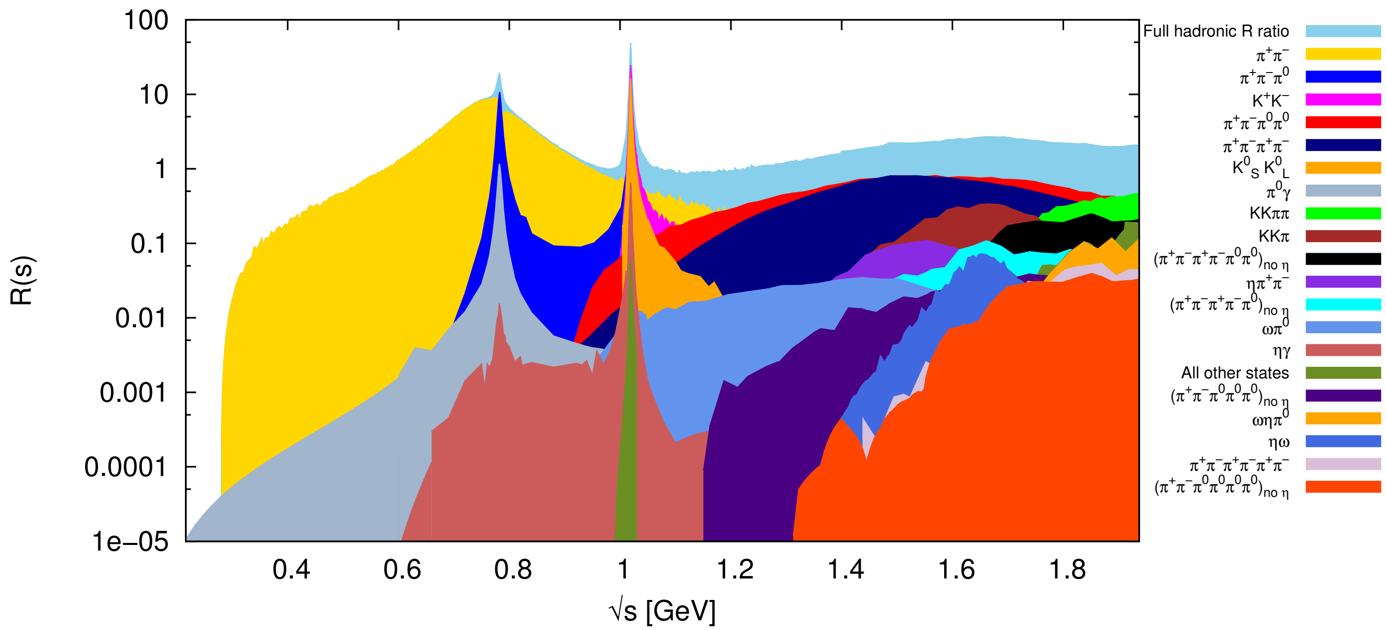
Muon g-2: data-driven HVP from KNT



Alex Keshavarzi

@AlexKeshavarzi

The 13th International Workshop on e+e- collisions from Phi to Psi

17th August 2022

MANCHESTER 1824

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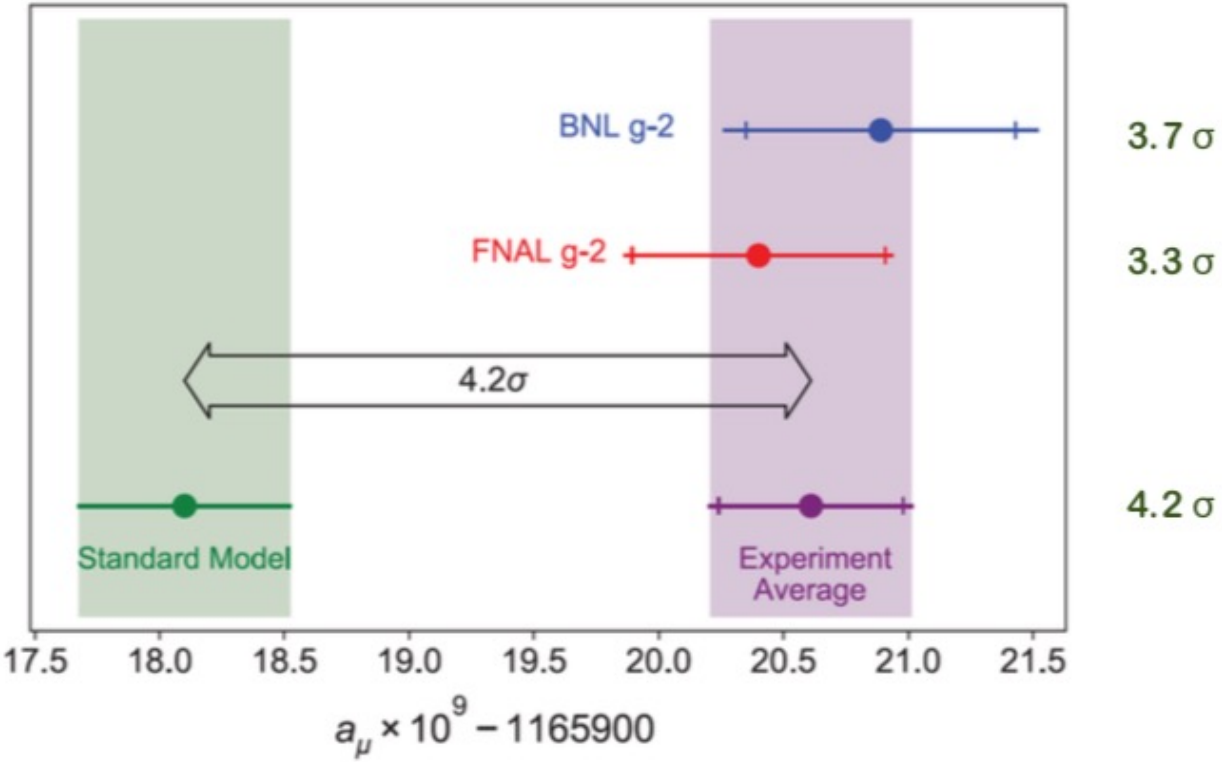
Muon g-2: FNAL confirms BNL



$a_\mu^{EXP} = (116592089 \pm 63) \times 10^{-11}$ [0.54ppm] BNL E821

$a_\mu^{EXP} = (116592040 \pm 54) \times 10^{-11}$ [0.46ppm] FNAL E989 Run 1

$a_\mu^{EXP} = (116592061 \pm 41) \times 10^{-11}$ [0.35ppm] WA

Magnetic moments

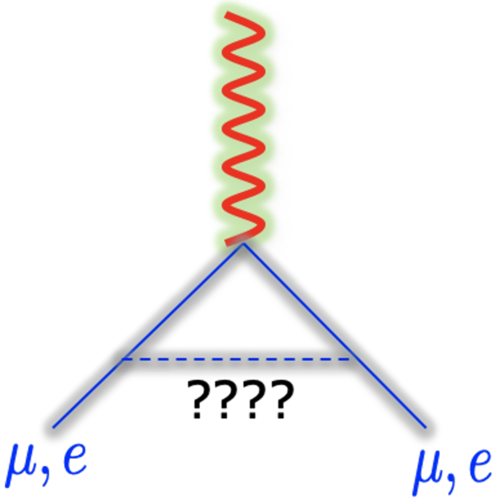
The muon has an intrinsic magnetic moment that is coupled to its spin via the gyromagnetic ratio *g*:

