EW AMPLITUDES WITHOUT GAUGE CANCELLATION

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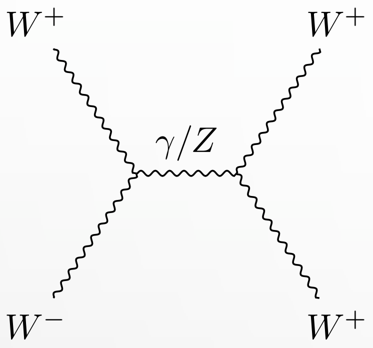
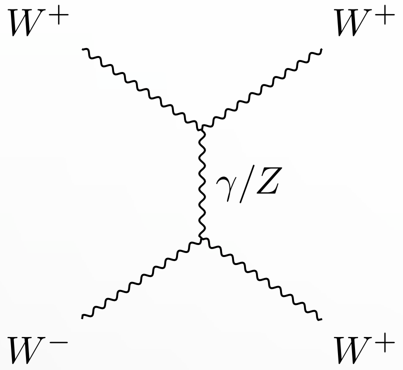
BASED MAINLY ON 2203.10440

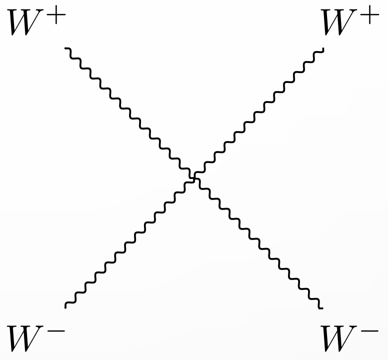
IN COLLABORATION WITH KAORU HAGIWARA, JUNICHI KANZAKI, KENTAROU MAWATARI

See also 1611.00788, 1902.06738

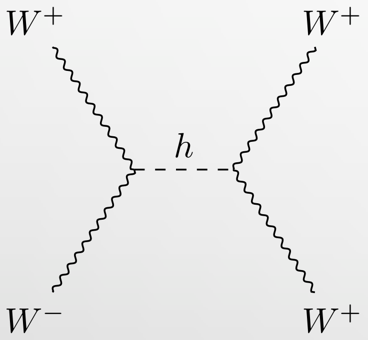
- LONGITUDINAL VECTOR BOSON: $\epsilon_L^{\mu} \sim k^{\mu}/\mathrm{M} \sim E/\mathrm{M}$
  - Ex: WW>WW

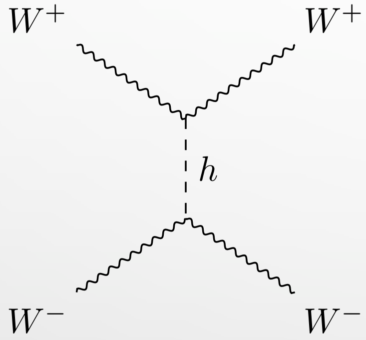






$\propto (E/m_V)^4$





$\propto -(E/m_W)^2$

- Final Amplitude: $\mathcal{A} \propto \mathcal{O}(1)$

- Unphysical energy increase for single (Feynman) diagrams
- Large cancellation between diagrams.

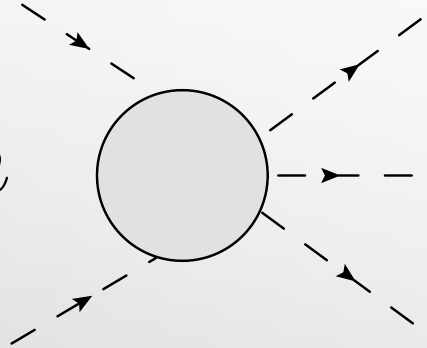
- Theoretical Problems
  - Ex: SMEFT and Higgs Couplings Measurement
  - Measuring Higgs self-couplings through $W_L^{\pm}/Z_L \sim \phi^{\pm}/\phi_0$

(Mainstream: double Higgs production)

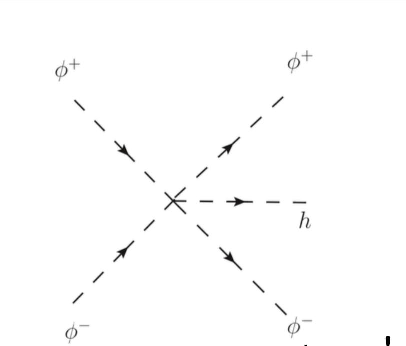
$$W_L W_L \to W_L W_L h/hhh$$

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{v+h+i\phi^0}{\sqrt{2}} \end{pmatrix}$$

$$E \gg m$$



$$\simeq$$



$$+\mathcal{O}\left(\frac{m}{E}\right)$$

$$\mathcal{O}_6 = \frac{c_6}{\Lambda^2}(\Phi^\dagger\Phi)^3$$

Importance is of the same level as hh production in muon colliders

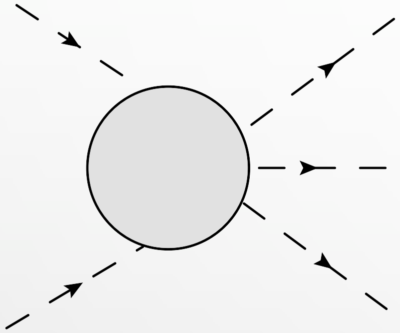
Junmou Chen, Chih-Ting Ting, Yongcheng Wu, Tong Li PhysRevD.105.053009
Chang-Yuan Yao JHEP10(2021)099

■ Theoretical Problems

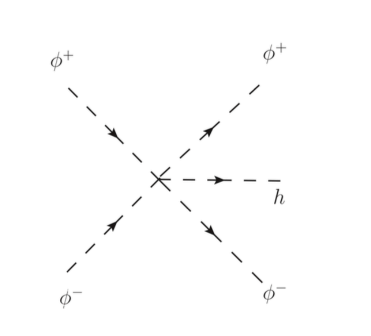
■ Ex.: SMEFT and Higgs Couplings Measurement

$$W_L W_L \to W_L W_L h / hhh \qquad W_L^{\pm}/Z_L \sim \phi^{\pm}/\phi_0$$

$E \gg m$



$\simeq$



$+\mathcal{O}(\frac{m}{E})$

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{v+h+i\phi^0}{\sqrt{2}} \end{pmatrix}$$

