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Running quark masses at future e^+e^- colliders

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The running of the Standard Model parameters

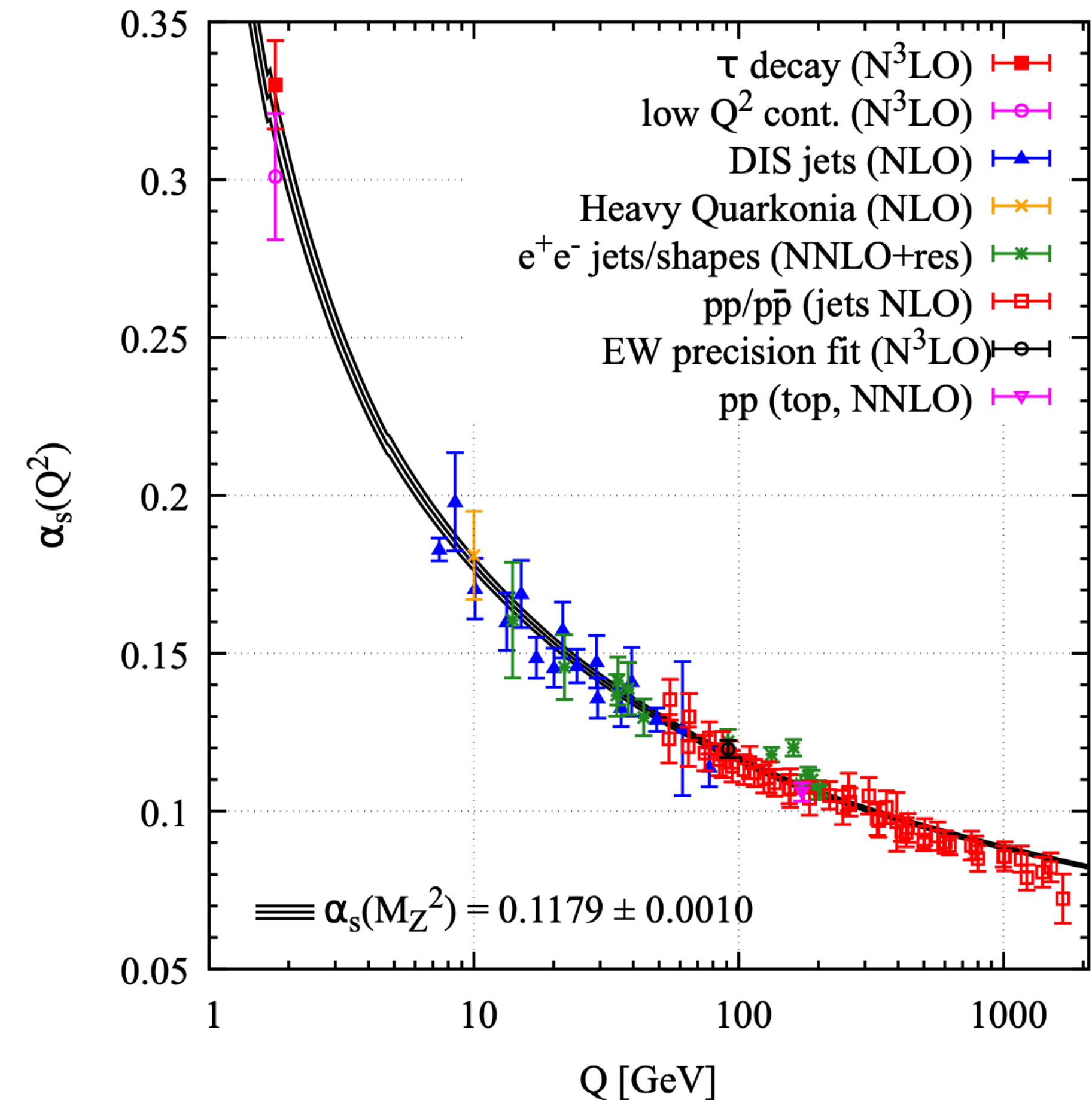
- Standard Model **parameters are subject to renormalization**, so they depend on the renormalization scale μ_R according to the Renormalization Group Equations:

$$\beta(\alpha) = \mu_R^2 \frac{\partial \alpha}{\partial \mu_R^2} = - \sum_{i=0} (\beta_i \alpha^{i+2})$$

$$\gamma(\alpha) = - \mu_R \frac{1}{m} \frac{\partial m}{\partial \mu_R^2} = \sum_{k=0} (\gamma_k \alpha^{k+1})$$

- They must be **determined experimentally**, and the observed value is tied to the characteristic energy of the process it is involved with
- Calculations at 5-loop precision enable a **precise determination** of the **evolution of the strong coupling**

