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T-odd asymmetry in radiative two-pion tau decay

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Outline:

1. Introduction and Motivation
2. Resonance chiral theory in the $\tau \to \pi\pi\gamma\nu_\tau$ process
3. Phenomenological discussions
4. Summary

Introduction

$\tau \to \pi\pi\gamma\nu_\tau$: <u>good place to probe T-odd triple-product asymmety</u>

A typical T-odd variable:

$$\xi \equiv \varepsilon_{\mu\nu\rho\sigma} a^\mu b^\nu c^\rho d^\sigma \;\underset{\text{of particle } a}{\overset{\text{rest frame}}{=\!=\!=}}\; \vec{b} \cdot (\vec{c} \times \vec{d})\, m_a / s_a$$

a, b, c, d*: either *momentum* or *spin

T transformation $(t \to -t, \vec{p} \to -\vec{p}, \cdots)$: $\bar{\xi} \to -\xi$

- **When spin is involved, measurement of polarization is needed.**

[Nelson, et al., PRD'94] [Tsai, PRD'95] [Datta, PRD'07] ...

- **We focus on the situation with four momenta, *i.e.***

$$\xi = \varepsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma \;\underset{\text{of particle } 1}{\overset{\text{rest frame}}{=\!=\!=}}\; \vec{p_2} \cdot (\vec{p_3} \times \vec{p_4})\, m_1$$

There should be at least four particles in the final state !

Early proposals to search T-odd triple-product asym in $K_{l3\gamma}$, i.e. $K \to \pi\gamma l\nu_l$

[Braguta et al., PRD'02 '03] [Muller et al., EPJC'06][Rudenko, PRD'11] ......

$$T(K^+_{\ell 3\gamma}) = \frac{G_F}{\sqrt{2}} e V^*_{us} \varepsilon^\mu(q)^* \left[(V_{\mu\nu} - A_{\mu\nu}) \overline{\nu}(p_\nu)\gamma^\nu(1-\gamma_5)\ell(p_\ell) + \frac{F_\nu}{2p_\ell q} \overline{\nu}(p_\nu)\gamma^\nu(1-\gamma_5)(m_\ell - \not{p}_\ell - \not{q})\gamma_\mu \ell(p_\ell) \right]$$

$$\mathcal{N}^{-1} \sum_{\text{spins}} |T|^2 = \sum_{f,f'} a_{ff'} f f' + \sum_{f,I} b_{fI} f \,\mathrm{Re}\{I\} + \xi \sum_{f,I} c_{fI} f \,\mathrm{Im}\{I\} + \mathcal{O}\left(V_i^2, A_i^2, A_i V_j\right)$$

c_{fI}: pure kinematical factors

f: form factors of K_{l3}

I: form factors of V_i or A_i entering in $V_{\mu\nu}$ and $A_{\mu\nu}$

- Conventional way to construct the T-odd asymmetry A_ξ :

$$A_\xi = \frac{N_+ - N_-}{N_+ + N_-} \quad \text{with} \quad N_\pm = \int_{\xi \gtrless 0} \mathrm{d}\Gamma$$

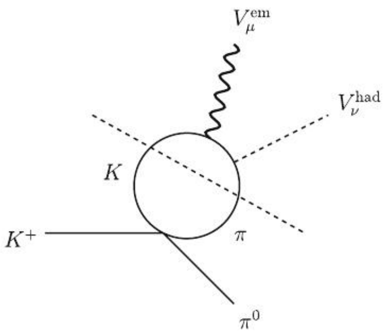
$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\xi} = \varrho_{\text{even}} + \xi \varrho_{\text{odd}}$$

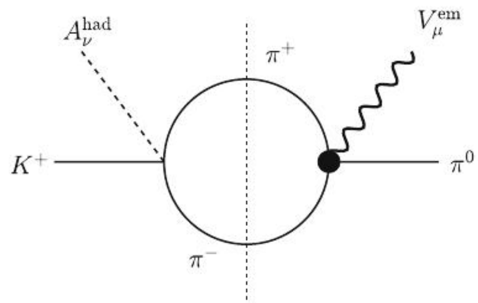
Need nonvanishing imaginary part of form factors

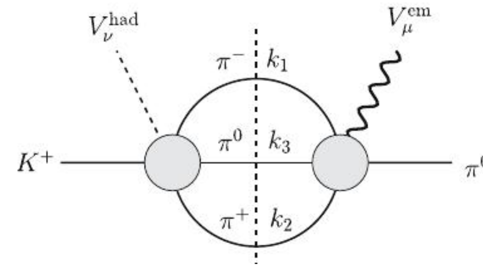
Important conclusions:

- **Strong interactions are suppressed for the T-odd asymmetry in $K_{l3\gamma}$**

[Muller et al., EPJC'06]

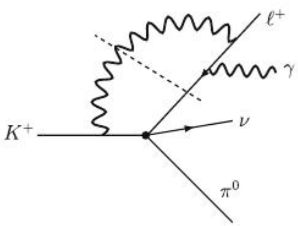






- **Photon loops play the dominant role in the T-odd asymmetry for $K_{l3\gamma}$**

[Braguta et al., PRD'02] [Rudenko, PRD'11]



- **Order of magnitudes of A_ξ for $K_{l3\gamma}$**

[Braguta et al., PRD'02] [Rudenko, PRD'11]

$$A_\xi(K^+ \to \pi e^+ \nu_e \gamma) = -0.59 \times 10^{-4} \qquad A_\xi(K^+ \to \pi \mu^+ \nu_\mu \gamma) = 1.14 \times 10^{-4}$$

TABLE II. A_ξ in $K^0 \to \pi^- l^+ \nu_l \gamma$ decays ($\omega \geq 30$ MeV and $\theta_{l\gamma} \geq 20°$).

	$l = \mu$	$l = e$
Group I ($l - \gamma$)	-0.54×10^{-4}	-1.32×10^{-4}
Group II ($\pi - \gamma$)	-3.6×10^{-4}	-3.2×10^{-4}
Group III ($\pi - l$)	1.73×10^{-3}	8.6×10^{-4}
Group IV ($\pi - l - \gamma$)	-1.41×10^{-3}	-8.6×10^{-4}
Total	-1×10^{-4}	-4.5×10^{-4}

How about A_ξ in $\tau \to \pi\pi\gamma\nu_\tau$?