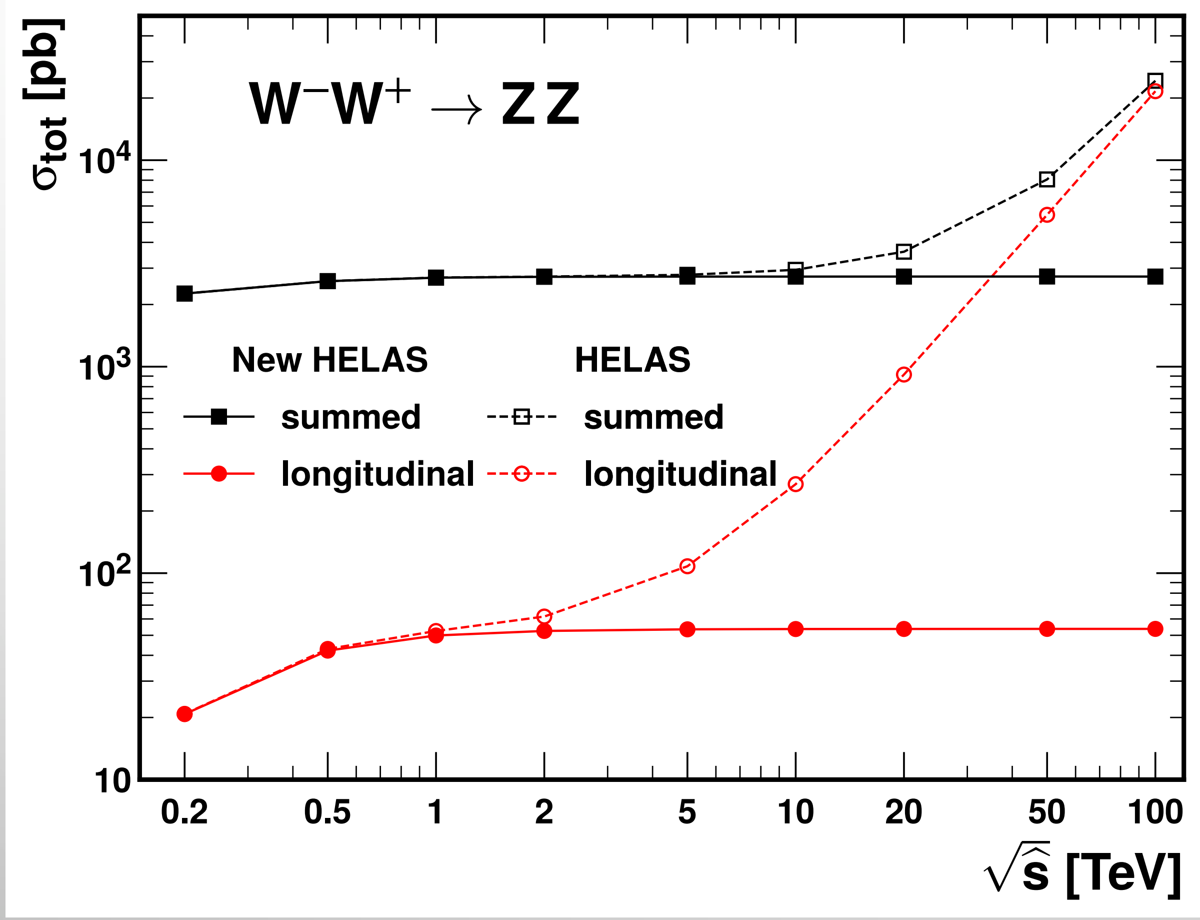
$$\mathcal{O}_6 = \frac{c_6}{\Lambda^2}(\Phi^\dagger \Phi)^3$$

At what exactly range is on this picture(GET) valid? Corrections?

Even simpler amplitude WW>WW?

<u>We need a method that manifests GET, but computes amplitudes exactly.</u>

- Numerical Problem
  - Cancellation between large numbers(diagrams) easily leads to errors



Total cross section of WW > ZZ when t/u channel propagators are given physical decay width

(Madgraph before v3_** versions)

1. Motivation: subtle gauge cancellation

- Numerical Problem: Cancellation between large numbers(diagrams)
  - Computing efficiency

Madgraph v3_2_0

pp > jj w+{0}w-{0}h:

Event no: 147/10000

run_02	p p 7000.0 x 7000.0 GeV	tag_1	0.0001979 ± 2.3e−06	147	parton madevent	LHE	remove run	launch detector simulation

pp > jj w+{T}w-{T}h:

Run	Collider	Banner	Cross section (pb)	Events	Data	Output	Action	
run_01	p p 7000.0 x 7000.0 GeV	tag_1 ERROR	0.001458 ± 4.3e−06	10000	parton madevent	LHE	remove run	launch detector simulation

- Goldstone equivalence theorem(GET)

$$\mathcal{M} \equiv \epsilon_L^\mu \mathcal{M}_\mu \simeq -i\mathcal{M}(\phi)$$

See, e.g. Nucl. Phys. B 261, 379 (1985)

M. S. Chanowitz and M. K. Gaillard

- Can be extended to full equality

$$k^\mu \mathcal{M}_\mu = -im_V \mathcal{M}(\phi)$$ (From gauge symmetry)

$$\epsilon_L^\mu = \frac{k^\mu}{m_W} - \frac{m}{E+|\vec{k}|}n^\mu : \mathcal{M}(W_L) = -im_V\mathcal{M}(\phi) - \frac{m}{E+\vec{k}}n^\mu \mathcal{M}_\mu$$

GET

$\mathcal{O}(\frac{m}{E})$

- N-point amplitudes:

$$\mathcal{M}(W_L, W_L, W_L, ..., W_L) = (-i)^n \mathcal{M}(\phi, \phi, \phi, ..., \phi) + (-i)^{n-1}\mathcal{M}(W_n, \phi, \phi, ..., \phi)$$
$$+.... + \mathcal{M}(W_n, W_n, W_n, ..., W_n) \quad (2)$$

The terms (diagrams) on the RH are 2^N times of LH (Naïve est.)

