LFV within little Higgs models realizing a low-scale see-saw

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The International Workshop e^+e^- Collisions From Phi to Psi 2022

August 18, 2022

The Center for Research and Advanced Studies of the National Polytechnic Institute

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Littlest Higgs Model with T-parity

THE MODEL

The Littlest Higgs with T-parity (LHT) is a non-linear σ model based on the coset space $SU(5)/SO(5)$, where $SU(5)$ is the global symmetry, so guarantees 14 Nambu-Goldstone bosons. The global symmetry is broken by the vev ($\sim \mathcal{O}$(TeV)).

The gauge group is taken to be $G_1 \times G_2 = [SU(2) \times U(1)]^2$, subgroup of the $SU(5)$ global symmetry.

$$\begin{array}{ccccc} SU(5) & \xrightarrow{f} & SO(5) & & \\ \bigcup & & \bigcup & & \\ [SU(2) \times U(1)]^2 & \xrightarrow{f} & SU(2)_w \times U(1)_Y & \xrightarrow{v_h} & U(1)_{\mathrm{EM}} \end{array}$$

Figure 1: The global SU(5) contains two copies of local $[SU(2) \times U(1)]^2$ that are diagonally broken to one copy $SU(2) \times U(1)$ contained in SO(5).

f is the energy scale where the symmetry breaking $[SU(2) \times U(1)]^2 \rightarrow SU(2)_W \times U(1)_Y$ occurs. The global symmetry breaking scale, f, is constrained on the order of a TeV.

T-parity is a natural symmetry of most little Higgs models (those that are product group models) where SM particles are even under this symmetry (T-even), while the new particles at the TeV scale are odd (T-odd). T-parity explicitly forbids any tree-level contributions from the heavy gauge bosons to the observables involving only SM particles as external states, as a result, the corrections to EWPO are generated exclusively at loop level.

A natural action of T-parity on the gauge fields is defined as

$$G_1 \leftrightarrow G_2, \tag{1}$$

its action on the gauge fields G_i exchanges the two gauge groups $SU(2)_i \times U(1)_i$. Then, T invariance requires that the gauge couplings associated to both factors are equal

$$g_1 = g_2 = \sqrt{2} g_W, \qquad g'_1 = g'_2 = \sqrt{2} g', \tag{2}$$

where g_W is the $SU(2)_W$ coupling constant and g' is the $U(1)_Y$ coupling constant.

The Scalar Lagrangian for the light and heavy gauge bosons sector is

$$\begin{aligned}\mathcal{L}_S = &\frac{1}{2}\left[\frac{g_W^2 v_h^2}{4}\left(1-\frac{v_h^2}{6f^2}\right)\right]W_L^+ W_L^- \\ &+\frac{1}{2}\left[f^2 g_W^2\left(1-\frac{v_h^2}{4f^2}\right)\right]W_H^+ W_H^- \\ &+\frac{1}{2}\left[f^2 g_W^2\left(1-\frac{v_h^2}{4f^2}\right)\right](Z_H)^2 \\ &+\frac{1}{2}\left[\frac{f^2 g'^2}{5}\left(1-\frac{5v_h^2}{4f^2}\right)\right](A_H)^2 \\ &+\frac{1}{2}\left[\frac{g_W^2 v_h^2}{4\cos^2\theta_W}\left(1-\frac{v_h^2}{6f^2}\right)\right]Z_L^2.\end{aligned} \tag{3}$$

If we pay attention, we will realize that there are no mixings between SM and heavy bosons.

The light gauge sector includes the $W_L^{\pm}$, Z_L, and A_L bosons, that we identify as the SM gauge bosons with masses

$$\begin{aligned}
M_{W_L^{\pm}} &= \frac{g_W v_h}{2}\left(1-\frac{v_h^2}{6f^2}\right)^{1/2} \approx \frac{g_W v_h}{2}\left(1-\frac{v_h^2}{12f^2}\right), \\
M_{Z_L} &= \frac{g_W v_h}{2\cos\theta_H}\left(1-\frac{v_h^2}{6f^2}\right)^{1/2} = \frac{M_{W_L^{\pm}}}{\cos\theta_W}, \quad (\rho \text{ is conserved}), \\
M_{A_L} &= 0,
\end{aligned} \tag{4}$$

and the mass of the heavy bosons are

$$\begin{aligned}
M_{W_H^{\pm}} = M_{Z_H} = f g_W\left(1-\frac{v_h^2}{4f^2}\right)^{1/2} &\approx f g_W\left(1-\frac{v_h^2}{8f^2}\right), \\
M_{A_H} = \frac{f g'}{\sqrt{5}}\left(1-\frac{5v_h^2}{4f^2}\right)^{1/2} &\approx \frac{f g'}{\sqrt{5}}\left(1-\frac{5v_h^2}{8f^2}\right).
\end{aligned} \tag{5}$$

FERMION SECTOR

We embed two $SU(5)$ incomplete quintuplets and introduce a right-handed $SO(5)$ multiplet Ψ_R [1]

$$\Psi_1 = \begin{pmatrix} i\psi_1 \\ 0 \\ 0 \end{pmatrix}, \quad \Psi_2 = \begin{pmatrix} 0 \\ 0 \\ i\psi_2 \end{pmatrix}, \quad \Psi_R = \begin{pmatrix} \psi_R^c \\ \chi_R \\ \psi_R \end{pmatrix}, \tag{6}$$

where the superscript (^{c}) denotes a partner lepton field, not to be confused with charge conjugation, and χ_R is a lepton singlet. Above (σ^2 is the second Pauli matrix)

$$\psi_i = -\sigma^2 \begin{pmatrix} \nu_{L_i} \\ \ell_{L_i} \end{pmatrix} \qquad (i = 1, 2), \qquad \psi_R = -\sigma^2 \begin{pmatrix} \nu_{HR} \\ \ell_{HR} \end{pmatrix}. \tag{7}$$

[1] Monika Blanke, *et al.*, JHEP **01** (2007), 066.

The mirror (T-odd) and partner leptons can be given $\mathcal{O}(f)$ masses via [2]

$$\mathcal{L}_{Y_H} = -\kappa f \left(\overline{\Psi}_2 \xi + \overline{\Psi}_1 \Sigma_0 \xi^\dagger \right) \Psi_R + \kappa_2 \tilde{\Psi}_R^T \Psi_R + \text{h.c.}, \tag{8}$$

where $\xi = e^{i\Pi/f}$, being Π the Goldstone bosons matrix, f is the new physics (NP) energy scale of $\mathcal{O}$(TeV) and κ_2 is a Dirac mass. The T-odd leptons thus acquire masses after EWSB, given by[3] [4]

$$m_{\ell_H^i} = \sqrt{2}\kappa_{ii} f \equiv m_{Hi}, \quad m_{\nu_H^i} = m_{Hi} \left(1 - \frac{v^2}{8f^2} \right), \tag{9}$$

where κ_{ii} are the eigenvalues of the mass matrix κ. The mass eigenstates are defined as the T-odd combination which turn out to be

$$\psi_H = \frac{1}{\sqrt{2}} \left(\psi_1 + \psi_2 \right), \tag{10}$$

and the T-even combination given by

$$\psi_{SM} = \frac{1}{\sqrt{2}} \left(\psi_1 - \psi_2 \right). \tag{11}$$

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[2] Francisco del Aguila, *et al.*, JHEP **12**, (2019), 154.
[3] Monika Blanke, *et al.*, JHEP **01** (2007), 066.
[4] Jay Hubisz, *et al.*, JHEP **06**, (2006), 041.