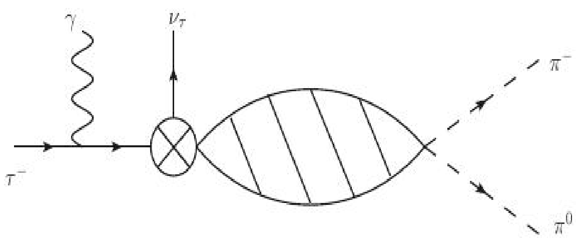
$$\tau^-(P) \to \pi^-(p_1)\pi^0(p_2)\nu_\tau(q)\gamma(k)$$

$$\mathcal{M} = e\,G_F\,V_{ud}^{*}\epsilon^{*\mu}(k)\Big\{F_\nu \bar{u}(q)\gamma^\nu(1-\gamma_5)(m_\tau + \not{P} - \not{k})\gamma_\mu u(P) + (V_{\mu\nu} - A_{\mu\nu})\bar{u}(q)\gamma^\nu(1-\gamma_5)u(P)\Big\}$$

$$\frac{1}{2}\sum_{spins}|\mathcal{M}|^2 = \hat{M}_0 + \xi \hat{M}_1$$

$$N^{-1}\hat{M}_1 = \sum_{f,f'}\tilde{C}_{ff'}\,\mathrm{Im}(f^* f') + \sum_{f,I}\tilde{C}_{f,I}\,\mathrm{Im}(f^* I) + \sum_{I,I'}\tilde{C}_{II'}\mathrm{Im}(I^* I')$$

***f / f'*: two-pion vector form factors F_V(t) or F_V(u)**

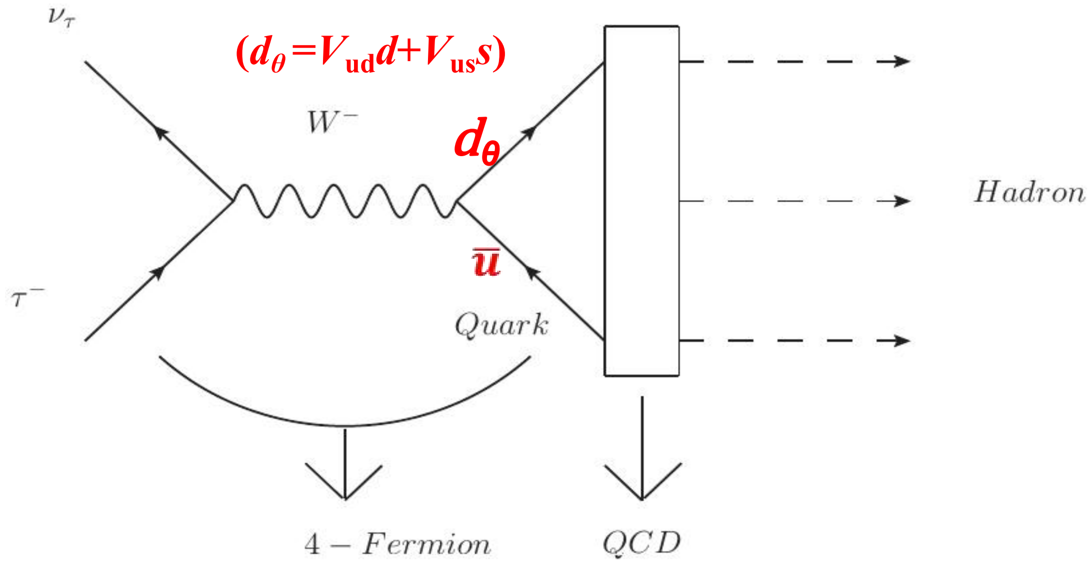


***I / I'*: form factors of V_i or A_i entering in $V_{\mu\nu}$ and $A_{\mu\nu}$**

The most important lesson:

Hadronic interactions are NOT suppressed !

Resonance chiral theory & $\tau \to \pi\pi\gamma\nu_\tau$ decay



Hadronic V-A currents

$$\mathbf{H}_\mu = \langle H^- | \bar{u}\gamma_\mu(1-\gamma_5) d_\theta \, e^{iL_{QCD}} | 0 \rangle$$

Chiral EFT is the low energy realization of QCD:

$$e^{i\mathcal{Z}(v_\mu, a_\mu, s, p)} = \int \mathcal{D}q\mathcal{D}\bar{q}\mathcal{D}G_\mu \; e^{i\int d^4x \mathcal{L}_{QCD}(v_\mu, a_\mu, s, p)} = \int \mathcal{D}u \; e^{i\int d^4x \mathcal{L}_{EFT}(v_\mu, a_\mu, s, p)}$$

$$\mathcal{L}^{QCD} = \mathcal{L}_0^{QCD} + \bar{q}\gamma^\mu(v_\mu + a_\mu\gamma_5)q - \bar{q}(s - i\gamma_5 p)q$$

v_μ, a_μ, s, p are the external source fields .

Leading order [Weinberg, '79]

$$\mathcal{L}_2 = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle$$

Higher orders [Gasser,Leutwyler,'83 '84] [Bijnens et al., '99]

$\mathcal{O}(p^4)$:

$$\mathcal{L}_4^{\chi PT} = L_1\langle u_\mu u^\mu\rangle^2 + L_2\langle u_\mu u^\nu\rangle\langle u^\mu u_\nu\rangle + L_3\langle u_\mu u^\mu u_\nu u^\nu\rangle + L_4\langle u_\mu u^\mu\rangle\langle\chi_+\rangle$$
$$+L_5\langle u_\mu u^\mu \chi_+\rangle + L_6\langle\chi_+\rangle^2 + L_7\langle\chi_-\rangle^2 + \frac{L_8}{2}\langle\chi_+^2 + \chi_-^2\rangle + \cdots$$

$\mathcal{O}(p^6)$:

$$\mathcal{L}_6^{\chi PT} = C_1\langle u_\rho u^\rho h_{\mu\nu}h^{\mu\nu}\rangle + C_2\langle u_\beta u^\beta\rangle\langle h_{\mu\nu}h^{\mu\nu}\rangle + C_3\langle h_{\mu\nu}u_\rho h^{\mu\nu}u^\rho\rangle$$
$$+\ldots 94 \text{ terms in total in SU(3) case}$$

Alternatively, one could explicitly introduce heavy dynamical degrees of freedom, i.e., resonances, in the chiral EFT.

