Three-dimensional betatron oscillation and radiation reaction in plasma accelerators

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Introduction

The Lorentz–Abraham–Dirac equation for the RR four-force:

$$F_\mu^{\mathrm{rad}} = \frac{2}{3} r_e \left[\frac{d^2 P_\mu}{d\tau^2} + \left(\frac{dP_\nu}{d\tau} \frac{dP^\nu}{d\tau} \right) P_\mu \right].$$

The electromagnetic field provided by the wake:

$$\begin{aligned} E_z &= E_{z0} + \lambda\zeta_1, \\ \vec{E}_\perp &= \kappa^2 (1-\lambda) \vec{r}, \\ B_\theta &= -\kappa^2 \lambda r, \end{aligned}$$

Introduction

The Lorentz-Newton equation :

$$\dot{\gamma} = -E_{z0}\beta_{z0} + \left(\frac{\lambda\beta_{z0}}{4} + \kappa^2\lambda - \kappa^2\right)(x\beta_x + y\beta_y) - \frac{2}{3}r_e\gamma^2\kappa^4\left(x^2+y^2\right),$$

$$\dot{p_z} = -E_{z0} + \lambda\left(\frac{1}{4} + \kappa^2\right)(x\beta_x + y\beta_y) - \frac{2}{3}r_e\gamma^2\kappa^4\left(x^2+y^2\right),$$

$$\dot{p_x} = -\kappa^2 x + \frac{\kappa^2\lambda}{2}\left(\langle\gamma\rangle^{-2} + \beta_x^2 + \beta_y^2\right)x - \frac{2}{3}r_e\gamma^2\kappa^4\left(x^2+y^2\right)\beta_x,$$

$$\dot{p_y} = -\kappa^2 y + \frac{\kappa^2\lambda}{2}\left(\langle\gamma\rangle^{-2} + \beta_x^2 + \beta_y^2\right)y - \frac{2}{3}r_e\gamma^2\kappa^4\left(x^2+y^2\right)\beta_y,$$

Introduction

The long-term equation:

$$\langle\dot{\gamma}\rangle = -E_{z0}\beta_{z0} - \frac{1}{3}r_e\kappa^3\langle\gamma\rangle^{\frac{3}{2}}(S_x+S_y),$$

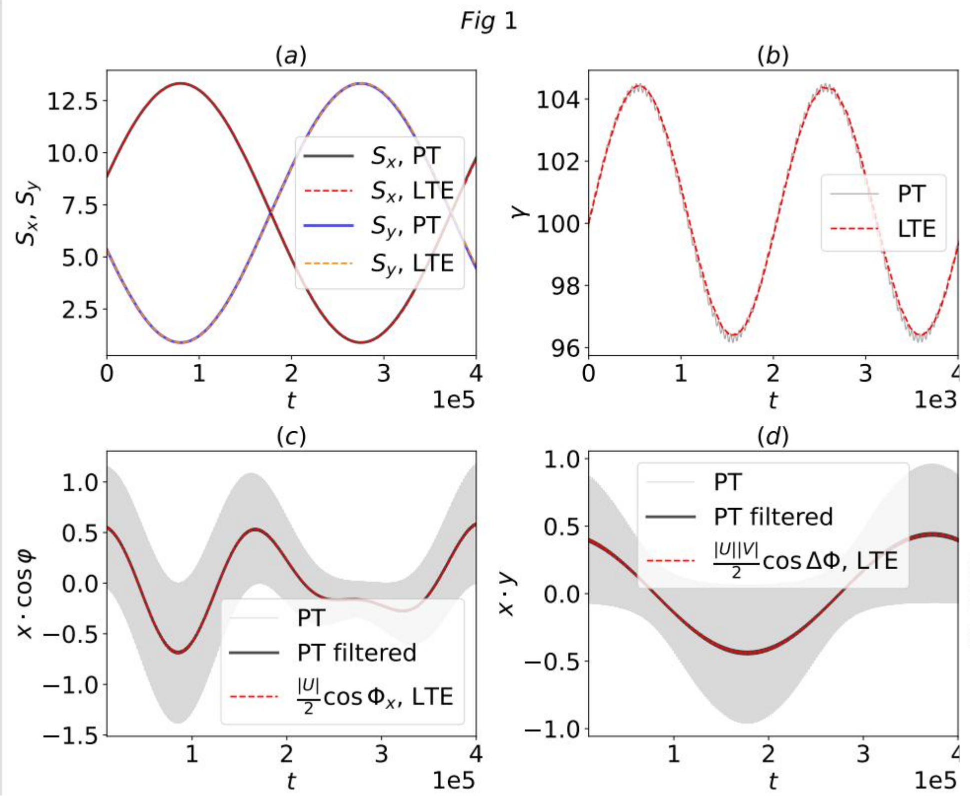
$$\langle\dot{\zeta}\rangle = \frac{1}{2}\gamma_w^{-2} - \frac{1}{2}\langle\gamma\rangle^{-2} - \frac{1}{4}\kappa\langle\gamma\rangle^{-\frac{3}{2}}(S_x+S_y),$$