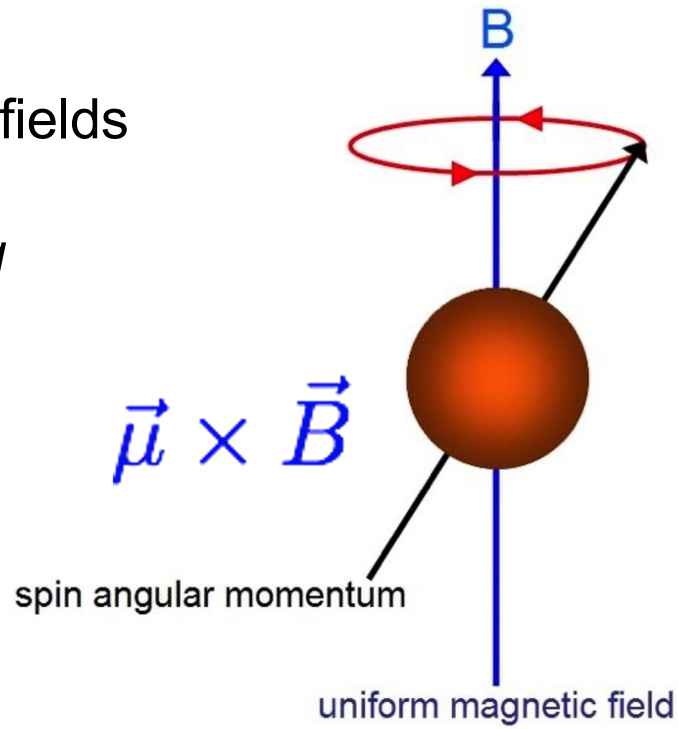
$$\vec{\mu} = g\frac{e}{2m_\mu}\vec{S}$$

Magnetic moment (spin) interacts with external B-fields

Makes spin precess at frequency determined by *g*

$$a_\mu = \frac{g-2}{2}$$





Muon g-2 Theory

arXiv.org > hep-ph > arXiv:2006.04822

Search...
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High Energy Physics – Phenomenology

[Submitted on 8 Jun 2020]

The anomalous magnetic moment of the muon in the Standard Model

T. Aoyama, N. Asmussen, M. Benayoun, J. Bijnens, T. Blum, M. Bruno, I. Caprini, C. M. Carloni Calame, M. Cè, G. Colangelo, F. Curciarello, H. Czyż, I. Danilkin, M. Davier, C. T. H. Davies, M. Della Morte, S. I. Eidelman, A. X. El-Khadra, A. Gérardin, D. Giusti, M. Golterman, Steven Gottlieb, V. Gülpers, F. Hagelstein, M. Hayakawa, G. Herdoíza, D. W. Hertzog, A. Hoecker, M. Hoferichter, B.-L. Hoid, R. J. Hudspith, F. Ignatov, T. Izubuchi, F. Jegerlehner, L. Jin, A. Keshavarzi, T. Kinoshita, B. Kubis, A. Kupich, A. Kupść, L. Laub, C. Lehner, L. Lellouch, I. Logashenko, B. Malaescu, K. Maltman, M. K. Marinković, P. Masjuan, A. S. Meyer, H. B. Meyer, T. Mibe, K. Miura, S. E. Müller, M. Nio, D. Nomura, A. Nyffeler, V. Pascalutsa, M. Passera, E. Perez del Rio, S. Peris, A. Portelli, M. Procura, C. F. Redmer, B. L. Roberts, P. Sánchez-Puertas, S. Serednyakov, B. Shwartz, S. Simula, D. Stöckinger, H. Stöckinger-Kim, P. Stoffer, T. Teubner, R. Van de Water, M. Vanderhaeghen, G. Venanzoni, G. von Hippel, H. Wittig, Z. Zhang, M. N. Achasov, A. Bashir, N. Cardoso, B. Chakraborty, E.-H. Chao, J. Charles, A. Crivellin, O. Deineka, A. Denig, C. DeTar, C. A. Dominguez, A. E. Dorokhov, V. P. Druzhinin, G. Eichmann, M. Fael, C. S. Fischer, E. Gámiz, Z. Gelzer, J. R. Green, S. Guellati-Khelifa, D. Hatton, N. Hermansson-Truedsson et al. (32 additional authors not shown)

The Muon g-2 Theory Initiative

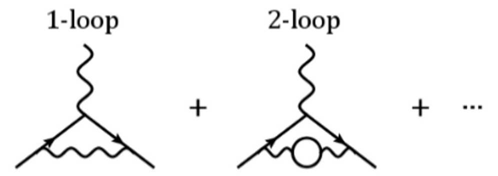
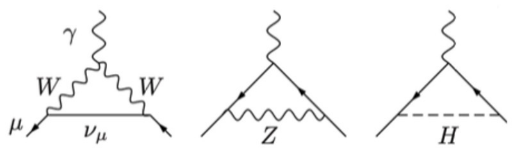
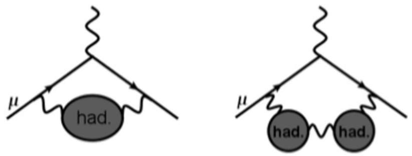
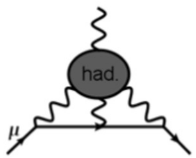
Next meeting, Edinburgh, 05/09/22 - 09/09/22: https://indico.ph.ed.ac.uk/event/112/

Muon g-2 in the SM

$$\Delta a_\mu = 279(76) \times 10^{-11} \rightarrow 2.39(0.65)\ \text{ppm}$$

- a_μ arises due to quantum corrections / higher order interactions / loop contributions
- All SM particles contribute → Calculate and sum all sectors of the SM:

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{HVP}} + a_\mu^{\text{HLbL}}$$

			a_μ^{SM} portion	δa_μ^{SM} portion
QED		Perturbative (Known to five-loop)	$\sim$ 99.99%	$\sim$0.001%
EW		Perturbative (Known to two-loop)	$\sim$ 1 ppm	$\sim$0.2%
HVP		Non-perturbative (Data-driven & lattice)	$\sim$ 59 ppm	$\sim$**84%**
HLbL		Non-perturbative (Data-driven & lattice)	$\sim$ 1 ppm	$\sim$**16%**

Muon g-2 in the SM

$\Delta a_\mu = 279(76) \times 10^{-11} \rightarrow 2.39(0.65)$ ppm

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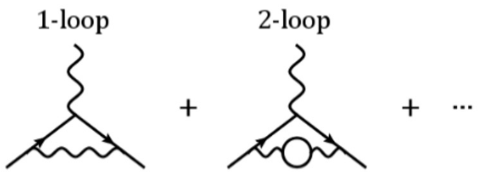
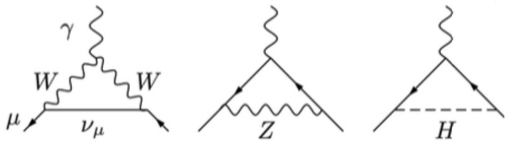
- a_μ arises due to quantum corrections / higher order interactions / loop contributions
- All SM particles contribute → Calculate and sum all sectors of the SM:

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{HVP}} + a_\mu^{\text{HLbL}}$$