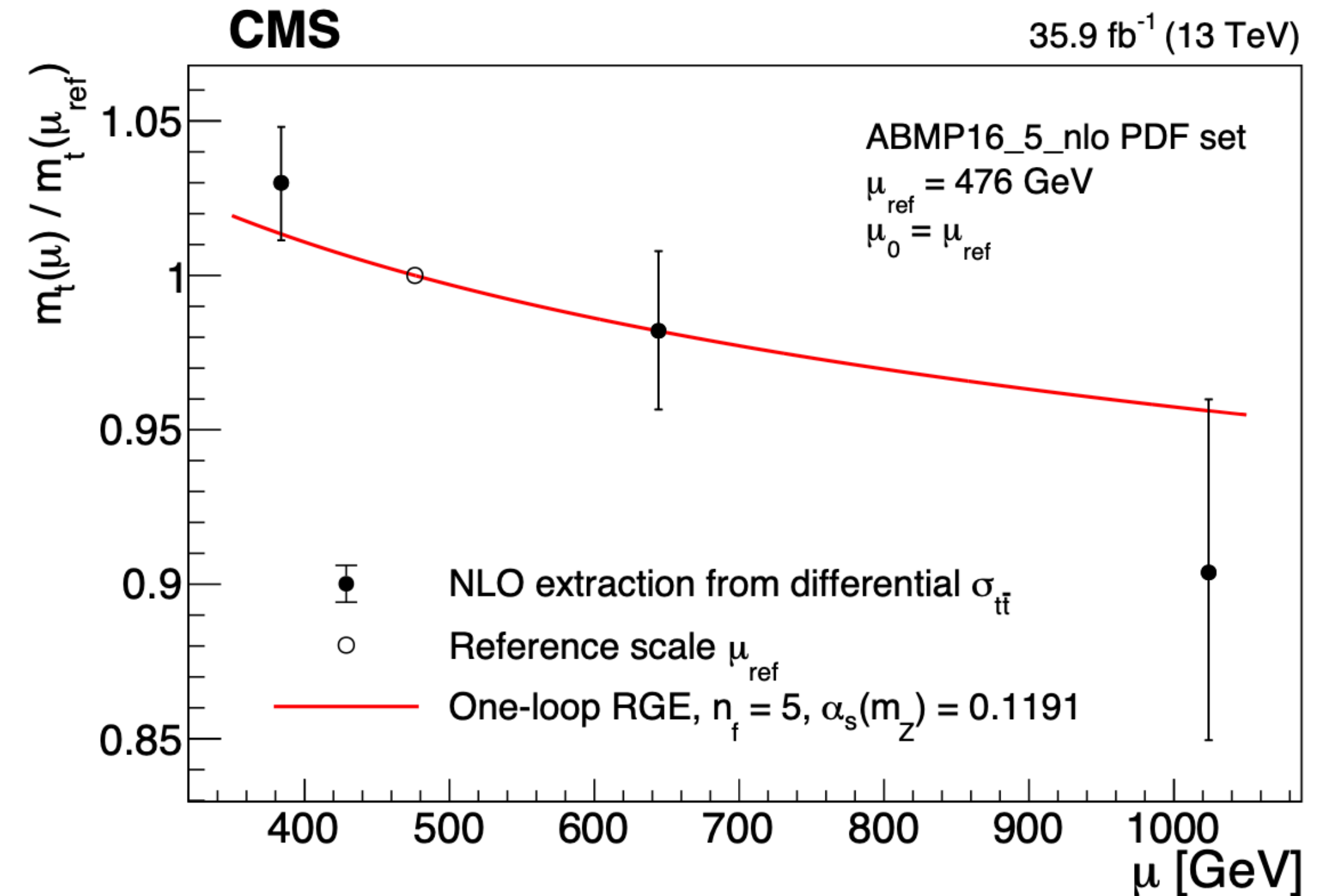
Figure from [Particle Data Group](#)



Quark masses running: the top quark

<https://doi.org/10.1016/j.physletb.2020.135263>

- **First experimental investigation of the running of the top \overline{MS} mass** with pp collision data at $\sqrt{s} = 13 \text{ TeV}$, collected by CMS experiment at the LHC
- Running extracted from measurements of the **differential $\sigma_{t\bar{t}}$ as a function of $m_{t\bar{t}}$**
- NLO QCD calculations to simultaneously **extract $m_t(\mu)$ at each bin at parton level**
- The running hypothesis is tested **considering $\mu = m_{t\bar{t}}$**



➔ Top mass running in agreement with SM RGE within 1.1σ

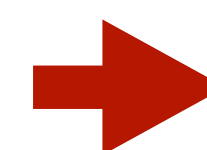
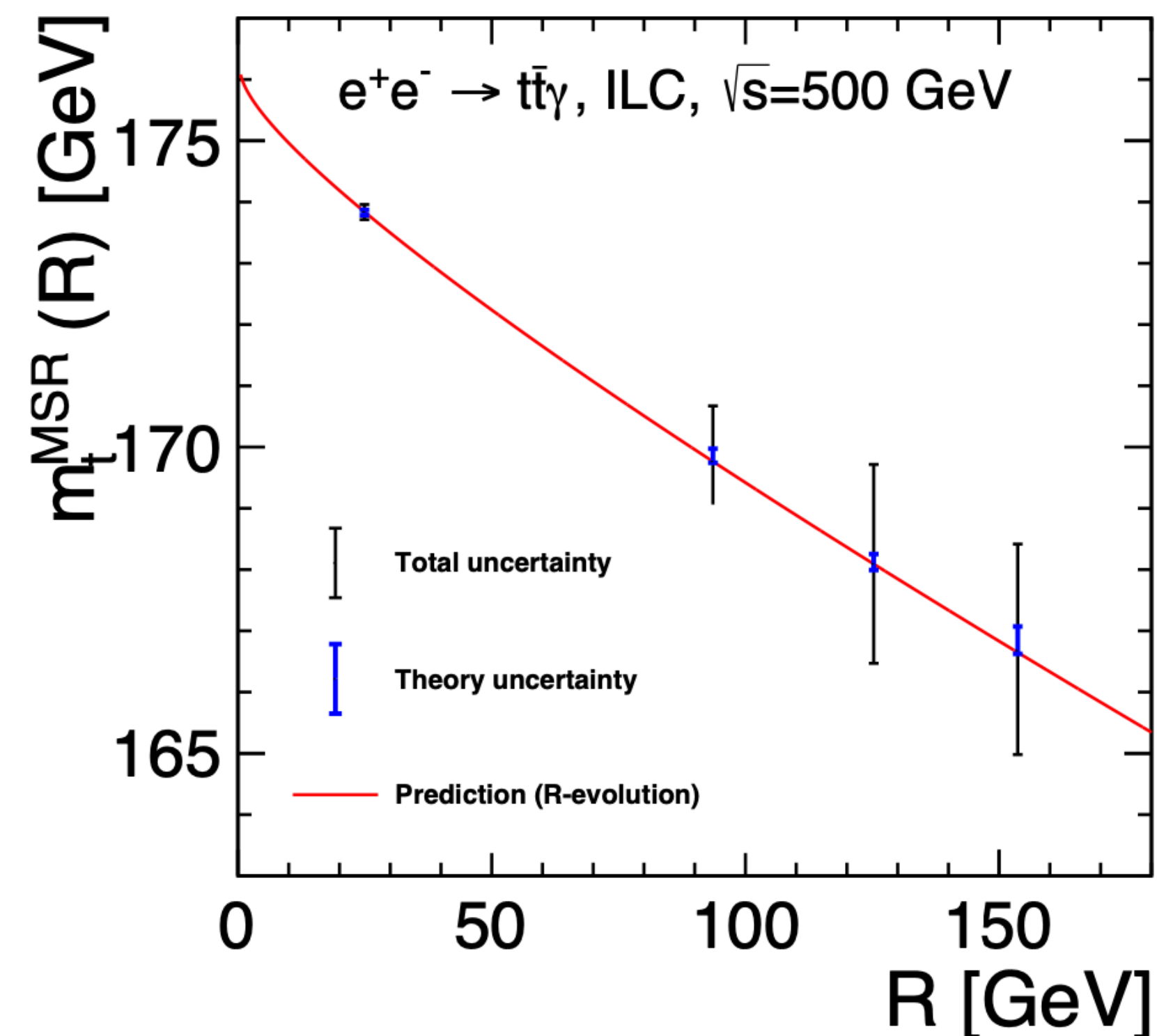
➔ No-running scenario ruled out at above $\% 95 \text{ CL}$

