Charmed baryon Λ_c^+ decays at BESIII

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- Introduction
- Decays involve neutron
 - $\Lambda_c^+ \rightarrow n\pi^+$
 - $\Lambda_c^+ \to n\pi^+\pi^0$, $n\pi^+\pi^-\pi^+$ and $nK^-\pi^+\pi^+$
- Partial wave analysis
 - $\Lambda_c^+ \to \Lambda \pi^+ \pi^0$
- Singly-Cabbibo-Suppressed decays
 - $\Lambda_c^+ \to \Sigma^0 K^+$ and $\Sigma^+ K_S^0$
- $\Sigma^+ h^+ h^-(\pi^0)$ decays
 - $\Lambda_c^+ \to \Sigma^+ K^+ K^-, \Sigma^+ K^+ \pi^- \text{ and } \Sigma^+ K^+ \pi^- \pi^0$
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 - $\Lambda_c^+ \rightarrow p K^- e^+ \nu_e$
 - $\Lambda_c^+ \to \Lambda e^+ \nu_e$
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- Summary

Introduction -- Charmed baryon physics

Why Λ_c^+ is interesting?

Charmed baryon $(\Lambda_c[udc])$

(qq)

(Q)

 $m_u, m_d \ll m_c \rightarrow diquark + quark$

- An important intermediate particle
 - Cornerstone of the charmed baryon spectra
 - Many b-baryons decay to Λ_c^+
- Its decays reveal information of strongand weak-interactions in charm region, complementary to charmed mesons



 $\rightarrow \underline{Charmed meson} (D^+[c\overline{d}]) \\ m_d \iff m_c \rightarrow \underline{quark} + \underline{heavy quark} \\ \underline{(q)} (Q)$

Non-trivial nonperturbative effects



Introduction -- BEPCII





Beijing Spectrometer(BESIII) Experiment





• Data produced near threshold without accompanying hadrons

Data samples	\sqrt{s} (GeV)	Int. $\mathcal{L}\left(pb^{-1} ight)$	•
	4.600	586.9	
	4.612	103.8	
	4.628	521.5	•
$\Lambda_{c}^{+}\overline{\Lambda}_{c}^{-}$	4.641	552.4	•
	4.661	529.6	•
	4.682	1669.3	
	4.699	536.1	

Single tag (ST): $\mathcal{B}(\Lambda_c \to f) = \frac{N_{\rm ST}}{2 \times N_{\rm pair}^{\rm tot} \times \epsilon_{\rm ST}}$ ϵ_{ST} : ST detection efficacy e Only reconstruct one of the pair Relative higher backgrounds Higher efficiencies $M_{\rm BC} \equiv \sqrt{E_{\rm beam}^2 - p_{\Lambda_c}^2}$ $\Delta E \equiv E_{\Lambda_c} - E_{\rm beam}$ miss

• Double tag (DT): reconstruct both particles in the pair $\mathcal{B}(\Lambda_c \to f) = \frac{N_{\text{DT}}}{N_{\text{ST}} \cdot \epsilon_{\text{DT}}}$ • Clean bkg in DT method • Be able to extract missing particle $\epsilon_{\text{DT}} = \frac{\sum N_{\text{ST}}^i \times \epsilon_{\text{DT}}^i / \epsilon_{\text{ST}}^i}{\sum N_{\text{ST}}^i}$: Averaged DT efficiency e^+ $E_{\text{miss}} = E_{\text{beam}} - \sum_f E_f$ $\vec{p}_{\text{miss}} = \vec{p}_{\Lambda_c} - \sum_f \vec{p}_f$ $M_{\text{miss}}^2 = E_{\text{miss}}^2 - p_{\text{miss}}^2$ $U_{\text{miss}} = E_{\text{miss}} - |\vec{p}_{\text{miss}}|$

 $\overline{\Lambda}_c^-$

₿€SШ

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First observation of $\Lambda_c^+ \rightarrow n\pi^+$

- ₿€SШ
- $\Lambda_c^+ \rightarrow n\pi^+$ is Singly-Cabbibo-Suppressed decay, containing non-factorizable contributions



