Coexistence of cluster states and mean-field-type states

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Int. Inst. for Adv. Studies

1. Introduction
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1. Introduction

Coexistence of clustering dynamics and mean-field dynamics and their relation are discussed.

(1) Coexistence of cluster and mean-field-type states.
Actual features of coexistence in many nuclei are discussed.

(2) Mechanism of the coexistence and the structure change.

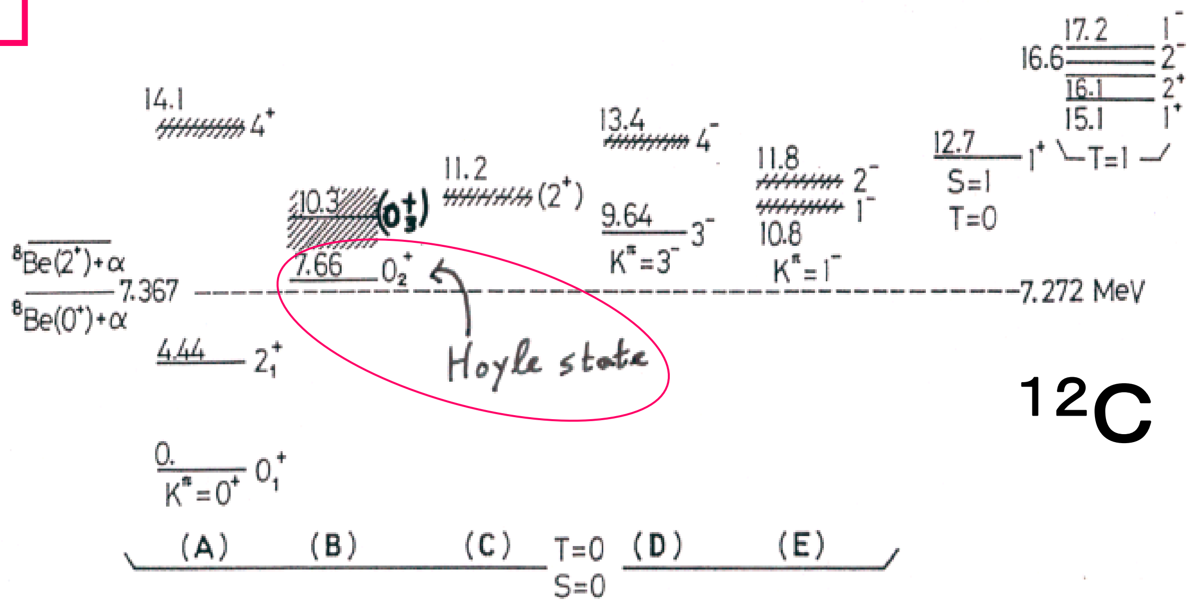
Duality of cluster wave function and shell-model wave function is the basis of the mechanism.
Coexistence of cluster and mean-field-type states is of logical necessity and hence inevitable.

------Janus nature of the nucleus

(3) Observations which verify the Janus nature.
Energy spectra of coexistence, Electric transitions

2. Actual features of coexistence of cluster and mean-field-type states.

^{12}C



M.Itoh et al.

M. Freer et al. ,

M. Gal et al.

10.3 MeV $(0^+)_3$

10.0 Mev $(0^+)_3$ Γ :2.7 MeV

9.9 MeV $(2^+)_2$ Γ:1.0 MeV

3 α calculations reproduce data well .

	Exp.	Theor.
Excitation energy (0_2^+) (MeV)	7.65	7.74
Width (0_2^+) (eV)	8.7±2.7	7.7
$M(0_2^+ \rightarrow 0_1^+)$ (fm^2)	5.4±0.2	6.7
B(E2: $0_2^+ \rightarrow 2_1^+$) (e^2fm^4)	13±4	5.6
B(E2: $2_1^+ \rightarrow 0_1^+$) (e^2fm^4)	7.8	9.3
$R_{rms}(0_1^+)$ (fm)	2.43	2.4
$R_{rms}(0_2^+)$ (fm)		3.47

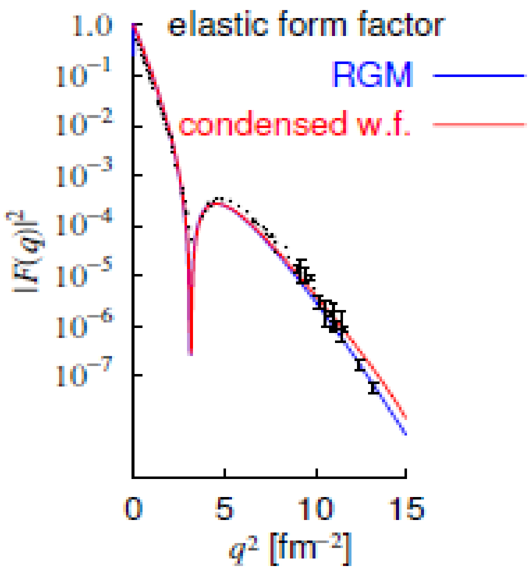
M. Kamimura et al.
Nucl. Phys. A351 (1981), 456.

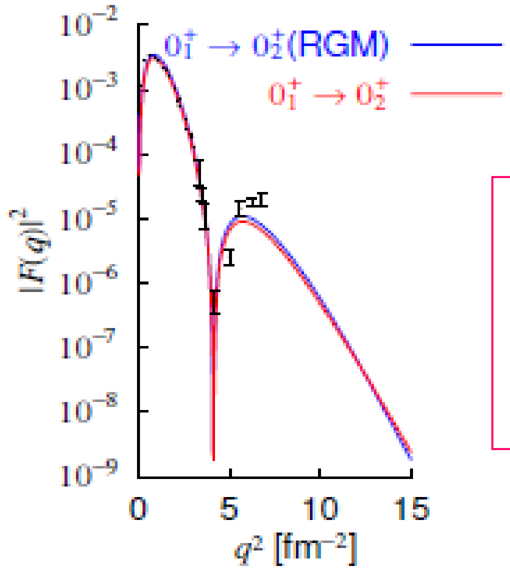
Uegaki et al.
Prog. Theor.Phys. Supple. 68 (1980).Chpt.2.

H. Horiuchi
Prog. Theor.Phys. 51 (1974), 1266

$(2.4 / 3.47)^3 = 0.33$

$$\rho(0_2^+) = 0.33\ \rho(0_1^+)$$





3 α wave function

$$\mathcal{A}\left[\chi(s,t)\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\right]$$

Hoyle state = ‘’ α -condensate-like state’ ’

$$|\langle \Phi^{3\alpha \mathrm{THSR}}(B(0_2^+))|\Psi_{3\alpha \mathrm{RGM}}(0_2^+)\rangle|^2 = 0.97$$

Y.Funaki, et al.,
Phys.Rev.C67 (2003), 051306(R)

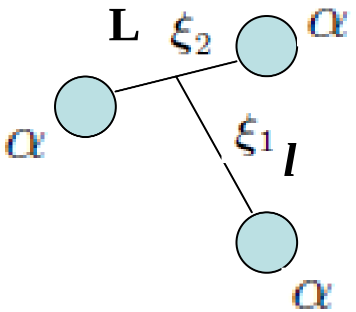
$$\Phi^{3\alpha\mathrm{THSR}}(B) = \mathscr{A}\{\exp[-\frac{2}{B^2}(X_1^2+X_2^2+X_3^2)]\ \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\}$$

$$= \exp(-\frac{6}{B^2}\xi_3^2)\mathscr{A}\{\underline{\exp(-\frac{4}{3B^2}\xi_1^2-\frac{1}{B^2}\xi_2^2)}\ \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\}$$

[L=0 × *l*=0]

$$\xi_3 = (X_1+X_2+X_3)/3$$

Dominance of
[L=0 × *l*=0] in 0_2^+

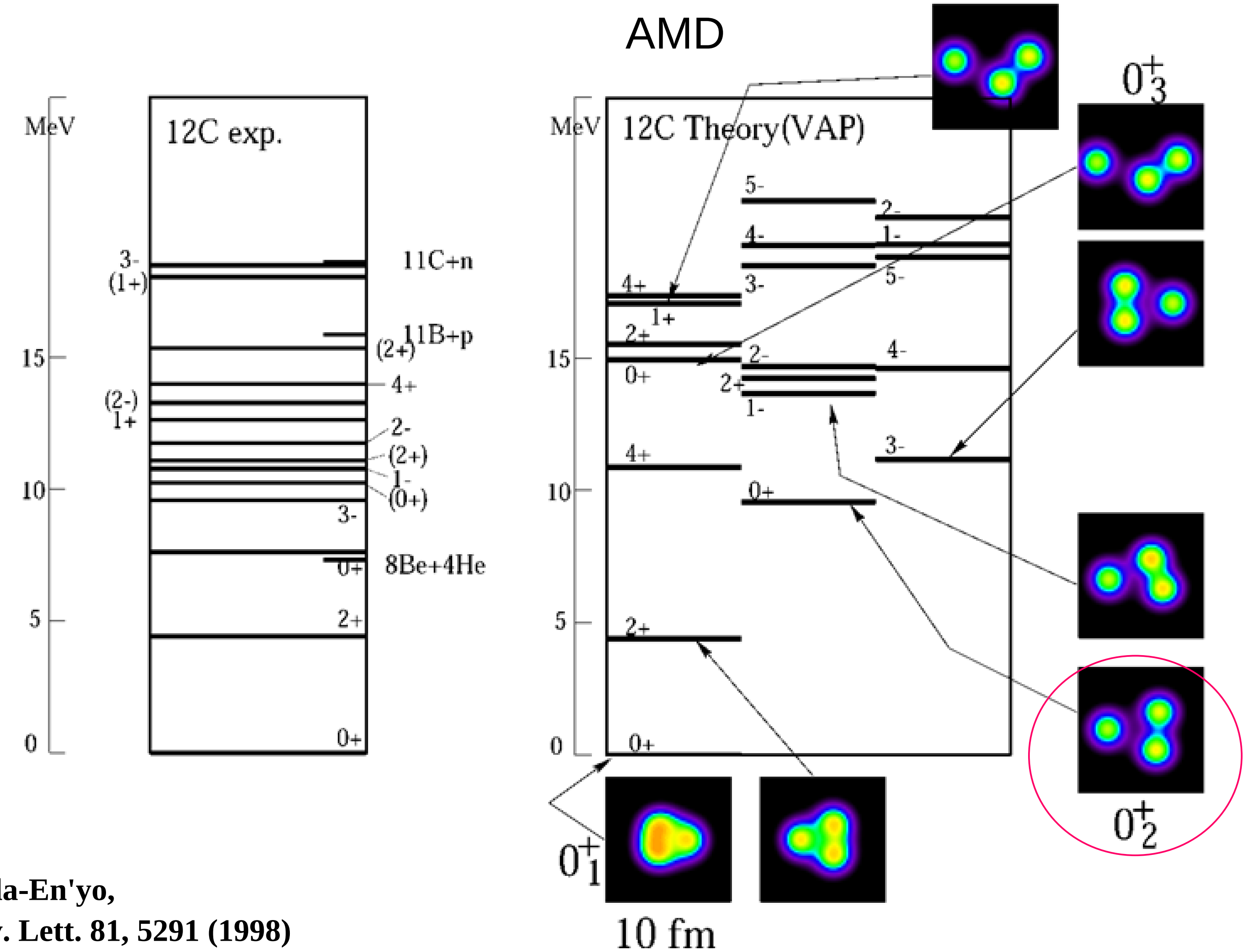


Three α clusters occupy the same S-orbit
$\exp(-(2/B^2)X^2)$ **(Bose-condensation-like)**

A.Tohsaki, H. Horiuchi, P. Schuck, and G. Roepke,
Phys. Rev. Lett. 87 (2001), 192501.