Top quark mass running in e^+e^- colliders

[arXiv:1912.01275](https://arxiv.org/abs/1912.01275)

- **Radiative** $e^+e^- \rightarrow t\bar{t} + X + \gamma$ events above the $t\bar{t}$ production threshold $> 2m_t$ allow:
 - **Precise determination** of m_t in the MSR (R) mass scheme (with $R < m_t$) with N^3LO QCD + N^2LL prediction
 - Measurements of m_t at **different renormalization scales**
- Study performed in two operating and realistic scenarios: **ILC and CLIC**

cms energy	CLIC, $\sqrt{s} = 380$ GeV		ILC, $\sqrt{s} = 500$ GeV	
luminosity [fb^{-1}]	500	1000	500	4000
statistical	140 MeV	90 MeV	350 MeV	110 MeV
theory	46 MeV		55 MeV	
lum. spectrum	20 MeV		20 MeV	
photon response	16 MeV		85 MeV	
total	150 MeV	110 MeV	360 MeV	150 MeV



R-evolution of top quark mass demonstrated for $R < m_t$ in the 500 GeV ILC program

Bottom quark mass measurements

- Running of the bottom quark mass also studied from low-energy measurements

Bottom mass scale

- ▶ Bottom quark mass production threshold
- ▶ Most precise determinations from mass of bottomonium bound states in $e^+e^- \rightarrow$ hadrons
- ▶ World combination from PDG in \overline{MS} scheme:

$$m_b(m_b) = 4.18^{+0.03}_{-0.02}$$

Z mass scale

- ▶ Hadronic decays of Z bosons
- ▶ Event-shapes and jet-rates sensitive to subleading mass effects
- ▶ A combination of the most precise measurements in \overline{MS} scheme yields:

$$m_b(m_Z) = 2.82 \pm 0.28 \text{ GeV}$$

m_b from Higgs boson decay rates (I)

Higgs discovery and observation of its decay into bottom quarks enabled a new measurement at the Large Hadron Collider

- ❖ We profit from the **ratio of branching ratios**:

$$\frac{\mathcal{B}(H \rightarrow b\bar{b})}{\mathcal{B}(H \rightarrow ZZ)} = \frac{\Gamma(H \rightarrow b\bar{b})}{\Gamma(H \rightarrow ZZ)}$$

- ❖ Dependence of $\Gamma(H \rightarrow b\bar{b})$ with m_b obtained **assuming SM Yukawa couplings** and $m_b \ll m_H$:

$$\Gamma(H \rightarrow b\bar{b}) = \frac{3G_F m_H}{4\sqrt{2}\pi} m_b(\mu)^2 (1 + \delta_{ew}) \times \left(1 + \delta_{QCD} + \delta_t + \delta_{mix}\right)$$

Excellent convergence of the perturbative calculation at $\mu_R = m_H$, so **small higher-order corrections** expected

m_b from Higgs boson decay rates (II)

- ❖ Individual **partial widths** $\Gamma(H \rightarrow b\bar{b}, ZZ)$ **estimated separately**. The ratio is built and parametrized for several values of $m_b(m_H)$:

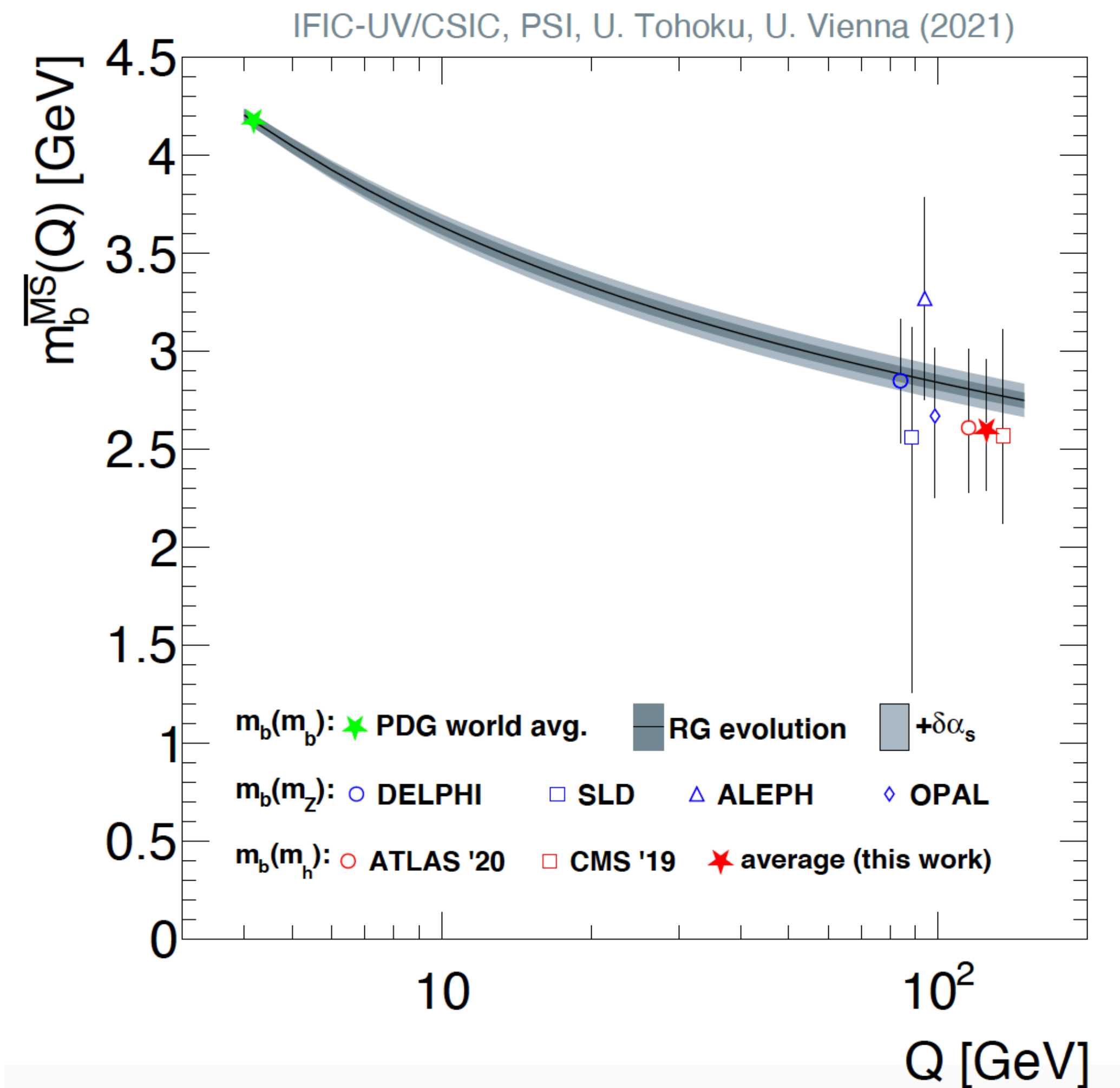
$$\frac{\Gamma(H \rightarrow b\bar{b})}{\Gamma(H \rightarrow ZZ)} = 2.82 \frac{m_b^2}{\text{GeV}^2} - 0.0014 \frac{m_b^4}{\text{GeV}^4} + \mathcal{O}(m_b^6)$$

- ⦿ **ATLAS** and **CMS** measurements at 139 fb^{-1} and 35 fb^{-1} :

$\Gamma^{b\bar{b}} / \Gamma^{ZZ}$	$m_b(m_H)$	
$0.87^{+0.22}_{-0.17} \text{ (stat)}^{+0.18}_{-0.12} \text{ (syst)}$	$2.61^{+0.32}_{-0.27} \text{ (stat)}^{+0.26}_{-0.19} \text{ (syst) GeV}$	}
$0.84^{+0.27}_{-0.21} \text{ (stat)}^{+0.26}_{-0.17} \text{ (syst)}$	$2.57^{+0.39}_{-0.35} \text{ (stat)}^{+0.37}_{-0.28} \text{ (syst) GeV}$	

