

The pion and its Importance in Nuclear Physics

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The pion is important in nuclear physics!!

- Yukawa predicted the pion as the mediator of nuclear force to form a nucleus (1934)
- Mayer-Jansen introduced the shell model (1949)--phenomenological
- Nambu found the origin of mass and the pion due to spontaneous breaking of chiral symmetry (1960)

Content

- QCD Lagrangian and chiral symmetry
- Linear sigma model and non-linear sigma model
- Relativistic chiral mean field model
- Conclusion and perspective

QCD Lagrangian and chiral symmetry

Quark and gluon confinement

(difficult--monopole condensation??)

Chiral symmetry breaking (u and d quark sector)

$$m_u \approx m_d \approx 5 MeV \ll 1 GeV$$

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

Y. NAMBU AND G. JONA-LASINIO†

The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois

(Received October 27, 1960)

Nobel prize (2008)



He was motivated by the BCS theory (1958).

$$E_i = (\epsilon_i^2 + \Delta^2)^{1/2} \quad \longleftrightarrow \quad E_p = (p^2 + m^2)^{1/2}$$

$$\epsilon_i \psi_i + \Delta \psi_{\bar{i}}^* = E_i \psi_i$$

$$-\epsilon_i \psi_{\bar{i}}^* + \Delta^* \psi_i = E_i \psi_{\bar{i}}^*$$

$$\vec{\sigma} \cdot \vec{p} \psi_R + m \psi_L = E_p \psi_R$$

$$-\vec{\sigma} \cdot \vec{p} \psi_L + m \psi_R = E_p \psi_L$$

Δ is the order parameter

Particle number

m is the order parameter

Chiral symmetry

Nambu-Jona-Lasinio Lagrangian(phenomenological)

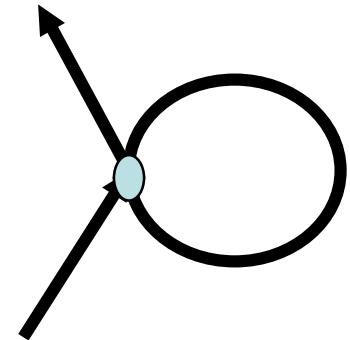
$$L_{NJL} = i\bar{\psi}\gamma_\mu\partial^\mu\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2]$$

$$L_{Dirac} = i\bar{\psi}\gamma_\mu\partial^\mu\psi + m\bar{\psi}\psi$$

Mean field approximation; Hartree approximation

$$[i\gamma_\mu\partial^\mu + 2G\langle\bar{\psi}\psi\rangle]\psi(x) = 0$$

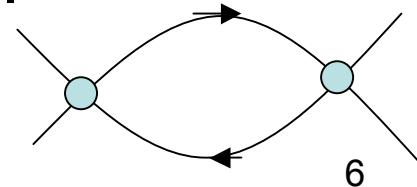
$$m = -2G\langle\bar{\psi}\psi\rangle \quad \text{Fermion gets mass.}$$



The chiral symmetry is spontaneously broken.

Pion appears as a Nambu-Goldstone boson.