• Using double tag and extract neutron through missing technique



- First observation, 7.3 σ significance
 - $\mathcal{B}(\Lambda_c^+ \to n\pi^+) = (6.6 \pm 1.2 \pm 0.4) \times 10^{-4}$
- $\mathcal{B}(\Lambda_c^+ \to n\pi^+)/\mathcal{B}(\Lambda_c^+ \to p\pi^0) > 7.2 \text{ at } 90\% \text{ CL}.$
 - $\mathcal{B}(\Lambda_c^+ \to p\pi^0) < 8.0 \times 10^{-5}$ at 90% CL. by Belle [Phys. Rev. D 103, 072004 (2021)]
 - Disagree with most theoretical predictions by phenomenological models
- Agree with previous results:
 - $\mathcal{B}(\Lambda_c^+ \to \Lambda \pi^+) = (1.31 \pm 0.08 \pm 0.05)\%$
 - $\mathcal{B}(\Lambda_c^+ \to \Sigma^0 \pi^+) = (1.22 \pm 0.08 \pm 0.07)\%$

Measurement of $\Lambda_c^+ \to n\pi^+\pi^0$, $n\pi^+\pi^-\pi^+$ and $nK^-\pi^+\pi^+$



• Firstly measured, no previous theoretical calculation







- Absolute BFs results:
 - $\mathcal{B}(\Lambda_c^+ \to n\pi^+\pi^0) = (0.64 \pm 0.09 \pm 0.02)\%$
 - $\mathcal{B}(\Lambda_c^+ \to n\pi^+\pi^-\pi^+) = (0.45 \pm 0.07 \pm 0.03)\%$
 - $\mathcal{B}(\Lambda_c^+ \to nK^-\pi^+\pi^+) = (1.90 \pm 0.08 \pm 0.09)\%$
- Relative BFs results: Previous slide

• $\mathcal{B}(\Lambda_c^+ \to n\pi^+\pi^0)/\mathcal{B}(\Lambda_c^+ \to n\pi^+) = 9.7 \pm 2.2$

- PDG $\mathcal{B}(\Lambda_c^+ \to n\pi^+\pi^-\pi^+)/\mathcal{B}(\Lambda_c^+ \to nK^-\pi^+\pi^+) = 0.24 \pm 0.04$
 - $\mathcal{B}(\Lambda_c^+ \to p\pi^-\pi^+)/\mathcal{B}(\Lambda_c^+ \to n\pi^+\pi^0) = 0.72 \pm 0.13$

Provide significant input for future theoretical calculations of Λ_c^+ decays involving neutron!

Significantly larger than one, reveals possible abundant intermediates in $n\pi^+\pi^0$ final states

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Partial wave analysis of $\Lambda_c^+ \to \Lambda \pi^+ \pi^0$



- First charmed baryon decays PWA at BESIII
- Intermediate processes in $\Lambda_c^+ \to \Lambda \pi^+ \pi^0$ are interesting in theoretical calculations



- Use new-developed TensorFlow based package TF-PWA* to perform the PWA fit
- Use helicity formalism to construct the full decay amplitude
- PWA is able to extract the intermediate processes explicitly

Alignment angles correctly considered

$$\begin{aligned} \mathcal{A}_{\lambda_{\Lambda_{c}^{+}},\lambda_{p}} &= (A_{\lambda_{\Lambda_{c}^{+}},\lambda_{p}}^{\rho} + A_{\lambda_{\Lambda_{c}^{+}},\lambda_{p}}^{NR}) \\ &+ \sum_{\lambda_{p}'} (\sum A_{\lambda_{\Lambda_{c}^{+}},\lambda_{p}'}^{\Sigma^{*+}}) D_{\lambda_{p}',\lambda_{p}}^{1/2}(\alpha_{p},\beta_{p},\gamma_{p}) \\ &+ \sum_{\lambda_{p}'} (\sum A_{\lambda_{\Lambda_{c}^{+}},\lambda_{p}'}^{\Sigma^{*0}}) D_{\lambda_{p}',\lambda_{p}}^{1/2}(\alpha_{p}',\beta_{p}',\gamma_{p}') \end{aligned}$$

Partial wave analysis of $\Lambda_c^+ \to \Lambda \pi^+ \pi^0$



- Use single tag method, extract around 10k signal candidates with purities > 80%
- Fit results on invariant mass spectra:



• Results of BFs and weak decays asymmetry α parameters:

	Theoretical c	This work	PDG	
$10^2 \times \mathcal{B}(\Lambda_c^+ \to \Lambda \rho(770)^+)$	4.81 ± 0.58 [13]	$4.0 \ [14, \ 15]$	4.06 ± 0.52	< 6
$10^3 \times \mathcal{B}(\Lambda_c^+ \to \Sigma(1385)^+ \pi^0)$	2.8 ± 0.4 [16]	$2.2 \pm 0.4 \; [17]$	5.86 ± 0.80	
$10^3 \times \mathcal{B}(\Lambda_c^+ \to \Sigma(1385)^0 \pi^+)$	2.8 ± 0.4 [16]	$2.2 \pm 0.4 \; [17]$	6.47 ± 0.96	
$lpha_{\Lambda ho(770)^+}$	-0.27 ± 0.04 [13]	-0.32 [14, 15]	-0.763 ± 0.066	
$lpha_{\Sigma(1385)^+\pi^0}$	$-0.91\substack{+0.4\\-0.5}$	$^{45}_{10}$ [17]	-0.917 ± 0.083	
$lpha_{\Sigma(1385)^0\pi^+}$	$-0.91\substack{+0.4\\-0.2}$		-0.79 ± 0.11	

 α extracted through results of internal partial wave amplitudes

First measurements

Ref. [13]: Phys. Rev. D 101 (2020) 053002.
Ref. [14,15]: Phys. Rev. D 46 (1992) 1042; Phys. Rev. D 55 (1997) 1697.
Ref. [16]: Eur. Phys. J. C 80 (2020) 1067.
Ref. [17]: Phys. Rev. D 99 (2019) 114022.

BESIII Preliminary; Definition of α see in backup



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Measurement of $\Lambda_c^+ \to \Sigma^0 K^+$ and $\Sigma^+ K_S^0$

- Many theoretical calculations arises on these two SCS decays
- Precise measurement can provide crucial information for test various theoretical models

	$\mathcal{B}(\Lambda_c^+ \to \Sigma^0 K^+)$	$\mathcal{B}(\Lambda_c^+ o \Sigma^+ K_{ m S}^0)$
QCD corrections	2(8)	2(4)
MIT bag model	7.2 ± 1.8	7.2 ± 1.8
Diagrammatic analysis	5.5 ± 1.6	9.6 ± 2.4
$SU(3)_F$ flavor symmetry	5.4 ± 0.7	5.4 ± 0.7
IRA method	5.0 ± 0.6	1.0 ± 0.4
PDG 2020	5.2 ± 0.8	/

Relative BFs extracted through simultaneous fits: ullet



Submitted to PRD, pre-print as arXiv.2207.10906

Improved

Measurement of $\Lambda_c^+ \to \Sigma^+ K^+ K^-$, $\Sigma^+ K^+ \pi^-$ and $\Sigma^+ K^+ \pi^- \pi^0$

- Several $\Lambda_c^+ \to \Sigma^+ h^+ h^-(\pi^0)$ modes also measured
- $\Sigma^+ K^+ K^-$ final states also containing $\Sigma^+ \phi$ contributions
- Relative BFs extracted, $\Sigma^+\pi^+\pi^-$ as reference



Results	Relative Branching Fraction to $\Lambda_c^+ \rightarrow \Sigma^+ \pi^+ \pi^-$ (×10 ⁻²)		Branching Fraction (%)			
Decay mode	RBF (Belle)	RBF (This work)	This work	PDG	Theoretical Prediction	
$\Lambda_c^+ \to \Sigma^+ K^+ K^-$	$7.6\pm0.7\pm0.9$	$7.33 \pm 0.80 \pm 0.31$	$0.330 \pm 0.036 \pm 0.014 \pm 0.018$	0.35 ± 0.04	0.25 ± 0.03	
$\Lambda_c^+ \to \Sigma^+ K^+ \pi^-$	$4.7\pm1.1\pm0.8$	$4.51 \pm 0.52 \pm 0.23$	$0.202 \pm 0.023 \pm 0.011 \pm 0.011$	0.21 ± 0.06	0.25 ± 0.03	
$\Lambda_c^+\to \Sigma^+ K^+ \pi^- \pi^0$	-	< 2.4	< 0.11	-	- M	ore precise
$\Lambda_c^+\to \Sigma^+\phi$	$8.5 \pm 1.2 \pm 1.2$	9.2 ± 1.8 ± 0.6	$0.414 \pm 0.080 \pm 0.027 \pm 0.023$	0.39 ± 0.06	- Fi	rst measurement
$\Lambda_c^+ \to \Sigma^+ K^+ K^- (\text{non} - \phi)$	-	$4.38 \pm 0.79 \pm 0.16$	$0.197 \pm 0.036 \pm 0.007 \pm 0.011$	-	- Co	onsistent