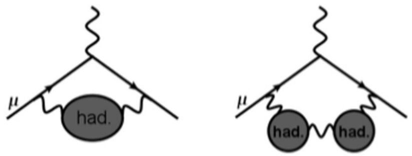
QED



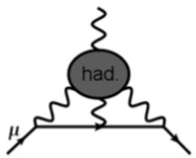
EW



HVP



HLbL

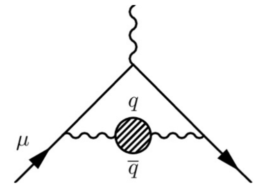


		a_μ^{SM} portion	δa_μ^{SM} portion
QED	Perturbative (Known to five-loop)	$\sim$ 99.99%	~0.001%
EW	Perturbative (Known to two-loop)	~ 1 ppm	~0.2%
HVP	Non-perturbative (Data-driven & lattice)	~ 59 ppm	**~84%**
HLbL	Non-perturbative (Data-driven & lattice)	~ 1 ppm	**~16%**

The 2021 status of the HVP

$$\Delta a_\mu = 279(76) \times 10^{-11} \to 2.39(0.65)\ \text{ppm}$$

- Hadronic Vacuum Polarisation - hadronic blob coupled to 2 photons.
- Two-point function - in principle, much easier than HLbL.
- Most precisely calculated from $e^+e^- \to$ hadrons cross section data.



Lattice (error ~ 1.6 ppm of a_μ^{SM})

- Uncertainties dominated by finite volume, discretisation and isospin breaking systematics.

Data-driven (error ~ 0.3 ppm of a_μ^{SM})

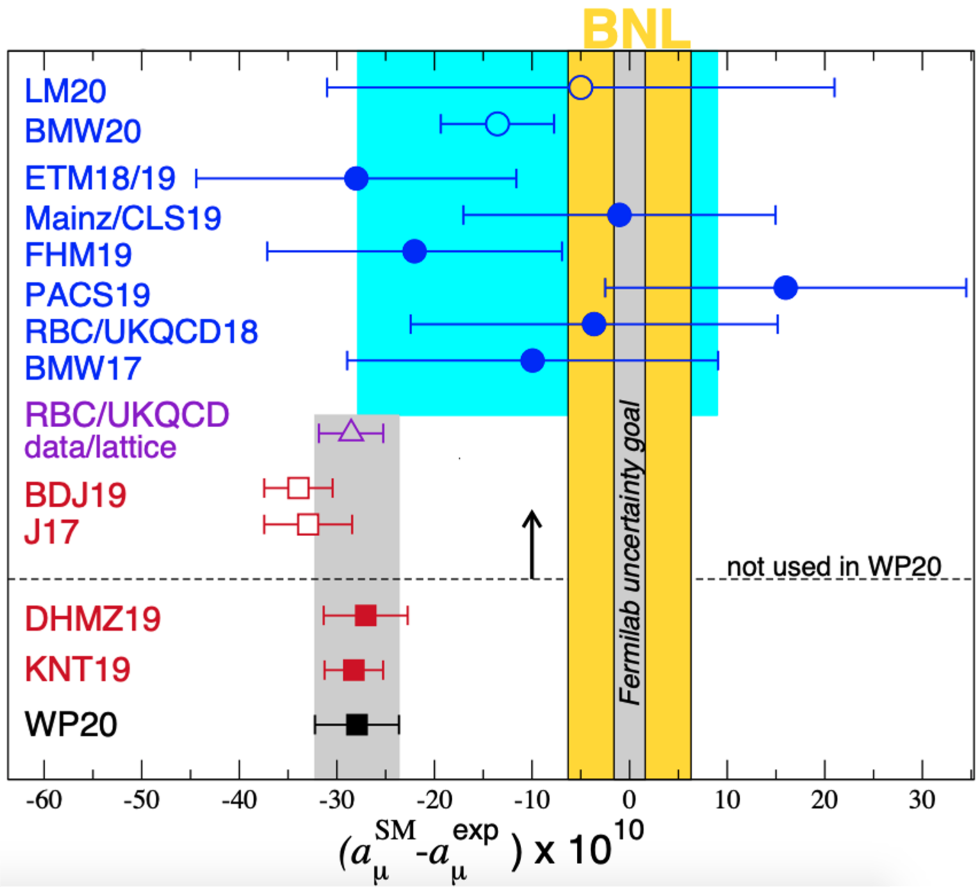
- Cross section data consistently combined and input into dispersion integral:

$$a_\mu^{\mathrm{LO\,HVP}} = \frac{1}{4\pi^3}\int_{s_{th}}^{\infty} \mathrm{d}s\, K(s)\, \sigma_{\mathrm{had}}(s)$$

- Several groups have achieved this.

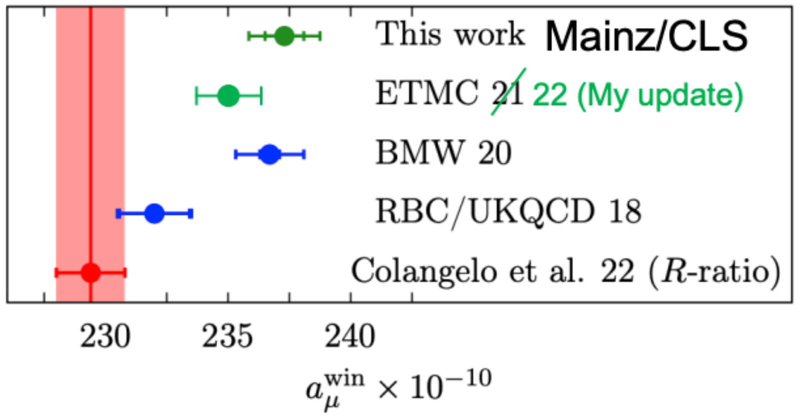
Recommended Muon g-2 TI value from data-driven result:

$$a_\mu^{\mathrm{HVP}} = 6845(40) \times 10^{-11}$$

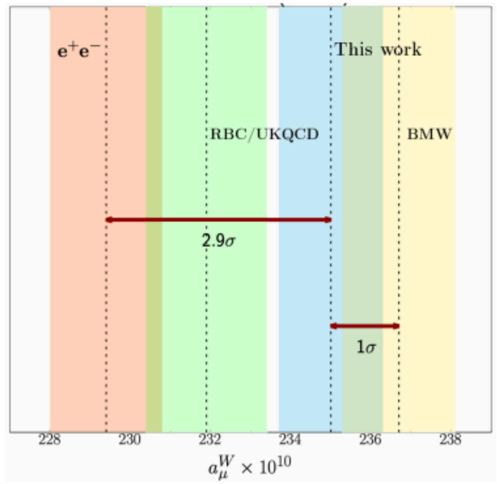


Current status of the HVP

https://arxiv.org/abs/2206.06582 Jun 14th 2022



ETMC Talk at SchwingerFest

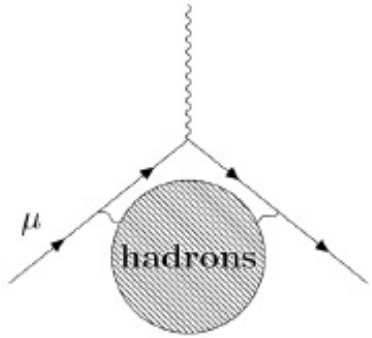


- New value from **Mainz/CLS** agrees with **BMW20**.
- **ETMC 21** closer to data-driven. Update moves it over by ~1σ to match BMW better if I'm understanding the talk slides correctly?
- **RBC/UKQCD (2018)** seems to agree more with data-driven
- New results expected soon from **FNAL/MILC & RBC/UKQCD**

