How to identify compact multiquarks in the heavy quark sector

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PhiPsi 2022, August 19th, 2022

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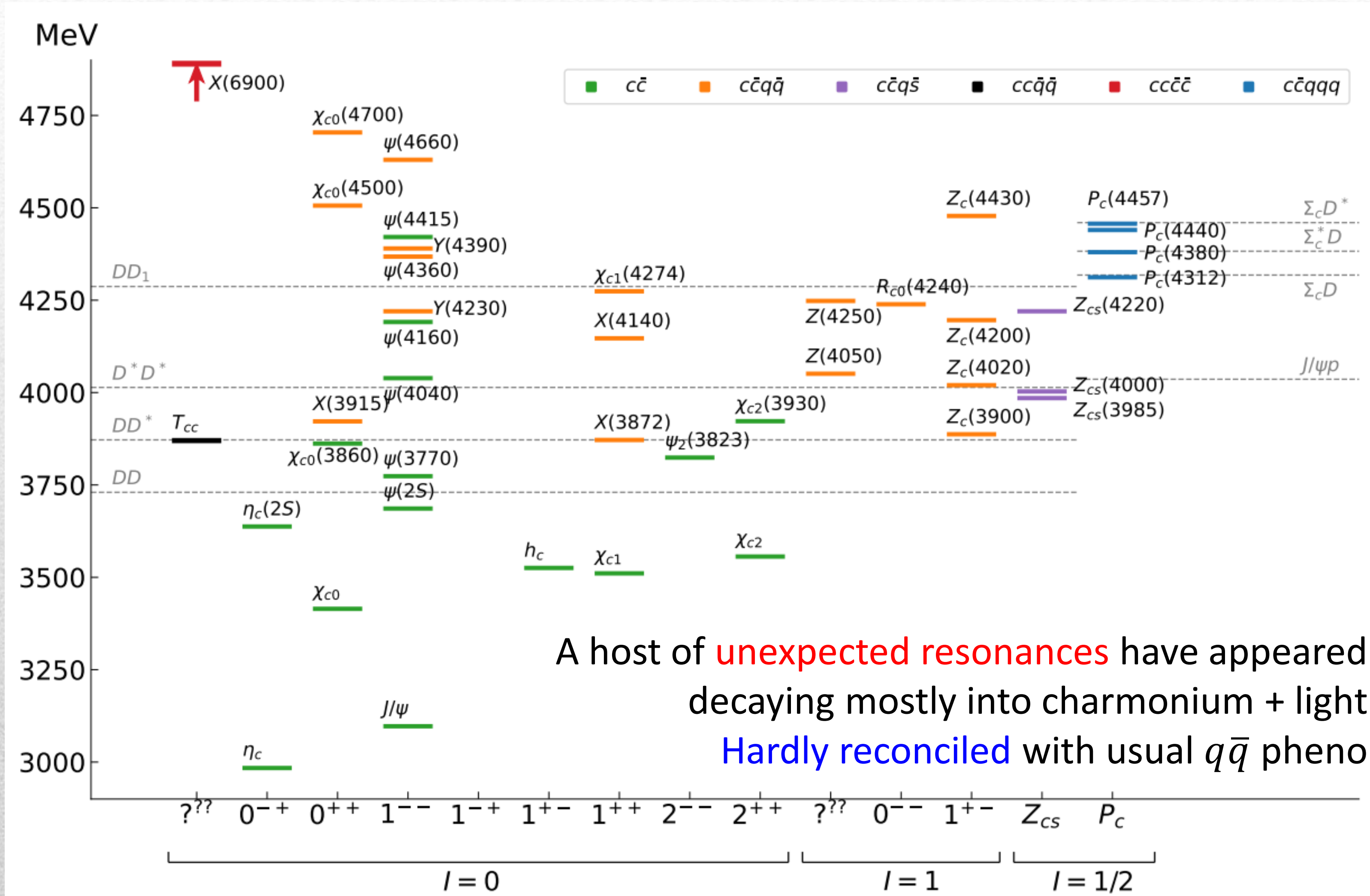
Exotic landscape at $c\bar{c}$

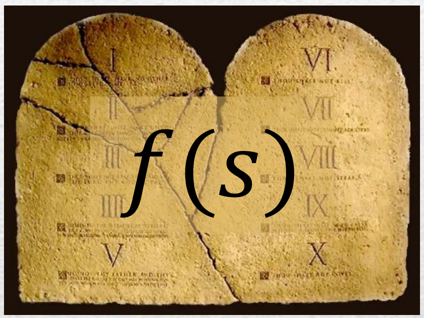
Esposito, AP, Polosa, Phys.Rept. 668
JPAC, arXiv:2112.13436



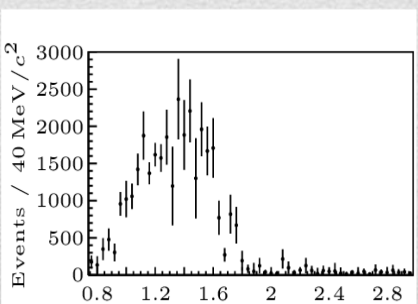
A host of unexpected resonances have appeared decaying mostly into charmonium + light
Hardly reconciled with usual $q\bar{q}$ pheno

The flowchart(s)

1) You are given a model/theory



2) You calculate the amplitude

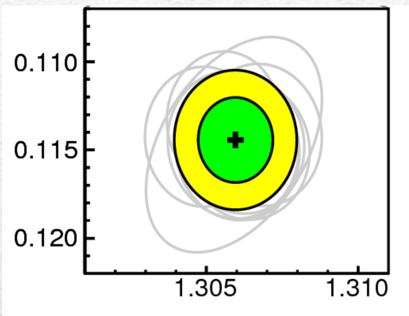


3) You compare with data.
   Or you don't.

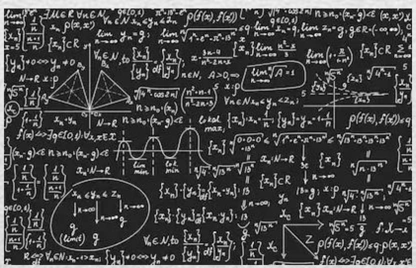
Predictive power ✓
Physical interpretation ✓
(within the model! ✘)
Biased by the input ✘

The flowchart(s)

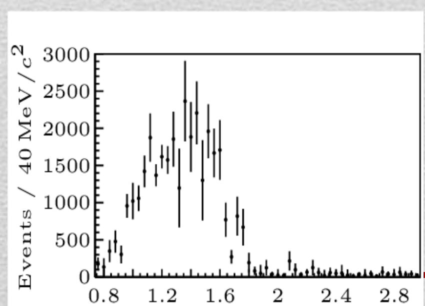
Less predictive power ✘
Some physical interpretation ✘
Minimally biased ✓



