High Energy THeory Forum (HETH-Forum)
高能理论论坛 / 中科院高能物理研究所

The challenge of the high accuracy Lattice QCD calculation in China

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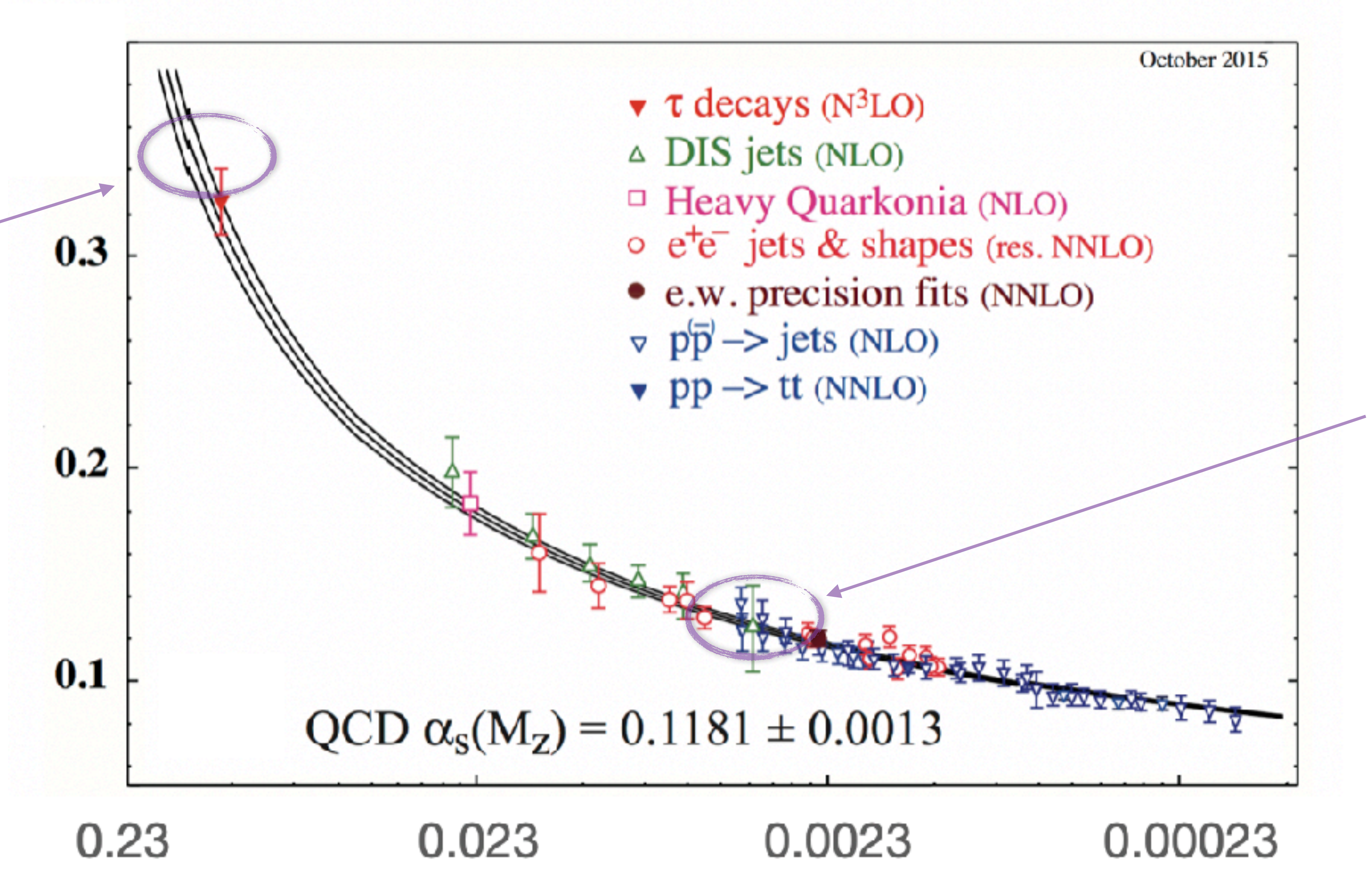
Quantum Chromo dynamics (QCD)

The feature of QCD at ~0.2GeV or nucleon radius scale can not be calculated analytically, which corresponds to one of seven Millennium Prize Problems.



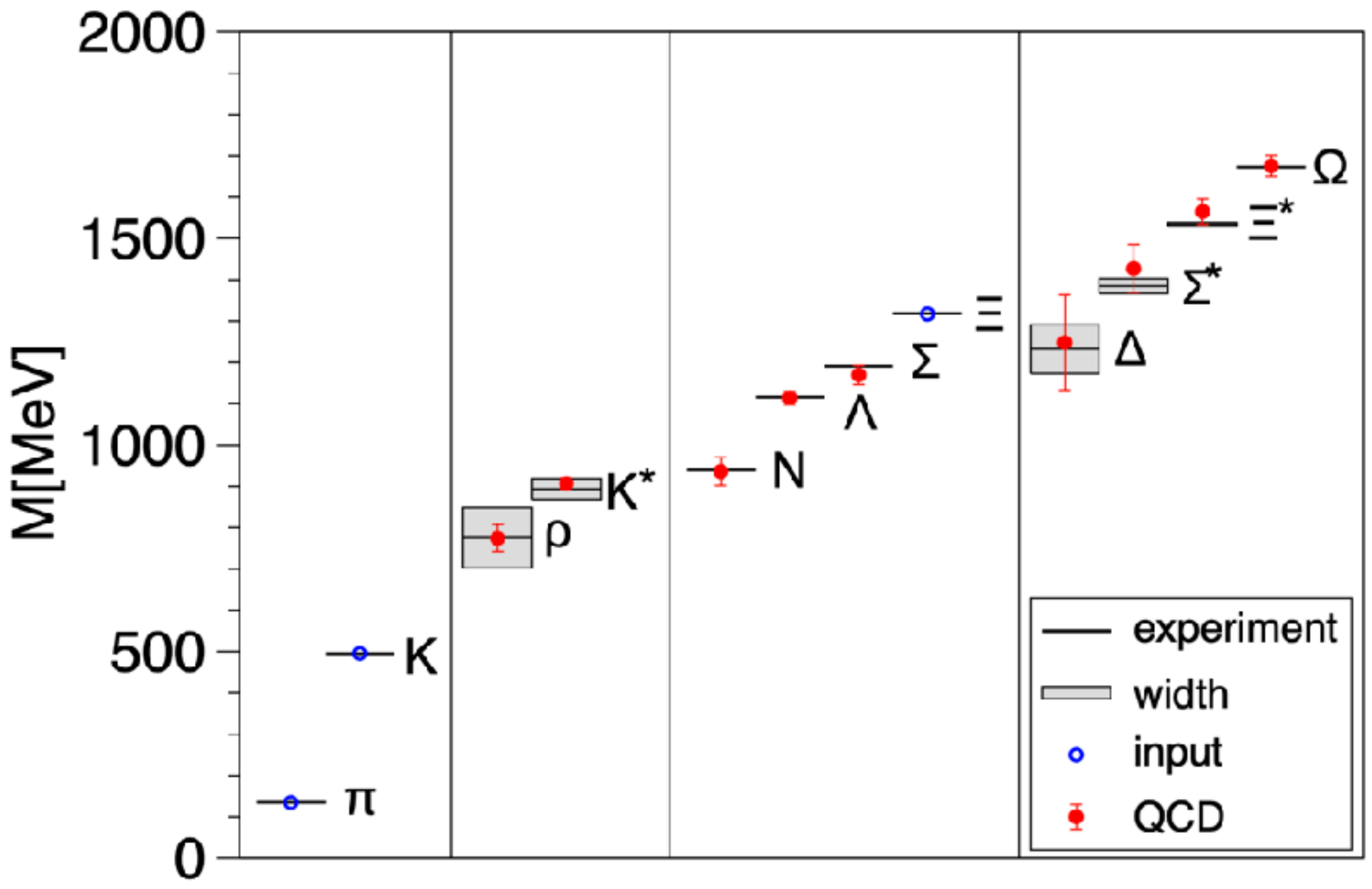
Relative scale comparing to the nucleon radius

- QCD at ~100 GeV scale has been tested accurately;
- And won the Nobel prize at 2004.

Yang-Mills existence and mass gap：

- Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on 4-D euclidean space and has a mass gap。
- For QCD, this mass gap is the nucleon mass.

Quantum Chromo dynamics (QCD)



BMWc, Science 322(2008)1224

- The QCD coupling diverges at $\Lambda_{\rm QCD} \sim 300$ MeV;
- It corresponds to massive hadron mass $\geq n\Lambda_{\rm QCD}$, where n is the number of the valence quarks,
- …except the pseudo Goldstone boson likes pion with $m_\pi = 135$ MeV.

Lattice QCD

QCD can be defined on a discretized Euclidean 4D lattice, while it is very hard to calculate analytically!

High Energy Physics – Experiment (**hep-ex** new, recent, search)
High Energy Physics – Lattice (**hep-lat** new, recent, search)
High Energy Physics – Phenomenology (**hep-ph** new, recent, search)
High Energy Physics – Theory (**hep-th** new, recent, search)

One of the four directions of HEP

Taking the gluon propagator as an example:

$$G_{\mu\nu}(k) = \frac{1}{k^4}\left(\alpha k_\mu k_\mu + (g_{\mu\nu}k^2 - k_\mu k_\mu)\right) \xrightarrow[\text{To lattice QFT}]{\text{From continuum QFT}}$$

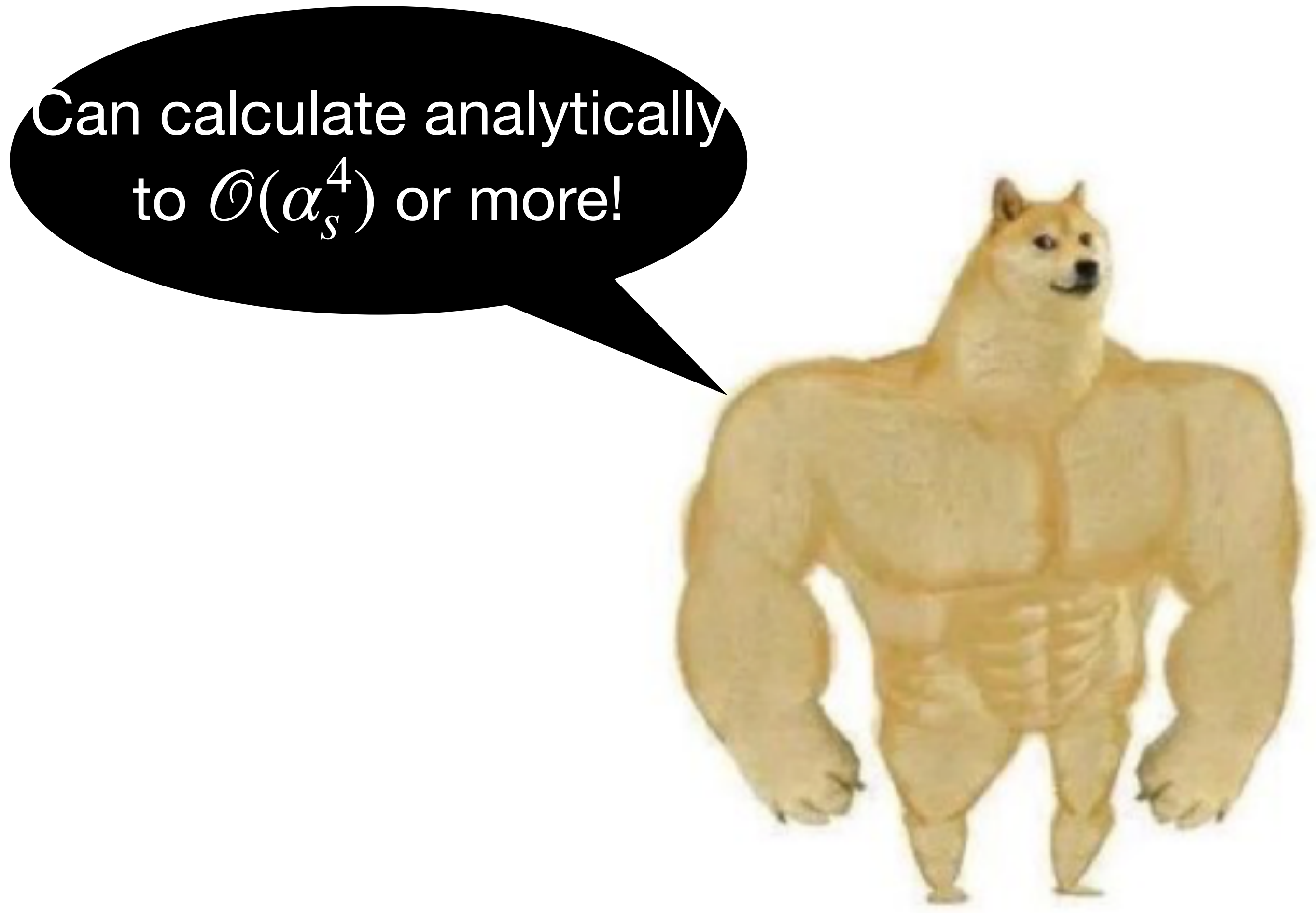
$$G_{\mu\nu}(k) = \frac{1}{(\hat{k}^2)^2}\left(\alpha\hat{k}_\mu\hat{k}_\nu + \sum_\sigma(\hat{k}_\sigma\delta_{\mu\nu} - \hat{k}_\nu\delta_{\mu\sigma})\hat{k}_\sigma A_{\sigma\nu}(k)\right),$$

with

$$\begin{aligned} A_{\mu\nu}(k) &= A_{\nu\mu}(k) = (1-\delta_{\mu\nu})\,\Delta(k)^{-1}\left[(\hat{k}^2)^2 - c_1\hat{k}^2\left(2\sum_\rho\hat{k}_\rho^4 + \hat{k}^2\sum_{\rho\neq\mu,\nu}\hat{k}_\rho^2\right)\right. \\ &\left. + c_1^2\left(\left(\sum_\rho\hat{k}_\rho^4\right)^2 + \hat{k}^2\sum_\rho\hat{k}_\rho^4\sum_{\tau\neq\mu,\nu}\hat{k}_\tau^2 + (\hat{k}^2)^2\prod_{\rho\neq\mu,\nu}\hat{k}_\rho^2\right)\right], \end{aligned}$$