RPA treats $(\bar{\psi}i\gamma_5\psi)^2$ → Pion
10.10.14 toki@beihang



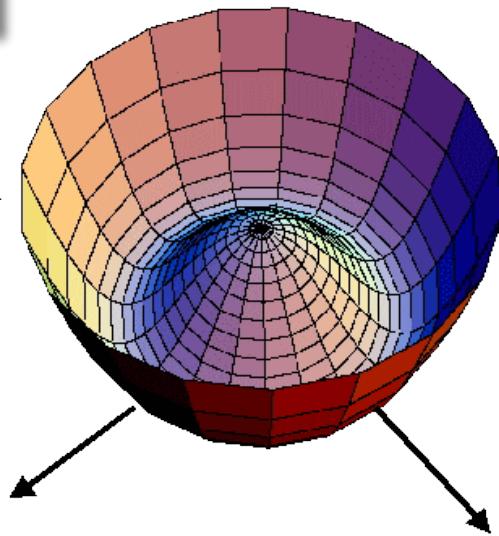
Chiral linear sigma model

Y. Ogawa et al. PTP (2004)

Pion is the Nambu boson of chiral symmetry

Linear Sigma Model Lagrangian

$$\begin{aligned}\mathcal{L}_{\sigma\omega} = & \bar{\Psi}(i\gamma_\mu\partial^\mu - g_\sigma(\sigma + i\gamma_5\boldsymbol{\tau} \cdot \boldsymbol{\pi}) - g_\omega\gamma_\mu\omega^\mu)\Psi \\ & + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} - \frac{\mu^2}{2}(\sigma^2 + \boldsymbol{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \boldsymbol{\pi}^2)^2 \\ & - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}\tilde{g}_\omega^2(\sigma^2 + \boldsymbol{\pi}^2)\omega_\mu\omega^\mu \\ & + \mathcal{L}_b, \quad \mathcal{L}_b = \varepsilon\sigma, \quad \text{where } \tilde{g}_\omega = \eta g_\omega.\end{aligned}$$
$$\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



Polar coordinate

$$\sigma + i\boldsymbol{\tau} \cdot \boldsymbol{\pi} = \rho U,$$

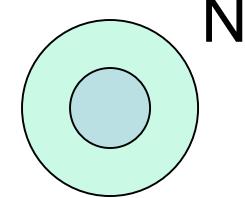
$$U = e^{i\boldsymbol{\tau} \cdot \boldsymbol{\pi}}$$

$$N = \sqrt{U_5}\Psi$$

$$\sigma + i\gamma_5\boldsymbol{\tau} \cdot \boldsymbol{\pi} = \rho U_5,$$

$$U_5 = e^{i\gamma_5\boldsymbol{\tau} \cdot \boldsymbol{\pi}}$$

Non-linear sigma model



Lagrangian $\rho = f_\pi + \varphi$

$$\begin{aligned}
 \mathcal{L}'_{\sigma\omega} = & \bar{N}(i\gamma_\mu\partial^\mu - M - g_\sigma\varphi - \frac{1}{2f_\pi}\gamma_5\gamma_\mu\boldsymbol{\tau}\cdot\partial^\mu\boldsymbol{\pi} - g_\omega\gamma_\mu\omega^\mu)N \\
 & + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m_\sigma^2\varphi^2 - \lambda f_\pi\varphi^3 - \frac{\lambda}{4}\varphi^4 \\
 & + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} - \frac{1}{2}m_\pi^2\boldsymbol{\pi}^2 \\
 & - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \tilde{g}_\omega^2 f_\pi\varphi\omega_\mu\omega^\mu + \frac{1}{2}\tilde{g}_\omega^2\varphi^2\omega_\mu\omega^\mu
 \end{aligned}$$