3) You extract physics

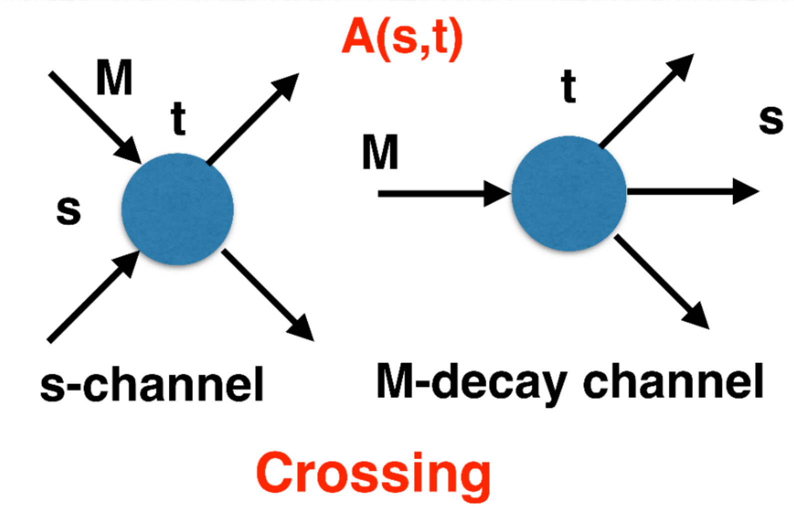


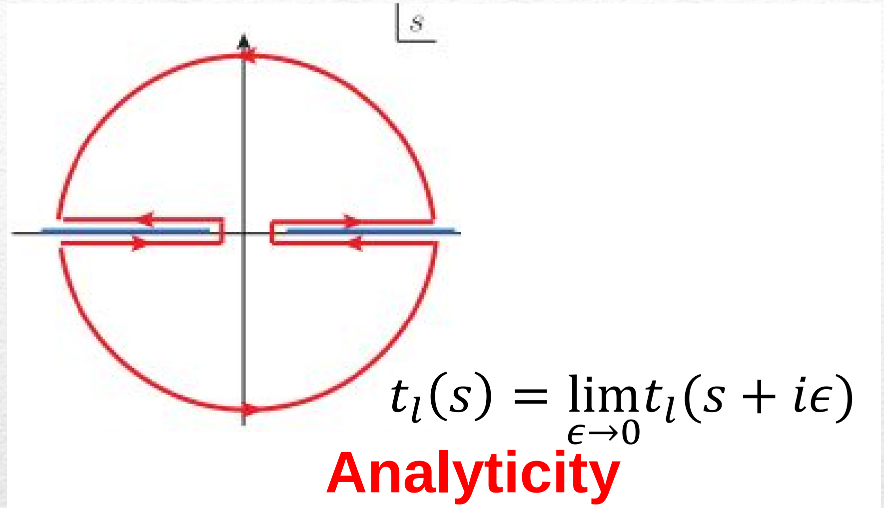
2) You choose a set of generic amplitudes

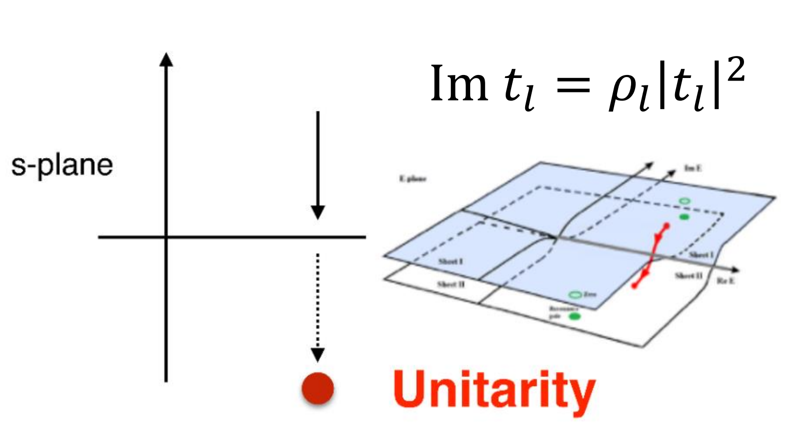


1) You start with data

S-Matrix principles







+ Lorentz, discrete & global symmetries

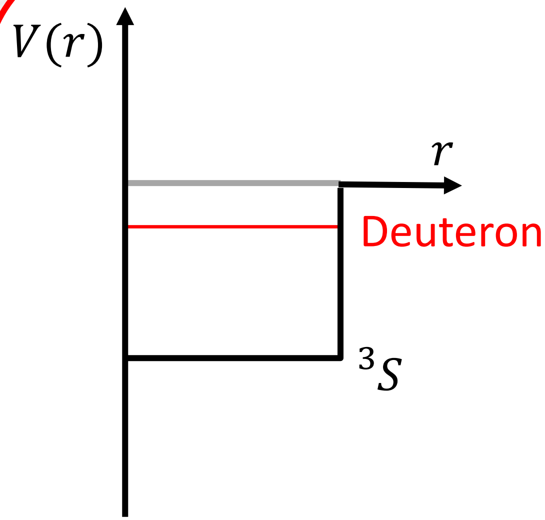
These are constraints the amplitudes have to satisfy, but do not fix the dynamics

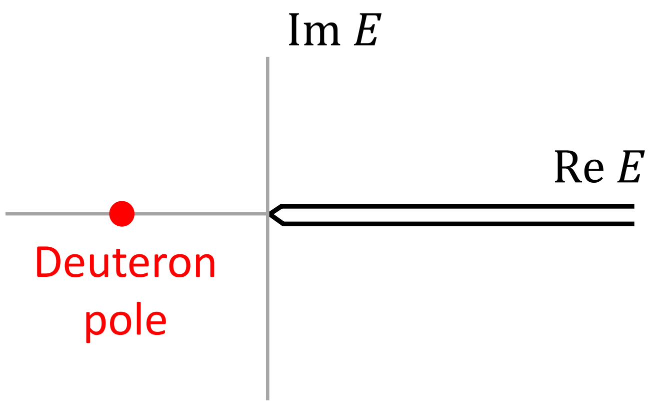
They can be imposed with an increasing amount of rigor, to extract robust physics information

The «background» phenomena can be effectively parameterized in a controlled way

Bound and virtual states

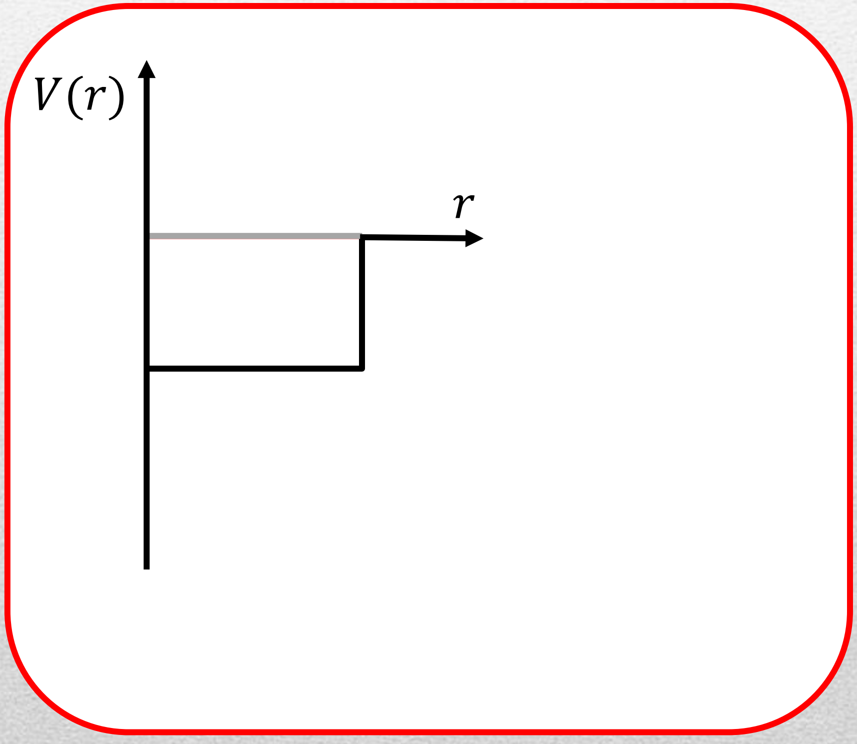
Example from pn scattering
Bound state on the real axis 1st sheet (deuteron)