where

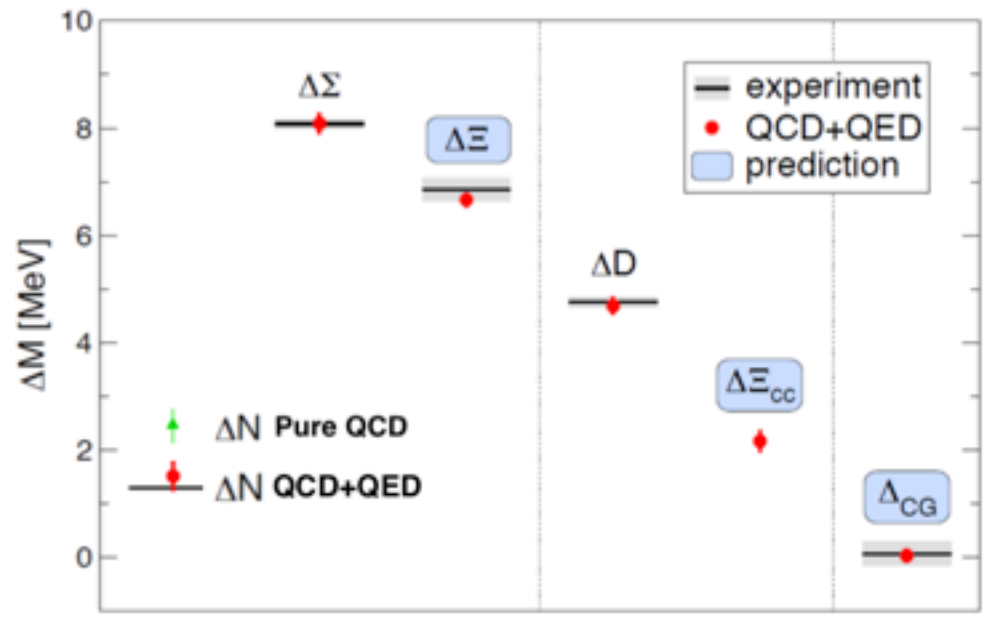
$$\begin{aligned} \Delta(k) &= \left(\hat{k}^2 - c_1\sum_\rho\hat{k}_\rho^4\right)\left[\hat{k}^2 - c_1\left((\hat{k}^2)^2 + \sum_\tau\hat{k}_\tau^4\right) + \frac{1}{2}c_1^2\left((\hat{k}^2)^3 + 2\sum_\tau\hat{k}_\tau^6 - \hat{k}^2\sum_\tau\hat{k}_\tau^4\right)\right] \\ &- 4c_1^3\sum_\rho\hat{k}_\rho^4\sum_{\tau\neq\rho}\hat{k}_\tau^2. \end{aligned}$$



Lattice QCD

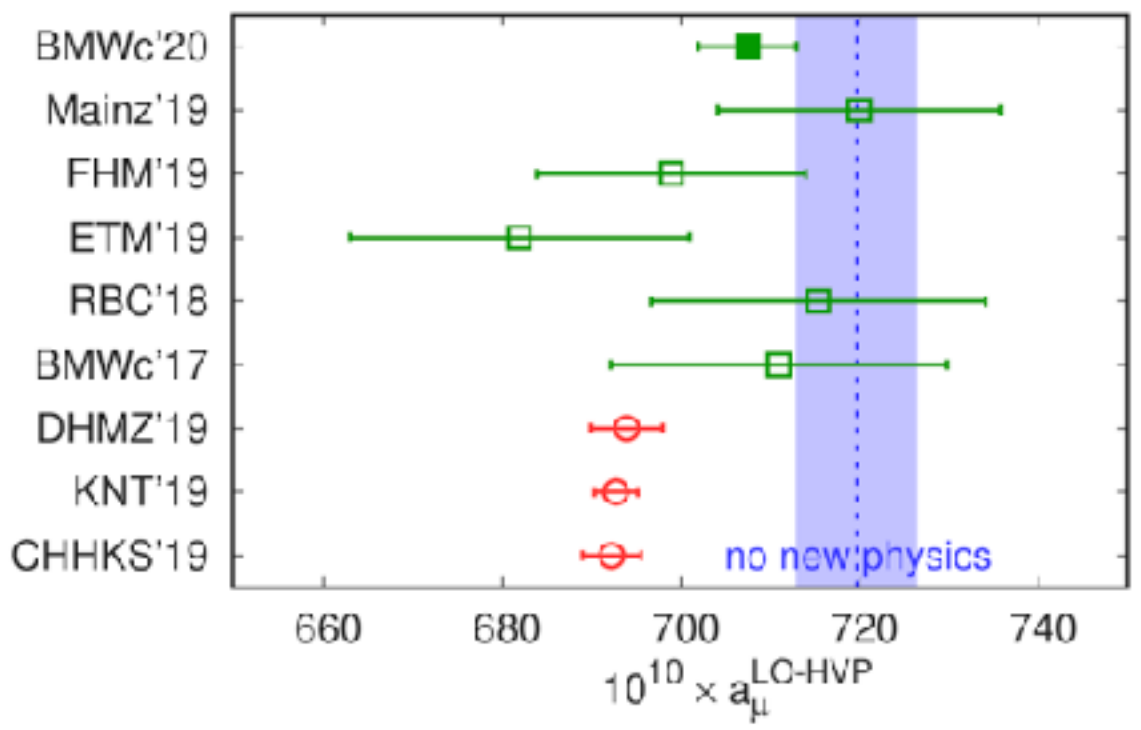
QCD can be defined on a discretized Euclidean 4D lattice, while it is very hard to calculate analytically!

……but we don't have to.



