Towards the establishment of the $J^{P(C)} = 1^{-(+)}$ hybrid nonet

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Based on L. Qiu and Q. Zhao, Chin. Phys. C 46, 051001 (2022) [2202.00904[hep-ph]]

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Outline

1. The lowest hybrid nonet and phenomenology
2. Production of the $J^{P(C)} = 1^{-(+)}$ hybrid nonet in J/ψ radiative decays
3. Predictions for $J/\psi \to VH$
4. Discussions and summary

1. The lowest hybrid nonets and phenomenology

Conventional QM states: Quantum numbers accessible by the quark-antiquark scenario:

States in **natural spin-parity**: if $P = (-1)^{L+1} = (-1)^{J}$.

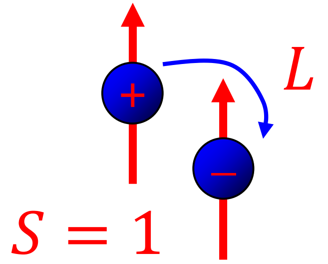
Then with $S = 1$, one has $CP = (-1)^{(L+S)+(L+1)} = +1$

→ Mesons with **natural spin-parity** but $CP = -1$ will be forbidden:

$\mathbf{0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \ldots}$

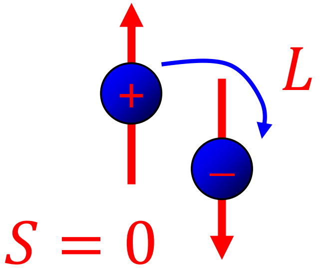
Exotic mesons:

Quantum numbers cannot be accessed by the spin-orbital couplings between a pair of $q\bar{q}$.



Natural: $0^{++}, 1^{--}, 2^{++}, 3^{--}, \ldots$

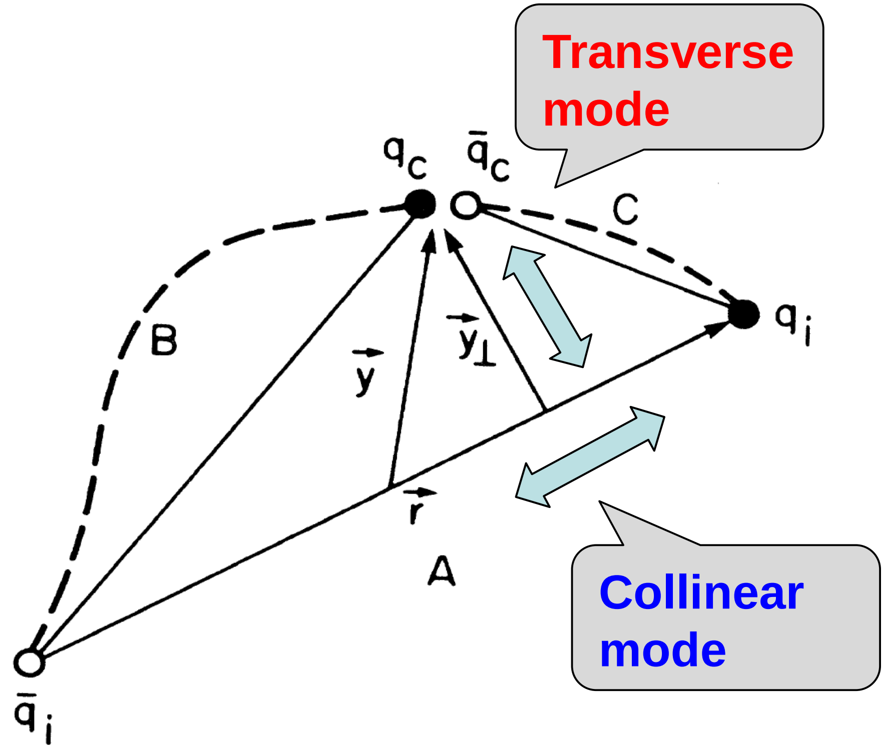
Unnatural: (0^{--}), $1^{++}, 2^{--}, 3^{++}, \ldots$



Unnatural: $0^{-+}, 1^{+-}, 2^{-+}, 3^{+-}, \ldots$

A brief introduction to the flux-tube model

- Lowest gluon fields generate adiabatic potential on which the quark motion can be described.
- The flux tube may be excited on which the quark motion in the adiabatic potential of such excited gluon field configurations will give access to hybrid states.
- The decays of both conventional and exotic hadrons can be well described by the flux tube breaking mechanism.



Flux tube model Hamiltonian:

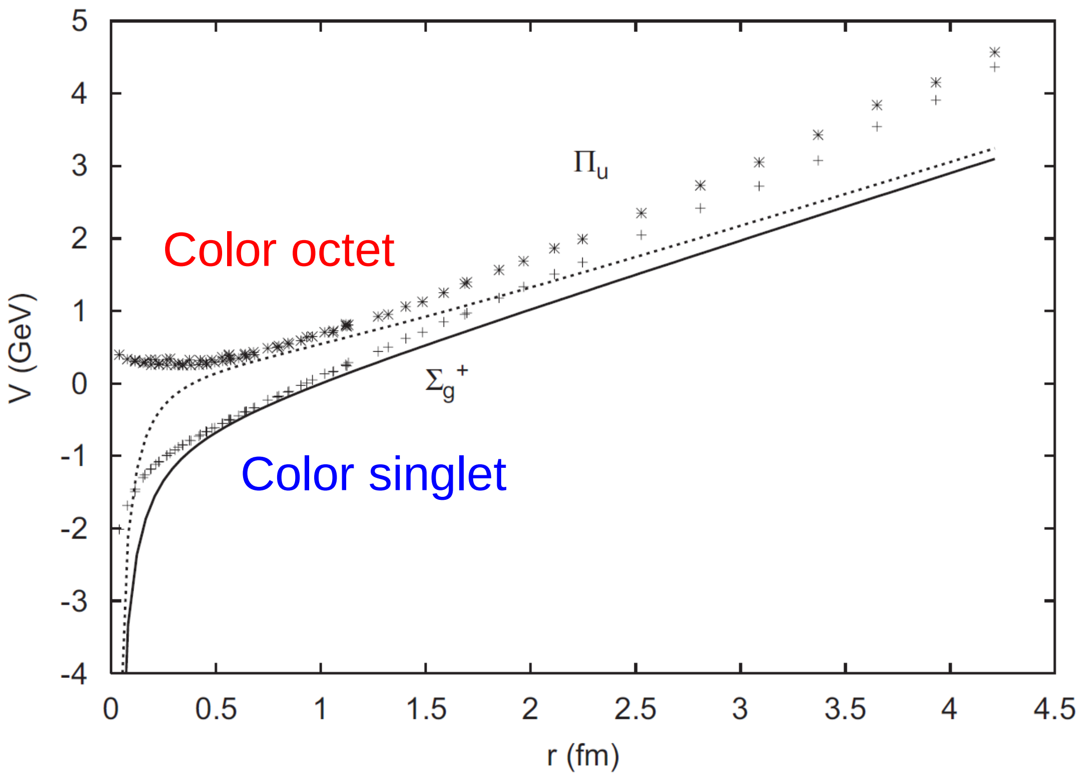
$$H = H_{\text{quarks}} + H_{\text{flux tube}},$$

$$H_{\text{quarks}} = -\frac{1}{2m_q}\vec{\nabla}_q^2 - \frac{1}{2m_{\bar{q}}}\vec{\nabla}_{\bar{q}}^2 + V_{q\bar{q}},$$

$$H_{\text{fluxtube}} = b_0 R + \sum_n \left[\frac{p_n^2}{2b_0 a} + \frac{b_0}{2a}(y_n - y_{n+1})^2\right]$$

N. Isgur and J. Paton, PLB124 (1983) 247-251; PRL54 (1985) 869; PRD31 (1985) 2910

Adiabatic gluonic potential for the gluonlump in the flux tube model compared with the LQCD simulation



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G.S. Bali and A. Pineda, PRD69 (2004) 094001;
C.A. Meyer and E.S. Swanson, PPNP 82 (2015) 21
E. Klempt and A. Zaitsev, Phys. Rept. 454, 1-202 (2007)

Quantum numbers of the hybrid mesons:

- Parity: $P = \epsilon(-1)^{L+\Lambda+1}$
- Charge conjugation: $C = \epsilon\eta(-1)^{L+\Lambda+S}$

  where $\eta \equiv (PC)_g$; Λ $(= 0,1,2,\ldots \rightarrow \Sigma, \Pi, \Delta, \ldots)$ is the projection of the gluonic angular momentum onto the $q\bar{q}$ axis.
- The adiabatic gluonic potential for the gluonlump is denoted as Λ_η^Y ($Y = \pm$ is the so-called Y-parity).

J^{PC}	Mass (GeV)	Adiabatic surface quantum numbers
1^{+-}	0.87(15)	Σ_u^-, Π_u
1^{--}	1.25(16)	Π_g, $\Sigma_g^{+\prime}$
2^{--}	1.45(17)	Σ_g^-, Π_g', Δ_g
2^{+-}	1.86(19)	Σ_u^+, Π_u', Δ_u
3^{+-}	1.86(18)	$\Sigma_u^{-\prime}$, Π_u'', $\Delta_u' i$, Φ_u
0^{++}	1.98(18)	$\Sigma_g^{+\prime\prime}$
4^{--}	2.13(18)	$\Sigma_g^{-\prime}$, Π_g'', $\Delta_g' i$, Φ_g, Γ_g
1^{-+}	2.15(20)	$\Sigma_u^{+\prime}$, Π_u'''