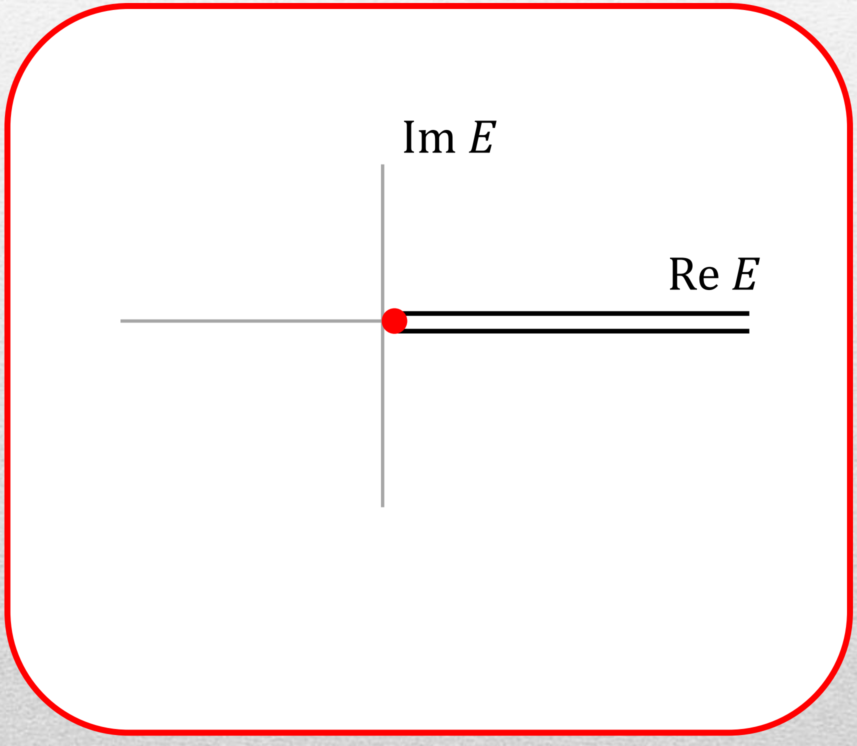




Bound and virtual states

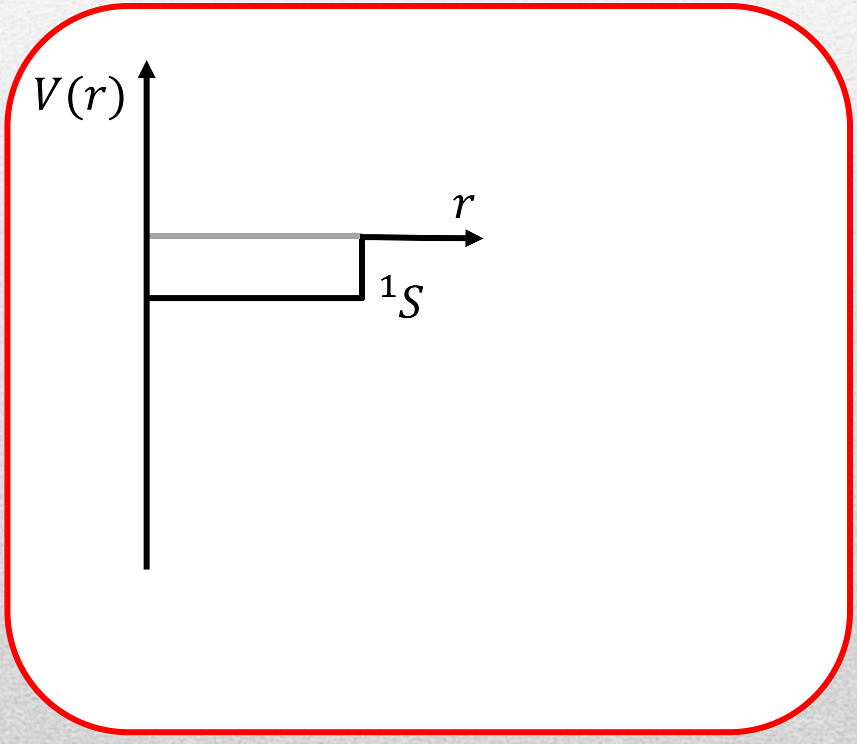
Decreasing the potential strength,
the pole reaches threshold