12C

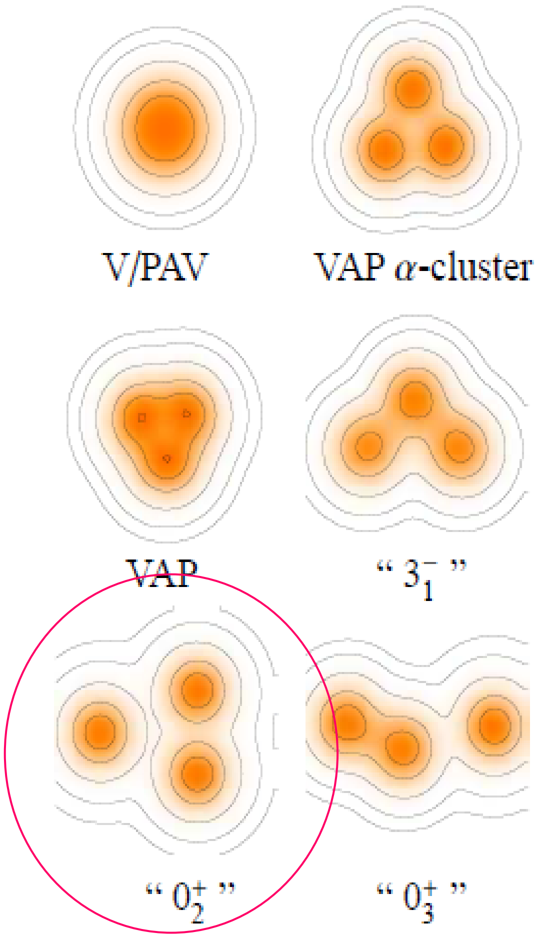
12 nucleon calculation without assuming alpha clustering



Y. Kanada-En'yo,
Phys. Rev. Lett. 81, 5291 (1998)

12C FMD

12 nucleon calculation without assuming alpha clustering and by using realistic nuclear force



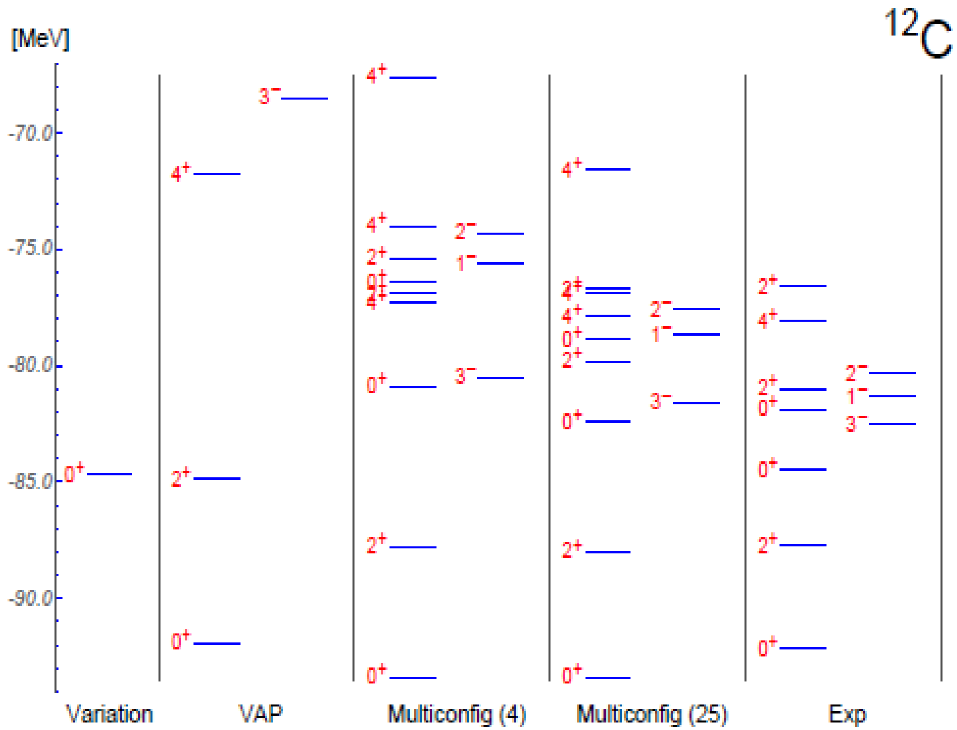


Figure 2. Intrinsic shapes used in the ^{12}C calculations (l.h.s.). Calculated and experimental spectrum of ^{12}C (r.h.s.)

M. Chernykh, H. Feldmeier, T. Neff, P. von Neumann-Cosel, and A. Richter, Phys. Rev. Lett. 98, 032501 (2007).

3 α cluster states have the wave function of the form

$$\mathcal{A}\left[\chi(\boldsymbol{s},\boldsymbol{t})\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\right]$$

The ground state has the large SU_3 component of

$$|(0s)^4(0p)^8;SU_3(04)J=0\rangle = C_0\mathcal{A}\{R^{8,J=0}_{4,4,L=0}(\boldsymbol{s},\boldsymbol{t})(\phi(\alpha))^3\}\phi_G(\boldsymbol{r}_G)$$

$$R^{N_1+N_2,J=0}_{N_1,N_2,L=0}(\boldsymbol{s},\boldsymbol{t}) = R_{N_1,0}(\boldsymbol{s},(8/3)\nu_N)R_{N_2,0}(\boldsymbol{t},2\nu_N)$$

Bayman-Bohr theorem

Thus the formation of the 3 α cluster states are due to the excitation of the 3 α relative motion embedded in the gound state:

$$R^{8,J=0}_{4,4,L=0}(\boldsymbol{s},\boldsymbol{t}) \quad \Longrightarrow \quad \chi(\boldsymbol{s},\boldsymbol{t})$$

^{16}O

There are many states with ^{12}C + α structure

Y. Suzuki, Prog. Theor. Phys. 55 (1976), 1751; 56 (1976), 111.

Reproduction of energy levels of ^{16}O by ^{12}C + α model

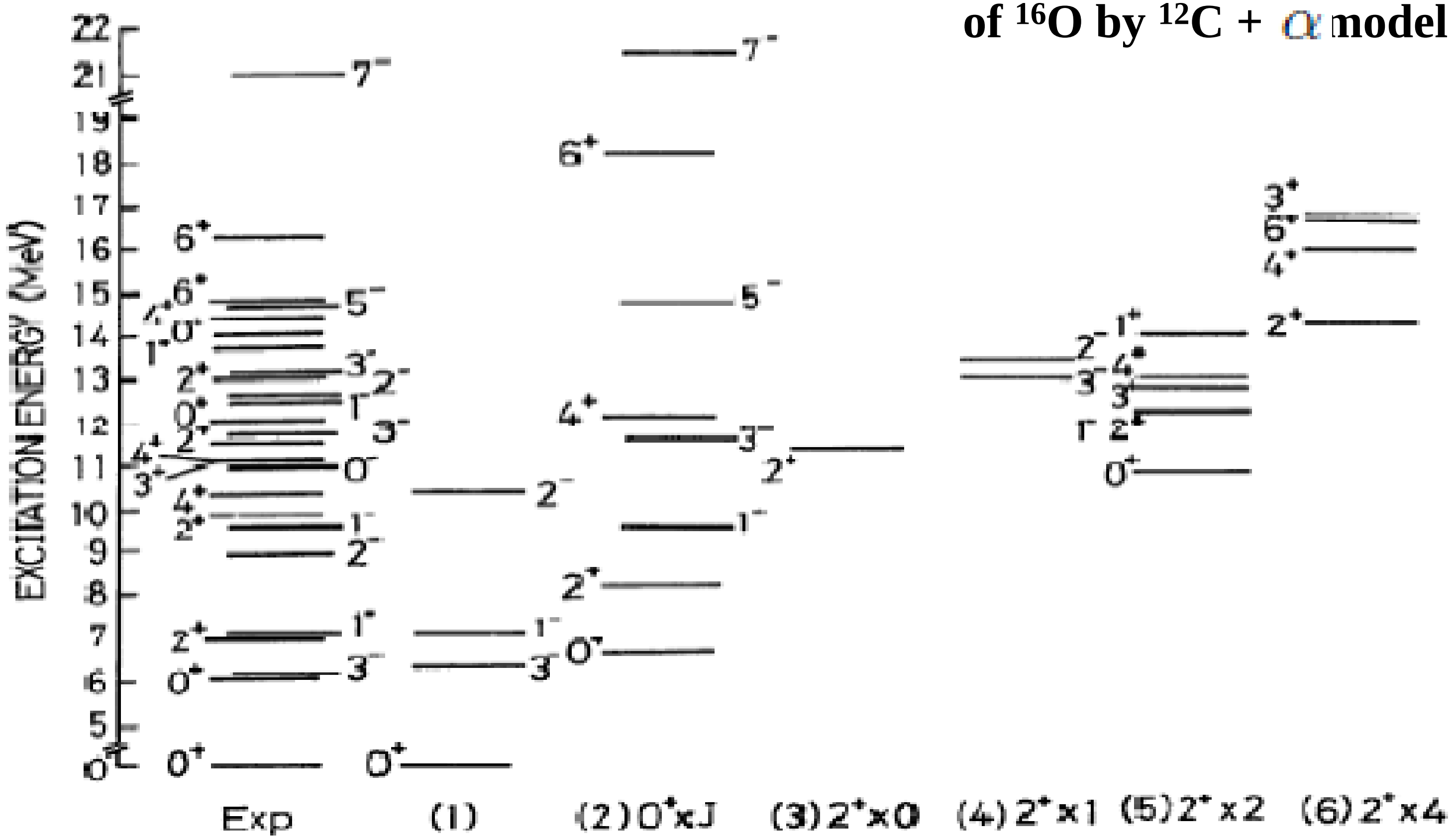


Fig. 8 Energy spectra by ^{12}C $+ \alpha$ OCM [33] classified by dominant component $L^+ \times \ell$ (^{12}C$(L^+) + \alpha(\ell)$). Classification denoted as (1) is for the mean-field-type states including the ground state and 1p - 1h type states.