Lowest hybrid in the flux-tube model

Collinear mode

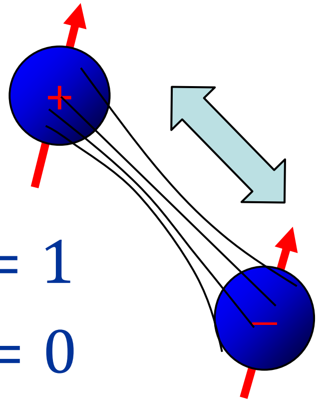
Transverse mode

$J_q^{P(C)} = 1^{-(-)}$

$\boldsymbol{J}_g = \boldsymbol{S}_g + \boldsymbol{L}_g,$

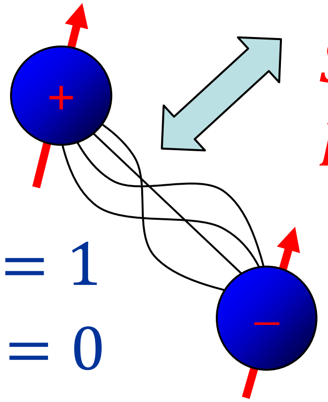
$J_g^{PC} = 1^{+-}$

$$J^{P(C)} = (0, 1, 2)^{-(+)}$$



$S_q = 1$

$L_q = 0$



$S_g = 1$

$L_g > 0$

$S_q = 1$

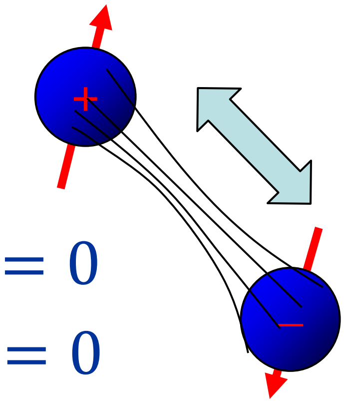
$L_q = 0$

$J_q^{P(C)} = 0^{-(+)}$

$\boldsymbol{J}_g = \boldsymbol{S}_g + \boldsymbol{L}_g,$

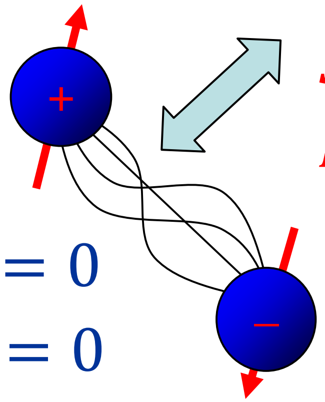
$J_g^{PC} = 1^{+-}$

$$J^{P(C)} = 1^{--}$$



$S_q = 0$

$L_q = 0$



$S_g = 1$

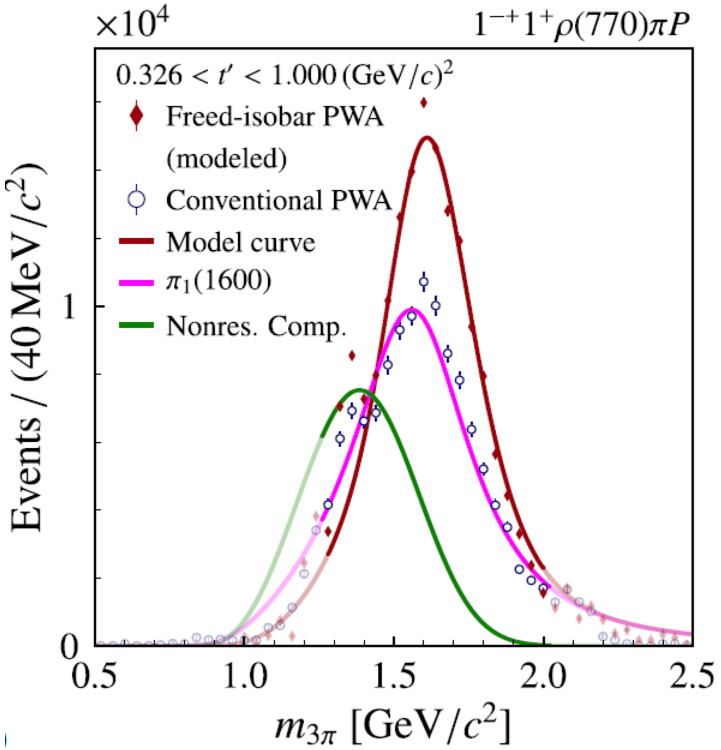
$L_g > 0$

$S_q = 0$

$L_q = 0$

N. Isgur and J. Paton, PLB124 (1983) 247-251; PRL54 (1985) 869; PRD31 (1985) 2910

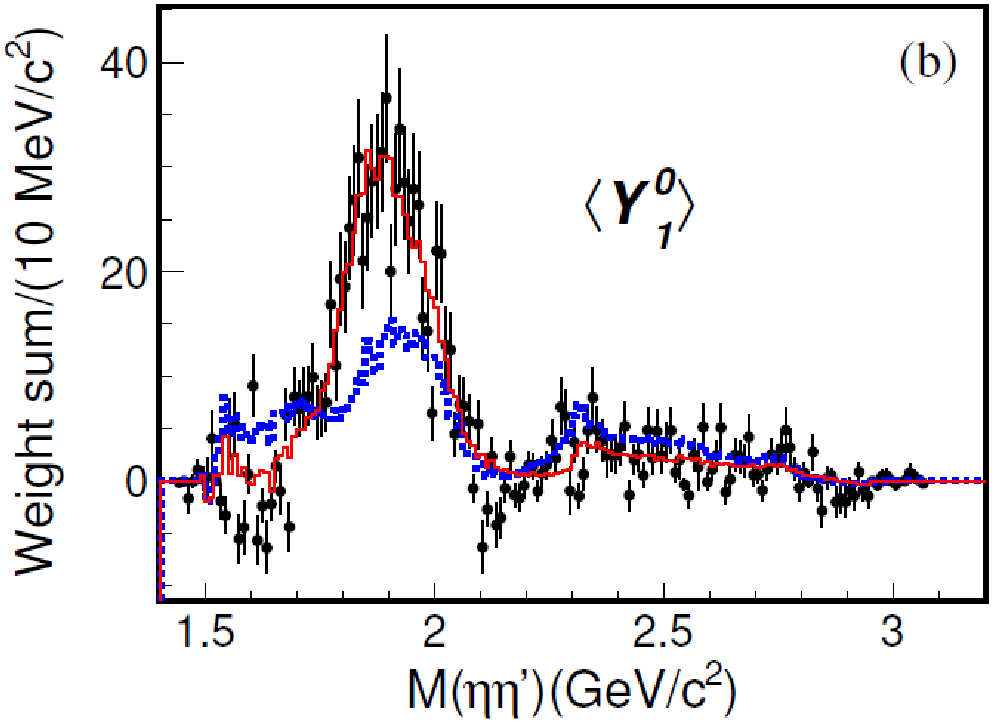
Recent Compass confirmation of $\pi_1(1600)$



- $\pi_1(1600)$ has been seen in $\eta'\pi$, $f_1(1285)\pi$, $b_1(1235)\pi$, and $\rho\pi$ in various experiments.
- The Compass results favor a single $\pi_1(1600)$, and do not support the existence of $\pi_1(1400)$.

See Compass Collab., PRD105 (2022) 012005

Recent BESIII observation of $\eta_1(1855)$



- $\eta_1(1855)$ is seen in $J/\psi \to \gamma\eta\eta'$ at BESIII with $m_{\eta_1} = (1855 \pm 9^{+6}_{-1})$ MeV and $\Gamma_{\eta_1} = (188 \pm 18^{+3}_{-8})$ MeV.
- Only one $(I = 0, J^{PC} = 1^{-+})$ state is seen.