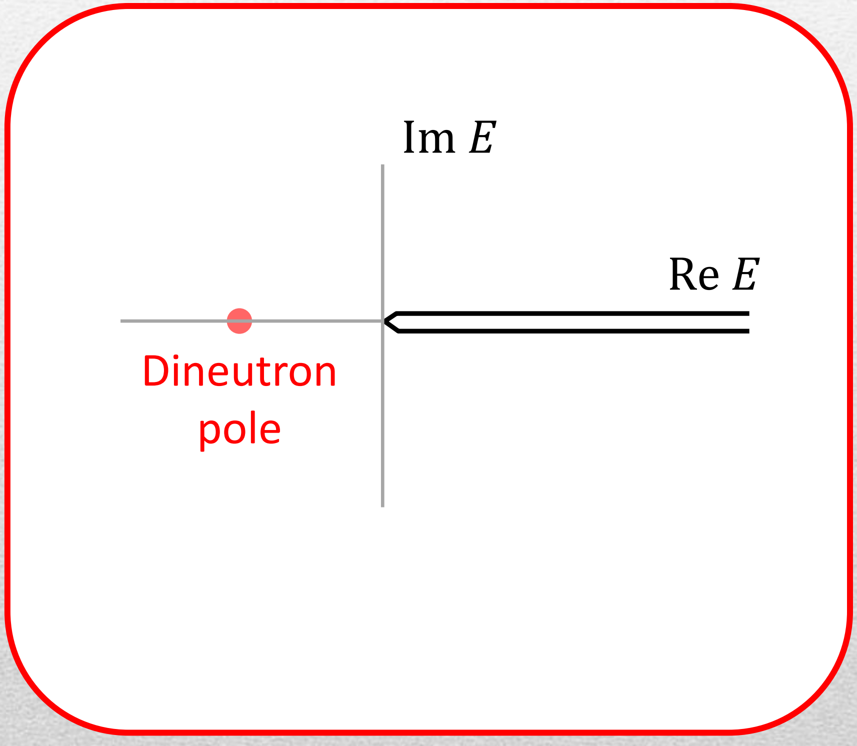




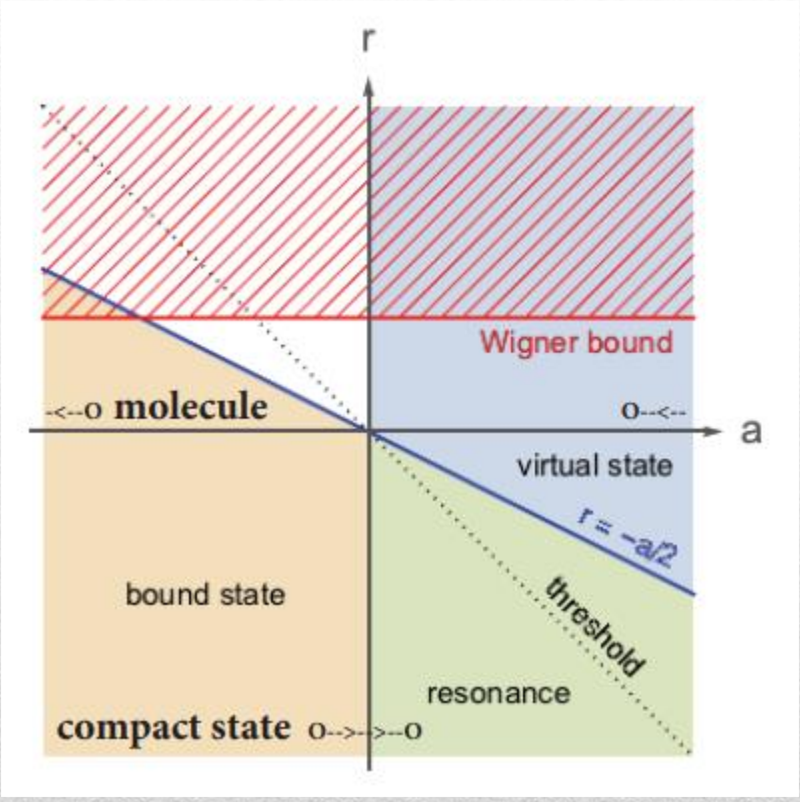
Bound and virtual states

The pole jumps on the 2nd sheet (dineutron),
it becomes a virtual state





Bound and virtual states



Matuschek *et al.* EPJA57 (2021) 3, 101

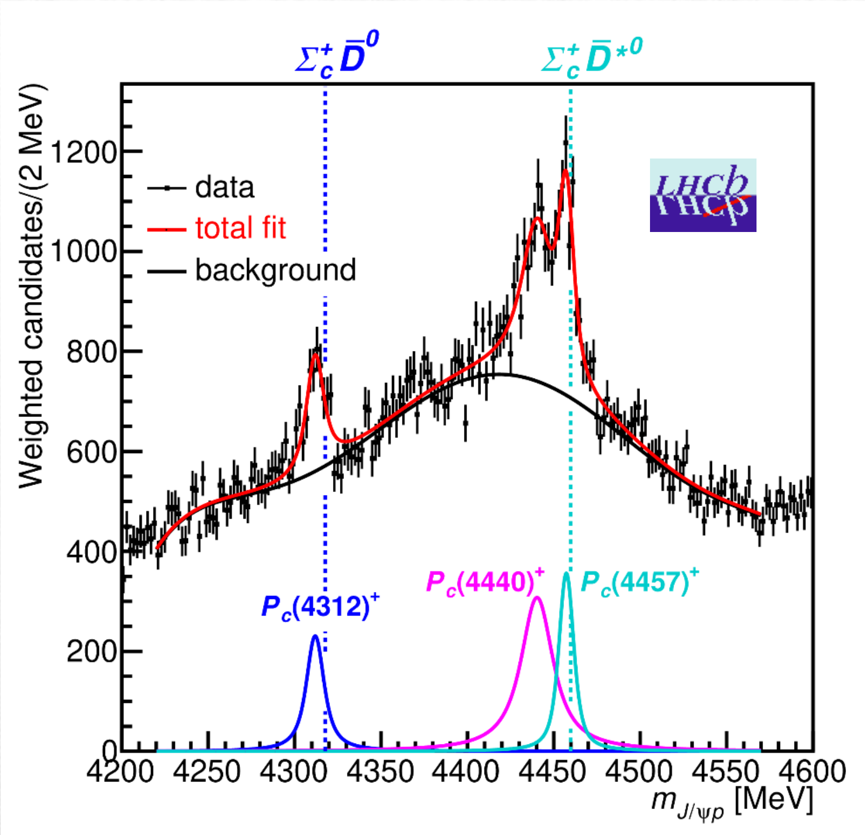
The amplitude close to threshold can be expanded as

$$A(E) = \frac{1}{\frac{1}{a} + \frac{1}{2} r_0 k^2 - ik + O(k^4)}$$

a is the scattering length
r_0 is the effective range

The sign of a controls whether we have a bound or virtual state

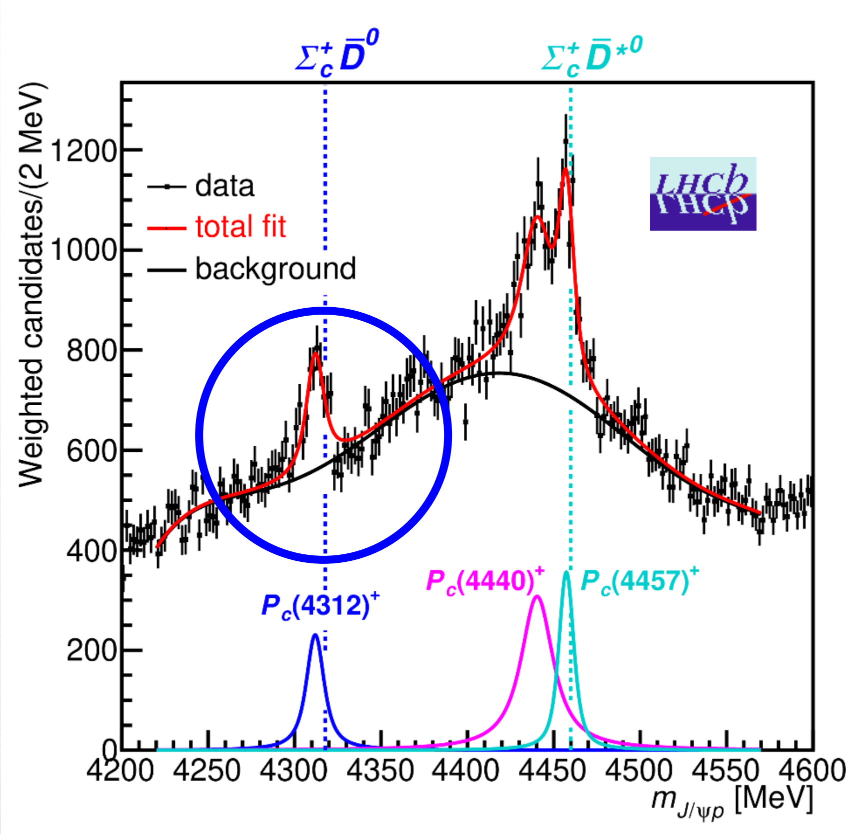
New pentaquarks discovered



The lowest $P_c(4312)$ appears
as an isolated peak
at the $\Sigma_c^+ \bar{D}^0$ threshold

A detailed study of the lineshape provides insight on its nature

New pentaquarks discovered



The lowest $P_c(4312)$ appears
as an isolated peak
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A detailed study of the lineshape provides insight on its nature

Bottom-up:
DON'T YOU DARE describing everything!!!
Focus on the peak region

Minimal(istic) model for $P_c(4312)$

(LHCb data)

Fernandez-Ramirez, AP *et al.* (JPAC), PRL 123, 092001

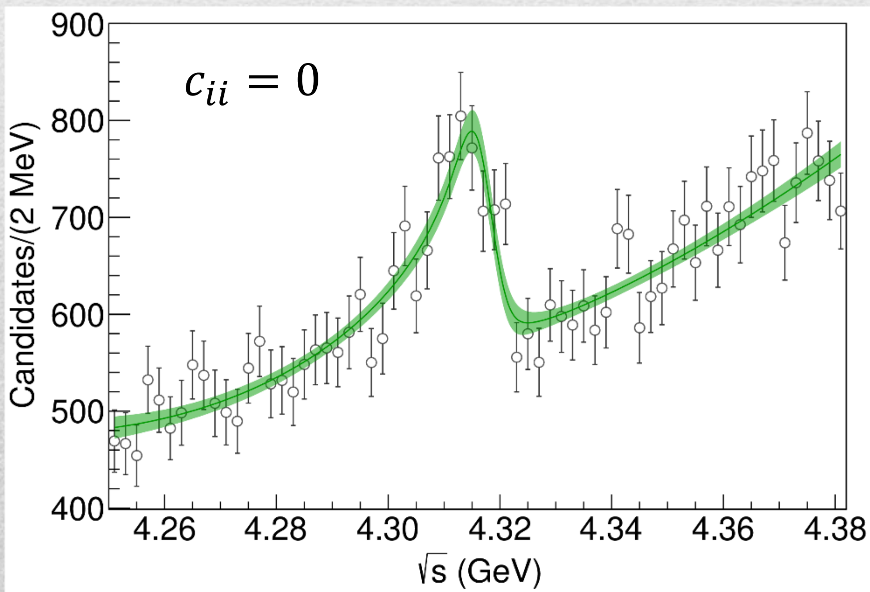
$$\frac{dN}{d\sqrt{s}} = \rho(s)\left[|F(s)|^2 + b_0 + b_1 s\right]$$

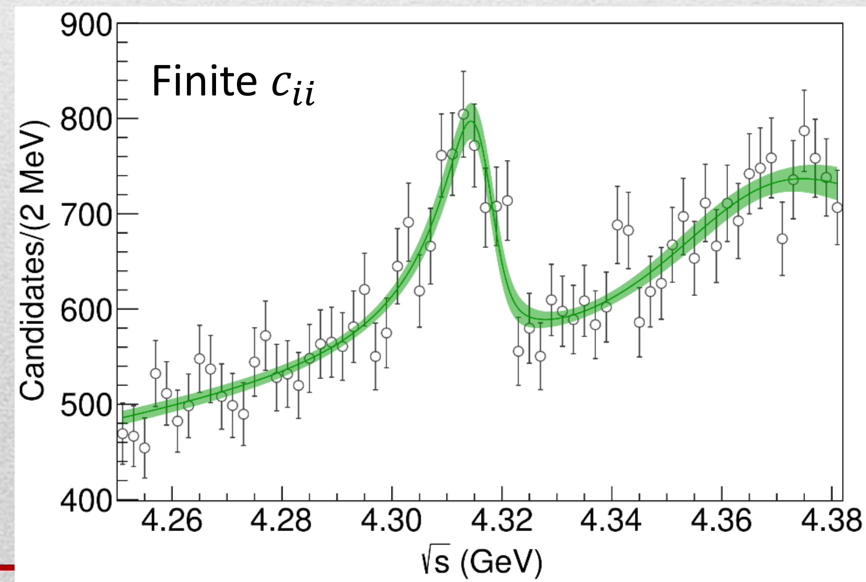
Effective range expansion

$$F(s) = (N_1 + N_2 s)\, T_{11}(s)$$

We can set $c_{ii} = 0$ to reduce to the scattering length approximation

$$T(s) = \begin{pmatrix} m_{11} - c_{11}s - i\rho_1(s) & m_{12} \\ m_{12} & m_{22} - c_{22}s - i\rho_2(s) \end{pmatrix}^{-1}$$





