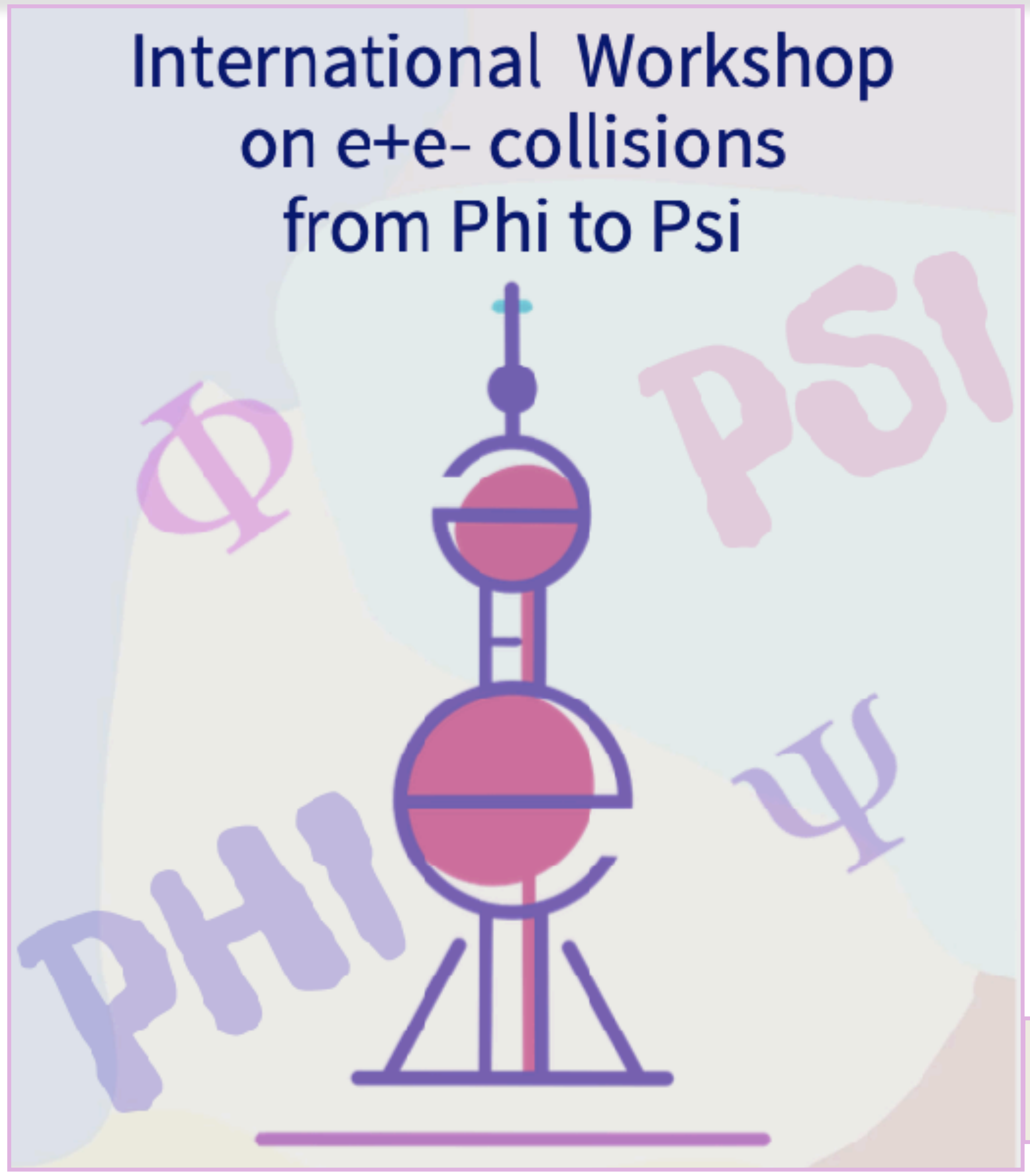
Triangle Singularities in the Production of $X(3872)$ and $T_{cc}^{+}(3875)$



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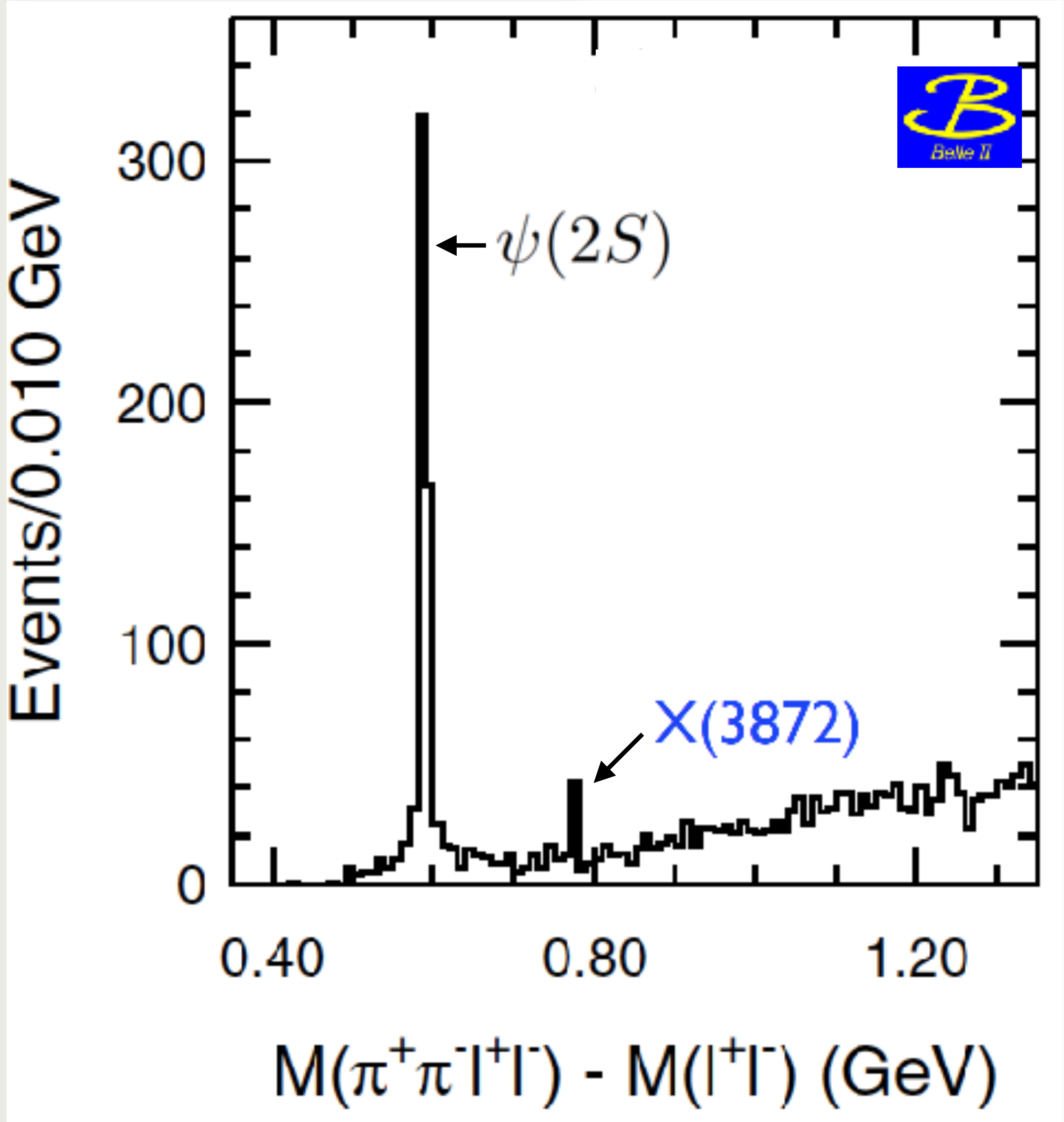
Outline

- Brief review of $X(3872)$ and $T_{cc}^{+}(3875)$
- Charm-meson triangle singularity
- Production of $X(3872)$
  - $X+\pi$ from B meson decay [PRD100, 074028(2019)]
  - $X+\pi$ in prompt production at hadron colliders [PRD100, 094006(2019)]
  - $X+\gamma$ in e^+e^- annihilation [PRD100, 031501(2019), PRD101, 014021(2020), PRD101, 096020(2020)]
- Production of $T_{cc}^{+}(3875)+\pi$ at hadron colliders [arXiv: 2202.03900]
- Summary

Brief review of $X(3872)$ [$\equiv \chi_{c1}(3872)$]

- discovery at e^+e^- collider [Belle (2003)]:

$$B^+ \to K^+ + X \qquad X \to J/\psi\pi^+\pi^-$$



- confirmation at $p\bar{p}$ collider [CDF (2003)]:

$$p\bar{p} \to X + \text{anything}$$

- quantum numbers [LHCb (2013)]:

$J^{PC} = 1^{++}$

- mass [LHCb (2020)]:

$E_X = M_X - (M_{D^{*0}} + M_{D^0}) = (-0.07 \pm 0.12)$ MeV
$|E_X| < 0.22$ MeV at 90% CL

- first measurement of width (Breit-Wigner)

[LHCb (2020) average]:

$\Gamma_X = (1.19 \pm 0.19)$ MeV

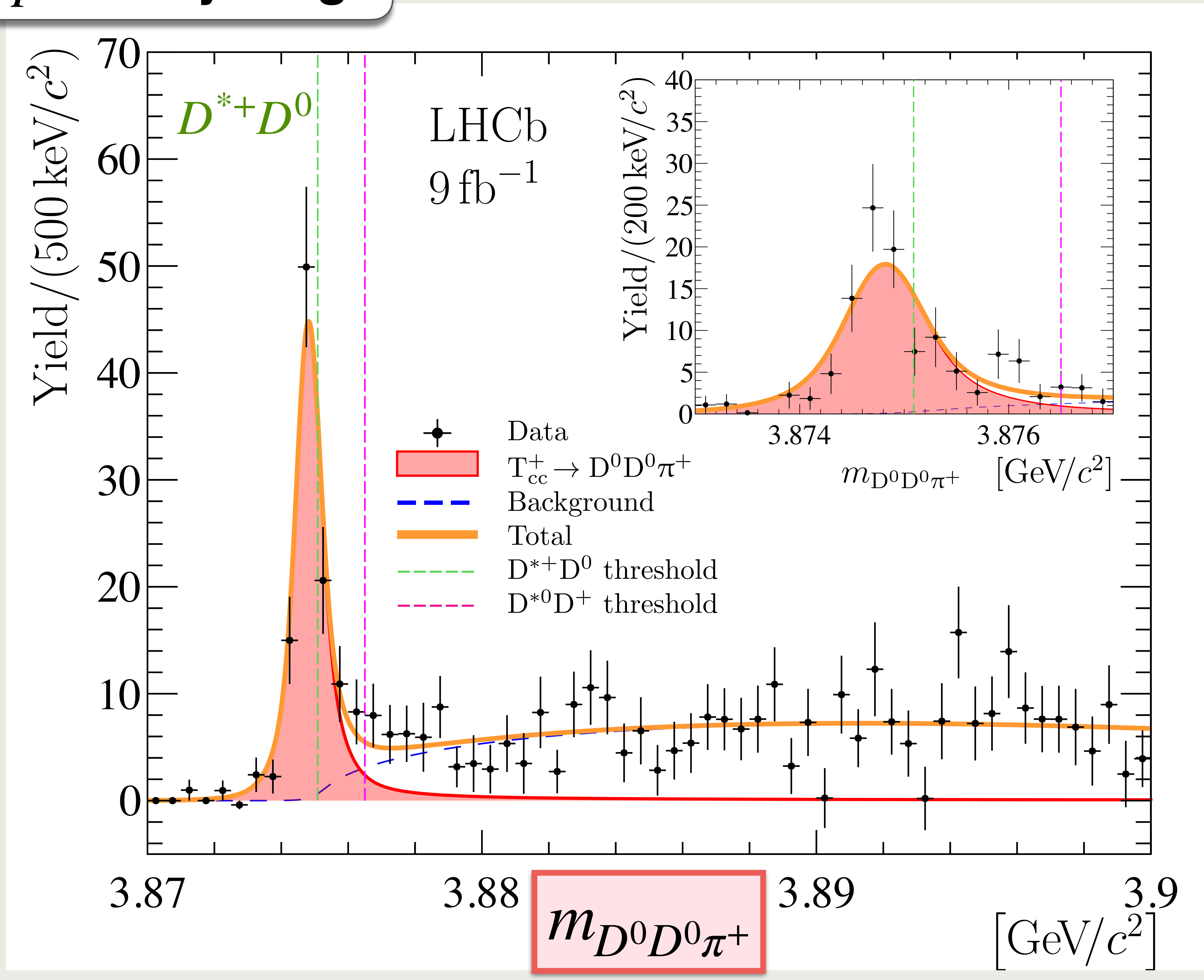
- 7 observed decay modes: $J/\psi\,\pi^+\pi^-$, $J/\psi\pi^+\pi^-\pi^0$, $J/\psi\,\gamma$, $\psi(2S)\gamma$, $D^0\bar{D}^0\pi^0$, $D^0\bar{D}^0\gamma$, $\chi_{c1}\pi^0$

numerous studies in the literature
[more than 2000 citations for the Belle's discovery paper]

Brief review of $T_{cc}^{+}(3875)$

discovery by LHCb [arXiv:2109.01038, arXiv:2109.01056]

$pp \to$ **anything**



many theoretical studies in the literature
[more than 100 citations for the LHCb papers]

- **quark contents:** $cc\bar{u}\bar{d}$
- **quantum numbers:**

  I (J^P)= 0 (1$^+$)
- **mass (Breit-Wigner):**

  $\varepsilon_T = M_T - (M_{D^{*+}} + M_{D^0}) = (-273 \pm 63)$ keV
- **mass (pole energy)** [$D^{*+}D^0$ threshold effect]**:**

  $\varepsilon_T = M_T - (M_{D^{*+}} + M_{D^0}) = (-360 \pm 40)$ keV
- **width:**

  $\Gamma_{BW} = 410$ keV, $\Gamma_{pole} = 48$ keV
- **decay modes:** $D^+D^0\pi^0$, $D^0D^0\pi^+$, $D^+D^0\gamma$

See "*An updated review of the new hadron states*":
Chen *at al.*, arXiv: 2204.02649

Brief review of $X(3872)$ and $T_{cc}^{+}(3875)$

What is the X(3872)?

$J^{PC} = 1^{++}$ → S-wave coupling to $D^{*0}\bar{D}^0 / \bar{D}^{*0}D^0$

$|E_X| < 0.22$ MeV → resonant coupling

S-wave loosely bound charm-meson molecule!!

$$X = \frac{1}{\sqrt{2}}\left(D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}\right)$$

What is the $T_{cc}^{+}(3875)$?

$J^{P} = 1^{+}$ → S-wave coupling to $D^{*+}D^0$

$|\varepsilon_T| = 0.36 \pm 0.04$ keV → resonant coupling

S-wave loosely bound charm-meson molecule!!

$$T_{cc}^{+} = D^{*+}D^0$$

dominant component

Brief review of $X(3872)$ and $T_{cc}^{+}(3875)$

What is the X(3872)?

$J^{PC} = 1^{++}$ → S-wave coupling to $D^{*0}\bar{D}^0 / \bar{D}^{*0}D^0$

$|E_X| < 0.22$ MeV → resonant coupling

} **S-wave loosely bound charm-meson molecule!!**

$$X = \frac{1}{\sqrt{2}}\left(D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}\right)$$

other components of wave functions have small probabilities at short distances:

- ✦ P wave charmonium $\chi_{c1}(2P)$?
- ✦ charged charm mesons $D^{*+}D^- + D^+D^{*-}$?
- ✦ compact tetraquark $[cq][\bar{c}\bar{q}]$?

What is the $T_{cc}^{+}(3875)$?