where

$$M = g_\sigma f_\pi$$

$$M^* = M + g_\sigma \varphi$$

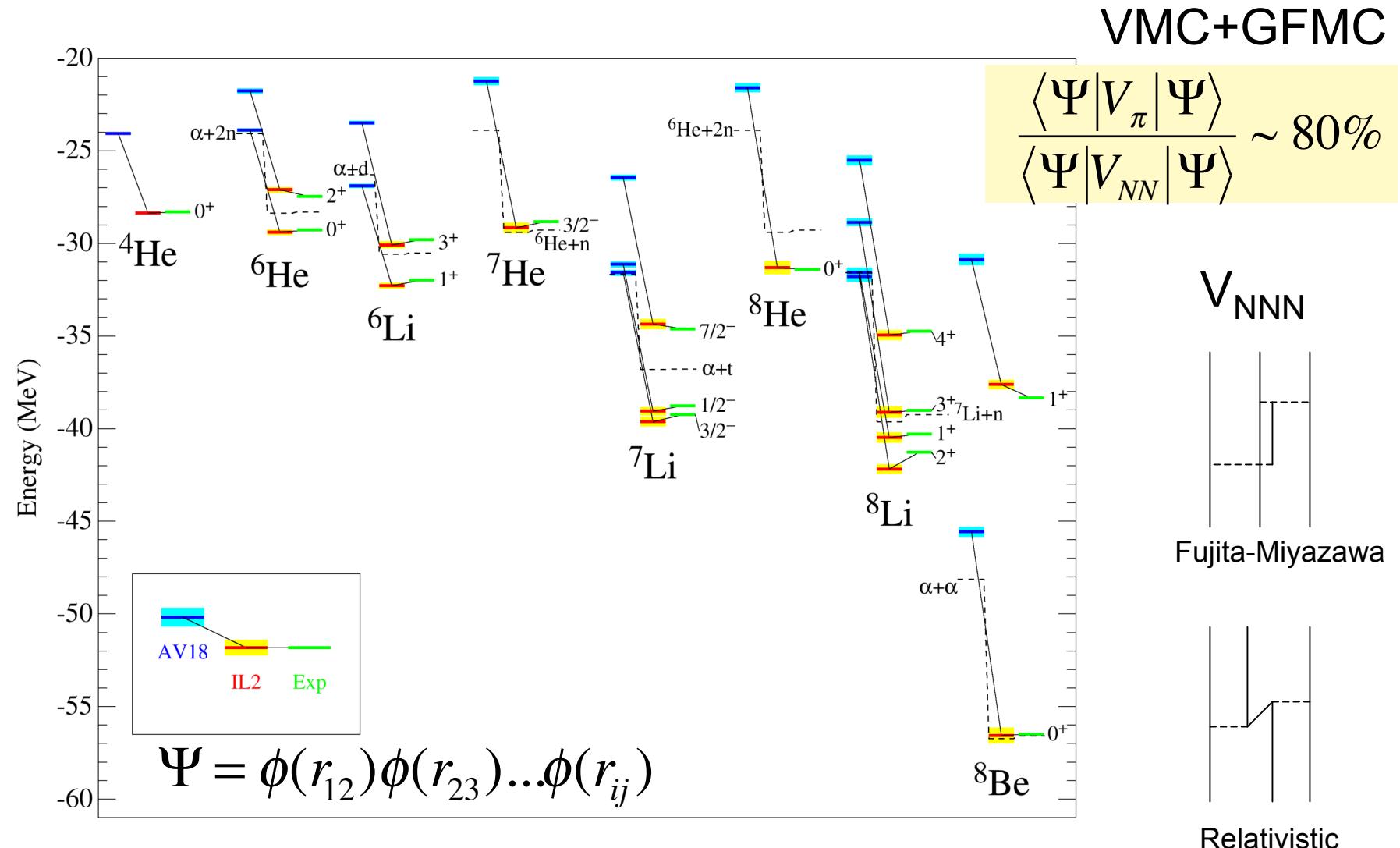
$$m_\pi^2 = \mu^2 + \lambda f_\pi$$

$$m_\sigma^2 = \mu^2 + 3\lambda f_\pi$$

$$m_\omega = \tilde{g}_\omega f_\pi$$

$$m_\omega^* = m_\omega + g_\omega \varphi$$

核力を使った少数多体系計算法による変分計算



C. Pieper and R. B. Wiringa, Annu. Rev. Nucl. Part. Sci. 51(2001)

重い核を計算したい(スーパー・モデル)

パイオノの扱いが鍵

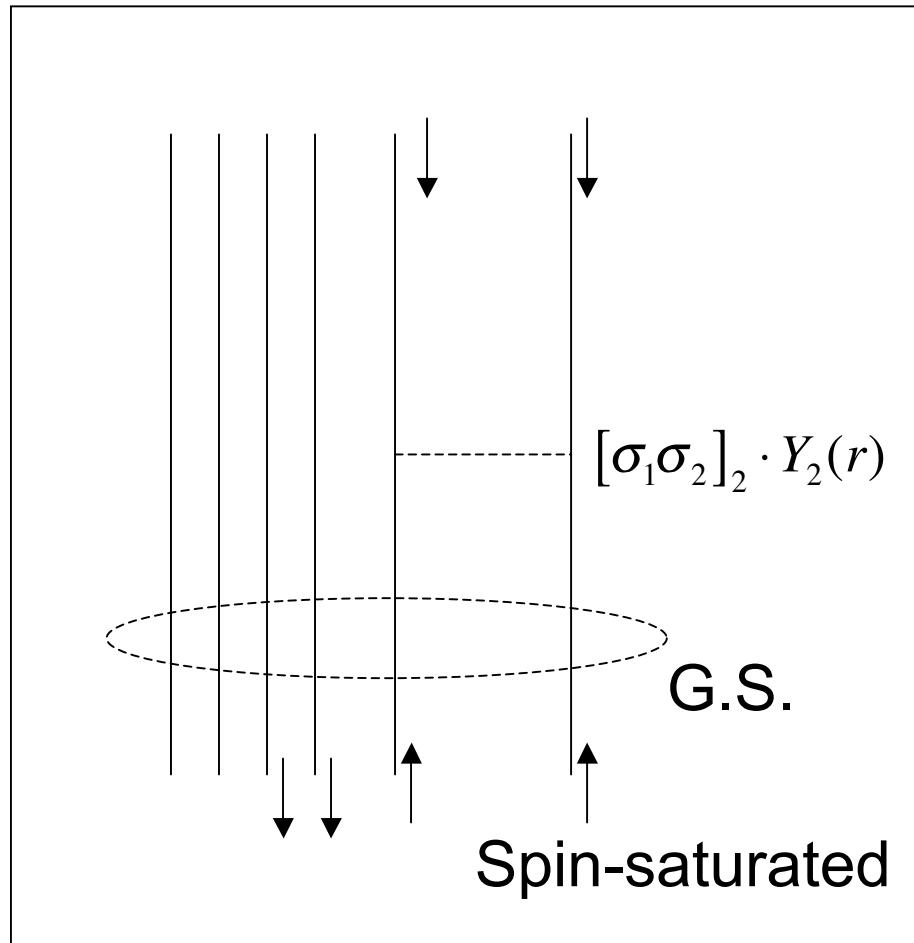
The pion plays an important role for formation of nucleus

