On the $\eta_1(1855)$ as a hadronic molecular state and its SU(3) partners

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Introduction

Regarding the newly observed $\eta_1\,(1855)$ [3] is interpreted to be $K_1\,(1400)\,\bar{K}$ bound state in OBE [5], $SU(3)$ symmetry is introduced to $K_1(1400)\bar{K}$ scattering in the framework of Chiral unitary approach. In the low energy scattering involving $\phi^8\,(\pi,\ K\,\eta)$ mesons, the interaction are described in chiral perturbation theory, where the leading order in S-wave scattering is the famous Weinberg-Tomozawa term (WT),

$$\mathcal{L}_I = -\frac{1}{4f_\pi^2}\langle[\Phi^\mu, \partial^\nu\Phi_\mu]\left[\phi^8, \partial_\nu\phi^8\right]\rangle, \tag{1}$$

with $\Phi = (A_1,\, B_1)$ and $f_\pi = 92.1\,\text{MeV}$, and Φ reads

$$A_1\,(1^{++}) = \begin{pmatrix} \frac{a_1^0}{\sqrt{2}}+\frac{f_1^8}{\sqrt{6}} & a_1^+ & K_{1A}^+ \\ a_1^- & -\frac{a_1^0}{\sqrt{2}}+\frac{f_1^8}{\sqrt{6}} & K_{1A}^0 \\ K_{1A}^- & \bar{K}_{1A}^0 & -\frac{2f_1^8}{\sqrt{6}} \end{pmatrix},\quad B_1\,(1^{+-}) = \begin{pmatrix} \frac{b_1^0}{\sqrt{2}}+\frac{h_1}{\sqrt{6}} & b_1^+ & K_{1B}^+ \\ b_1^- & -\frac{b_1^0}{\sqrt{2}}+\frac{h_1}{\sqrt{6}} & K_{1B}^0 \\ K_{1B}^- & \bar{K}_{1B}^0 & -\frac{2}{\sqrt{6}}h_1 \end{pmatrix}. \tag{2}$$

From this Lagrangian we obtain the S-wave transition amplitude among the channels that is

$$V_{ij}(s) = -\frac{\epsilon\cdot\epsilon'}{8F_\pi^2}C_{ij}\left[3s - \left(M^2+m^2+M'^2+m'^2\right) - \frac{1}{s}\left(M^2-m^2\right)\left(M'^2-m'^2\right)\right], \tag{3}$$

where $\epsilon\,(\epsilon')$ stands for the polarization four-vector of the incoming (outgoing) axial-vector meson. The masses $M\,(M')\,, m\,(m')$ correspond to the initial (final) axial-vector mesons and initial (final) pseudoscalar mesons respectively. The indices i and j represent the initial and final PA states, respectively.

The unitarization procedure of the scattering amplitude (V in Eq. 3) is referenced to ChUA,

$$T = (1 - V\,G)^{-1}\,V \tag{4}$$

where the G loop function is dimensional regularized and the subtraction is matched at threshold from loop function regularized by three-momentum cut-off ($q_{max} = 0.7\,\text{GeV}$).

Flavor symmetries in mesons

The physical axial-vector mesons are mixtures of singlet and octet,

$$\eta = \cos\theta_P\eta^8 - \sin\theta_P\eta^1, \quad \eta' = \sin\theta_P\eta^8 + \cos\theta_P\eta^1,$$
$$f_1(1285) = \cos\theta_{^3P_1}f_1^1 + \sin\theta_{^3P_1}f_1^8, \quad f_1(1420) = -\sin\theta_{^3P_1}f_1^1 + \cos\theta_{^3P_1}f_1^8,$$
$$h_1(1170) = \cos\theta_{^1P_1}h_1^1 + \sin\theta_{^1P_1}h_1^8, \quad h_1(1415) = -\sin\theta_{^1P_1}h_1^1 + \cos\theta_{^1P_1}h_1^8,$$
$$K_1(1270) = K_{1A}\sin\theta_{K_1} + K_{1B}\cos\theta_{K_1}, \quad K_1(1400) = K_{1A}\cos\theta_{K_1} - K_{1B}\sin\theta_{K_1},$$

where the mixing angles are listed as follows,

	θ_{K_1}	$\theta_{^3P_1}$	$\theta_{^1P_1}$	θ_P
$Set-A$	57°	52.0°	-17.5°	-11.5°
$Set-B$	34°	23.1°	28.0°	-11.5°

Table 1. The mixing angles $\theta_{K1,^1P_1,^3P_1}$ are correlated in Ref.[4].

Isoscalar scattering

The isoscalar coupled scattering begins with axial-vector meson are in zero width, where the thresholds are listed as follows

Channel	$a_1\pi$	$K_1(1270)\bar{K}$	$f_1(1285)\eta$	$K_1(1400)\bar{K}$	$f_1(1420)\eta$
Threshold	1368	1748	1829	1898	1973

Isoscalar scattering

Solving the Bethe-Salpeter equation in Eq. 4, there are poles in the T-matrix, where the poles and corresponding coupling with respect to channels are presented in Table. 1.