Table 5 Electric transition rates in ^{16}O compared with ^{12}C+α OCM calculation [33]

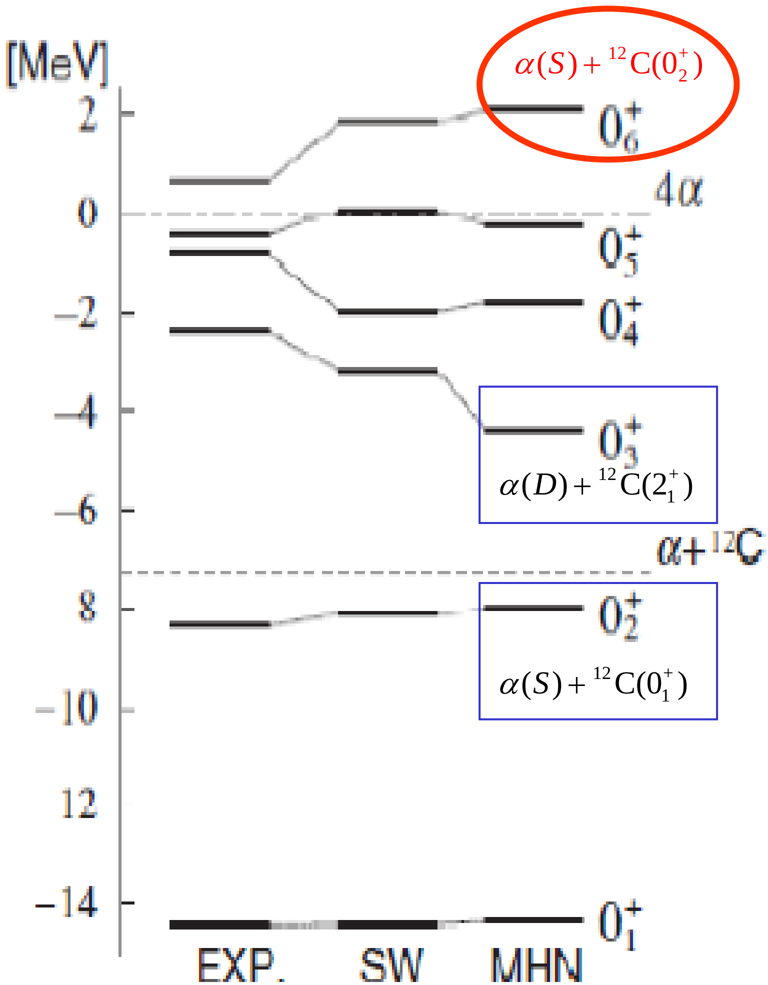
Transition	exp.	^{12}C+α	Transition	exp.	^{12}C+α
$B(E2)$ (e^2fm^4)			$M(E0)$ (fm^2)		
$2_1^+ \to 0_1^+$	7.8 ± 0.3	2.20	0_1^+ - 0_2^+	3.55 ± 0.21	3.88
$2_1^+ \to 0_2^+$	76 ± 13	60.2	0_1^+ - 0_3^+	4.03 ± 0.09	3.50
$1_1^- \to 3_1^-$	51 ± 10	25.5	$B(E3)$ (e^2fm^6)		
$2_1^- \to 3_1^-$	14^{+3}_{-4}	13.8	$3_1^- \to 0_1^+$	213 ± 11	130
$2_1^- \to 1_1^-$	36 ± 5	15.1			
$2_2^+ \to 0_1^+$	0.082 ± 0.007	0.247			
$2_2^+ \to 0_2^+$	3.0 ± 0.7	9.68			
$4_1^+ \to 2_1^+$	150 ± 18	102			
$4_2^+ \to 2_1^+$	2.4 ± 0.6	0.0405			
$2_3^+ \to 0_1^+$	3.7 ± 0.1	1.21			
$2_3^+ \to 0_2^+$	7.6 ± 2	1.20			

Table 6 Alpha decay data in ^{16}O compared with $^{12}\mathrm{C} + \alpha$ OCM calculation [33]. Reduced widths θ^2 are at the channel radius $a = 5.2$ fm.

J^π (Ex (MeV))	Γ (keV)	Decay	θ^2_{exp}	θ^2_{cal}
1_2^- (9.63)	510 ± 60	α_0	0.71	0.59
2_2^+ (9.85)	0.9 ± 0.3	α_0	0.0019	0.0079
4_1^+ (10.35)	27 ± 4	α_0	0.37	0.42
4_2^+ (11.10)	0.28 ± 0.05	α_0	0.0011	0.047
2_3^+ (11.52)	74 ± 4	α_0	0.033	0.048
3_2^- (11.60)	800 ± 100	α_0	0.63	0.51
0_3^+ (12.05)	1.5 ± 0.5	α_0	0.00037	0.097
1_3^- (12.44)	98 ± 7	α_0	0.024	0.000064
	0.025	α_1	0.084	0.18
2_2^- (12.53)	≤ 0.5	α_1	≤ 0.59	0.13
2_4^+ (13.02)	150 ± 11	α_0	0.039	0.069
3_3^- (13.13)	90 ± 14	α_0	0.032	0.091
	≈ 20	α_1	≈ 0.36	0.41
1_1^+ (13.66)	64 ± 3	α_1	0.54	0.54
5_1^- (14.67)	530 ± 71	α_0	0.38	0.30
	28 ± 4	α_1	0.10	0.074
6_1^+ (14.82)	22	α_0	0.043	0.025
	48	α_1	0.62	0.41
6_2^+ (16.29)	490 ± 40	α_0	0.42	0.42
7_2^- (20.88)	650 ± 75	α_0	0.27	0.34

full 4 α calculation

(full 4-body calculation)



The calculation reproduced well the excited states with ^{12}C + α structure.

	CAL	EXP
M(E0, $0^+_2 - 0^+_1$)	4.59	3.55±0.21
M(E0, $0^+_3 - 0^+_1$)	4.52	4.03±0.09
M(E0, $0^+_4 - 0^+_1$)	3.00	

Y. Funaki, T. Yamada, H. Horiuchi, G. Roepke, P. Schuck, and A. Tohsaki, Phys. Rev. Lett. 101, 082502 (2008).

^{12}C(L=0,2)+ α cluster states have the wave function of the form

$$\mathcal{A}\left[[\chi_\ell(\boldsymbol{r}_{C-\alpha})\phi_L(^{12}\mathrm{C})]_J\phi(\alpha)\right]$$

0_4^+ 0_5^+ 0_6^+ states have the wave function of the form

$$\mathcal{A}\left[\hat{\chi}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{v})\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4)\right] \qquad \boldsymbol{v}\equiv\boldsymbol{r}_{C-\alpha}$$

The ground state has the large component of the double closed shell

$$\begin{aligned}
\det|(0s)^4(0p)^{12}| &= C_L\mathcal{A}\left[R_{4,L}(\boldsymbol{r}_{C-\alpha})[Y_L(\hat{r}_{C-\alpha})\phi_L(^{12}\mathrm{C})]_{J=0}\phi(\alpha)\right]\phi_G(\boldsymbol{r}_G)\\
&= D_{L_1,L_2,L}\mathcal{A}\left[R^{12,J=0}_{L_1,L_2,L}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{v})\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4)\right]\phi_G(\boldsymbol{r}_G)\\
R^{12,J=0}_{L_1,L_2,L}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{v}) &= [[R_{4,L_1}(\boldsymbol{s},(8/3)\nu_N)R_{4,L_2}(\boldsymbol{t},2\nu_N)]_L R_{4,L}(\boldsymbol{v},3\nu_N)]_{J=0}
\end{aligned}$$

Formation of 12C(L=0,2)+ α cluster states :

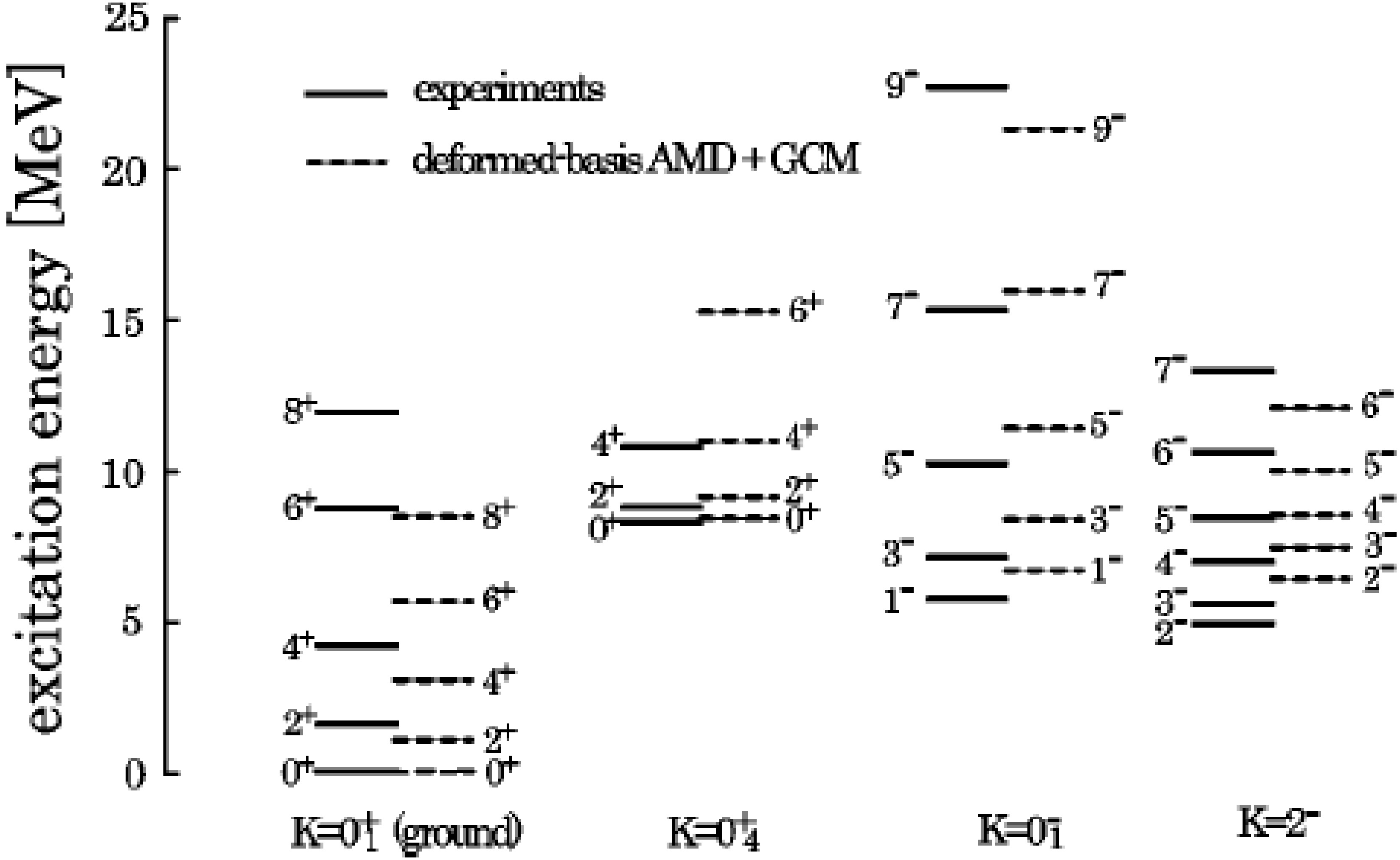
$$R_{4,L}(\boldsymbol{r}_{C-\alpha})Y_L(\hat{r}_{C-\alpha}) \Longrightarrow \chi_\ell(\boldsymbol{r}_{C-\alpha})$$

Formation of 0_4^+ 0_5^+ 0_6^+ **states :**

$$R^{12,J=0}_{L_1,L_2,L}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{v}) \Longrightarrow \hat{\chi}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{v})$$

^{20}Ne

M. Kimura
Phys. Rev. C 69, 044319 (2004)



^{20}Ne

W^J stands for the amount of the ^{16}O + α component in the wave function

TABLE II. The squared amplitude of the $\alpha + {}^{16}$O component W^J (see the Appendix) and the expectation value of the spin-orbit force $\langle \hat{V}_{ls} \rangle$ (in MeV) of the GCM wave function for each state.