It happens similarly for the masses of T-odd quarks, with κ^q_{ii} instead of κ_{ii}, and replacing d_H-quark by ℓ_H, u_H-quark by ν_H, and κ^q_2 by κ_2 for the mass matrix of partner quarks of u and d types.

Neutrino masses in the LHT and new contributions to LFV processes

NEUTRINO MASSES IN THE LHT

Recalling that the T-odd fermions are given masses of order of $\mathcal{O}(f)$ by the following Lagrangian

$$\mathcal{L}_{Y_H} = -\kappa f \left(\overline{\Psi}_2 \xi + \overline{\Psi}_1 \Sigma_0 \xi^\dagger\right) \Psi_R + h.c. \approx \sqrt{2}\kappa f\, \overline{\psi}_{HL} \psi_{HR}, \tag{12}$$

approaching $\xi = \exp\left(i\Pi/f\right) \approx \mathbb{I}$. Furthermore, let us recall the structure of Ψ_R:

$$\Psi_R = \begin{pmatrix} \psi_R^c \\ \chi_R \\ -i\sigma^2 l_{HR} \end{pmatrix}, \quad \Psi_R \xrightarrow{T} \Omega \Psi_R, \tag{13}$$

where χ_R stands for lepton singlets. The lepton singlets χ_R must also get a large (vector–like) mass by combining with a LH singlet χ_L through a direct mass term without further couplings to the Higgs. Thus, its mass term is written

$$\mathcal{L}_M = -M \overline{\chi}_L \chi_R + \text{h.c.}. \tag{14}$$

χ_L is an $SU(5)$ singlet and it is therefore nature to include a small Majorana mass for it. Once LN is assumed to be only broken by small Majorana masses μ in the heavy LH neutral sector,

$$\mathcal{L}_\mu = -\frac{\mu}{2}\overline{\chi_L^c}\chi_L + h.c., \quad (15)$$

the resulting (T-even) neutrino mass matrix reduces to the inverse see-saw one [5]:

$$\mathcal{L}_M^\nu = -\frac{1}{2}\left(\overline{\nu_l^c}\ \overline{\chi_R}\ \overline{\chi_L^c}\right)\mathcal{M}_\nu^{T-even}\begin{pmatrix}\nu_L\\ \chi_R^c\\ \chi_L\end{pmatrix} + h.c., \quad (16)$$

where

$$\mathcal{M}_\nu^{T-even} = \begin{pmatrix} 0 & i\kappa^* f sin\left(\frac{v}{\sqrt{2}f}\right) & 0 \\ i\kappa^\dagger f sin\left(\frac{v}{\sqrt{2}f}\right) & 0 & M^\dagger \\ 0 & M^* & \mu \end{pmatrix}, \quad (17)$$

with each entry standing for a 3×3 matrix to take into account the 3 lepton families. The κ entries are given by the Yukawa Lagrangian in eq. (12) and M stands for the direct heavy Dirac mass matrix from eq. (14), and μ is the mass matrix of small Majorana masses in eq.(15).

[5] Francisco del Aguila, José Ignacio Illana, José María Pérez-Poyatos, and José Santiago, Inverse see–saw neutrino masses in the Littlest Higgs model with T–parity, arXiv:1910.09569v2 [hep-ph].

On considering the hierarchy $\mu << \kappa f << M$ (inverse see-saw), the mass eigenvalues for M are ~ 10 TeV, of the order of $4\pi f$ with $f \sim TeV$, as required by current EWPD if we assume the κ eigenvalues to be order 1 [6]. On the contrary, the μ eigenvalues shall be much smaller than the GeV.

[6] We have considered lighter, O(4 TeV), Majorana neutrino masses in *I. Pacheco and P. Roig. Rev. Mex. Fis. Suppl.* ***3*** *(2022) no.2, 1-10.*

After diagonalizing (Majorana) mass matrix of eq. (17), The Lagrangian can be read

$$\mathcal{L}_M^{\nu} = -\frac{1}{2}\left(\sum_{i=1}^{3}(\mathcal{M}_{\nu}^{l})_i\overline{\nu_{Li}^{l}}\nu_{Ri}^{l} + \sum_{j=4}^{9}(\mathcal{M}_{\chi}^{h})_j\overline{\Psi_{Lj}^{h}}\Psi_{Rj}^{h}\right). \tag{18}$$

We notice $\mathcal{M}_{\nu}^{l}$ is a 3×3 matrix and $\mathcal{M}_{\chi}^{h}$ is a 6×6 matrix.

Applying $(\mu \ll \kappa f \ll M)$ hierarchy, the eigenstates of light and heavy Majorana neutrinos transform as [7]

$$\begin{aligned}\sum_{j=1}^{3}(U_{PMNS})_{ij}\nu_{Lj}^{l} &= \sum_{j=1}^{3}[\mathbf{1}_{3\times 3} - \frac{1}{2}(\theta\theta^{\dagger})]_{ij}\nu_{Lj} - \sum_{j=7}^{9}\theta_{ij}\chi_{Lj},\\ \chi_{Li}^{h} &= \sum_{j=7}^{9}[\mathbf{1}_{3\times 3} - \frac{1}{2}(\theta^{\dagger}\theta)]_{ij}\chi_{Lj} + \sum_{j=1}^{3}\theta_{ij}^{\dagger}\nu_{Lj},\end{aligned} \tag{19}$$

where θ matrix mixes light with heavy Majorana neutrinos.

[7] Francisco del Aguila, José Ignacio Illana, José María Pérez-Poyatos, and José Santiago, Inverse see–saw neutrino masses in the Littlest Higgs model with T–parity, arXiv:1910.09569v2 [hep-ph].

The charged and neutral SM Lagrangians read

$$\mathcal{L}_W^l = \frac{g}{\sqrt{2}} W_\mu^+ \sum_{i,j=1}^{3} \overline{\nu_i^l} W_{ij} \gamma^\mu P_L \ell_j + h.c., \quad \text{with} \quad W_{ij} = \sum_{k=1}^{3} (U_{PMNS}^\dagger)_{ik} [\mathbf{1}_{3\times 3} - \frac{1}{2}(\theta\theta^\dagger)]_{kj},$$
$$\mathcal{L}_W^{lh} = \frac{g}{\sqrt{2}} W_\mu^+ \sum_{i,j=1}^{3} \overline{\chi_i^h} \theta_{ij}^\dagger \gamma^\mu P_L \ell_j + h.c.. \tag{20}$$

and

$$\mathcal{L}_Z^l = \frac{g}{2\cos\theta_W} Z_\mu \sum_{i,j=1}^{3} \overline{\nu_i^l} \gamma^\mu (X_{ij} P_L - X_{ij}^\dagger P_R) \nu_j^l, \quad \text{with} \quad X_{ij} = \sum_{k=1}^{3} \left(U_{PMNS}^\dagger [\mathbf{1}_{3\times 3} - (\theta\theta^\dagger)] \right)_{ik} (U_{PMNS})_{kj},$$
$$\mathcal{L}_Z^{lh} = \frac{g}{2cos\theta_W} Z_\mu \sum_{i,j=1}^{3} \overline{\chi_i^h} \gamma^\mu (Y_{ij} P_L - Y_{ij}^\dagger P_R) \nu_j^l + h.c., \quad \text{with} \quad Y_{ij} = \sum_{k=1}^{3} \theta_{ik}^\dagger (U_{PMNS})_{kj},$$
$$\mathcal{L}_Z^h = \frac{g}{2\cos\theta_W} Z_\mu \sum_{i,j=1}^{3} \overline{\chi_i^h} \gamma^\mu (S_{ij} P_L - S_{ij}^\dagger P_R) \chi_j^h, \quad \text{with} \quad S_{ij} = \sum_{k=1}^{3} \theta_{ik}^\dagger \theta_{kj}. \tag{21}$$