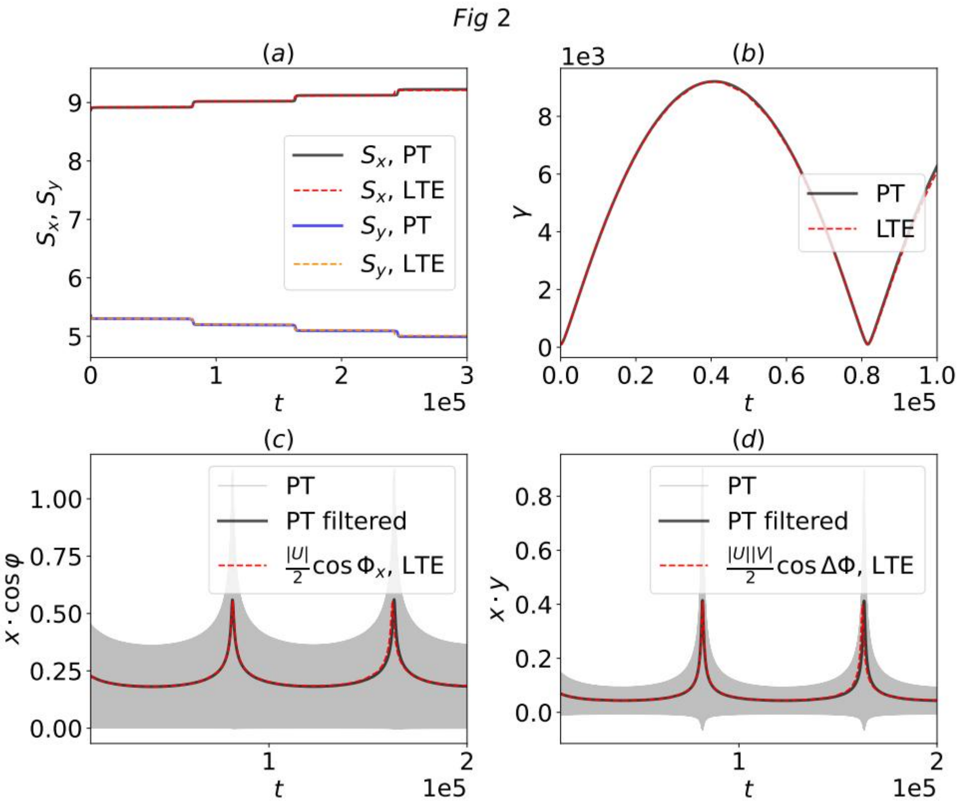
$$\frac{dS_x}{dt} = -\frac{1}{4}r_e\kappa^3\langle\gamma\rangle^{\frac{1}{2}}\left(S_x^2+\frac{4-\cos 2\Delta\Phi}{3}S_xS_y\right) - \frac{1}{8}\left[\frac{1}{4}\lambda\beta_{z0}-\kappa^2(1-2\lambda)\right]\langle\gamma\rangle^{-2}S_xS_y\sin 2\Delta\Phi,$$