K^π	J^π	W^J	$\langle \hat{V}_{ls} \rangle$	K^π	J^π	W^J	$\langle \hat{V}_{ls} \rangle$
0_1^+	0_1^+	0.70	−5.2	0_1^-	1_1^-	0.95	−0.8
	2_1^+	0.68	−5.3		3_2^-	0.93	−0.8
	4_1^+	0.54	−5.9		5_2^-	0.88	−0.7
	6_1^+	0.34	−8.4		7_2^-	0.71	−0.9
	8_1^+	0.28	−10.9		9_2^-	0.70	−1.3
0_4^+	0_4^+	0.82 (0.71)	−3.2	2^-	2_1^-		−12.9
	2_4^+	0.81 (0.71)	−3.0		3_1^-		−13.0
	4_4^+	0.79 (0.57)	−4.9		4_1^-		−14.1
	6_4^+	0.67 (0.37)	−6.8		5_1^-		−14.4
	8_4^+	0.55 (0.38)	−7.4		6_1^-		−16.5

^{20}Ne

E2 transition probabilities in $K^{\pi}=2^{-}$ band

TABLE V. Observed and calculated intraband $E2$ transition probabilities $B(E2;J_i^{\pi}\rightarrow J_f^{\pi})$ [e^2 fm^4] within the $K^{\pi}=2^{-}$ band. For comparison, the results of the j-j shell model (SM) [21] and (α + ^{16}O)+(^{8}Be+^{12}C) coupled channel OCM (OCM) [20] are also shown. The effective charges which were used in each calculation are given in the bottom line.

$J_i^{\pi}\rightarrow J_f^{\pi}$	EXP	SM	OCM	AMD+GCM
$3_1^-\rightarrow 2^-$	113 ± 29	97	108	102.8
$4^-\rightarrow 3_1^-$	77 ± 16	75	77	77.8
$4^-\rightarrow 2^-$	34 ± 6	36	34	38.5
$5_1^-\rightarrow 4^-$	<808	44	45	84.5
$5_1^-\rightarrow 3_1^-$	84 ± 19	48	49	56.6
$6^-\rightarrow 5_1^-$	32 ± 13	32	34	29.9
$6^-\rightarrow 4^-$	55^{+23}_{-13}	51	67	64.0
$\delta e/e$		0.8	0.069	0

$K^{\pi}=2^{-}$

effective charge

<u>The ground band</u>:

The reason why W^J is as large as 70% in spite of the density distribution of non-clear clustering is due to the Bayman-Bohr theorem,

$$|(0s)^4(0p)^{12}(1s,0d)^4; SU_3(8,0)J\rangle = C_J \mathcal{A}\{R_{8,J}(\boldsymbol{r}_{\alpha-{}^{16}\mathrm{O}})\phi(\alpha)\phi({}^{16}\mathrm{O})\}\,\phi_G(\boldsymbol{r}_G)$$

<u>K^π = 0^- band</u>

Almost pure ^{16}O+α clustering for low spins:

$$\mathcal{A}\{\chi_L(r)Y_L(\hat{r})\phi({}^{16}\mathrm{O})\phi(\alpha)\}$$ with 2n + L = 9

<u>K^π = $(0^+)_4$ band</u> :

^{16}O+α component is about 82% for low spins

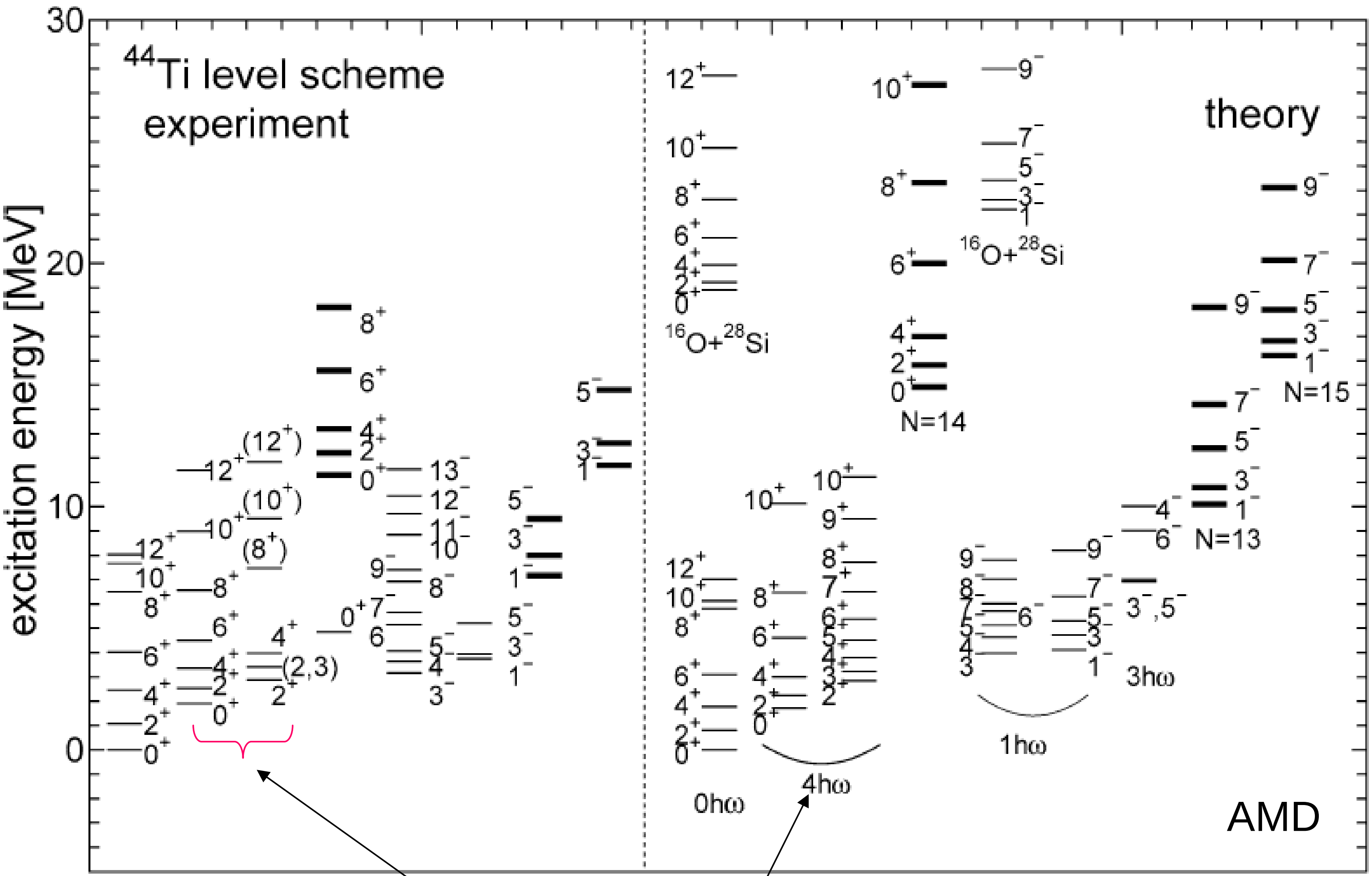
$$\mathcal{A}\{\chi_L(r)Y_L(\hat{r})\phi({}^{16}\mathrm{O})\phi(\alpha)\}$$ with 2n + L = 10

Formation of **^{16}**O-α cluster states :

$$R_{8,J}(r_{\alpha-{}^{16}\mathrm{O}} \longrightarrow \chi_L(r$$

^{44}Ti

^{40}Ca + α component is contained much in bold line levels



superdeformed bands

M. Kimura and H. Horiuchi
Nucl. Phys. A767, 58 (2006)

^{44}Ti

W_J stands for the amount of the ^{40}Ca + α component in the wave function

Table 4

Amount of the $\alpha + {}^{40}$Ca component W_J and α spectroscopic factor S_α of the obtained states (see text and Appendix A). The full GCM results of $N = 13$ and 15 band members and the experimental value of $N = 13$ band members show the sum of the fragmented states

ground	$0\hbar\omega + (\alpha + {}^{40}\mathrm{Ca})$ GCM		full GCM		Exp	$N = 14$	$0\hbar\omega + (\alpha + {}^{40}\mathrm{Ca})$ GCM		full GCM	
	W_J	S_α	W_J	S_α	S_α		W_J	S_α	W_J	S_α
0^+	0.40	0.14	0.39	0.14	0.20	0^+	0.48	0.22	0.46	0.22
2^+	0.36	0.14	0.34	0.12	0.20	2^+	0.43	0.23	0.42	0.23
4^+	0.33	0.14	0.32	0.12	0.18	4^+	0.38	0.19	0.38	0.19
6^+	0.25	0.14	0.25	0.09	0.16	6^+	0.32	0.17	0.30	0.17
8^+	0.21	0.13	0.21	0.08	0.13	8^+	0.23	0.14	0.21	0.13
10^+	0.06	0.01	0.06	0.01		10^+	0.14	0.08	0.12	0.08
12^+	0.05	0.00	0.06	0.00						
$N = 13$	$0\hbar\omega + (\alpha + {}^{40}\mathrm{Ca})$ GCM		full GCM		Exp	$N = 15$	$0\hbar\omega + (\alpha + {}^{40}\mathrm{Ca})$ GCM		full GCM	
	W_J	S_α	W_J	S_α	S_α		W_J	S_α	W_J	S_α
1^-	0.52	0.18	0.56	0.20	0.25	1^-	0.63	0.34	0.63	0.34
3^-	0.48	0.16	0.50	0.18	0.37	3^-	0.56	0.31	0.59	0.32
5^-	0.41	0.14	0.43	0.16	0.30	5^-	0.54	0.31	0.56	0.31
7^-	0.30	0.10	0.38	0.12		7^-	0.40	0.26	0.48	0.28
9^-	0.28	0.09	0.32	0.10		9^-	0.30	0.18	0.35	0.20

^{44}Ti

E2 transition probabilities

Table 2
Observed (EXP) [28–30] and calculated (AMD) intra-band E2 transition probabilities $B(\mathrm{E2}; J_i^\pi \to J_f^\pi)$ [e^2 fm^4] within the ground band. For the comparison, $\alpha + {}^{40}$Ca RGM (RGM) [8] and $\alpha + {}^{40}\mathrm{Ca}(I^\pi)$ OCM (OCM) [6] results are also shown

$J_i^\pi \to J_f^\pi$	EXP	RGM	OCM	AMD
$2_1^+ \to 0_1^+$	120 ± 30	107	166	142
$4_1^+ \to 2_1^+$	280 ± 60	146	231	222
$6_1^+ \to 4_1^+$	160 ± 30	140	212	167
$8_1^+ \to 6_1^+$	$14 >$	118	185	172
$10_1^+ \to 8_1^+$	140 ± 30	75	138	99
$12_1^+ \to 10_1^+$	40 ± 8	34	73	69

$K^\pi = 0_2^+ \to K^\pi = 0_2^+$	EXP	OCM	AMD
$2_2^+ \to 0_2^+$	220 ± 50	157	320
$4_2^+ \to 2_2^+$	268 ± 50	268	361
$K^\pi = 2^+ \to K^\pi = 2^+$	EXP	OCM	AMD
$3_3^+ \to 2_3^+$	< 590	185	298
$4_3^+ \to 2_3^+$	175^{+100}_{-60}	148	220
$4_3^+ \to 3_1^+$	$< 785 \pm 650$	11	302
$K^\pi = 2^+ \to K^\pi = 0_2^+$	EXP	OCM	AMD
$2_3^+ \to 0_2^+$	$43 <$	3.04	24

^{40}Ca+α cluster states $\mathcal{A}\left[\chi(\boldsymbol{r}_{Ca-\alpha})\phi({}^{40}\mathrm{Ca})\phi(\alpha)\right]$

K^{π}= 0- band : **^{40}**Ca+α component : about 55% for low spins,
about 30 % for high spins.