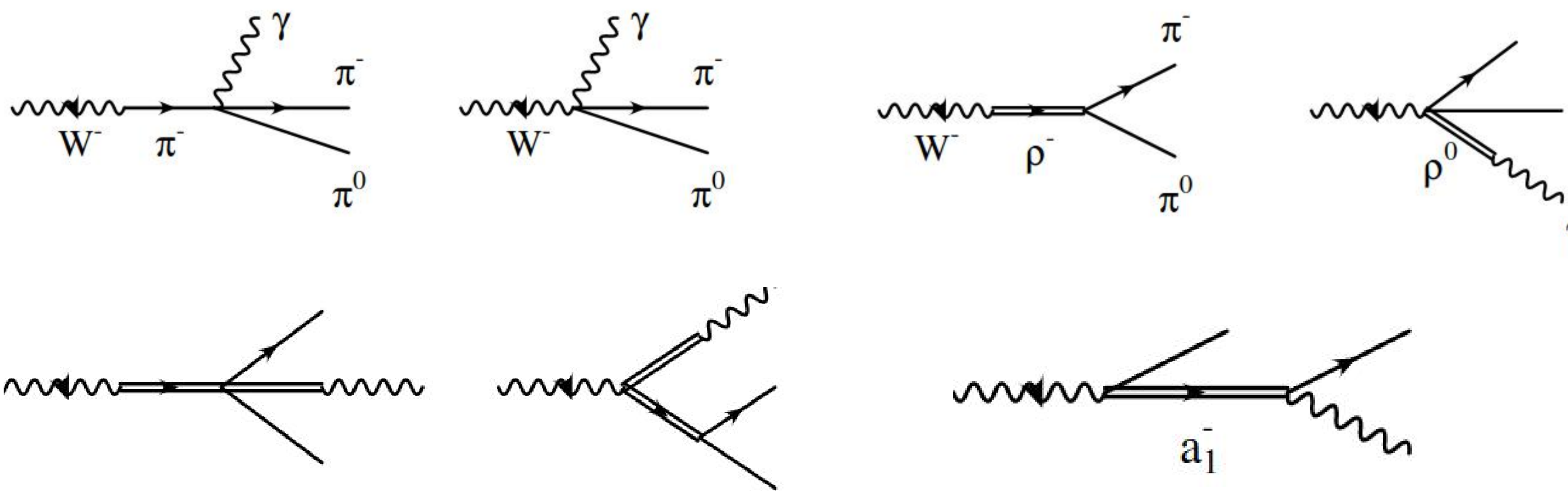
Minimal RχT Lagrangian [Ecker, *et al.*, '89]

$$\mathcal{L}_{kin}(V) = -\frac{1}{2} < \nabla^\lambda V_{\lambda\mu}\nabla_\nu V^{\nu\mu} - \frac{1}{2}V^{\mu\nu}V_{\mu\nu} >$$

$$\mathcal{L}_{2V} = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle \qquad \mathcal{L}_{2A} = \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu}f_-^{\mu\nu}\rangle$$

Minimal RChT contributions to $\tau \to \pi\pi\gamma\nu_\tau$

[Cirigliano et al., JHEP'02]



- **Other extentions by including anomalous vertices, such as the ρωπ types, and even-parity vertices of the $a_1\rho\pi$, are also studied.**
  [Flores-Tlalpa, et al., PRD'05] [Miranda, Roig, PRD'20]

- **Dedicated study of the isospin-breaing effect is considered to calibrate the tau data in the estimation of muon g-2.**

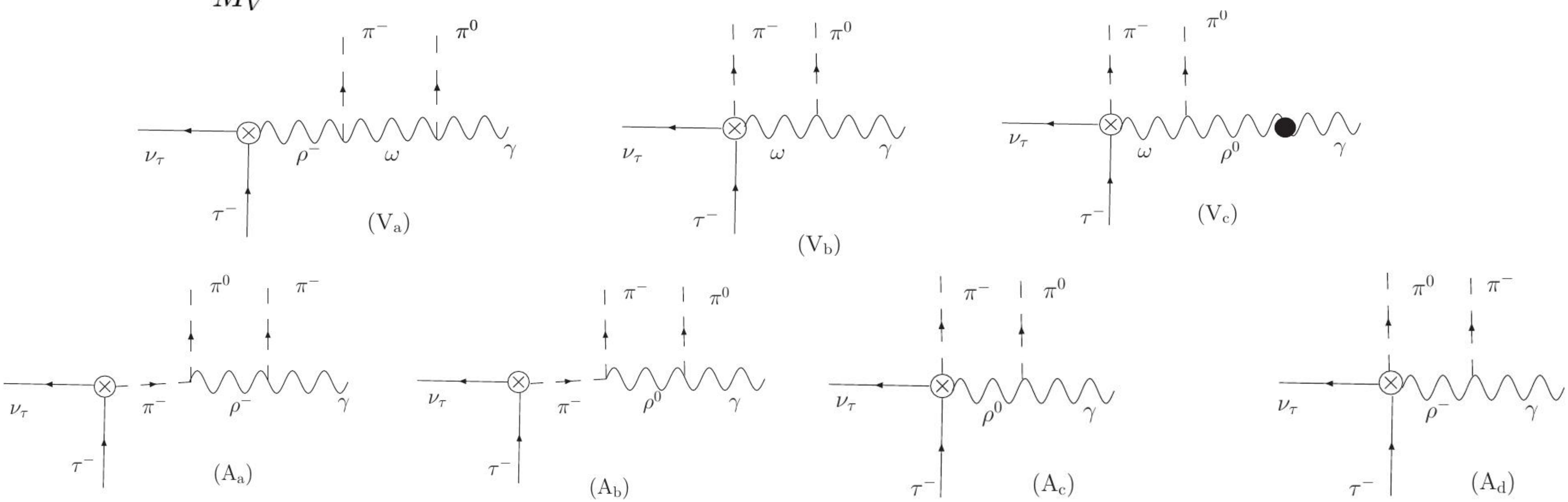
[Cirigliano,et al.,JHEP'02][Flores-Baez,et al.,PRD'06][Davier,et al.,EPJC'10][Miranda,Roig,PRD'20]