BESIII, 2202.00621 [hep-ex], 2202.00623 [hep-ex]

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Hybrid nonet and Gell-Mann-Okubo relation

- Flavor-blindness of the strong interaction

$$\pi_1^+,\ \pi_1^-,\ \pi_1^0 : u\bar{d}\tilde{g},\ d\bar{u}\tilde{g},\ \frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})\tilde{g}$$

$$\eta_1^{(8)} : \frac{1}{\sqrt{6}}(u\bar{u}+d\bar{d}-2s\bar{s})\tilde{g}$$

$$\eta_1^{(1)} : \frac{1}{\sqrt{3}}(u\bar{u}+d\bar{d}+s\bar{s})\tilde{g}$$

$$K^{*+},\ K^{*0},\ K^{*-},\ \bar{K}^{*0} : u\bar{s}\tilde{g},\ d\bar{s}\tilde{g},\ s\bar{u}\tilde{g},\ s\bar{d}\tilde{g}$$

- Flavor octet and singlet mixing in the $I=0$ sector

$$\begin{pmatrix}\eta_{1L}\\ \eta_{1H}\end{pmatrix}=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\eta_1^{(8)}\\ \eta_1^{(1)}\end{pmatrix}=\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}n\bar{n}\tilde{g}\\ s\bar{s}\tilde{g}\end{pmatrix}$$

$$n\bar{n}\equiv(u\bar{u}+d\bar{d})/\sqrt{2}$$

Ideal mixing: $\theta=0^\circ, \alpha=\arctan(\sqrt{2})\simeq 54.7^\circ$

- Gell-Mann-Okubo relation:

$$\tan\theta=\frac{4m_{K^*}-m_{\pi_1}-3m_{\eta_{1L}}}{2\sqrt{2}(m_{\pi_1}-m_{K^*})}$$

and

$$(m_{\eta_{1H}}+m_{\eta_{1L}})(4m_{K^*}-m_{\pi_1})-3m_{\eta_{1H}}m_{\eta_{1L}} =8m_{K^*}^2-8m_{K^*}m_{\pi_1}+3m_{\pi_1}^2\ ,$$

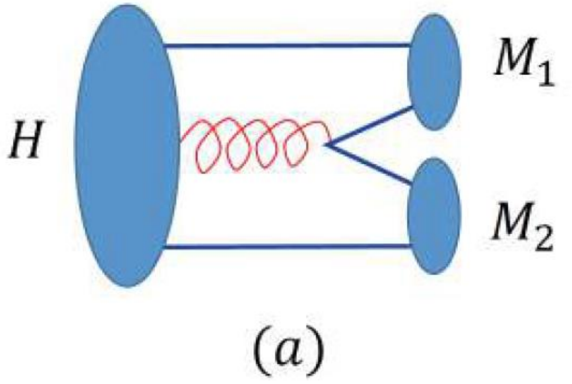
Note: With the input of m_{π_1} and $\eta_1(1855)$ it is still insufficient for the determination of $\theta, m_{\eta_{1L}}/m_{\eta_{1H}}, m_{K^*}$.

Decays of the $1^{-(+)}$ hybrid nonet into pseudoscalar meson pair

Decays via the quark pair creation (QPC) model ⇒ $\tilde{g} \to (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$.

Longitudinal transition mode:

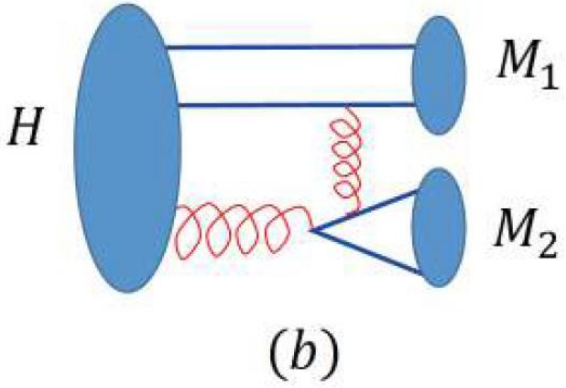
$$\mathcal{M}_a = \langle (q_1\bar{q}_4)_{M_1}(q_3\bar{q}_2)_{M_2}|\hat{V}_\mathrm{L}|q_1\bar{q}_2\tilde{g}\rangle \equiv g_1|\boldsymbol{k}|$$



Collinear process

Transverse transition mode:

$$\mathcal{M}_b = \langle (q_1\bar{q}_2)_{M_1}(q_3\bar{q}_4)_{M_2}|\hat{V}_\mathrm{T}|q_1\bar{q}_2\tilde{g}\rangle \equiv g_2|\boldsymbol{k}|$$



Recoiling process: Only contribute to the production of M_2 as an isoscalar.

- For a conventional $q\bar{q}$ meson decay, $M_a \gg M_b$.
- For a hybrid meson decay, $M_a \simeq M_b$.

Table 1. Coupling constants for the $1^{-(+)}$ hybrid nonet decays into pseudoscalar meson pairs. The couplings for the negative charge states are implied. The $SU(3)$ flavor symmetry breaking parameter R is also included.