$m_b(m_H) = 2.60^{+0.36}_{-0.31} \text{ GeV}$

The running of the bottom quark mass



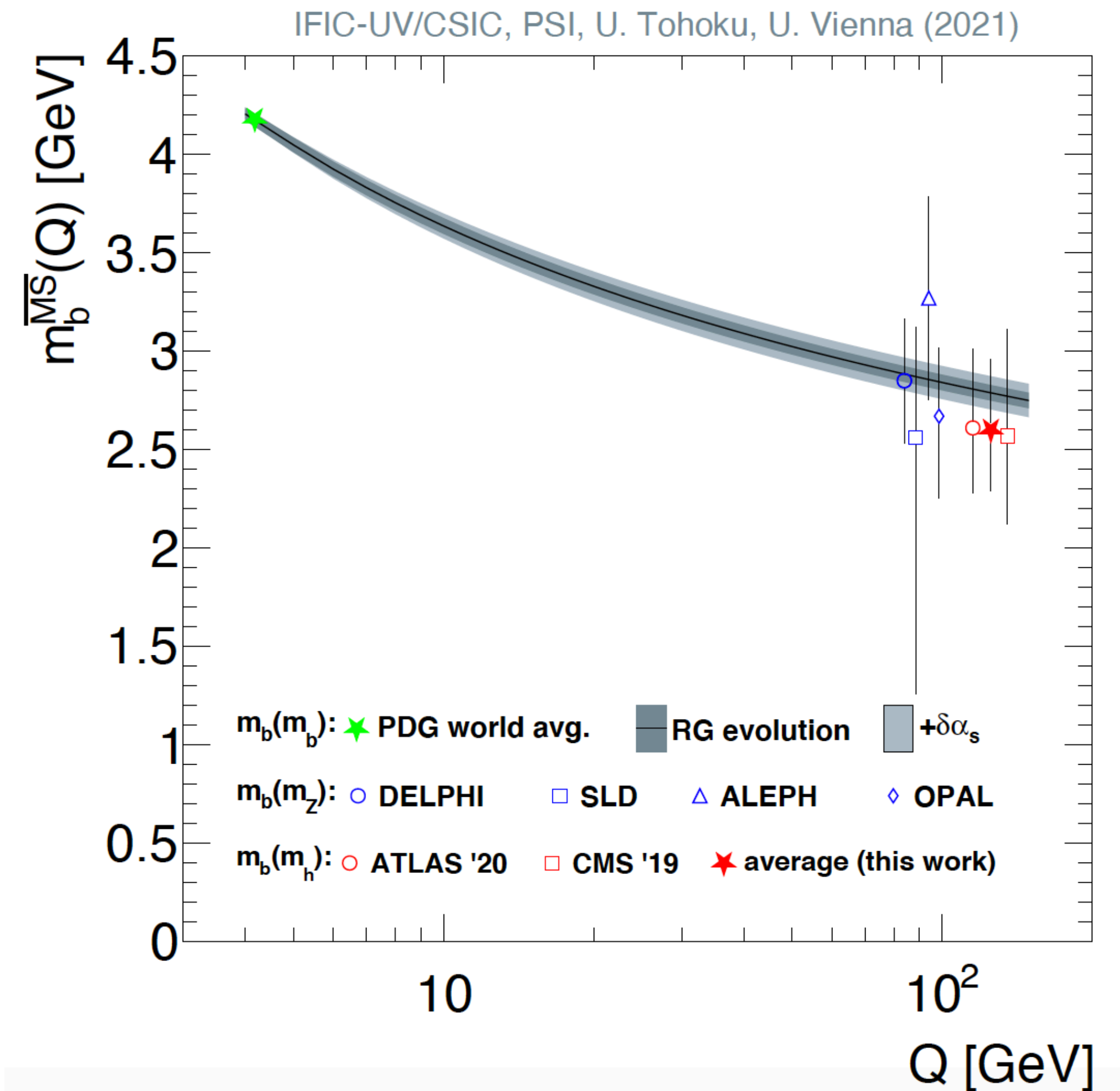
The running of the bottom quark mass

❖ A modified RGE SM evolution through x parameter:

$$m(\mu; x, m_b(m_b)) = x \left[m_b^{RGE}(\mu, m_b(m_b)) - m_b(m_b) \right] + m_b(m_b)$$

$x = 0 \rightarrow$ No-running scenario

$x = 1 \rightarrow$ SM scenario



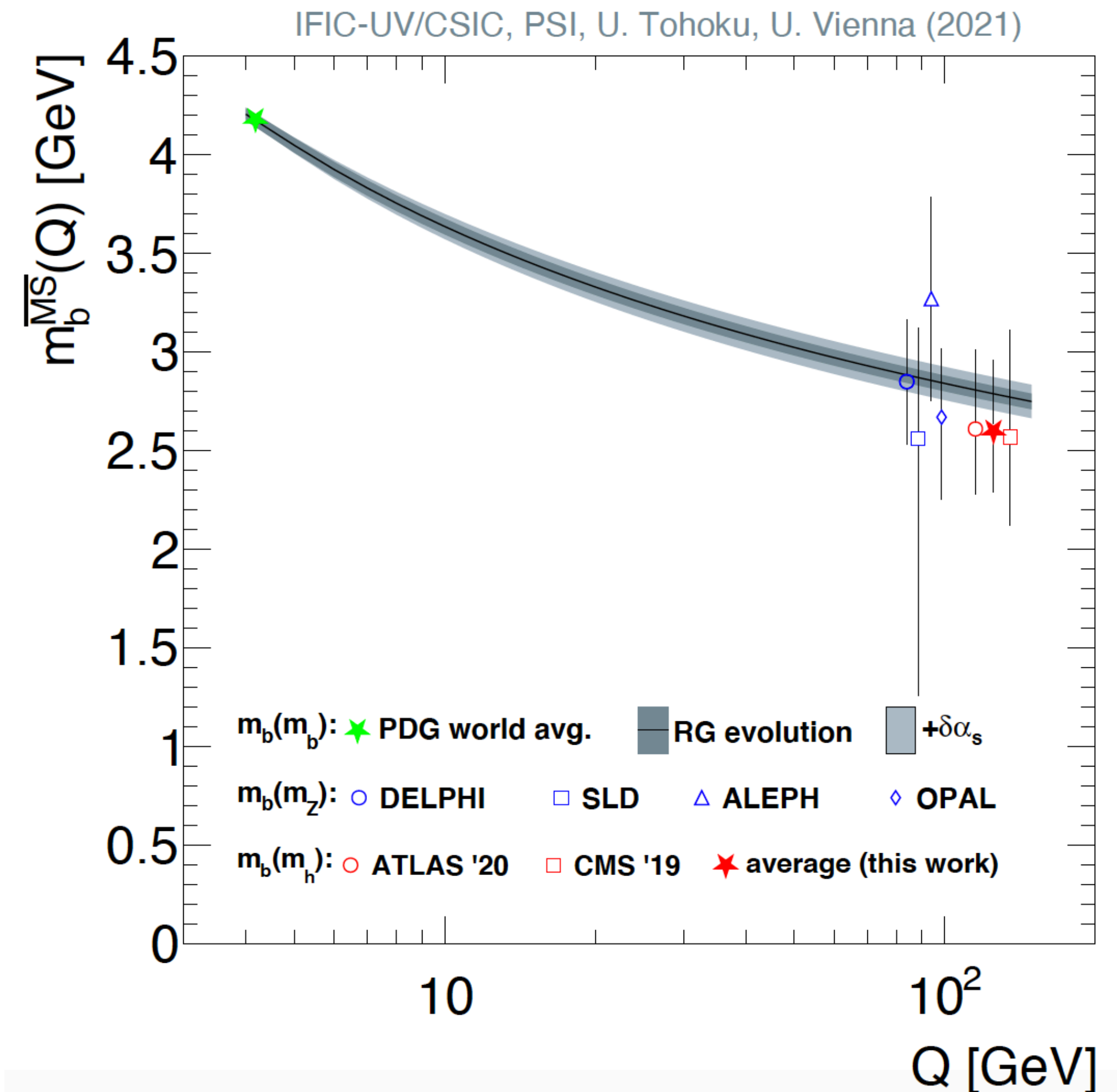
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- ❖ Taking $\mu_0 = m_b$, calculate $m_b(m_Z)$ and $m_b(m_H)$ from different starting values of $m_b(m_b)$ and x at 5-loop precision