PARTICLE CONTENT

Particle Content of LHT with Majorana neutrinos	
Nambu–Goldstone bosons	
SM Higgs	$\left(-i\pi^{+}/\sqrt{2},\ (v+h+i\pi^{0})/2\right)$
Longitudinal modes of the heavy gauge fields	$\omega^{\pm}, \omega^{0}, \eta$
Complex $SU(2)_L$ triplet	$\begin{pmatrix} i\Phi^{--} & i\Phi^{-}/\sqrt{2} \\ i\Phi^{-}/\sqrt{2} & (i\Phi^{0}+\Phi^{P})/\sqrt{2} \end{pmatrix}$
Gauge bosons	
SM gauge bosons (T-even)	$\{W_L^{\pm}, Z_L, \gamma\}$
Heavy gauge bosons (T-odd)	$\{W_H^{\pm}, Z_H, A_H\}$
Fermions (i = 1,2,3)	
SM Fermions (T-even)	$\{\ell_L^i, \nu_L^i, u_L^i, d_L^i\}$
Mirror/Heavy/T-odd Fermions	$\{\ell_H^i, \nu_H^i, u_H^i, d_H^i\}$
Partner Fermions	$\{\ell_i^c, \nu_i^c, u_i^c, d_i^c\}$
Majorana neutrinos ($i = 1, 2, 3$)	
Heavy Majorana neutrinos	χ_{Li}^{h}

Table 1: The full content of particles of LHT with Majorana neutrinos.

BOUNDS ON LFV PROCESSES

We present various LFV processes in this section: $\ell \to \ell' \gamma$ and $\ell \to \ell' \ell'' \bar{\ell}'''$, where the last one has three possible channels that are shown in Table 2

Type	Flavor	$\ell \to \ell' \ell'' \bar{\ell}'''$		
1	$\ell \neq \ell' = \ell'' = \ell'''$	$\mu \to ee\bar{e}$	$\tau \to ee\bar{e}$	$\tau \to \mu\mu\bar{\mu}$
2	$\ell \neq \ell' \neq \ell'' = \ell'''$		$\tau \to e\mu\bar{\mu}$	$\tau \to \mu e\bar{e}$
3	$\ell \neq \ell' = \ell'' \neq \ell'''$		$\tau \to ee\bar{\mu}$	$\tau \to \mu\mu\bar{e}$

Table 2: Three diferent decay channels of the $\ell \to \ell' \ell'' \bar{\ell}'''$ processes.

All of them involve the effective interaction of a neutral vector boson with a pair of on-shell fermions, through a loop with Majorana neutrinos.

$\ell \to \ell' \gamma$ DECAYS

The complete branching ratio to $\mu \to e\gamma$ decay considering active and heavy Majorana neutrinos yields

$$\mathrm{Br}(\mu \to e\gamma) = \frac{3\alpha}{2\pi} \left| \theta_{ej}\theta^{\dagger}_{\mu j} F^{\chi}_{M}(x) + W_{ej} W^{\dagger}_{\mu j} F^{\nu}_{M}(y) \right|^2, \tag{22}$$

with $x = \frac{M_W^2}{M_j^2}$, $y = \frac{m_{\nu_i}^2}{M_W^2}$ and W_{ij} matrix is given by the eq. (20).

It is important to note that the contribution to the $\mathrm{Br}(\mu \to e\gamma)$ coming from ν_H neutrinos behaves as

$$\mathrm{Br}(\mu \to e\gamma) = \frac{3\alpha}{2\pi} \left| \frac{v^2}{4f^2} V^{ie*}_{H\ell} V^{i\mu}_{H\ell} F_W(x) \right|^2, \tag{23}$$

this contribution is neglected because of $v^2/4f^2 << 1$ term.

Finally, after some simplifications

$$\mathrm{Br}(\mu \to e\gamma) \approx \frac{3\alpha}{8\pi} \left| \theta_{ej}\theta^{\dagger}_{\mu j} \right|^2. \tag{24}$$

We know that $Br(\mu \to e\gamma) < 4.2 \times 10^{-13}$, $\mathrm{Br}(\tau \to e\gamma) < 3.3 \times 10^{-8}$, $\mathrm{BR}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$ (at 90% C.L.) [8], hence

$$\left|\theta_{ej}\theta^{\dagger}_{\mu j}\right| < 0.14 \times 10^{-4}. \quad (25)$$

$$|\theta_{ej}\theta^{\dagger}_{\tau j}| < 0.95 \times 10^{-2}, \quad (26)$$

$$|\theta_{\mu j}\theta^{\dagger}_{\tau j}| < 0.011. \quad (27)$$

We notice that our result from the eq. (25) matches with the reported one in the Table 1 of [[9]] but the results from the eqs. (26) and (27) do not. For tau decays in the 0.17 factor is missing.

[8]R.L. Workman et al.(Particle Data Group), to be published (2022)

[9] Francisco del Aguila, José Ignacio Illana, José María Pérez-Poyatos, and José Santiago, Inverse see–saw neutrino masses in the Littlest Higgs model with T–parity, arXiv:1910.09569v2 [hep-ph].

Limits on LFV processes driven by $\mathcal{O}$(TeV) Majorana neutrinos

GLOBAL ANALYSIS

In this subsection we do a global analysis of the 10 processes above: LFV Z decays $Z \to \bar{\mu}e$, $Z \to \bar{\tau}e$, and $Z \to \bar{\tau}\mu$; LFV Type I $\mu \to ee\bar{e}$, $\tau \to ee\bar{e}$ and $\tau \to \mu\mu\bar{\mu}$; LFV Type II $\tau \to e\mu\bar{\mu}$ and $\tau \to \mu e\bar{e}$; $\mu - e$ conversion in nuclei $^{48}_{22}$Ti and $^{197}_{79}$Au.

We do the analysis through a single Monte Carlo simulation where the 10 processes are run simultaneously. The peculiarity of all these LFV processes is that they share the same free parameters: three heavy neutrino masses $\mathrm{M_i}$ with $i = 1, 2, 3$ and the neutral couplings given by $(\theta S \theta^\dagger)$ matrices.

After several attempts we decided to take the heavy neutrino masses interval from 15 to 20 TeV, since in this interval is where there are the greatest number of results that satisfy the limit of the branching ratios and conversion rates.