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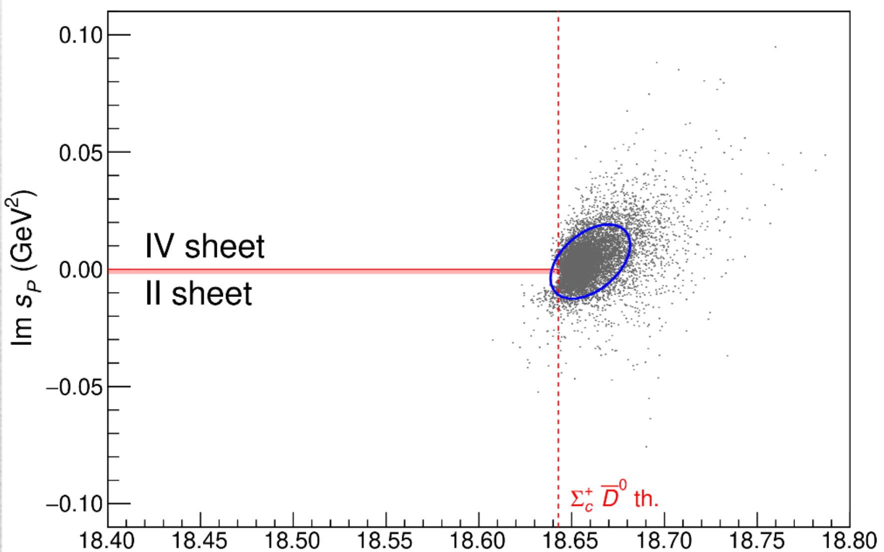
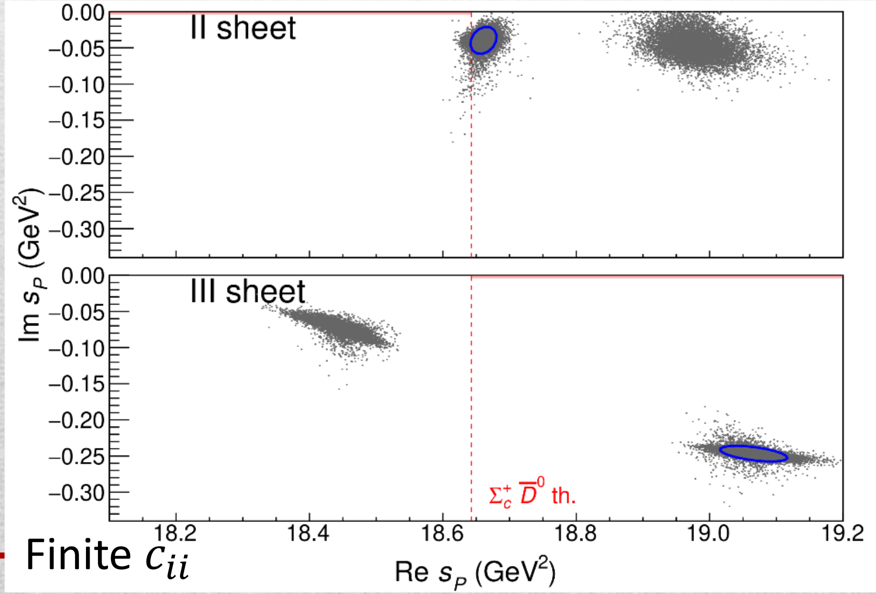


$c_{ii} = 0$



Finite c_{ii}

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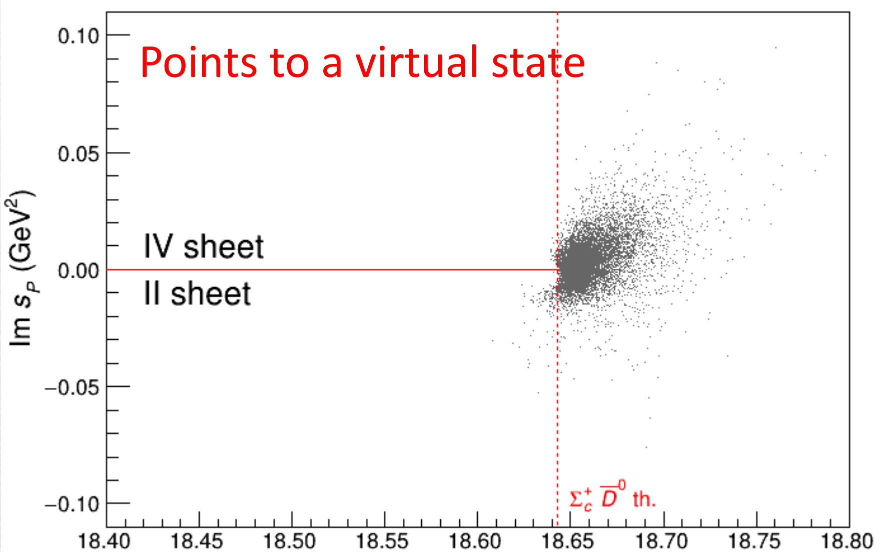

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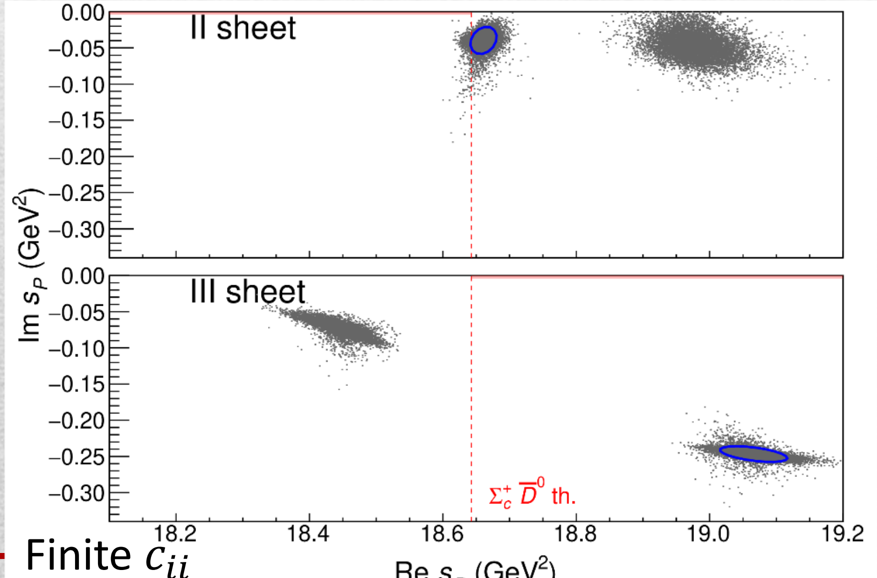

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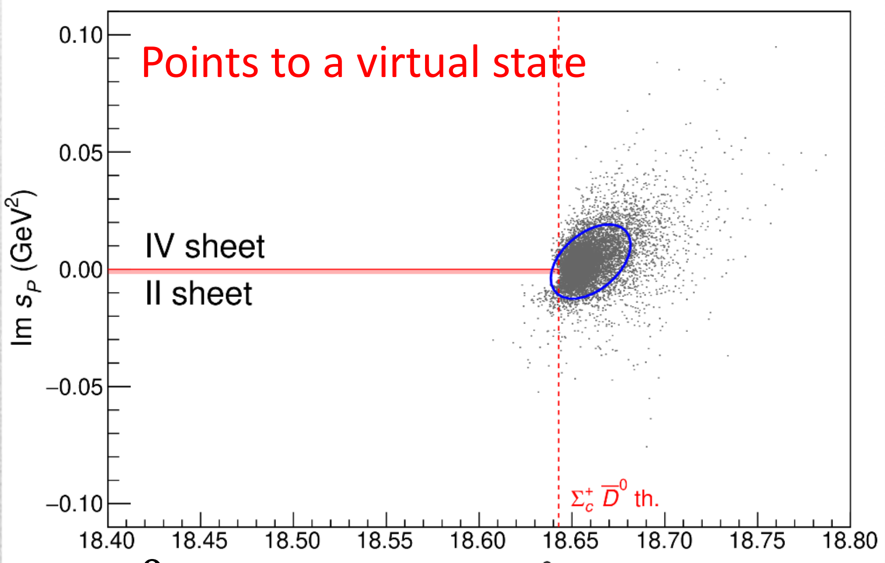