Contributions from VVP and VJP operators in RChT

[Ruiz-Femenia, Pich, Portoles, JHEP'03]

$$\begin{aligned}\mathcal{L}_{VVP} = \quad & d_1 \varepsilon_{\mu\nu\rho\sigma} \langle \{ V^{\mu\nu}, V^{\rho\alpha} \} \nabla_\alpha u^\sigma \rangle + i d_2 \varepsilon_{\mu\nu\rho\sigma} \langle \{ V^{\mu\nu}, V^{\rho\sigma} \} \chi_- \rangle \\ & + d_3 \varepsilon_{\mu\nu\rho\sigma} \langle \{ \nabla_\alpha V^{\mu\nu}, V^{\rho\alpha} \} u^\sigma \rangle + d_4 \varepsilon_{\mu\nu\rho\sigma} \langle \{ \nabla^\sigma V^{\mu\nu}, V^{\rho\alpha} \} u_\alpha \rangle\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{VJP} = \quad & \frac{c_1}{M_V} \varepsilon_{\mu\nu\rho\sigma} \langle \{ V^{\mu\nu}, f_+^{\rho\alpha} \} \nabla_\alpha u^\sigma \rangle + \frac{c_2}{M_V} \varepsilon_{\mu\nu\rho\sigma} \langle \{ V^{\mu\alpha}, f_+^{\rho\sigma} \} \nabla_\alpha u^\nu \rangle \\ & + \frac{i c_3}{M_V} \varepsilon_{\mu\nu\rho\sigma} \langle \{ V^{\mu\nu}, f_+^{\rho\sigma} \} \chi_- \rangle + \frac{i c_4}{M_V} \varepsilon_{\mu\nu\rho\sigma} \langle V^{\mu\nu} [f_-^{\rho\sigma}, \chi_+] \rangle \\ & + \frac{c_5}{M_V} \varepsilon_{\mu\nu\rho\sigma} \langle \{ \nabla_\alpha V^{\mu\nu}, f_+^{\rho\alpha} \} u^\sigma \rangle + \frac{c_6}{M_V} \varepsilon_{\mu\nu\rho\sigma} \langle \{ \nabla_\alpha V^{\mu\alpha}, f_+^{\rho\sigma} \} u^\nu \rangle \\ & + \frac{c_7}{M_V} \varepsilon_{\mu\nu\rho\sigma} \langle \{ \nabla^\sigma V^{\mu\nu}, f_+^{\rho\alpha} \} u_\alpha \rangle \, .\end{aligned}$$



[Chen, Duan, ZHG , JHEP'22]

High energy contraints to the resonance couplings

$$\int d^4x \int d^4y e^{i(p\cdot x+q\cdot y)} \langle 0|T[V_\mu^a(x)V_\nu^b(y)P^c(0)]|0\rangle$$
$$= d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \Pi_{\mathrm{VVP}}(p^2,q^2,r^2),$$

$$\lim_{\lambda\to\infty}\Pi^{(8)}_{\mathrm{VVP}}[(\lambda p)^2,(\lambda q)^2,(\lambda p+\lambda q)^2]$$
$$=\lim_{\lambda\to\infty}\Pi^{(0)}_{\mathrm{VVP}}[(\lambda p)^2,(\lambda q)^2,(\lambda p+\lambda q)^2]$$
$$=-\frac{\langle\bar\psi\psi\rangle_0}{2\lambda^4}\frac{p^2+q^2+r^2}{p^2q^2r^2}[1+\mathcal{O}(\alpha_S)]+O\left(\frac{1}{\lambda^6}\right).$$

$$c_1+4c_3=0 \qquad c_1-c_2+c_5=0 \qquad c_5-c_6=\frac{N_C M_V}{64\sqrt{2}\pi^2 F_V}$$

$$d_1+8d_2=-\frac{N_c M_V^2}{(8\pi F_V)^2}+\frac{F^2}{4F_V^2} \qquad d_3=-\frac{N_c M_V^2}{(8\pi F_V)^2}+\frac{F^2}{8F_V^2}$$

Other constraints from scattering and form factors

$$F_A=F_\pi,\quad F_V=\sqrt{2}F_\pi,\quad G_V=F_\pi/\sqrt{2}.$$

Or

$$F_A=\sqrt{2}F_\pi,\quad F_V=\sqrt{3}F_\pi,\quad G_V=F_\pi/\sqrt{3}$$

On-shell approxiamtion to the $J\omega\pi$ vertex and additional input from the $\omega \to \pi^0\pi^0\gamma$ decay width

$$T_{\omega\to\pi^0\pi^0\gamma} = \frac{2}{F}\Big\{ d_1(\epsilon_{\lambda\delta\mu\sigma}p_{1\nu}p_1^\sigma + \epsilon_{\mu\nu\lambda\sigma}p_{1\delta}p_1^\sigma) + 4d_2 m_\pi^2 \epsilon_{\mu\nu\lambda\delta}$$

$$+d_3[\,\epsilon_{\lambda\delta\mu\sigma}(k+p_2)_\nu p_1^\sigma - \epsilon_{\mu\nu\lambda\sigma}q_\delta p_1^\sigma\,] + d_4[\,\epsilon_{\lambda\delta\mu\sigma}(k+p_2)^\sigma p_1^\nu - \epsilon_{\mu\nu\lambda\sigma}q^\sigma p_{1\delta}]\Big\}$$

$$D^{\lambda\delta,\beta\theta}(k+p_2, M_V^2) g_{\rm v}\epsilon_{\beta\theta\rho\alpha}k^\rho \varepsilon_\gamma^\alpha(k)\,\frac{q^\mu\varepsilon_\omega^\nu(q) - q^\nu\varepsilon_\omega^\mu(q)}{M_\omega} \quad + \Big(p_1 \leftrightarrow p_2\Big)$$

$$\Gamma^{Exp}_{\omega\to\pi^0\pi^0\gamma} = (5.8 \pm 1.0)\times 10^{-5}\ \mathrm{MeV}$$

$$\Longrightarrow \quad \begin{aligned} d_4 &= -0.12 \pm 0.05\,, \\ d_4 &= 0.82 \pm 0.05\,. \end{aligned}$$

Important: we are left with a parameter free theoretical amplitude for the $\tau \to \pi\pi\gamma\nu_\tau$ process !

