Three-dimensional betatron oscillation and radiation reaction in plasma accelerators

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Introduction

Plasma-based accelerator offers high acceleration gradient and strong transverse focusing electromagnetic field to accelerate electrons. The electrons in the wake emit axial synchrotron radiation because of betatron oscillation (BO), which leads to the energy loss of electrons and in return affects beam properties [4]. Such effect is called the radiation reaction (RR). To study this process, we reestablish the BO model with RR [2, 3, 4, 5] by a considering the oscillations in two transverse and one longitudinal directions. In our three-dimensional model, long-term equations (LTE), with information in the betatron time scale being averaged out, are obtained and compared with the original equations by numerical methods and the particle tracking code PTracker (PT)[6]. Use our model, the energy evolution, the BO amplitude, as well as the BO phase shift, with arbitrary initial polarization, can be predicted without resolving the BO period. This work can be of great value for future plasma based high-energy accelerators and colliders.

Theoretical model and long-term equations

We use the plasma normalization units. The classical expression of radiation reaction (RR) is the Lorentz–Abraham–Dirac equation [1],

$$F_\mu^{\mathrm{rad}} = \frac{2}{3} r_e \left[\frac{d^2 P_\mu}{d\tau^2} + \left(\frac{dP_\nu}{d\tau} \frac{dP^\nu}{d\tau} \right) P_\mu \right].$$

Neglect the interaction between the beam particles, the electromagnetic field provided by the wake can be modeled as

$$\begin{aligned} E_z &= E_{z0} + \lambda \zeta_1, \\ \vec{E}_\perp &= \kappa^2 (1-\lambda) \vec{r}, \\ B_\theta &= -\kappa^2 \lambda r. \end{aligned}$$

Then, from the Lorentz-Newton equation, we derive the long-term equations (LTE) about the relativistic factor of electron γ, the amplitude of BO, phase and angular momentum

$$\begin{aligned} \langle \dot{\gamma} \rangle &= -E_{z0}\beta_{z0} - \frac{1}{3} r_e \kappa^3 \langle \gamma \rangle^{\frac{3}{2}} (S_x + S_y), \\ \langle \dot{\zeta} \rangle &= \frac{1}{2}\gamma_w^{-2} - \frac{1}{2} \langle \gamma \rangle^{-2} - \frac{1}{4}\kappa \langle \gamma \rangle^{-\frac{3}{2}} (S_x + S_y), \\ \frac{dS_x}{dt} &= -\frac{1}{4} r_e \kappa^3 \langle \gamma \rangle^{\frac{1}{2}} \left(S_x^2 + \frac{4-\cos 2\Delta\Phi}{3} S_x S_y \right) - \frac{1}{8}\left[\frac{1}{4}\lambda\beta_{z0} - \kappa^2 (1-2\lambda) \right] \langle \gamma \rangle^{-2} S_x S_y \sin 2\Delta\Phi, \end{aligned}$$

$$\dot{\Phi}_x = \frac{1}{24} r_e \kappa^3 \langle \gamma \rangle^{\frac{1}{2}} S_y \sin 2\Delta\Phi + \frac{1}{64}\lambda\beta_{z0} \langle \gamma \rangle^{-2} (S_x + S_y \cos 2\Delta\Phi) - \frac{1}{16}\kappa^2 \langle \gamma \rangle^{-2} \left[S_x + 2\lambda S_y + (1-2\lambda) S_y \cos 2\Delta\Phi \right] - \frac{1}{4}\kappa\lambda \langle \gamma \rangle^{-\frac{5}{2}},$$

$$\frac{d\Delta\Phi}{dt} = -\frac{1}{24} r_e \kappa^3 \langle \gamma \rangle^{\frac{1}{2}} (S_y + S_x) \sin 2\Delta\Phi + \frac{1}{8}\left[\frac{\lambda\beta_{z0}}{4} - \kappa^2 (1-2\lambda) \right] \langle \gamma \rangle^{-2} (S_y - S_x) \sin^2 \Delta\Phi.$$