From the form factors of all those LFV decays we can see that they receive two contributions: one is coming from charged couplings $(\theta\theta^\dagger)$ and the second one comes from neutral couplings $(\theta S \theta^\dagger)$. It implies that there is an interference between them.

LFV Z decays	Our mean values	Present limits [PDG]
$\mathrm{Br}(Z \to \bar{\mu} e)$	1.20×10^{-14}	3.7×10^{-7}
$\mathrm{Br}(Z \to \bar{\tau} e)$	1.46×10^{-8}	4.9×10^{-6}
$\mathrm{Br}(Z \to \bar{\tau} \mu)$	1.09×10^{-8}	0.6×10^{-5}
LFV Type I		
$\mathrm{Br}(\mu \to e e \bar{e})$	1.85×10^{-14}	1.0×10^{-12}
$\mathrm{Br}(\tau \to e e \bar{e})$	4.16×10^{-9}	2.7×10^{-8}
$\mathrm{Br}(\tau \to \mu \mu \bar{\mu})$	4.24×10^{-9}	2.1×10^{-8}
LFV Type II		
$\mathrm{Br}(\tau \to e \mu \bar{\mu})$	3.60×10^{-9}	2.7×10^{-8}
$\mathrm{Br}(\tau \to \mu e \bar{e})$	2.48×10^{-9}	1.8×10^{-8}
$\mu - e$ conversion rate		
$\mathcal{R}(\mathrm{Ti})$	6.21×10^{-14}	4.3×10^{-13}
$\mathcal{R}(\mathrm{Au})$	7.82×10^{-14}	7.0×10^{-12}
Heavy neutrino masses		
M_1 (TeV)	17.186	
M_2 (TeV)	17.185	
M_3 (TeV)	17.187	

Table 3: Mean values for branching ratios, conversion rates and three heavy neutrino masses compared to current upper limits.

The modulus of the $(\theta S\theta^\dagger)_{e\mu}$ elements are all smaller than 7.5×10^{-10}, while for the other flavor combinations we get $|(\theta S\theta^\dagger)_{e\tau}| < 5.13\times10^{-7}$ and $|(\theta S\theta^\dagger)_{\mu\tau}| < 6.2\times10^{-7}$.

Bounds for wrong sign processes, Type III: $\ell \to \ell'\ell''\bar{\ell}'''$ with $\ell \neq \ell' = \ell'' \neq \ell'''$

In this subsection we are going to study two tau decays which are known as wrong processes: $\tau \to ee\bar{\mu}$ and $\tau \to \mu\mu\bar{e}$.

We will have 9 free parameters for each process: M_i, $(\theta_{\mu i}\theta_{\tau i})^\dagger$ $((\theta_{ei}\theta_{\tau i})^\dagger)$, and $\theta_{ei}\theta_{ei}$ $(\theta_{\mu i}\theta_{\mu i})$ with $i = 1, 2, 3$.

We bind the coupling terms as follows [10]

$$\begin{aligned} &|\theta_{\mu 1}\theta_{\tau 1}| + |\theta_{\mu 2}\theta_{\tau 2}| + |\theta_{\mu 3}\theta_{\tau 3}| < 0.32 \times 10^{-3}, \\ &|\theta_{e1}\theta_{e1}| + |\theta_{e2}\theta_{e2}| + |\theta_{e3}\theta_{e3}| < 0.01, \end{aligned} \tag{28}$$

from the equations above we limit each term

$$\begin{aligned} &-0.32 \times 10^{-3} \leq (\theta_{\mu 1}\theta_{\tau 1})^\dagger, (\theta_{\mu 2}\theta_{\tau 2})^\dagger, (\theta_{\mu 3}\theta_{\tau 3})^\dagger \leq 0.32 \times 10^{-3}, \\ &-0.01 \leq (\theta_{e1}\theta_{e1}), (\theta_{e2}\theta_{e2}), (\theta_{e3}\theta_{e3}) \leq 0.01, \end{aligned} \tag{29}$$

and the product of them must satisfy that

$$|\theta_{\mu i}\theta_{\tau i}||\theta_{ej}\theta_{ej}| < 0.32 \times 10^{-5}. \tag{30}$$

[10] Enrique Fernández-Martínez, Josu Hernández-García, Jacobo López-Pavón, Global constraints on heavy neutrino mixing, JHEP 1608 (2016) 033, [arXiv:1605.08774v2 [hep-ph]].

We are analysing the wrong sign processes which were computed simultaneously through a single Monte Carlo simulation where the free parameters are the heavy neutrino masses $(\mathrm{M}_{i=1,2,3})$ and the LNV couplings.

The heavy neutrino masses $\mathrm{M_i}$ $(i = 1, 2, 3)$ run from 15 to 20 TeV, we decided to take this interval based on the experience gained in the previous processes as in this one data are more concentrated.

Branching Ratios	Our mean values
$\mathrm{Br}(\tau \to ee\bar{\mu})$	1.8×10^{-9}
$\mathrm{Br}(\tau \to \mu\mu\bar{e})$	1.9×10^{-9}
Heavy neutrino masses	
M_1 (TeV)	17.170
M_2 (TeV)	17.166
M_3 (TeV)	17.166
LNV couplings	
$(\theta_{e1}\theta_{\tau 1})^{\dagger}$	$(2.24 \pm 9.59) \times 10^{-7}$
$(\theta_{e2}\theta_{\tau 2})^{\dagger}$	$(14.82 \pm 9.69) \times 10^{-7}$
$(\theta_{e3}\theta_{\tau 3})^{\dagger}$	$(1.84 \pm 9.79) \times 10^{-7}$
$\lvert\theta_{ei}\theta_{\tau i}\rvert$	2.76×10^{-4}
$(\theta_{\mu 1}\theta_{\mu 1})$	$(0.75 \pm 2.98) \times 10^{-5}$
$(\theta_{\mu 2}\theta_{\mu 2})$	$-(8.78 \pm 2.99) \times 10^{-5}$
$(\theta_{\mu 3}\theta_{\mu 3})$	$(1.02 \pm 2.98) \times 10^{-5}$
$\lvert\theta_{\mu i}\theta_{\mu i}\rvert$	8.5×10^{-3}
$(\theta_{\mu 1}\theta_{\tau 1})^{\dagger}$	$(2.08 \pm 2.26) \times 10^{-6}$
$(\theta_{\mu 2}\theta_{\tau 2})^{\dagger}$	$-(0.39 \pm 2.27) \times 10^{-6}$
$(\theta_{\mu 3}\theta_{\tau 3})^{\dagger}$	$(0.55 \pm 2.25) \times 10^{-6}$
$\lvert\theta_{\mu i}\theta_{\tau i}\rvert$	6.52×10^{-4}
$(\theta_{e1}\theta_{e1})$	$(3.88 \pm 1.95) \times 10^{-5}$
$(\theta_{e2}\theta_{e2})$	$(4.59 \pm 1.96) \times 10^{-5}$
$(\theta_{e3}\theta_{e3})$	$-(5.04 \pm 1.95) \times 10^{-5}$
$\lvert\theta_{ei}\theta_{ei}\rvert$	5.65×10^{-3}

Table 4: Mean values for the free parameters and branching ratios in the wrong sign processes considering Majorana neutrinos.