Higher nodal K^{π}= 0+ : ^{40}Ca+α about 45% for low spins,
about 10 % for high spins.

Higher nodal K^{π}= 0- : 40Ca+α about 65% for low spins,
about 35 % for high spins.

Ground band : **^{40}**Ca+α component: about 40 % for low spins,
about 5 % for high spins.

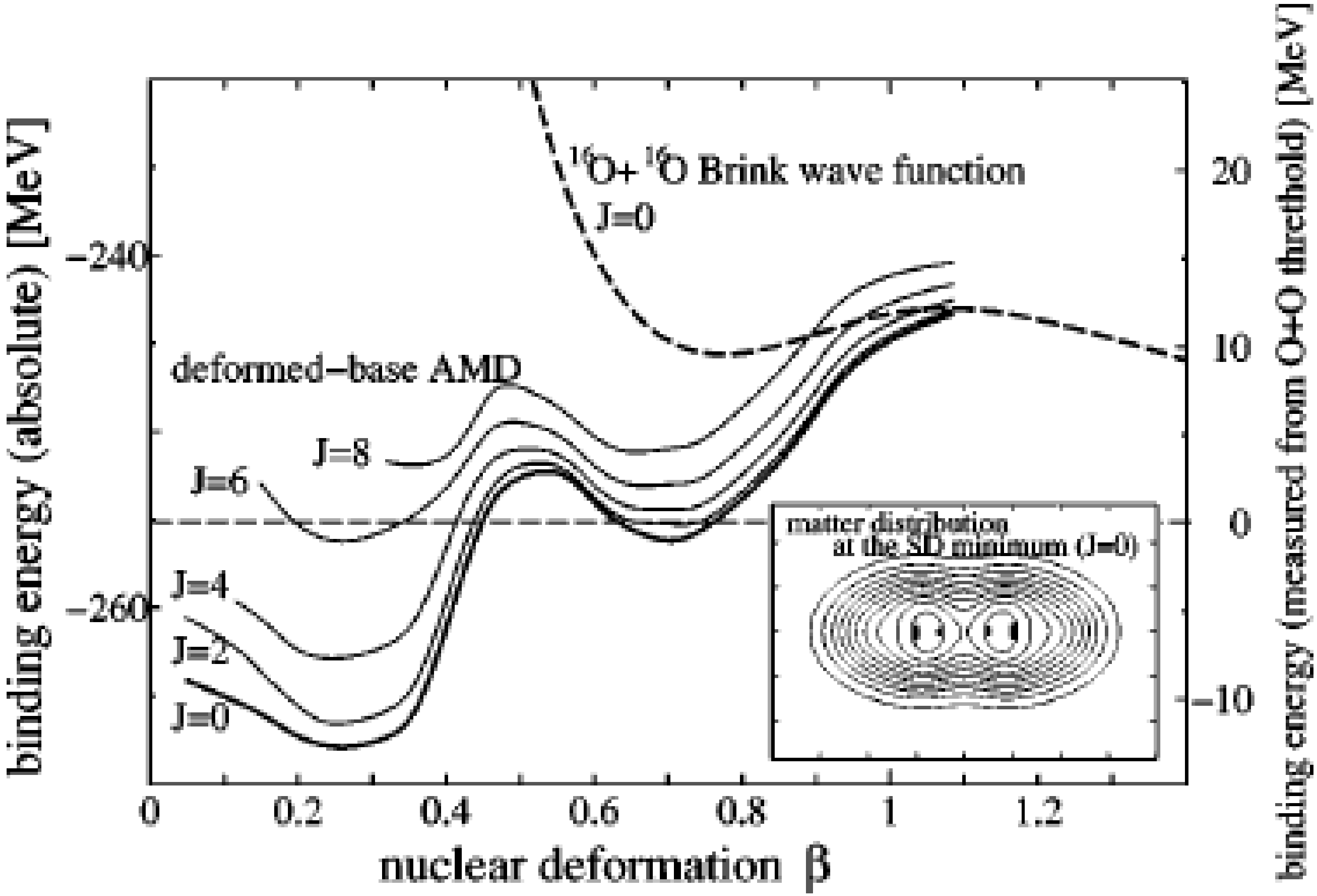
The reason of getting W^J= 40% is due to the Bayman-Bohr theorem,

$$|{}^{40}\mathrm{Ca}\ (0f,1p)^4; SU_3(12,0)J\rangle = D_J\mathcal{A}\{R_{12,J}(\boldsymbol{r}_{\alpha-{}^{40}\mathrm{Ca}})\phi(\alpha)\phi({}^{40}\mathrm{Ca})\}\phi_G(\boldsymbol{r}_G)$$

Formation of **^{40}**Ca + α cluster states **:**

$$R_{12,J}(\boldsymbol{r}_{Ca-\alpha}, (40/11)\nu_N) \Longrightarrow \chi(\boldsymbol{r}_{Ca-\alpha})$$