$J^P = 1^+$ → S-wave coupling to $D^{*+}D^0$

$|\varepsilon_T| = 0.36 \pm 0.04$ keV → resonant coupling

} **S-wave loosely bound charm-meson molecule!!**

$$T_{cc}^{+} = D^{*+}D^0$$

dominant component

at short distances

- $D^{*0}D^+$ component with small probability, isospin = 0 → $\left(D^{*+}D^0 - D^{*0}D^+\right)/\sqrt{2}$
- other possible components of wave functions: compact tetraquark $cc\bar{q}\bar{q}$, $\bar{q}\bar{q}$ bound to heavy diquark (cc)

Brief review of $X(3872)$ and $T_{cc}^{+}(3875)$

Universal properties determined by the binding energy

- large scattering length: a = $1/\sqrt{2\mu\,|E_X|}$, a >> range
- large mean separation: $\langle r \rangle$ = a/2, |Ex| < 0.22 MeV implies $\langle r \rangle$ > 5 fm
- scattering amplitude at $E \ll 1/(2\mu\,\mathrm{range}^2)$: $f(E) = 1/\left(-1/a + i\sqrt{2\mu E}\right)$
- universal wave function: $\psi(k) = \sqrt{8\pi\gamma}/(k^2+\gamma^2)$ ➔ $\psi(r=0) = \int d^3k/(2\pi)^3 \psi(k) = +\infty$

model wave function (more physical qualitative behavior at large k):

$$\psi^{(\Lambda)}(k) = \frac{\sqrt{8\pi(\Lambda+\gamma)\Lambda\gamma}}{\Lambda-\gamma}\left(\frac{1}{k^2+\gamma^2} - \frac{1}{k^2+\Lambda^2}\right)$$

XEFT effective field theory for charm mesons and pions

Fleming, Kusunoki, Mehen & van Kolck [PRD 76, 034006(2007)]

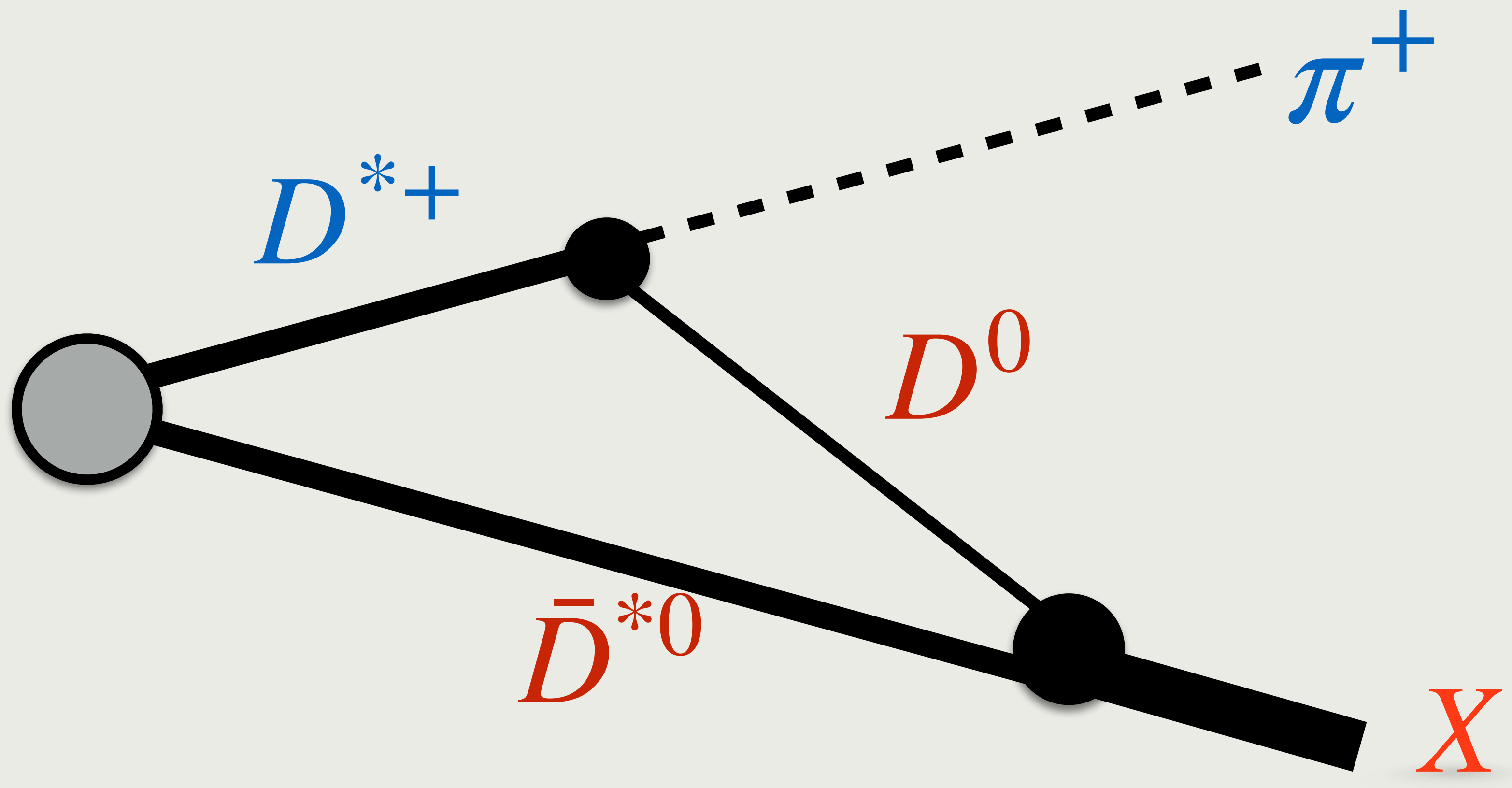
Galilean-invariant XEFT

Braaten [PRD 91, 114007(2015)]

Braaten, He & Jiang [PRD 103, 036014(2021)]

Charm-meson triangle singularity

review on TS: Guo, Liu, Sakai [Prog. Part. Nucl. Phys. 112, 103757 (2020)]



△ **triangle singularity:**
three charm mesons can be on shell simultaneously

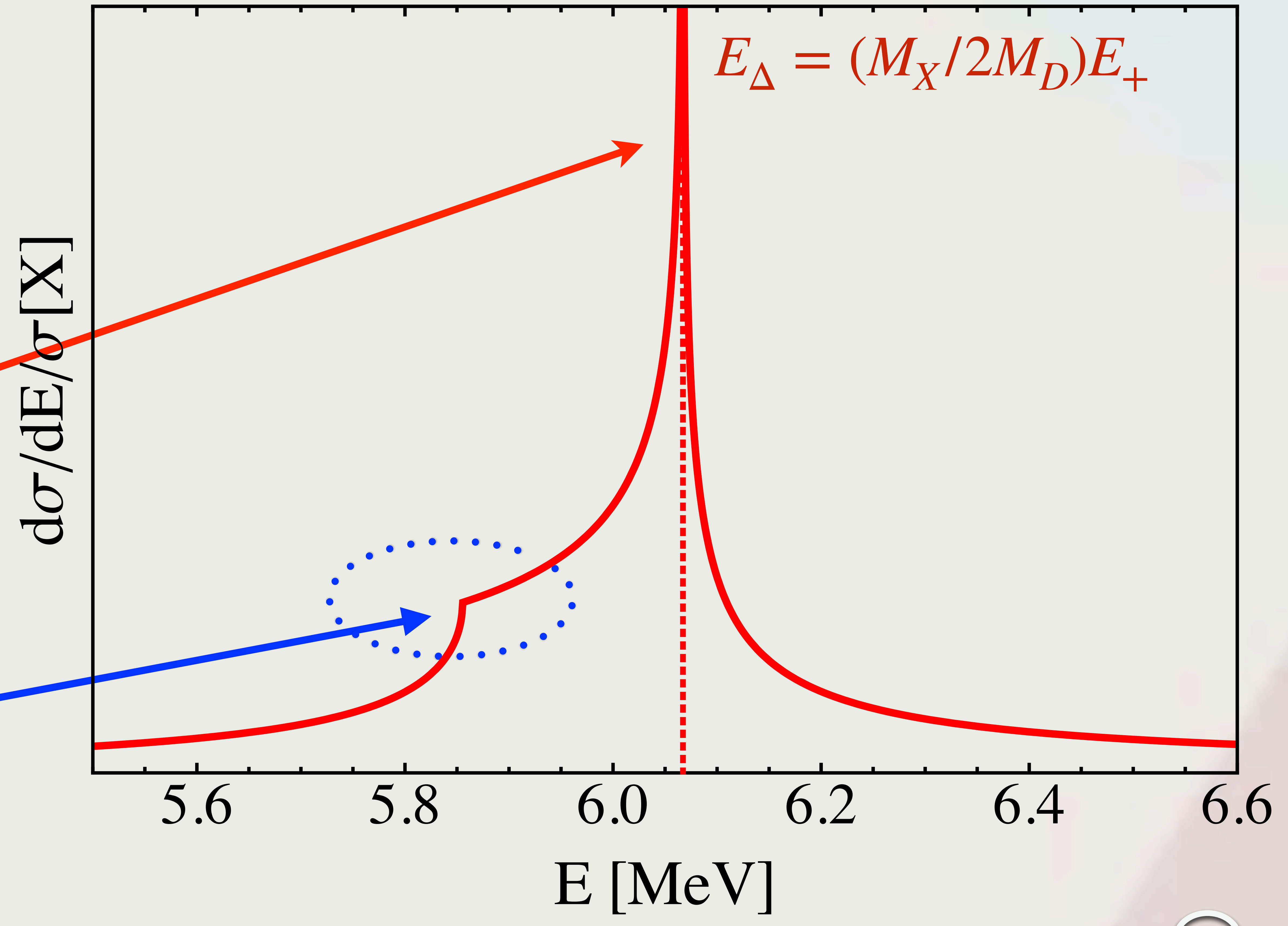
$\log^2(E - E_\Delta)$ **divergence in reaction rate at** E_Δ **determined by masses**

√ **square-root branch point at** $E = E_+$

cusp at $E = E_+$

$E_+ = M_{D^{*+}} - M_{D^0} - m_{\pi^0} = 5.9$ MeV

for $\varepsilon_X = 0,\ \Gamma_{D^*} = 0$



Charm-meson triangle singularity

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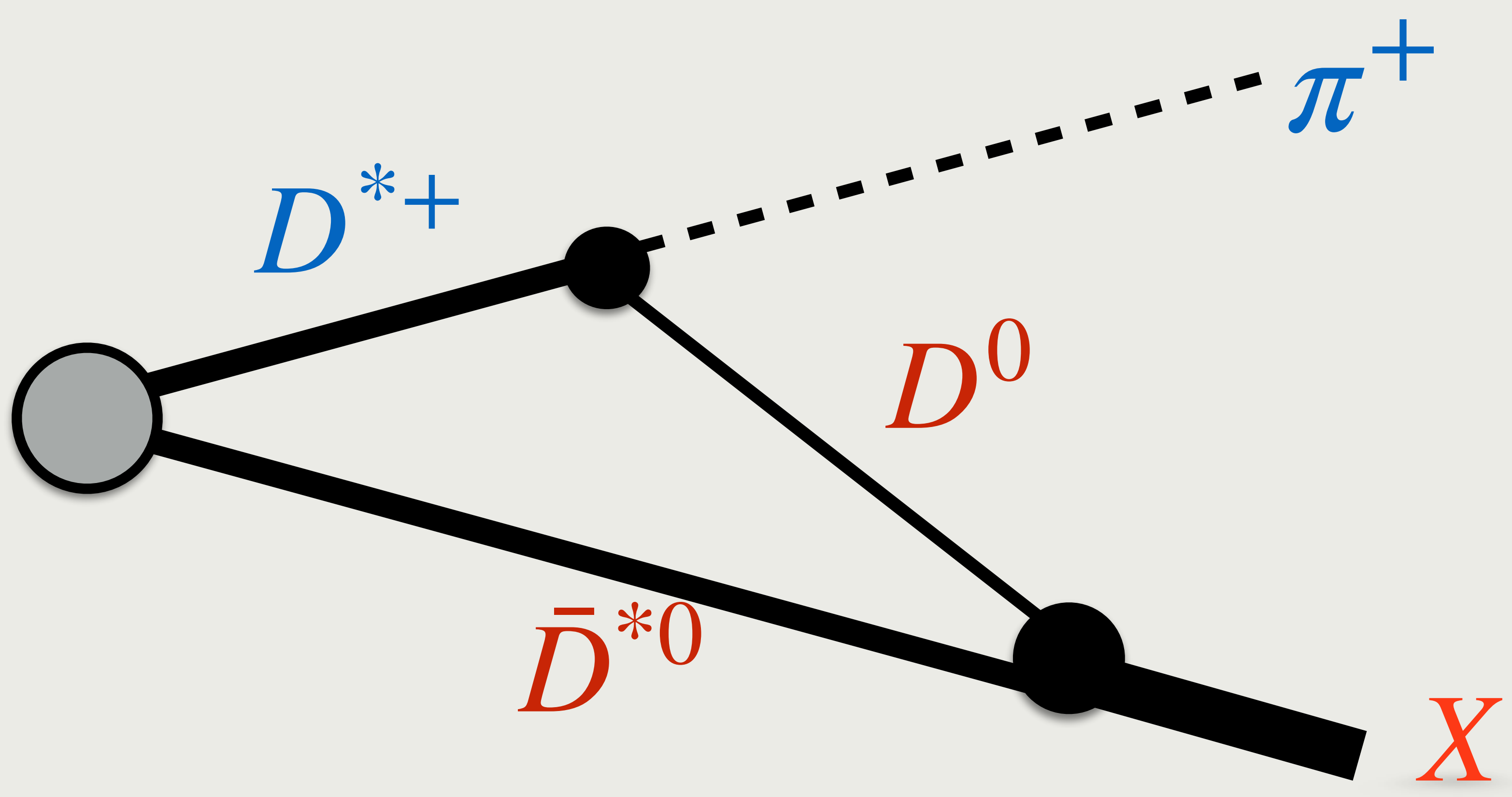


△ triangle singularity:

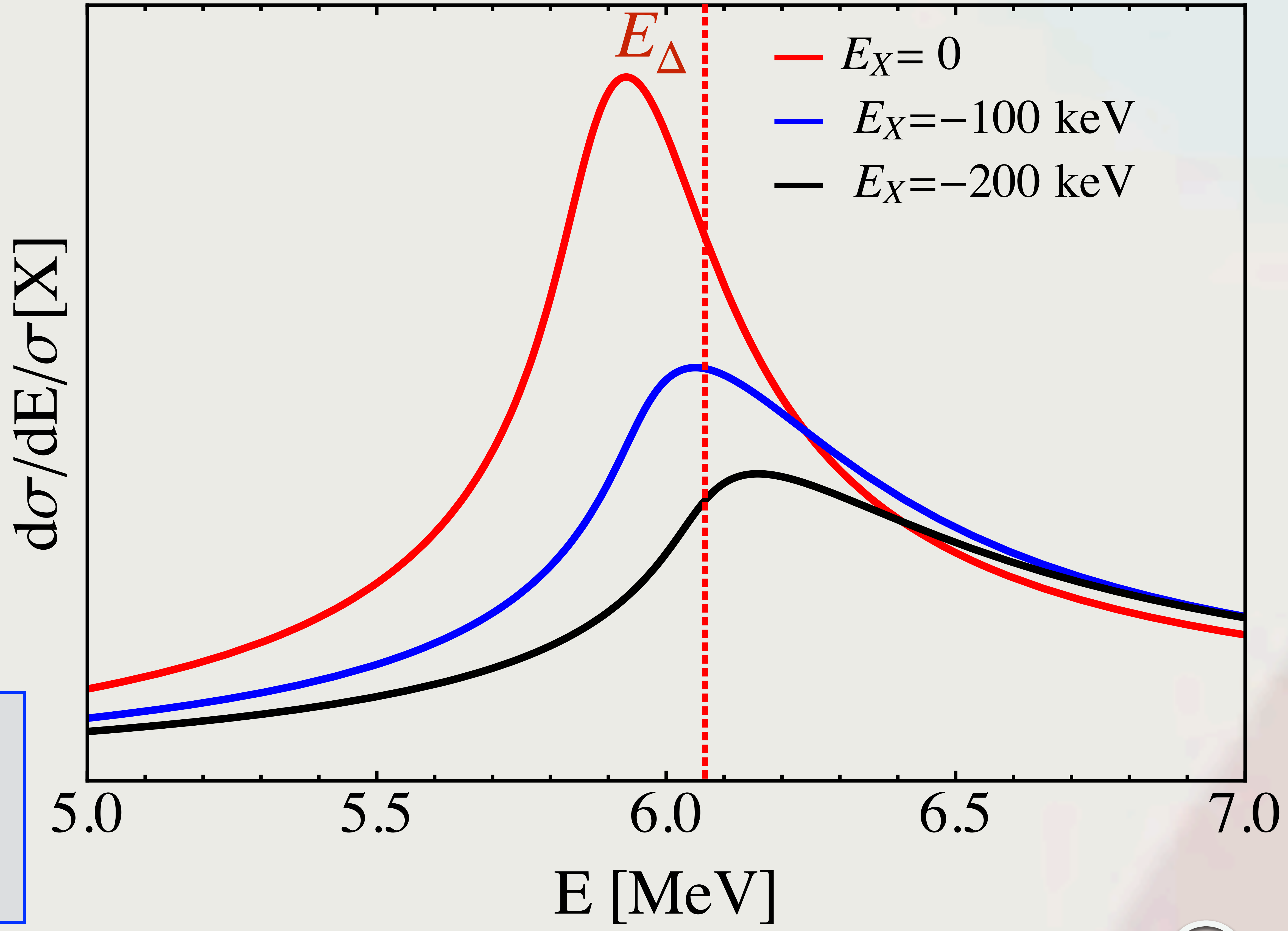
$\log^2(E - E'_\Delta)$ divergence in dσ/dE at complex E'_Δ