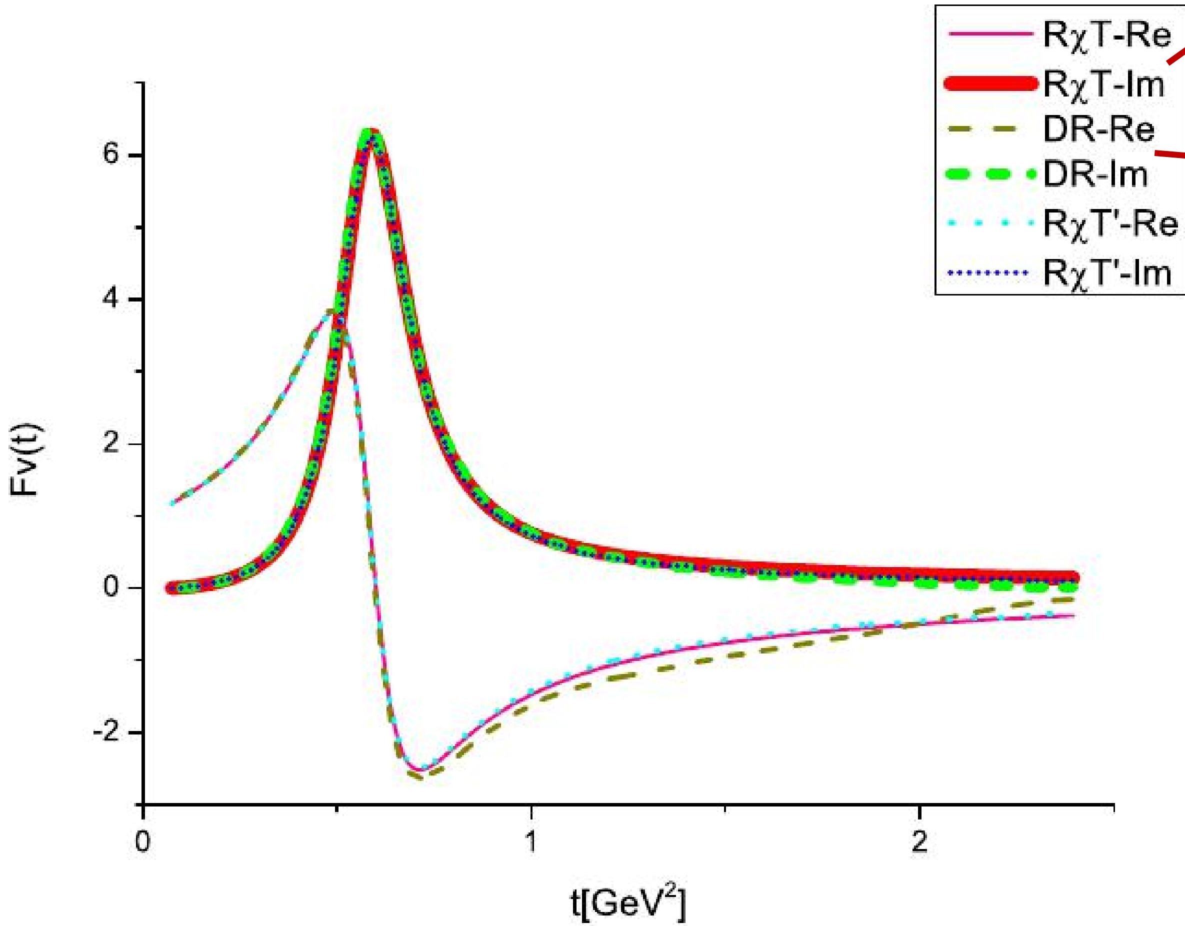
Phenomenological discussions

[Chen, Duan, ZHG, JHEP'22]

Inputs of the pion vector form factors

$$F_V(t) = M_\rho^2 D_\rho \exp\left\{\frac{-t}{96\pi^2 F_\pi^2}\mathcal{R}e\left[B(t,m_\pi^2) + \frac{1}{2}B(t,m_K^2)\right]\right\}$$

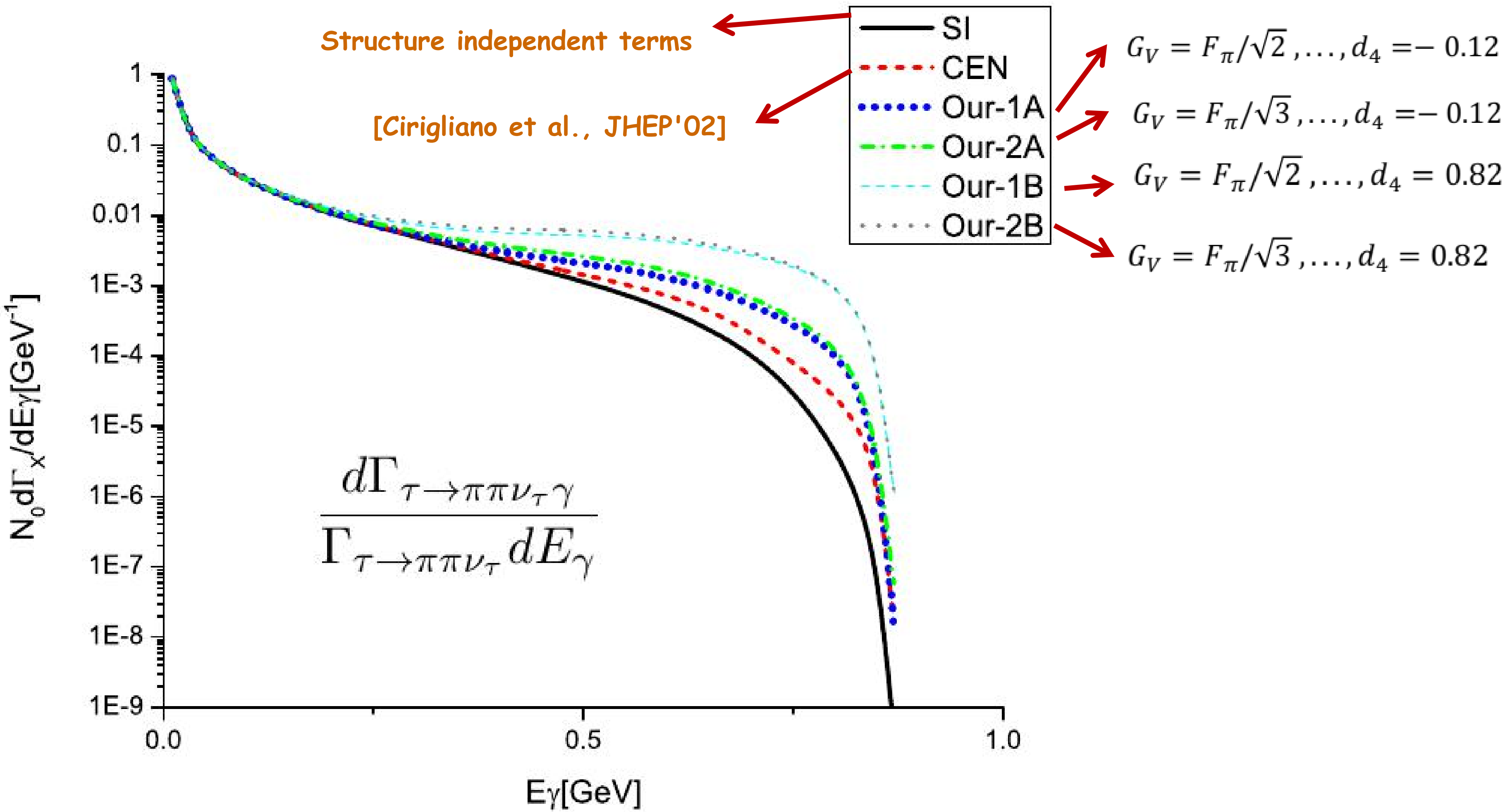
$$B(t,m_P^2) = \ln\left(\frac{m_P^2}{\mu^2}\right) + \frac{8m_P^2}{t} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P+1}{\sigma_P-1}\right)$$