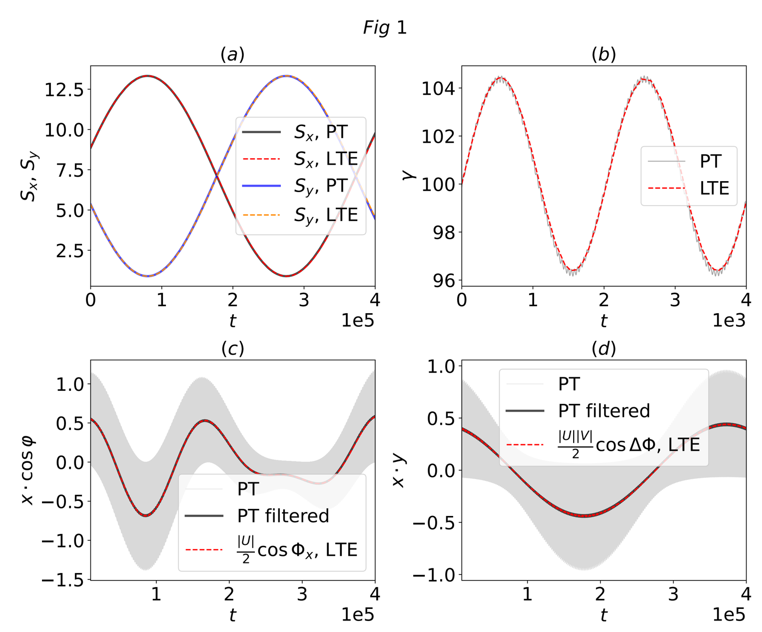
where, S_x is the areas (divided by 2π) of the ellipses encircled by the particle trajectory in $x - p_x$ phase space. Analogous expressions can be found for the y direction.

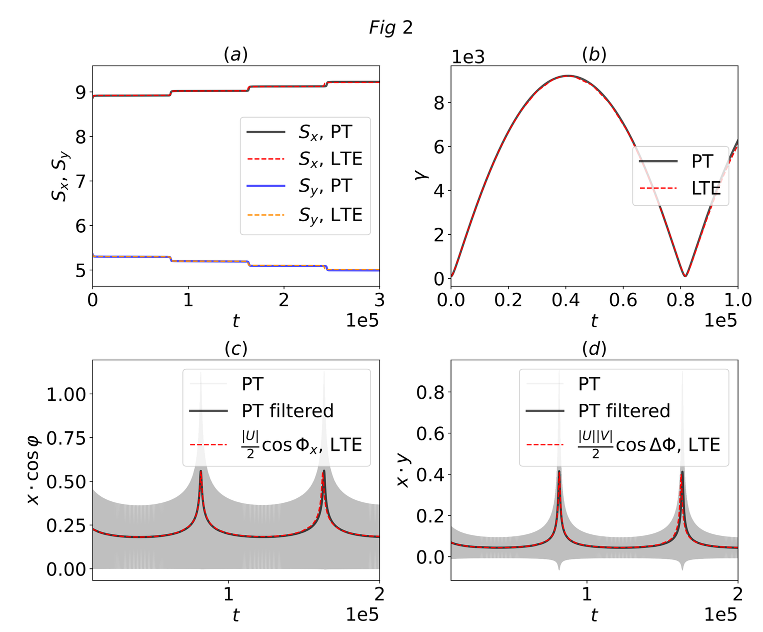
The value of parameters

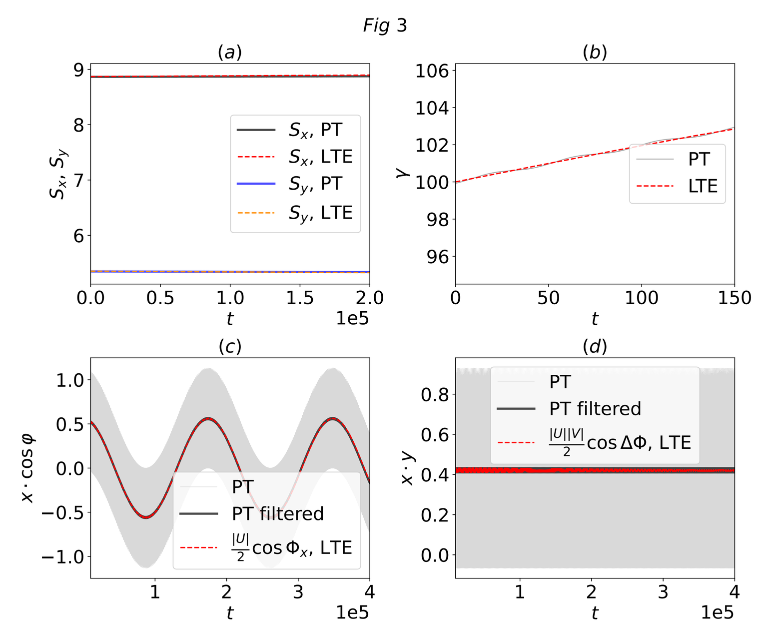
Case	Parameters			
	λ	κ	r_e	γ_w
Fig 1	0.25	$1/\sqrt{2}$	0	14
Fig 2	0.25	$1/\sqrt{2}$	0	100
Fig 3	0.40	$1/\sqrt{2}$	0	14
Fig 4	0.25	0.696	0	14
Fig 5	0.25	$1/\sqrt{2}$	$1.e^{-10}$	446
Fig 6	0.25	$1/\sqrt{2}$	$1.e^{-10}$	10^5
Fig 7	0.00	$1/\sqrt{2}$	$1.e^{-10}$	446
Fig 8	0.25	0.913	$1.e^{-10}$	446