Processes	Couplings
$\pi_1^0 \to \eta\pi^0$	$\frac{1}{\sqrt{2}}(g_1+g_2)\cos\alpha_P - Rg_2\sin\alpha_P$
$\pi_1^0 \to \eta'\pi^0$	$\frac{1}{\sqrt{2}}(g_1+g_2)\sin\alpha_P + Rg_2\cos\alpha_P$
$\pi_1^+ \to \eta\pi^+$	$\sqrt{2}(g_1+g_2)\cos\alpha_P - Rg_2\sin\alpha_P$
$\pi_1^+ \to \eta'\pi^+$	$\sqrt{2}(g_1+g_2)\sin\alpha_P + Rg_2\cos\alpha_P$
$\eta_{1L} \to \eta\eta'$	$\frac{1}{2}(g_1+g_2)\sin 2\alpha_P(\cos\alpha + R\sin\alpha) + g_2\cos 2\alpha_P(R\cos\alpha - \sin\alpha)$
$\eta_{1H} \to \eta\eta'$	$\frac{1}{2}(g_1+g_2)\sin 2\alpha_P(\sin\alpha - R\cos\alpha) + g_2\cos 2\alpha_P(R\sin\alpha + \cos\alpha)$
$K^{*+} \to K^+\pi^0$	$\frac{1}{\sqrt{2}}g_1$
$K^{*+} \to K^0\pi^+$	g_1
$K^{*+} \to K^+\eta$	$g_1\left(\frac{1}{\sqrt{2}}\cos\alpha_P - R\sin\alpha_P\right) + g_2(\sqrt{2}\cos\alpha_P - R\sin\alpha_P)$
$K^{*+} \to K^+\eta'$	$g_1\left(\frac{1}{\sqrt{2}}\sin\alpha_P + R\cos\alpha_P\right) + g_2(\sqrt{2}\sin\alpha_P + R\cos\alpha_P)$

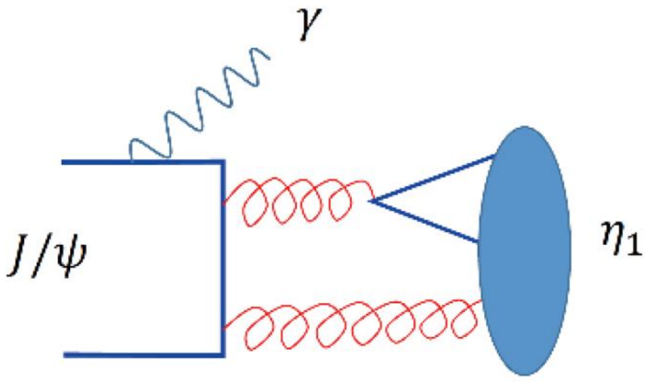
- $\alpha_P \simeq 42^\circ$ is the mixing angle between η and η'.
- $R \simeq f_\pi/f_K \simeq 0.93$ indicates the SU(3) flavor symmetry breaking effects in the production of the $s\bar{s}$ pair.
- g_1, g_2, α are the parameters to be determined.

Note: η_{1L} and η_{1H} decays into $\pi\pi$ and $K\bar{K}$ are forbidden by the Bose symmetry and G-parity conservation. They can only access $\eta\eta'$ via the octet and singlet mixing.

2. Production of the $J^{P(C)} = 1^{-(+)}$ hybrid nonet in J/ψ radiative decays

The P-wave coupling for three vectors can be described as:

$$\mathcal{L}_{VVV} = \mathrm{i}g_{VVV}(V_{1,\nu}\overleftrightarrow{\partial}^{\mu}V_2^{\nu}V_{3,\mu} + V_{1,\mu}V_2^{\nu}\overleftrightarrow{\partial}^{\mu}V_{3,\nu} + V_{2,\mu}V_3^{\nu}\overleftrightarrow{\partial}^{\mu}V_{1,\nu})$$



With the real photon the Lagrangian reduces to

$$\mathcal{L}_{J/\psi\to\gamma\eta_1} = \mathrm{i}g_{J/\psi\eta_1\gamma}F_{\mu\nu}V^{\mu}_{J/\psi}V^{\nu}_{\eta_1}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$

By defining the production potential for the hybrid via the J/ψ radiative decay, i.e.

$$g_0 \equiv \langle (q\bar{q}\tilde{g})_{1^{-+}}|\hat{H}_{em}|J/\psi\rangle$$

We have

$$\begin{cases} g_{J/\psi\eta_{1L}\gamma} = g_0(\sqrt{2}\cos\alpha - R\sin\alpha) \\ g_{J/\psi\eta_{1H}\gamma} = g_0(\sqrt{2}\sin\alpha + R\cos\alpha) \end{cases}$$

The relative production rate for η_{1L} and η_{1H} can be obtained

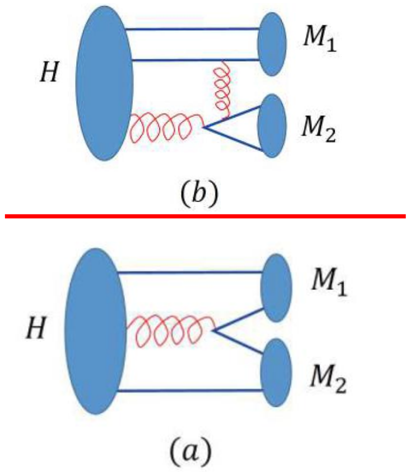
$$R_{\eta_{1L}/\eta_{1H}} = \left(\frac{|\boldsymbol{q}_L|}{|\boldsymbol{q}_H|}\right)^3 \frac{(\sqrt{2}\cos\alpha - R\sin\alpha)^2}{(\sqrt{2}\sin\alpha + R\cos\alpha)^2}\frac{m^2_{\eta_{1H}}(m^2_{J/\psi}+m^2_{\eta_{1L}})}{m^2_{\eta_{1L}}(m^2_{J/\psi}+m^2_{\eta_{1H}})} \times\left(\frac{|\boldsymbol{k}_L|}{|\boldsymbol{k}_H|}\right)^3\left(\frac{\Gamma_H m_{\eta_{1H}}}{\Gamma_L m_{\eta_{1L}}}\right)^2 \times\frac{[(1+\delta)\tan 2\alpha_P(\cos\alpha + R\sin\alpha)+2\delta(R\cos\alpha-\sin\alpha)]^2}{[(1+\delta)\tan 2\alpha_P(\sin\alpha - R\cos\alpha)+2\delta(R\sin\alpha+\cos\alpha)]^2}$$