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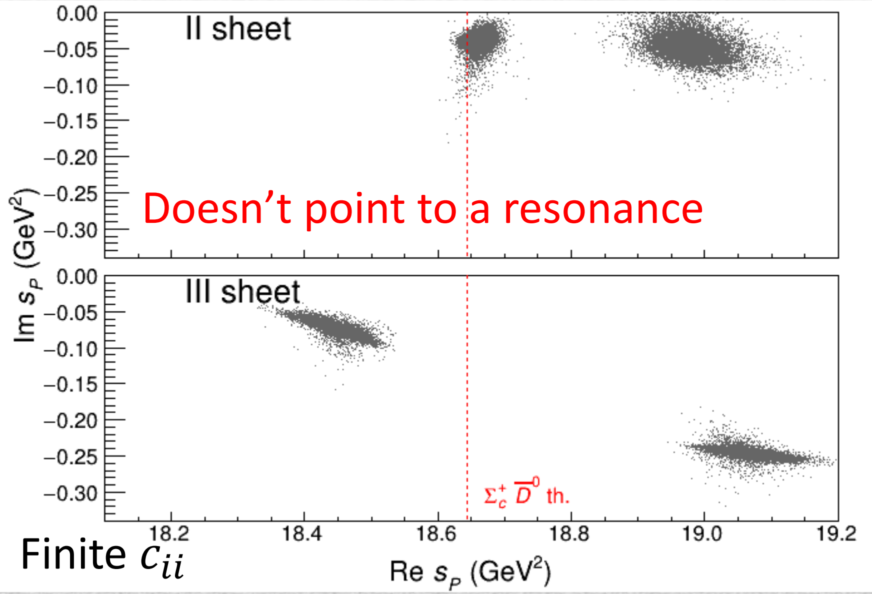
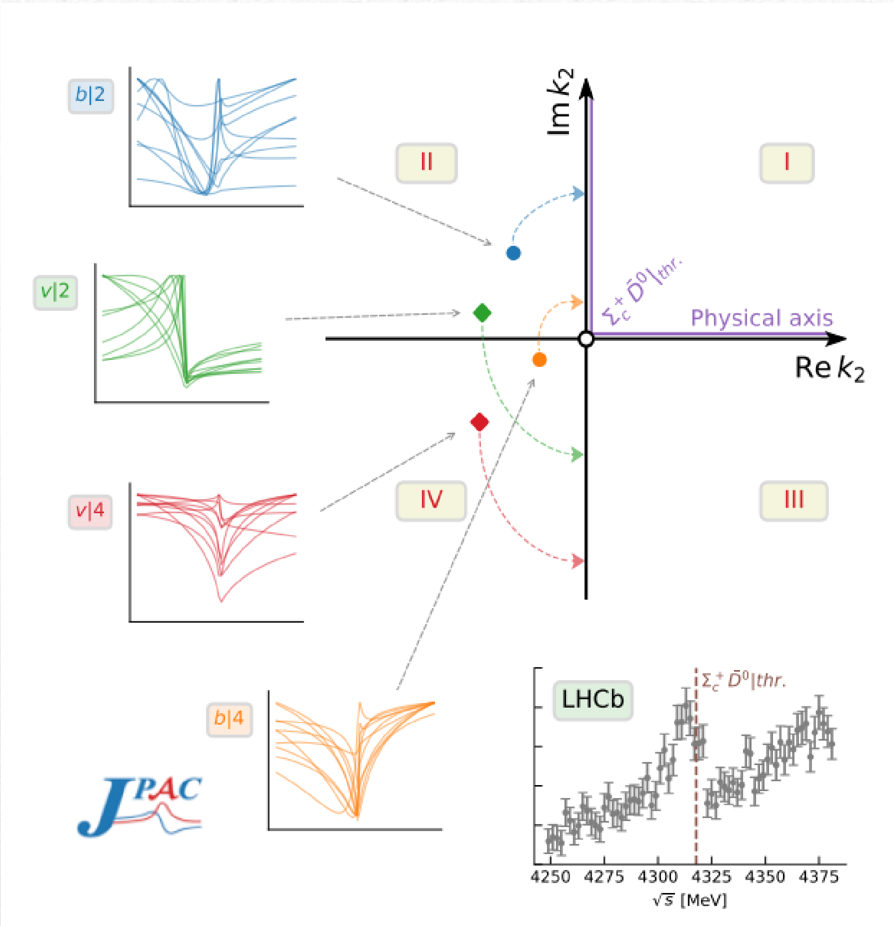