$$\chi^2(m_b(m_b), x) = \frac{\sum_{\mu_i} (m_b^{exp}(\mu_i) - m(\mu_i; x, m_b(m_b)))^2}{\sigma_i^2}$$

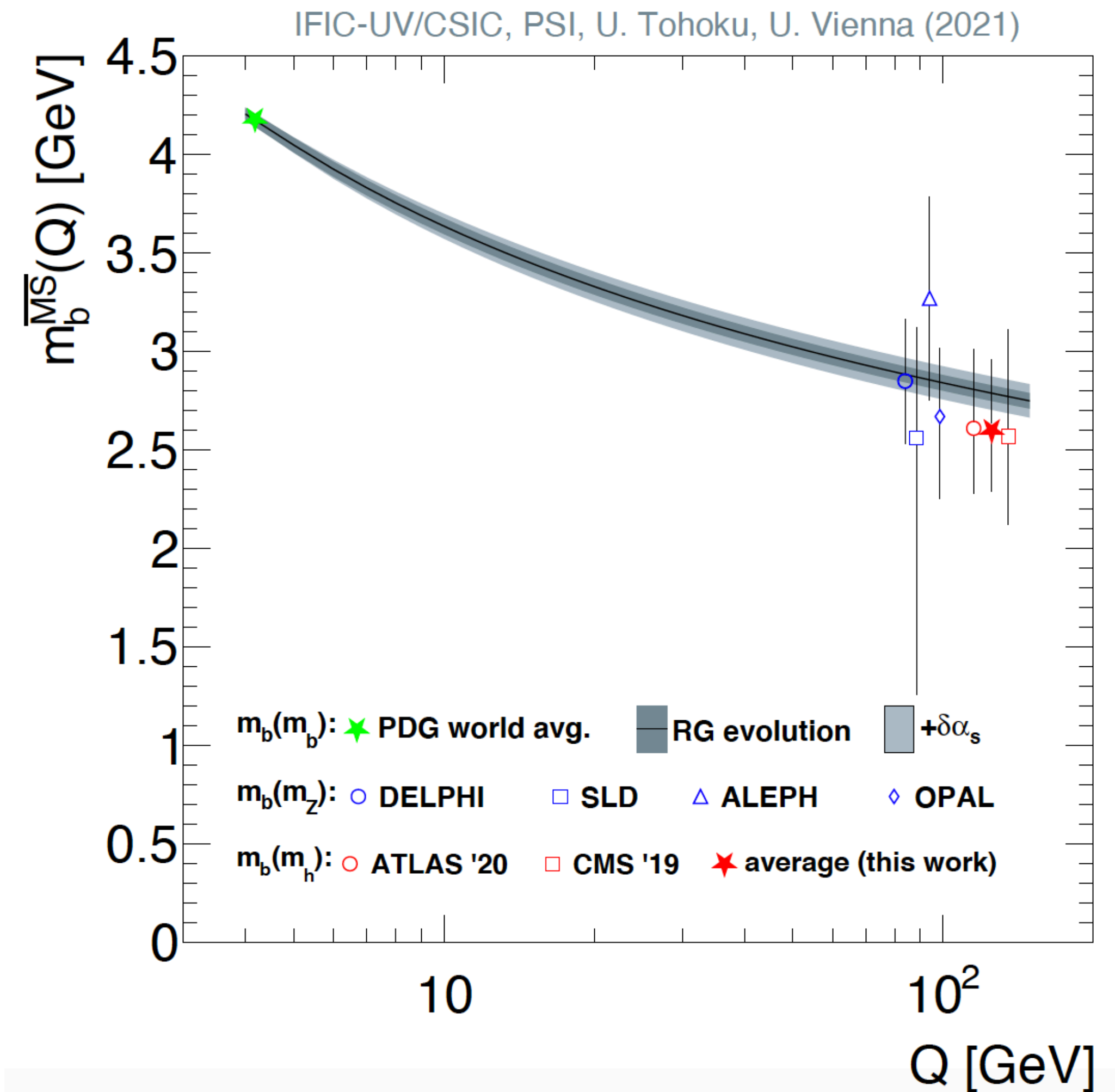
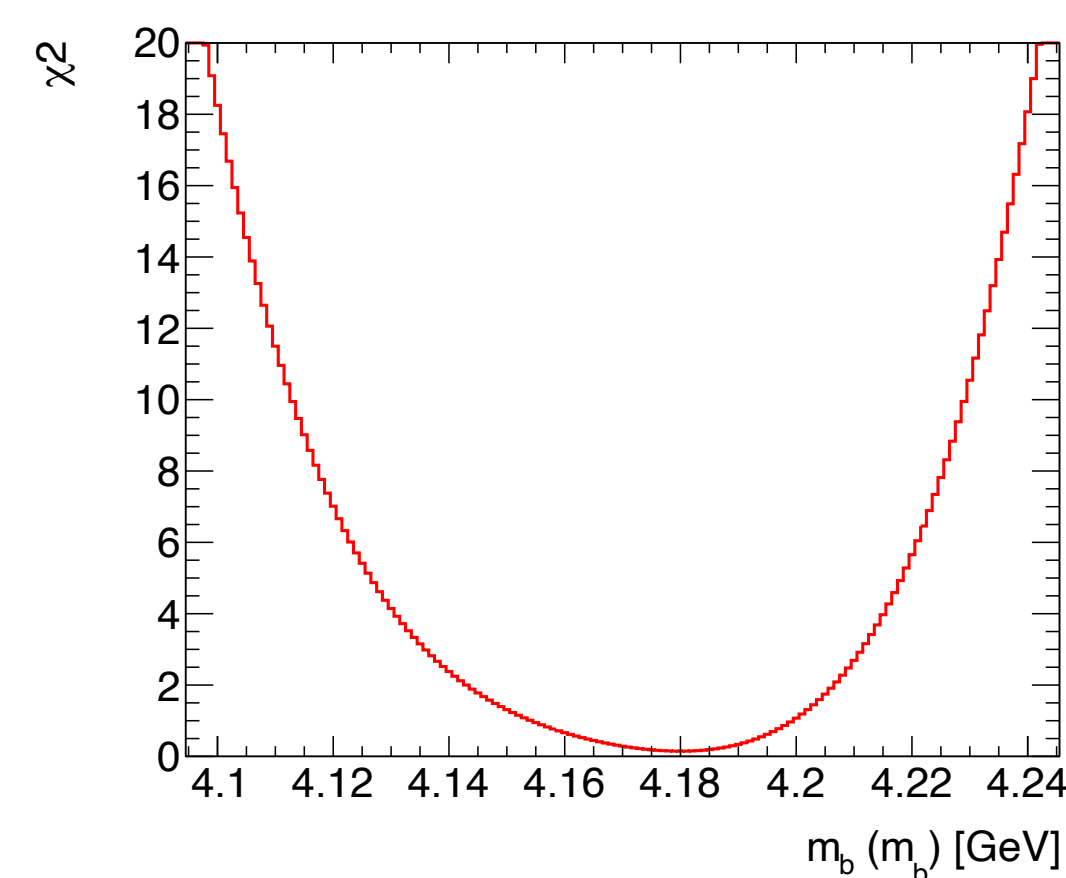
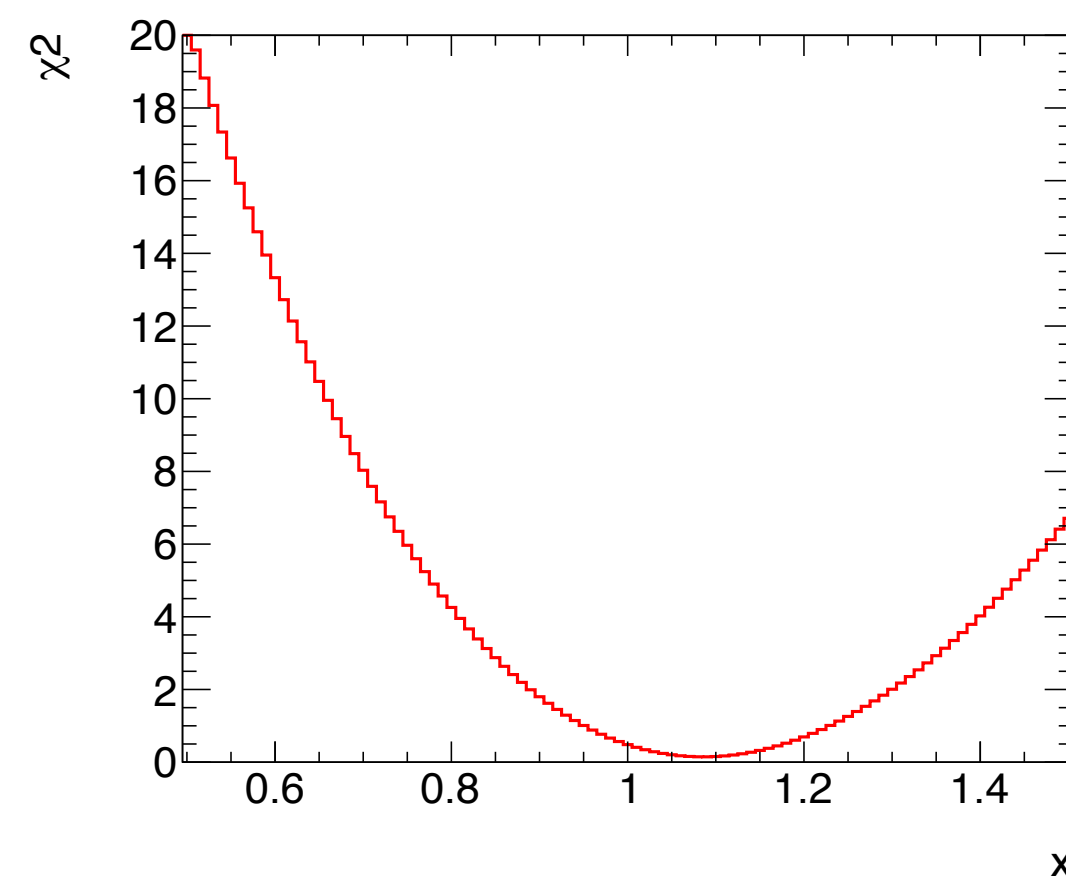