$$\begin{aligned}\dot{\Phi}_x =& \frac{1}{24}r_e\kappa^3\langle\gamma\rangle^{\frac{1}{2}}S_y\sin 2\Delta\Phi + \frac{1}{64}\lambda\beta_{z0}\langle\gamma\rangle^{-2}(S_x+S_y\cos 2\Delta\Phi)\\ &-\frac{1}{16}\kappa^2\langle\gamma\rangle^{-2}[S_x+2\lambda S_y+(1-2\lambda)S_y\cos 2\Delta\Phi] - \frac{1}{4}\kappa\lambda\langle\gamma\rangle^{-\frac{5}{2}},\end{aligned}$$

$$\frac{d\Delta\Phi}{dt} = -\frac{1}{24}r_e\kappa^3\langle\gamma\rangle^{\frac{1}{2}}(S_y+S_x)\sin 2\Delta\Phi + \frac{1}{8}\left[\frac{\lambda\beta_{z0}}{4}-\kappa^2(1-2\lambda)\right]\langle\gamma\rangle^{-2}(S_y-S_x)\sin^2\Delta\Phi.$$

Results of LTE compared with PTracke





Case	Parameters			
	λ	κ	r_e	γ_w
Fig 1	0.25	$1/\sqrt{2}$	0	14
Fig 2	0.25	$1/\sqrt{2}$	0	100
Fig 3	0.40	$1/\sqrt{2}$	0	14
Fig 4	0.25	0.696	0	14
Fig 5	0.25	$1/\sqrt{2}$	$1.e^{-10}$	446
Fig 6	0.25	$1/\sqrt{2}$	$1.e^{-10}$	10^5
Fig 7	0.00	$1/\sqrt{2}$	$1.e^{-10}$	446
Fig 8	0.25	0.913	$1.e^{-10}$	446

Case	Initial Values						
	$\lvert U\rvert$	$\lvert V\rvert$	Φ_y	Φ_x	E_{z0}	$\langle\gamma\rangle$	$\langle\zeta\rangle$
Fig 1	1.12	0.87	$\pi/6$	0	0	100	-0.05
Fig 2	1.12	0.87	$\pi/6$	0	0	100	0.00
Fig 3	1.12	0.87	$\pi/6$	0	0	100	-0.05
Fig 4	1.12	0.87	$\pi/6$	0	0	100	-0.05
Fig 5	1.12	0.87	$\pi/6$	0	0	10^5	-0.80
Fig 6	1.12	0.87	$\pi/6$	0	0	10^5	0.00
Fig 7	1.12	0.87	$\pi/6$	0	0	10^5	-0.80
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Results of LTE compared with PTracke