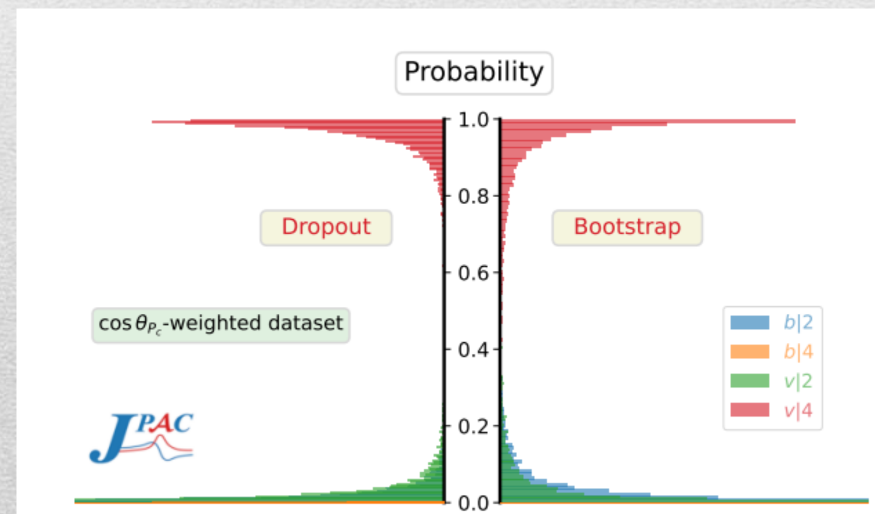
Finite c_{ii}

Minimal(istic) model with ANN

Ng, *et al.* (JPAC), PRD 105, L091501



Same conclusion reached if the analysis is performed with a Deep Neural Network



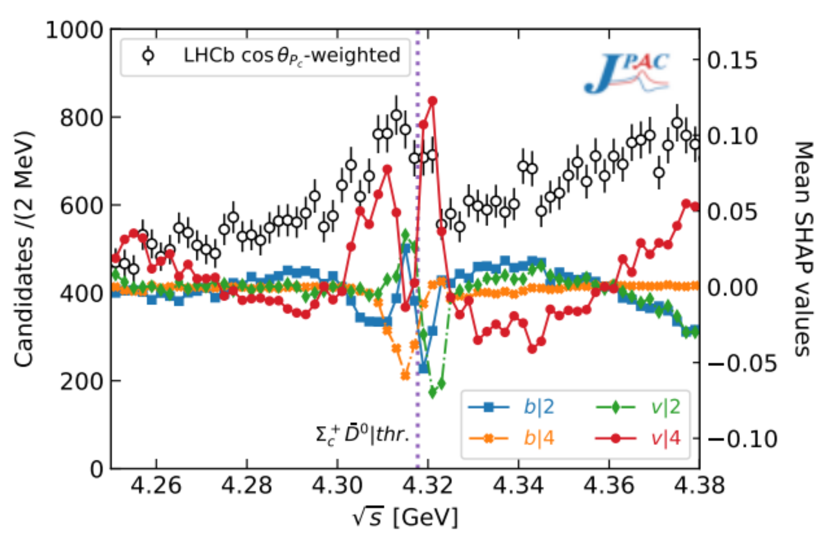
Minimal(istic) model with ANN

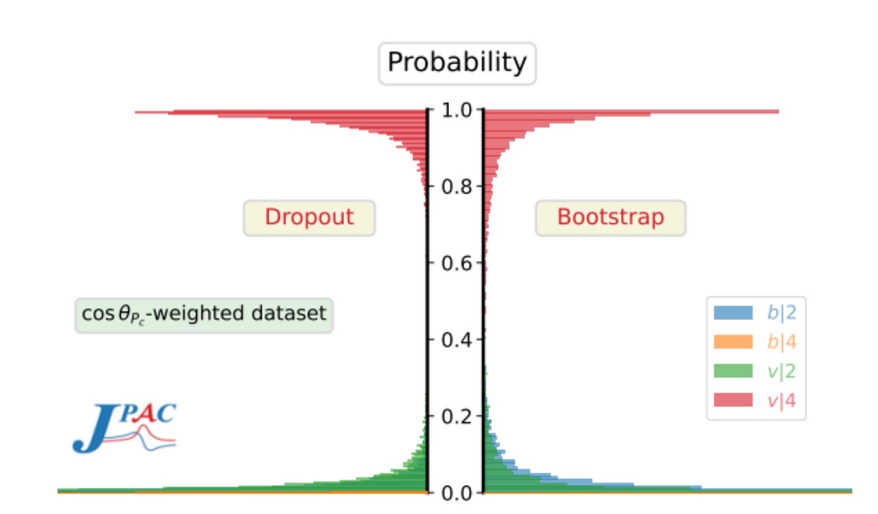
Ng, *et al.* (JPAC), PRD 105, L091501

	$b\|2$	$b\|4$	$v\|2$	$v\|4$
$\cos\theta_{P_c}$-weighted	0.6%	< 0.01%	1.1%	98.3%
$m_{Kp} > 1.9\,\mathrm{GeV}$	1.4%	< 0.1%	1.6%	97.0%
m_{Kp} all	5.4%	< 0.1%	21.0%	73.6%

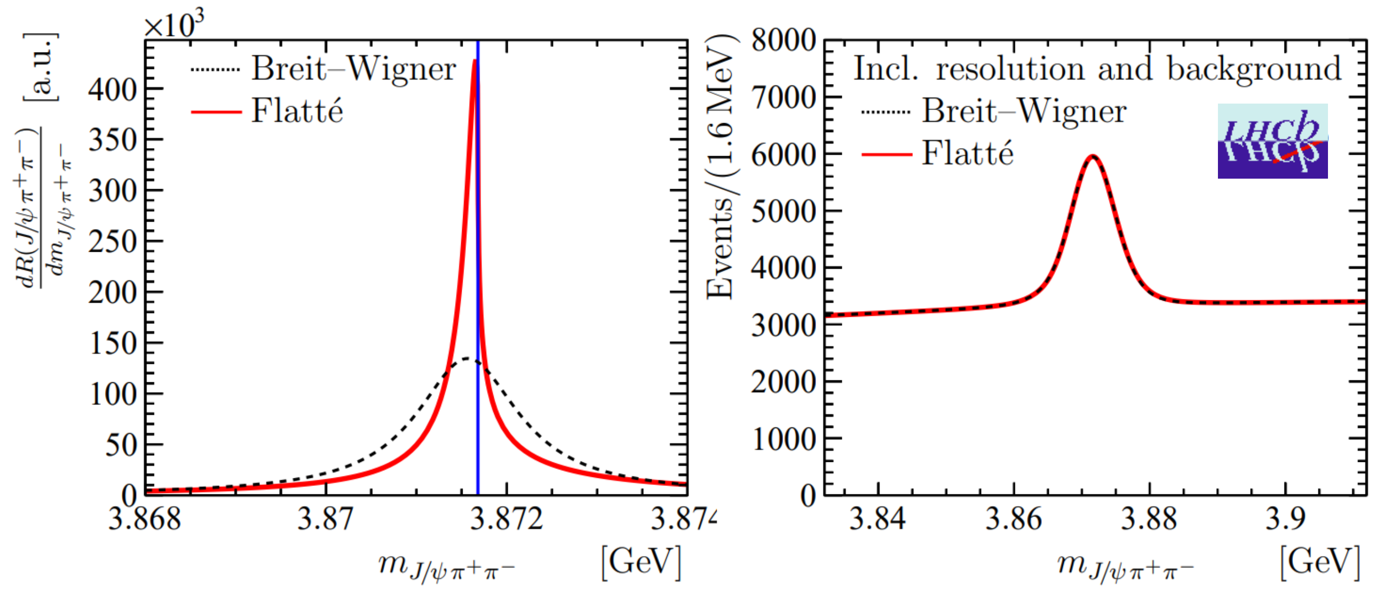
The peak region has the largest impact for the decision

Highest probability for a virtual state in the IV sheet





The lineshape of the $X(3872)$



LHCb, PRD 102, 092005

Blue line is $D^0\overline{D^{*0}}$, D^+D^{*-} is $\delta = 8.2$ MeV heavier

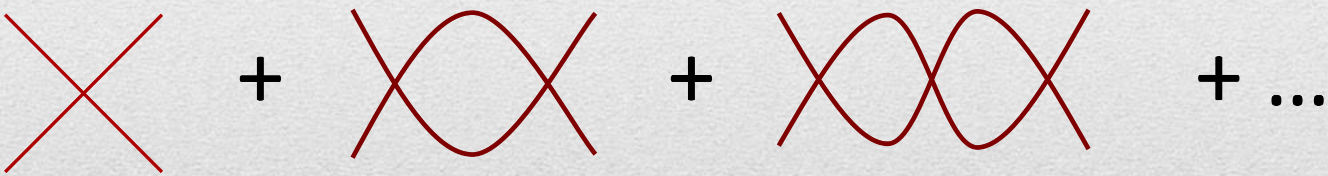
Because of experimental resolution, different lineshapes are indistinguishable

Unitary parametrizations tend to be narrower,
$\Gamma_{BW} = 1.39 \pm 0.24 \pm 0.10$ MeV, $\Gamma_{Fl} = 0.22^{+0.07+0.11}_{-0.06-0.13}$ MeV

Weinberg's criterion and lineshapes

Let us imagine to have a theory with a bound state with a binding momentum much smaller than the inverse of the range of the potential

The potential is just a delta function,
we calculate the $2 \to 2$ scattering amplitude

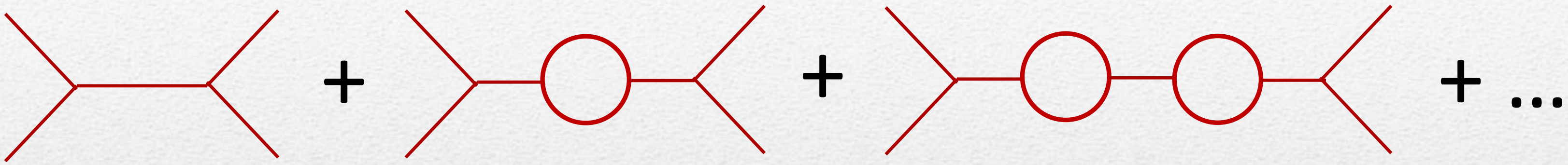


$$A(E) = \frac{1}{1/a - i\sqrt{2\mu E}}$$

This has a pole at $E_B = -\frac{1}{2\mu a^2}$ and residue $g^2 = \sqrt{\frac{2B}{\mu}}$

Weinberg's criterion and lineshapes

Now let us consider the propagation of a bare intermediate state



$$A(E) = \frac{g_0^2}{E_0 - E - i g_0^2 \sqrt{2\mu E}} = \frac{g_0^2}{-E_B - E - i g_0^2 \sqrt{2\mu E} - g_0^2 \sqrt{2\mu E_B}}$$

This has a pole at E_B and residue $g^2 = \sqrt{\frac{2B}{\mu}}(1 - Z)$ where Z is the w.f. renormalization,

$Z = \left(1 + g_0^2 \sqrt{\frac{\mu}{2B}}\right)^{-1}$ = overlap between the bare state and the continuum

Weinberg's criterion and lineshapes

The amplitude can be rewritten as

$$A(E) = \frac{1}{\frac{1}{a} + \frac{1}{2} r_0 k^2 - ik}$$

Thus identifying $a = -2\,\frac{1-Z}{2-Z}\frac{1}{\sqrt{2\mu E_B}}, r_0 = -\frac{Z}{1-Z}\frac{1}{\sqrt{2\mu E_B}}$