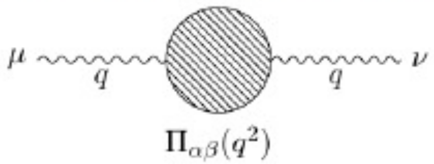
Dispersive HVP: theoretical setup

The University of Manchester

⇒ We want to calculate the leading order hadronic vacuum polarisation (HVP) contribution

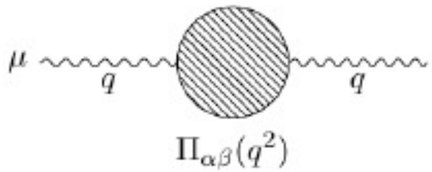


1) Feynman rules for HVP insertion to photon propagator:



$$= \frac{-ig^{\mu\alpha}}{(q^2 - i\varepsilon)}(-ie)i\Pi_{\alpha\beta}(q^2)(-ie)\frac{-ig^{\beta\nu}}{(q^2 - i\varepsilon)}$$

2) Employ analyticity:



$$= \frac{ie^2 g_{\mu\nu}}{(q^2 - i\varepsilon)^2}\frac{q^4}{\pi}\int_{s_{th}}^{\infty} \mathrm{d}s \frac{\mathrm{Im}\,\Pi(s)}{s(s - q^2 - i\varepsilon)}$$

3) Insert to vertex correction, solve for a_μ: $a_\mu^{\mathrm{had,\,LO\,VP}} = \frac{\alpha}{\pi^2}\int_{s_{th}}^{\infty}\frac{\mathrm{d}s}{s}\mathrm{Im}\,\Pi_{\mathrm{had}}(s)K(s)$

4) Utilise optical theorem:

5) Arrive at equation for $a_\mu^{\mathrm{had,\,LO\,VP}}$:

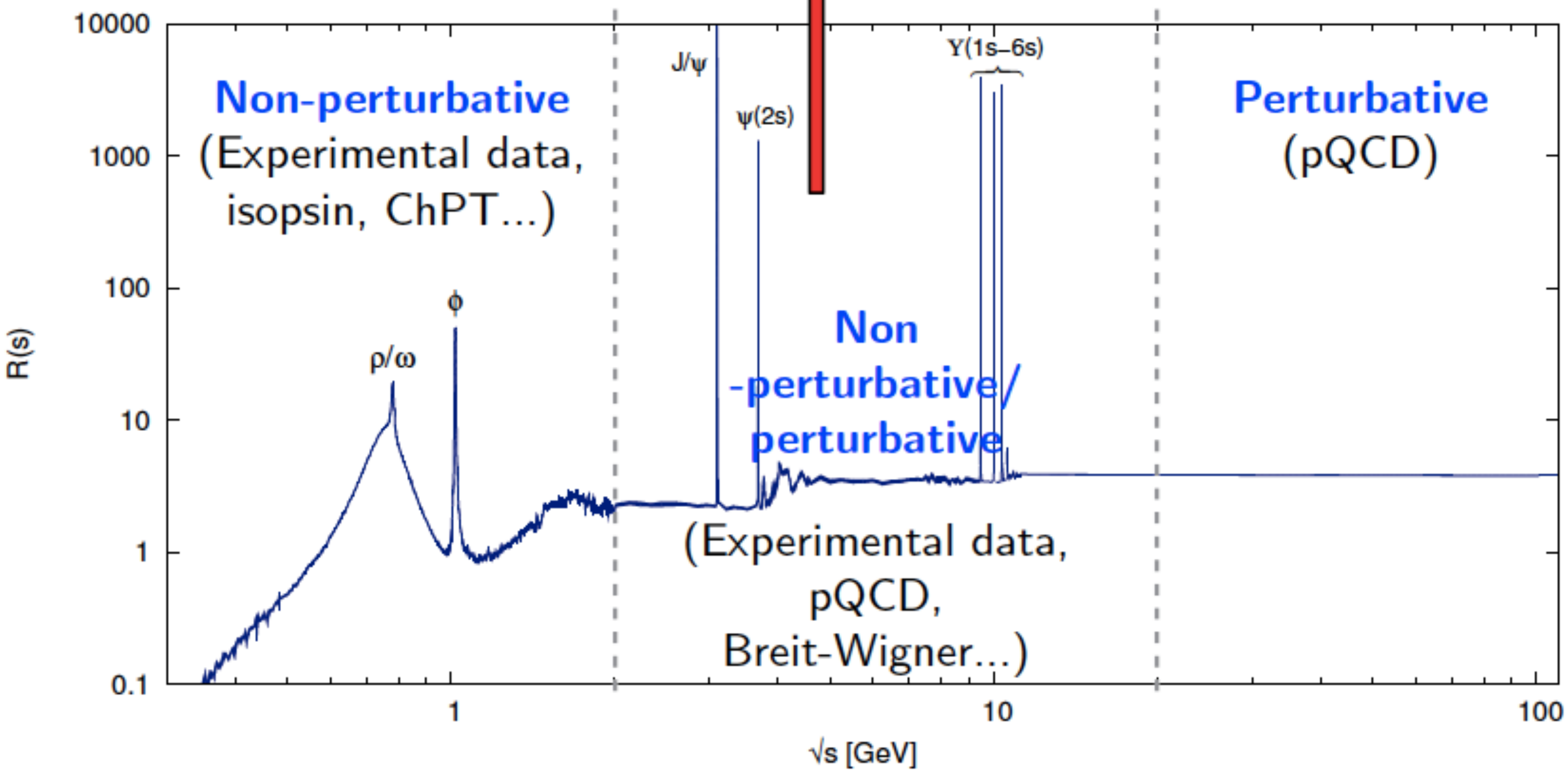
$$a_\mu^{\mathrm{had,\,LO\,VP}} = \frac{1}{4\pi^3}\int_{s_{th}}^{\infty} \mathrm{d}s\, \sigma^0_{\mathrm{had},\gamma}(s) K(s)$$

$\sigma^0_{\mathrm{had},\gamma}$ = bare cross section, FSR included

⇒ Similar dispersion integrals for NLO and NNLO HVP

Building the hadronic R-ratio

$$a_\mu^{\mathrm{had,\,LO\,VP}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{ds}{s}R(s)K(s), \text{ where } R(s) = \frac{\sigma^0_{\mathrm{had},\gamma}(s)}{4\pi\alpha^2/3s}$$



Dispersive HVP

Slide content by Aida El-Khadra.

