





# R value measurement at BESIII

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# **Introduction: Definition of R value**

#### Theoretical definition

The *R* value is defined as the leading-order production cross section ratio of hadron and muon pairs in the annihilation of electron-positron:

$$R \equiv \frac{\sigma^{0}(e^{+}e^{-} \rightarrow \text{hadrons})}{\sigma^{0}(e^{+}e^{-} \rightarrow \mu^{+}\mu^{-})} \equiv \frac{\sigma^{0}_{\text{had}}}{\sigma^{0}_{\mu\mu}}$$

That is, according to Feynman diagrams



A direct result from the QED theory:

$$\sigma^{0}_{\mu\mu}(s) = \frac{4\pi\alpha^2}{3s} \frac{\beta_{\mu}(3-\beta^2_{\mu})}{2}$$
, with  $\beta_{\mu} = \sqrt{1-4m^2_{\mu}/s}$ 

#### **Introduction: R** value by pQCD and experiments

PDG



R value were measured by many Collaborations:

BESII, KEDR, VEPP, DAΦNE, DM2, DASP, PLUTO, Crystal-Ball, MARKI, MARKII, CLEO-c, AMY, JADE, TASSO, CUSB, MD-1, MARKJ, SLAC-LBL, MAC, γγ2, ..... BESIII

# **Introduction:** Why R value important

# QED running coupling constant $\Delta \alpha(s)$ – one of the most important QED parameters

The contributions to  $\Delta \alpha(s)$  can be distinguished to three pieces:

$$\Delta \alpha(s) = 1 - \alpha(0) / \alpha(s) = \Delta \alpha_{\text{lepton}}(s) + \Delta \alpha_{\text{had}}^{(5)}(s) + \Delta \alpha_{\text{top}}(s)$$

- $\Delta \alpha_{\text{lepton}}(s)$  can be calculated analytically using the perturbative theory.
- Since the top quark is heavy,  $\Delta \alpha_{top}(s)$  is small ( $10^{-7} \sim 10^{-10}$  for BESIII region).
- $\Delta \alpha_{had}^{(5)}(s)$  should be calculated by using the *R* value:

$$\Delta \alpha_{\text{had}}^{(5)}(s) = -\frac{\alpha s}{3\pi} \operatorname{Re} \int_{E_{\text{th}}}^{\infty} \mathrm{d}s' \frac{R(s')}{s'(s'-s-i\varepsilon)}$$





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Source	Contribution( $\times 10^{-4}$ )
$\Delta \alpha_{ m lepton}(M_Z^2)$	$314.979 \pm 0.002$
$\Delta lpha_{ m had}^{(5)}(M_Z^2)$	$276.0 \pm 1.0$
$\Delta \alpha_{\rm top}(M_Z^2)$	$-0.7180 \pm 0.0054$

 $\Delta \alpha_{had}^{(5)}(s)$  is sensitive with the *R* value over all energy region!

# **Introduction:** Why R value important

Muon anomalous magnetic moment  $a_u$  – the most precision test of SM

- Magnetic moment of the muon:  $\vec{\mu} = g_{\mu} \frac{e}{2m_{\mu}} \vec{S}$
- Dirac theory:  $g_{\mu} = 2 \Rightarrow$  Quantum Field Theory:  $a_{\mu} = \frac{|g_{\mu} 2|}{2}$

#### **Anomalous Magnetic Moment:**

Standard model prediction:

 $a^{ extsf{SM}}_{\mu} = a^{ extsf{QED}}_{\mu} + a^{ extsf{QCD}}_{\mu} + a^{ extsf{weak}}_{\mu}$ 

Phys. Rep. 887, 1 (2020)Direct measurement

(Exp. average BNL & FNAL) Phys. Rev. Lett. 126, 141801 (2021)  $\Rightarrow$  Discrepancy of 4.2  $\sigma$ !

- Hadronic contributions dominate uncertainty of  $a_{\mu}^{\rm SM}$ 
  - Hadronic Light-by-Light Scattering (HLbL) & Hadronic Vacuum Polarization (HVP)

The HVP contribution, i.e.,  $a_{\mu}^{\text{LO-HVP}}$ , is calculated in terms of *R* value with the dispersion relation:

$$a_{\mu}^{\text{LO-HVP}} = \left(\frac{\alpha m_{\mu}}{3\pi}\right)^2 \int_{4m_{\pi}^2}^{\infty} \mathrm{d}s \frac{R(s)K(s)}{s^2}$$



⇒ Muon Anomaly



fractions of  $\delta(a_{\mu})$  caused by error of R value from different region.