Fig 3

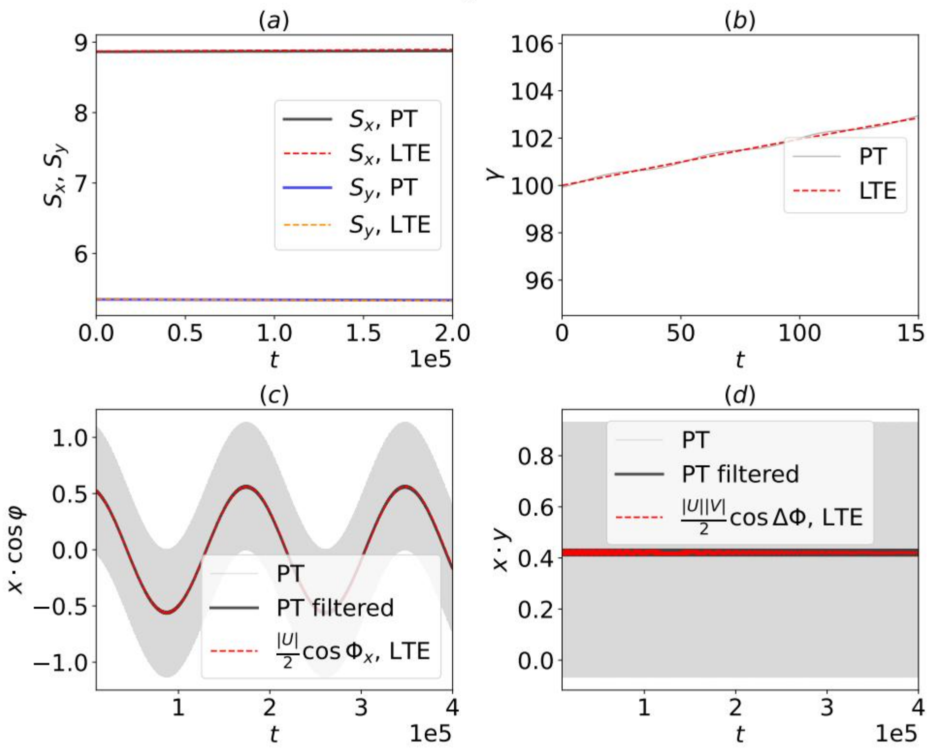
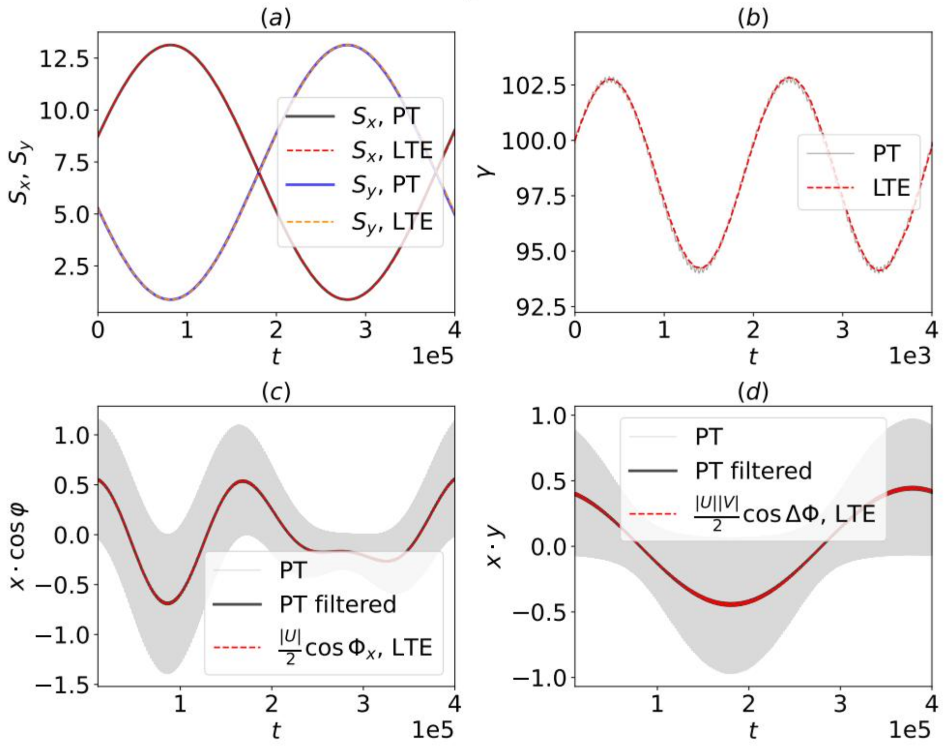


Fig 4



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Results of LTE compared with PTracke