⇒ narrow peak in reaction rate

√ square-root branch point at $E = E'_+$ (complex)

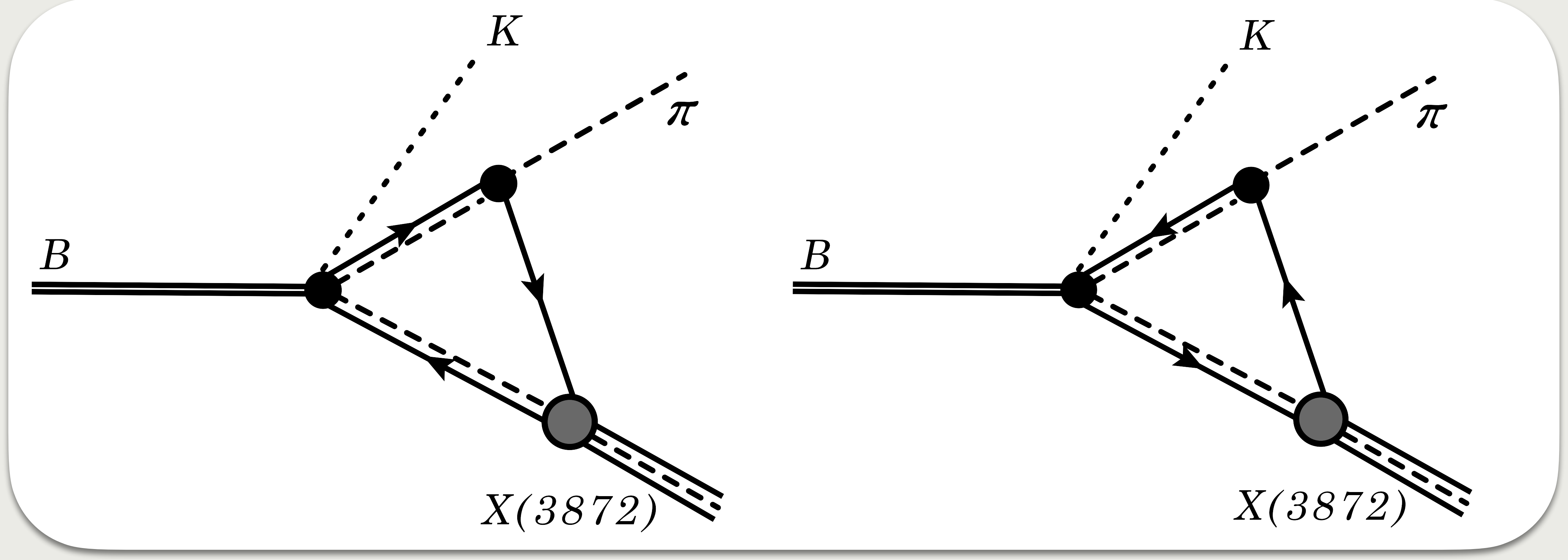
The shape of dσ/dE near the peak is determined by the interplay between the logarithmic singularity and the square-root singularity in the triangle amplitude.

for $\varepsilon_X < 0,\ \Gamma_{D^*} > 0$



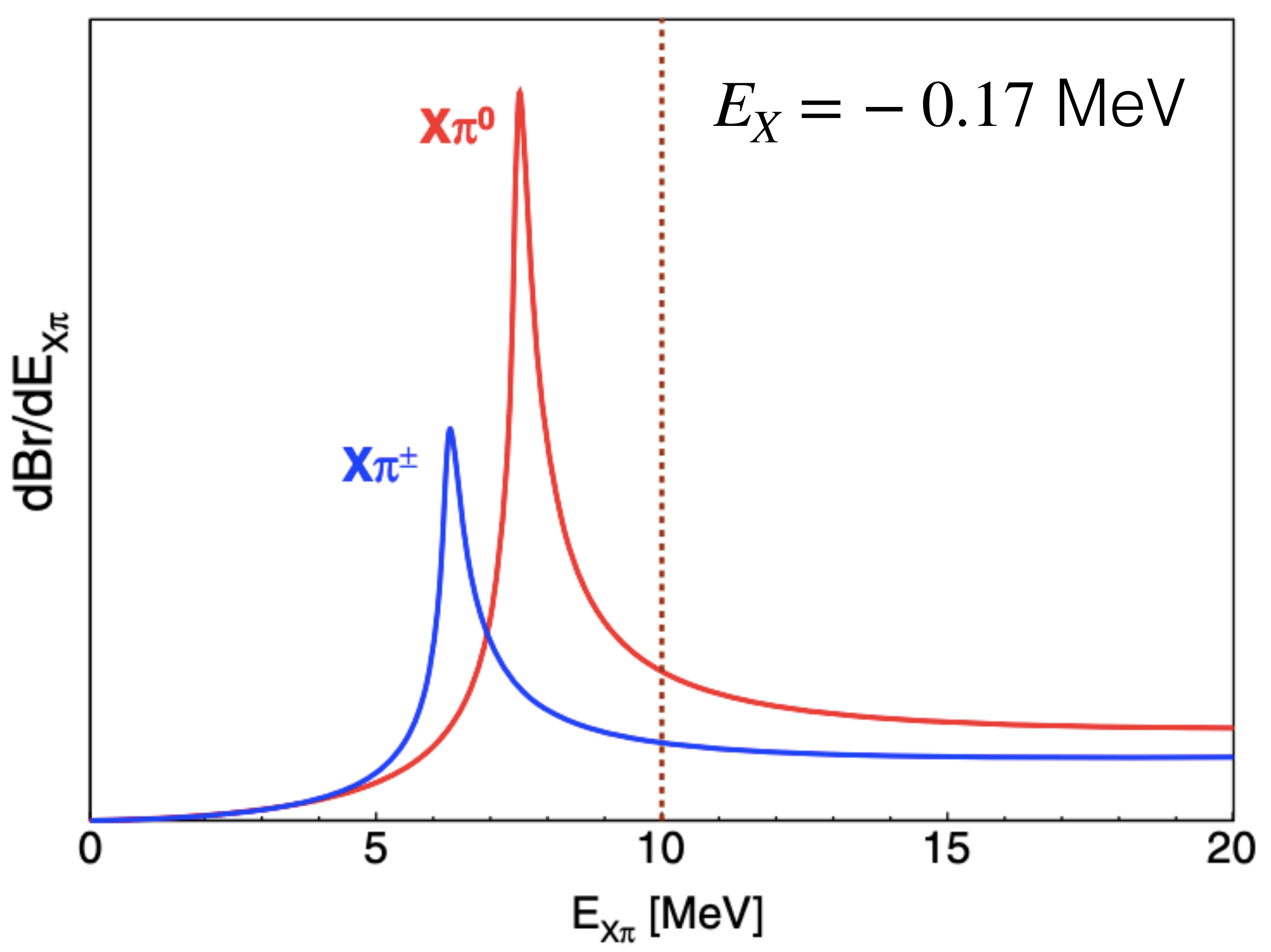
Production of $X(3872) + \pi$ from B meson decay

B $\to$ K $D^*\bar{D}^*$ $\to$ K $X\pi$ decay of B meson into K+$D^*\bar{D}^*$, rescattering of virtual $D^*\bar{D}^*$ into $X\pi$



BHI (2019): narrow **peaks** in dBr[B→K Xπ]

- $X\pi^{\pm}$: near 6.1 MeV above $X\pi^{+}$ threshold with width about 1 MeV
- $X\pi^{0}$: near 7.3 MeV above $X\pi^{0}$ threshold with width about 1 MeV



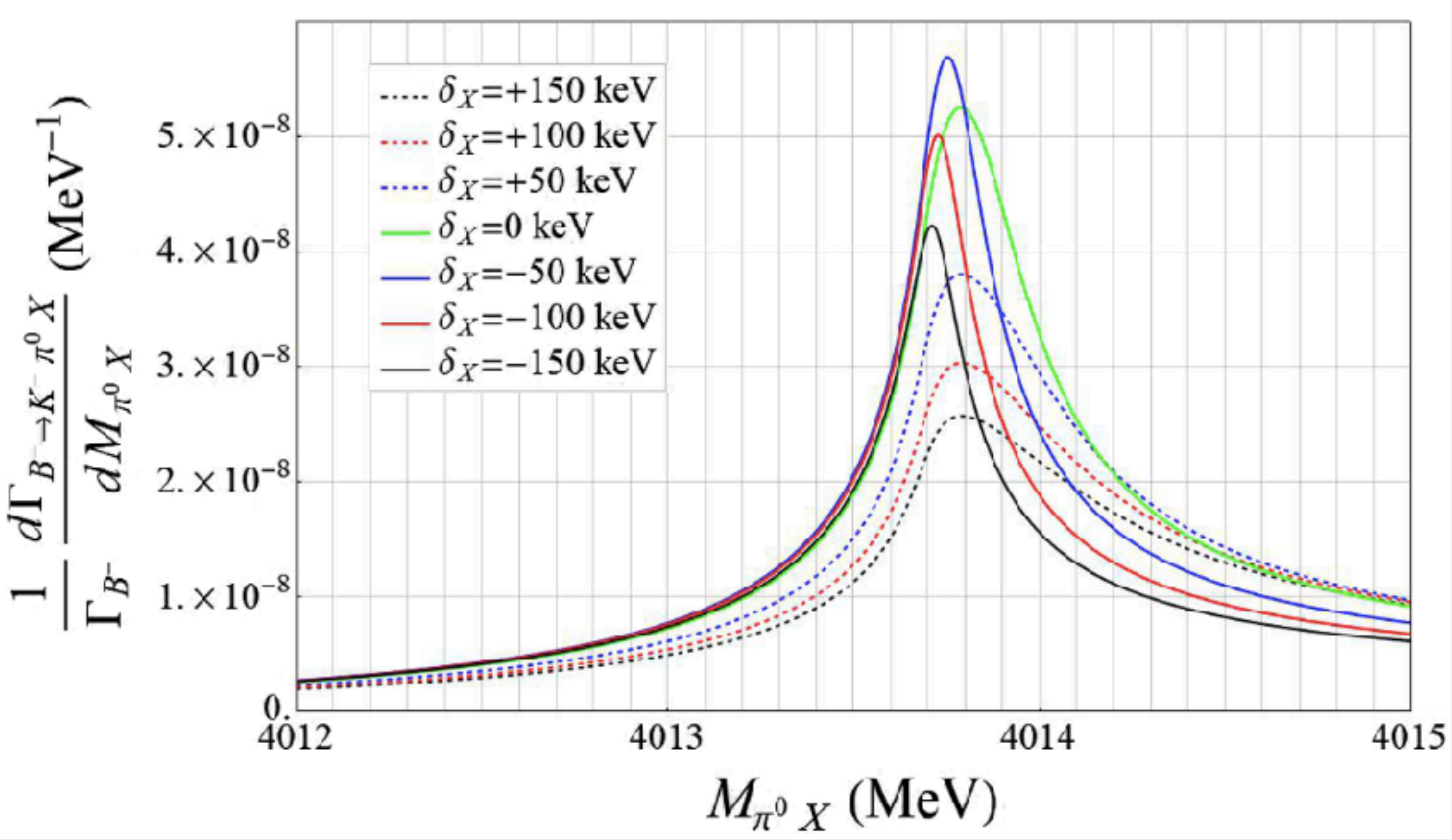
TS could contribute an observable fraction to the decay of $B^0 \to K^+ X \pi^-$ $\sim 6 \times 10^{-7}$

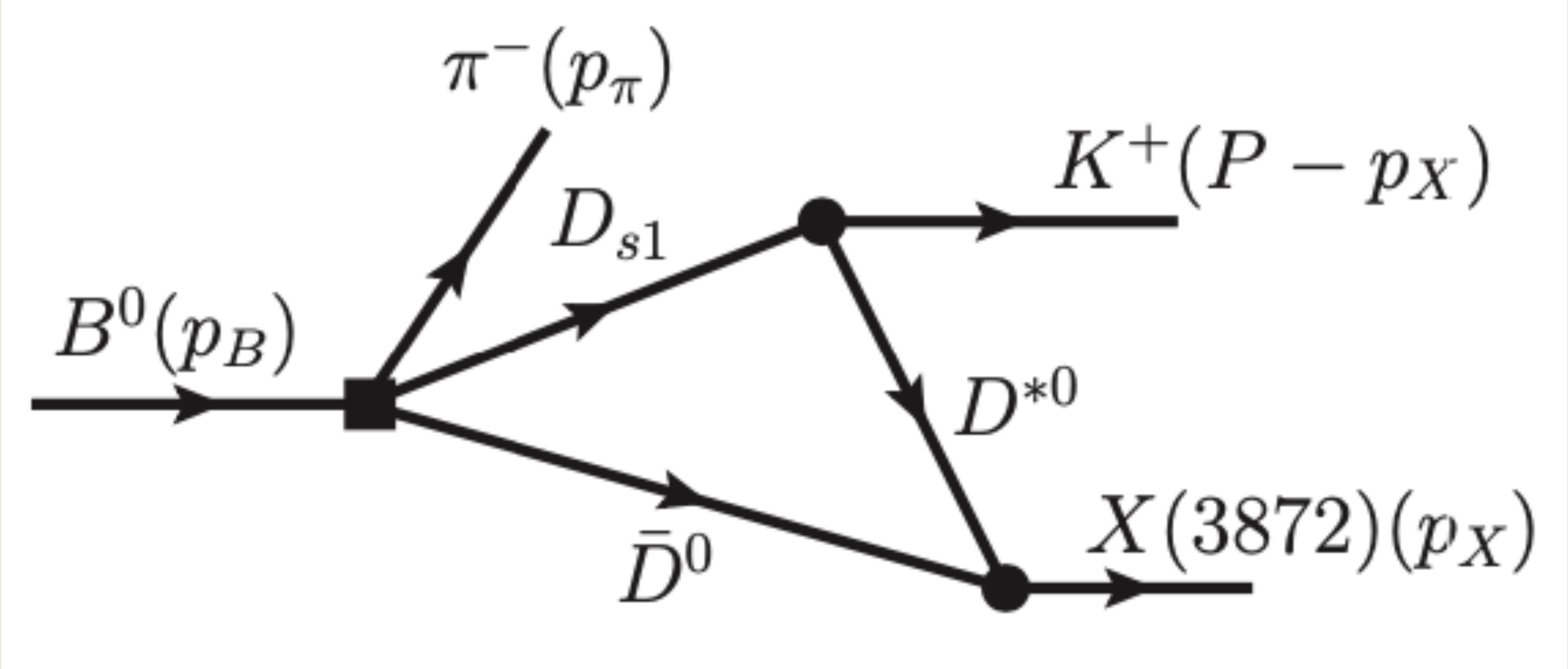
Production of $X(3872)+\pi$ from B meson decay

- **Sakai, Oset & Guo** [PRD 101, 054030(2020)]
  $B^- \to K^- D^{*0}\bar{D}^{*0} \to K^- X\pi^0$
  Ex (= -δx) may be extracted from the asymmetry of the Xπ line shape