- 80% of attractive interaction originates from the pion
- In particular the tensor interaction contributes about 50% of attraction.

$$\vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q} = \frac{1}{3} q^2 S_{12}(\hat{q}) + \frac{1}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 q^2 \quad S_{12}(\hat{q}) = \sqrt{24\pi} [Y_2(\hat{q}) [\sigma_1 \sigma_2]_2]_0$$

Pion Tensor spin-spin

2p-2h excitation is essential to treat tensor interaction (TOSM) Myo, Ikeda, Toki

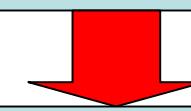


$$\left\langle 0 \left| \sum_{ij} \left[Y_2(r_{ij}) \cdot [\sigma_i \sigma_j]_2 \right]_2^0 \right| 0 \right\rangle = 0$$

$$\Psi = C_0 |0\rangle + \sum_i C_i |2p - 2h : i\rangle_T$$

$$\left\langle \Psi \left| \sum_{ij} \left[Y_2(r_{ij}) \cdot [\sigma_i \sigma_j]_2 \right]_2^0 \right| \Psi \right\rangle \neq 0$$

Tensor operator is
the order parameter



Tensor condensation

Relativistic Brueckner Hartree-Fock theory is successful

Relativistic chiral mean field model

Energy minimization with respect to
meson and nucleon fields

$$\delta \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} = 0 \quad \frac{\delta E}{\delta \sigma} = 0 \quad \frac{\delta E}{\delta \omega} = 0$$

(Mean field equation)

$$\Psi = C_0 |0\rangle + \sum_i C_i |2p - 2h : i\rangle_T$$

$$\frac{\delta E}{\delta \psi_i(x)} = 0 \quad \frac{\delta E}{\delta C_i} = 0$$

Numerical calculation is done self-consistently.
The HF equation contains the contribution of the tensor term.

Relativistic chiral mean field model Ogawa Toki (2010)

$$E = \langle 0 | H | 0 \rangle + \sum_{\alpha, \beta} C_{\alpha}^* C_{\beta} \langle 2p - 2h_{\alpha} | H | 2p - 2h_{\beta} \rangle'$$

全エネルギー
Total energy

$$- \sum_{\alpha} C_0^* C_{\alpha} N \hat{J} \langle [ij]_{JM} | V | [kl]_{JM} \rangle_A$$

$$- \sum_{\alpha} C_0 C_{\alpha}^* N \hat{J} \langle [kl]_{JM} | V | [ij]_{JM} \rangle_A$$

変分

Variation

$$\frac{\delta E}{\delta \psi_{im}^*(x)} - \varepsilon_{im} \frac{\delta \langle im | im \rangle}{\delta \psi_{im}^*(x)} \quad \langle 0 | V | 2p - 2h_{\alpha} \rangle = \hat{J} \langle [ij]_{JM} | V | [kl]_{JM} \rangle$$

$$= T \psi_{im}(x) + \sum_{jn} \int d^3 x' \psi_{jn}^*(x') V(x - x') [\psi_{im} \psi_{jn}]_A$$

$$+ \sum_{\alpha, \beta} C_{\alpha}^* C_{\beta} \frac{\delta}{\delta \psi_{im}^*(x)} \langle 2p - 2h_{\alpha} | H | 2p - 2h_{\beta} \rangle'$$

$$- \sum_{\alpha} C_0^* C_{\alpha} N \hat{J} \frac{\delta}{\delta \psi_{im}^*(x)} \langle [ij]_{JM} | V | [kl]_{JM} \rangle_A - \varepsilon_{im} \psi_{im}(x) = 0$$