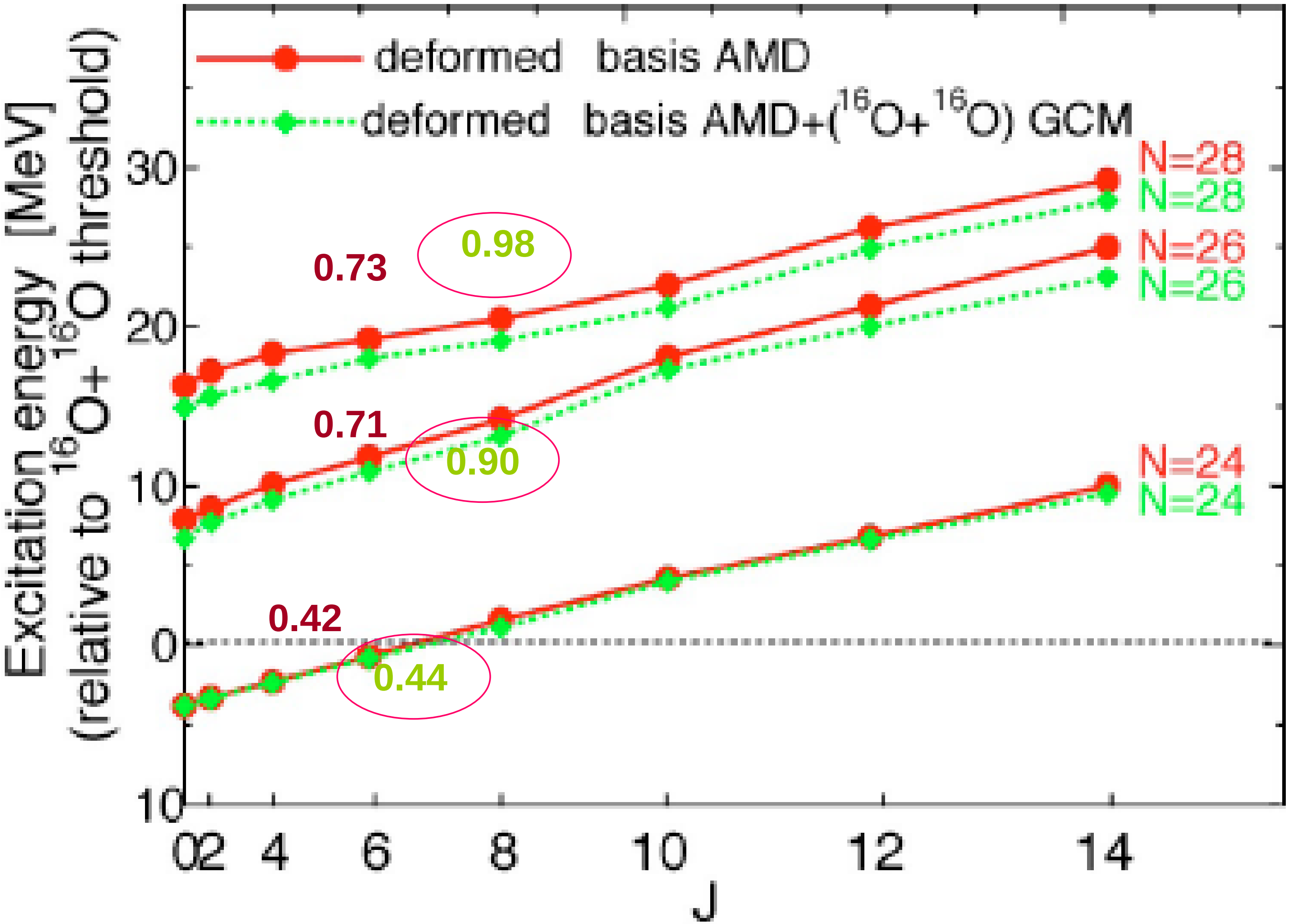
^{32}S



M. Kimura and H. Horiuchi,
Phys. Rev. C69, 051304 (R) (2004)

^{32}S

Numbers are amount of $^{16}O + {}^{16}O$ component in the wave function



$^{16}O+^{16}O$ molecular resonances

$^{16}O+^{16}O$ molecular resonances

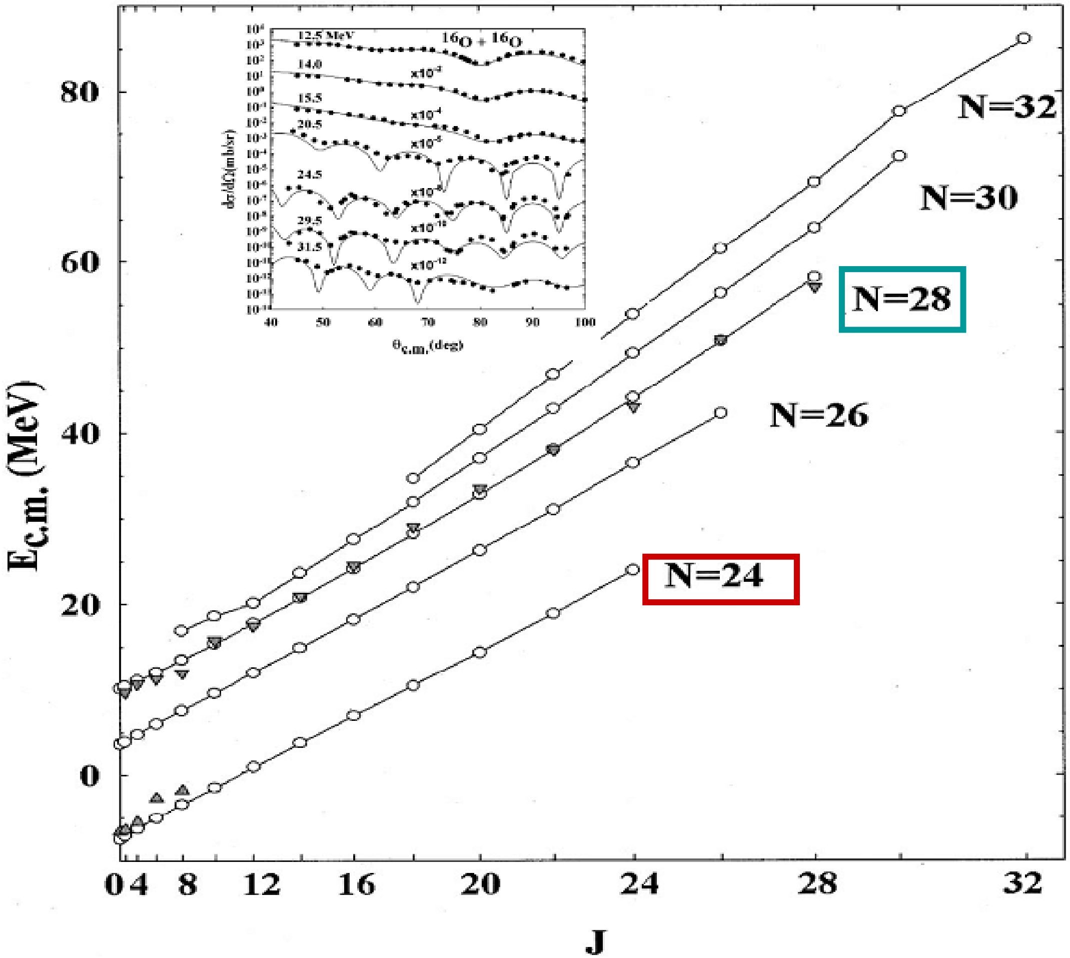
T. Ando, et al., PTP61 101, (1979).
, PTP64 1608, (1980).
Y. Kondo, et al., PLB227 310, (1989).
S. Ohkubo, et al., PRC66 021301(R).

Observed states are denoted by <u>triangles.</u>
Calculated states are denoted by <u>circles</u>.

Calculation is based on the unique optical potential for ^{16}O-^{16}O.

Observed molecular band can be identified with $\boxed{N=28}$ band.

The lowest band of the unique potential with $\boxed{N=24}$ starts at about 8MeV above the g.s.



S. Ohkubo, et al., PRC66 021301(R)

The magnitude of ^{16}O + ^{16}O component of the superdeformed band which amounts to about 44% is largely due to the Bayman-Bohr theorem with respect to the *4p-4h* shell model state

$$\begin{aligned}
&4p-4h \text{ excitation} \\
&= (0,0,0)^4(1,0,0)^4(0,1,0)^4(0,0,1)^4(1,0,1)^4(0,1,1)^4(0,0,2)^4(0,0,3)^4 \\
&= n\mathcal{A}\{X_{(0,0,24)}(r_{\mathrm{O-O}})\phi(^{16}\mathrm{O})\phi(^{16}\mathrm{O})\}\ \phi_G(r_G)
\end{aligned}$$

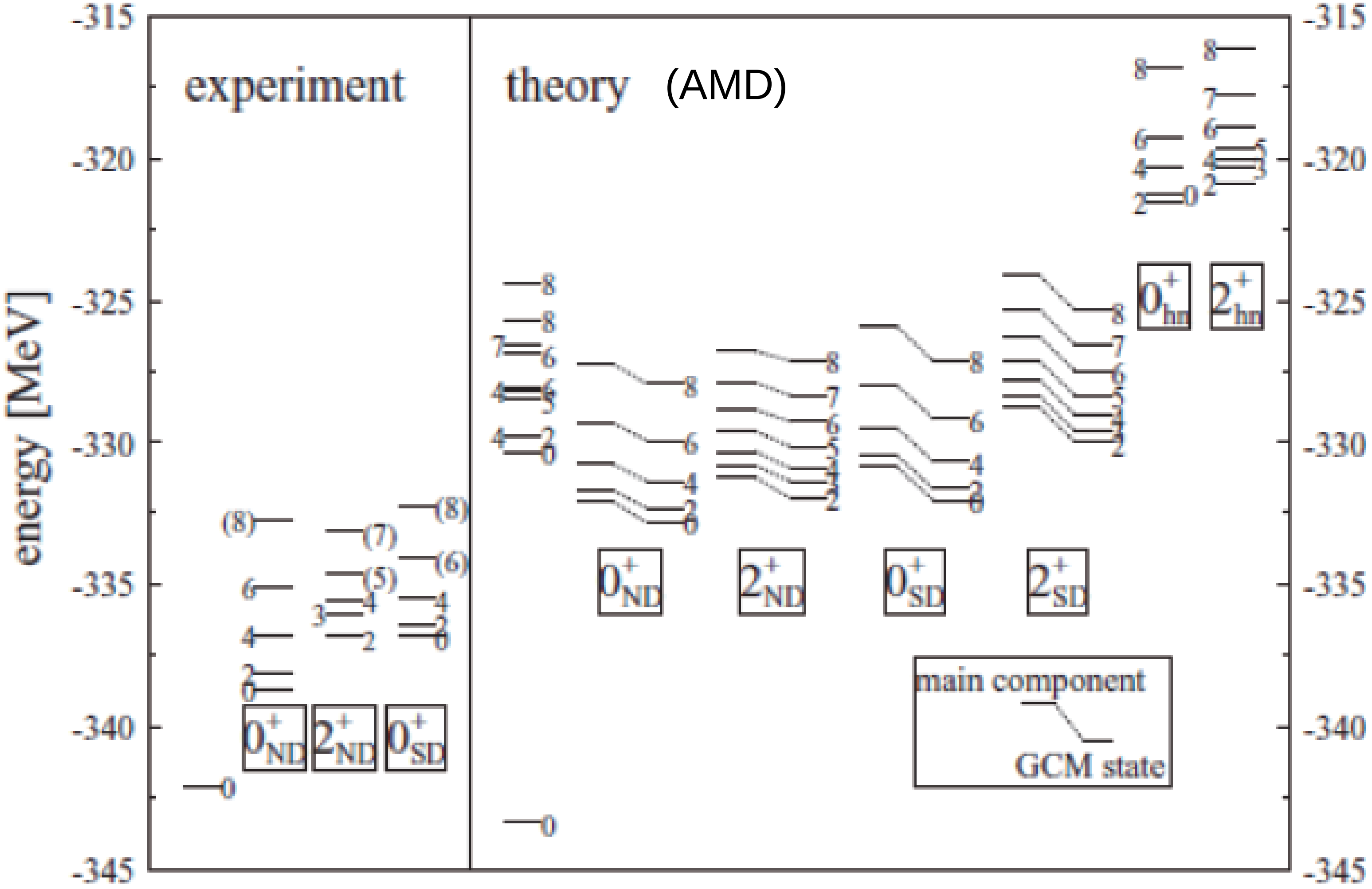
Formation of ^{16}O+^{16}O molecular resonance states:

$$X_{(0,0,24)}(r_{\mathrm{O-O}}, 8\nu) \Longrightarrow \chi_{\mathrm{Mol.Res.}}(r_{\mathrm{O-O}})$$

The ^{32}S ground state (*0p-0h* state) does not have ^{16}O + ^{16}O feature.

^{40}Ca

Y.Taniguchi et al.,
Phys. Rev. C76, 044317 (2007)



Magnitude of the wave function's components which belong to the functional spaces spanned by the β-constraint ($W(\beta)$), $d(\alpha$ - ^{36}Ar)-constraint ($W(\alpha$-Ar)), and $d(^{12}$C - ^{28}Si)-constraint (W(C-Si)) basis wave functions.

K^π	J^π	$W(\beta)$	$W(\alpha\text{-Ar})$	$W(\text{C-Si})$
0^+_{ND}	0^+	0.99	0.37	
	2^+	0.99	0.40	
	4^+	0.99	0.41	
2^+_{ND}	2^+	0.99	0.41	
	3^+	0.99	0.45	
	4^+	0.99	0.42	
0^+_{SD}	0^+	0.95		0.59
	2^+	0.95		0.59
	4^+	0.95		0.58
2^+_{SD}	2^+	0.95		0.61
	3^+	0.95		0.61
	4^+	0.95		0.60
0^+_{hn}	0^+	0.50	0.40	
	2^+	0.49	0.47	
	4^+	0.63	0.33	
2^+_{hn}	2^+	0.54	0.39	
	3^+	0.50	0.47	
	4^+	0.55	0.41	

The ground state has the large component of the double closed shell

$$
\begin{aligned}
&\frac{1}{\sqrt{40!}}\det|(0s)^4(0p)^{12}(1s0d)^{24}| \\
&= e_L \mathcal{A}\left[R_{8,L}(r_{\mathrm{Ar}-\alpha},(18/5)\nu)[Y_L(\hat{r}_{\mathrm{Ar}-\alpha})\phi_L(^{36}\mathrm{Ar})]_{J=0}\phi(\alpha)\right] g(\boldsymbol{X}_G,40\nu) \\
&= f_{L.L_1,L_2}\mathcal{A}\left[R_{16,L}(r_{\mathrm{Si-C}},(42/5)\nu)[Y_L(\hat{r}_{\mathrm{Si-C}})[\phi_{L_1}(^{28}\mathrm{Si})\phi_{L_2}(^{12}\mathrm{C})]_L]_{J=0}\right] \\
&\qquad \times g(\boldsymbol{X}_G,40\nu),
\end{aligned}
$$

Formation of ^{36}Ar + α cluster states :

