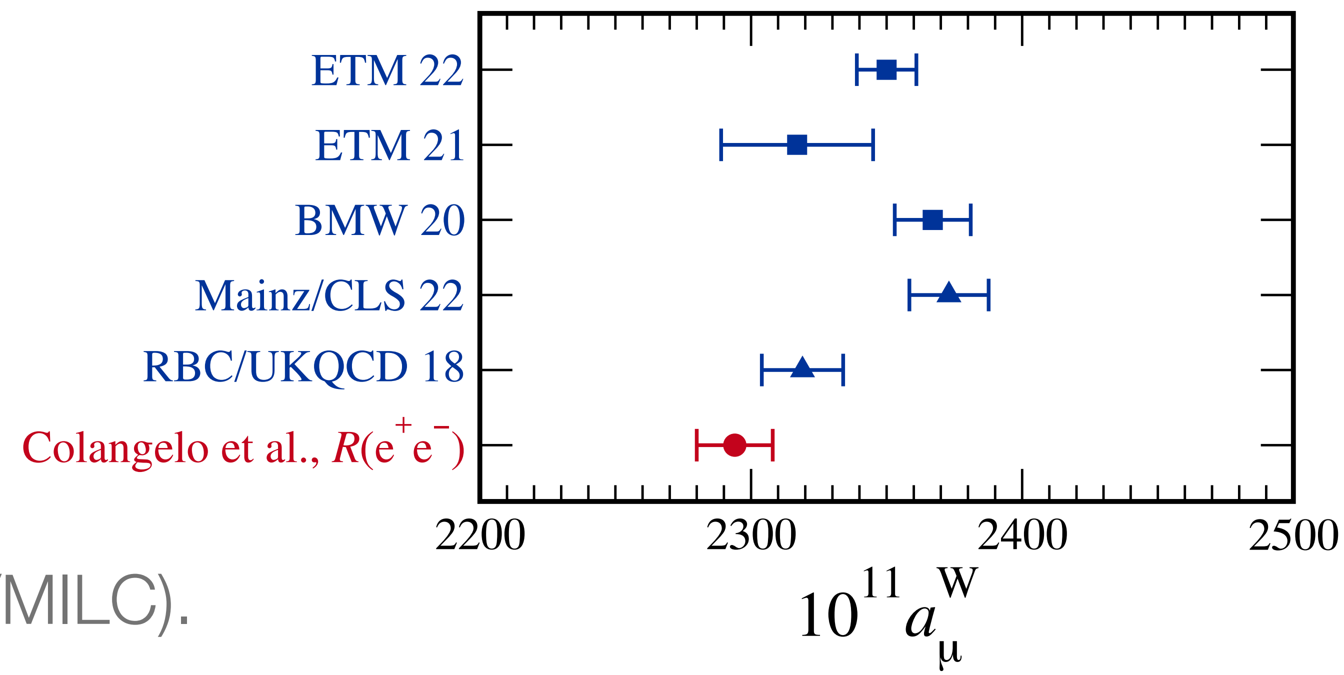

Outlook: Enabling Precision

Enabling Precision Lattice QCD

- The push beyond 1% requires QED ($q_d \neq q_u$) and strong isospin breaking ($m_d \neq m_u$). QED requires theoretical development as well as computational.
- Lattice QCD is (mostly) carried out by collaborations with a mix of skills:
  - computing, phenomenology, theory;
  - on the computing side, generous support (in U.S.) from the Exascale Computing Project (ECP from DOE ASCR) until end 2023;
  - phenomenology and theory need students, postdocs, junior faculty.
- Lattice gauge theory (QCD and BSM) is a tool valuable in a wide variety of topics in particle physics, nuclear physics, and astrophysics.

Thank you for your attention!

LGT @ Snowmass: arXiv:2209.10758
USQCD @ TF: arXiv:2207.07641
USQCD @ CompF: arXiv:2204.00039