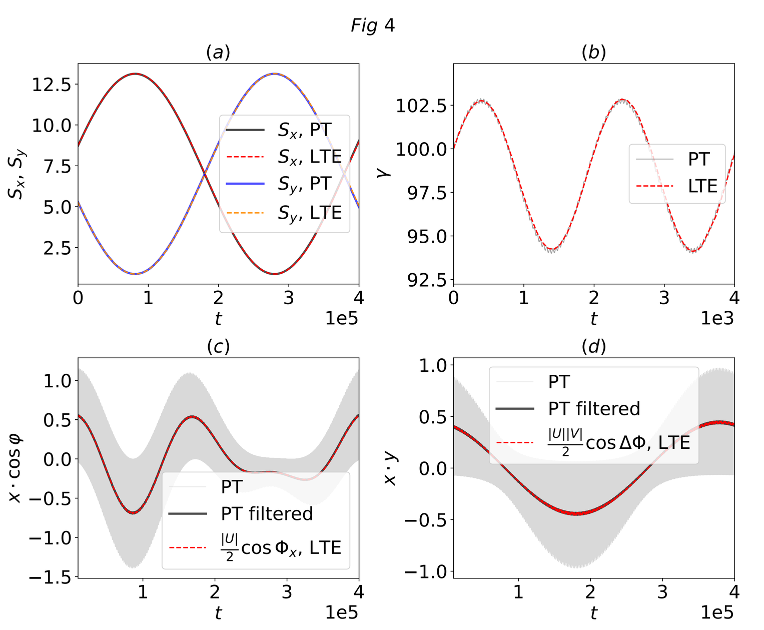
Case	Initial Values						
	$\lvert U\rvert$	$\lvert V\rvert$	Φ_y	Φ_x	E_{z0}	$\langle\gamma\rangle$	$\langle\zeta\rangle$
Fig 1	1.12	0.87	$\pi/6$	0	0	100	-0.05
Fig 2	1.12	0.87	$\pi/6$	0	0	100	0.00
Fig 3	1.12	0.87	$\pi/6$	0	0	100	-0.05
Fig 4	1.12	0.87	$\pi/6$	0	0	100	-0.05
Fig 5	1.12	0.87	$\pi/6$	0	0	10^5	-0.80
Fig 6	1.12	0.87	$\pi/6$	0	0	10^5	0.00
Fig 7	1.12	0.87	$\pi/6$	0	0	10^5	-0.80
Fig 8	1.12	0.87	$\pi/6$	0	0	10^5	-0.80

Results of LTE compared with PTracker ($r_e = 0$)