The mean values for the heavy neutrino masses from the studies in the previous section differ only slightly from the 'Wrong Sign' analysis, $\sim 0.12\%$ in all cases.

Lepton Flavour Violation in Hadron Decays of the Tau Lepton in LHT

$\tau \to \ell q\bar{q}$ ($\ell = e, \mu$)

Two generic topologies are involved in this amplitude: i) penguin-like diagrams, namely $\tau \to \ell\{\gamma, Z\}$, followed by $\{\gamma, Z\} \to q\bar{q}$ and ii) box diagrams. We will assume, for simplicity, that light quarks and leptons (ℓ) are massless in our calculation.

The full amplitude is given by two contributions: one of them comes from T-odd particles and the other from heavy Majorana neutrinos, then

$$\mathcal{M} = \mathcal{M}^{\mathrm{T-odd}} + \mathcal{M}^{\mathrm{Maj}}, \tag{31}$$

where each one is written

$$\begin{aligned} \mathcal{M}^{\mathrm{T-odd}} &= \mathcal{M}_{\gamma}^{\mathrm{T-odd}} + \mathcal{M}_{Z}^{\mathrm{T-odd}} + \mathcal{M}_{\mathrm{box}}^{\mathrm{T-odd}}, \\ \mathcal{M}^{\mathrm{Maj}} &= \mathcal{M}_{\gamma}^{\mathrm{Maj}} + \mathcal{M}_{Z}^{\mathrm{Maj}} + \mathcal{M}_{\mathrm{box}}^{\mathrm{Maj}}, \end{aligned} \tag{32}$$

we will use the 't Hooft-Feynman gauge along the calculation and will write $\ell = \mu$ for definiteness.

$\tau \to \mu P$

These decays where $P = \{\pi^0, \eta, \eta'\}$ are mediated only by axial-vector current (Z gauge boson) and box diagrams. Thus, the amplitude is given by

$$\mathcal{M}_{\tau\to\mu P} = \mathcal{M}_Z^P + \mathcal{M}_{\text{Box}}^P. \tag{33}$$

Each contribution reads

$$\begin{aligned}\mathcal{M}_Z^P &= -i\frac{g^2}{2c_W}\frac{F}{M_Z^2}C(P)\sum_i \overline{\mu}(p')\left[\not{Q}F_L^Z P_L\right]\tau(p),\\ \mathcal{M}_{\text{Box}}^P &= -ig^2F\sum_i B_i(P)\overline{\mu}(p')\left[\not{Q}P_L\right]\tau(p),\end{aligned} \tag{34}$$

where $C(P)$ and $B_j(P)$ functions are given as follows

$$\begin{aligned}C(\pi^0) &= 1,\\ C(\eta) &= \frac{1}{\sqrt{6}}\left(\sin\theta_\eta + \sqrt{2}\cos\theta_\eta\right),\\ C(\eta') &= \frac{1}{\sqrt{6}}\left(\sqrt{2}\sin\theta_\eta - \cos\theta_\eta\right).\end{aligned} \tag{35}$$

The box amplitude is composed by the following $B_j(P)$ factors

$$
\begin{aligned}
B_j(\pi^0) &= \frac{1}{2}(B_d^j - B_u^j), \\
B_j(\eta) &= \frac{1}{2\sqrt{3}}\left[(\sqrt{2}\sin\theta_\eta - \cos\theta_\eta)B_u^j + (2\sqrt{2}\sin\theta_\eta + \cos\theta_\eta)B_d^j\right], \\
B_j(\eta') &= \frac{1}{2\sqrt{3}}\left[(\sin\theta_\eta - 2\sqrt{2}\cos\theta_\eta)B_d^j - (\sin\theta_\eta + \sqrt{2}\cos\theta_\eta)B_u^j\right],
\end{aligned} \tag{36}
$$

where the B_q^j functions are the form factors from box diagrams and the angle $\theta_\eta \simeq -18^\circ$.

The branching ratio reads

$$
\mathrm{Br}(\tau \to \mu P) = \frac{1}{4\pi}\frac{\lambda^{1/2}(m_\tau^2, m_\mu^2, m_P^2)}{m_\tau^2\Gamma_\tau}\frac{1}{2}\sum_{i,f}|\mathcal{M}_{\tau\to\mu P}|^2, \tag{37}
$$

where $\Gamma_\tau \approx 2.267\times 10^{-12}$ GeV and $\lambda(x,y,z) = (x+y-z)^2 - 4xy$.

Thus,

$$\sum_{i,f} |\mathcal{M}_{\tau\to\mu P}|^2 = \frac{1}{2m_\tau} \sum_{k,l} \left[(m_\tau^2 + m_\mu^2 - m_P^2)(a_P^k a_P^{l*} + b_P^k b_P^{l*}) + 2m_\mu m_\tau (a_P^k a_P^{l*} - b_P^k b_P^{l*}) \right], \tag{38}$$

with $k, l = Z, B$. Defining $\Delta_{\tau\mu} = m_\tau - m_\mu$ and $\Sigma_{\tau\mu} = m_\tau + m_\mu$ we get

$$\begin{aligned} a_P^Z &= -\frac{g^2}{2c_W} \frac{F}{2} \frac{C(P)}{M_Z^2} \Delta_{\tau\mu} (F_L^Z + F_R^Z), \\ b_P^Z &= \frac{g^2}{2c_W} \frac{F}{2} \frac{C(P)}{M_Z^2} \Sigma_{\tau\mu} (F_R^Z - F_L^Z), \\ a_P^B &= -\frac{g^2 F}{2} \Delta_{\tau\mu} B_j(P), \\ b_P^B &= -\frac{g^2 F}{2} \Sigma_{\tau\mu} B_j(P). \end{aligned} \tag{39}$$

$\tau \to \mu PP$

In this part we will consider the decays into the pairs $P\overline{P} = \{\pi^+\pi^-, K^+K^-, K^0\overline{K^0}\}$. The contributions to this kind of decays come from $\gamma-$, $Z-$ penguins and box diagrams. Thus, the total amplitude can be written as follows

$$\mathcal{M}_{\tau\to\mu PP} = \mathcal{M}_{\gamma}^{P\overline{P}} + \mathcal{M}_{Z}^{P\overline{P}} + \mathcal{M}_{\text{Box}}^{P\overline{P}}. \tag{40}$$

The next step is to hadronize the quark bilinears appearing in each amplitude. They turn out to be

$$\begin{aligned}
\mathcal{M}_\gamma^{P\overline{P}} =& \frac{e^2}{Q^2} F_V^{P_1P_2}(s) \times \\
& \sum_j V_\ell^{j\mu *} V_\ell^{j\tau} \overline{\mu}(p')[Q^2(\not{p}_q - \not{p}_{\overline{q}}) F_L^\gamma(Q^2) P_L + 2im_\tau p_q^\mu \sigma_{\mu\nu} p_{\overline{q}}^\nu F_M^\gamma(Q^2) P_R]\tau(p), \\
\mathcal{M}_Z^{P\overline{P}} =& g^2 \frac{2s_W^2 - 1}{2c_W M_Z^2} F_V^{P_1P_2}(s) \sum_j V_\ell^{j\mu *} V_\ell^{j\tau} \overline{\mu}(p')(\not{p}_q - \not{p}_{\overline{q}})[\gamma^\mu(F_L^Z P_L + F_R^Z P_R)]\tau(p), \\
\mathcal{M}_{\text{Box}}^{P\overline{P}} =& \frac{g^2}{2} F_V^{P_1P_2}(s) \sum_j V_\ell^{j\mu *} V_\ell^{j\tau} (B_u^j - B_d^j) \overline{\mu}(p')(\not{p}_q - \not{p}_{\overline{q}}) P_L \tau(p).
\end{aligned} \tag{41}$$