■ 2. Solution: “New Feynman Diagram”

$$g_{MN} = diag(1, -1, -1, -1, -1)$$

- 5-component formalism
  - GET $k^M = \begin{pmatrix} k^\mu \\ im_V \end{pmatrix}$ $\quad k^M \mathcal{M}_M = k^\mu \mathcal{M}_\mu - i\mathcal{M}(\phi) = 0$
  - Polarization vector

$$\epsilon_L^M = \begin{pmatrix} \epsilon_L^\mu \\ 0 \end{pmatrix} - k^M = \begin{pmatrix} \tilde{\epsilon}_n^\mu \\ i \end{pmatrix}$$

Well behaved in high energy and a single object

  - Gauge and Goldstone components combined together

$$\mathcal{M} = \epsilon^M \mathcal{M}_M = \tilde{\epsilon}^\mu \mathcal{M}_\mu - i\mathcal{M}(\phi)$$

  - N-point amplitudes(pol.vec., propagators, vertices)

$$\mathcal{M}(k_1, k_2, \dots, k_n) = \epsilon^{M_1}\epsilon^{M_2}\dots\epsilon^{M_N}\mathcal{M}_{M_1,M_2,\dots,M_N}(k_1, k_2, \dots, k_n)$$

- Gauge Choice and Propagator: 5-components formalism
  - Gauge cancellation exists also on the propagator (Rxi gauge)
  - Degrees of freedom in Feynman gauge

$$-g^{\mu\nu} + \frac{k^\mu k^\nu}{m_V^2} = \sum_{s=\pm,L} \epsilon_L^\mu \epsilon_L^\nu$$

energy increase

$$-g^{\mu\nu} = \sum_{s=\pm} \epsilon_s^\mu \epsilon_s^\nu + \epsilon_L^\mu \epsilon_L^\nu - \frac{k^\mu k^\nu}{m_V^2}$$

Physical degrees of freedom

$$= \sum_{s=\pm} \epsilon_s^\mu \epsilon_s^\nu - \frac{n^\mu k^\nu + k^\mu n^\nu}{n \cdot k} + \frac{m^2 n^\mu n^\nu}{(n \cdot k)^2},$$

Even Feynman gauge gives unphysical energy increase!

Gauge Choice and Propagators: 5-components formalism

Goldstone equivalence gauge(GEG)

proposed in JHEP11(2017)093

Junmou Chen, Tao Han & Brock Tweedie

$$\epsilon_L^\mu = \frac{k^\mu}{m_W} - \frac{m}{E+|\vec{k}|} n^\mu$$

Gauge condition: $n \cdot V = 0$ or $n \cdot \epsilon_s = 0$

Transverse: $n \cdot \epsilon_\pm = 0$

$\epsilon_\pm$ satisfies

Longitudinal: $n \cdot \epsilon_L \neq 0$

$\epsilon_L^\mu = \frac{k^\mu}{m_V} + \tilde{\epsilon}_n^\mu$ does not satisfy

$\tilde{\epsilon}_n^\mu = -\frac{m_V}{n \cdot k} n^\mu$ with $n^\mu = (1, -\frac{\vec{k}}{|\vec{k}|})$

$$n \cdot n = 0 \to n \cdot \tilde{\epsilon}_n = 0$$

So where is k^μ term?

“Scalarized” into Goldstone mode! $\cancel{k^\mu / m_V} \to \phi$

Gauge Choice and Propagators: 5-components formalism

Goldstone equivalence gauge(GEG)

$$\mathcal{L}_\xi = \frac{1}{2\xi}(n\cdot V)^2 \qquad \tilde{\epsilon}_n^\mu = -\frac{m_V}{n\cdot k}n^\mu$$

$$\langle (V_a,\phi_a),(V_b,\phi_b)\rangle = \frac{i\delta_{ab}}{k^2-m_V^2}\begin{pmatrix} -g^{\mu\nu}+\frac{n^\mu k^\nu+k^\mu n^\nu}{n\cdot k} & -i\tilde{\epsilon}_n^\mu \\ i\tilde{\epsilon}_n^\nu & 1 \end{pmatrix}$$

When on-shell, $-g^{\mu\nu}+\dfrac{n^\mu k^\nu+k^\mu n^\nu}{n\cdot k} = \displaystyle\sum_{s=\pm,n}\epsilon_s^\mu\epsilon_s^\nu$

$$\begin{pmatrix} -g^{\mu\nu}+\frac{n^\mu k^\nu+k^\mu n^\nu}{n\cdot k} & -i\tilde{\epsilon}_n^\mu \\ i\tilde{\epsilon}_n^\nu & 1 \end{pmatrix} = \sum_{s=\pm,L}\epsilon_s^M\epsilon_s^{N*}$$

Gauge-Goldstone mixing terms

Pol. Vec. : $\epsilon_\pm^M = \begin{pmatrix}\epsilon_\pm^\mu \\ 0\end{pmatrix} \qquad \epsilon_L^M = \begin{pmatrix}\tilde{\epsilon}_n^\mu \\ i\end{pmatrix}$

■ 2. Solution: “New Feynman Diagram”

■ Vertices(SM)

Table 2. Vertex for VVV: WWZ & WWA. All momenta are outgoing, particles are final states.