The relative production rate for η_{1L} and η_{1H} in $J/\psi \to (\gamma\eta_{1L}/\gamma\eta_{1H}) \to \gamma\eta\eta'$ can be expressed as

$$R_{\eta_{1L}/\eta_{1H}} = \left(\frac{|\boldsymbol{q}_L|}{|\boldsymbol{q}_H|}\right)^3 \frac{(\sqrt{2}\cos\alpha - R\sin\alpha)^2}{(\sqrt{2}\sin\alpha + R\cos\alpha)^2} \frac{m_{\eta_{1H}}^2 (m_{J/\psi}^2 + m_{\eta_{1L}}^2)}{m_{\eta_{1L}}^2 (m_{J/\psi}^2 + m_{\eta_{1H}}^2)} \times \left(\frac{|\boldsymbol{k}_L|}{|\boldsymbol{k}_H|}\right)^3 \left(\frac{\Gamma_H m_{\eta_{1H}}}{\Gamma_L m_{\eta_{1L}}}\right)^2$$
$$\times \frac{[(1+\delta)\tan 2\alpha_P(\cos\alpha + R\sin\alpha) + 2\delta(R\cos\alpha - \sin\alpha)]^2}{[(1+\delta)\tan 2\alpha_P(\sin\alpha - R\cos\alpha) + 2\delta(R\sin\alpha + \cos\alpha)]^2}$$

Note that the phase space factor actually enhances the ratio.

$\delta \equiv g_2/g_1$ ⇒



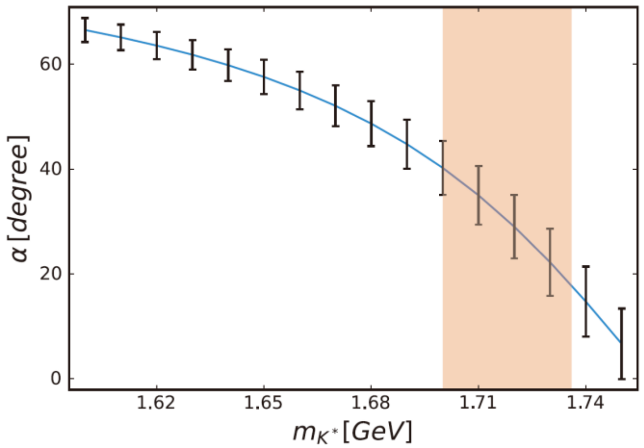
Hybrid: $|\delta| \sim 1$

Conventional meson: $|\delta| \ll 1$

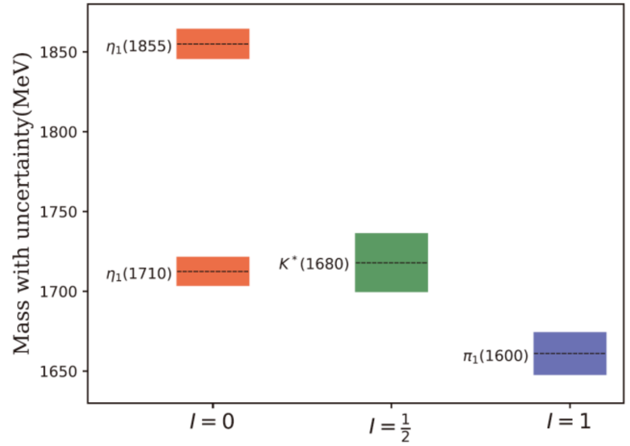
- Mixing angle α is correlated with the undetermined masses of K^* and η_{1L}/η_{1H}.

Two schemes for the nonet

Correlation between α and m_{K^*}



Hybrid nonet

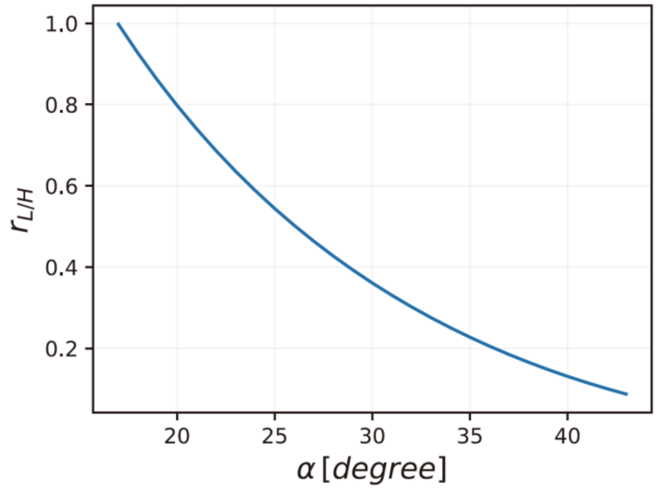


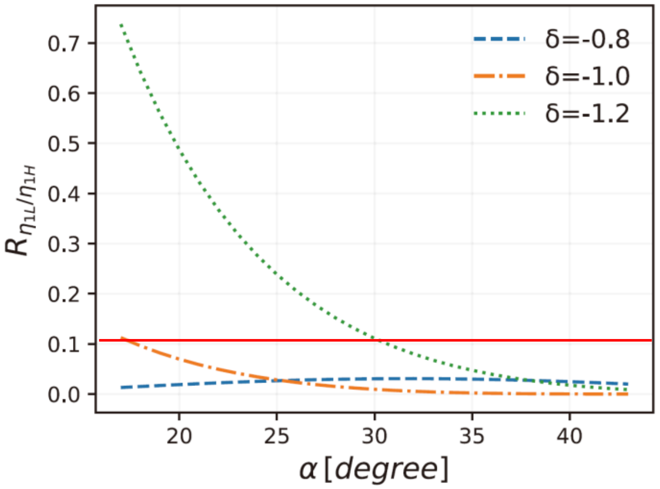
Scheme-I:

Assuming that $\eta_1(1855)$ is the higher state, η_{1H}, and $K^*(1680)$ is the strange partner with $m_{K^*} = 1718 \pm 18$ MeV. We have $m_{\eta_{1L}} = 1712.5 \pm 8.7$ MeV.