# **Introduction:** Why R value important

# Muon anomalous magnetic moment $a_{\mu}$ – the most precision test of SM New calculation of $a_{\mu}^{\text{LO-HVP}}$ from the lattice QCD approach decreases the discrepancy between $a_{\mu}^{\text{SM}}$ and $a_{\mu}^{\exp}$ :



**The** *R***-ratio approach still gives the best prediction to**  $a_{\mu}^{\text{LO-HVP}}$ 

### **BEPCII and BESIII: BEPCII**



#### **BEPCII:** Beijing electron-positron collider II

Bird's-eye view of **BEPCII** and **BESIII** 

## **BEPCII and BESIII: BESIII**



**BESIII: BEijing Spectrometer III** 

## Methodology: R value measurement

Experimentally, the *R* value is determined by

$$R = \frac{N_{had}^{obs} - N_{bkg}}{\sigma_{\mu\mu}^0 \mathcal{L}_{int.} \varepsilon_{trig} \varepsilon_{had} (1 + \delta)}$$

- $N_{had}^{obs}$ : Numbers of observed hadronic events.
- *N*<sub>bkg</sub>: Number of the residual background events.
- $\sigma^0_{\mu\mu}(s) = 86.85 \text{ nb}/s$ : Leading order QED cross section for  $e^+e^- \rightarrow \mu^+\mu^-$ .
- $\mathcal{L}_{int.}$ : Integrated luminosity is measured by analyzing Bhabha events.
- $\varepsilon_{\text{trig}}$ : Trigger efficiency ~ 100%.
- $\varepsilon_{had}$ : Detection efficiency of the hadronic events.
- $(1 + \delta)$ : ISR correction factor.
- Determination of ε<sub>had</sub> is the most challenging task!
- Two different signal simulation models are developed and investigated intensively.

9

#### Methodology: data analysis strategy



#### Signal simulation: hadronic generator

#### 

At BEPC energy region, Lund area law formula:

$$d\wp_{n}(q\bar{q} \rightarrow m_{1}, m_{2}, \cdots m_{n}) = (2\pi)^{4} \delta(1 - \sum_{j=1}^{n} \frac{m_{\perp j}^{2}}{sz_{j}}) \delta(1 - \sum_{j=1}^{n} z_{j}) \delta(\sum_{j=1}^{n} \vec{k}_{j}) \sum |\mathcal{M}_{LUND}|^{2} d\Phi_{n}$$

$$z_{j} \equiv (E_{j} \pm p_{zj})/(E_{0} \pm P_{z0}) \qquad \vec{k}_{j} \equiv \frac{\vec{p}_{\perp j}}{2\sigma}$$
Transverse momentum (Gaussian-like form):  $\mathcal{M}_{\perp} = \exp(-\sum_{j=1}^{n} \vec{k}_{j}^{2})$ 
Longitudinal momentum (Lund area law):  $\mathcal{M}_{//} = \exp(i\xi\mathcal{A}_{n}) \qquad \xi = \frac{1}{2\kappa} + i\frac{b}{2}$ 

# Signal simulation: LUARLW

#### Few-Body States in Lund String Fragmentation Model

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#### Abstract

The well-known Monte Carlo simulation packet JETSET is not built in order to describe few-body states (in particular at the few GeV level in  $e^+e^-$  annihilation as in BEPC). In this note we will develop the formalism to use the basic Lund Model area law directly for Monte Carlo simulations.

Main features of the **LUARLW** model:

- A self-consistent inclusive generator.
- Initial-state radiation (ISR) process is implemented from  $2m_{\pi}$  to 5 GeV.
- Kinematic quantities of initial hadrons are sampled by the Lund area law.
- Phenomenological parameters are tuned based on comparisons between data and MC.