$V_{VVV}^{M_1M_2M_3}$			W^-W^+Z	W^-W^+A
$V^{\mu_1\mu_2\mu_3}$		$-ia(g^{\mu_1\mu_2}(p_1-p_2)^{\mu_3}+g^{\mu_2\mu_3}(p_2-p_3)^{\mu_1}+g^{\mu_3\mu_1}(p_3-p_1)^{\mu_2})$	$a=gc_W$	$a=gs_W$
$V^{4\mu_i\mu_j}$	$V^{4\mu_2\mu_3}$	$b_1\eta^{\mu_2\mu_3}$	$b_1=-\frac{g^2{s_W}^2v}{2c_W}$	$b_1=\frac{g^2vs_W}{2}$
	$V^{\mu_14\mu_3}$	$b_2g^{\mu_1\mu_3}$	$b_2=\frac{g^2{s_W}^2v}{2c_W}$	$b_2=-\frac{g^2vs_W}{2}$
	$V^{\mu_1\mu_24}$	$b_3g^{\mu_1\mu_2}$	$b_3=0$	$b_3=0$
$V^{44\mu_i}$	$V^{44\mu_3}$	$ic_1(p_1-p_2)^{\mu_3}$	$c_1=\frac{gc_{2W}}{2c_W}$	$c_1=gs_W$
	V^{μ_144}	$ic_2(p_2-p_3)^{\mu_1}$	$c_2=\frac{g}{2}$	$c_2=0$
	$V^{4\mu_24}$	$ic_3(p_3-p_1)^{\mu_2}$	$c_3=\frac{g}{2}$	$c_3=0$
V^{444}			0	0

Full List see 2203.10440

3.1 Numerical Implementation

(Old) HELAS(HELicity Amplitudes Subroutines) Introduction

HELAS compute amplitudes for Madgraph See e.g. 1106.0522

HELAS Basic setup: subroutines

Vertices

External Lines

External line	Subroutine
Flowing-In Fermion	IXXXXX
Flowing-Out Fermion	OXXXXX
Vector Boson	VXXXXX
Scalar Boson	SXXXXX

Table 2.3: List of the vertex subroutines in HELAS system.

Vertex	Inputs	Output	Subroutine
FFV	FFV	Amplitude	IOVXXX
	FF	V	JIOXXX, J3XXXX
	FV	F	FVIXXX, FVOXXX
FFS	FFS	Amplitude	IOSXXX
	FF	S	HIOXXX
	FS	F	FSIXXX, FSOXXX
VVV	VVV	Amplitude	VVVXXX
	VV	V	JVVXXX
VVS	VVS	Amplitude	VVSXXX
	VS	V	JVSXXX
	VV	S	HVVXXX

■ New HELAS (implementing “New Feynman Diagrams”)

1. Replace polarization vectors of W/Z to 5-component form

2. Replace propagators of W/Z to GEG in 5-component form

3. Extend vertices involving W/Z to incorporate Goldstone components

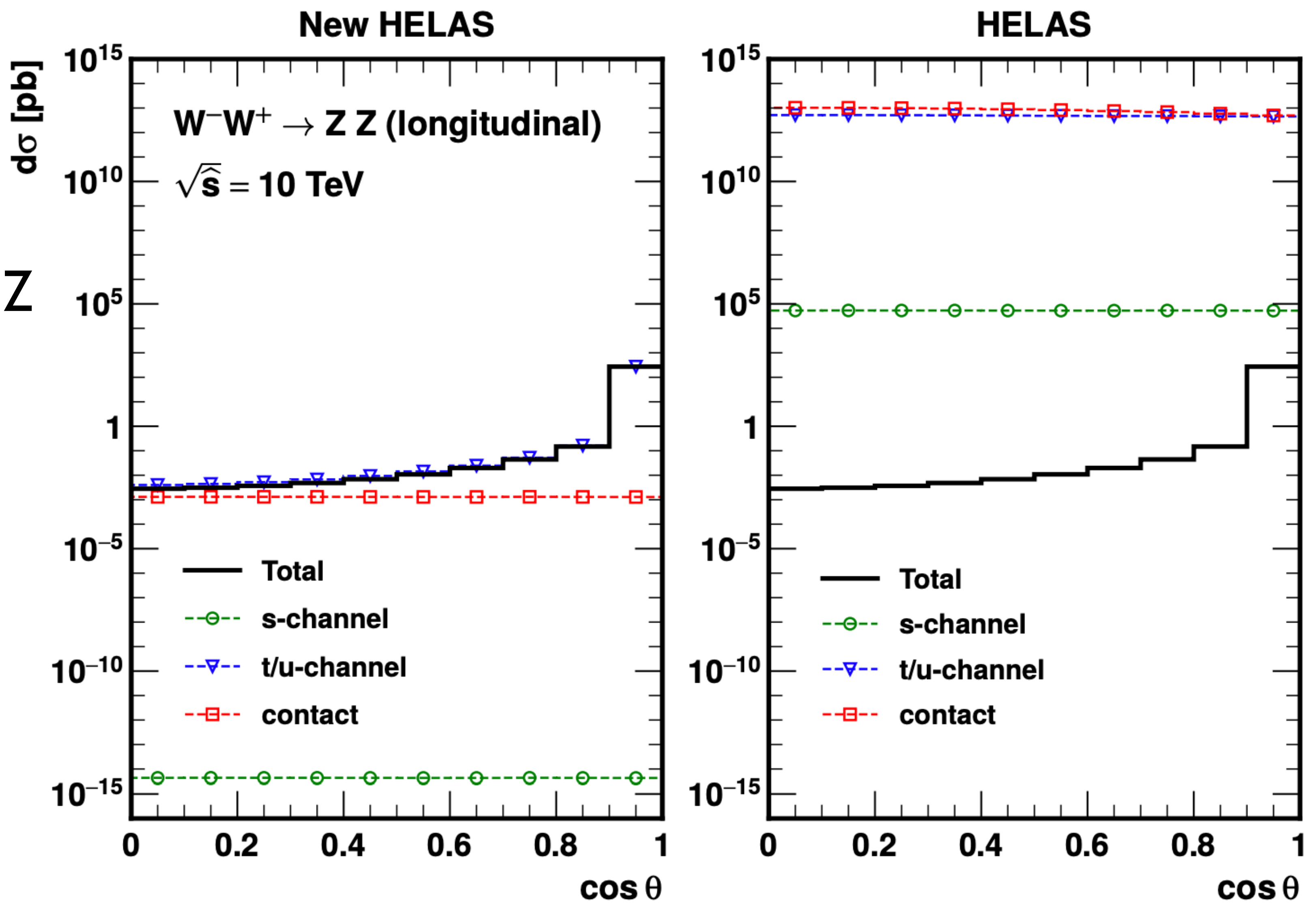
(4). Replace propagators of photon and gluons to parton shower gauge (counter part of GEG for massless case.)