- **Nakamura** [PRD 102, 074004(2020)]
  $B^0 \to K^+ D^{*0}D^{*-} \to K^+ (J/\psi\ \rho)\pi^-$
  TS could produce narrow peak in J/ψρ invariant mass near 3872 MeV even without X resonance

- **Yan, Ge & Liu** [arXiv:2208.03943]
  $B^0 \to \pi^- \bar{D}^0 D_{s1}(2536) \to \pi^- K^+ X$
  Lineshape from TS is sensitive to X mass if its width is relatively small (< 1 MeV)





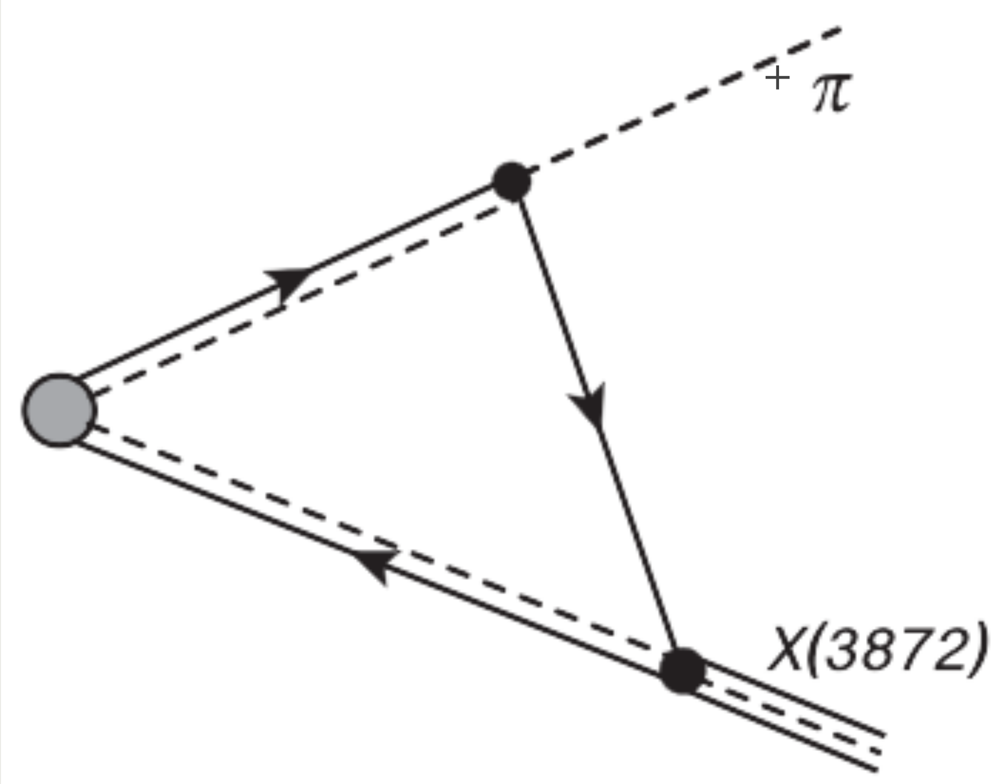
Production of $X(3872) + \pi$ at hadron colliders

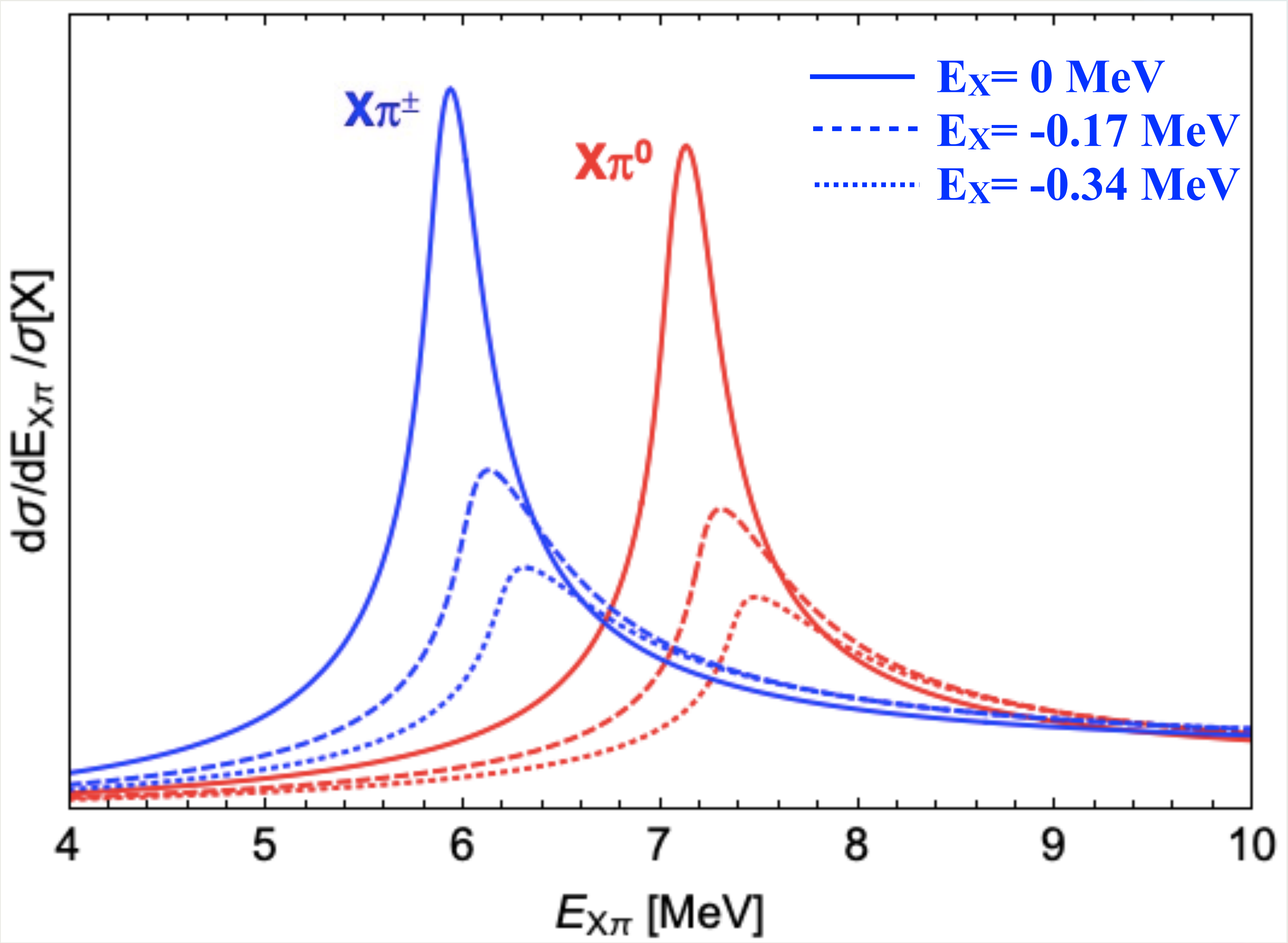
$D^{*+}\bar{D}^{*0} \to X(3872)\pi^+$

Braaten, He & Ingles [PRD 100, 094006(2019)]

triangle singularity produces narrow peak in $X\pi^{\pm}$ invariant mass

peak near 6.1 MeV above $X\pi^+$ threshold





- creation of $D^{*+}\bar{D}^{*0}$ at short distance
- rescattering of virtual $D^{*+}\bar{D}^{*0}$ into $X\pi^+$

a small fraction of events are from triangle singularity, but all within 1 MeV of the peak

Experimental observation of $X(3872)+\pi$ in $p\bar{p}$ collisions

D0 Collaboration [PRD 102, 072005 (2020)]

prompt and b-hadron decay production of X(3872) + soft $\pi^{\pm}$

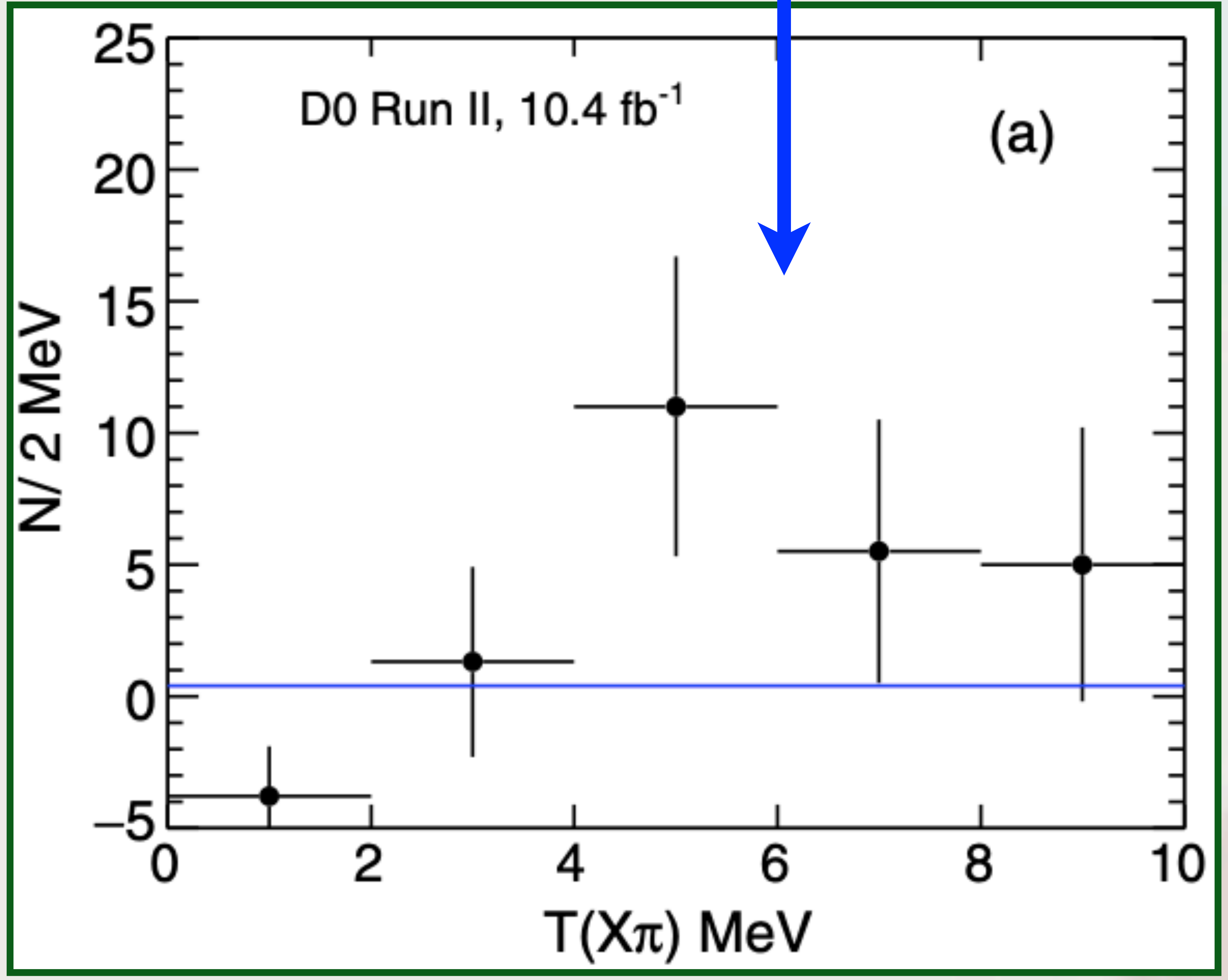
a small excess in small $T(X\pi)$ region, significance of 2σ

$T(X\pi) < 11.8$ MeV	observed events	X + random π
prompt production:	18 ± 16	6
b-decay:	27 ± 12	2

conclusions:

* prompt production: no evidence for an enhancement as expected from the triangle singularity
* b-decay: no “significant” evidence for an enhancement as expected from the triangle singularity

Δ

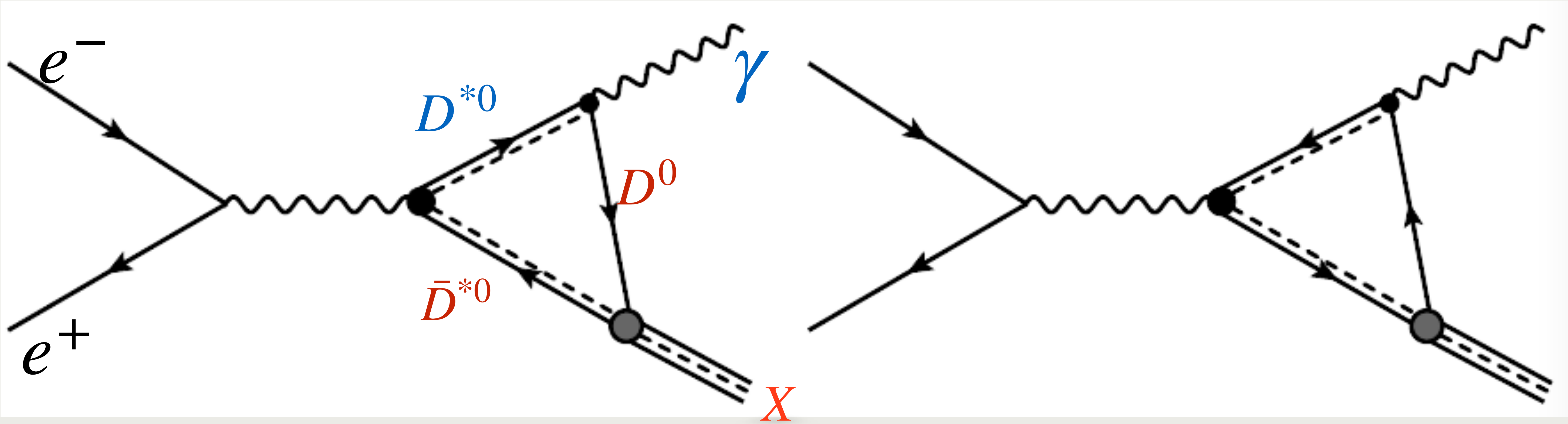


questions:

no peak in prompt production??
first hint of narrow peak from triangle singularity in b-decay??