[Guerrero,Pich,PLB'97]

[Dumm,Roig,EPJC'13]

Differential decay widths as a function of photon energies



When the photon energy cutoff is around 300 MeV, the absolute branching ratio is predicted to be around 10^{-4}.

Variant predictions for the branching ratios ($\times 10^{-4}$):

E_γ^{cut}	SI	CEN	Our-1A	Our-2A	Our-1B	Our-2B
100MeV	7.9	8.3	8.7/9.6/8.6/9.4	9.5/10/9.2/9.7	13/9.6/12/9.4	14/10/13/9.7
300MeV	1.5	1.8	2.4/3.0/2.3/2.8	2.9/3.3/2.6/3.0	5.6/3.0/5.2/2.8	6.3/3.3/5.5/3.0
500MeV	0.26	0.40	0.73/1.0/0.68/0.90	0.93/1.1/0.81/0.91	2.6/1.0/2.4/0.90	2.9/1.1/2.4/0.91

$\omega(\surd), a_1(\surd)$

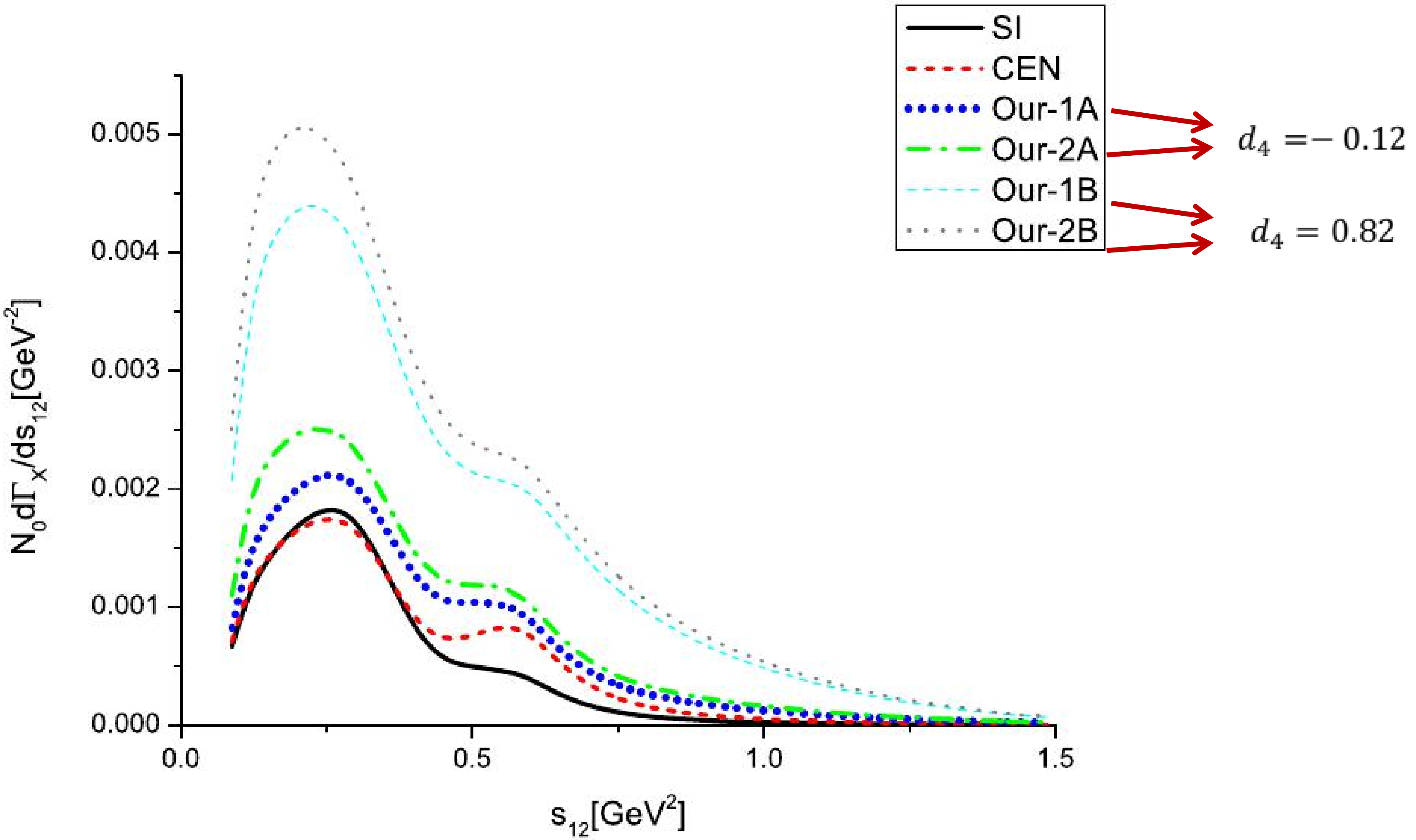
$\omega(\times), a_1(\surd)$

$\omega(\surd), a_1(\times)$

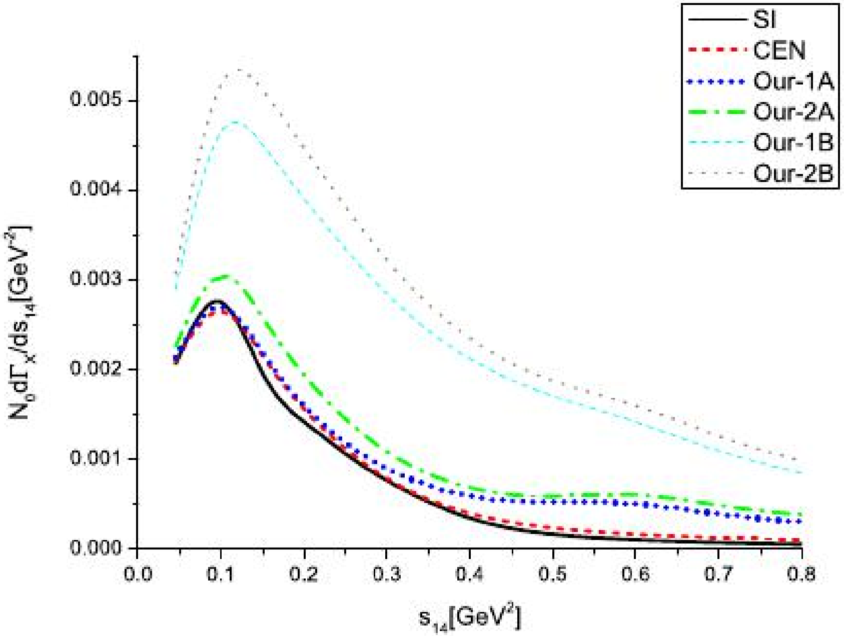
$\omega(\times), a_1(\times)$

- **45 billion pairs of tau lepton are expected at Belle-II. Therefore $\tau \rightarrow \pi\pi\gamma\nu_\tau$ has good chance to be well measured with reasonable photon energy cuts.**