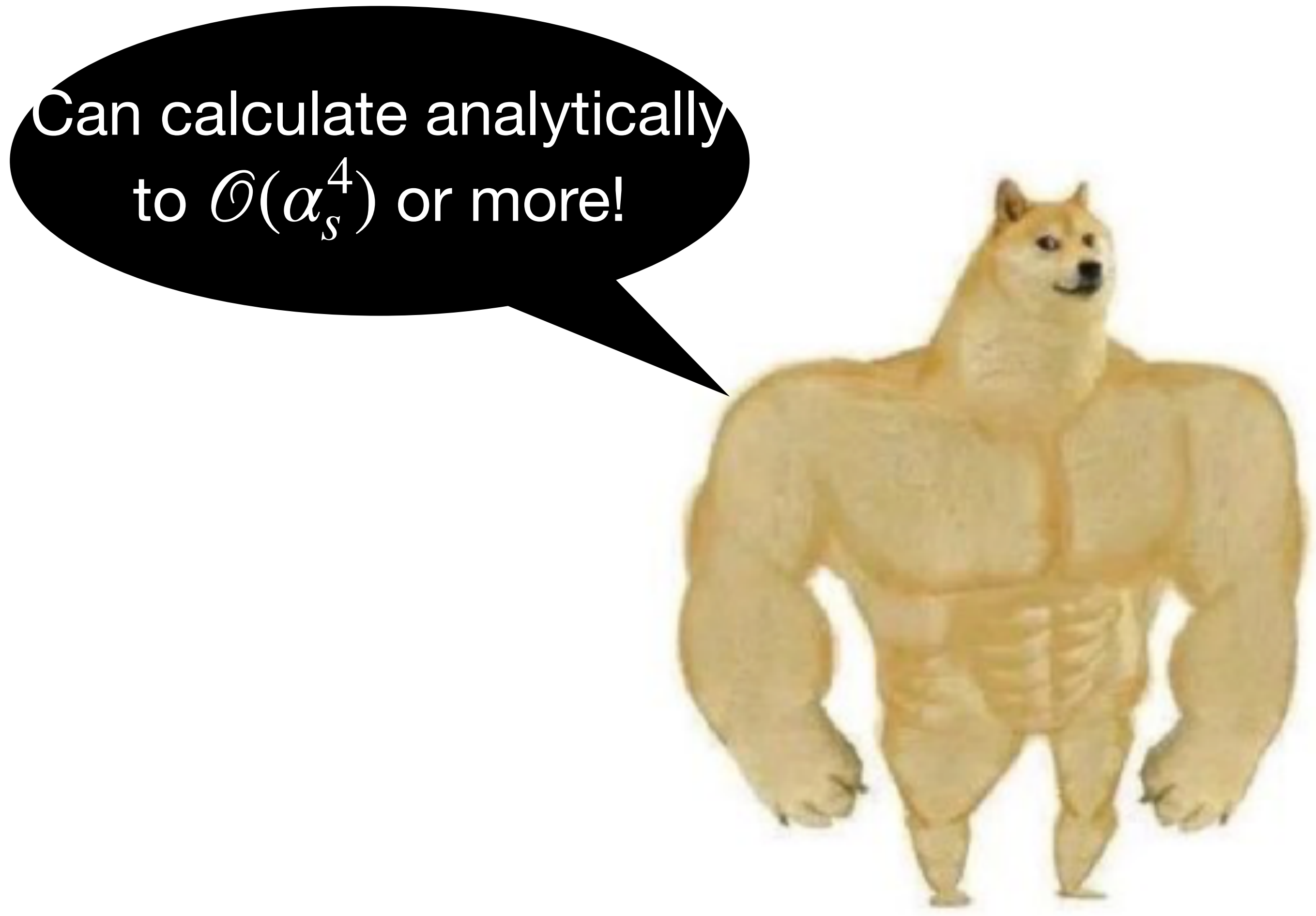
Neutron-proton mass difference

BMWc, Science 347(2015)1452



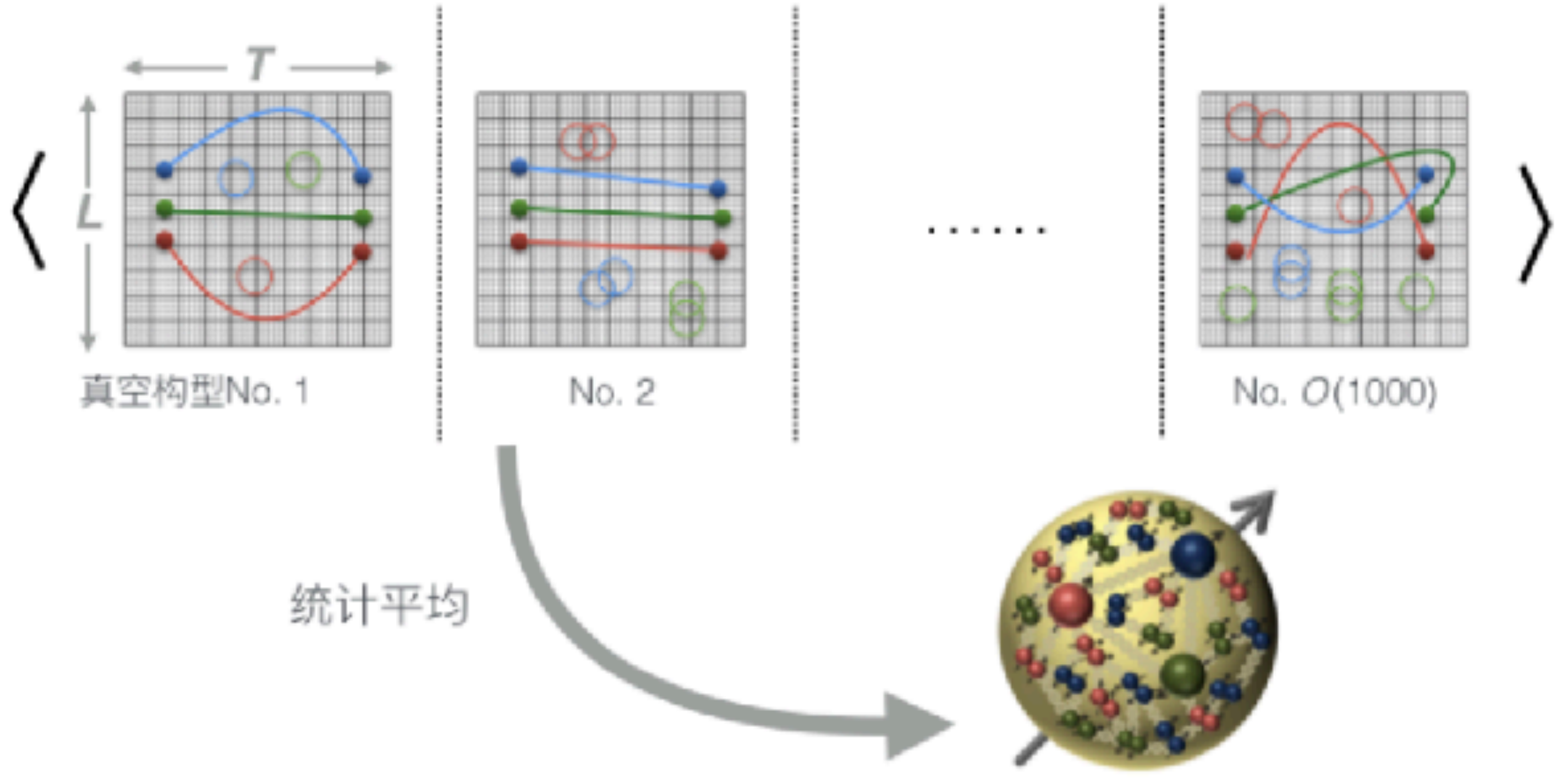
LO-HVP contribution to g-2

BMWc, Nature 593(2021)7857



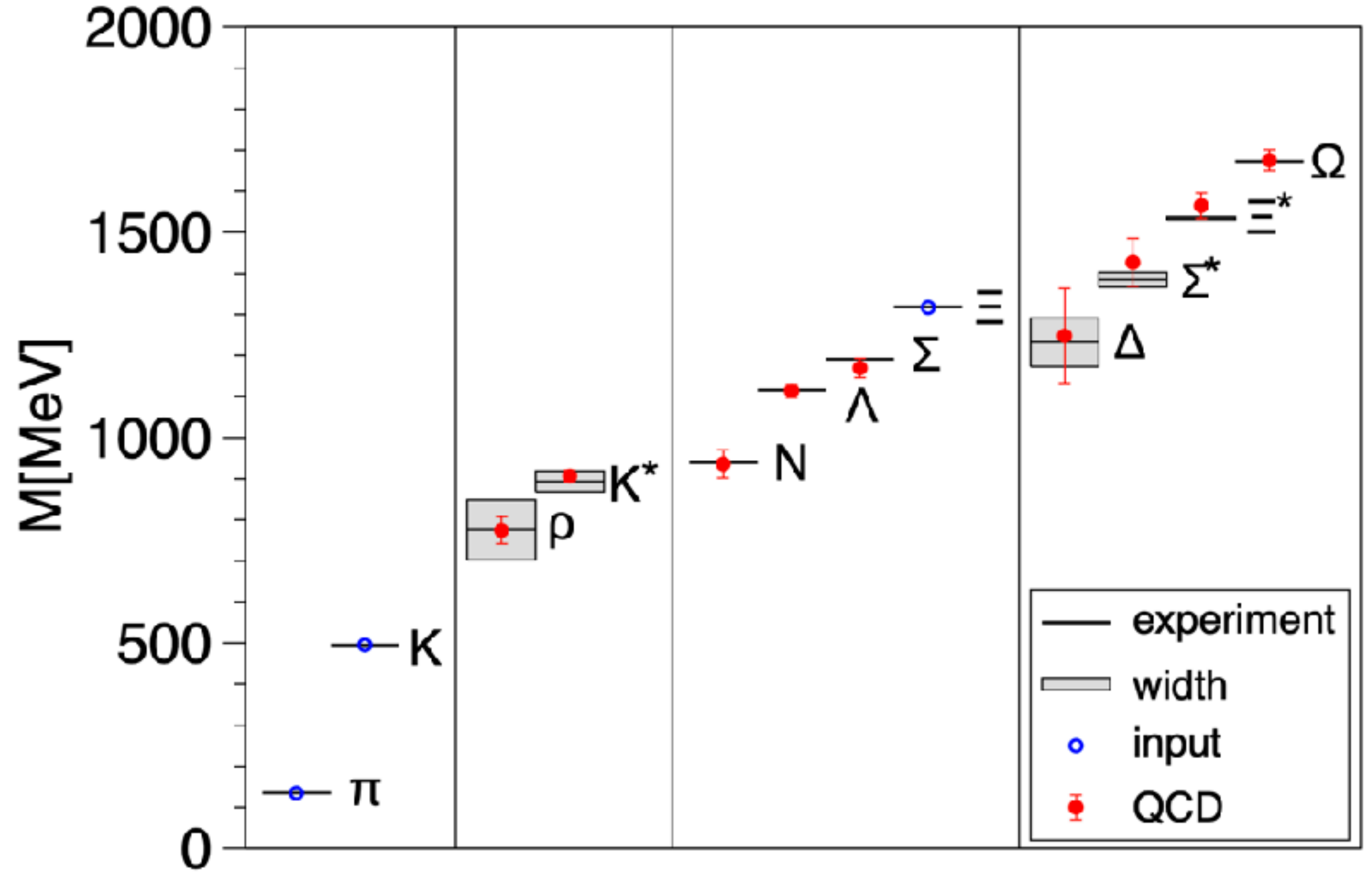
Outline

Basic idea of LQCD



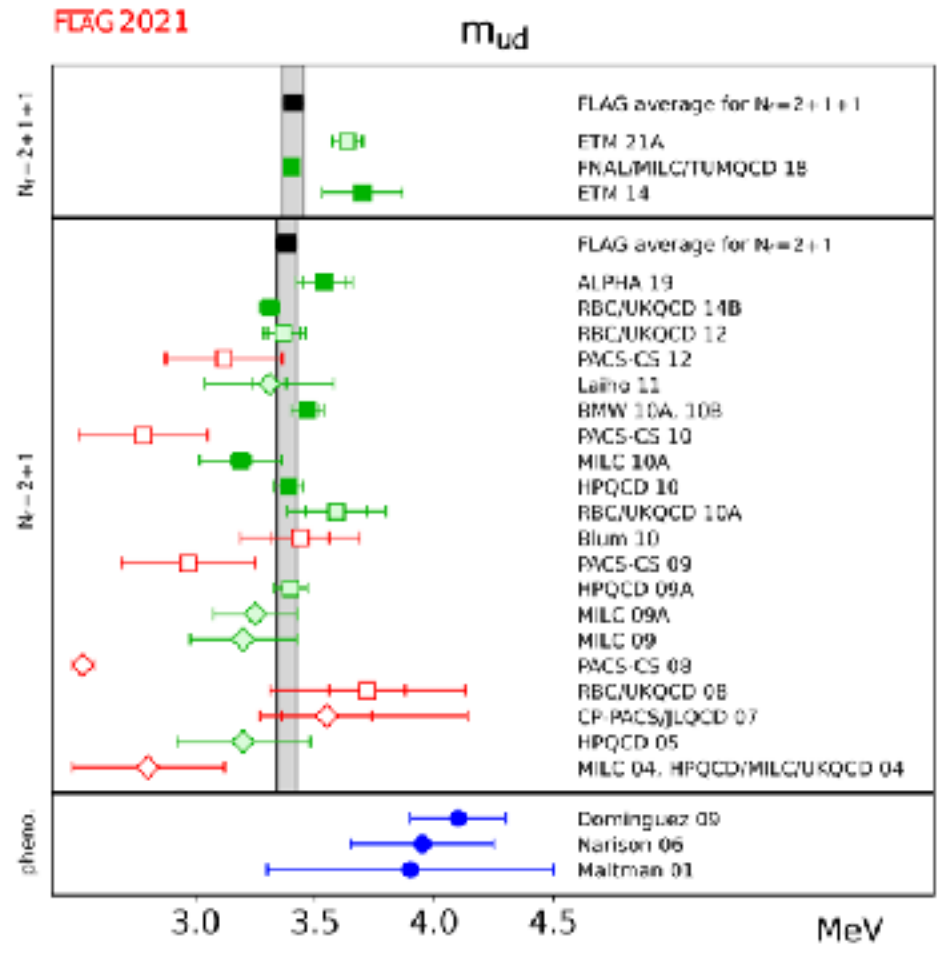
Hadron spectrum

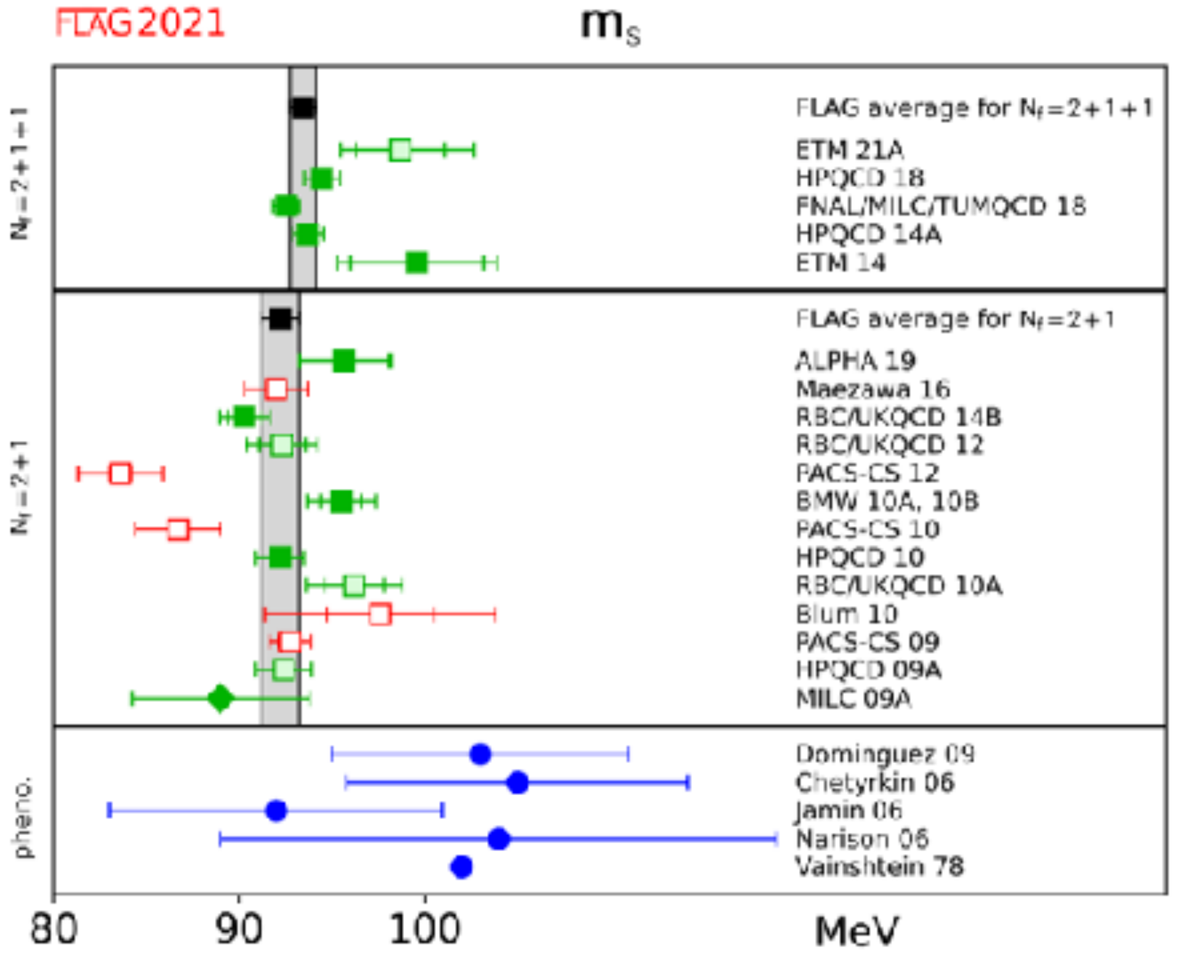
BMWc, Science 322(2008)1224



Quark mass determination

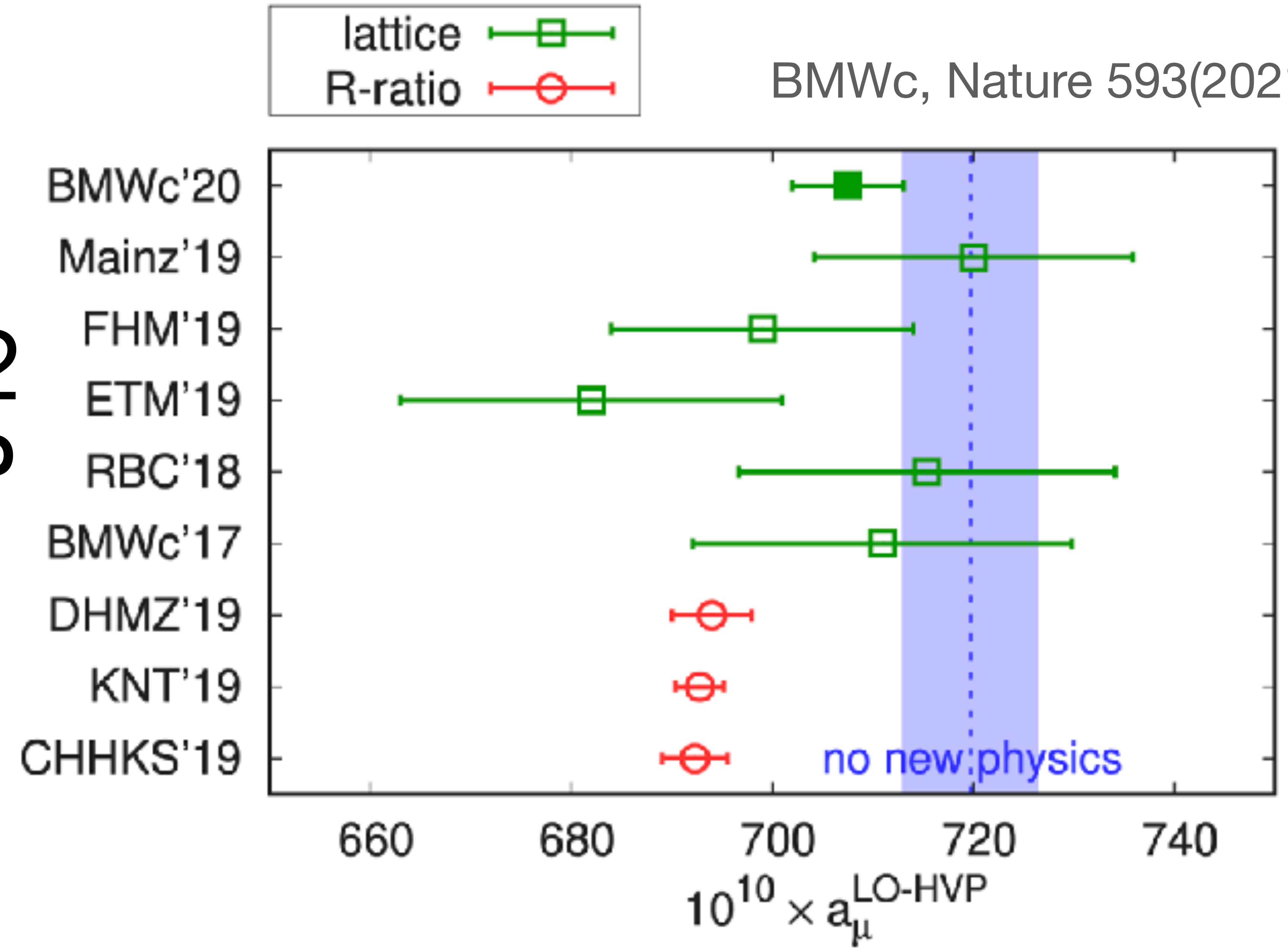
FLAG, EPJC80(2020)113





Muon g-2 from HVP

BMWc, Nature 593(2021)51



Discretized quantum field theory

	Continuum	Lattice
EOM	$(\partial_\mu \partial^\mu + m^2)\phi = 0$	$\sum_\mu \left(\phi(x+\hat{n}_\mu) + \phi(x-\hat{n}_\mu)\right) + (m^2 a^2 - 2)\phi(x) = 0$
Propagator	$S = \dfrac{1}{p^2 + m^2}$	$S_L = \dfrac{1}{4\sum_\mu \mathrm{Sin}^2(\frac{a p_\mu}{2})/a^2 + m^2}$
Loop Integral	$\int_{-\infty}^{\infty} \mathrm{d}^4 p$	$\int_{-\pi/a}^{\pi/a} \mathrm{d}^4 p$
	Require additional regulation to be finite	The divergence has been regularized into the $1/a^n$ and $\log(a)$ terms