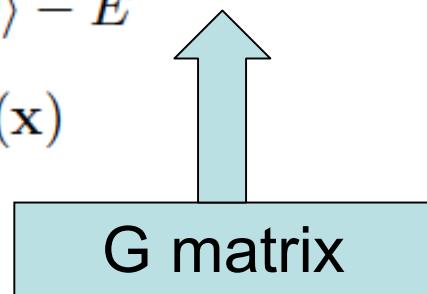
$$\sum_{\beta} C_{\beta} \langle 2p - 2h_{\alpha} | H | 2p - 2h_{\beta} \rangle' + C_{\alpha} \langle 0 | H | 0 \rangle$$

$$- C_0 N \hat{J} \langle [kl]_{JM} | V | [ji]_{JM} \rangle_A - E C_{\alpha} = 0$$

Brueckner theory is naturally expressed in the variational Method. --- beautiful theory

Approximate 2p-2h matrix element is diagonal.

$$\langle 2p - 2h_\alpha | H | 2p - 2h_\beta \rangle' \sim E_{2p-2h}^\alpha \delta_{\alpha\beta} = (\varepsilon_k + \varepsilon_l - \varepsilon_i - \varepsilon_j) \delta_{\alpha\beta}$$

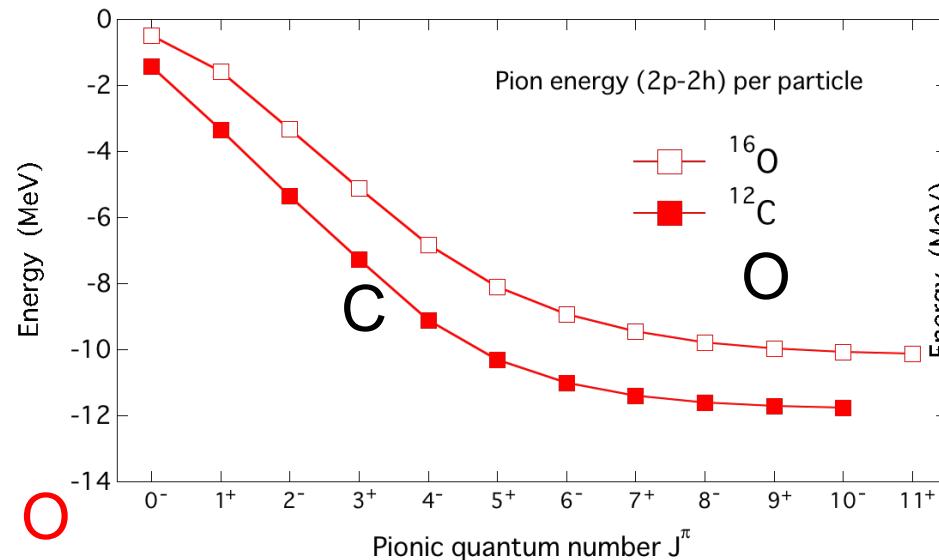
$$\begin{aligned}
 & T\psi_{im}(\mathbf{x}) + \sum_{jn} \int d^3x' \psi_{jn}^*(\mathbf{x}') V(\mathbf{x} - \mathbf{x}') [\psi_{im} \psi_{jn}]_A \\
 & - |C_0|^2 \sum_{\alpha} N^2 (\hat{J})^2 \frac{[\frac{\delta}{\delta \psi_{im}^*(\mathbf{x})} \langle [ij]_{JM} | V | [kl]_{JM} \rangle_A] \langle [kl]_{JM} | V | [ij]_{JM} \rangle_A}{E_{2p-2h}^\alpha + \langle 0 | H | 0 \rangle - E} \\
 & + \sum_{\alpha} |C_{\alpha}|^2 (\varepsilon_k + \varepsilon_l - \varepsilon_i - \varepsilon_j) \psi_{im}(\mathbf{x}) = \varepsilon_{im} \psi_{im}(\mathbf{x})
 \end{aligned}$$