5. Add vertices not existing in Old Feynman Rules(ZZZZ, WWZh,WWAh)

Codes can be found in https://madgraph.ipmu.jp/IPMU/Softwares/HELAS/index.html***(HELAS V42)***

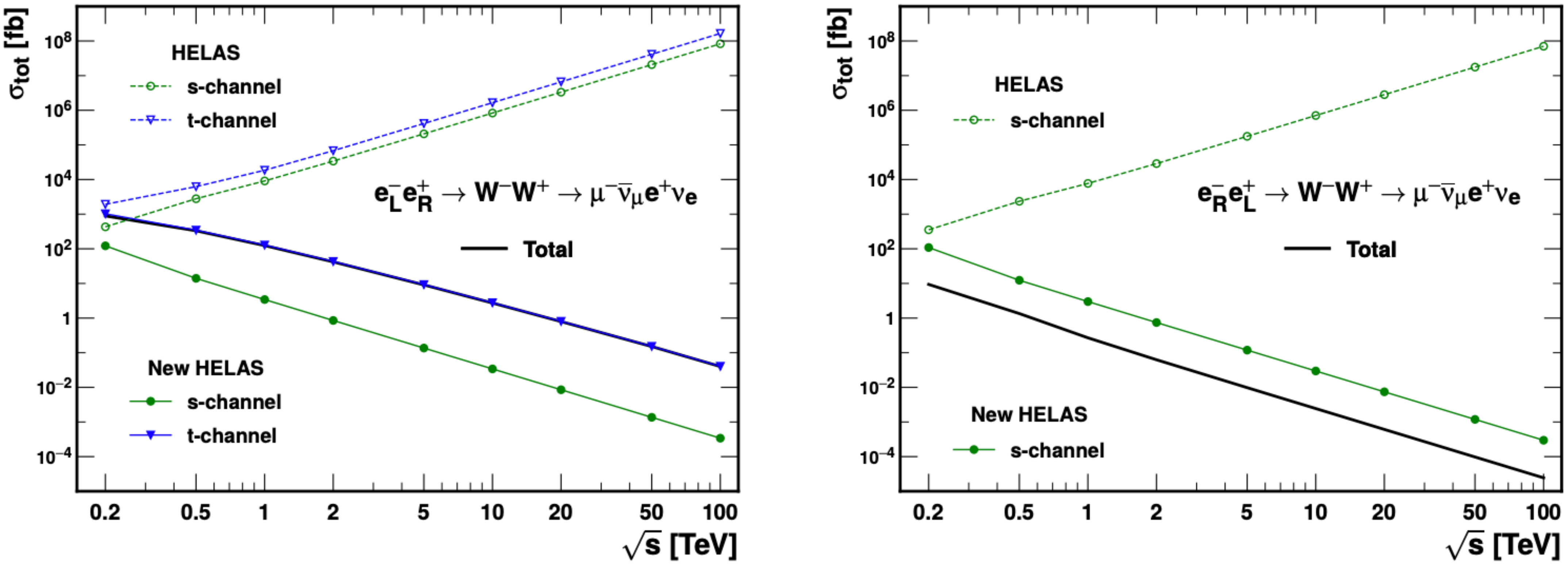
■ New HELAS Results: 2→2 VBS

WW>ZZ



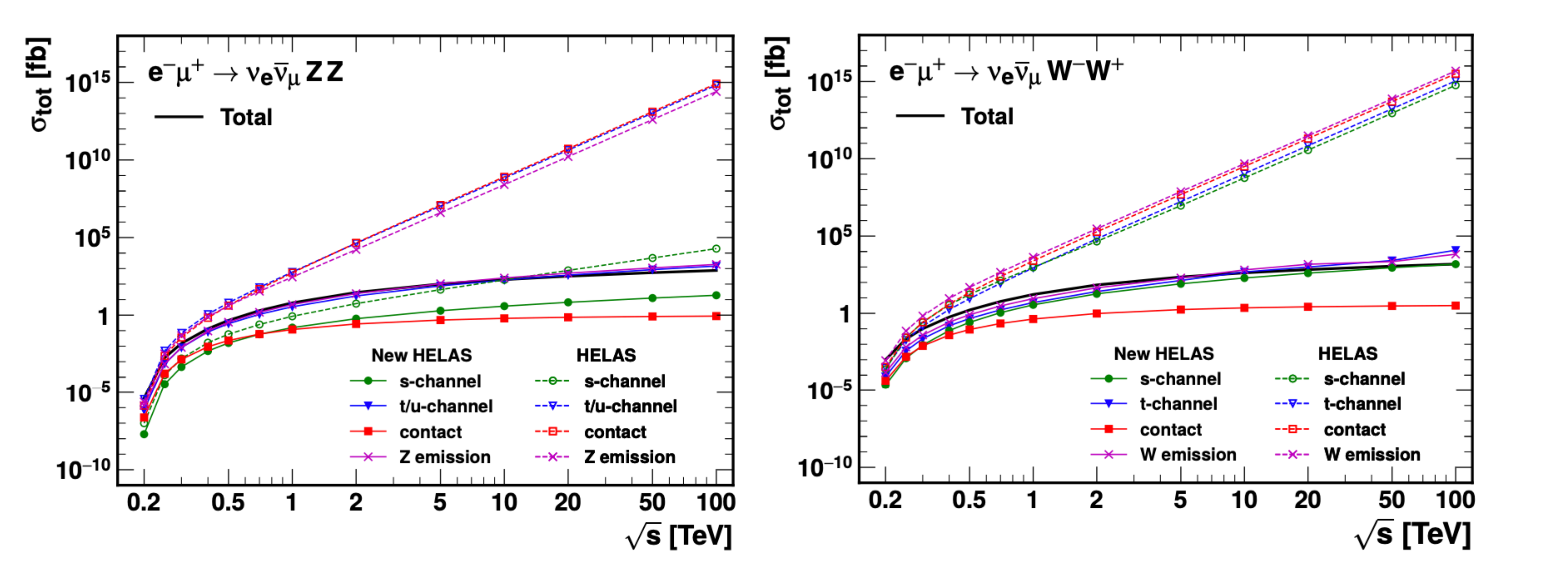
New HELAS Results: WW production at lepton colliders