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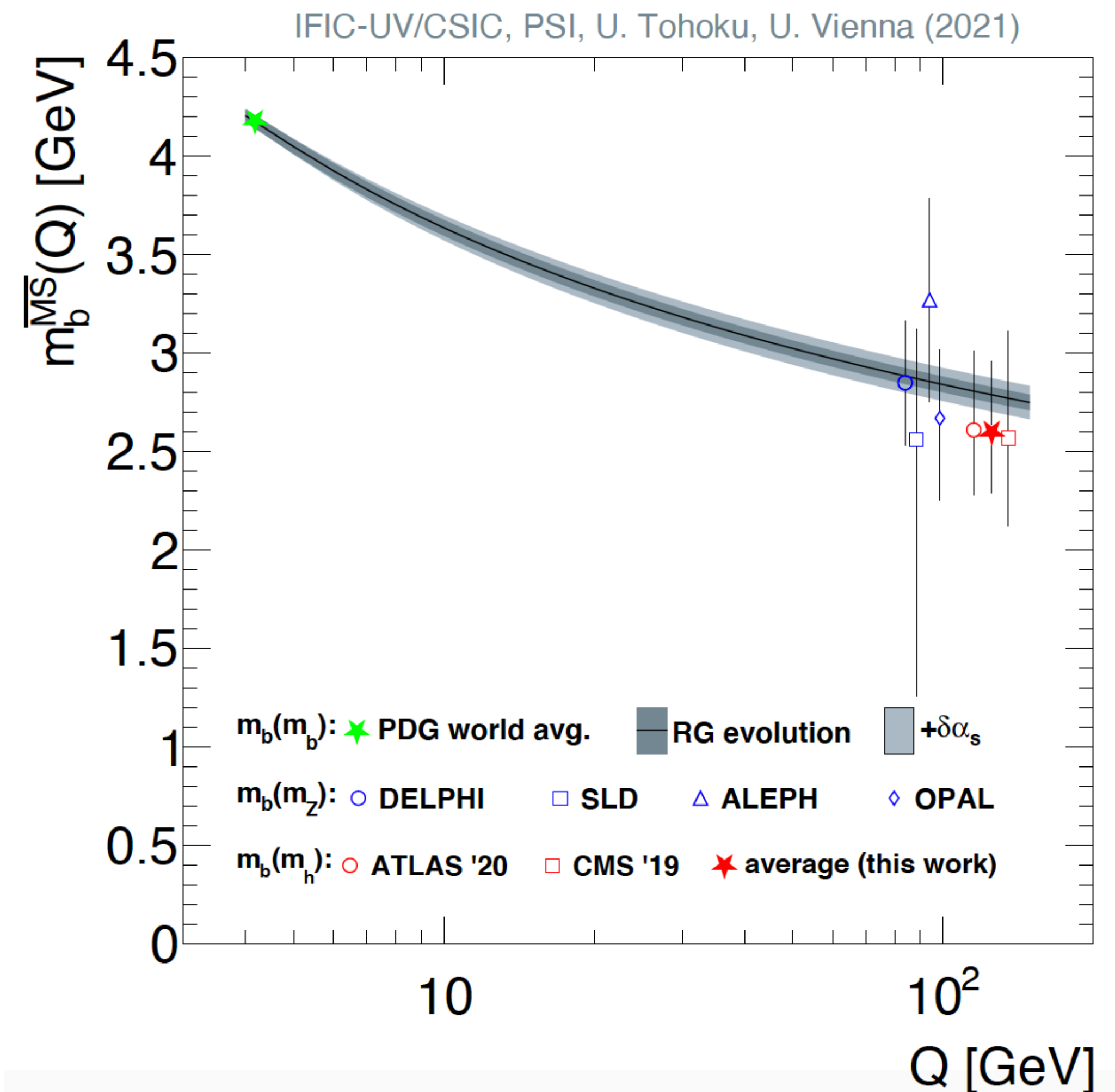
- ❖ Taking $\mu_0 = m_b$, calculate $m_b(m_Z)$ and $m_b(m_H)$ from different starting values of $m_b(m_b)$ and x at 5-loop precision

$$m_b(m_b) = 4.18^{+0.03}_{-0.02} \text{ GeV} ,$$

$$x = 1.08 \pm 0.15 \text{ (exp)} \pm 0.05 \text{ (}\alpha_s\text{)}$$

→ **No-running scenario ruled out with $> 5\sigma$**

→ **SM running confirmed within 1σ**



Prospects in future colliders

Collider	Channel	Expected experimental unc. on channel meas.	Expected experimental unc. on $m_b(m_H)$
HL-LHC	$BR(H \rightarrow b\bar{b})$	4 %	$\pm 63 \text{ MeV}$
ILC:250	$\frac{BR(H \rightarrow b\bar{b})}{BR(H \rightarrow WW)}$	0.86 %	$\pm 12 \text{ MeV}$
ILC:250+500		0.47 %	$\pm 6 \text{ MeV}$

→ Very competitive measurements are possible with this method. The prospects for theory uncertainties need to be carefully assessed.

Summary

Quark masses evolution with the energy scale subject of study at hadron and e^+e^- colliders

- **Running of the top quark** mass inspected at the LHC with NLO QCD precision
- Study from radiative events at e^+e^- **colliders shows a better precision in top quark mass measurement and provides observation of its running**
- **First determination of the bottom quark mass** at the Higgs mass scale at the LHC presented. The **running is observed with $> 5\sigma$** including this measurement to the existing low-energy ones
- Excellent prospects for this method at future e^+e^- colliders: **uncertainty reduced by one order of magnitud in the ILC program**

Bonus slides

$m_b(m_Z)$ combination from LEP and SLC

experiment	$m_b(m_Z)$ [GeV]
ALEPH[14]	3.27 ± 0.22 (stat.) ± 0.44 (syst.) ± 0.16 (theo.)
DELPHI[16]	2.85 ± 0.18 (stat.) ± 0.23 (syst.) ± 0.12 (theo.)
OPAL[15]	2.67 ± 0.03 (stat.) $^{+0.29}_{-0.37}$ (syst.) ± 0.19 (theo.)
SLD[12, 13]	2.56 ± 0.27 (stat.) $^{+0.28}_{-0.38}$ (syst.) $^{+0.49}_{-1.48}$ (theo.)