BESIII Preliminary

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2D fit $M_{\rm BC}$ vs. M_{KK}



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Measurement of $\Lambda_c^+ \rightarrow p K^- e^+ \nu_e$



- First study of Λ_c^+ decaying into semi-leptonically into inclusive pK^- system
- Double tag method and missing technique



- Fit to U_{miss}
 - $\mathcal{B}(\Lambda_c^+ \to pK^-e^+\nu_e) = (0.82 \pm 0.15 \pm 0.06) \times 10^{-3}$
 - First observation with 8.2 σ significance
 - Semi-leptonic decays not saturated by $\Lambda l^+ \nu_l!$

	$ \mathcal{B}(\Lambda_c^+ \to \Lambda(1520)e^+\nu_e) \times 10^{-3}]$
Constituent quark model	1.01
Nonrelativistic quark model	0.60
Lattice QCD	$0.512 \pm 0.082 \pm 0.008$
Measurement	$1.36 \pm 0.56 \pm 0.14$

- Also study with M_{pK} spectrum
- 2D fit to M_{pK} vs. U_{miss}
 - Evidence with 3.3 σ for $\Lambda(1520)e^+\nu_e$
 - $\mathcal{B}(\Lambda_c^+ \to \Lambda(1520)e^+\nu_e) = (1.36 \pm 0.56 \pm 0.14) \times 10^{-3}$

Invaluable in extending the understanding of semi-leptonic decays of Λ_c^+ !

Submitted to PRL, pre-print as arXiv.2207.11483

Measurement of $\Lambda_c^+ \to \Lambda e^+ \nu_e$

- ₿€SШ
- $\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e$ measured more precisely at BESIII with larger data samples than 2015
- Double tag method and missing technique
- Most precise measurement
 - $\hat{\mathcal{B}}(\Lambda_c^+ \to \Lambda e^+ \nu_e) = (3.56 \pm 0.11 \pm 0.07)\%$
- Comparison with theoretical models:

Disfavor at C.L. more than 95%

	$\mathcal{B}(\Lambda_c^+ \to \Lambda e^+ \nu_e) \ [\%]$
Constituent quark model (HONR)	4.25
Light-front approach	1.63
Covariant quark model	2.78
Relativistic quark model	3.25
Non-relativistic quark model	3.84
Light-cone sum rule	3.0 ± 0.3
Lattice QCD	3.80 ± 0.22
SU(3)	3.6 ± 0.4
Light-front constituent quark model	3.36 ± 0.87
MIT bag model	3.48
Light-front quark model	4.04 ± 0.75
This work	$3.56 \pm 0.11 \pm 0.07$

- Form factors $\Lambda_c^+ \to \Lambda$ firstly measured
- Provide first direct comparisons to LQCD
- Provide important inputs in understanding the Λ_c^+ semi-leptonic decays



Submitted to PRL, pre-print as arXiv.2207.14149

Search for $\Lambda_c^+ \to \Lambda \pi^+ \pi^- e^+ \nu_e$ and $p K_S^0 \pi^- e^+ \nu_e$



- Also search for other non- $\Lambda l^+ \nu_l$ semi-leptonic modes
- Double tag method and missing technique
- $\Lambda \pi^+ \pi^-$ and $p K_S^0 \pi^-$ are able to tag Λ^*

Phys. Rev. D 95, 053005 (2017).
 Phys. Rev. C 72, 035201 (2005).
 Phys. Rev. D 93, 014021 (2016).
 Phys. Rev. D 104, 013005 (2021).
 Phys. Rev. D 105, L051505 (2022).