Poles (Set A)	Channels				
1.84 $-i0.00$ $(----++)$	$a_1\pi$	$K_1(1270)\bar{K}$	$f_1(1285)\eta$	$K_1(1400)\bar{K}$	$f_1(1420)\eta$
g_l	$-0.02-i0.01$	$0.05-i0.2$	$0.77-i0.07$	$23.63-i0.72$	$0.36-i0.04$
Poles (Set B)	**Channels**				
1.84 $-i0.00$ $(----++)$	$a_1\pi$	$K_1(1270)\bar{K}$	$f_1(1285)\eta$	$K_1(1400)\bar{K}$	$f_1(1420)\eta$
g_l	$-0.10-i0.05$	$0.01-i0.05$	$0.91-i0.09$	$22.24-i0.81$	$3.93-i0.41$

Comparing to the scattering without including axial-vector meson width, the inclusion of meson width modifies the line shapes of $|T_{44}|^2$ and nontrivial line shapes appear around 1900 MeV.

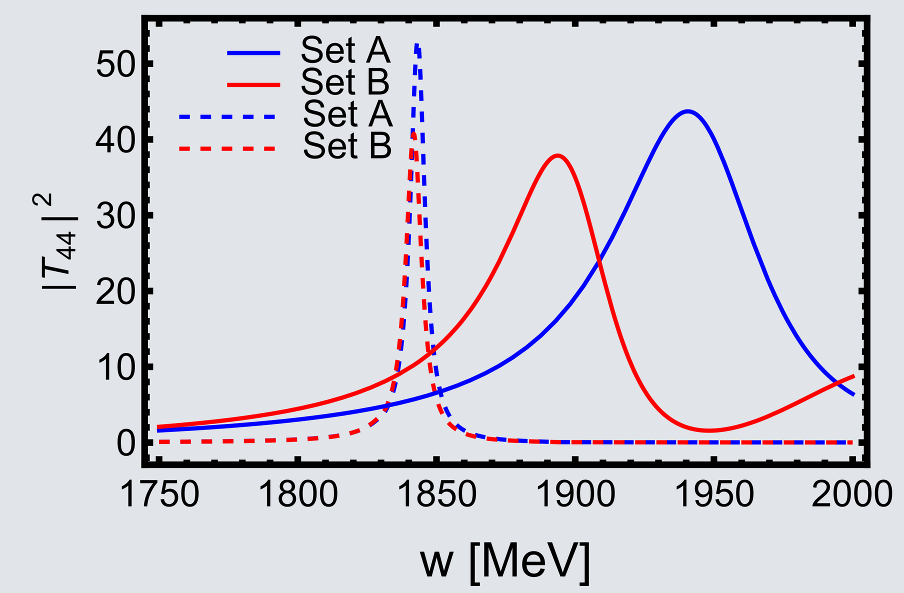
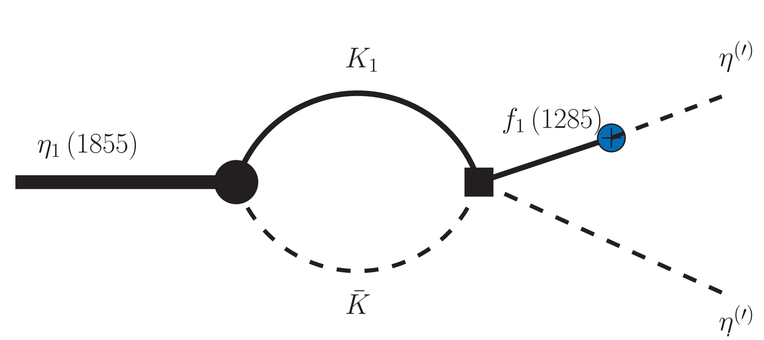


Figure 1. The dashed and solid lines correspond to zero and full widths ($\omega_i \to \omega_i - i\Gamma_i/2$) of the axial-vector mesons in G^{Cut}.

Isoscalar scattering: $a_1 - \pi$ mixing in two-body decay of $\eta_1(1855)$

Considering $\eta_1(1855)$ is observed in $\eta\eta'$ invariant mass distribution, the decay width of $\eta_1 \to \eta\eta'$ can be predicted via $a_1 - \pi$ in Fig. 1.



Referencing the mixing,

$$\partial_\mu\pi \to \partial_\mu\pi + \lambda_1\,m_{a1}\,a_{1\mu}\,,$$

the transition between $K_1(1400)\bar{K}$ and η'/η reads

$$\mathcal{A}^p \propto -\lambda_1\frac{C_{ij}}{4f_\pi^2}\epsilon_{K_1}\left(2m_{K_1}E_K + 2m_{f_1}E_\eta\right)\vec{\epsilon}_{f_1},$$

which contributes to the decay width,

$$\Gamma_{\eta\eta'} = \left(29.76^{-8.67}_{+9.78}\,\text{MeV}\right)^A \ or\ \left(15.72^{-4.30}_{+5.01}\,\text{MeV}\right)^B,\ \ \lambda_1 = 1.6.$$

Isovector scattering

In the coupled channel scattering in $I = 1$ sector, there are poles being candidates of $\pi_1\,(1400/1600)$.