After computing each amplitude, we get the following branching ratio

$$\text{Br}(\tau \to \mu PP) = \frac{\kappa_{PP}}{64\pi^3 m_\tau^2 \Gamma_\tau} \int_{s_-}^{s_+} ds \int_{t_-}^{t_+} dt \frac{1}{2} \sum_{i,f} |\mathcal{M}_{\tau\to\mu PP}|^2, \tag{42}$$

where κ_{PP} is 1 for $PP = \pi^+\pi^-, K^+K^-, K^0\bar{K}^0$ and $1/2$ for $PP = \pi^0\pi^0$. In terms of the momenta of the particles participating in the process, $s = (p_q + p_{\overline{q}})^2$ and $t = (p - p_{\overline{q}})^2$, so that

$$\begin{aligned}
t_-^+ &= \frac{1}{4s}\left[\left(m_\tau^2 - m_\mu^2\right)^2 - \left(\lambda^{1/2}(s, m_{P_1}^2, m_{P_2}^2) \mp \lambda^{1/2}(m_\tau^2, s, m_\mu^2)\right)^2\right], \\
s_- &= 4m_P^2, \\
s_+ &= (m_\tau - m_\mu)^2.
\end{aligned} \tag{43}$$

$\tau \to \mu V$

When an experiment measures a final state with a vector resonance, the experiment reconstructs its structure from the pair of pseudoscalar mesons with a squared total mass approaching m_V^2, where $V = \rho, \phi$. For these cases the branching ratio of $\tau \to \mu V$ is related with the $\tau \to \mu PP$ by trying to implement the experimental procedure as follows

$$\mathrm{Br}(\tau \to \mu V) = \sum_{P_1, P_2} \mathrm{Br}(\mu P_1 P_2)\Big|_V . \qquad (44)$$

In the above equation the s limits are now restricted to

$$s_- = M_V^2 - \frac{1}{2} M_V \Gamma_V, \quad s_+ = M_V^2 + \frac{1}{2} M_V \Gamma_V . \qquad (45)$$

Therefore, when $V = \rho, \phi$ their branching ratios are given by

$$\begin{aligned} \mathrm{Br}(\tau \to \mu\rho) &= \mathrm{Br}(\tau \to \mu\pi^+\pi^-)\big|_\rho , \\ \mathrm{Br}(\tau \to \mu\phi) &= \mathrm{Br}(\tau \to \mu K^+K^-)\big|_\phi + \mathrm{Br}(\tau \to \mu K^0\bar{K}^0)\Big|_\phi . \end{aligned} \qquad (46)$$

NUMERICAL ANALYSIS

First of all, we recall the masses of particles which come from LHT that are involved in the processes under study:

$$M_W = \frac{v}{2s_W}\left(1-\frac{v^2}{12f^2}\right), \quad M_Z = M_W/c_W, \quad v \simeq 246\text{ GeV}, \quad (\rho \text{ factor is conserved}),$$
$$M_{W_H} = M_{Z_H} = \frac{f}{s_W}\left(1-\frac{v^2}{8f^2}\right) \approx 0.65f, \quad M_{A_H} = \frac{f}{\sqrt{5}c_W}\left(1-\frac{5v^2}{8f^2}\right) \approx 0.16f, \quad M_\Phi = \sqrt{2}M_h\frac{f}{v},$$
$$m_{\ell^i_H} = \sqrt{2}\kappa_{ii}f \equiv m_{Hi}, \quad m_{\nu^i_H} = m_{Hi}\left(1-\frac{v^2}{8f^2}\right), \quad m_{\ell^c,\nu^c} = \kappa_2. \tag{47}$$

We can approximate

$$M_\Phi = \sqrt{2}M_h\frac{f}{v} \approx \frac{\sqrt{2}}{2}f. \tag{48}$$

Considering just mixing between two lepton families the mixing matrix of T-odd leptons $(V_{H\ell}^{i\mu *} V_{H\ell}^{i\tau})$ and the mixing matrix among partner leptons $(W_{ij}^{\dagger} W_{jk})$ can be parameterized as follows [11]

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_V & \sin\theta_V \\ 0 & -\sin\theta_V & \cos\theta_V \end{pmatrix} , \quad W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_W & \sin\theta_W \\ 0 & -\sin\theta_W & \cos\theta_W \end{pmatrix} , \tag{49}$$

where $\theta_V, \theta_W \in [0, \pi/2)$ is the physical range for the mixing angles and θ_W must not be confused with the weak-mixing ('Weinberg') angle. We have assumed $\mu - \tau$ mixing, similarly for the evaluation of processes with $\tau - e$.

We will assume no extra quark mixing and degenerate heavy quarks, then $V_{H\ell}^{q}$ and W^q will be equal to the identity. Therefore, the other free parameter are: θ_V, θ_W and neutral couplings of heavy Majorana neutrinos: $(\theta S \theta^{\dagger})_{\mu\tau}$.

[11]Francisco del Aguila, Lluis Ametller, José Ignacio Illana, José Santiago, Pere Talavera, and Roberto Vega-Morales, The Full Lepton Flavor of the Littlest Higgs Model with T–parity, doi 10.1007/JHEP07(2019)154, arXiv:1901.07058.

For completeness, we include two analyses: first we do not assume heavy Majorana neutrinos contributions and the second case involves the presence of heavy Majorana neutrinos raising from the Inverse See Saw mechanism seen in previous sections. We have decided to include the first case because there is not a previous analysis without contributions coming from Majorana neutrinos for those processes.

These processes are computed in a single Monte Carlo simulation which runs them simultaneously.

$\tau \to \ell P$ $(\ell = e, \mu)$ WITHOUT MAJORANA NEUTRINOS

$\tau \to \ell P$ $(\ell = e, \mu)$ (C.L. $= 90\%$) without Majorana neutrinos contribution.			
New physics (NP) scale (TeV)		Mixing angles	
f	1.49	θ_V	42.78°
Branching ratio		θ_W	42.69°
Br($\tau \to e\pi^0$)	5.24×10^{-9}	Masses of partner leptons ($m_{\nu^c} = m_{\ell^c}$)(TeV)	
Br($\tau \to \mu\pi^0$)	3.42×10^{-9}	$m_{\nu_1^c}$	3.12
Br($\tau \to e\eta$)	2.32×10^{-9}	$m_{\nu_2^c}$	3.15
Br($\tau \to \mu\eta$)	1.91×10^{-9}	$m_{\nu_3^c}$	3.37
Br($\tau \to e\eta'$)	2.20×10^{-8}	Masses of partner quarks ($m_{u^c} = m_{d^c}$) (TeV)	
Br($\tau \to \mu\eta'$)	1.79×10^{-8}	$m_{u_i^c}$	3.55
Masses of T-odd leptons (TeV)			
$m_{\ell_H^1}$	2.11		
$m_{\ell_H^2}$	2.11		
$m_{\ell_H^3}$	2.12		
$m_{\nu_H^1}$	2.10		
$m_{\nu_H^2}$	2.11		
$m_{\nu_H^3}$	2.11		
Masses of T-odd quarks (TeV)			
$m_{d_H^i}$	2.71		
$m_{u_H^i}$	2.70		

Table 5: Mean values obtained by Monte Carlo simulation of $\tau \to \ell P$ $(\ell = e, \mu)$ processes where Majorana neutrinos contribution is not considered.