Discretized QCD

Wilson link

The QCD Lagrangian is the following：

$$\bar{\psi}(\gamma_\mu(\partial_\mu - igA_\mu) - m)\psi - \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

with basic variable ψ and A_μ

The lattice gauge theory replaces the basic variable A_μ into the gauge link (or Wilson link) U_μ:

$$A_\mu(x + \frac{1}{2}\hat{n}_\mu) = a^4\frac{U_\mu(x) - U^\dagger_\mu(x)}{2ig_0a} + \mathcal{O}(a^2g^2), U_\mu(x) \equiv e^{ig_0\int_x^{x+a\hat{\mu}} dyA_\mu(y)};$$

$$U_\mu(x) \equiv e^{ig_0\int_x^{x+a\hat{\mu}} dyA_\mu(y)}, \ U^\dagger_\mu(x) \equiv e^{-ig_0\int_{x+a\hat{\mu}}^{x} dyA_\mu(y)} = e^{ig_0\int_{x+a\hat{\mu}}^{x} dyA_\mu(y)}$$

$$x \longrightarrow x + a\hat{\mu}$$

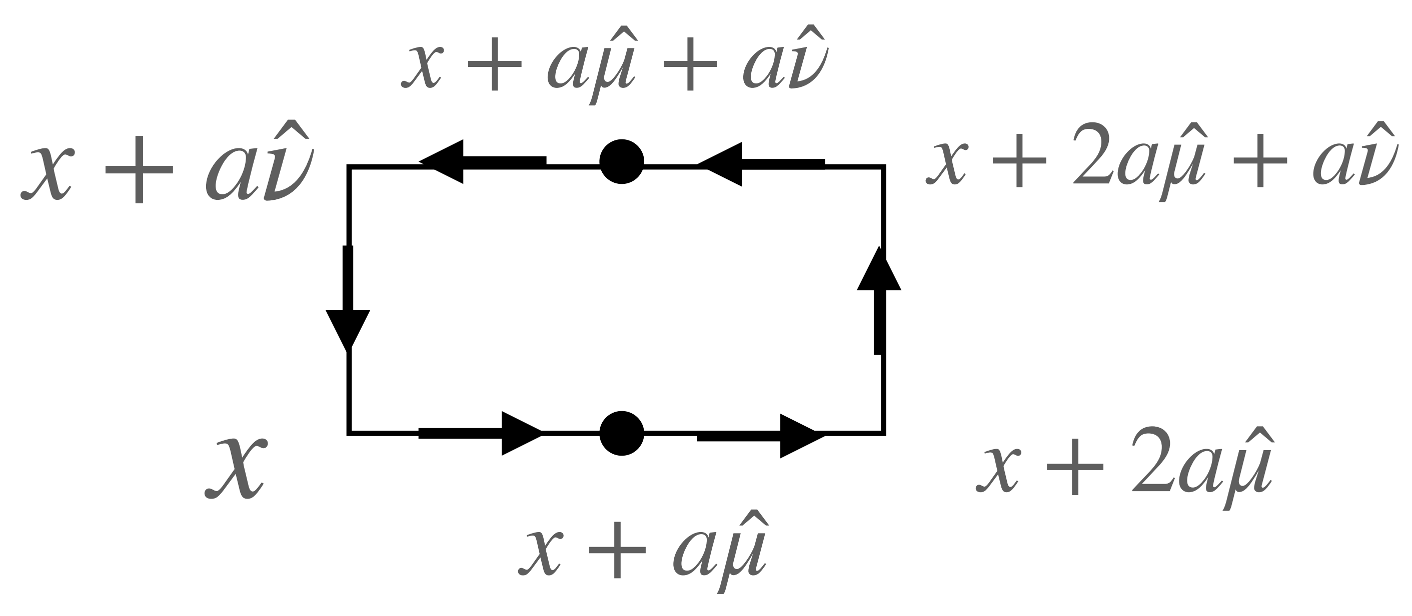
Discretized QCD

gauge action

- $\mathscr{P}_{\mu\nu}(x) = U_\mu(x)U_\nu(x+a\hat{\mu})U_\mu^\dagger(x+a\hat{\nu})U_\nu^\dagger(x) = 1 + ig_0a^2F_{\mu\nu}(x+\frac{a}{2}(\hat{\mu}+\hat{\nu})) - \frac{1}{2}a^4g_0^2F_{\mu\nu}(x+\frac{a}{2}(\hat{\mu}+\hat{\nu}))F_{\mu\nu}(x+\frac{a}{2}(\hat{\mu}+\hat{\nu})) + i\mathcal{O}(a^4)$
- it can also be used to define the gauge field tensor $F_{\mu\nu}$, and also gauge action：
  $\mathcal{S}_g = \frac{1}{2g_0^2}\sum_{x,\mu\nu}\mathrm{Re}\left[1 - \mathrm{Tr}[\mathscr{P}_{\mu\nu}(x)]\right] = \frac{1}{2}\mathrm{Tr}[\int \mathrm{d}^4xF_{\mu\nu}F_{\mu\nu}] + \mathcal{O}(a^2)$
  - Such an action has the $\mathcal{O}(a^2)$ discretization error.
  - It can combine with the 1x2 loop
    $\mathscr{P}_{\mu\nu}^{Rect}(x) = U_\mu(x)U_\mu(x+a\hat{\mu})U_\nu(x+2a\hat{\mu})U_\mu^\dagger(x+a\hat{\mu}+a\hat{\nu})U_\mu^\dagger(x+a\hat{\nu})U_\nu^\dagger(x)$
    to construct the Symanzik or Iwasaki action, to suppress the discretization error to $\mathcal{O}(a^4)$。

$$S_g^{\mathrm{Symanzik}} = \frac{5}{3}S_g^{1x1} - \frac{1}{12}S_g^{1x2}$$

$$S_g^{\mathrm{Iwasaki}} = (1+8\times 0.331)S_g^{1x1} - 0.331S_g^{1x2}$$



Discretized QCD

Naive and Staggered actions

- The naive discretization suffers from the doubling problem:
  - $\mathcal{S}_q^{Naive}(m) = \sum_{x,y} \bar{\psi}(x) D_{Naive}(m; x, y)\psi(y),\ D_{Naive}(m; x, y) = \frac{1}{2a}\sum_{\mu}\gamma_\mu\big(U_\mu(x)\delta_{y,x+a\hat{\mu}} - U_\mu^\dagger(x - a\hat{\mu})\delta_{y,x-a\hat{\mu}}\big) + m\delta_{y,x}$
  - The propagator has $1/m$ IR poles at $pa = (0/\pi, 0/\pi, 0/\pi, 0/\pi)$, which is different from the continuum theory.
- Staggered fermion:
  - $\psi^{\rm st}(x) = \gamma_4^{x_4}\gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}\psi(x),\ \{\gamma_1^{\rm st}, \gamma_2^{\rm st}, \gamma_3^{\rm st}, \gamma_4^{\rm st}\} = \{(-1)^{x_4}, (-1)^{x_1+x_4}, (-1)^{x_1+x_2+x_4}, 1\};$
  - 16 IR poles $\rightarrow$ 4 IR poles.
  - Mixing between IR poles can be suppressed with kinds of the improvement, likes the so-call highly-improved staggered quark (HISQ).

$\boxed{\text{Staggered/HISQ}} \xrightarrow{\text{Cost x10}} \boxed{\text{Wilson/Clover}} \xrightarrow{\text{Cost x10}} \boxed{\text{Domain wall}} \xrightarrow{\text{Cost x10}} \boxed{\text{Overlap}}$

Discretized QCD

Wilson and clover actions

- Wilson fermion action：
  - $D + m \rightarrow D + aD^2 + m$
  - It removes the unphysical IR pole at $p_i = \pi/a$, while introduce the additional chiral symmetry breaking at $\mathcal{O}(\alpha_s/a)$.
- Clover fermion action:
  - $D + m \rightarrow D + aD^2 + m + ac_{\text{sw}}\sigma_{\mu\nu}F^{\mu\nu}$
  - Suppress the additional chiral symmetry breaking at $\mathcal{O}(\alpha_s^2/a)$.
- The cost of either Wilson or Clover action is $\mathcal{O}(10)$ of the Staggered fermion.

$$\boxed{\text{Staggered/HISQ}} \xrightarrow{\text{Cost x10}} \boxed{\text{Wilson/Clover}} \xrightarrow{\text{Cost x10}} \boxed{\text{Domain wall}} \xrightarrow{\text{Cost x10}} \boxed{\text{Overlap}}$$

Discretized QCD

Ginsparg-Wilson action

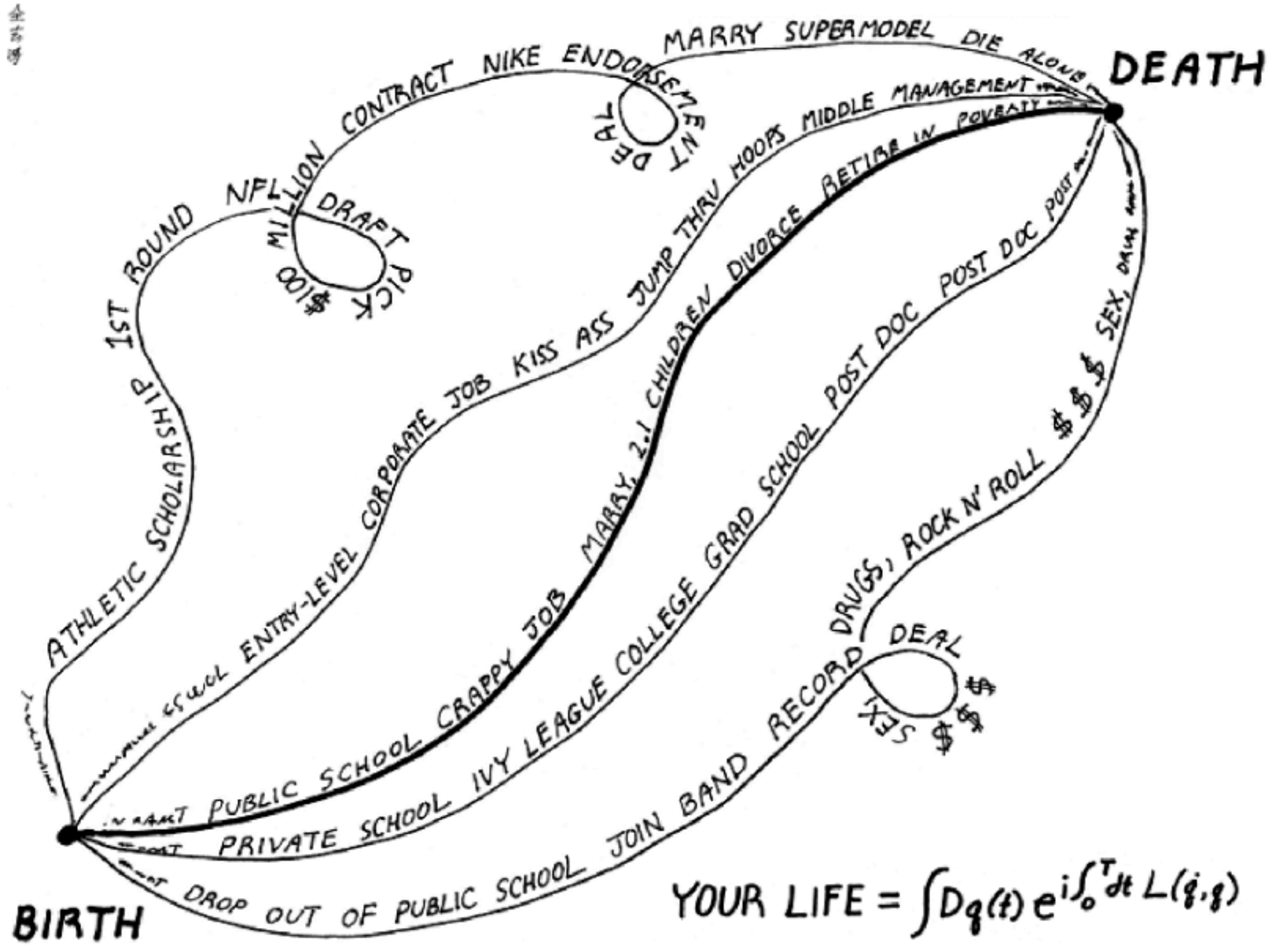
- Ginsparg-Wilson relation: $\gamma_5 D_{\rm GW} + D_{\rm GW}\gamma_5 = \frac{1}{\rho} D_{\rm GW}\gamma_5 D_{\rm GW}$.
- Overlap fermion as a possible solution:

$$\mathcal{S}_q^{ov}(m) = \sum_{x,y} \bar{\psi}(x)\big(\delta_{xy}m + \sum_z D_{ov}(\rho; x, z)\frac{\rho/a}{\delta_{zy} - D_{ov}(\rho; z, y)/2}\big)\psi(y),\ D_{ov}(\rho) = 1 + \frac{D_w(-\rho)}{\sqrt{D_w(-\rho)D_W^\dagger(-\rho)}}$$

- In $p \to 0$ region, $D_{ov} \to a\gamma_\mu p_\mu$;
- In $p \to \pi/a$ region, $D_{ov} \to \mathcal{O}(1)$.
- But approximate the sign function $\frac{\gamma_5 D_w(-\rho)}{\sqrt{D_w(-\rho)D_W^\dagger(-\rho)}} = \frac{\gamma_5 D_w(-\rho)}{|\gamma_5 D_w(-\rho)|}$ need $\mathcal{O}(100)$ cost of the Wilson/Clover action.
- Domain wall fermion action is an approximation of overlap fermion with $\mathcal{O}(10)$ cost of the Wilson/Clover action.