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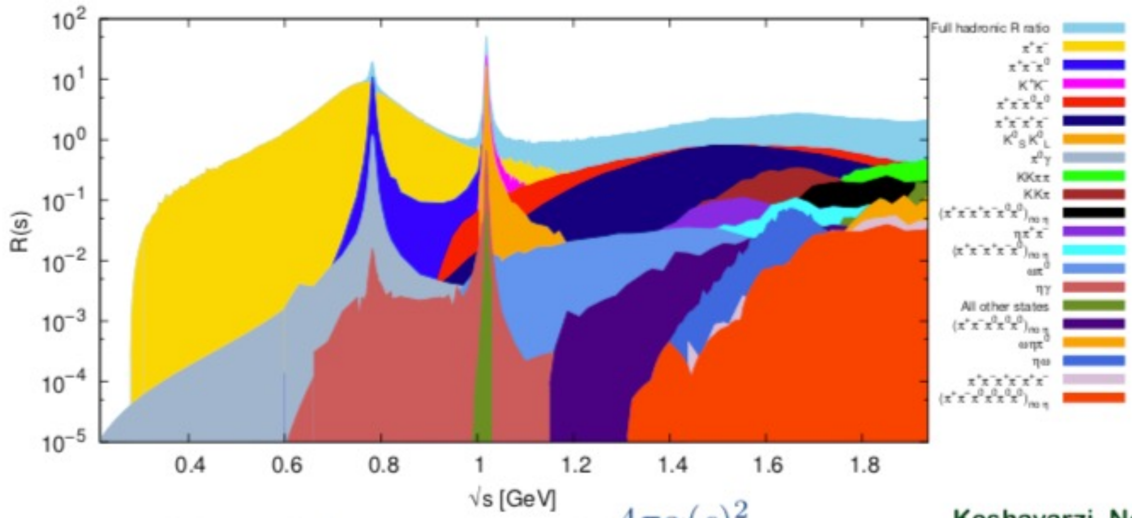
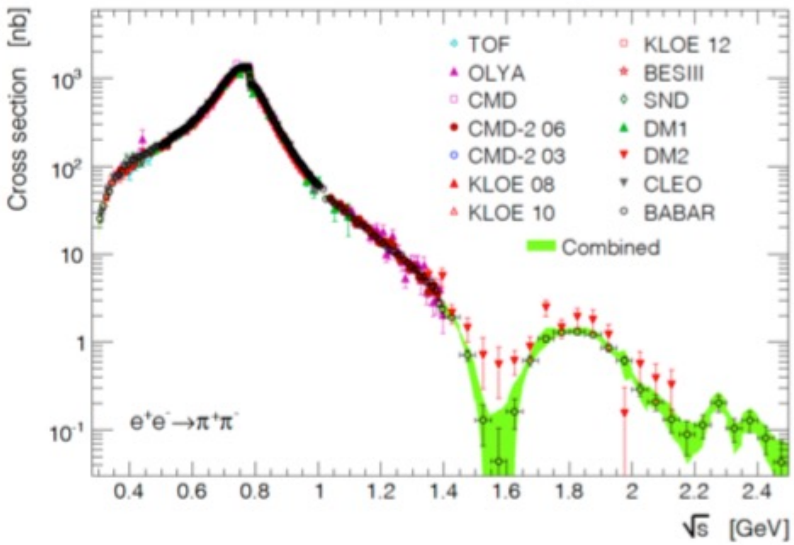
- Target: ~0.2% total error
- Dispersion relation + experimental data for $e^+e^- \to \mathrm{hadrons}$ (and τ data)
  - current uncertainty ~0.5%
  - can be improved with more precise experimental data
  - new experimental measurements expected/ongoing at BaBar, BES-III, Belle-II, CMD-3, SND, KEDR, KLOE,….
- Challenges:
  - below ~2 GeV: sum > 30 exclusive channels: 2π, 3π, 4π, 5π, 6π, $2K$, $2K\pi$, $2K2\pi$, $\eta\pi$,…. (use isospin relations for missing channels)
  - above ~1.8 GeV:
    inclusive, pQCD (away from flavor thresholds)
    + narrow resonances (J/ψ, Υ,..)
  - Combine data from different experiments/measurements:
    understanding correlations, sources of sys. error, tensions…
  - include FS radiative corrections

Low energy hadronic cross section



$$R(s) = \sigma(e^+e^- \to \text{hadrons}) / \frac{4\pi\alpha(s)^2}{3s}$$

Keshavarzi, Nomura Teubner
PRD 2018



Davier, Hoecker, Malaescu, Zhang
EPJC 2020

Dispersive HVP from KNT

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The muon $g-2$ and $\alpha(M_Z^2)$: a new data-based analysis

Alexander Keshavarzi[1], Daisuke Nomura[2,3] and Thomas Teubner[4]

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[2] *KEK Theory Center, Tsukuba, Ibaraki 305-0801, Japan*
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[4] *Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, United Kingdom*
Email: thomas.teubner@liverpool.ac.uk

Abstract

This work presents a complete re-evaluation of the hadronic vacuum polarisation contributions to the anomalous magnetic moment of the muon, $a_\mu^{\rm had,\,VP}$ and the hadronic contributions to the effective QED coupling at the mass of the Z boson, $\Delta\alpha_{\rm had}(M_Z^2)$, from the combination of $e^+e^-\to$ hadrons cross section data. Focus has been placed on the development of a new data combination method, which fully incorporates all correlated statistical and systematic uncertainties in a bias free approach. All available $e^+e^-\to$ hadrons cross section data have been analysed and included, where the new data compilation has yielded the full hadronic R-ratio and its covariance matrix in the energy range $m_\pi \le \sqrt{s} \le 11.2$ GeV. Using these combined data and perturbative QCD above that range results in estimates of the hadronic vacuum polarisation contributions to $g-2$ of the muon of $a_\mu^{\rm had,\,LO\,VP} = (693.26\pm 2.46)\times 10^{-10}$ and $a_\mu^{\rm had,\,NLO\,VP} = (-9.82\pm 0.04)\times 10^{-10}$. The new estimate for the Standard Model prediction is found to be $a_\mu^{\rm SM} = (11\,659\,182.04\pm 3.56)\times 10^{-10}$, which is 3.7σ below the current experimental measurement. The prediction for the five-flavour hadronic contribution to the QED coupling at the Z boson mass is $\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = (276.11\pm 1.11)\times 10^{-4}$, resulting in $\alpha^{-1}(M_Z^2) = 128.946\pm 0.015$. Detailed comparisons with results from similar related works are given.

← Major 2018 update to data combination methodology and data input.
Results for $a_\mu^{\rm had,\,VP}$ and $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$.
Phys.Rev.D 97 (2018) 114025.

The $g-2$ of charged leptons, $\alpha(M_Z^2)$ and the hyperfine splitting of muonium

Alexander Keshavarzi[1,2], Daisuke Nomura[3] and Thomas Teubner[4]

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[2] *Department of Physics and Astronomy, The University of Mississippi, Mississippi 38677, U.S.*
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[4] *Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, United Kingdom*
Email: thomas.teubner@liverpool.ac.uk

Abstract

Following updates in the compilation of $e^+e^-\to$ hadrons data, this work presents re-evaluations of the hadronic vacuum polarisation contributions to the anomalous magnetic moment of the electron (a_e), muon (a_μ) and tau lepton (a_τ), to the ground-state hyperfine splitting of muonium and also updates the hadronic contributions to the running of the QED coupling at the mass scale of the Z boson, $\alpha(M_Z^2)$. Combining the results for the hadronic vacuum polarisation contributions with recent updates for the hadronic light-by-light corrections, the electromagnetic and the weak contributions, the deviation between the measured value of a_μ and its Standard Model prediction amounts to $\Delta a_\mu = (28.02\pm 7.37)\times 10^{-10}$, corresponding to a muon $g-2$ discrepancy of 3.8σ.