So a negative r_0 points to a short range component in the wave function

This is true up to corrections of the order of the range of the potential, which btw are positive under general assumptions

Esposito, Maiani, Pilloni, Polosa, Riquer, 105 (2022) 3, L031503

The lineshape of the $X(3872)$

Esposito, Maiani, AP, Polosa, Riquer,
PRD 105 (2022) 3, L031503

LHCb data is fitted with the Flatté parametrization

The $D^0\overline{D^{*0}}$ threshold   The D^+D^{*-} threshold

$$t^{-1}(E) \propto E - m_X^0 + \frac{i}{2} g_{\text{LHCb}} \left(\sqrt{2\mu E} + \sqrt{2\mu_+ (E-\delta)} \right) + \frac{i}{2} \left(\Gamma_\rho^0(E) + \Gamma_\omega^0(E) + \Gamma_0^0 \right)$$

The $J/\psi\ \rho, \omega$, and other unknown channels

This considers coupled channel, but Weinberg's criterion applies to single channel bound states only

The lineshape of the $X(3872)$

Esposito, Maiani, AP, Polosa, Riquer,
PRD 105 (2022) 3, L031503

LHCb data is fitted with the Flatté parametrization

Two options: a) I set $\delta = 0$ b) I expand at threshold

$$t^{-1}(E) \propto E - m_X^0 + \frac{i}{2} g_{\rm LHCb} \left(\sqrt{2\mu E} + \sqrt{2\mu_+(E-\delta)} \right) + \frac{i}{2} \left(\Gamma_\rho^0(E) + \Gamma_\omega^0(E) + \Gamma_0^0 \right)$$

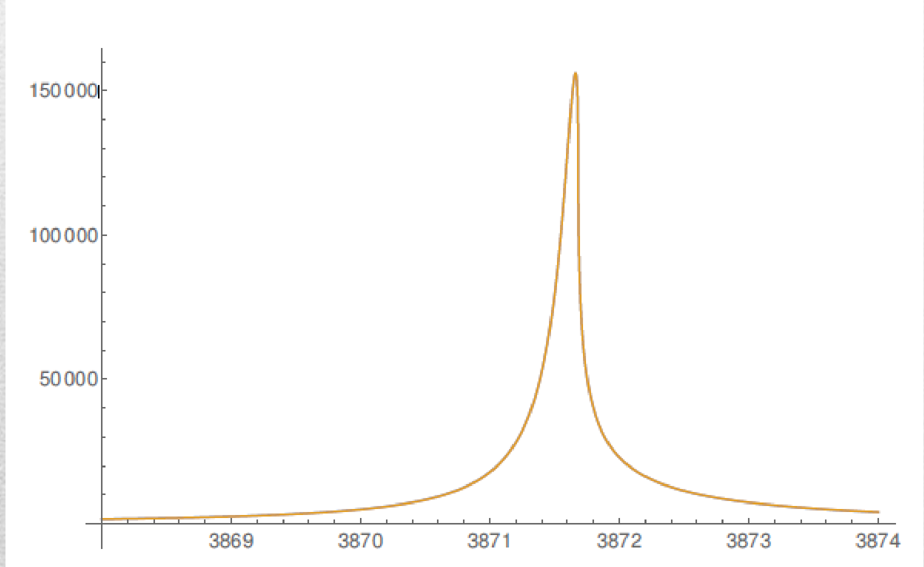
This considers coupled channel, but Weinberg's criterion applies to single channel bound states only

Isospin breaking

Isospin is badly broken. The question we want to ask is:
a) "Is the X a $D^0\overline{D^{*0}}$ molecule" rather than
b) "Is the X a $D\overline{D^*}$ molecule with $I = 0$ in the isospin limit"

For a) expanding $\sqrt{2\mu_+(E-\delta)}$ at $E = 0$ and get r_0 is the right thing to do

This is fine as can be seen by comparing the original curve with the expanded one



The lineshape of the $X(3872)$

Esposito, Maiani, AP, Polosa, Riquer, PRD 105 (2022) 3, L031503

Option b) $-5.34 \text{ fm} \lesssim r_0 \lesssim -1.56 \text{ fm}$

Option a) $-3.78 \text{ fm} \lesssim (r_0)_{\delta\to 0} \lesssim 0 \text{ fm}$

According to Weinberg's, the first result points to a sizeable short-range structure of the X(3872)

Still disagreement on how to perform the extraction though

Conclusions

- Study of lineshapes is informative about the nature of resonances
- Needs for high-statistics precise datasets, more yet to come!

Thank you

BACKUP

A little theorem (Landau-Smorodinski)

- Consider the Schroedinger's equation for the radial wave function of the molecular constituents

$$u_k''(r) + \left[k^2 - U(r)\right] u_k(r) = 0$$

with $U(r) = 2\mu V(r)$, $V(r) < 0$ is the potential, assumed to be attractive everywhere.

- We consider the wave function for two values of the momentum: $u_{k_{1,2}} \equiv u_{1,2}$

With simple manipulations we find the identity

$$u_2 u_1' - u_2' u_1 \Big|_0^R = (k_2^2 - k_1^2) \int_0^R dr\, u_2 u_1 \qquad \text{(A)}$$

R >> a_0, the range of the potential ($\simeq 1/m_\pi$).

- Consider now the free equation, $\psi_k''(r) + k^2 \psi_k(r) = 0$, from which we also obtain

$$\psi_2 \psi_1' - \psi_2' \psi_1 \Big|_0^R = (k_2^2 - k_1^2) \int_0^R dr\, \psi_2 \psi_1 \qquad \text{(B)}$$

- Normalizing to unity at r=0, the general expression for ψ_k is

$$\psi_k(r) = \frac{\sin(kr + \delta(k))}{\sin \delta(k)}, \quad \text{and:} \quad \psi_k'(0) = k \cot \delta(k).$$

- The radial wave function u_k vanishes at r=0, and we normalize so that it tends exactly to the corresponding ψ_k for large enough radii.
- Now, subtract (A) from (B) and let $R \to \infty$ (the integral now is convergent) to find

$$k_2 \cot \delta(k_2) - k_1 \cot \delta(k_1) = (k_2^2 - k_1^2) \int_0^\infty dr\, (\psi_2 \psi_1 - u_2 u_1)$$

L. Maiani

$$k_2 \cot\delta(k_2) - k_1 \cot\delta(k_1) = (k_2^2 - k_1^2)\int_0^\infty dr\,(\psi_2\psi_1 - u_2 u_1) \quad \text{(C)}$$

We compare (C) with the parameters of the scattering amplitude.

First we set $k_1 = 0$. Since $\lim_{k_1\to 0} k_1 \cot\delta(k_1) = -\kappa_0$

$$k_2 \cot\delta(k_2) = -\kappa_0 + k_2^2\int_0^\infty dr\,\left(\psi_2\psi_0 - u_2 u_0\right)$$

For small momenta: $k_2 \cot\delta(k_2) = -\kappa_0 + \frac{1}{2} r_0 k_2^2 + \ldots$ so that

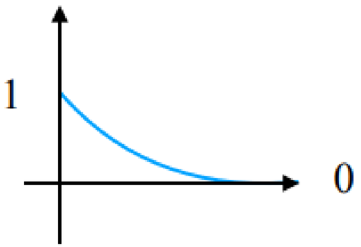
$$\boxed{r_0 = 2\int_0^\infty dr\,(\psi_0^2 - u_0^2)}$$

We know that $u_0(0) = 0$, $\psi_0(0) = 1$. Defining $\Delta(r) = \psi_0(r) - u_0(r)$ we have

$$\Delta(0) = +1,\ \Delta(\infty) = 0$$

The equations of motion imply $\Delta''(r) = -U(r)u_0(r)$. In presence of a single bound state, where u(r) has no nodes, we get

$$\Delta''(r) > 0 \rightarrow \psi_0(r) > u_0(r) \qquad \text{that is}$$



$$\boxed{r_0 > 0}$$

- reassuringly: $r_0(\text{deuteron}) = +1.75$ fm,
- conversely a negative value of $r_0 > 0$ implies $Z > 0$

L. Maiani