# hep-ph/9910285

#### Signal simulation: LUARLW

Function of LUARLW: it can simulate ISR inclusive continuous channels and resonant  $J^{PC} = 1^{-1}$  resonances from  $2m_{\pi}$  –5 GeV, parameters need tuning by data.

$$\begin{split} e^+e^- \Rightarrow \gamma^* \Rightarrow \rho(770), &\omega(782), \phi(1020), \omega(1420), \rho(1450), \omega(1650), \phi(1680), \rho(1700) \\ e^+e^- \Rightarrow \gamma^* \Rightarrow \begin{cases} q\bar{q} \Rightarrow \text{string} \Rightarrow \text{hadrons} \\ gq\bar{q} \Rightarrow \text{string} + \text{string} \Rightarrow \text{hadrons} \\ ggq\bar{q} \Rightarrow \text{string} + \text{string} \Rightarrow \text{hadrons} \\ e^+e^- \Rightarrow \gamma^* \Rightarrow \psi(2S) \Rightarrow \begin{cases} \gamma^* \Rightarrow e^+e^-, \ \mu^+\mu^-, \ \tau^+\tau^- \\ \gamma^* \Rightarrow q\bar{q} \Rightarrow \text{string} \Rightarrow \text{hadr} \\ ggg \Rightarrow \text{string} + \text{string} \Rightarrow \text{hadrons} \\ ggg \Rightarrow \gamma^+ \text{string} + \text{string} \Rightarrow \text{hadrons} \\ \gamma \varphi \Rightarrow \varphi = \gamma^* \Rightarrow \psi(2S) \Rightarrow \begin{cases} \gamma^* \Rightarrow e^+e^-, \ \mu^+\mu^-, \ \tau^+\tau^- \\ \gamma^* \Rightarrow q\bar{q} \Rightarrow \text{string} + \text{string} \Rightarrow \gamma + \text{hadrons} \\ \pi^+\pi^-J/\psi, \ \pi^0\pi^0J/\psi, \ \pi^0J/\psi, \ \eta J/\psi, \ \gamma \chi_{cJ}, \ \phi\eta \\ e^+e^- \Rightarrow \gamma^* \Rightarrow \psi(3770) \Rightarrow \end{cases} \begin{cases} \gamma^* \Rightarrow e^+e^-, \ \mu^+\mu^-, \ \tau^+\tau^- \\ D^0\bar{D}^0, \ D^+\bar{D}^- \\ \gamma^* \Rightarrow q\bar{q} \Rightarrow \text{string} + \text{string} \Rightarrow \text{hadrons} \\ ggg \Rightarrow \text{string} + \text{string} \Rightarrow \text{hadrons} \\ ggg \Rightarrow \text{string} + \text{string} \Rightarrow \text{hadrons} \\ e^+e^- \Rightarrow \gamma^* \Rightarrow \begin{cases} \psi(4040) \Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, \bar{D}D^*, D_s\bar{D}_s, \\ \psi(4160) \Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, D_s\bar{D}_s, \\ \psi(415) \Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, D_s\bar{D}_s, \\ D_s\bar{D}_s, D_s\bar{D}_s, \\ D_s\bar{D}_s, D_s\bar{D}_s, \\ D_s\bar{D}_s,$$

 $e^+e^- \Rightarrow \gamma^* \Rightarrow X(4160), X(4260) \cdots$  with  $J^{PC} = 1^{--}$ 

#### Good generator: MC simulations and data agree well for many final state distributions.

### **Methodology: Monte Carlo simulation**



 $N_{prg}$  is the multiplicity of good charged tracks at detector level



The LUARLW model can reasonably reproduce the  $N_{\rm prg}$  distributions in data <sup>15</sup>

 $\cos\theta$  is the polar angle of good charged tracks at detector level



Good agreement between data and MC means the simulated hadronic event efficiency is reliable <sup>16</sup>

 $N_{iso}^{2-prg}$  is the number of isolated photon in 2-prong events at detector level



17

E/(pc) is the ratio of deposited energy and momentum of charged tracks



**Background with** E/(pc) **close to 1 is significantly suppressed in data** 18

## Signal simulation: calculation of ISR correction factor in LUARLW



# Signal simulation: calculation of ISR correction factor in LUARLW

#### Line-shape of total hadronic cross section and $(1+\delta)$ by different schemes



• Scheme error:

for same input  $\sigma^0(s')$ , difference between different scheme is considered as theoretical uncertainty

• Input error:

for the nominal scheme, the change of  $(1+\delta)$  by error of  $\sigma^0(s')$  line-shape is accounted as input error