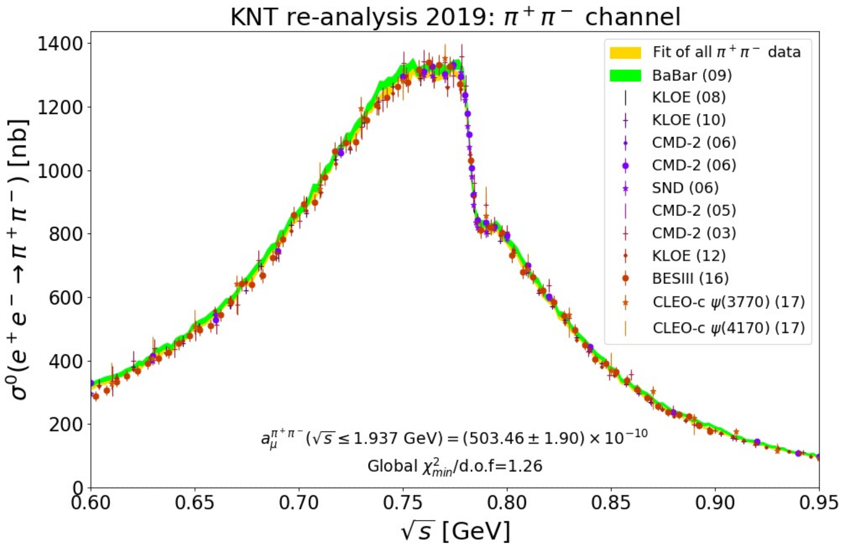
2019 data update and applications of data → compilation to other observables.
Results for a_e, a_μ, a_τ, $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$ and $\Delta\nu_{\rm Mu}^{\rm had,\,VP}$.
Phys.Rev.D 101 (2020) 014029.

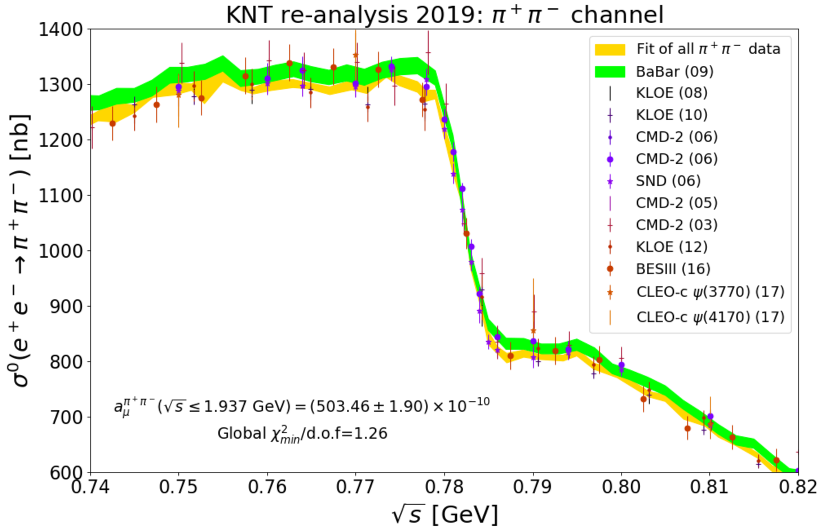
The $\pi^+\pi^-$ channel *Phys.Rev.D* 101 (2020) 014029.

The University of Manchester

$\pi^+\pi^-$ accounts for over 70% of $a_\mu^{\text{had, LOVP}}$

→ Combines ~30 measurement totalling over 1000 data points





→ Correlated & experimentally corrected $\sigma^0_{\pi\pi(\gamma)}$ data entirely dominant

$$a_\mu^{\pi^+\pi^-}[0.305 \leq \sqrt{s} \leq 1.937 \text{ GeV}] = 503.46 \pm 1.14_{stat} \pm 1.52_{sys} \pm 0.05_{vp} \pm 0.14_{fsr}$$
$$= 503.46 \pm 1.91_{tot}$$

→ 14% local χ^2_{min}/d.o.f. error inflation due to tensions in clustered data

The $\pi^+\pi^-$ channel *Phys.Rev.D* 101 (2020) 014029.

$\pi^+\pi^-$ accounts for over 70% of $a_\mu^{\text{had, LOVP}}$

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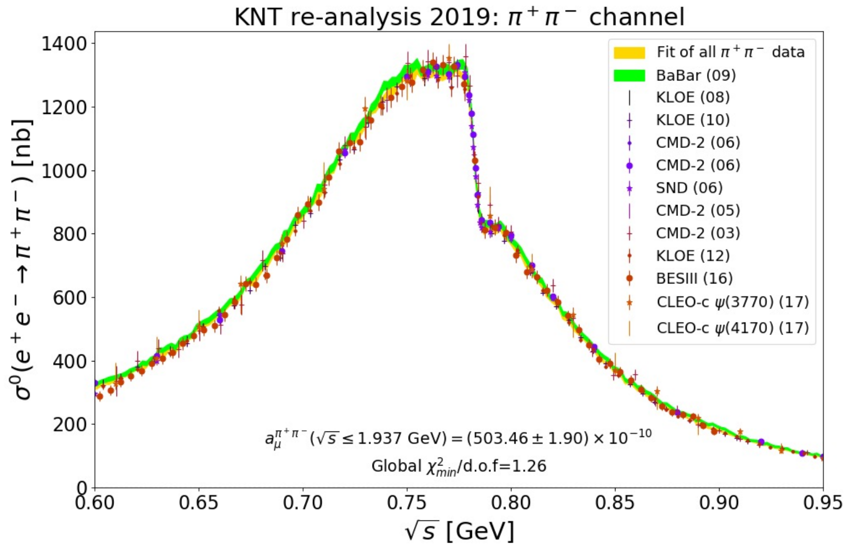

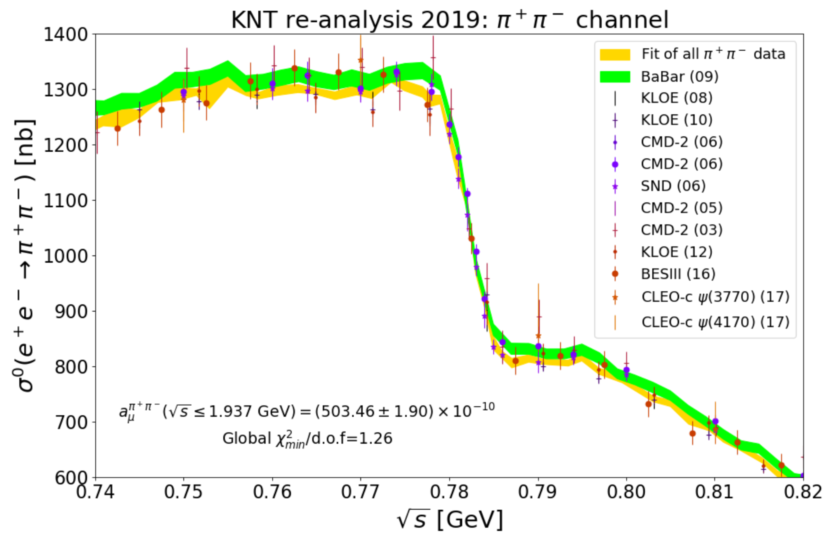

→ Correlated & experimentally corrected $\sigma^0_{\pi\pi(\gamma)}$ data entirely dominant

$$a_\mu^{\pi^+\pi^-}[0.305 \leq \sqrt{s} \leq 1.937 \text{ GeV}] = 503.46 \pm 1.14_{stat} \pm 1.52_{sys} \pm 0.05_{vp} \pm 0.14_{fsr}$$
$$= 503.46 \pm 1.91_{tot}$$

→ 14% local χ^2_{min}/d.o.f. error inflation due to tensions in clustered data

The $\pi^+\pi^-$ channel *Phys.Rev.D* 101 (2020) 014029.