Staggered/HISQ $\xrightarrow{\text{Cost x10}}$ Wilson/Clover $\xrightarrow{\text{Cost x10}}$ Domain wall $\xrightarrow{\text{Cost x10}}$ Overlap

Path integral approach of quantum field theory



○ Harmonic oscillator：

- $\langle q';t'|q;t\rangle = \sqrt{\frac{m\omega}{\pi}} e^{-\frac{q'^2+q^2}{2}m\omega}\left(e^{-\mathrm{i}\frac{\omega}{2}(t'-t)} + 2q'qm\omega e^{-\mathrm{i}\frac{3\omega}{2}(t'-t)} + \frac{-2(q'^2+q^2)m\omega + 4q'^2q^2m^2\omega^2+1}{2}e^{-\mathrm{i}\frac{5\omega}{2}(t'-t)} + \ldots.\right).$

- The paths which deviate from the classical one by a few $\hbar$ are curial for the quantized energy levels;

- All the information can be extracted from the above partition function.

○ QCD:

- The partition function is not accessible due to the infinite dimension integral of the functional phase space.

- But it is possible to sample in the phase space to estimate of the expectation value of given quantities!

Stochastic quantization approach of quantum gauge theory

- Stochastic quantization for the gauge theory:

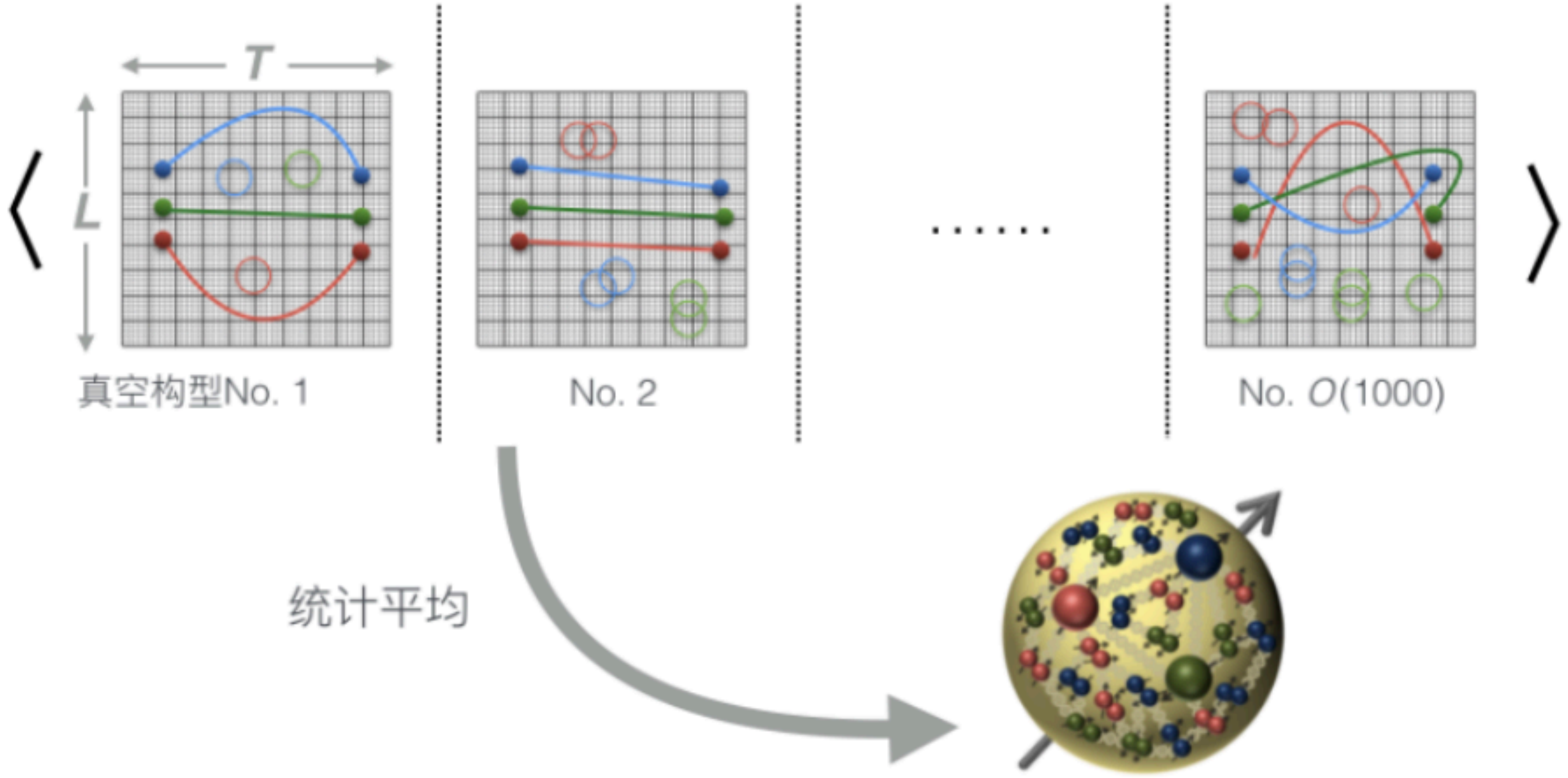
Parisi and Wu, Sci.Sin. 24(1981)483

- $H[\pi, \phi] = \sum_x \frac{\pi^2(x)}{2} + S[\phi], P(\pi(x,\tau)) = e^{-\pi^2/2};$
- $\frac{d\pi(x)}{d\tau} = -\frac{\partial H[\pi,\phi]}{\partial \phi(x)} = -\frac{\partial S[\phi]}{\partial \phi(x)}, \frac{d\phi(x)}{d\tau} = \frac{\partial H[\pi,\phi]}{\partial \pi(x)} = \pi(x)$
- Only the gauge invariant correlation functions will be finite when $\tau \to \infty$;
- Avoid the Faddeev-Popov ghost and Gribov copies.

- Hybrid Monte Carlo (HMC):

- Sample the functional phase space following stochastic quantization with step size $\delta\tau \sim \hbar$;
- The gauge dependent correlation function will be zero when we replace gauge field A by Wilson link $U = e^{\mathrm{i}g\int \mathrm{d}xA}$.

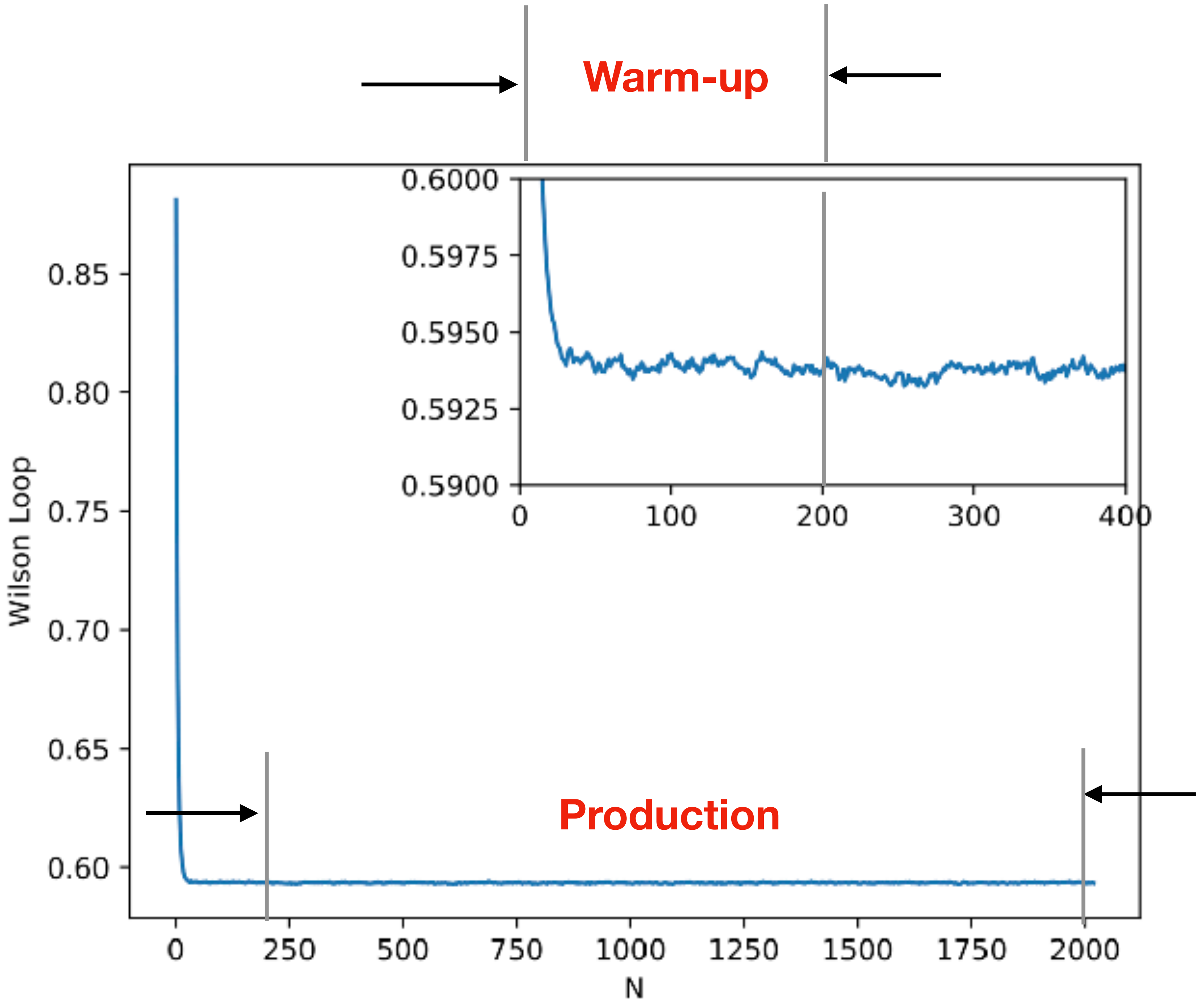
Stochastic quantization + path integral



$$\langle O[U]\rangle = \frac{\int [\Pi_y \mathrm{d}U(y)] O[U] e^{-S[U]}}{\int [\Pi_y \mathrm{d}U(y)] e^{-S[U]}} = \frac{1}{n}\sum_i O[U] + \mathcal{O}(\frac{1}{\sqrt{n}})$$

- Approximate the correlation functions defined by the path integral approach,
- with the samples during the stochastic quantization evolution at large enough τ.

Cost of HMC

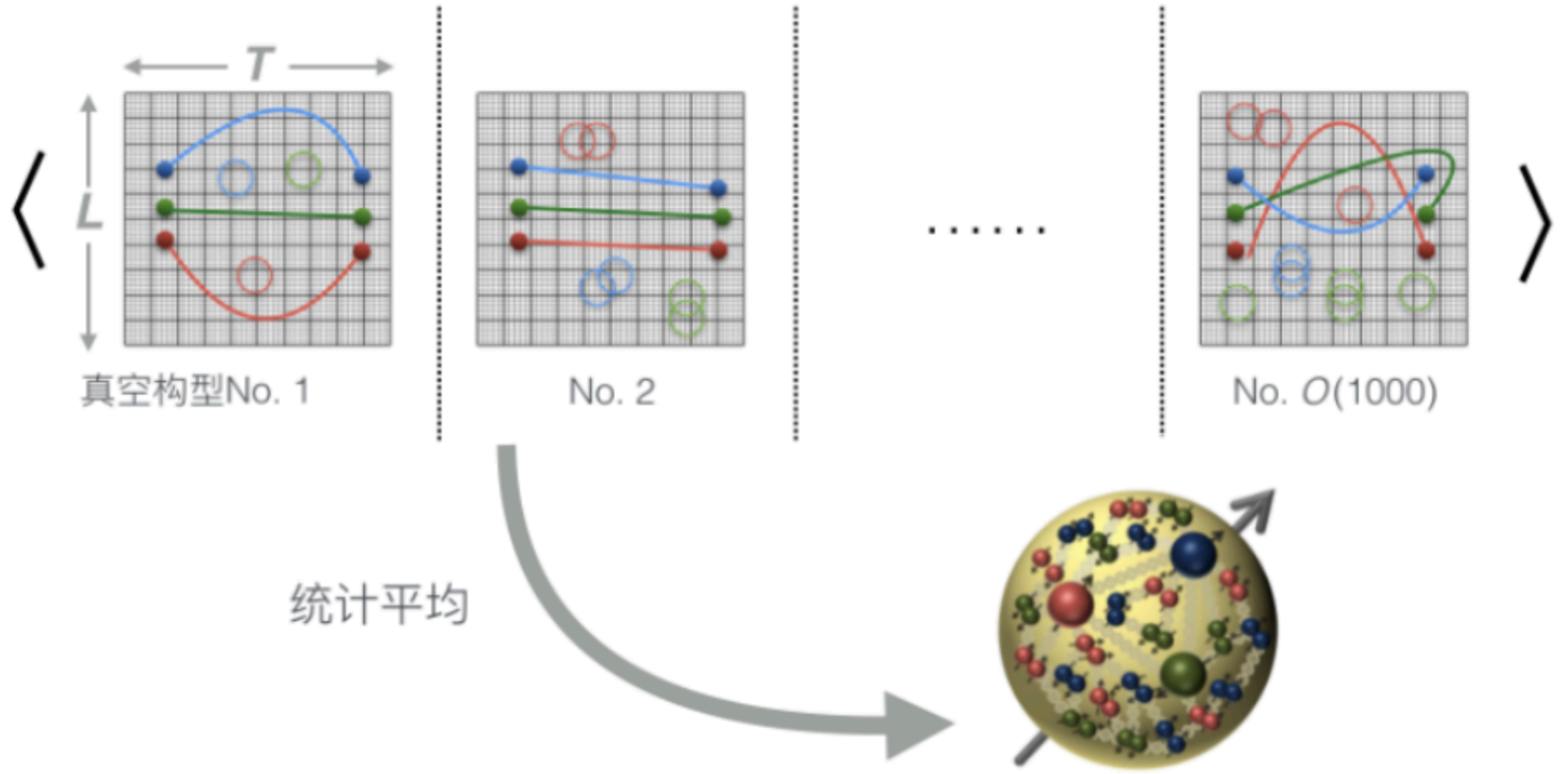


- Case 1:
  - Clover+Symanzik,
  - 24^3x72, a=0.108 fm, m_π=300 MeV,
  - 8 V100 GPUs:
  - One week for warn-up;
  - Another week for 200 configurations (5 traj. per conf.)
  - And 13 GB storage.