#### Systematic uncertainty

According to the experimental definition of *R*, its uncertainty is roughly expressed as

$$\left(\frac{\Delta R}{R}\right)_{\rm sys}^2 = \left(\frac{\Delta \tilde{N}}{\tilde{N}}\right)^2 + \left(\frac{\Delta \sigma_{\mu\mu}^0}{\sigma_{\mu\mu}^0}\right)^2 + \left(\frac{\Delta \mathcal{L}_{\rm int.}}{\mathcal{L}_{\rm int.}}\right)^2 + \left(\frac{\Delta \varepsilon_{\rm trig}}{\varepsilon_{\rm trig}}\right)^2 + \left(\frac{\Delta \varepsilon_{\rm had}}{\varepsilon_{\rm had}}\right)^2 + \left[\frac{\Delta (1+\delta)}{(1+\delta)}\right]^2.$$

where

$$ilde{N} = rac{N_{ ext{had}}^{ ext{net}}}{arepsilon_{ ext{had}}} = rac{N_{ ext{had}}^{ ext{obs}} - N_{ ext{bkg}}}{arepsilon_{ ext{had}}}$$

In practice, the uncertainties are addressed in following different aspects:

- Event selection: all implemented selection criteria are slightly varied.
- Background estimation: different methods and background simulation models are used.
- $\sigma^0_{\mu\mu}$ : uncertainty is negligible due to the high precision of QED.
- $\mathcal{L}_{int.}$ : uncertainty is directly cited from published results.
- $\varepsilon_{\text{trig}}$ :  $\varepsilon_{\text{trig}}$  approaches to 100% with an uncertainty less than 0.1%.
- Signal simulation: A different signal simulation model, HYBRID, is developed.
- ISR correction factor: considered in calculation precision and uncertainty in  $\sigma_{had}^0(s)$ .

# Systematic uncertainty: the hybrid model

## The first attempt of exclusive simulations in determination of hadronic efficiency



- Combination of THREE different well-established simulation models.
- As much as currently known experimental knowledges are implemented.
- Different ISR and VP correction schemes are adopted.

## Systematic uncertainty: the hybrid model

#### Construction of HYBRID generator

- HYBRID is consisted of **CONEXC**, **PHOKHARA** and **LUARLW** components.
- **PHOKHARA** is used to simulate 10 exclusive processes with known cross sections and intermediate states,  $e^+e^- \rightarrow 2\pi$ ,  $3\pi$ ,  $4\pi$  etc..
- **CONEXC** simulates 47 exclusive processes with known cross sections according to PHSP model, such as  $e^+e^- \rightarrow K^+K^-\pi^0$ ,  $K_S^0K^\pm\pi^\mp$ ,  $K^+K^-\pi\pi$ ,  $5\pi$ ,  $6\pi$  and  $\gamma_{\text{ISR}}J/\psi$ .
- As much as exclusive channels containing intermediate states are implemented in the **CONEXC** with their contributions to the related inclusive channels are excluded.
- LUARLW model is partially used to simulate remain unknown processes, in which a set of chosen parameters are tuned after comparing HYBRID simulations with data.
- Processes simulated by **PHOKHARA** or **CONEXC** are prohibited in **LUARLW** to avoid excessive generation of some specific processes.
- The residual double-generatings among the three components are negligible.

#### A lot of efforts have gone into engineering the HYBRID generator!

### Systematic uncertainty: the HYBRID model

#### $N_{prg}$ is the multiplicity of good charged tracks at detector level



The Hybrid model gives better reproduction of the  $N_{\rm prg}$  distributions. <sup>24</sup>

# Systematic uncertainty: comparison between LUARLW and HYBRID

### Comparison between LUARLW/HYBRID and data for a few observables

- *N*<sub>prg</sub>: The number of detected the good charged tracks (prong).
- $\cos \theta$ , *E*, and *p*: polar angle, deposited energy in EMC, and measured momentum in MDC.
- $N_{iso}^{2-prg}$ : number of isolated photons of two-prong events.



#### LUARLW

#### HYBRID



#### Both the two simulation models give good consistency with data!

Systematic uncertainty: comparison between LUARLW and HYBRID

Effective ISR energy distributions in LUARLW and HYBRID simulations The comparisons of the  $\sqrt{s'}$  spectrum between LUARLW and HYBRID



These two different simulation schemes result in consistent  $\sqrt{s'}$  spectra!