Production of $X(3872)+\gamma$ in e^+e^- annihilation

$e^+e^- \to D^*\bar{D}^*(P\ wave) \to X\gamma$



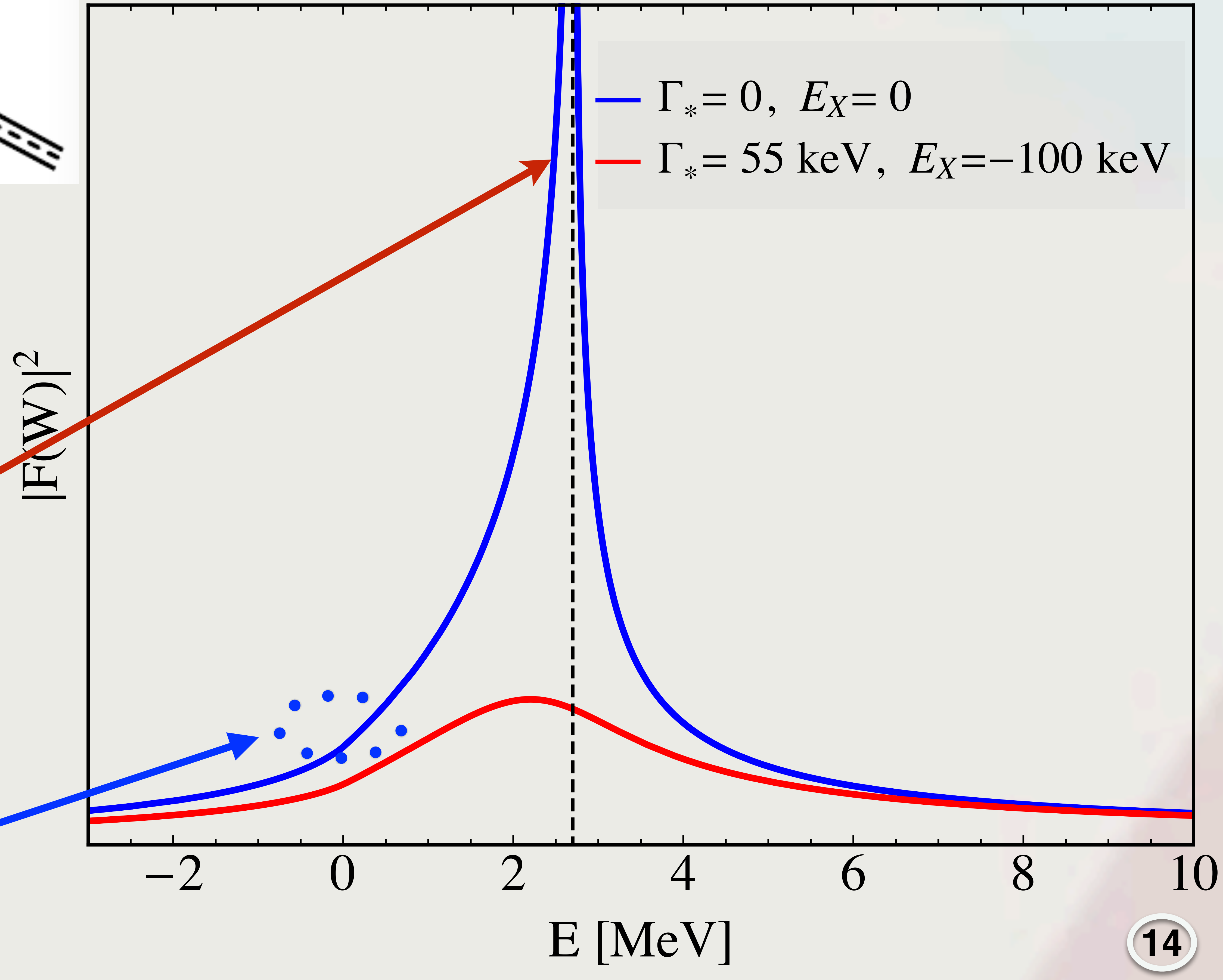
- e^+e^- annihilation creates $D^{*0}\bar{D}^{*0}$(P-wave)
- rescattering of virtual $D^{*0}\,\bar{D}^{*0}$ into $X\gamma$

△ **triangle singularity:**
three charm mesons can be on shell simultaneously

$\log^2(E - E_\Delta)$ **divergence in reaction rate at**
$E_\Delta = (M_{D^{*0}}/M_X^2)(M_{D^{*0}} - M_{D^0})^2 = 2.7$ MeV
above $D^*\bar{D}^*$ threshold

No square-root branch point at $E = 0$ because of cancellation

NO cusp at $E = 0$

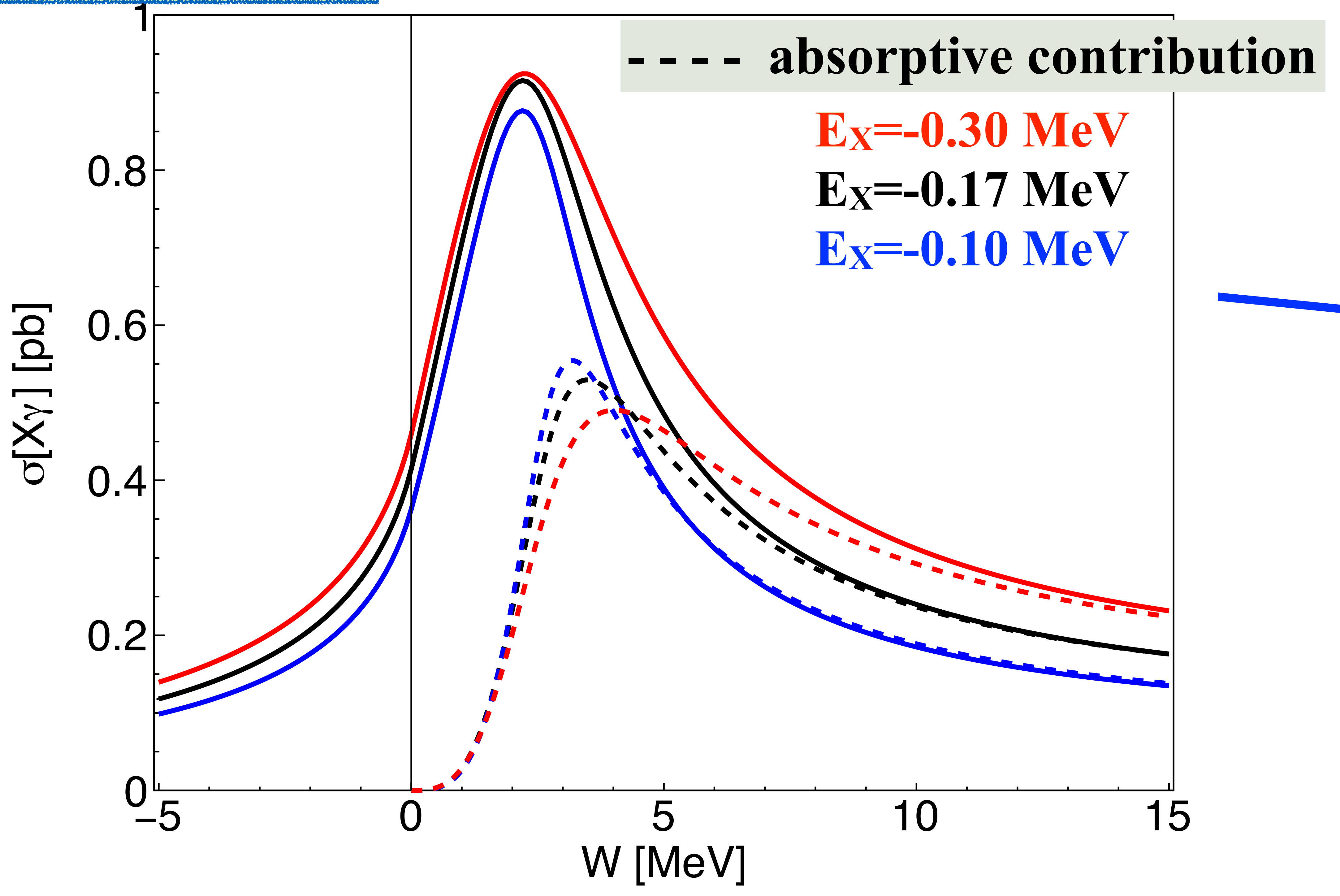


Production of $X(3872) + \gamma$ in e^+e^- annihilation

$e^+e^- \to X\gamma$

- Dubinskiy & Voloshin (2006) [only absorptive part]
- BHI (2019), BHIJ(2020):

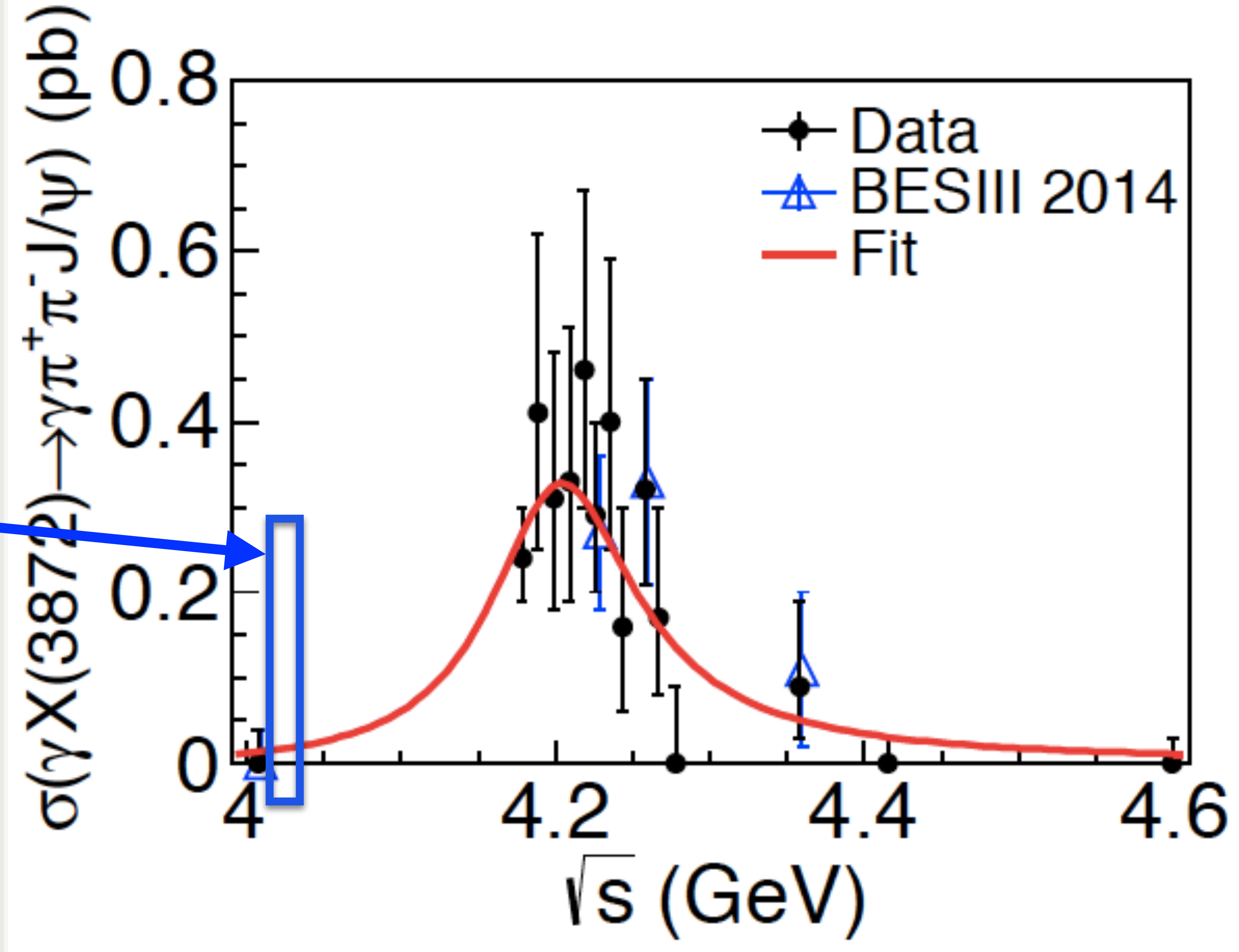
cross section for Xγ



absorptive contribution only is not a good approximation!

BESIII data

BESIII: $e^+e^- \to X\gamma$
[PRL122,232002 (2019)]

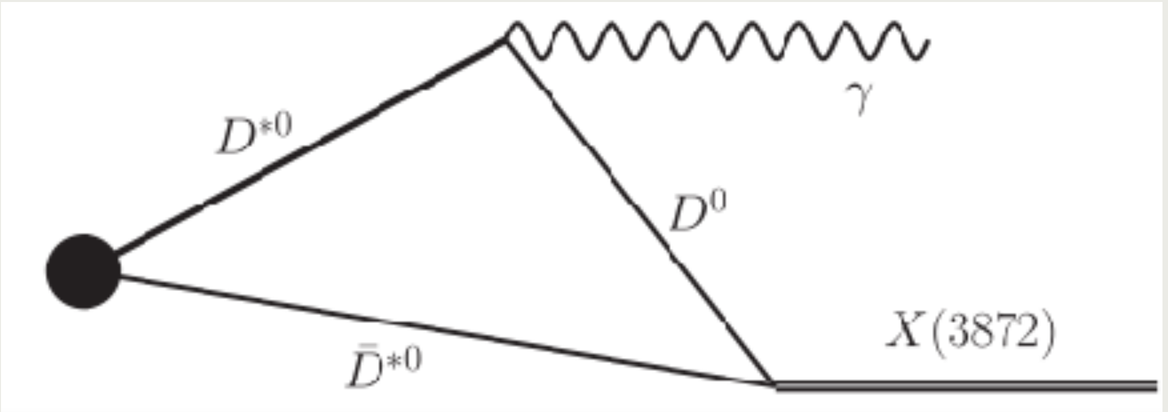


- triangle singularity gives narrow peak at 2.2 MeV above $D^{*0}D^{*0}$ threshold at 4013.7 MeV
- position of peak insensitive to binding energy
- may be observable by BESIII detector!

Production of $X(3872) + \gamma$ in e^+e^- annihilation

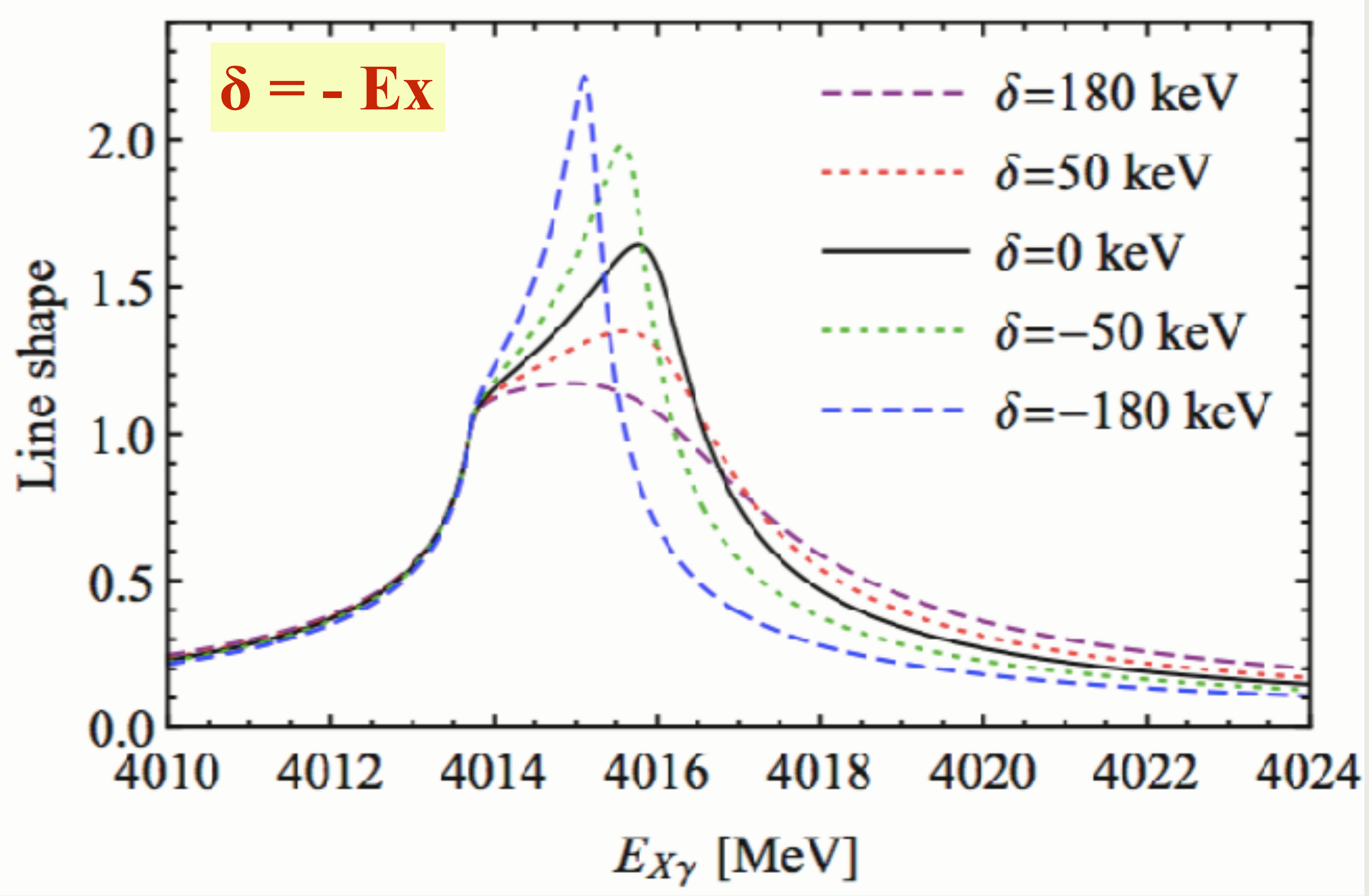
- **Guo** [PRL 112, 202002 (2019)]

- creation of $D^{*0}\bar{D}^{*0}$(S-wave) at short distance
- rescattering of virtual $D^{*0}\bar{D}^{*0}$ into $X\gamma$