- Case 2:
  - Mobius DWF+Iwasaki,
  - 96^3x192, a=0.071 fm, m_π=140 MeV,
  - 512 V100 GPUs:
  - One year for warn-up;
  - Another year for 200 configurations (5 traj. per conf.)
  - And 2,278 GB storage.

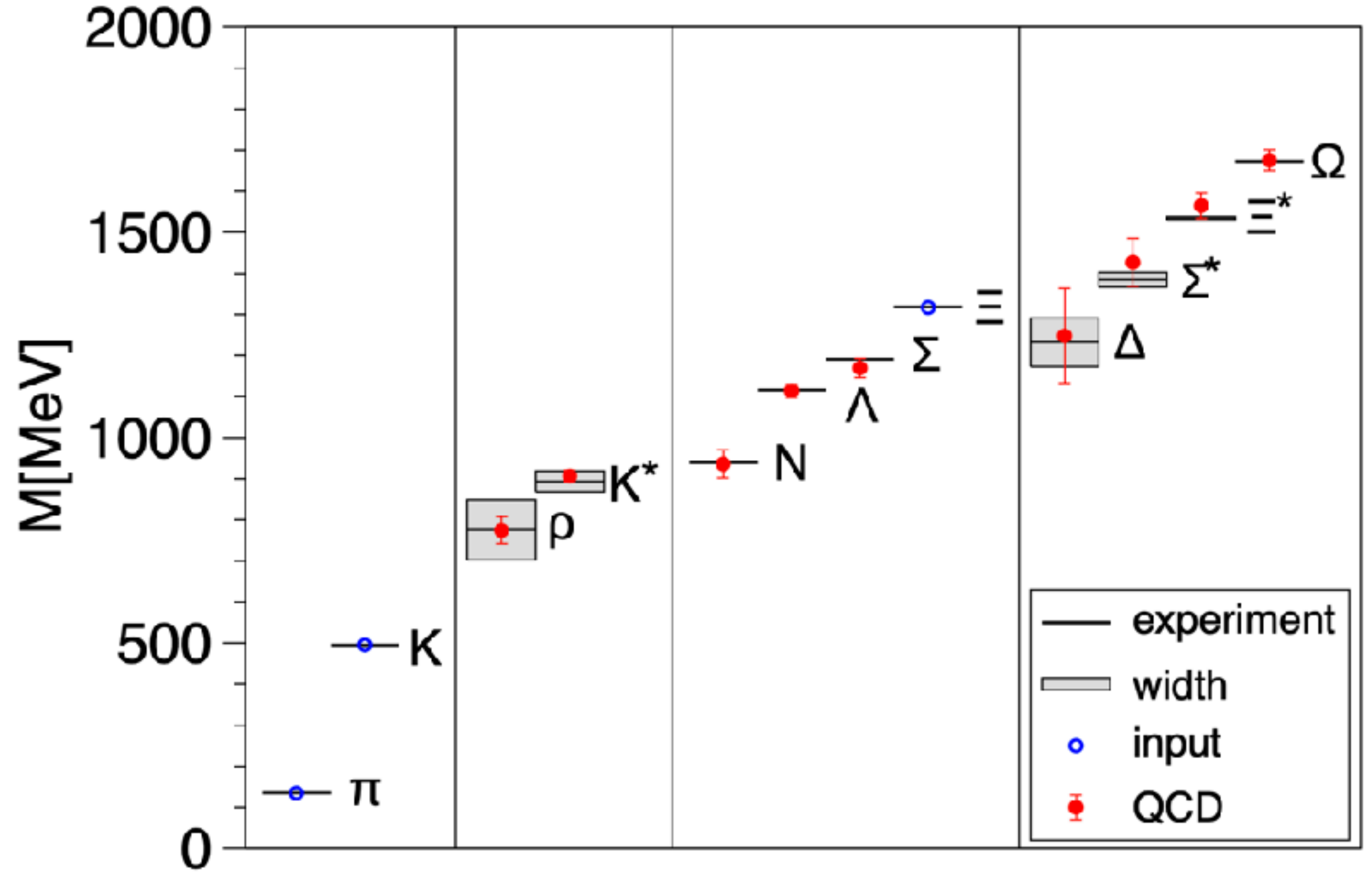
Outline

Basic idea of LQCD



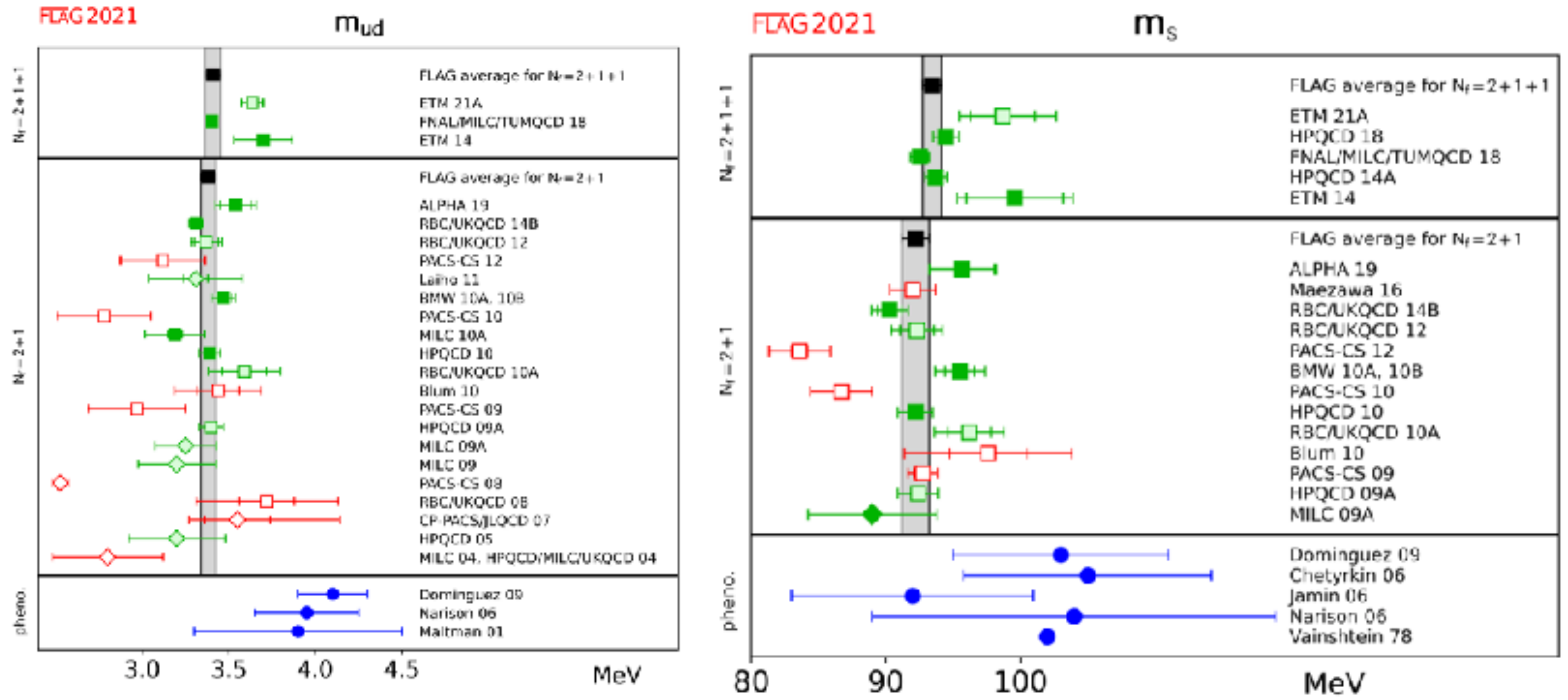
Hadron spectrum

BMWc, Science 322(2008)1224



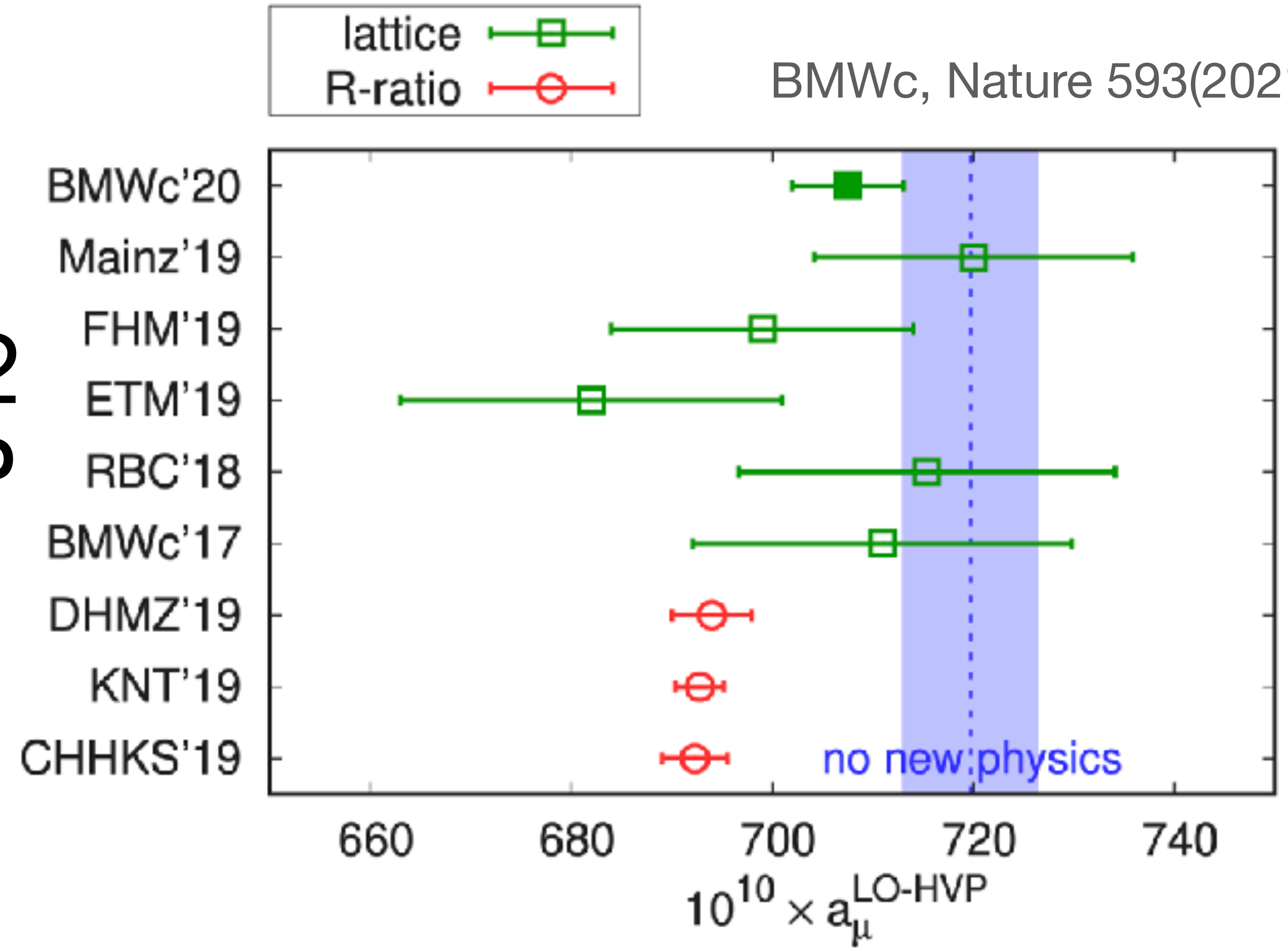
Quark mass determination

FLAG, EPJC80(2020)113



Muon g-2 from HVP

BMWc, Nature 593(2021)51



Hadron mass from Lattice QCD

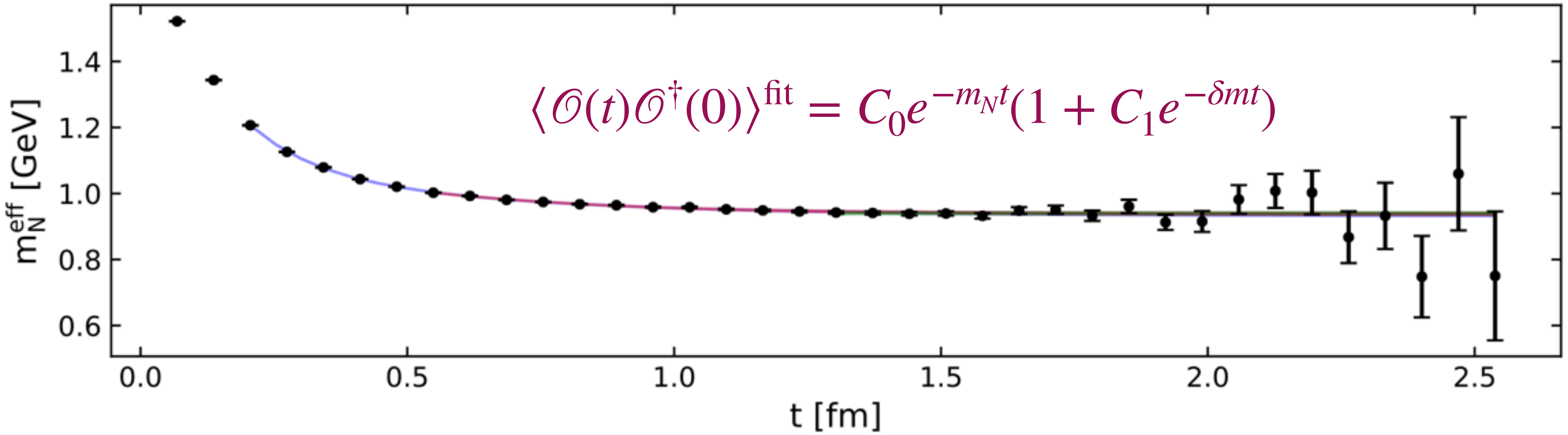
- From the time order product ($\mathcal{O} = \bar{\psi}\gamma_5\psi$):

$$\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \sum_n \langle \mathcal{O}(t)\,|\,n\rangle \frac{e^{-E_n t}}{2E_n}\langle n\,|\,\mathcal{O}^\dagger(0)\rangle \xrightarrow[t\to\infty]{} \frac{|\langle \mathcal{O}(t)\,|\,0\rangle|^2}{2E_0}e^{-E_0 t}$$