$e^+e^- \to W^+W^- \to \mu^-\bar{\nu}e^+\nu_e$: total cross section



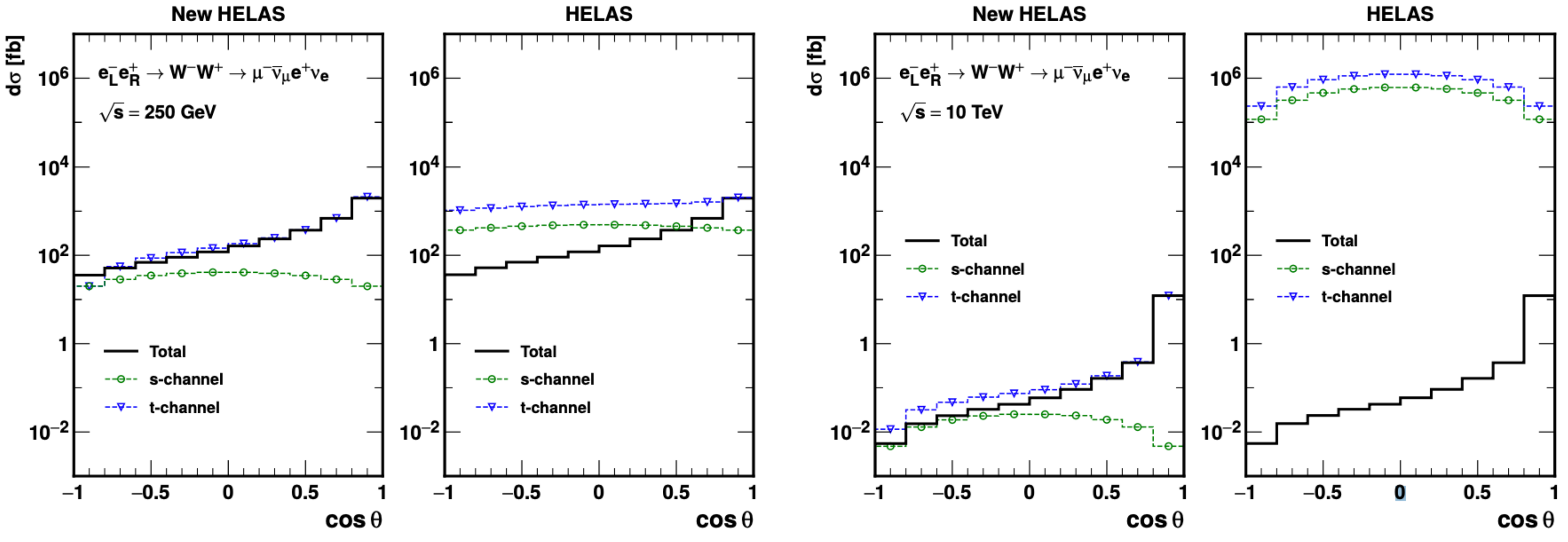
New HELAS Results: VBS at lepton colliders

$e^-\mu^+ \to \nu_e\bar{\nu}_\mu ZZ/WW\,(WW \to ZZ/WW)$: total cross section



New HELAS Results: WW production at lepton colliders

$e^+e^- \to W^+W^- \to \mu^-\bar{\nu}e^+\nu_e$: angular distribution



(a)

(b)

CONCLUSION

1. We propose a formalism (New Feynman Diagram) to solve gauge cancellation of weak scattering amplitudes, including:

   5-component pol. vec. , propagators (in GEG), vertices

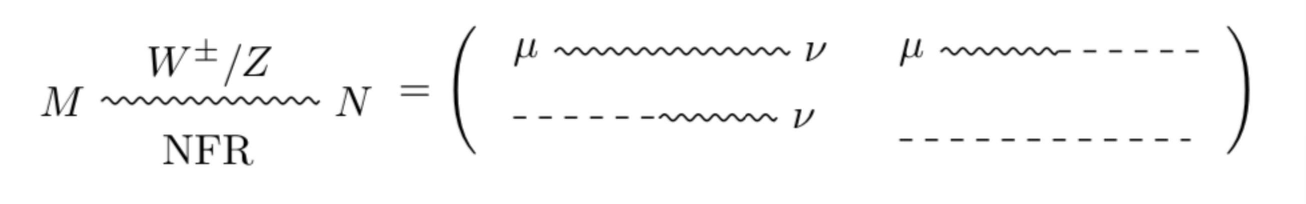
2. We implement NFD to HELAS(SM), and demonstrate some numerical results