	Constituent quark model					
State J ^P	Hussain and Roberts ^[1]	Pervin, Roberts and Capstick ^[2]	Chiral unitary approach ^[3]	Light-font quark model ^[4]	Lattice QCD ^[5]	
$\Lambda(1405) \frac{1}{2}$	2.4×10^{-3}	3.8×10 ⁻²	$(2-5) \times 10^{-5}$	$(3.1 \pm 0.8) \times 10^{-3}$		
$\Lambda(1520) \frac{3}{2}$	5.94×10^{-4}	1.00×10^{-3}			$(5.12 \pm 0.82) \times 10^{-4}$	
$\Lambda(1600) \frac{1}{2}^{+}$	1.26×10^{-4}	4.00×10^{-4}		$(7 \pm 2) \times 10^{-5}$		
$\Lambda(1890) \frac{3}{2}^{+}$	3.16×10^{-6}					
$\Lambda(1820) \frac{5}{2}^{+}$	1.32×10^{-6}					

• Upper limit @ 90% C.L. extracted for the first time through likelihood scan:



BESIII Preliminary

Summary



- Using data samples of $\Lambda_c^+ \overline{\Lambda}_c^-$ pair production with 4.5 fb⁻¹ integrated luminosity collected by BESIII, many significant measurements performed
- Λ_c^+ decays involve neutron $n\pi^+(\pi^0)$ and $nh^+h^-h^+$ firstly measured
- First PWA of $\Lambda_c^+ \to \Lambda \pi^+ \pi^0$ at BESIII performed
- SCS decays $\Lambda_c^+ \to \Sigma^0 K^+$ with improved measurement, and $\Lambda_c^+ \to \Sigma^+ K_S^0$ firstly measured
- $\Sigma^+ h^+ h^-(\pi^0)$ decays measured, consistent with PDG results
- Significant achievement on semi-leptonic decays
 - $\Lambda_c^+ \to p K^- e^+ \nu_e$ firstly observed with 8.2 σ significance
 - $\Lambda_c^+ \to \Lambda e^+ \nu_e$ with improved measurement, first direct comparison of $\frac{d\Gamma}{da^2}$
 - $\Lambda_c^+ \to \Lambda \pi^+ \pi^- e^+ \nu_e$ and $p K_S^0 \pi^- e^+ \nu_e$ upper limit firstly obtained
- BESIII has made pretty good progresses on the exploration of the charmed baryon Λ⁺_c decays! More results will be released in the future!

Thanks for listening!

Backup

Measurement of $\Lambda_c^+ \to n\pi^+\pi^0$, $n\pi^+\pi^-\pi^+$ and $nK^-\pi^+\pi^+$



• Firstly measured, no previous theoretical calculation



- Can also give results for $\Lambda_c^+ \hookrightarrow \Lambda \pi^+ \pi^0$ and $\Sigma^0 \pi^+ \pi^0$: Consist with PDG results:
 - $\mathcal{B}(\Lambda_c^+ \to \Lambda \pi^+ \pi^0) = (6.52 \pm 0.43_{\text{stat.}})\% \iff (7.1 \pm 0.4)\%$
 - $\mathcal{B}(\Lambda_c^+ \to \Sigma_c^0 \pi^+ \pi^0) = (3.77 \pm 0.47_{\text{stat.}})\% \longleftrightarrow (2.5 \pm 0.4)\%$

Partial wave analysis of $\Lambda_c^+ \to \Lambda \pi^+ \pi^0$ -- Definition of α

- Decay asymmetry parameters is defined under angular distribution
- Only considering angular distribution, LS-coupling formula reads:

$$H_{\lambda_1,\lambda_2} = \sum_{ls} g_{ls} \sqrt{\frac{2l+1}{2J_0+1}} \langle ls, 0 \ \delta | J_0, \delta \rangle \langle J_1 J_2, \lambda_1 \ -\lambda_2 | s, \delta \rangle$$

• For the decay $\Lambda_c^+ \to \Lambda \rho(770)^+$, the differential width formula reads:

$$rac{\mathrm{d}\Gamma}{\mathrm{d}\cos\Theta_{\Lambda}} \propto 1 + lpha_{\Lambda
ho(770)^+} \cdot lpha_{\Lambda} \cdot \cos\Theta_{\Lambda}$$