Fig 5

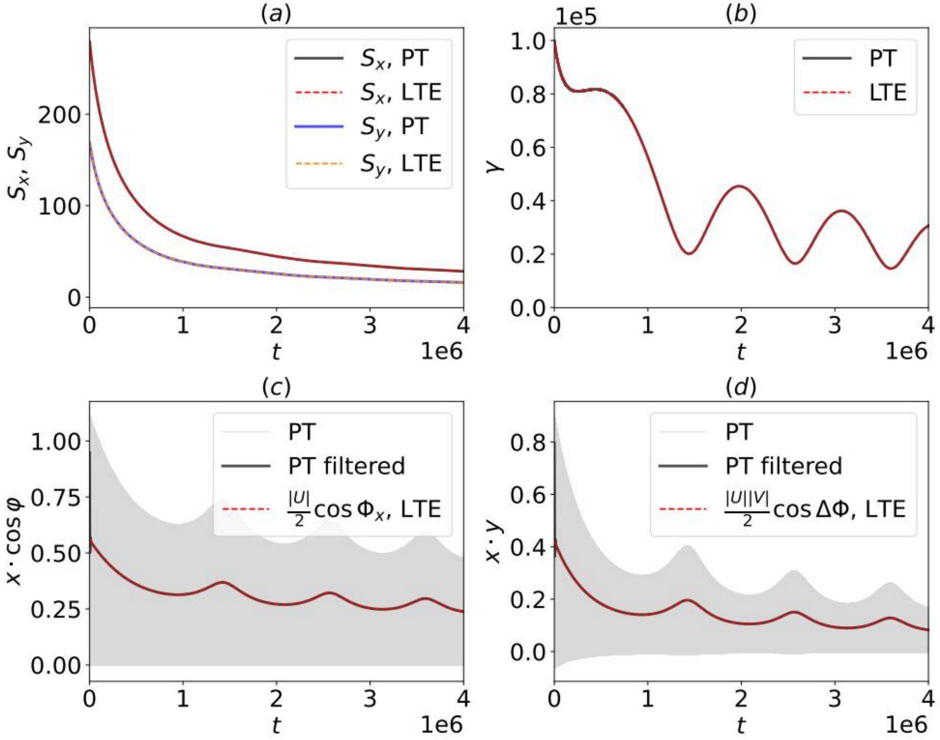
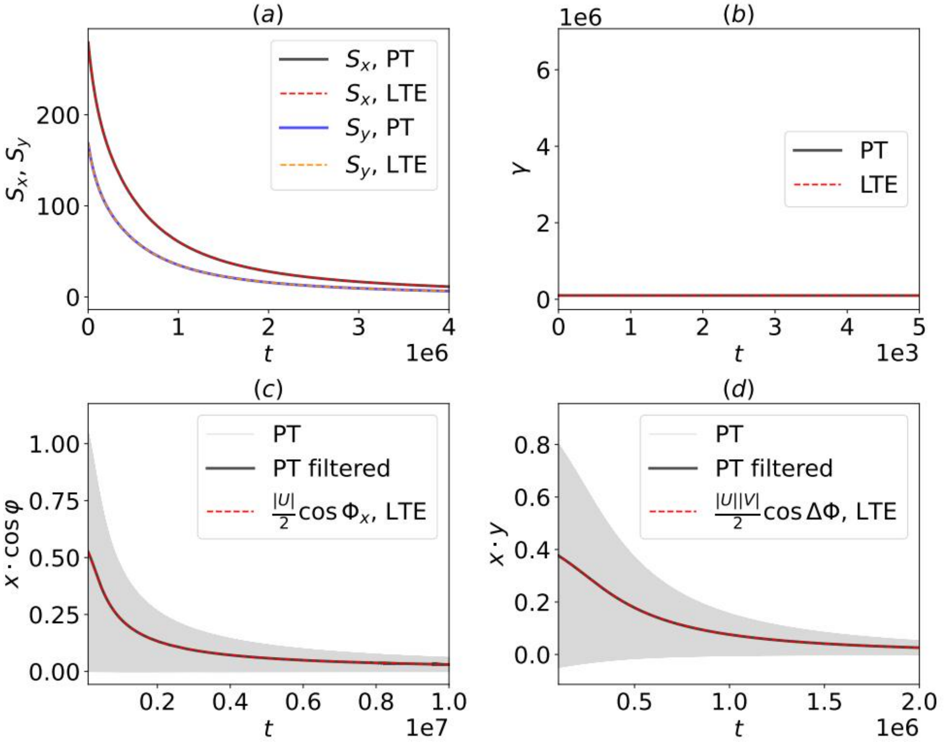


Fig 6



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Results of LTE compared with PTracke