3. Future: Needing deeper understanding, integrated into Madgraph(?), Other models(?)

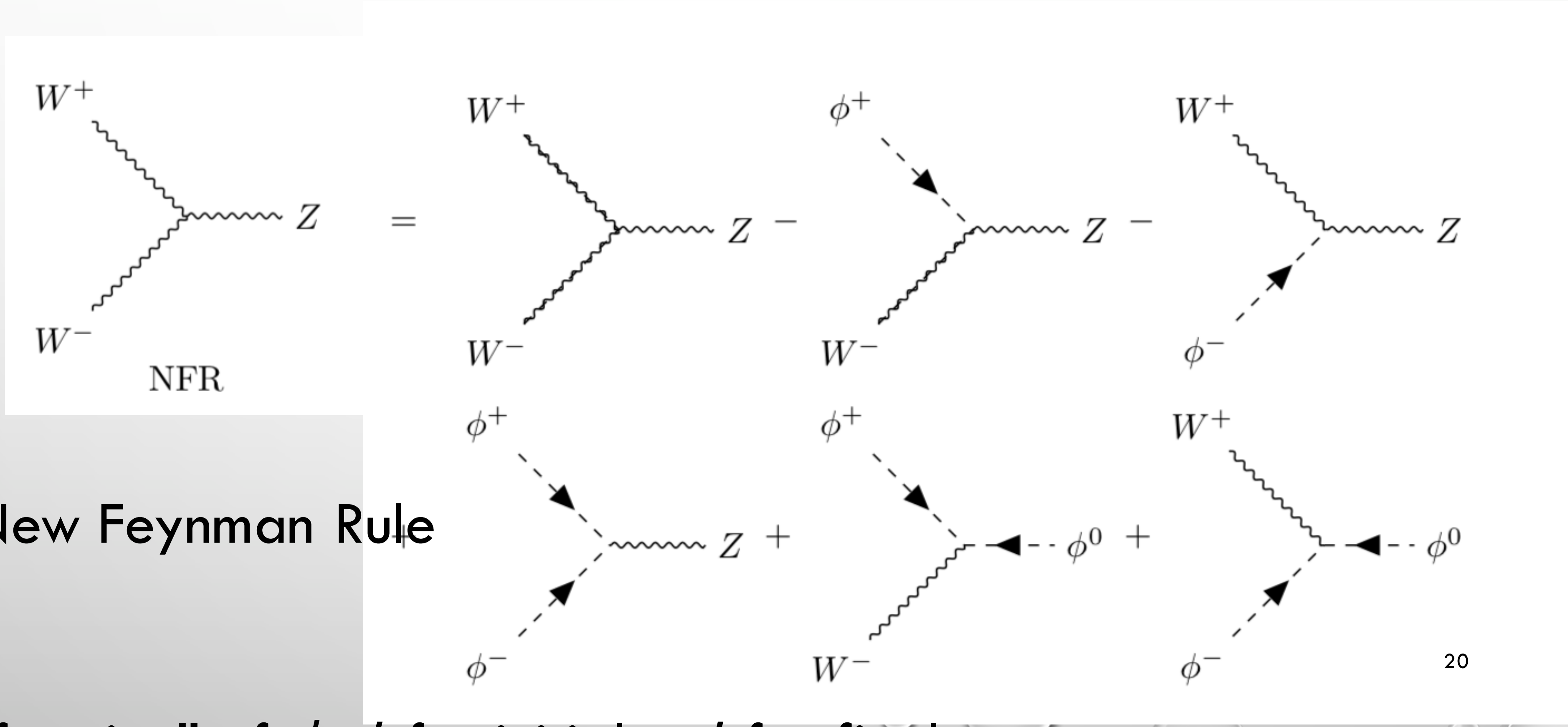
■ 2. Solution: “New Feynman Diagram”

■ New Feynman Rule(SM)

Propagator:



WWZ:



NFR=New Feynman Rule

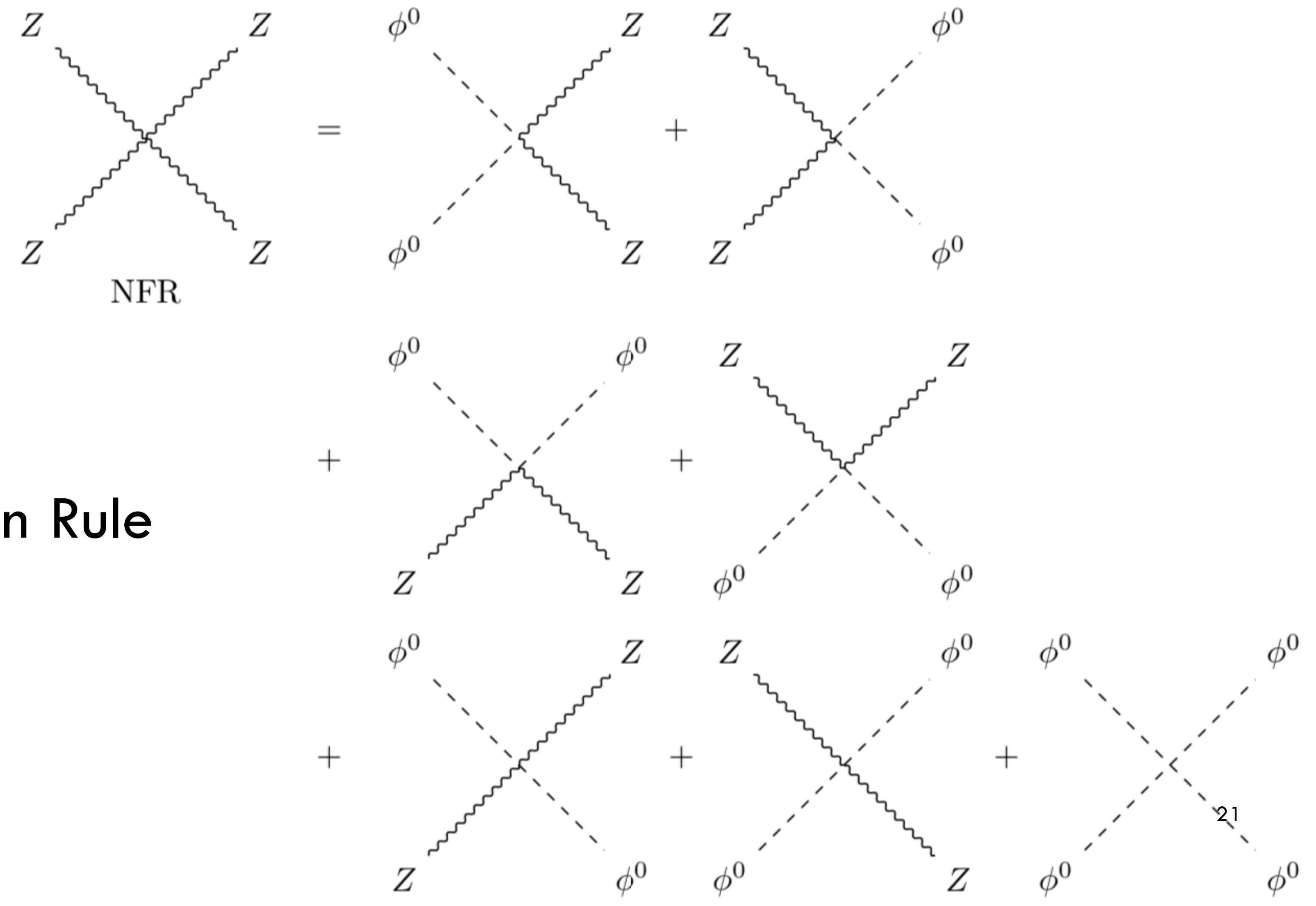
“wave function” of ϕ: i for initial, $-i$ for final

■ 2. Solution: "New Feynman Diagram"

■ New Feynman Rule(SM)