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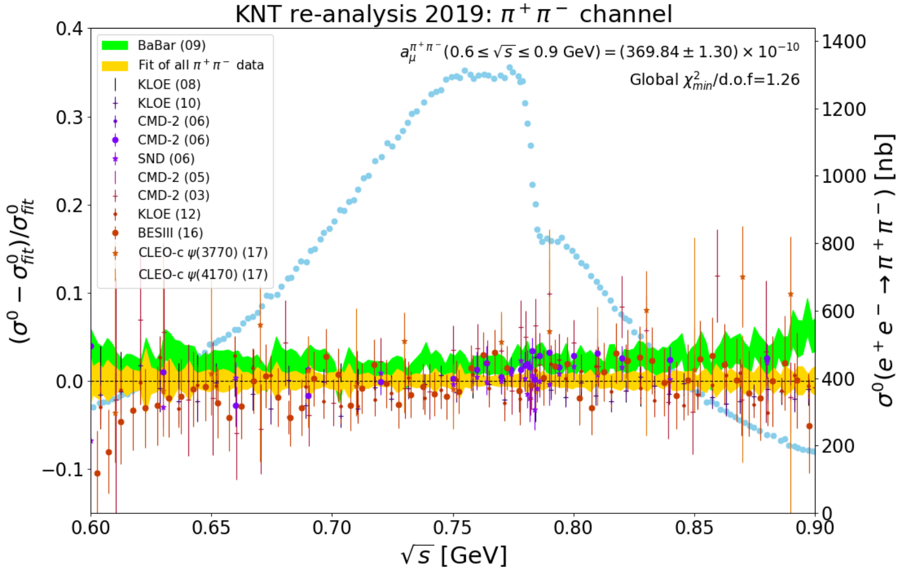
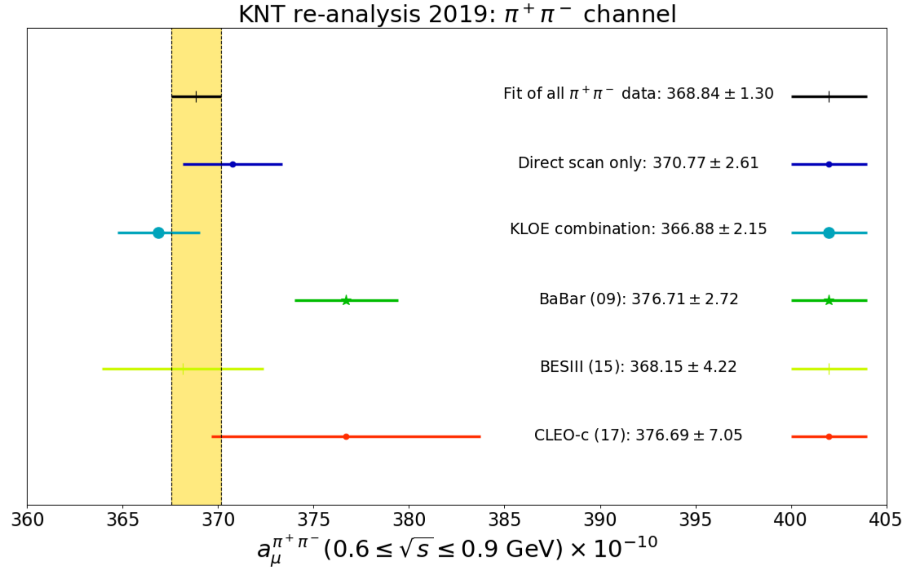
Large difference between KNT vs. BaBar and KLOE vs. BaBar is still evident.





Compared to $a_\mu^{\pi^+\pi^-} = 503.5 \pm 1.9 \rightarrow a_\mu^{\pi^+\pi^-}$(BaBar data only) = 513.2 ± 3.8

Simple weighted average of all data → $a_\mu^{\pi^+\pi^-}$(weighted average) = 509.2 ± 2.9
(i.e. – no correlations in determination of mean value)

BaBar data dominate when no correlations are accounted for in the mean value.

➢ Highlights the importance of incorporating available correlated uncertainties in fit.

KNT19 HVP

Phys.Rev.D 101 (2020) 014029.

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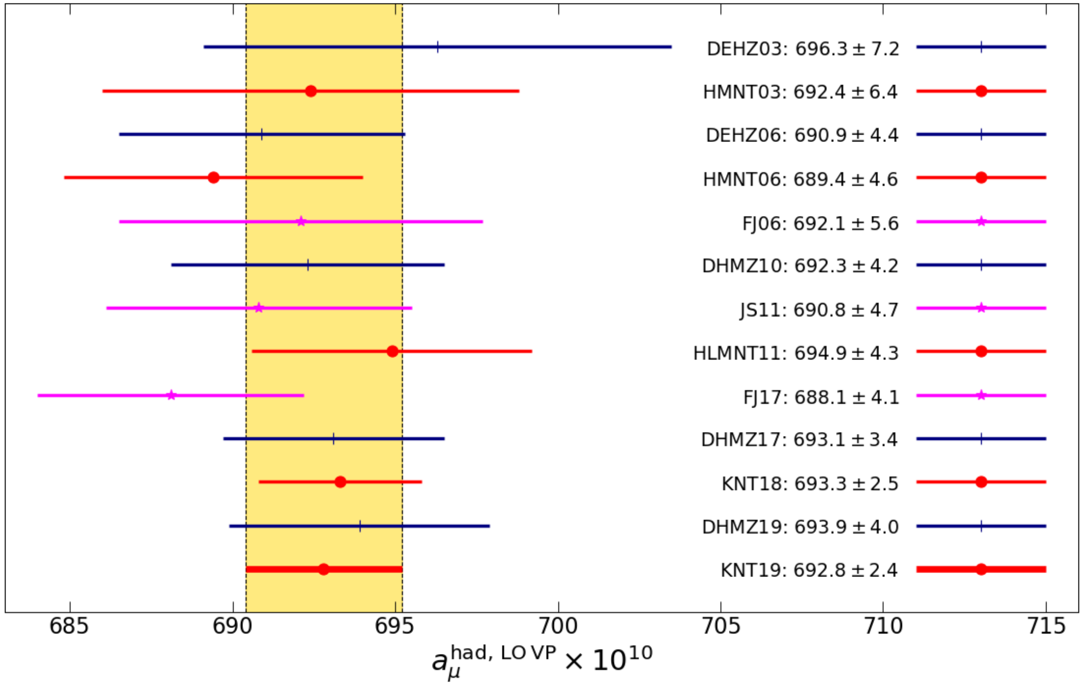
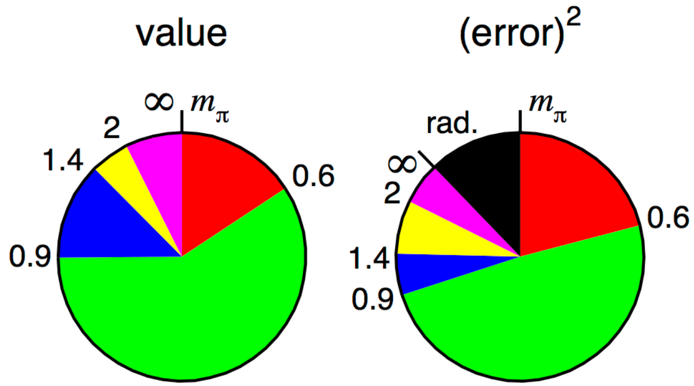
KNT18: $a_\mu^{\text{had, LOVP}} = 693.26 \pm 2.46_{tot}$

$$a_\mu^{\text{had, LOVP}} = 693.84 \pm 1.19_{stat} \pm 1.96_{sys} \pm 0.22_{vp} \pm 0.71_{fsr}$$
$$= 693.84 \pm 2.29_{exp} \pm 0.74_{rad}$$
$$= 692.78 \pm 2.42_{tot}$$