26

# Systematic uncertainty: LUARLW and HYBRID

Comparisons of resulted  $\varepsilon_{had}$  and  $(1 + \delta)$  between luarly and hybrid:

$\sqrt{s}$ (GeV)	luarlw ε <sub>had</sub> (%)	hybrid e <sub>had</sub> (%)	Diff. (%)	$\begin{array}{c} \text{luarlw} \\ (1+\delta) \end{array}$	$\begin{array}{c} \text{Hybrid} \\ (1+\delta) \end{array}$	Diff. (%)
2.2324	64.45	64.50	-0.09	1.1955	1.2016	-0.52
2.4000	67.29	67.62	-0.49	1.2043	1.2118	-0.62
2.8000	72.25	73.16	-1.25	1.2185	1.2276	-0.74
3.0500	73.91	74.54	-0.85	1.1929	1.2040	-0.93
3.0600	73.88	74.54	-0.90	1.1825	1.1940	-0.97
3.0800	73.98	74.11	-0.18	1.1228	1.1357	-1.15
3.4000	74.81	75.19	-0.50	1.3817	1.4009	-1.39
3.5000	75.32	75.88	-0.75	1.3509	1.3690	-1.33
3.5424	75.58	76.17	-0.78	1.3413	1.3587	-1.30
3.5538	75.50	76.23	-0.97	1.3384	1.3557	-1.29
3.5611	75.50	76.27	-1.02	1.3368	1.3542	-1.30
3.6002	75.73	76.52	-1.05	1.3285	1.3453	-1.26
3.6500	76.00	76.89	-1.16	1.3082	1.3234	-1.16
3.6710	76.11	77.11	-1.30	1.2597	1.2718	-0.96

Same input  $\sigma_{had}^0(s)$  but different ISR correction schems and VP operators  $\Pi(s)$ .

- The two simulation models are consistent both in  $\varepsilon_{had}$  and  $(1 + \delta)$ .
- Since there are correlations between ε<sub>had</sub> and (1 + δ), the differences in the final *R* values are taken as systematic uncertainties.

27

# Systematic uncertainty: summary of uncertainty terms

	Event	QED	Beam		Trigger	Signal	ISR	
$\sqrt{s}$ (GeV)	selection	background	background	Luminosity	efficiency	model	correction	Total
2.2324	0.41	0.23	0.28	0.80	0.10	0.60	1.15	1.62
2.4000	0.55	0.27	0.15	0.80	0.10	1.11	1.10	1.87
2.8000	0.58	0.28	0.34	0.80	0.10	1.97	1.06	2.48
3.0500	0.61	0.33	0.41	0.80	0.10	1.76	1.01	2.33
3.0600	0.60	0.34	0.48	0.80	0.10	1.84	1.00	2.39
3.0800	0.61	0.35	0.35	0.80	0.10	1.31	1.05	2.02
3.4000	0.65	0.33	0.16	0.80	0.10	1.86	1.24	2.49
3.5000	0.60	0.35	0.62	0.80	0.10	2.05	1.16	2.66
3.5424	0.61	0.37	0.01	0.80	0.10	2.05	1.14	2.58
3.5538	0.66	0.31	0.39	0.80	0.10	2.22	1.13	2.74
3.5611	0.74	0.34	0.34	0.80	0.10	2.28	1.12	2.81
3.6002	0.66	0.33	0.38	0.80	0.10	2.27	1.09	2.77
3.6500	0.53	0.35	0.69	0.80	0.10	2.28	1.13	2.83
3.6710	0.61	0.42	0.63	0.80	0.10	2.23	1.04	2.77

- Systematic uncertainties dominated by the uncertainty of the signal simulation model.
- Last four sources are regarded as completely correlated between different energies.

#### Summary: results

Comparing BESIII *R* values with previously published results: PRL128(2022)06200



- ► The accuracy is better than 2.6% below 3.1 GeV and 3.0% above.
- Larger than the pQCD prediction by 2.7 $\sigma$  between  $3.4 \sim 3.6$  GeV.

#### **Summary: prospect**

#### Data samples taken at BESIII



▶ BESIII has collected data from 2.00 to 4.95 GeV, which can be used for *R* measurement.

- ▶ *R* measurement both in the continuum and open-charm regions has significant impacts.
- ► Energy scan data samples below 2.0 GeV are desired at BESIII.

#### **Summary: prospect**

#### Different methods: inclusive, exclusive, ISR return



- R measured inclusively and exclusively at or below 2.0 GeV, and a comparison between them would be interesting.
- ► *R* measured via the ISR technique taking advantage of BESIII  $\psi$ (3770) data, the *R* value from  $\pi^+\pi^-$  threshold to continuum region can be accessed.
- Both of these two attempts will contribute to understanding the discrepancy of muon anomaly between SM calculation and experiment measurement.

#### **Summary and outlook**

- Improving the accuracy of *R* value is of great importance for precision test of standard model.
- The first round measurement of *R* value at BESIII is published and more results are coming soon.
- There are many possibilities of *R* measurement at BESIII.

# Thanks for your attention!