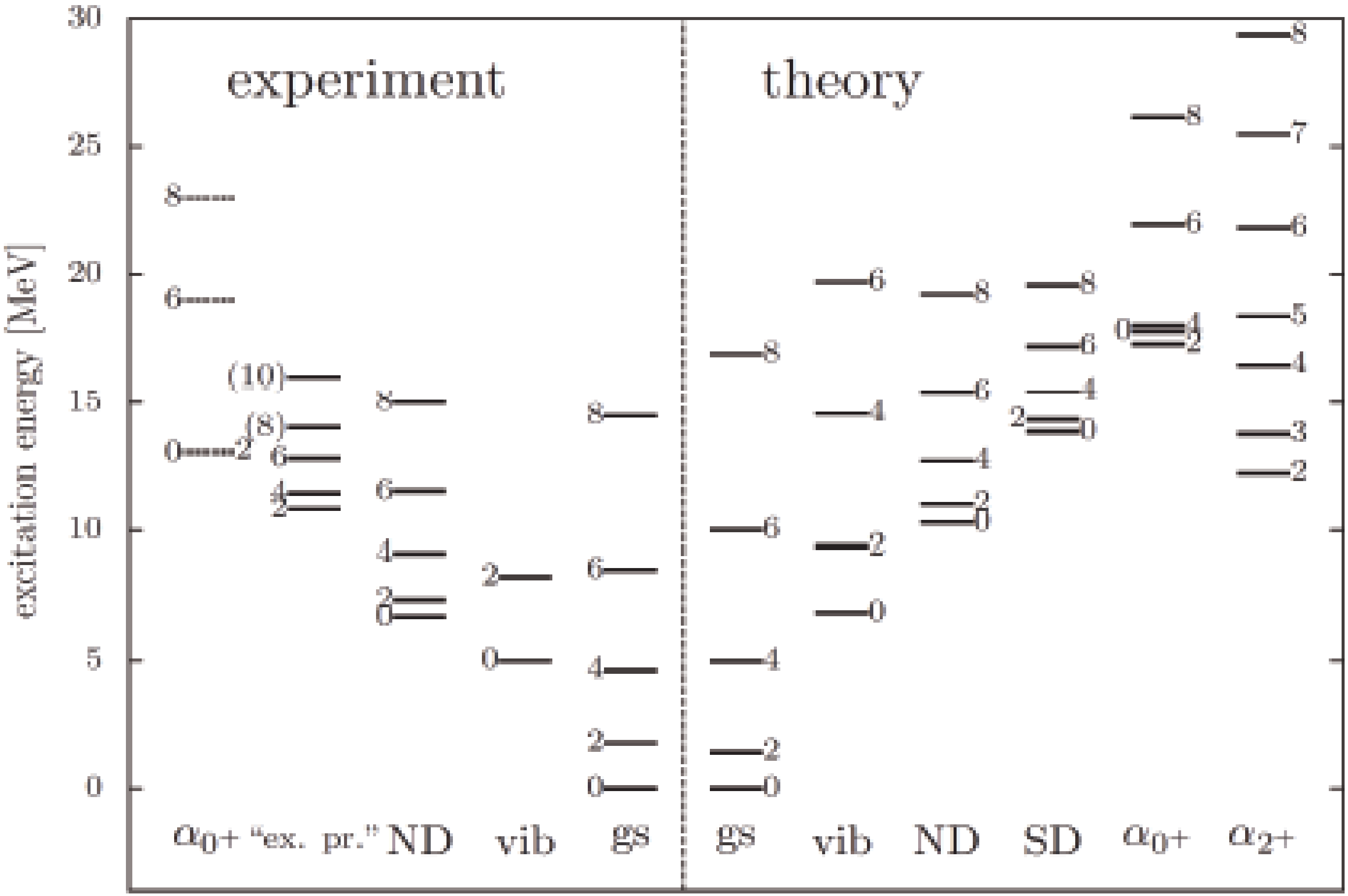
$$R_{8,L}(r_{\mathrm{Ar}-\alpha},(18/5)\nu) \Longrightarrow \chi(r_{\mathrm{Ar}-\alpha})$$

Formation of ^{28}Si + ^{12}C cluster states :

$$R_{16,L}(r_{\mathrm{Si-C}},(42/5)\nu) \Longrightarrow \chi(r_{\mathrm{Si-C}})$$

^{28}Si

Y.Taniguchi et al.,
Phys. Rev. C80, 044316 (2009)



Magnitudes of wave function's components which belong to the functional spaces of the groups, β_{OB}, β_{ND}, β_{SD}, α_T, α_A, C_T, C_A.

Band	J^π	β_{OB}	β_{ND}	β_{SD}	α_T	α_A	C_T	C_A
gs	0^+	0.97			0.97			
	2^+	0.97			0.97			
	4^+	0.97			0.93			
vib	0^+	0.97			0.89			
	2^+	0.95			0.86			
	4^+	0.94			0.53			
ND	0^+		0.99					0.89
	2^+		0.99					0.88
	4^+		0.99					0.88
SD	0^+			0.94		0.88	0.15	
	2^+			0.94		0.88	0.16	
	4^+			0.94		0.88	0.16	
α_{0^+}	0^+	0.13			0.72			
	2^+	0.25			0.76			
	4^+	0.05			0.81			
α_{2^+}	2^+	0.17			0.98			
	3^+				1.00			
	4^+	0.07			0.99			

The ground state has the large SU_3 oblate component of

$$
\begin{aligned}
&|(1s0d)^{12}[4](0,12)J, S=T=0\rangle \\
&= a_J \mathcal{A}\left\{[R_8(r_{\mathrm{Mg}-\alpha},(24/7)\nu)\phi_{(8,4)}(^{24}\mathrm{Mg})]_{(0,12),J}\phi(\alpha)\right\} g(X_G, 28\nu)
\end{aligned}
$$

ND band has the large SU_3 prolate component of

$$
\begin{aligned}
&|(1s0d)^{12}[4](12,0)J, S=T=0\rangle \\
&= b_J \mathcal{A}\left\{[R_{16}(r_{\mathrm{O}-\mathrm{C}},(48/7)\nu)\phi_{(0,4)}(^{12}\mathrm{C})]_{(12,0),J}\phi(^{16}\mathrm{O})\right\} g(X_G, 28\nu)
\end{aligned}
$$

^{24}Mg + α cluster states are not yet clearly identified in ^{28}Si.
But theoretically they are existent.

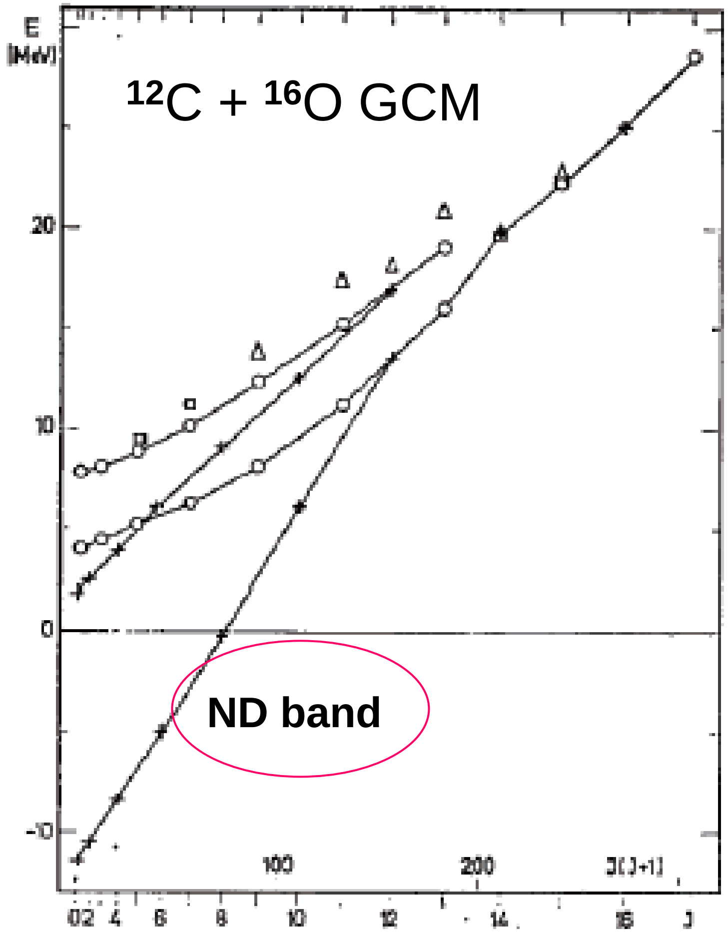
^{12}C + ^{16}O molecular states have been observed since long time ago.

Furthermore, it was theoretically indicated in early time that the ND band can be explained as having ^{12}C + ^{16}O structure with no spatial localization.

(Kato et al. were the first who mentioned it clearly, while Baye et al. reported 10 years before Kato the calculation indicating it without any clear statement.)

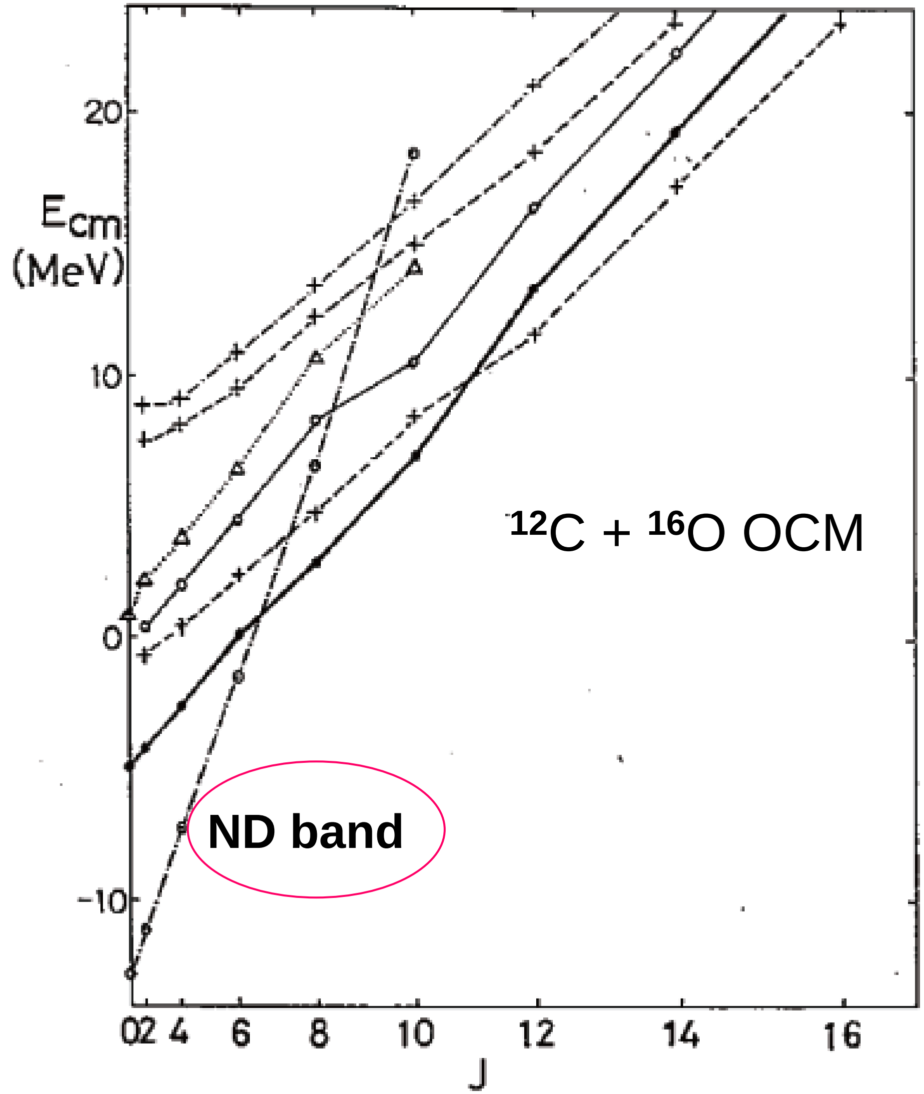
Formation of ^{12}C + ^{16}O molecular states :

$$R_{16}(r_{\mathrm{O-C}}, (48/7)\nu) \quad \Rightarrow \quad \chi(r_{\mathrm{C-O}})$$



D. Baye, Nucl. Phys. A **272**, 445 (1976)

D. Baye and P. H. Heenen, Nucl. Phys. A **283**, 176 (1977)



K. Katō, S. Okabe, and Y. Abe, Prog. Theor. Phys. **74**, 1053 (1985)

$E_{\rm th}(^{16}\mathrm{O} + {}^{12}\mathrm{C})$ is 16.75 MeV

3. Mechanism of the coexistence

Cluster states are formed when the degrees of freedom of inter-cluster relative motion embedded in the ground state are excited.