- Precision better than 0.4% (uncertainties include all available correlations and χ^2 inflation)



- Clear $\pi^+\pi^-$ dominance

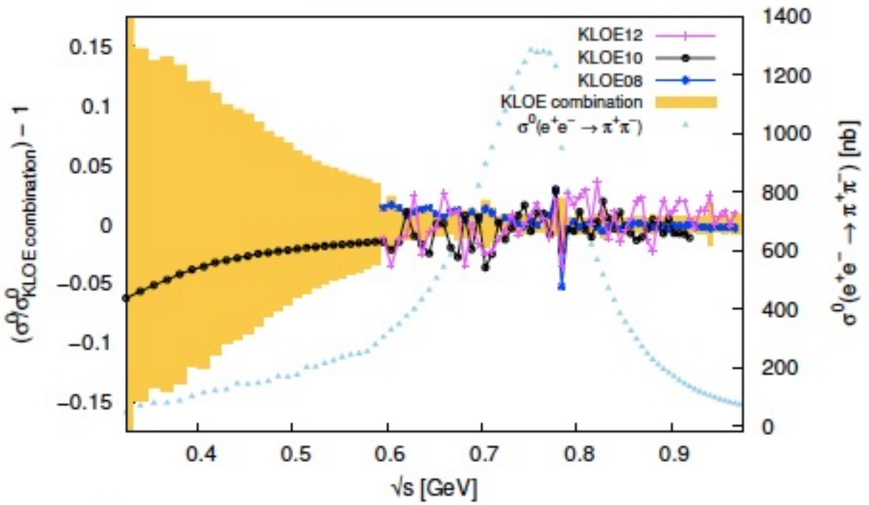
KNT vs. DHMZ: the use of correlations

Differences are dominated by the choices of how to use the correlations

⇒ Consider the combination of the KLOE $\pi^+\pi^-$ data sets:

KNT18

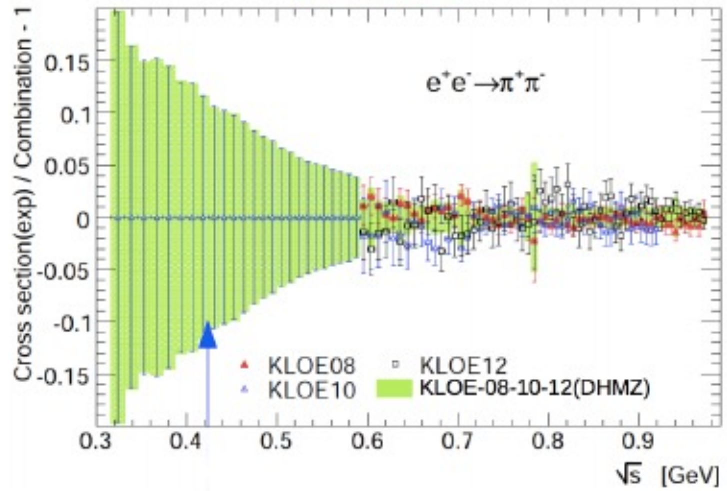
Use available uncertainty and correlation information to fully influence over entire energy range



$$a_\mu^{\pi^+\pi^-}(\sqrt{s} \leq 2.0 \text{ GeV}) = 503.74 \pm 1.96$$

DHMZ17

Only allow correlations to only influence the combination locally inside defined energy intervals



$$a_\mu^{\pi^+\pi^-}(\sqrt{s} \leq 2.0 \text{ GeV}) = 507.14 \pm 2.58$$

Take-home message: correlations are important and the choices of how to use them are not trivial

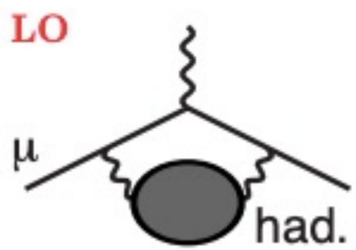
Data-driven HVP

Slide content by Aida El-Khadra.

First-time agreement between various groups…

Detailed comparisons by-channel and energy range between direct integration results:

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
[3.7, ∞) GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2



+ evaluations using unitarity & analyticity constraints for $\pi\pi$ and $\pi\pi\pi$ channels
[CHS 2018, HHKS 2019]

Conservative merging to obtain a realistic assessment of the underlying uncertainties:
account for differences in results from the same experimental inputs
include correlations between systematic errors

$$\Rightarrow a_\mu^{\text{HVP,LO}} = 693.1\,(4.0) \times 10^{-10}$$

New updates since KNT19

The University of Manchester

- pi+pi-pi0, BESIII (2019), arXiv:1912.11208
- pi+pi- [covariance matrix erratum], BESIII (2020), Phys.Lett.B 812 (2021) 135982 (erratum)
- K+K-pi0, SND (2020), Eur.Phys.J.C 80 (2020) 12, 1139
- etapi0gamma (res. only), SND (2020), Eur.Phys.J.C 80 (2020) 11, 1008
- pi+pi-, SND (2020), JHEP 01 (2021) 113
- etaomega → pi0gamma, SND (2020), Eur.Phys.J.C 80 (2020) 11, 1008
- pi+pi-pi0, SND (2020), Eur.Phys.J.C 80 (2020) 10, 993
- pi+pi-pi0, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112003
- pi+pi-2pi0omega, BaBar (2021), Phys. Rev. D 103, 092001
- etaetagamma, SND (2021), Eur.Phys.J.C 82 (2022) 2, 168
- etaomega, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- pi+pi-pi0eta, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- omegaetapi0, BaBar (2021), Phys. Rev. D 103, 092001
- pi+pi-4pi0, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- pi+pi-pi0pi0eta, BaBar (2021), Phys.Rev.D 103 (2021) 9, 092001
- pi+pi-3pi0eta, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- 2pi+2pi-3pi0, BaBar (2021), Phys. Rev. D 103, 092001
- omega3pi0, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- pi+pi-pi+pi-eta, BaBar (2021), Phys. Rev. D 103, 092001
- Inclusive R(s), BESIII (2021), Phys.Rev.Lett. 128 (2022) 6, 062004
- nnbar,SND (2022), arXiv:2206.13047
- K0sK3pi. CMD-3 (2022), arXiv:2207.04615
- KK3pi, BaBar (2022). arXiv:2207.10340

Plus, analysis updates to be presented at Edinburgh TI workshop…

Conclusions

- Fermilab's Muon g-2 Experiment has confirmed BNL's result: the discrepancy between experiment and SM increases to 4.2σ.
- All SM contributions other than HVP, including HLbL, now fully cross checked and understood to be under control.
- Data-driven HVP dominates theory uncertainty with 0.6% error.
- The BMW and other lattice QCD results weakens the exp-SM discrepancy.
- Improvements to come:
  - Updated HVP evaluation with new measurements of hadronic cross section data.
  - HVP comparisons for BMW result and between lattice groups/R-ratio as part of theory initiative.
  - HLbL uncertainty to reach ~10% .
  - New, full SM update from theory initiative before Fermilab's next result.