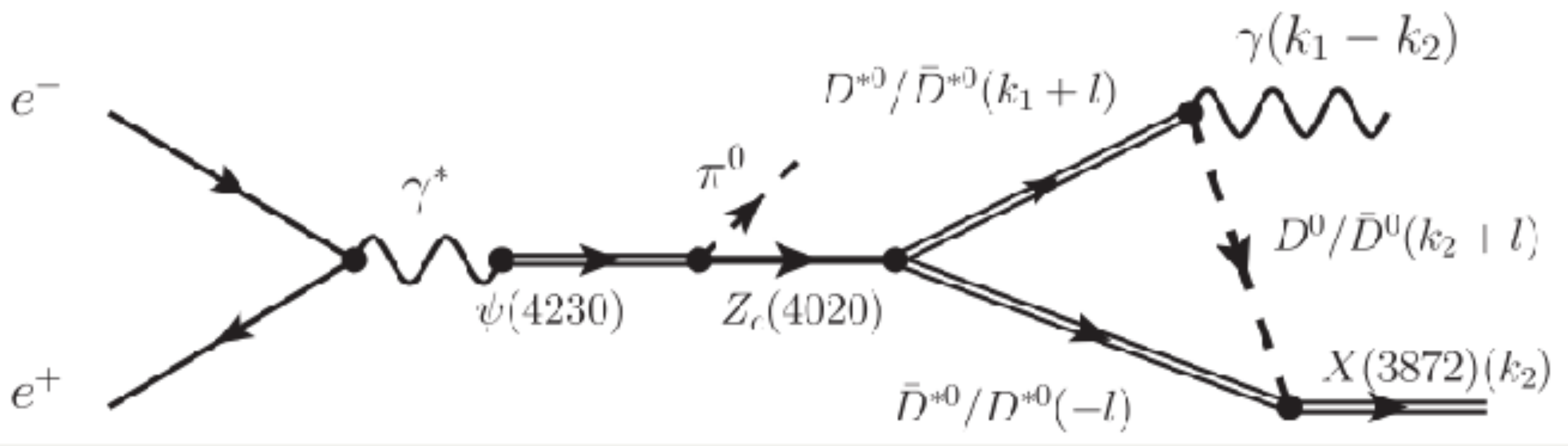
Line shape in Xγ:

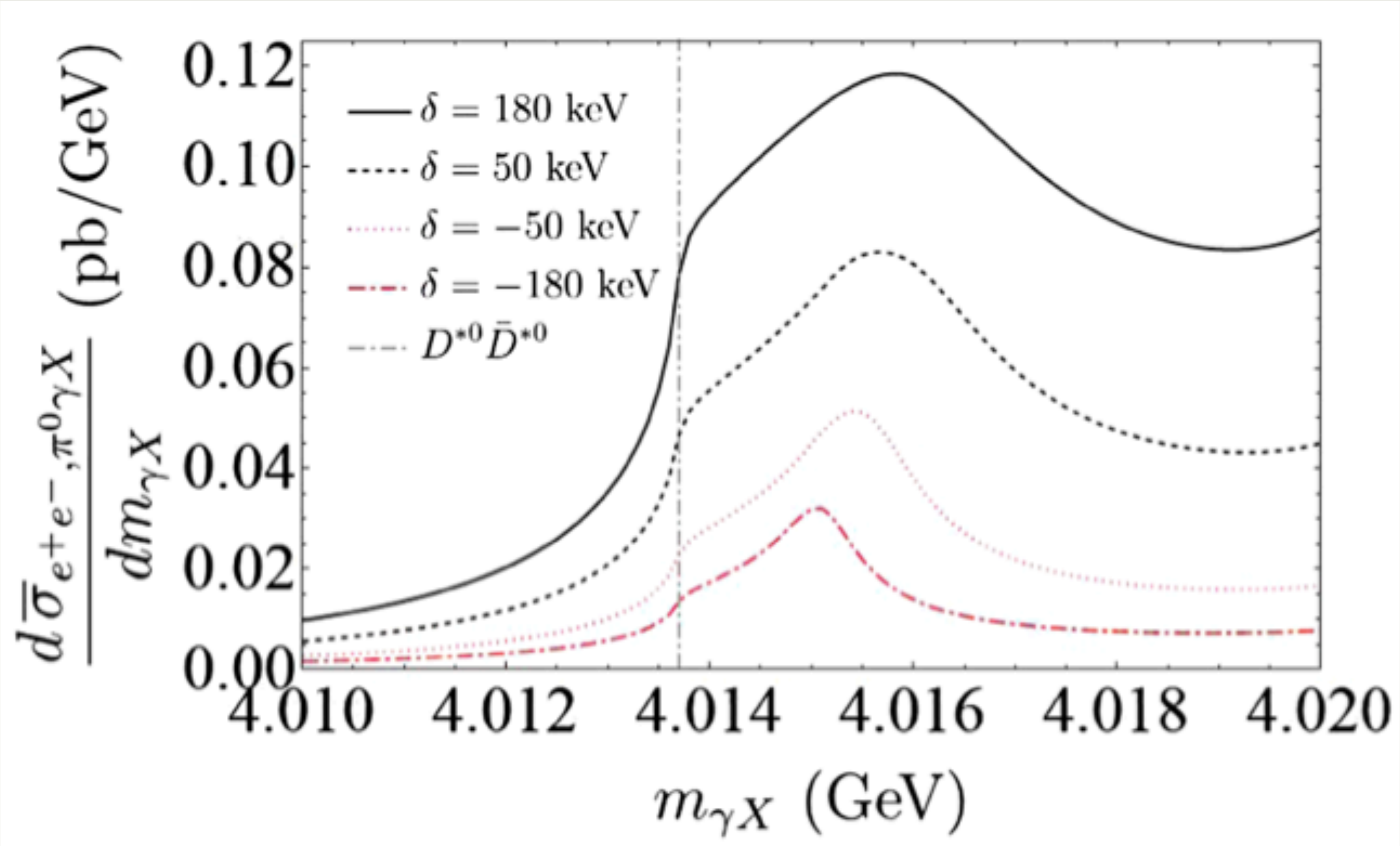
- ❖ peak a few MeV above $D^{*0}\bar{D}^{*0}$ threshold
- ❖ can be used to measure E_X



- **Sakai, Jing & Guo** [PRD 102, 114041(2020)]

$e^+e^- \to Z_c(4020)\,\pi^0$, $Z_c(4020) \to D^{*0}\bar{D}^{*0}$(S-wave) $\to X\gamma$



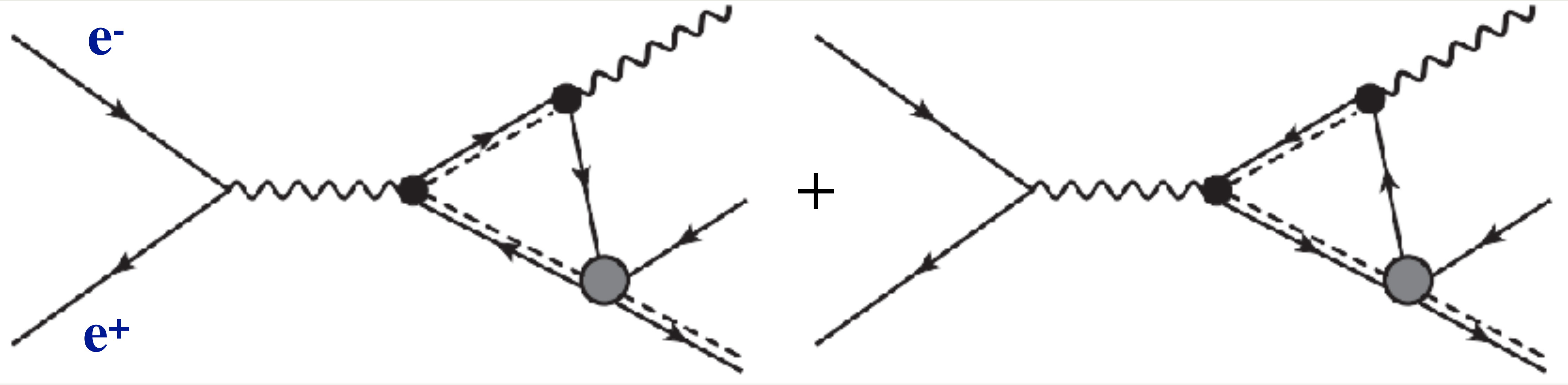


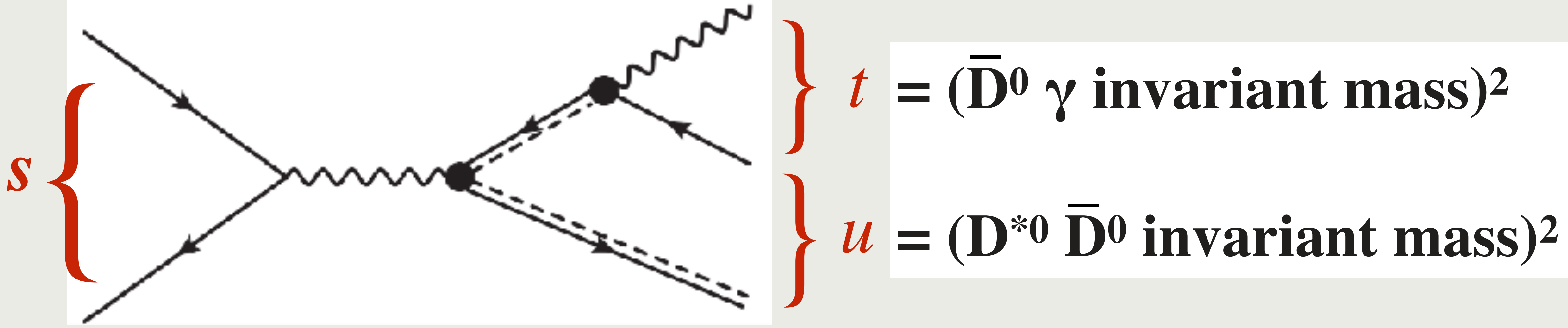
- **BESIII** [arXiv:2101.00644]: no significant signal

$e^+e^- \to Z_c(4020)\,\pi^0$, $Z_c(4020) \to D^{*0}\bar{D}^{*0}$(S-wave) $\to X\gamma$

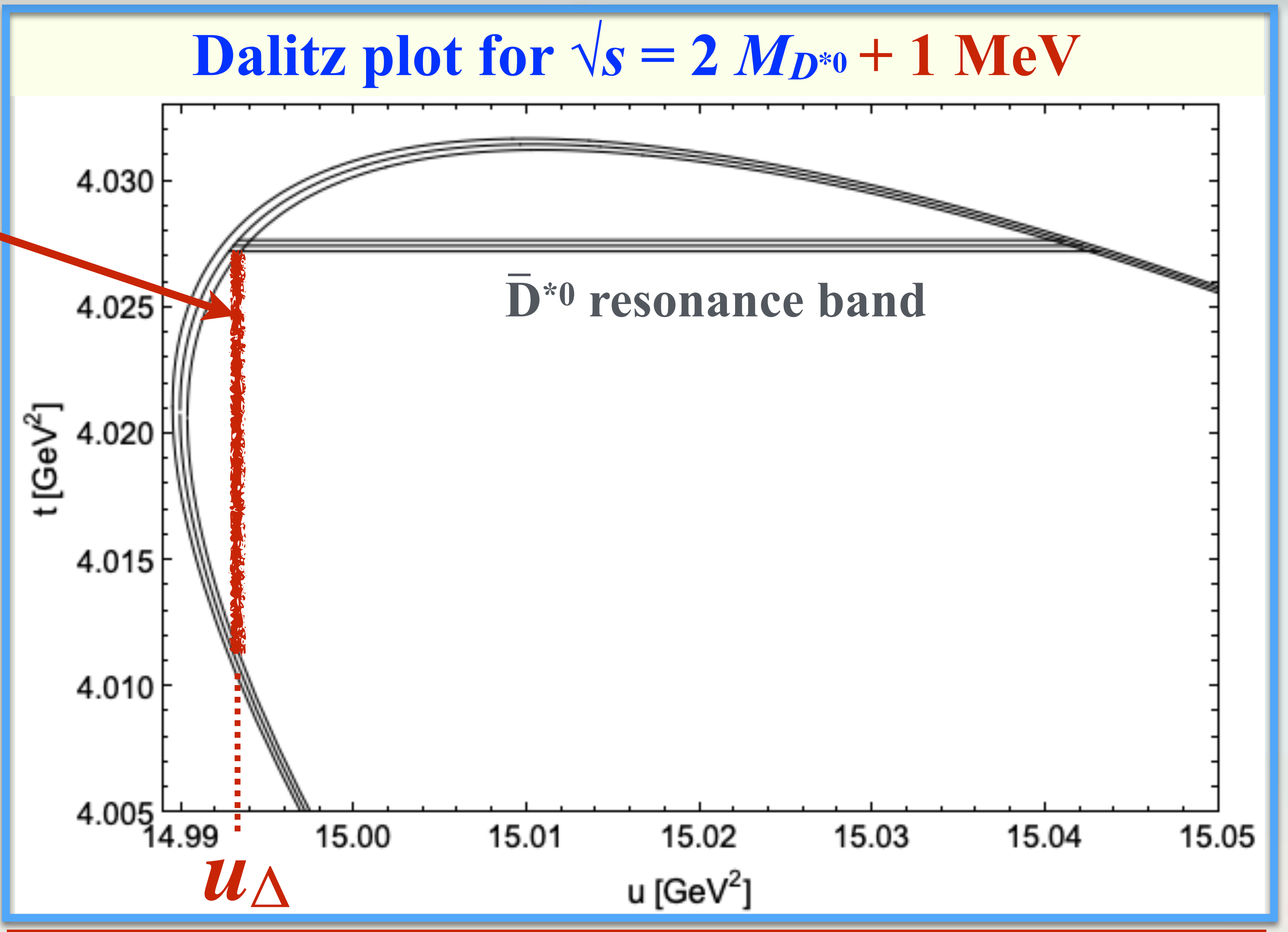
Schmid cancellation

Braaten, He, Ingles & Jiang [PRD 101, 096020(2020)]





triangle singularity at $u = u_\Delta$



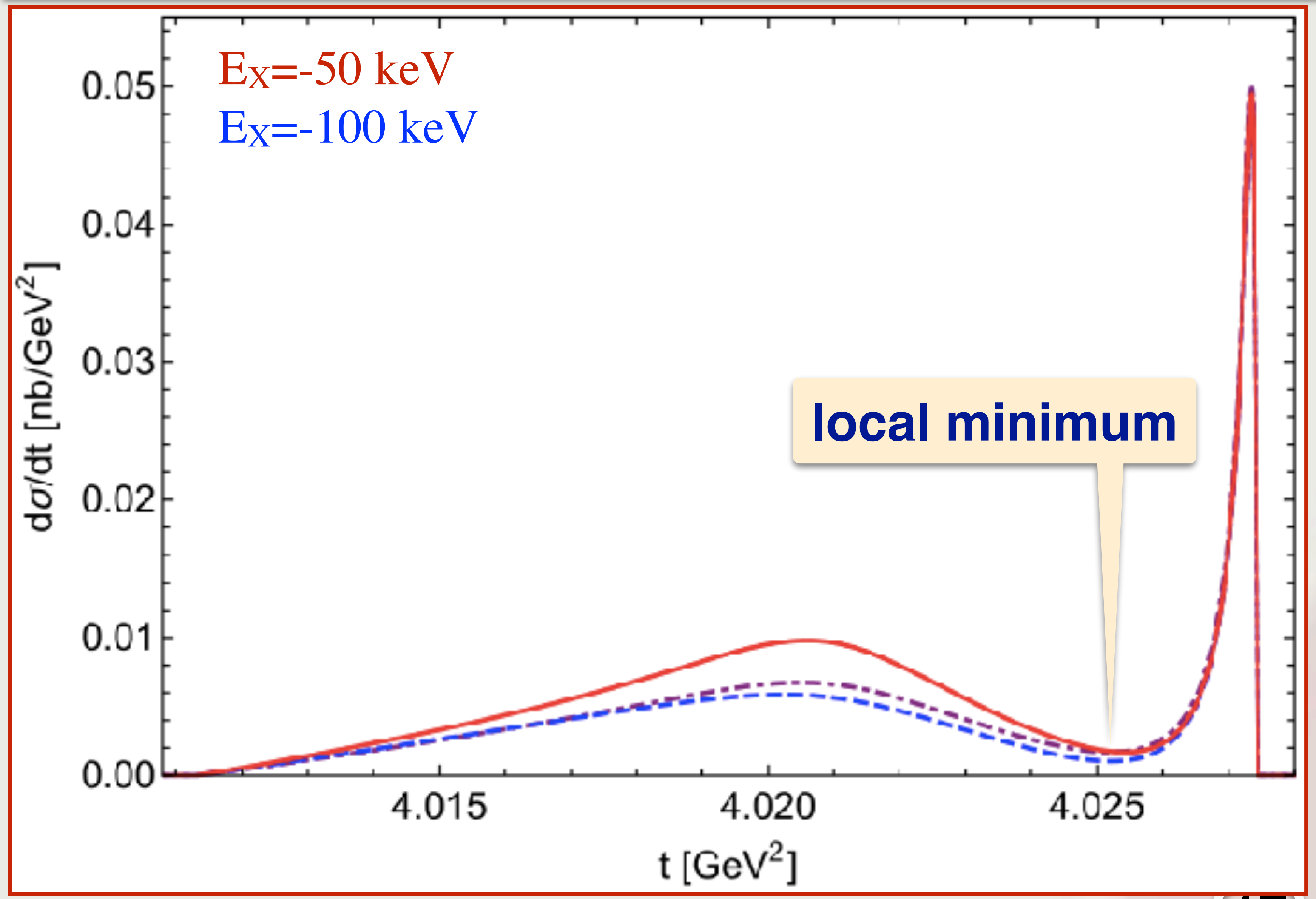
- **Schmid cancellation:**

  Schmid [PR154, 1363(1967)]
  Anisovich & Anisovich [PLB345, 321(1995)]