This is just in the similar way about the formation of the mea-field-type excited states which are due to the excitation of the degree of freedom of mean-field dynamics embedded in the ground state.

Duality of the ground state wave function implies that the coexistence of cluster and mean-field-type states is of logical necessity and hence inevitable.

Duality is exact for the harmonic oscillator many-body wave function.
The fact that the mean-field potential resembles the h.o. potential comes from the characters of nuclear force.

The characteristic feature of the nucleus which inevitablly shows the the coexistence of cluster and mean-field-type states due to the duality of the nuclear many-body wave function is called ‘’ Janus nature ‘’ of the nucleus. (Horiuchi, Ikeda, Kato, Yamada)

Janus (Roman god with two faces) stands at the door. He is also a symbol of a new start like January.

4. Observations which verify the Janus nature

(i) Actual existence of many cluster states which are indicated by the Bayman-Bohr theorem for the ground state
(ii) Monopole transition between many cluster states and the ground state (T.Kawabata insisted strong E0)
(iii) Cluster transfer reaction

Monopole transition between many cluster states and the ground state

^{16}O case

0^+ Cluster states *E0* transition $\longleftrightarrow$ **Ground state**

$$\mathcal{A}\{\chi_L(r)[Y_L(\hat{r})\phi_L(^{12}\mathrm{C})]_0\phi(\alpha)\} \qquad \mathcal{A}\{R_{4,L}(r)[Y_L(\hat{r})\phi_L(^{12}\mathrm{C})]_0\phi(\alpha)\}$$

***E0* transition is the transition of relative motion**

$\chi_L(r) \longleftrightarrow R_{4,L}(r)$ **(H.O. function, *4=2n+L*)**

^{16}O

E0 transition

0$^+$ Cluster states $\longleftrightarrow$ **Ground state**

$\mathcal{A}\{\chi_L(r)[Y_L(\hat{r})\phi_L(^{12}\mathrm{C})]_0\phi(\alpha)\}$ $\qquad$ $\mathcal{A}\{R_{4,L}(r)[Y_L(\hat{r})\phi_L(^{12}\mathrm{C})]_0\phi(\alpha)\}$

E0 **transition is the transition of relative motion**

$\chi_L(r) \longleftrightarrow R_{4,L}(r)$ **(H.O. function, *4=2n+L*)**

E0 **strength comes only from relative motion**

No contribution from ^{12}C part and α part

$$O(\mathrm{E0}) = \frac{1}{2}\sum_{i=1}^{16}(\mathbf{r}_i - \mathbf{r}_G)^2$$

^{12}C part $\qquad$ **causes $2\hbar\omega$ excitation**

$$= \frac{1}{2}\left\{\sum_{i\in{}^{12}C}(\mathbf{r}_i - \mathbf{r}_C)^2 + \sum_{i\in\alpha}(\mathbf{r}_i - \mathbf{r}_\alpha)^2 + \frac{12\times 4}{16}\mathbf{r}_{\alpha-C}^2\right\}$$

α part

$$M(E0, 0_2^+ \to 0_1^+) = \frac{1}{2}\sqrt{\frac{\tau_{0,4}}{\tau_{0,6}}}\eta_6 \langle R_{40}(r,\nu_N)|r^2|R_{60}(r,\nu_N)\rangle,$$

L=0

$$M(E0, 0_3^+ \to 0_1^+) = \frac{1}{2}\sqrt{\frac{\tau_{2,4}}{\tau_{2,6}}}\varsigma_6 \langle R_{42}(r,\nu_N)|r^2|R_{62}(r,\nu_N)\rangle,$$

L=2

$$\sqrt{\frac{\tau_{0,4}}{\tau_{0,6}}} \approx \sqrt{\frac{\tau_{2,4}}{\tau_{2,6}}} \approx 1.0$$

Antisymmetrization effect is very strong but it does not show up

$$\nu_N = \frac{m\omega}{2\hbar}$$

$$\tau_{L,N} = \langle \Psi_{L,N} | \mathcal{A}\{\Psi_{L,N}\}\rangle$$

$$\Psi_{L,N} = R_{N,L}(r_{\alpha-\mathrm{C}}, 3\nu)[Y_L(\hat{r}_{\alpha-\mathrm{C}})\phi_L(^{12}\mathrm{C})]_0\phi(\alpha)$$

$$|0_2^+\rangle = \sum_{N=6}^{\infty} \eta_N (C_N \mathcal{A}\{\Psi_{0,N}\}), \quad ||C_N \mathcal{A}\{\Psi_{0,N}\}|| = 1,$$

$$|0_3^+\rangle = \sum_{N=6}^{\infty} \zeta_N (D_N \mathcal{A}\{\Psi_{2,N}\}), \quad ||D_N \mathcal{A}\{\Psi_{2,N}\}|| = 1,$$

M(E0) have the magnitude similar to single-nucleon strength.

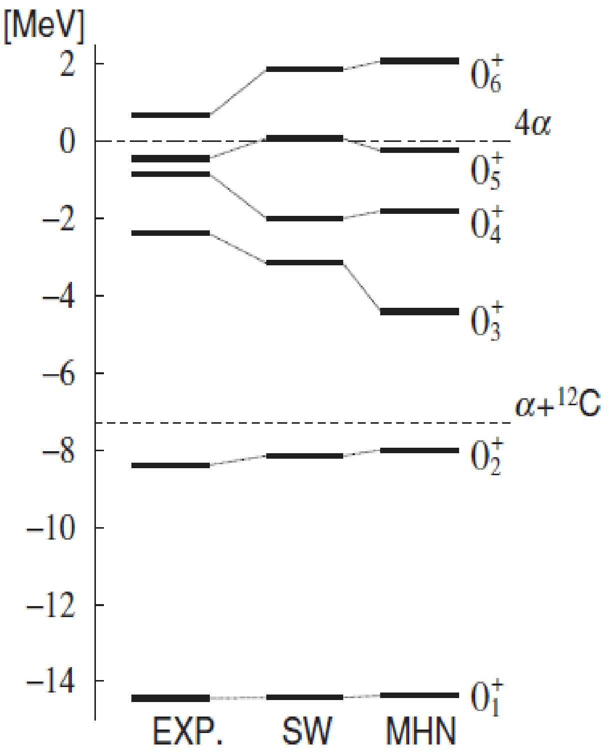
This explains why the many-particle many-hole states have large M(E0) similar to single-nucleon strength (about 5 fm^2).

	Cal.	Obs.
$M(E0, 0_2^+ \rightarrow 0_1^+)$	**1.97**	**3.55**
$M(E0, 0_3^+ \rightarrow 0_1^+)$	**3.89**	**4.03**

Y.Funaki et al.
Phys. Rev. Lett. 101, 082502 (2008)

4 α OCM

	R		M(E0)		R_{expt}	$M(\mathrm{E0})_{\mathrm{expt}}$
	SW	MHN	SW	MHN		
0_1^+	2.7	2.7			2.71 ± 0.02	
0_2^+	3.0	3.0	4.1	3.9		3.55 ± 0.21
0_3^+	2.9	3.1	2.6	2.4		4.03 ± 0.09
0_4^+	4.0	4.0	3.0	2.4		No data
0_5^+	3.1	3.1	3.0	2.6		3.3 ± 0.7
0_6^+	5.0	5.6	0.5	1.0		No data



Order of magnitude of all M(E0) for 5 excited 0^+ states
is that of single-nucleon strength.
This is impossible to explain in many-particle many-hole picture.

^{12}C case

M(E0) **: Hoyle state ⇔ *SU$_3$* shell model ground state**

≈ 1.0

$$M(E0, 0_2^+ - 0_1^+) = \sqrt{\frac{7}{6}} \sqrt{\frac{\langle F_4 \rangle}{\langle F_5 \rangle}} \xi_5 \langle R_{40}(r, \nu_N) | r^2 | R_{60}(r, \nu_N) \rangle,$$

Antisymmetrization effect is very strong but it does not show up

$$\langle F_n \rangle = \langle Q_n | \; Q_n = F_n(s,t) \phi(\alpha_1) \phi(\alpha_2) \phi(\alpha_3),$$

$$F_n(s,t) = \sum_{n_1+n_2=n} \sqrt{\frac{(2n_1+1)!!(2n_2+1)!!}{(2n_1)!!(2n_2)!!}} R^{2n,J=0}_{2n_1,2n_2,L=0}(s,t),$$

$$\Phi_{0_2^+} = \sum_{n=5}^{\infty} \xi_n (e_n \mathcal{A}\{Q_n\}), \quad ||e_n \mathcal{A}\{Q_n\}|| = 1.$$

SU$_3$ **shell model ground state ⟹ *M(E0)* = 1.3 fm^2**

Order of magnitude of the observed *M(E0)* = 5.4 fm^2 is already obtained without G.S. correlation

With G.S. correlation we get *M(E0)* = 6.7 fm^2

Cluster transfer reaction

Alpha transfer reaction

Theoretical description is always based on the duality of the ground state wave function:

(^{6}Li, d): ^{6}Li = α + d,
(^{7}Li, t): ^{7}Li = α + t,
(^{16}O, ^{12}C): ^{16}O = ^{12}C + α ,
etc.

Related reaction processes:

i. **breakup reaction into alpha + others,**
ii. **knockout reaction of alpha(s) such as quasi-elastic process,**
iii. **dynamical fragmentation reaction (non-statistical),**

(Takemoto et al. Phys. Rev. C54, 266 (1996))

5. Discussions

Formation of cluster states via mean-field dynamics ?

^{32}S ground state $\Rightarrow$ Superdeformed state $\Rightarrow$ ^{16}O+^{16}O state

mean-field dynamics **Janus nature**

$^{32}S \rightarrow {}^{28}Si + \alpha \rightarrow {}^{24}Mg + 2\alpha \rightarrow {}^{20}Ne + {}^{12}C \rightarrow {}^{16}O + {}^{16}O$

Janus nature

<Formation of cluster states in unit of alpha clusters>

In heavy nuclei, this type of structure change may be important

mean-field dynamics **Janus nature**

Ground state of heavy nucleus $\Rightarrow$ **Mean-field-type state with large cluster component** $\Rightarrow$ **Cluster state**

6. Summarizing comments

Janus nature is based on the duality of model wave function of
of nuclei which is purely of quantum mechanical nature.

Duality is for the model wave functions of nuclei.
How is duality for the realistic nuclear wave function ?

Janus narure Is not equivalent to the dynamical structure change
itself but is partly the stage on which the dynamical change
from mean-field-like structure to cluster structure takes place.

How is the structure change realized dynamically by the action
of realistic nuclear force with strong tensor component ?