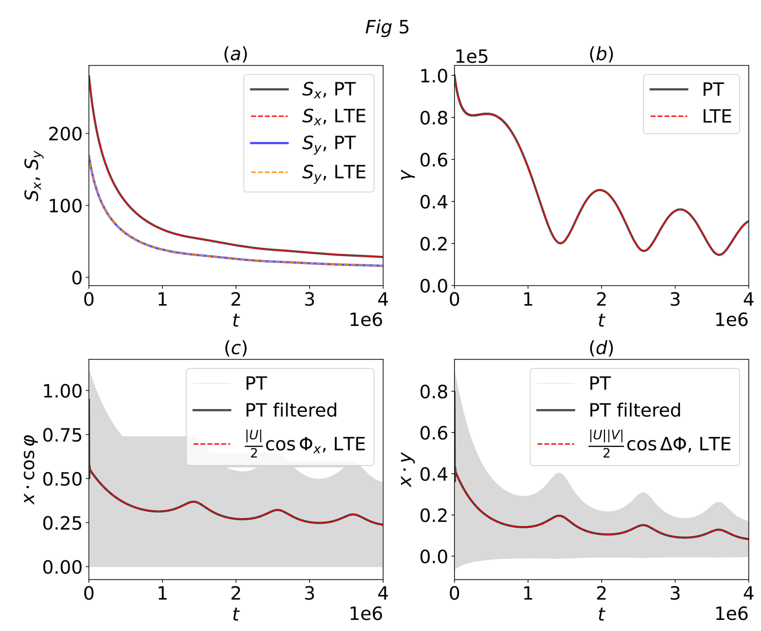


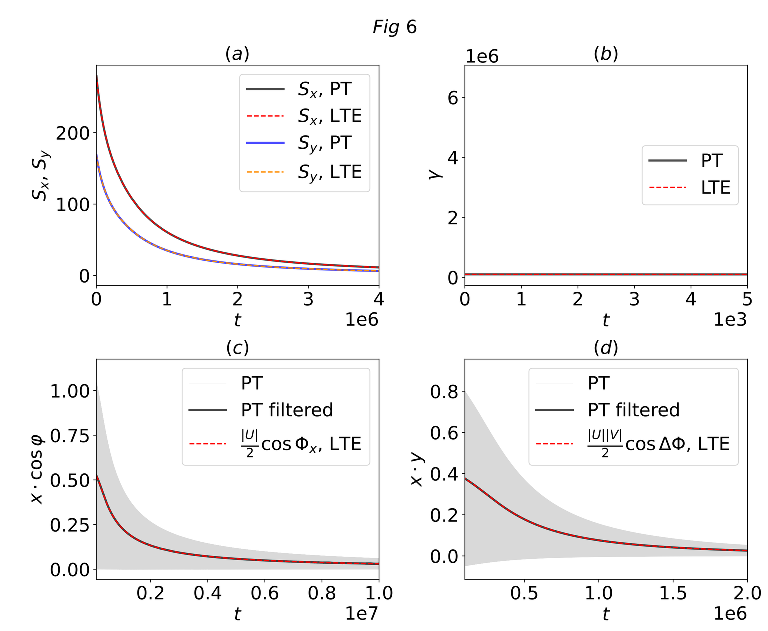


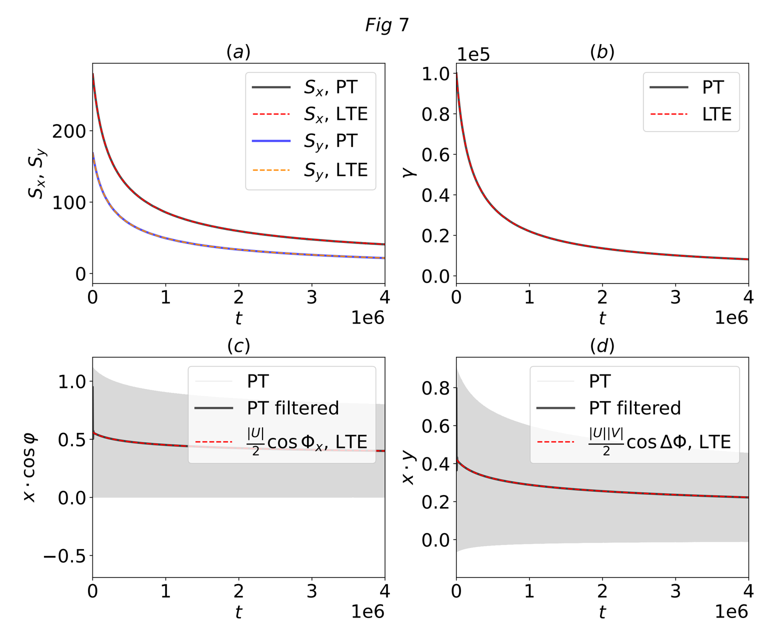


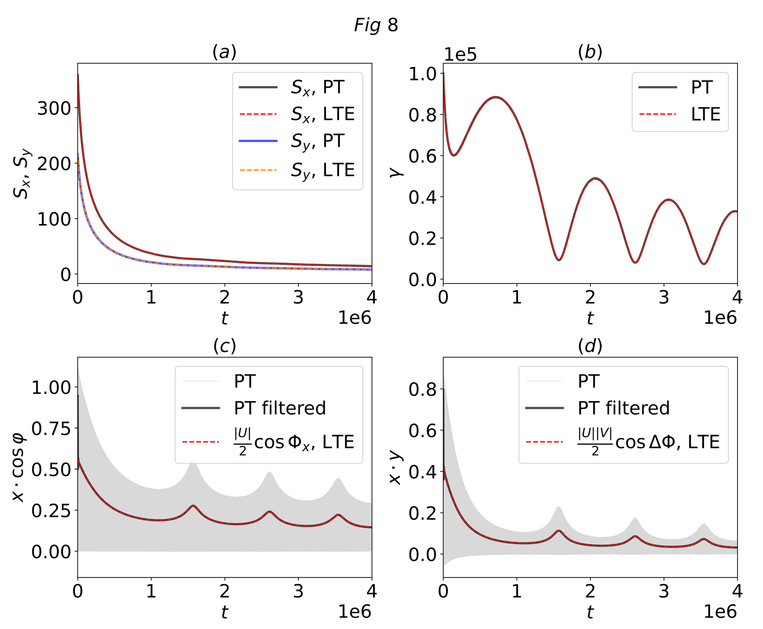


Results of LTE compared with PTracker ($r_e = e^{-10}$)









Conclusion

γ and ζ have the same drift frequency

In this study, three-dimensional betatron oscillation in non-linear plasma wakefield associated with radiation reaction force is discussed. The long-term equations obtained in our model are proved by comparing with the original equations by numerical methods. These equations provide a powerful tool for predicting the long-term behavior of the betatron oscillation, and have a great potential in future high-energy plasma accelerators.

References

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