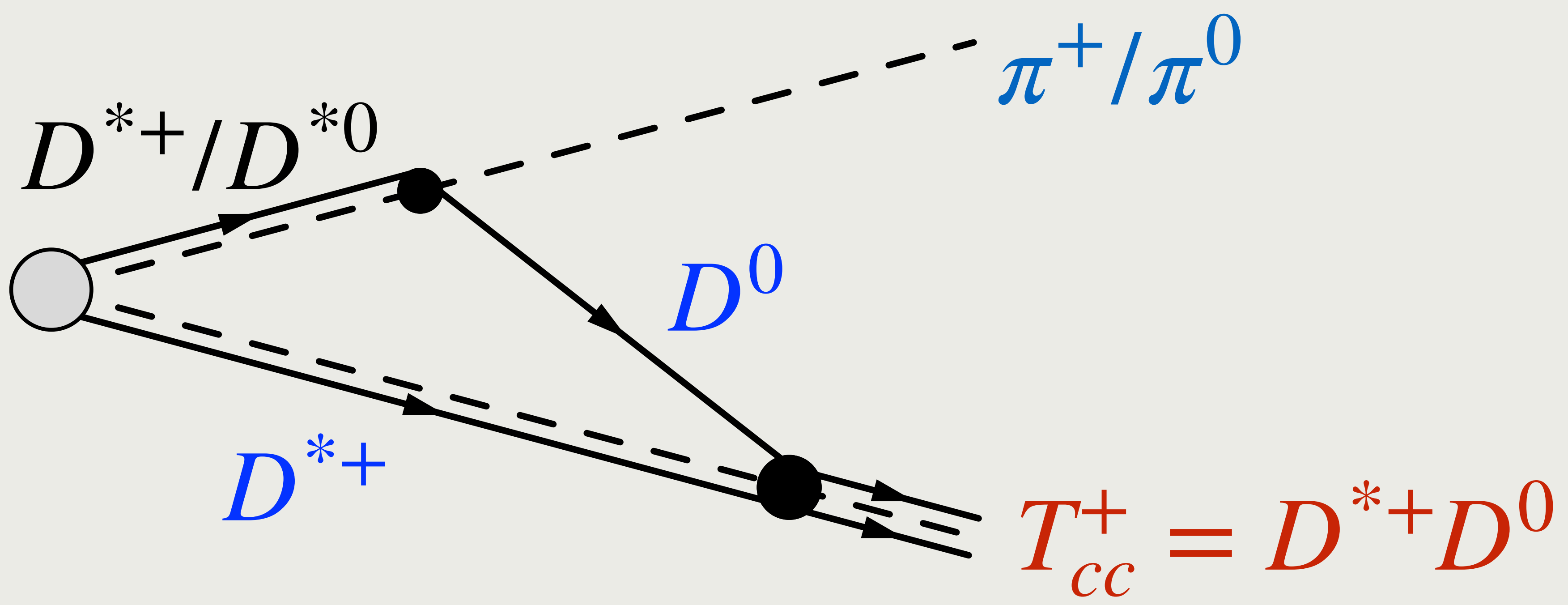
  - $d\sigma/(du\,dt)$ at fixed t: log^2 divergence
  - $d\sigma/du$ integrated over t: log divergence

- **indirect way to observe triangle singularity:**

  $d\sigma/dt$ integrated over $u < u_\Delta$ has local minimum in t

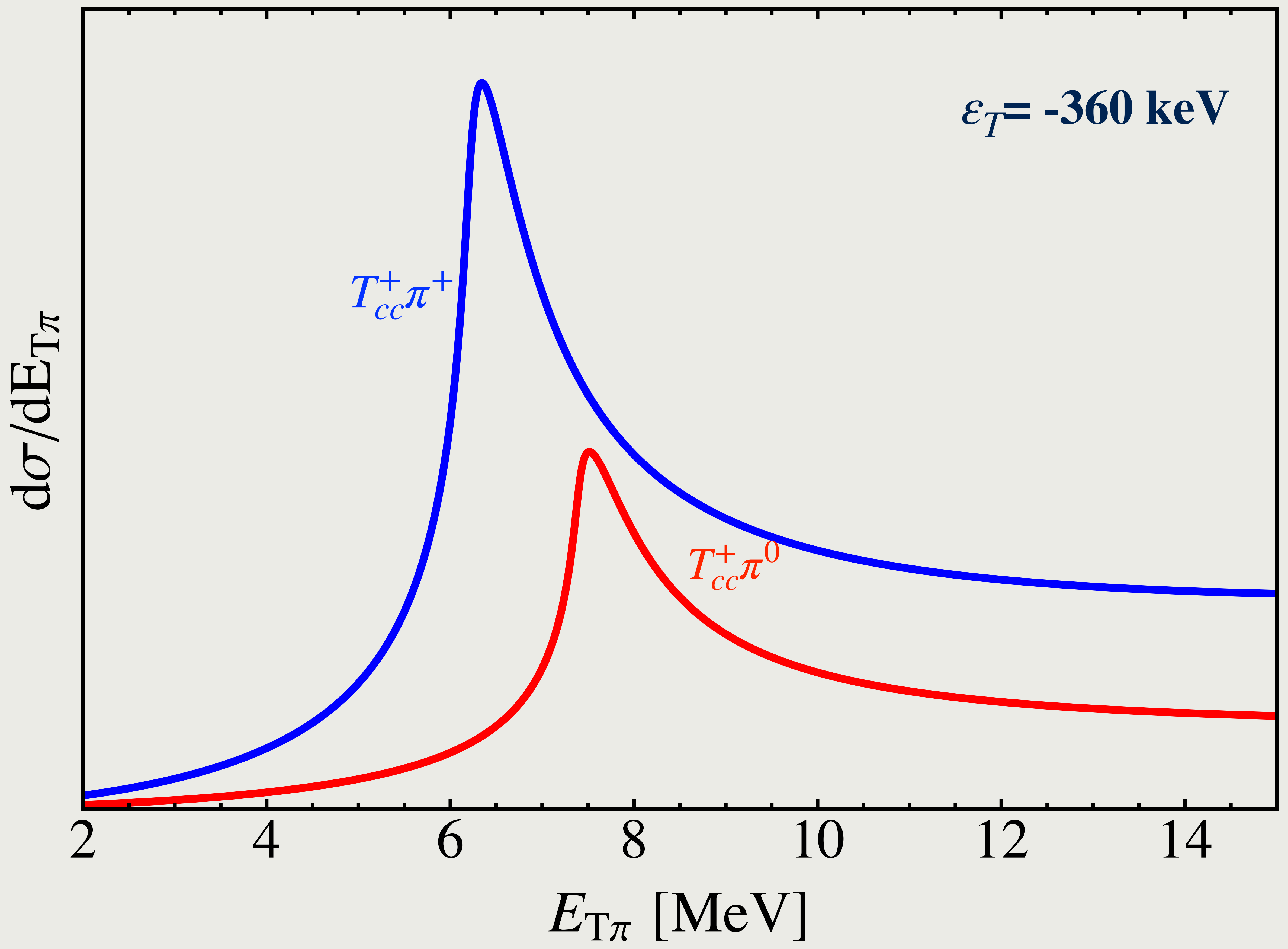


Production of $T_{cc}^{+}(3875) + \pi$ at hadron colliders



peak in dσ/dE from interplay between the Triangle singularity and the square-root singularity

At large energy, $d\sigma/dE \propto E^{1/2}$ ⟷ unphysical behavior [an artifact of using the universal approximation for T_{cc}^+ beyond its range of applicability]



coupled-channel model (including $D^{*0}D^+$)

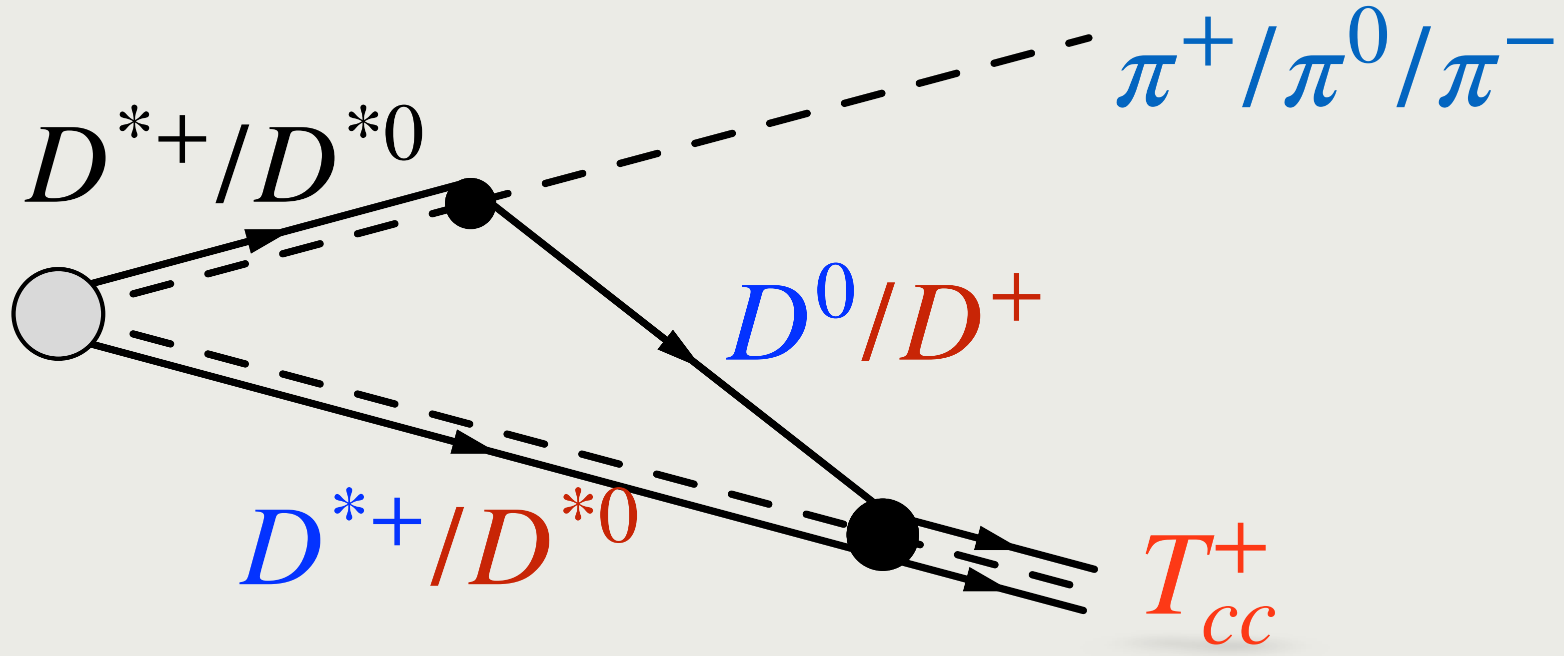
using model wave function

→ more physical qualitative behavior at large energy

relative probability for the $D^{*0}D^+$ channel:

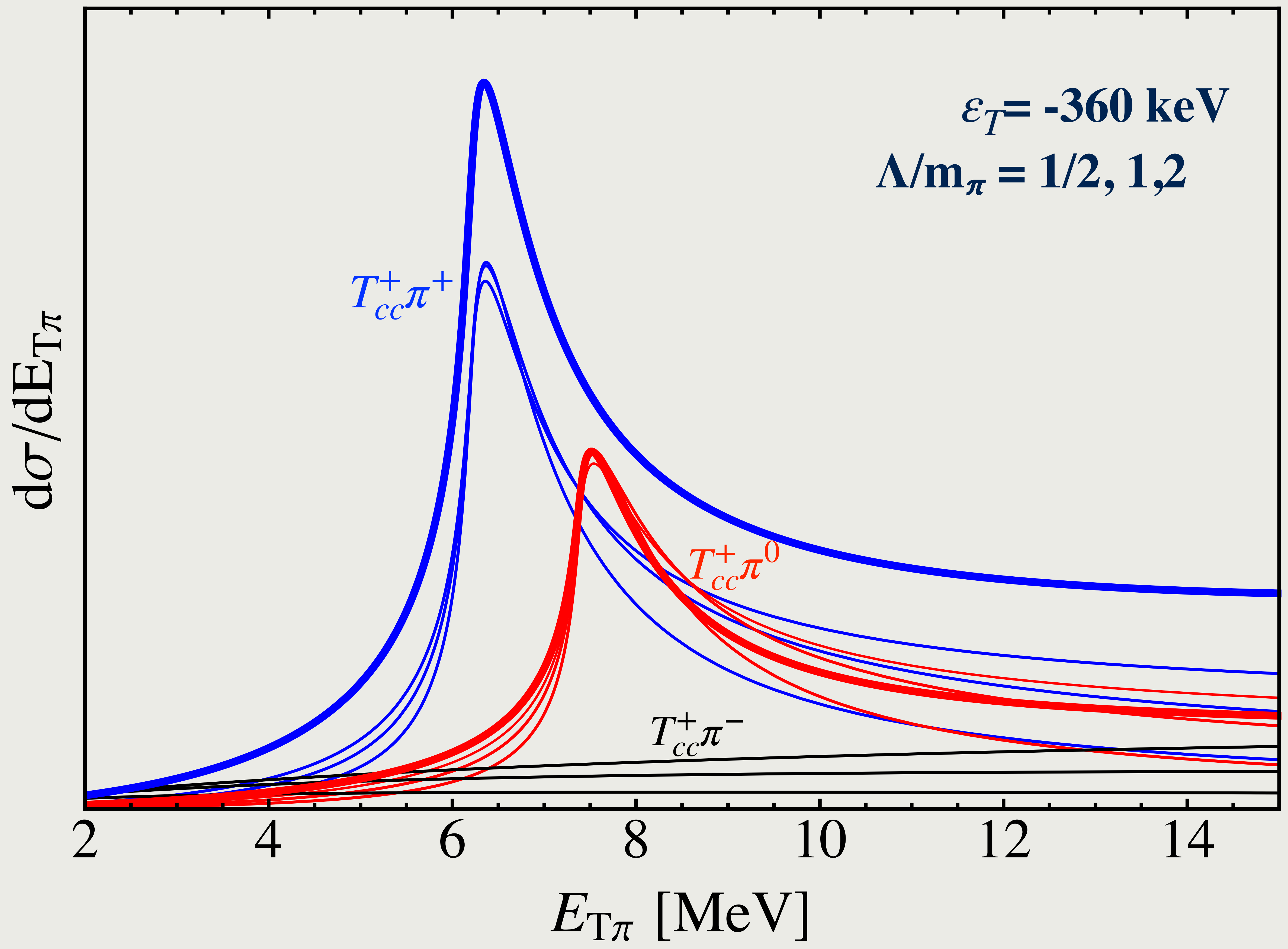
$$Z_{0+} = \frac{(\Lambda + \gamma)\gamma}{(\Lambda + \gamma_{0+})\gamma_{0+}} < 1$$