- From the path integral ($S(x;y) = (D(x;y)+m)^{-1}$):

$$\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \sum_{\vec{x}} \langle \mathrm{Tr}[S(\vec{0},0;\vec{x},t)\gamma_5 S(\vec{x},t;\vec{0},0)\gamma_5]\rangle = \sum_{\vec{x}} \langle \mathrm{Tr}[S^\dagger(\vec{x},t;\vec{0},0)S(\vec{x},t;\vec{0},0)]\rangle$$

- All the ground state hadron masses can be obtained with different $\mathcal{O}$ and m.



$$m_N^{\text{eff}} = \frac{1}{a}\log\frac{\langle \mathcal{O}(t-a)\mathcal{O}^\dagger(0)\rangle}{\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle}$$

C. Alexandrou, et,al. ETMC, PRD104(2021)074515

The light quark masses

From lattice QCD

P.Zyla et,al, PTEP(2020)083C01 (PDG2020):

- $m_p = 938.27\ \mathrm{MeV} = m_{p,\mathrm{QCD}} + 1.00(16)\ \mathrm{MeV} + \ldots;$

Sz. Borsanyi, et,al. Science347(2015)1452

- $\boxed{m_n = 939.57\ \mathrm{MeV};}$
- $\boxed{m_\pi^0 = 134.98\ \mathrm{MeV};}$

$(m_{p,\mathrm{QCD}} + m_n)/2 = 938.4(1)\ \mathrm{MeV}$

- $m_\pi^+ = 139.57\ \mathrm{MeV} = m_\pi^0 + 4.53(6)\ \mathrm{MeV} + \ldots;$

X. Feng, et,al. PRL128(2022)062003

- $\boxed{m_K^0 = 497.61(1)\ \mathrm{MeV} = m_{K,\mathrm{QCD}}^0 + 0.17(02)\ \mathrm{MeV} + \ldots;}$
- $\boxed{m_K^+ = 493.68(2)\ \mathrm{MeV} = m_{K,\mathrm{QCD}}^+ + 2.07(15)\ \mathrm{MeV} + \ldots}$

D. Giusti, et,al. PRD95(2017)114504

$\boxed{\alpha_s^{\mathrm{bare}}}$

$\downarrow$

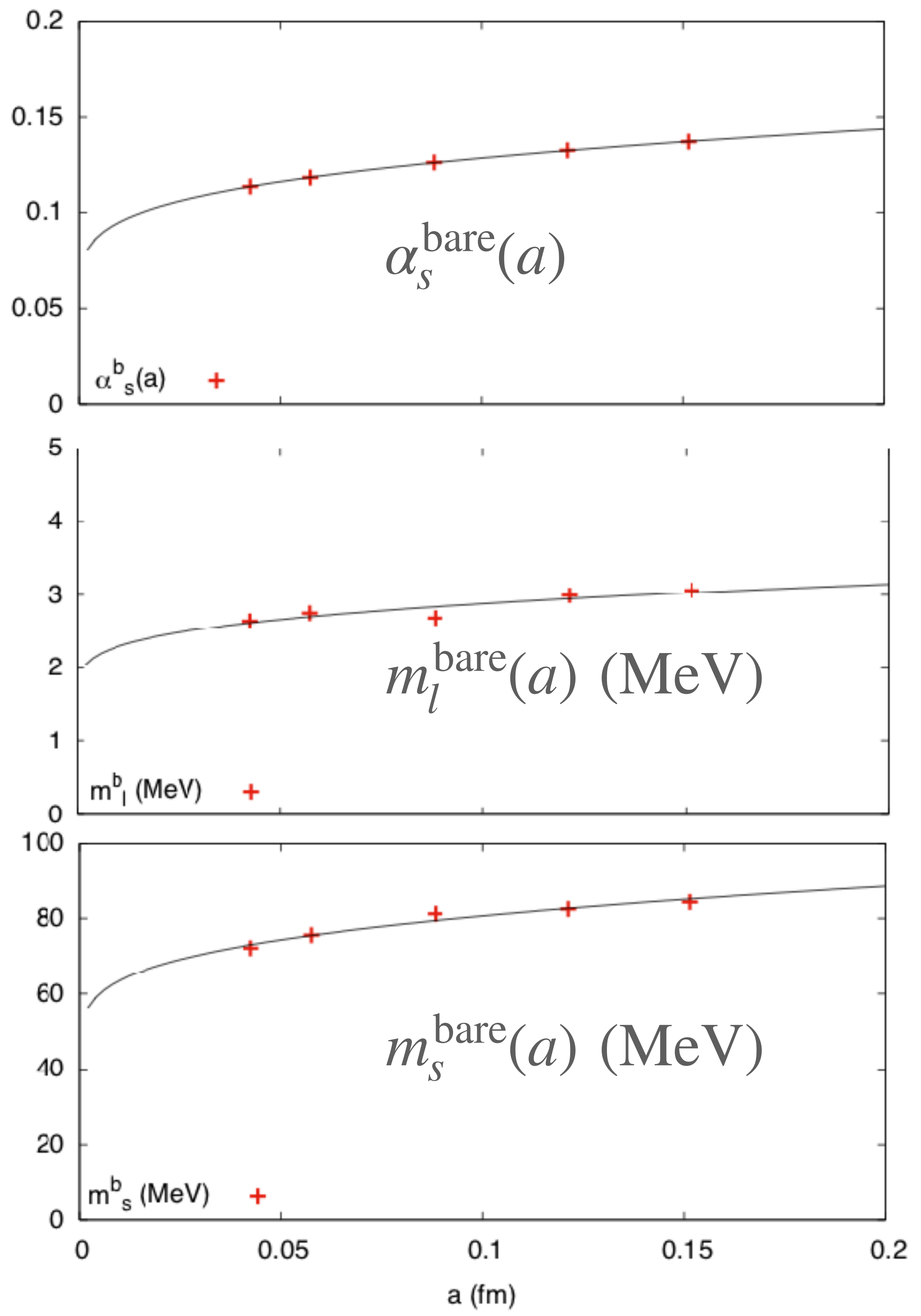
$\boxed{(m_u^{\mathrm{bare}} + m_d^{\mathrm{bare}})/2 \text{ and } a}$ (lattice spacing)

$\downarrow$

$\boxed{m_u^{\mathrm{bare}},\ m_d^{\mathrm{bare}} \text{ and } m_s^{\mathrm{bare}}}$

The light quark masses

Lattice spacing dependence



A. Bazavov, et,al., MILC, PRD87(2013)054505

- The lattice spacing a is very sensitive to the bare coupling;
- The light quark mass to satisfy the condition is very small;
- Renormalization is needed to convert the result to MS-bar.

$\boxed{\alpha_s^{\text{bare}}}$

$\downarrow$

$\boxed{(m_u^{\text{bare}} + m_d^{\text{bare}})/2 \text{ and } a}$ (lattice spacing)

$\downarrow$

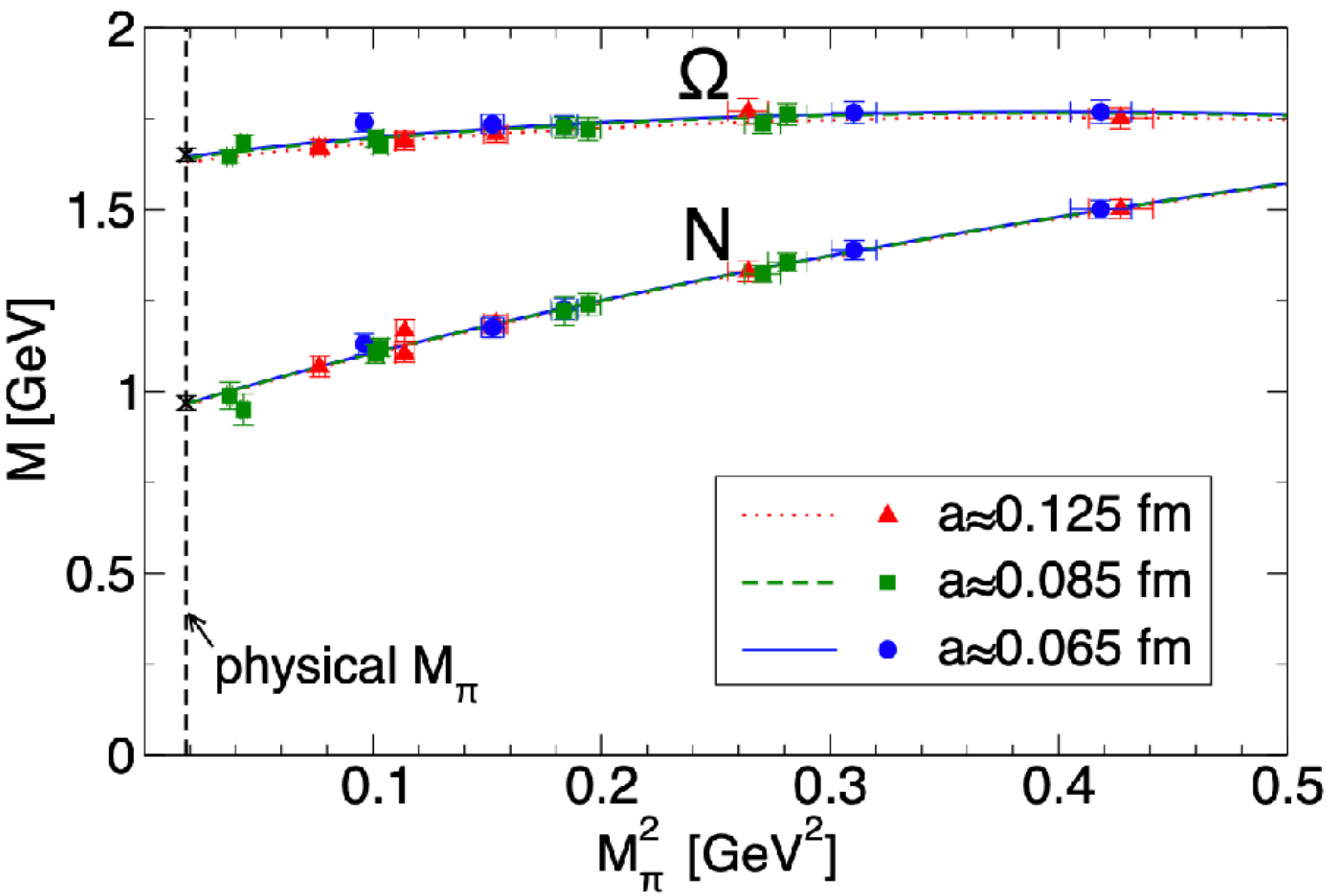
$\boxed{m_u^{\text{bare}},\ m_d^{\text{bare}} \text{ and } m_s^{\text{bare}}}$

Ensembles used by Science 322(2008)1224

- $2^3 \times (4-6)\ \mathrm{fm}^4$ box in most of the cases
- Check the finite volume dependence using a few ensembles.