The mean values for mixing angles θ_V and θ_W are 42.78° and 42.69°, respectively. If these values are transformed to rad (perhaps it is a more comfortable way to read this information), they become $\theta_V \approx \theta_W \approx \pi/4.2$. This result is close to maximize the LFV effects, since this happens when $\theta_V = \theta_W = \pi/4$.

$\tau \to \ell P$ $(\ell = e, \mu)$ with Majorana neutrinos

$\tau \to \ell P$ $(\ell = e, \mu)$ (C.L. $= 90\%$) with Majorana neutrinos contribution.			
New physics (NP) scale (TeV)		Mixing angles	
f	1.51	θ_V	43.07°
Branching ratio		θ_W	42.82°
Br($\tau \to e\pi^0$)	8.69×10^{-9}	Masses of partner leptons $(m_{\nu^c} = m_{\ell^c})$(TeV)	
Br($\tau \to \mu\pi^0$)	6.96×10^{-9}	$m_{\nu_1^c}$	3.26
Br($\tau \to e\eta$)	6.19×10^{-9}	$m_{\nu_2^c}$	3.26
Br($\tau \to \mu\eta$)	5.19×10^{-9}	$m_{\nu_3^c}$	3.30
Br($\tau \to e\eta'$)	2.19×10^{-8}	Masses of partner quarks $(m_{u^c} = m_{d^c})$ (TeV)	
Br($\tau \to \mu\eta'$)	1.94×10^{-8}	$m_{u_i^c}$	3.31
Masses of T-odd leptons (TeV)		Masses of heavy Majorana neutrinos (TeV)	
$m_{\ell_H^1}$	3.06	M_1	19.18
$m_{\ell_H^2}$	3.03	M_2	19.07
$m_{\ell_H^3}$	3.03	M_3	19.25
$m_{\nu_H^1}$	3.05	Neutral couplings of heavy Majorana neutrinos	
$m_{\nu_H^2}$	3.02	$\lvert(\theta S\theta^\dagger)_{e\tau}\rvert$	3.32×10^{-7}
$m_{\nu_H^3}$	3.02	$\lvert(\theta S\theta^\dagger)_{\mu\tau}\rvert$	3.90×10^{-7}
Masses of T-odd quarks (TeV)			
$m_{d_H^i}$	2.78		
$m_{u_H^i}$	2.78		

Table 6: Mean values for $\tau \to \ell P$ $(\ell = e, \mu)$ processes.

In contrast to the analysis done previously, here the heavy Majorana neutrinos are barely correlated among them. Recalling the results obtained in Section 3 the mean value for heavy Majorana masses is around 17.2 TeV, differing slightly $(\sim 0.12\%)$ in all cases. In this analysis the mean value for heavy Majorana neutrinos is ~ 19.16 TeV. Thus, the difference between these new results and the previous ones is just $\sim 10.23\%$.

The two neutral couplings $|(\theta S \theta^\dagger)_{\ell\tau}|$ $(\ell = e, \mu)$ have the same order of magnitude, $\mathcal{O}(10^{-7})$.

$\tau \to \ell PP,\ \ell V\ (\ell = e, \mu)$ WITHOUT MAJORANA NEUTRINOS

$\tau \to \ell PP,\ \ell V\ (\ell = e, \mu)$ (C.L. = 90%) without Majorana neutrinos contribution

New physics (NP) scale (TeV)	
f	1.50
Branching ratio	
Br($\tau \to e\pi^+\pi^-$)	3.92×10^{-9}
Br($\tau \to \mu\pi^+\pi^-$)	3.96×10^{-9}
Br($\tau \to eK^+K^-$)	2.38×10^{-9}
Br($\tau \to \mu K^+K^-$)	2.85×10^{-9}
Br($\tau \to eK^0\overline{K^0}$)	1.15×10^{-9}
Br($\tau \to \mu K^0\overline{K^0}$)	1.33×10^{-9}
Br($\tau \to e\rho$)	1.10×10^{-9}
Br($\tau \to \mu\rho$)	1.12×10^{-9}
Br($\tau \to e\phi$)	1.77×10^{-9}
Br($\tau \to \mu\phi$)	1.87×10^{-9}
Masses of T-odd leptons (TeV)	
$m_{\ell_H^1}$	3.13
$m_{\ell_H^2}$	2.99
$m_{\ell_H^3}$	3.10
$m_{\nu_H^1}$	3.12
$m_{\nu_H^2}$	2.98
$m_{\nu_H^3}$	3.09
Masses of T-odd quarks (TeV)	
$m_{d_H^i}$	2.92
$m_{u_H^i}$	2.91

Mixing angles	
θ_V	43.36°
θ_W	41.50°
Masses of partner leptons ($m_{\nu^c} = m_{\ell^c}$)(TeV)	
$m_{\nu_1^c}$	3.20
$m_{\nu_2^c}$	3.15
$m_{\nu_3^c}$	3.31
Masses of partner quarks ($m_{u^c} = m_{d^c}$) (TeV)	
$m_{u_i^c}$	3.32

$\tau \to \ell PP,\ \ell V\ (\ell = e, \mu)$ WITH MAJORANA NEUTRINOS

$\tau \to \ell PP,\ \ell V\ (\ell = e,\ \mu)$ (C.L. = 90%) with Majorana neutrinos contribution			
New physics (NP) scale (TeV)		Mixing angles	
f	1.54	θ_V	43.61°
Branching ratio		θ_W	42.21°
Br($\tau \to e\pi^+\pi^-$)	4.75×10^{-9}	Masses of partner leptons ($m_{\nu^c} = m_{\ell^c}$)(TeV)	
Br($\tau \to \mu\pi^+\pi^-$)	4.90×10^{-9}	$m_{\nu_1^c}$	2.91
Br($\tau \to eK^+K^-$)	2.53×10^{-9}	$m_{\nu_2^c}$	2.99
Br($\tau \to \mu K^+K^-$)	3.38×10^{-9}	$m_{\nu_3^c}$	2.95
Br($\tau \to eK^0\overline{K^0}$)	1.16×10^{-9}	Masses of partner quarks ($m_{u^c} = m_{d^c}$) (TeV)	
Br($\tau \to \mu K^0\overline{K^0}$)	1.50×10^{-9}	$m_{u_i^c}$	3.08
Br($\tau \to e\rho$)	1.33×10^{-9}	Masses of heavy Majorana neutrinos (TeV)	
Br($\tau \to \mu\rho$)	1.39×10^{-9}	M_1	18.82
Br($\tau \to e\phi$)	1.78×10^{-9}	M_2	19.33
Br($\tau \to \mu\phi$)	2.08×10^{-9}	M_3	18.92
Masses of T-odd leptons (TeV)		Neutral couplings of heavy Majorana neutrinos	
$m_{\ell_H^1}$	3.49	$\lvert(\theta S\theta^\dagger)_{e\tau}\rvert$	2.20×10^{-7}
$m_{\ell_H^2}$	3.47	$\lvert(\theta S\theta^\dagger)_{\mu\tau}\rvert$	3.14×10^{-7}
$m_{\ell_H^3}$	3.21		
$m_{\nu_H^1}$	3.48		
$m_{\nu_H^2}$	3.46		
$m_{\nu_H^3}$	3.20		
Masses of T-odd quarks (TeV)			
$m_{d_H^i}$	3.73		
[illegible]	3.71		

The branching ratios for all processes, when Majorana neutrinos contribution is considered, are greater than when this contribution is excluded. All our results for $\tau \to \ell P, PP, V$ $(\ell = e, \mu)$ are very promising, as they are only, at most, 2 orders of magnitude smaller than current bounds[12].