Poles (Set A)	Channels					
1.47 $-i0.12$ $(-+++++)$	$b_1\pi$	$f_1(1285)\pi$	$f_1(1420)\pi$	$K_1(1270)\bar{K}$	$a_1\eta$	$K_1(1400)\bar{K}$
g_l	$3.94-i15.31$	$-0.00+i0.00$	$-0.00+i0.00$	$0.09-i1.05$	$-0.00+i0.00$	$-0.20-i1.50$
1.52 $-i0.04$ $(----+-)$	$b_1\pi$	$f_1(1285)\pi$	$f_1(1420)\pi$	$K_1(1270)\bar{K}$	$a_1\eta$	$K_1(1400)\bar{K}$
g_l	$2.14-i2.98$	$4.27-i0.93$	$1.36+i0.43$	$7.07+i2.37$	$7.52-i1.51$	$0.95-i2.15$
Poles (Set B)	**Channels**					
1.47 $-i0.12$ $(-+++++)$	$b_1\pi$	$f_1(1285)\pi$	$f_1(1420)\pi$	$K_1(1270)\bar{K}$	$a_1\eta$	$K_1(1400)\bar{K}$
g_l	$4.31-i15.18$	$-0.00+i0.00$	$-0.00+i0.00$	$3.67-i2.41$	$-0.00+i0.00$	$-0.05-i0.66$
1.57 $-i0.02$ $(----+-)$	$b_1\pi$	$f_1(1285)\pi$	$f_1(1420)\pi$	$K_1(1270)\bar{K}$	$a_1\eta$	$K_1(1400)\bar{K}$
g_l	$-0.36-i0.96$	$1.26-i4.33$	$3.94-i1.51$	$3.96-i1.07$	$8.93-i2.96$	$4.28+i2.81$

Besides the poles in T-matrix, the ratio of $\pi_1(1600)$ decay into $f_1(1285)\pi$ and $\eta'\pi$ can be evaluated via $a_1 - \pi$ mixing, which is similar to the one in Fig. 1, where the predicted ratio is

$$\mathcal{R}_1 = \left(3.92^{+0.38}_{-0.31}\right)^A,\ \left(4.76^{+0.31}_{-0.28}\right)^B$$

and the one listed in PDG is 3.80 ± 0.78.

Scattering in $I = 1/2$ sector

In $f_1K - K_1(1270)\eta\ (\Gamma_i = 0)$ coupled channel scattering may relate to the jump around 1770 MeV in ϕK^+ invariant mass distribution in $B \to J/\psi\phi K$ [1, 2].

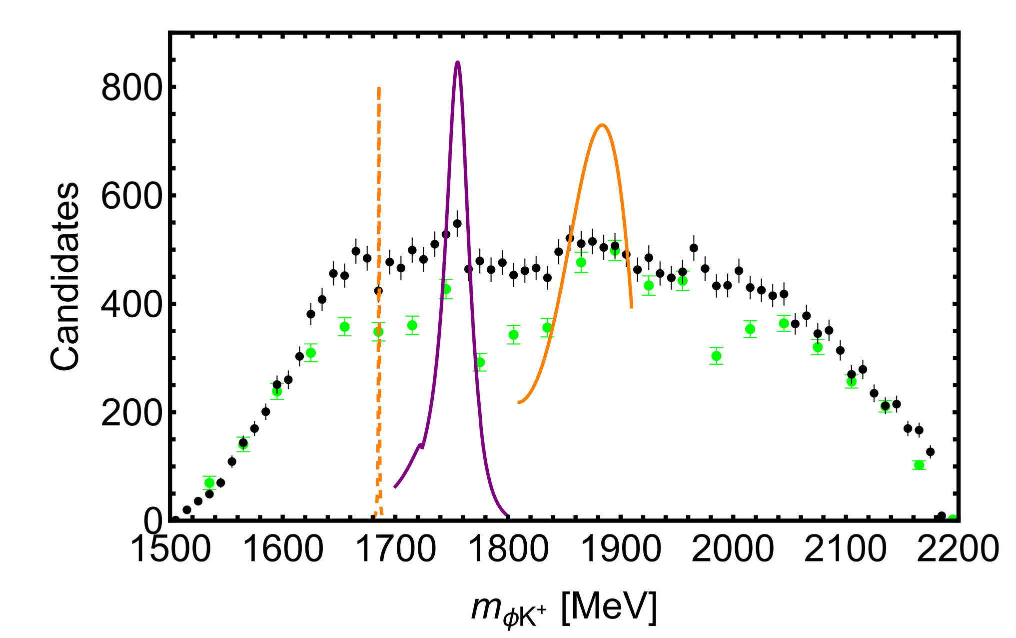


Figure 2. In set-B, b_1K, $K_1(1270)\eta - f_1K$ and $K_1\,(1270)\,\eta - h_1\,(1415)\,K$

References

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