Production of $T_{cc}^{+}(3875) + \pi$ at hadron colliders



amplitude in XEFT for the coupled-channel model

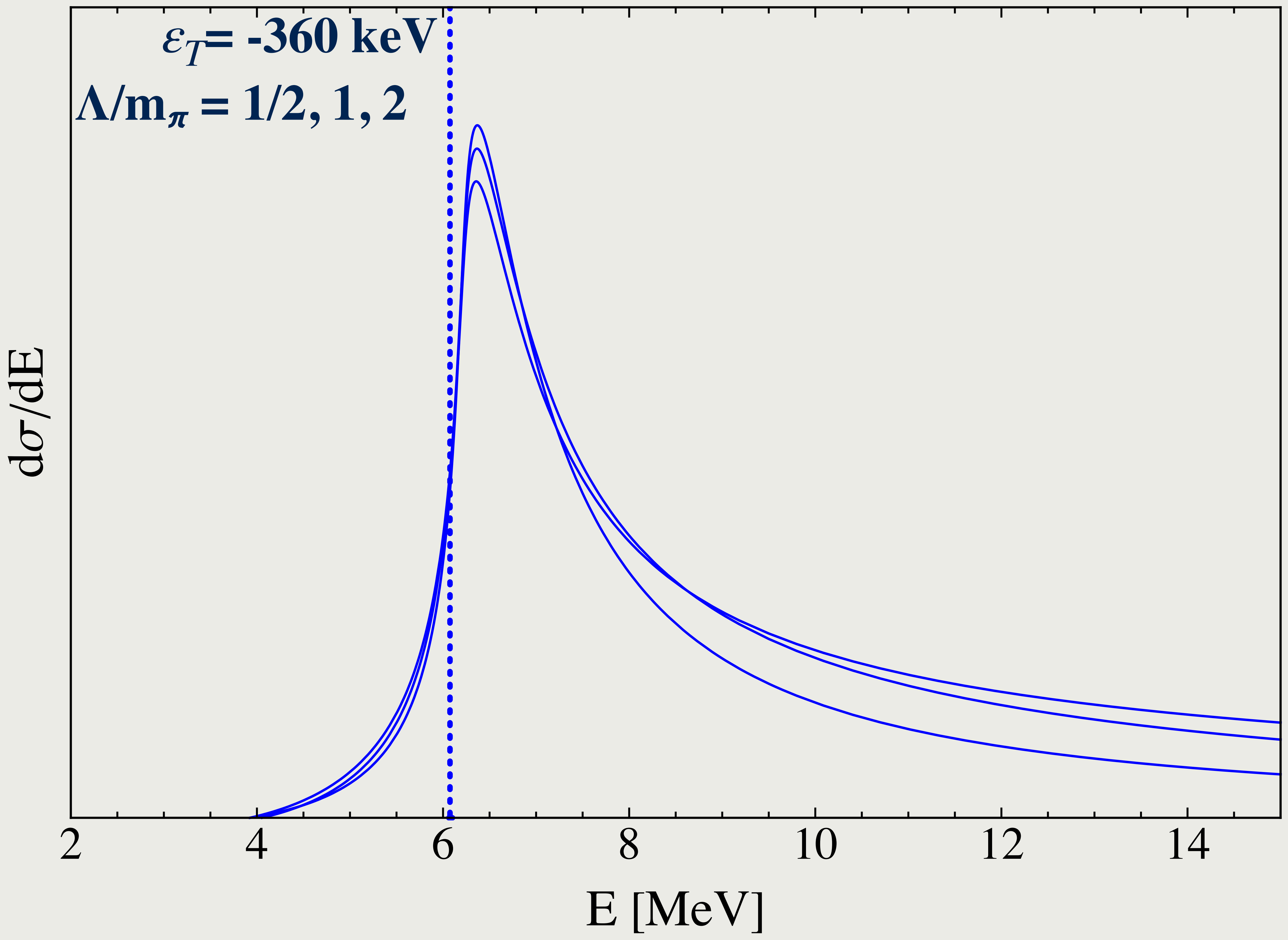
$$\frac{1}{k^2+\gamma^2} \to \frac{1}{\sqrt{1+Z_{0+}}}\frac{\psi^{(\Lambda)}(k)}{\sqrt{8\pi\gamma}}, \qquad \frac{1}{k^2+\gamma_{0+}^2} \to -\frac{1}{\sqrt{1+Z_{0+}}}\frac{\psi_{0+}^{(\Lambda)}(k)}{\sqrt{8\pi\gamma}}$$



- ✱ $T_{cc}^+\pi^-$: no triangle singularity peak
- ✱ $T_{cc}^+\pi^+(\pi^0)$: same shape as those with the universal triangle amplitudes
- ✱ $T_{cc}^+\pi^-$ approaches $T_{cc}^+\pi^+$ at large energy
- ✱ At large energy, $d\sigma/dE \propto E^{-1/2}$

Production of $T_{cc}^{+}(3875) + \pi$ at hadron colliders

difference between $d\sigma/dE$ for $T_{cc}^{+}\pi^{+}$ and $T_{cc}^{+}\pi^{-}$ near triangle-singularity peak



difference between $\sigma[T_{cc}^{+}\pi]$ for $T_{cc}^{+}\pi^{+}$ and $T_{cc}^{+}\pi^{-}$

$$\sigma\left[T_{cc}^{+}\pi^{+}\right] - \sigma\left[T_{cc}^{+}\pi^{-}\right] \approx \left(1.3^{+1.5}_{-0.8}\right) \times 10^{-2} \sigma^{(\Lambda)}\left[T_{cc}^{+}, \text{no } \pi\right]$$

- **independent of E_{max}**
- **dominated by the triangle-singularity peak**
- **$T_{cc}^{+}\pi^{-}$ can be used to measure background**

Production of $T_{cc}^+(3875) + \pi$ at hadron colliders

subtraction of $T_{cc}^+\pi^-$ subtracts the background for $T_{cc}^+\pi^+$ but keeps the peak from the triangle singularity.

fraction of $T_{cc}^+\pi^+$ events with $T_{cc}^+\pi^+$ in the peak from triangle singularity: 1.2%

a small fraction of events are from triangle singularity, but all within 1 MeV of the peak

LHCb observed 117 ± 16 events

more statistics to observe the triangle-singularity peak

Summary

given $J^P = 1^+$, $|E_X|$ < 0.22 MeV, $\varepsilon_T = (-360 \pm 40)$ keV

X(3872) and $T_{cc}^+(3875)$ **must be** loosely bound charm-meson molecules

- ☑ charm-meson triangle singularities

  produce narrow peaks in $X\pi, X\gamma$ and $T_{cc}^+\pi$ invariant mass near *D*D** threshold

  smoking gun for X and T_{cc}^+ as charm-meson molecules !!

  compact tetraquark would have suppressed coupling to charm mesons

- ☑ production of $X + \gamma$:
  - ✦ $\sigma[e^+e^- \to X\gamma]$: narrow peak at 4016 MeV

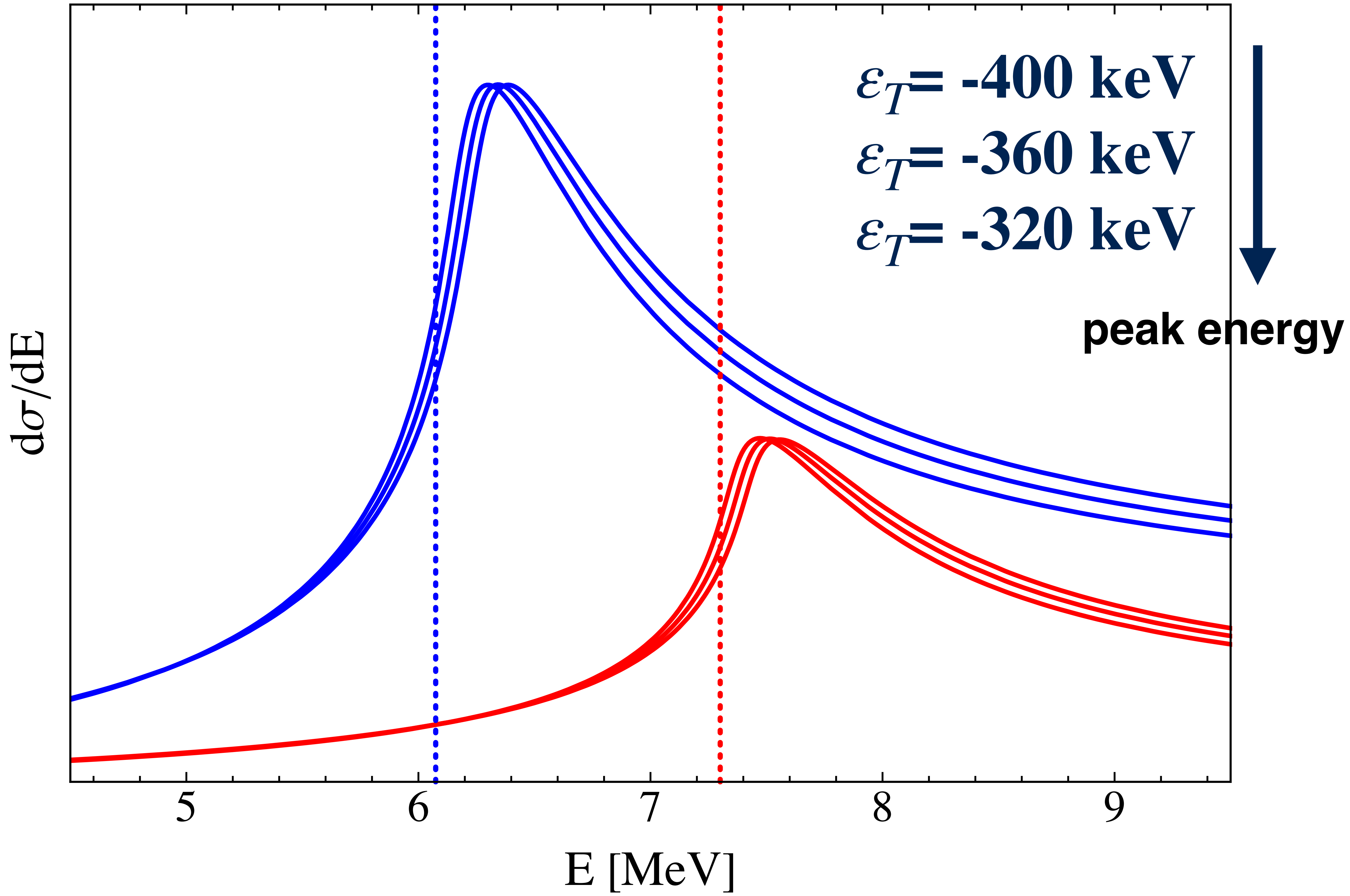
- ☑ production of $X/T_{cc}^+ + \pi^+$:
  - ✦ dσ/dE: narrow peak near 6.1 MeV above $X/T_{cc}^+ + \pi^+$ threshold
  - ✦ A small fraction of events are from triangle singularity, but within 1 MeV of the peak

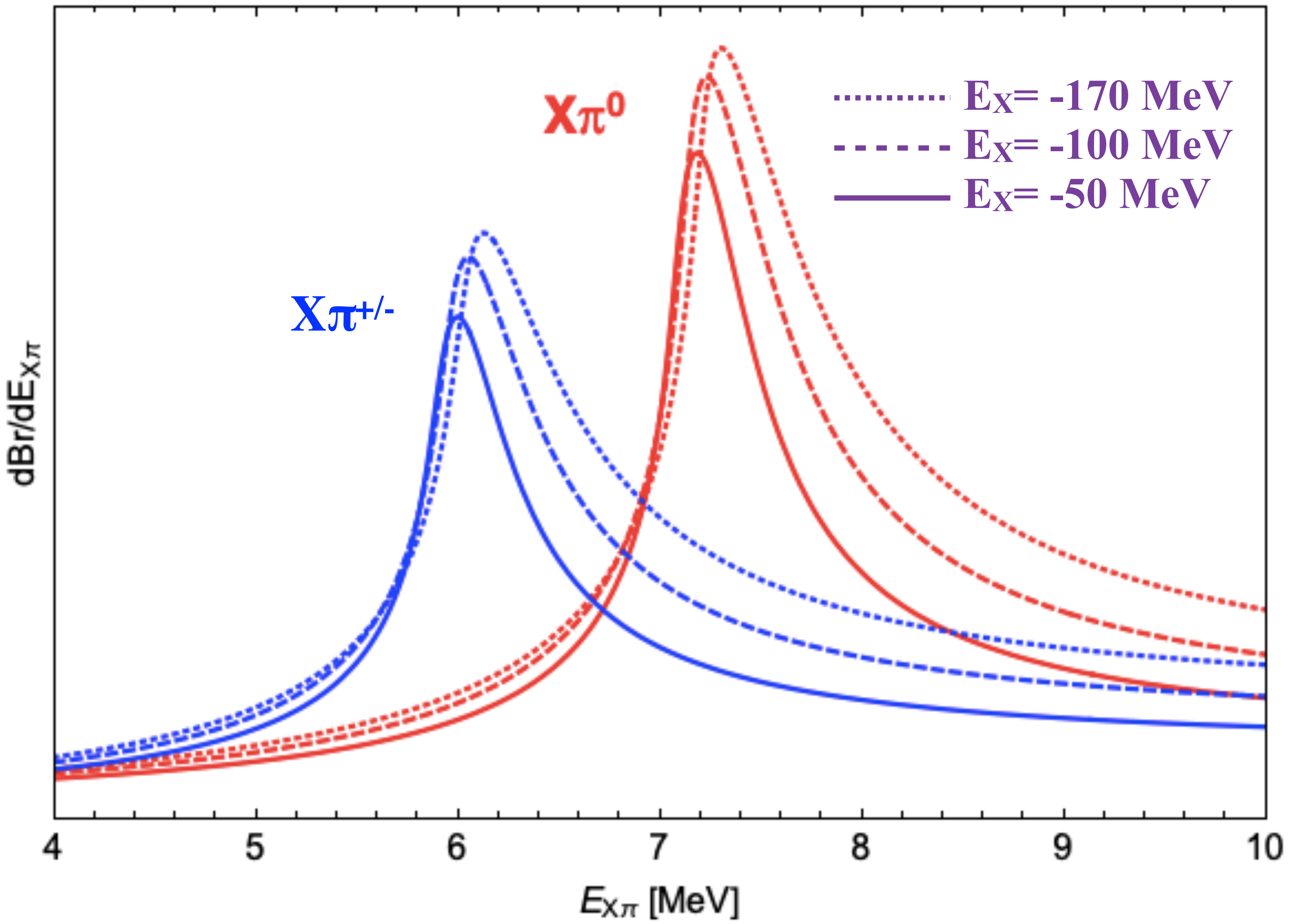
Backup

triangle amplitude

$$T_+(q^2,\gamma^2) = \left(1+\frac{mb}{2M_T c}\right)\frac{1}{\sqrt{c}}\log\frac{\sqrt{a}+\sqrt{c}+\sqrt{a+b+c}}{\sqrt{a}-\sqrt{c}+\sqrt{a+b+c}} + \frac{m}{M_T c}\left(\sqrt{a}-\sqrt{a+b+c}\right)$$

$$F(W) = -i\frac{\mu\sqrt{\pi\gamma_X}}{4\pi M_0}q\left(\frac{b}{2c}\log\frac{\sqrt{a}+\sqrt{a+b+c}+\sqrt{c}}{\sqrt{a}+\sqrt{a+b+c}-\sqrt{c}}+\frac{\sqrt{a}-\sqrt{a+b+c}}{\sqrt{c}}\right)$$





- At higher energy, $d\sigma/dE$ **decreases** as $E^{-1/2}$ for molecule

  $d\sigma/dE$ **increases** as $E^{3/2}$ for compact tetraquark