Fig 7

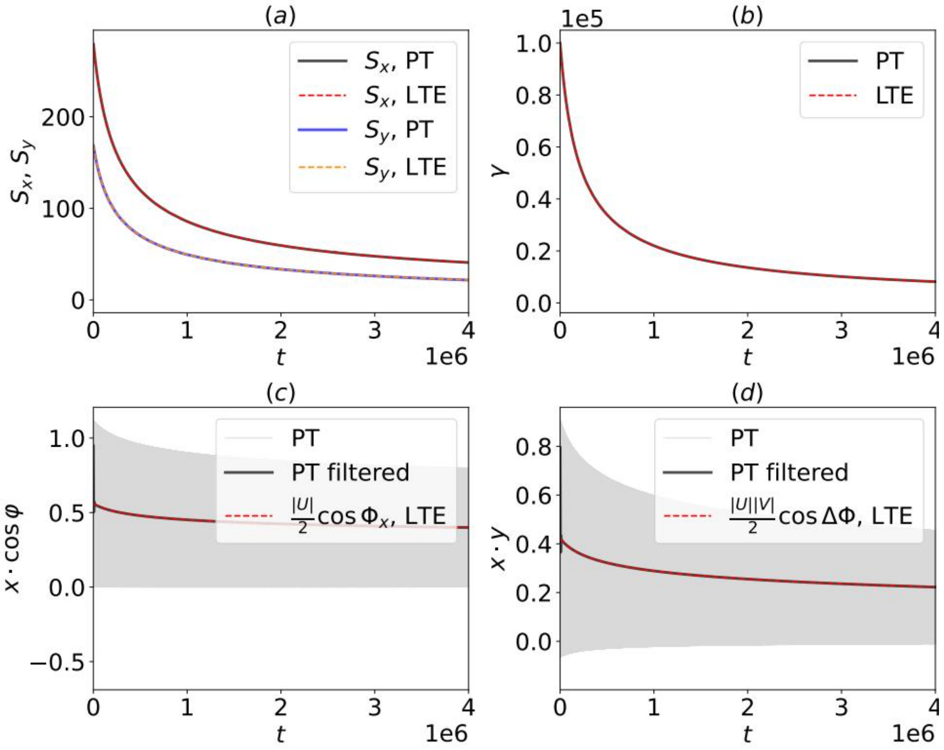
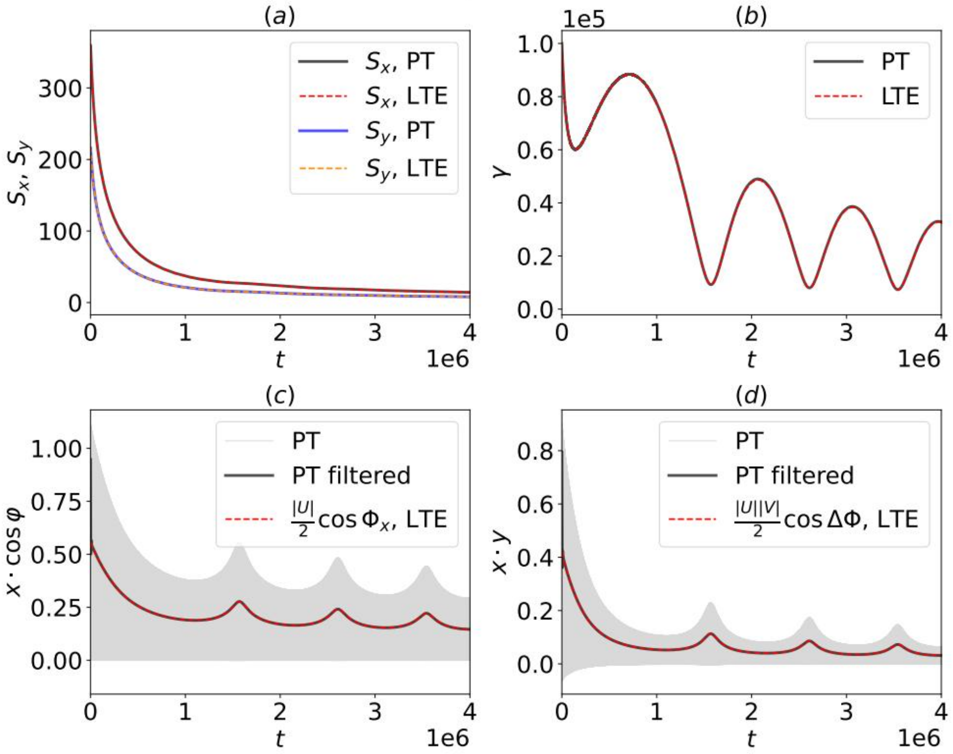


Fig 8



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Conclusion

- Betatron oscillation in y direction is considered.
- When λ=0 and κ^2=0.5, the changes of S_x, γ, ζ and L_z are consistent with traditional models.
- When λ and κ^2 get arbitrary value, our model can extend to every kind of polarization.
- Phase shift is proved.