[12] R.L. Workman et al.(Particle Data Group), to be published (2022)

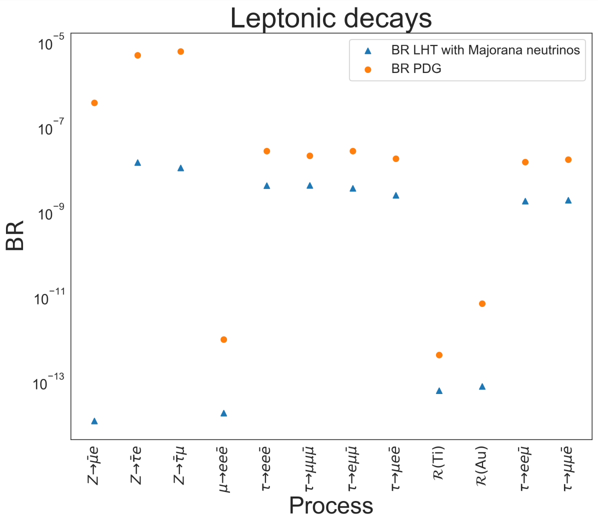

Figure 2: Leptonic decays considering Majorana neutrinos

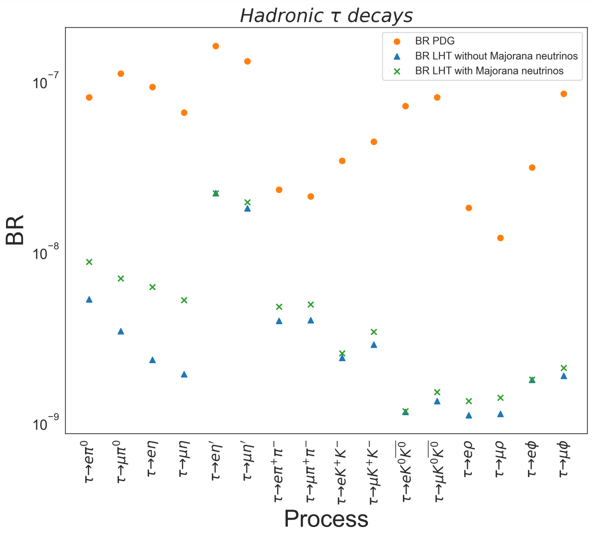

Figure 3: Hadronic τ decays considering Majorana neutrinos

CONCLUSION

In this work we developed all the needed tools in order to get the numerical predictions of branching ratios, particle masses, couplings, etc., coming from our model.

Our effort was focused firstly on studying purely leptonic decays involved in Section 3. Afterwards, the analysis was extended to LFV hadronic decays of the tau lepton, in Section 4. The most important results can be summarized as follows:

- From the numerical analysis done in Section 3 the new physics (NP) energy scale is around $f \sim 1.36$ TeV, whereas in Section 4 $f \sim 1.50$ TeV. The difference between these two figures is $\sim 9.34\%$, which is reasonable at this stage (a global analysis of all LFV processes is needed and will be presented elsewhere).
- LHT with Majorana neutrinos extension enables to bind the LNV couplings shown in Table 4. This is a novel result since they were not restricted in ref. [[13]]

[13] Enrique Fernández-Martínez, Josu Hernández-García, Jacobo López-Pavón, Global constraints on heavy neutrino mixing, JHEP 1608 (2016) 033, [arXiv:1605.08774v2 [hep-ph]].

- Regarding new couplings that we first encounter, there arise the neutral couplings of heavy Majorana neutrinos, denoted as $(\theta S \theta^\dagger)_{\ell\ell'}$. The magnitude of these couplings agree in both sets of analyses, as reported in Sections 3 and 4.
- Masses of particles coming from LHT, T-odd and partner fermions, are below 4 TeV, almost 5 times lighter than heavy Majorana neutrinos. This is consistent with the first item.
- The masses of heavy Majorana neutrinos in hadronic τ decays are $\mathrm{M}_i \sim 19$ TeV (we recall that $\mathrm{M}_i \sim 4\pi f$). Compared to previous analyses in leptonic decays, these masses of heavy Majorana neutrinos are heavier by ~ 2 TeV ($\sim 10\%$ of difference) and f is fully consistent.
- In all $\tau \to \ell\ell'\bar{\ell}''$ (including wrong-sign) decays and in $\mu \to e$ conversion in Ti, the mean values of our simulated events satisfying all present bounds are only one order of magnitude smaller than current limits. In $\mu \to ee\bar{e}$, $Z \to \bar{\tau}\ell$ and conversion in Au, our mean values are around two orders of magnitude smaller than current limits (only $Z \to \bar{\mu}e$ does not have the potential for probing our results in the near future).

All these results have been published in [14] and [15] and they offer us rosy prospects in a near future:

- Due to mean values of new physics (NP) energy scale f, masses of heavy Majorana neutrinos M_i and neutral couplings $(\theta S \theta^\dagger)_{\ell\ell'}$ matching in all our studies, we plan to undertake a global analysis (including purely leptonic processes and conversions in nuclei, see e.g. refs.[16 17], which is required and will be presented elsewhere.
- All our results are very promising and will be probed in current and near future measurements, as they lie approximately only one order of magnitude below currents bounds.

[14] I. Pacheco and P. Roig, "Lepton flavor violation in the Littlest Higgs Model with T parity realizing an inverse seesaw," JHEP **02** (2022), 054, doi:10.1007/JHEP02(2022)054.

[15] I. Pacheco and P. Roig Garces, "Lepton Flavour Violation in Hadron Decays of the Tau Lepton within the Littlest Higgs Model with T-parity," [arXiv:2207.04085 [hep-ph]].

[16] Tomas Husek, Kevin Monsálvez-Pozo, Jorge Portolés, Lepton-flavour violation in hadronic tau decays and $\mu - \tau$ conversion in nuclei, J. High Energ. Phys. 2021, 59 (2021). doi: 10.1007/JHEP01(2021)059.

[17] F. del Aguila, J. I. Illana and M. D. Jenkins, "Muon to electron conversion in the Littlest Higgs model with T-parity," JHEP **09** (2010), 040, doi:10.1007/JHEP09(2010)040.

Thank you!