• Decay asymmetry parameter defined as:

 Θ_{Λ} is helicity angle of $\Lambda \rightarrow p\pi^{-}$ in this decay chain

$$\begin{split} \alpha_{\Lambda\rho(770)^{+}} &= \frac{|H^{\rho}_{\frac{1}{2},1}|^{2} - |H^{\rho}_{-\frac{1}{2},-1}|^{2} + |H^{\rho}_{\frac{1}{2},0}|^{2} - |H^{\rho}_{-\frac{1}{2},0}|^{2}}{|H^{\rho}_{\frac{1}{2},1}|^{2} + |H^{\rho}_{-\frac{1}{2},-1}|^{2} + |H^{\rho}_{\frac{1}{2},0}|^{2} + |H^{\rho}_{-\frac{1}{2},0}|^{2}} \\ &= \frac{\sqrt{\frac{1}{9}} \cdot 2 \cdot \Re\left(g^{\rho}_{0,\frac{1}{2}} \cdot \bar{g}^{\rho}_{1,\frac{1}{2}} - g^{\rho}_{1,\frac{3}{2}} \cdot \bar{g}^{\rho}_{2,\frac{3}{2}}\right) - \sqrt{\frac{8}{9}} \cdot 2 \cdot \Re\left(g^{\rho}_{0,\frac{1}{2}} \cdot \bar{g}^{\rho}_{1,\frac{3}{2}} + g^{\rho}_{1,\frac{1}{2}} \cdot \bar{g}^{\rho}_{2,\frac{3}{2}}\right)}{|g^{\rho}_{0,\frac{1}{2}}|^{2} + |g^{\rho}_{1,\frac{1}{2}}|^{2} + |g^{\rho}_{1,\frac{3}{2}}|^{2} + |g^{\rho}_{2,\frac{3}{2}}|^{2}} \end{split}$$

Partial wave analysis of $\Lambda_c^+ \to \Lambda \pi^+ \pi^0$ -- Definition of α

For the decay $\Lambda_c^+ \rightarrow \Sigma(1385)\pi$, the differential width formula reads: ٠

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\cos\theta_{ee}\mathrm{d}\cos\theta_{\Lambda_c^+}\mathrm{d}\cos\theta_{\Sigma^*}\mathrm{d}\phi_{\Lambda_c^+}^{ee}} \propto (7+9\cos(2\theta_{\Sigma^*})) \cdot \left(1+\alpha_0\cos^2(\theta_{ee})+\alpha_{\Sigma^*\pi}\sqrt{1-\alpha_0^2}\sin\Delta_0\cos\theta_{ee}\sin\theta_{ee}\sin\theta_{\Lambda_c^+}\sin\phi_{\Lambda_c^+}^{ee}\right)$$

Decay asymmetry parameter defined as:

$$\alpha_{\Sigma(1385)\pi} = \frac{|H_{0,\frac{1}{2}}^{\Sigma(1385)}|^{2} - |H_{0,-\frac{1}{2}}^{\Sigma(1385)}|^{2}}{|H_{0,\frac{1}{2}}^{\Sigma(1385)}|^{2} + |H_{0,-\frac{1}{2}}^{\Sigma(1385)}|^{2}} = \frac{2\Re\left(g_{1,\frac{3}{2}}^{\Sigma(1385)} \cdot \bar{g}_{2,\frac{3}{2}}^{\Sigma(1385)}\right)}{|g_{1,\frac{3}{2}}^{\Sigma(1385)}|^{2} + |g_{2,\frac{3}{2}}^{\Sigma(1385)}|^{2}}$$

$$\prod_{\substack{e^{+} \\ e^{+} \\ CM \text{ frame}}} \int_{\Sigma^{*}} \int_{\Sigma^{*}} \int_{\Sigma^{*}} \int_{\Sigma^{*}} \int_{\Lambda^{+}_{c} \text{ rest frame}} \int_{\Lambda^{+}_{c} \text{ rest frame}} \int_{\Lambda^{+}_{c} \text{ rest frame}} \int_{\Lambda^{+}_{c} \text{ rest frame}} \int_{\Sigma^{*} \text{ rest frame}} \int_{$$

1