■ Vertices doesn't exist in "Old" Feynman Diagrams

ZZZZ:

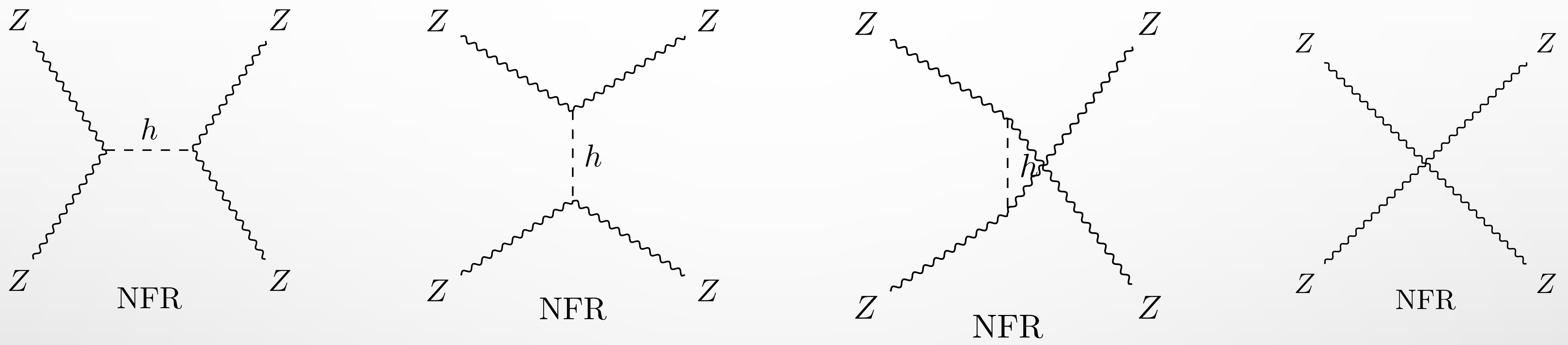


NFR=New Feynman Rule

■ 2. Solution: “New Feynman Diagram”

■ New Feynman Rule(SM)

■ Ex2: ZZ>ZZ

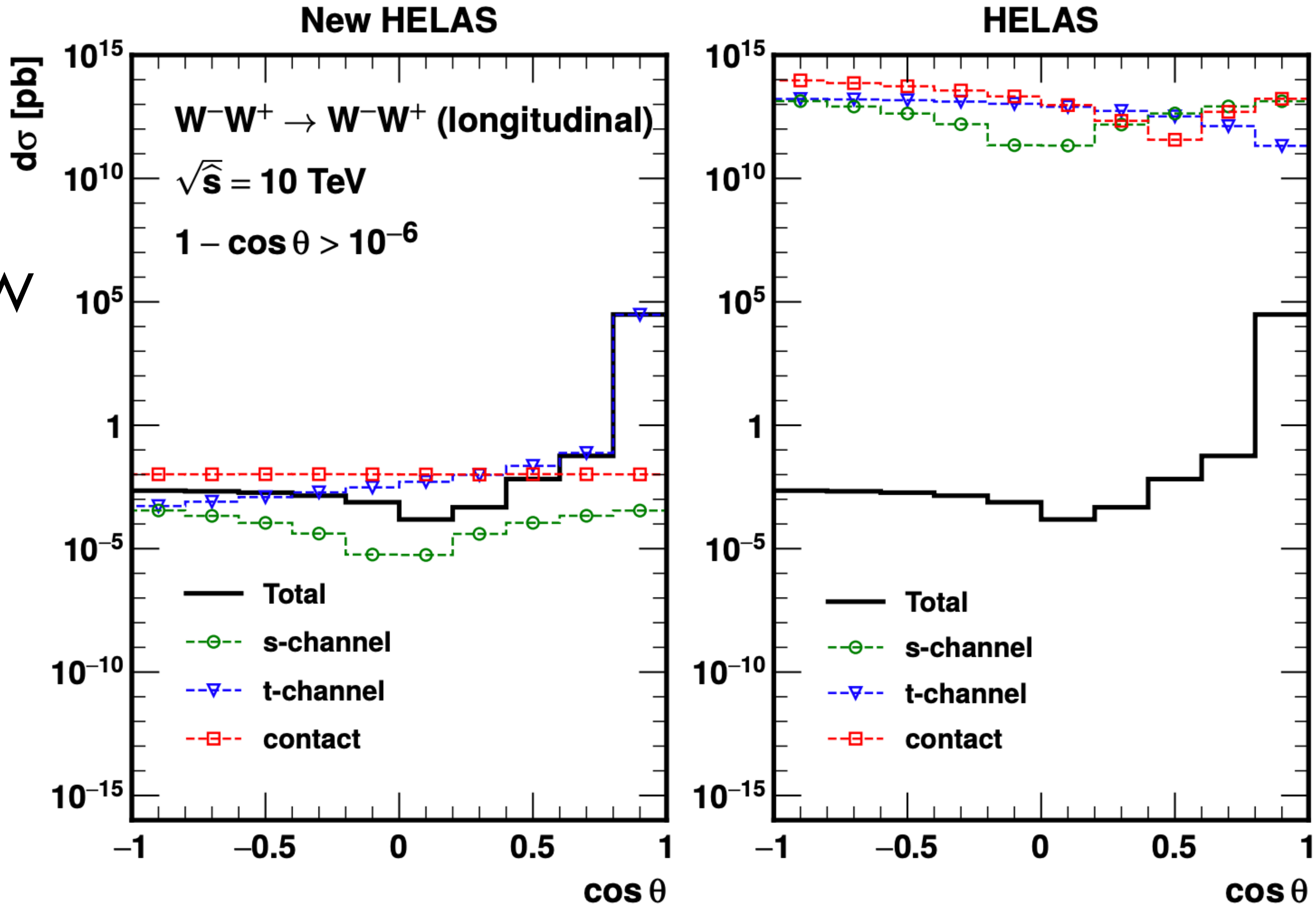


Doesn’t exist in
“Old” Feynman Rules

NFR=New Feynman Rule

■ New HELAS Results: 2→2 VBS

WW>WW



4. Numerical Results

New HELAS Results: 2→2 VBS

ZZ>ZZ