Imposing the experimental observation,

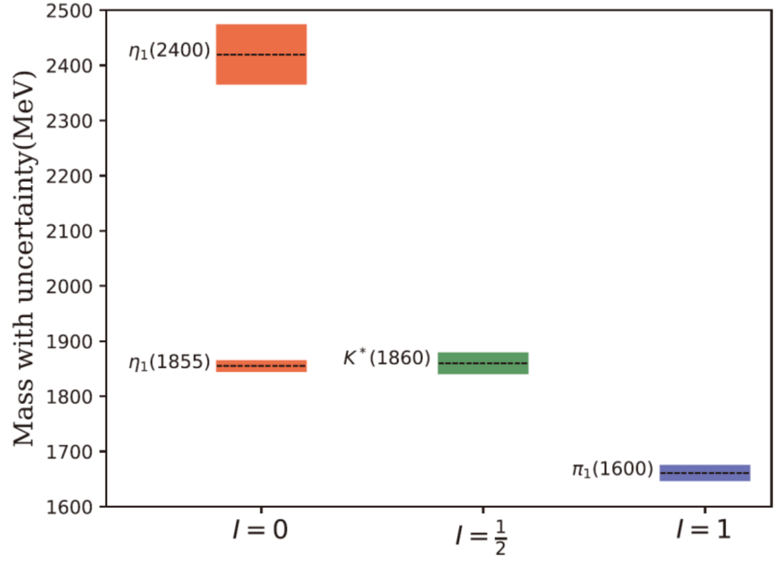
$$R_{\eta_{1L}/\eta_1(1855)} \equiv \frac{BR(J/\psi \to \gamma\eta_{1L} \to \gamma\eta\eta')}{BR(J/\psi \to \gamma\eta_1(1855) \to \gamma\eta\eta')} < 10\%$$





$$\begin{cases} \alpha = 30^\circ \pm 13^\circ \\ \delta \simeq -1.0 \end{cases}$$

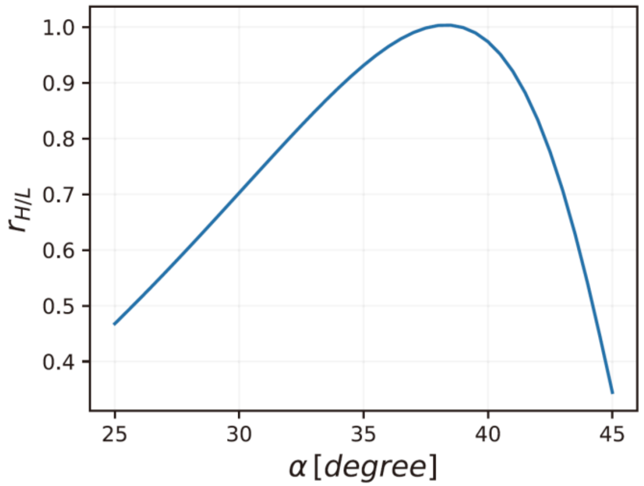
Hybrid nonet

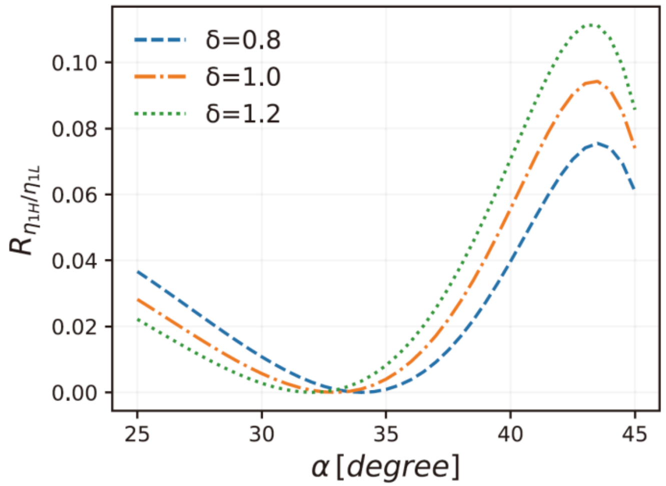


Scheme-II:
Assuming that $\eta_1(1855)$ is the lower state, η_{1L}, one finds that $K^*(1680)$ is no longer suitable to be the strange partner. We thus impose the experimental observation as a constraint:

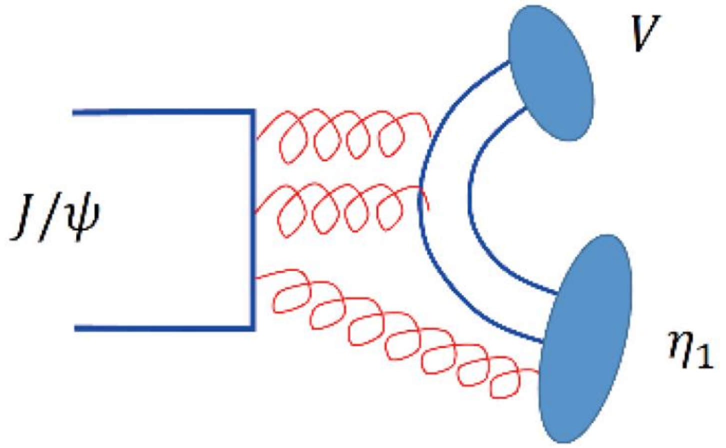
$$R_{\eta_{1\mathrm{H}}/\eta_1(1855)} \equiv \frac{BR(J/\psi \to \gamma\eta_{1\mathrm{H}} \to \gamma\eta\eta')}{BR(J/\psi \to \gamma\eta_1(1855) \to \gamma\eta\eta')} < 10\%$$

$$\begin{cases} \alpha \simeq 35^\circ \pm 10^\circ \\ m_{K^*} = 1.83 \sim 1.90\ \mathrm{GeV} \\ m_{\eta_{1H}} \simeq 2.4\ \mathrm{GeV} \\ \delta \simeq 1.0 \end{cases}$$