Invariant-mass distributions of the ππ system

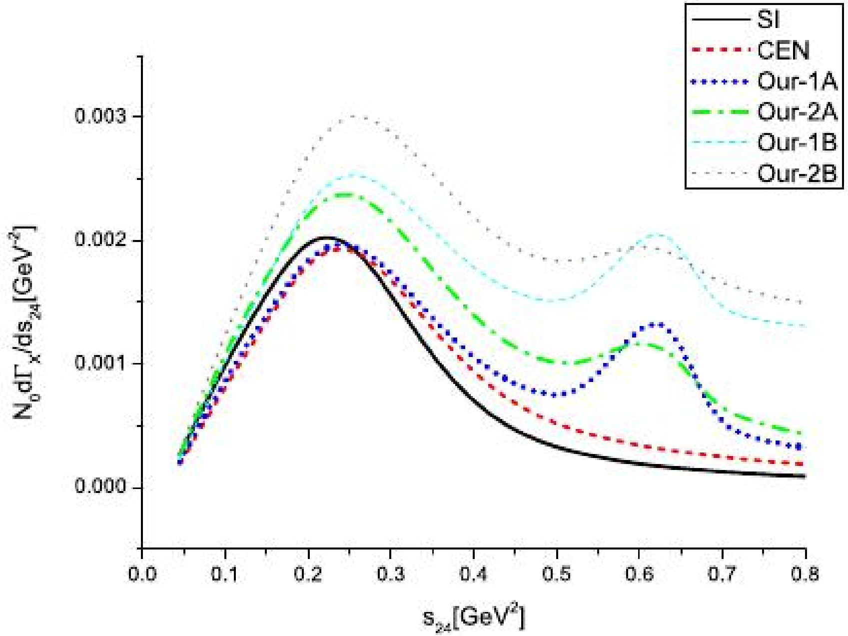


Invariant-mass distributions of the $\pi^- \gamma$ and $\pi^0 \gamma$ systems



$\pi^- \gamma$

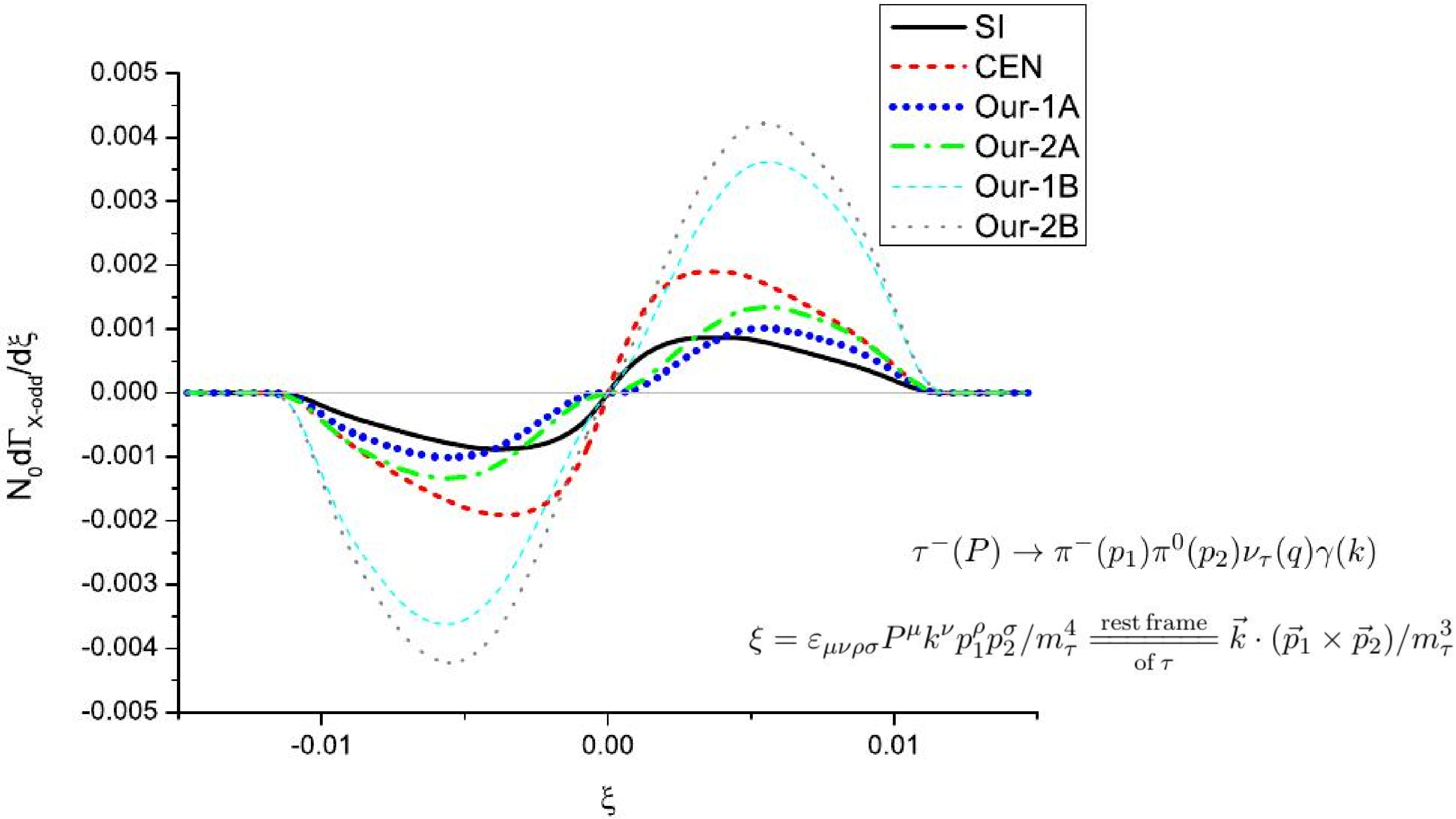
(ρ)



$\pi^0 \gamma$

(ω)

Predicitons of the T-odd asymmetry distributions with respect to ξ



Predictions to the rates of the T-odd asymmetries

(numbers are multiplied by 10^{-2})

E_γ^{cut}	A_ξ(Our-1A)	A_ξ(Our-2A)	A_ξ(Our-1B)	A_ξ(Our-2B)
100 MeV	1.2/1.7/1.0/1.6	1.3/1.8/0.98/1.4	1.6/1.7/1.4/1.6	1.7/1.8/1.3/1.4
300 MeV	1.5/2.6/1.0/2.2	1.6/2.5/0.73/1.6	2.3/2.6/2.0/2.2	2.4/2.5/1.7/1.6
500 MeV	0.98/1.4/0.58/0.88	0.91/1.4/0.68/0.43	2.1/1.4/1.8/8.8	2.1/1.4/1.5/4.3

- **The magnitudes of A_ξ for $\tau \to \pi\pi\gamma\nu_\tau$ are around two orders larger than those in $K_{l3\gamma}$. It has the good chance to be measured in Belle-II and super tau-charm facilities.**

Prospects of revealing the genuine CPV signals

- **Both the CP-conserving and CPV interactions can contribute to A_ξ for $\tau \to \pi\pi\gamma\nu_\tau$**

$$A_\xi = \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-} \qquad \Gamma_+ = \frac{(2\pi)^4}{2m_\tau}\int_{\xi>0} d\Phi\,(\hat{M}_0 + \xi\hat{M}_1)\,, \qquad \Gamma_- = \frac{(2\pi)^4}{2m_\tau}\int_{\xi<0} d\Phi\,(\hat{M}_0 + \xi\hat{M}_1)$$

- **CPV signals can be probed by taking the differences of A_ξ in $\tau^- \to \pi^-\pi^0\gamma\nu_\tau$ and $\tau^+ \to \pi^+\pi^0\gamma\nu_\tau$**