G matrix is expressed in the variational method

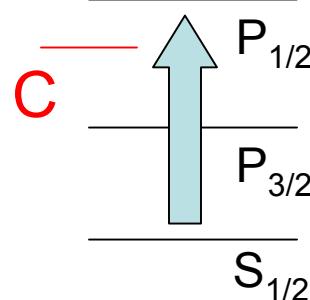
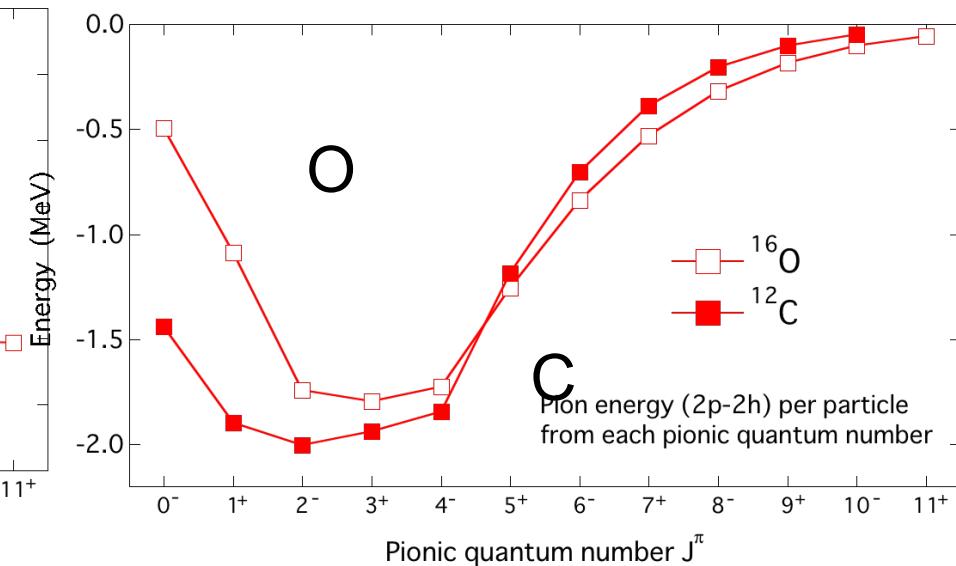
相対論的カイラル平均場近似

Ogawa-Toki

パイオンの寄与



個々の2p-2h励起の寄与



The difference between ^{12}C and ^{16}O is 2MeV/N.

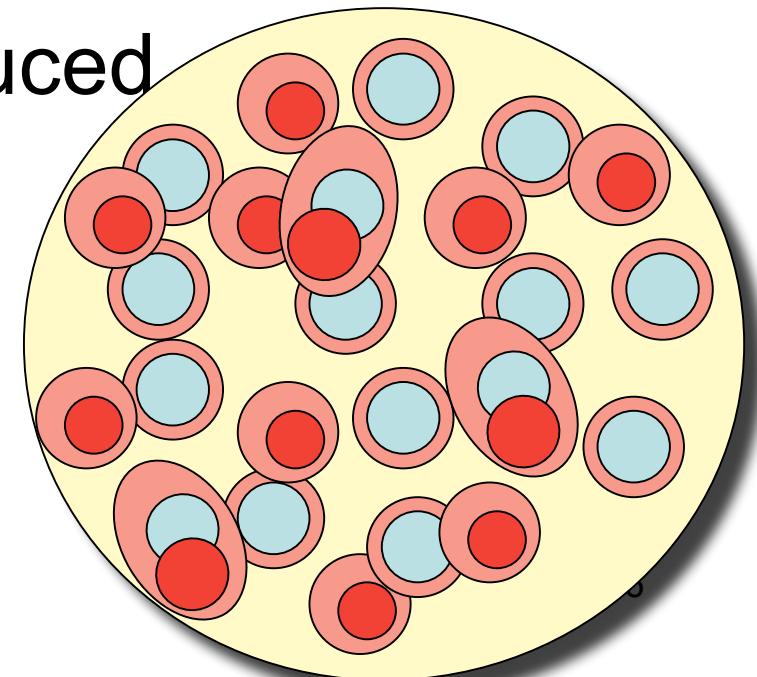
The difference comes from low pion spin states ($J < 2$). This is the Pauli blocking effect.

Pion energy

Pion provides larger attraction for ^{12}C than ^{16}O

Nuclear structure caused by pion

- 2p–2h excitation is 20%
- High momentum components
- Low momentum components
(Shell model) are reduced
by 20%



Conclusion and perspective

- We have a (plausible) path to relate QCD with nuclear physics.
- We can formulate the relativistic chiral mean field model for finite nuclei.
- The TOSM framework is essential for the treatment of pion in nuclear physics.
- We should calculate many nuclei and many observables.