3. Predictions for $J/\psi \to VH$



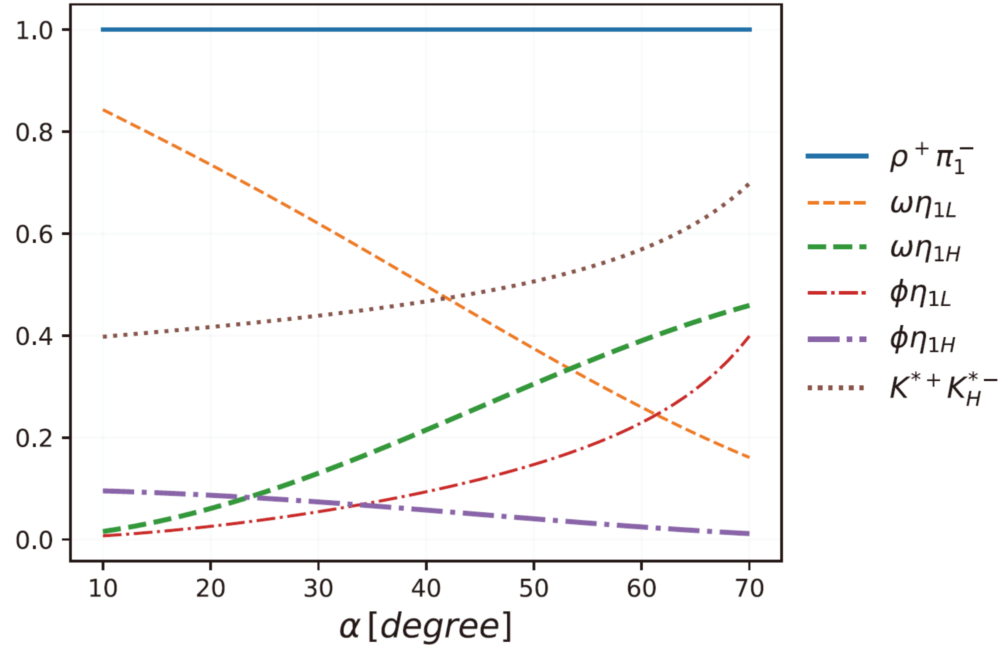
The coupling for $J/\psi \to VH$ can be parametrized as:

$$g_P \equiv \langle [q\bar{q}]_1 [q\bar{q}\tilde{g}]_{1^{-+}} | \hat{V}_P | J/\psi \rangle$$

Then, the coupling constants for different decay channels can be expressed as:

$$\begin{aligned}
g_{J/\psi \rho^+ \pi_1^-} &= g_P, \\
g_{J/\psi \omega \eta_{1L}} &= g_P \cos\alpha\ , \\
g_{J/\psi \omega \eta_{1H}} &= g_P \sin\alpha\ , \\
g_{J/\psi \phi \eta_{1L}} &= -g_P R^2 \sin\alpha\ , \\
g_{J/\psi \phi \eta_{1H}} &= g_P R^2 \cos\alpha\ , \\
g_{J/\psi K^{*+} K_H^{*-}} &= g_P R\ ,
\end{aligned}$$

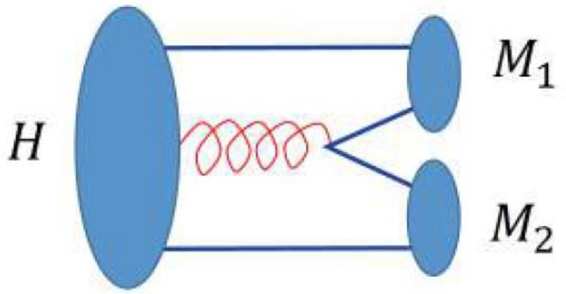
$$\rho^+\pi_1^- : \omega\eta_{1L} : \omega\eta_{1H} : \phi\eta_{1L} : \phi\eta_{1H} : K^{*+}K_H^{*-}$$
$$= 1 : \cos^2\alpha : \sin^2\alpha : R^4\sin^2\alpha : R^4\cos^2\alpha : R^2$$



4. Discussions and summary

- The gluonlumps will finally convert into quark-antiquark pairs. Phenomenologically, how to distinguish these effective degrees of freedom from other scenarios?

➢ The hybrid also favors strong couplings to an axial vector plus a pseudoscalar [Burns, Close and Dudek, PRD 77 (2008) 034008]



(a)

- Why only one state $\eta_1(1855)$ is seen? Are we expecting its partner in the nonet?

➢ If $\eta_1(1855)$ is associated with $K_1(1400)\bar{K}$, should we expect to have $\eta_1(1763)$ associated with $K_1(1270)\bar{K}$?

➢ Broad width of $K_1(1400)$ may cause problems in the molecular scenario.

- The width of $\eta_1(1855)$ is about 188 MeV. What are the dominant decay channels?

➢ In the hybrid scenario $\eta_1(1855)$ cannot be a pure $s\bar{s}\tilde{g}$ or $n\bar{n}\tilde{g}$. The S-wave decays into vector and pseudoscalar pairs may not be suppressed [Close and Dudek, PRD70 (2004) 094015, The 'Forbidden' decays of hybrid mesons to pi rho can be large]

- More open questions …

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Thanks for your attention!