$$\overline{A}_{\overline{\xi}} = \frac{\overline{\Gamma}_+ - \overline{\Gamma}_-}{\overline{\Gamma}_+ + \overline{\Gamma}_-} \qquad \overline{\Gamma}_+ = \frac{(2\pi)^4}{2m_\tau}\int_{\overline{\xi}>0} d\Phi\,(\overline{\hat{M}_0} + \overline{\xi}\overline{\hat{M}_1})\,, \qquad \overline{\Gamma}_- = \frac{(2\pi)^4}{2m_\tau}\int_{\overline{\xi}<0} d\Phi\,(\overline{\hat{M}_0} + \overline{\xi}\overline{\hat{M}_1})$$

$$\mathcal{M} = e\,G_F\,V_{ud}^{*}\epsilon^{*\mu}(k)\,\Big\{(1+g_V)F_\nu\bar{u}(q)\gamma^\nu(1-\gamma_5)(m_\tau + \not{P} - \not{k})\gamma_\mu u(P) + \big[(1+g_V)V_{\mu\nu} - (1-g_A)A_{\mu\nu}\big]\bar{u}(q)\gamma^\nu(1-\gamma_5)u(P)\Big\}$$

$$\mathcal{A}_\xi = A_\xi - \overline{A}_{\overline{\xi}} \supset \mathrm{Im}(g_V^* g_A)\mathrm{Re}[F_V(t/u)^* A_i],\ \mathrm{Im}(g_V^* g_A)\mathrm{Re}(V_j^* A_i)$$

- **Generally speaking, sizable hadronic contributions are also expected to enhance the CPV signals.**

Summary

- **Rich phenomenologies in $\tau^- \to \pi^-\pi^0\gamma\nu_\tau$: photon spectrum (useful inputs for the estimation of muon g-2 from tau data), different resonance interactions in the $\pi^-\pi^0$, $\pi^-\gamma$, $\pi^0\gamma$ spectra.**

- **We give a promising prediction of the triple-product T-odd asymmetry in tau decay. It could provide a useful guide for future experimental measurements, especially in Belle-II and super-charm facilities.**

- **CPV signals in the T-odd asymmetries are also expected to be enhanced due to the large hadron contributions, compared to the situation in $K_{l3\gamma}$.**

Thanks for your patience!