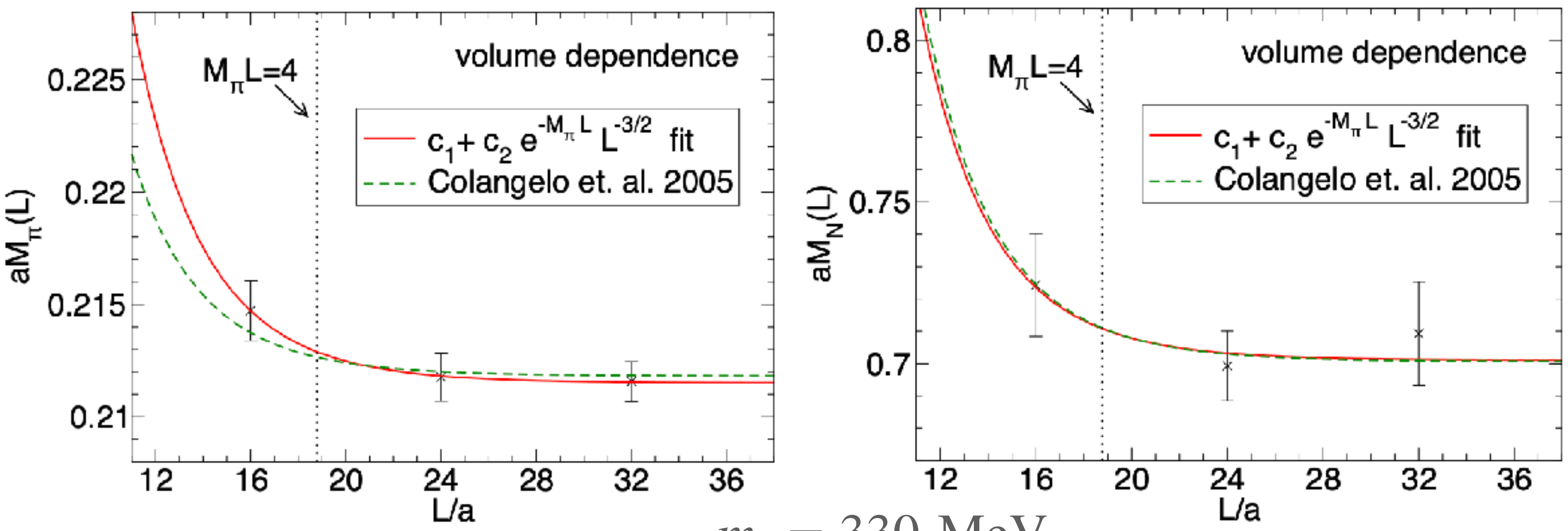
β	am_{ud}	am_s	$L^3 \cdot T$	# traj.
3.3	-0.0960	-0.057	$16^3 \cdot 32$	10000
	-0.1100	-0.057	$16^3 \cdot 32$	1450
	-0.1200	-0.057	$16^3 \cdot 64$	4500
	-0.1233	-0.057	$16^3 \cdot 64$ / $24^3 \cdot 64$ / $32^3 \cdot 64$	5000 / 2000 / 1300
	-0.1265	-0.057	$24^3 \cdot 64$	2100
3.57	-0.0318	0.0 / -0.01	$24^3 \cdot 64$	1650 / 1650
	-0.0380	0.0 / -0.01	$24^3 \cdot 64$	1350 / 1550
	-0.0440	0.0 / -0.007	$32^3 \cdot 64$	1000 / 1000
	-0.0483	0.0 / -0.007	$48^3 \cdot 64$	500 / 1000
3.7	-0.0070	0.0	$32^3 \cdot 96$	1100
	-0.0130	0.0	$32^3 \cdot 96$	1450
	-0.0200	0.0	$32^3 \cdot 96$	2050
	-0.0220	0.0	$32^3 \cdot 96$	1350
	-0.0250	0.0	$40^3 \cdot 96$	1450

BMWc, Science 322(2008)1224



- The lattice spacing dependence is mild given the precision required.
- The m_π dependence of m_Ω is relatively weak and can be used to set the scale.

Finite volume effect



$m_\pi = 330$ MeV

β	am_{ud}	am_s	$L^3 \cdot T$	# traj.
3.3	-0.0960	-0.057	$16^3 \cdot 32$	10000
	-0.1100	-0.057	$16^3 \cdot 32$	1450
	-0.1200	-0.057	$16^3 \cdot 64$	4500
	$\boxed{-0.1233}$	$\boxed{-0.057}$	$\boxed{16^3 \cdot 64 / 24^3 \cdot 64 / 32^3 \cdot 64}$	5000 / 2000 / 1300
	-0.1265	-0.057	$24^3 \cdot 64$	2100
3.57	-0.0318	0.0 / -0.01	$24^3 \cdot 64$	1650 / 1650
	-0.0380	0.0 / -0.01	$24^3 \cdot 64$	1350 / 1550
	-0.0440	0.0 / -0.007	$32^3 \cdot 64$	1000 / 1000
	-0.0483	0.0 / -0.007	$48^3 \cdot 64$	500 / 1000
3.7	-0.0070	0.0	$32^3 \cdot 96$	1100
	-0.0130	0.0	$32^3 \cdot 96$	1450
	-0.0200	0.0	$32^3 \cdot 96$	2050
	-0.0220	0.0	$32^3 \cdot 96$	1350
	-0.0250	0.0	$40^3 \cdot 96$	1450

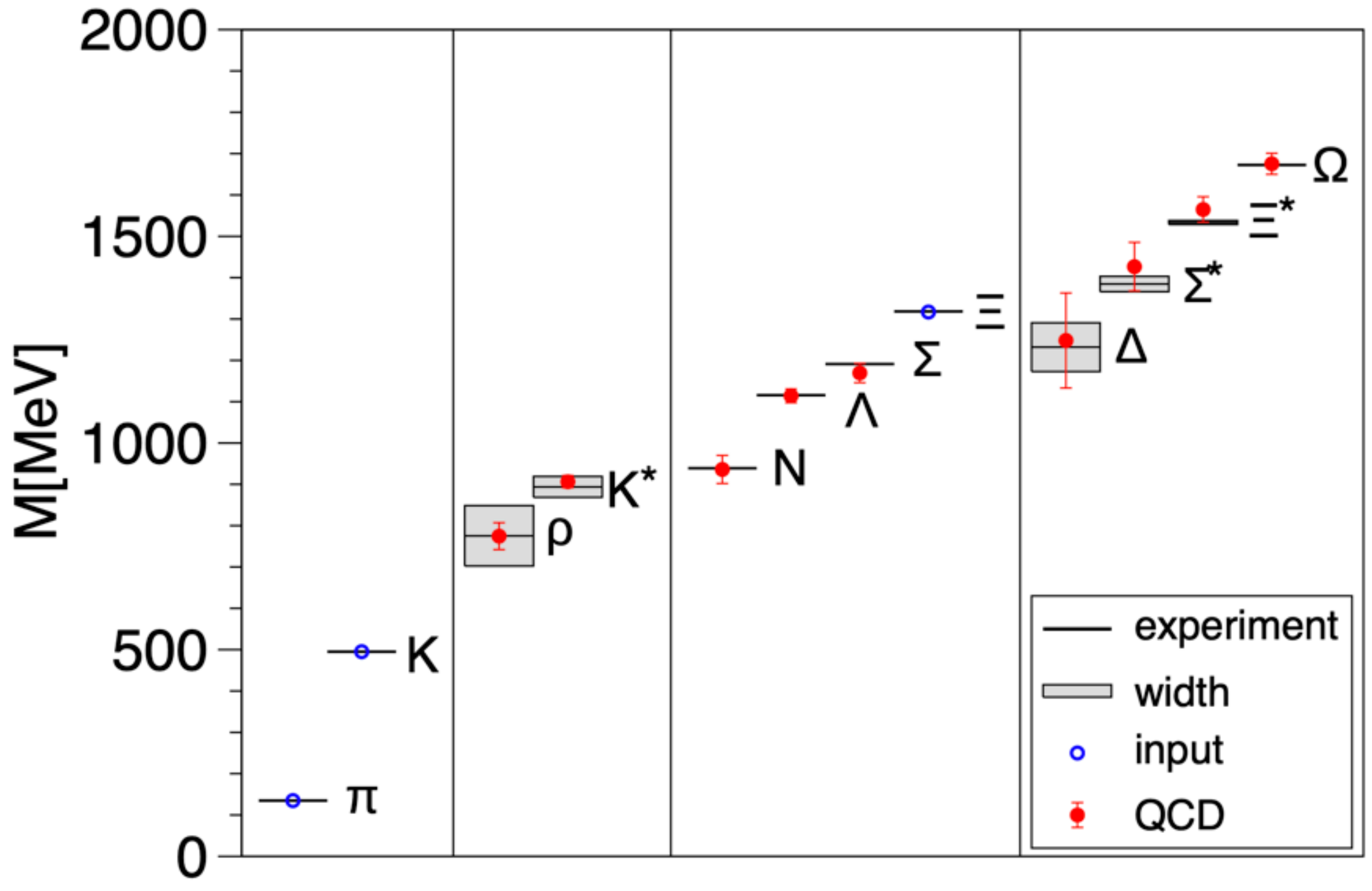
BMWc, Science 322(2008)1224

- One would need $m_\pi L \sim 4$ to be free of the finite volume effect.
- Most of the ensembles can have obvious finite volume correction based on above criteria.

Final results

X	Exp. (27)	M_X (Ξ set)	M_X (Ω set)
ρ	0.775	0.775(29)(13)	0.778(30)(33)
K^*	0.894	0.906(14)(4)	0.907(15)(8)
N	0.939	0.936(25)(22)	0.953(29)(19)
Λ	1.116	1.114(15)(5)	1.103(23)(10)
Σ	1.191	1.169(18)(15)	1.157(25)(15)
Ξ	1.318	1.318	1.317(16)(13)
Δ	1.232	1.248(97)(61)	1.234(82)(81)
Σ^*	1.385	1.427(46)(35)	1.404(38)(27)
Ξ^*	1.533	1.565(26)(15)	1.561(15)(15)
Ω	1.672	1.676(20)(15)	1.672

BMWc, Science 322(2008)1224



- They used the physical value of either m_Ξ or m_Ω as input to set the lattice spacing.
- The results are consistent with each other within the uncertainty.

From 2008 to 2015

- N_f = 1+1+1+1 instead of N_f = 2+1;
- $\beta = 6/g^2$ becomes smaller at the same lattice spacing, due to additional dynamical charm flavor;
- Much larger box at the same lattice spacing, and then finite volume effect is highly suppressed.

β	am_{ud}	am_s	$L^3 \cdot T$		# traj.
3.3	-0.0960	-0.057	$16^3 \cdot 32$	~0.125 fm	10000
	-0.1100	-0.057	$16^3 \cdot 32$		1450
	-0.1200	-0.057	$16^3 \cdot 64$		4500
	-0.1233	-0.057	$16^3 \cdot 64$ / $24^3 \cdot 64$ / $32^3 \cdot 64$		5000 / 2000 / 1300
	-0.1265	-0.057	$24^3 \cdot 64$		2100
3.57	-0.0318	0.0 / -0.01	$24^3 \cdot 64$	~0.085 fm	1650 / 1650
	-0.0380	0.0 / -0.01	$24^3 \cdot 64$		1350 / 1550
	-0.0440	0.0 / -0.007	$32^3 \cdot 64$		1000 / 1000
	-0.0483	0.0 / -0.007	$48^3 \cdot 64$		500 / 1000
3.7	-0.0070	0.0	$32^3 \cdot 96$	~0.065 fm	1100
	-0.0130	0.0	$32^3 \cdot 96$		1450
	-0.0200	0.0	$32^3 \cdot 96$		2050
	-0.0220	0.0	$32^3 \cdot 96$		1350
	-0.0250	0.0	$40^3 \cdot 96$		1450

BMWc, Science 322(2008)1224

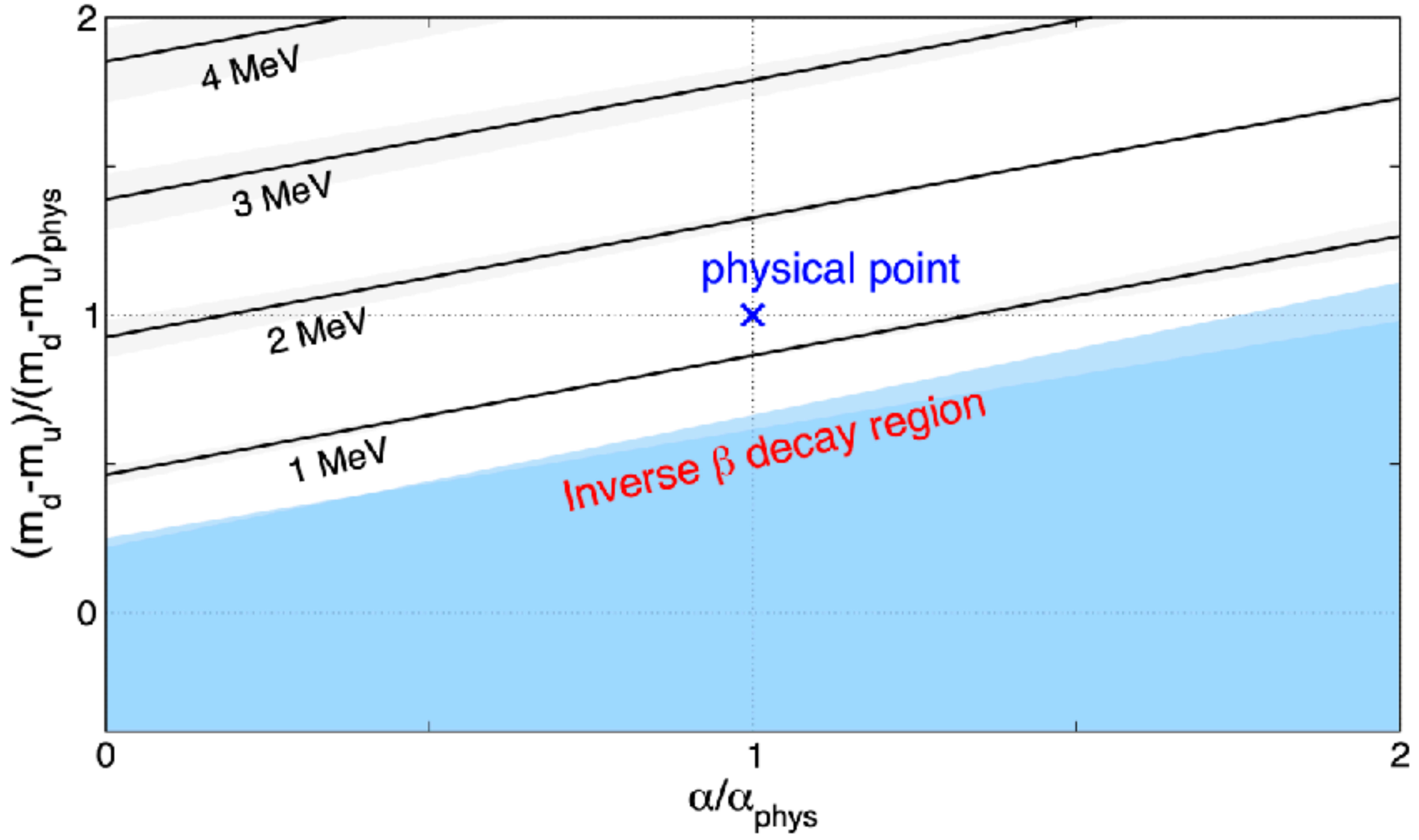
	$6/g^2$	am_u	am_d	am_s	$L^3 \times T$	m_π[MeV]	$m_\pi L$	×1000 trajectories
~0.102 fm	3.2	-0.0686	-0.0674	-0.068	$32^3 \times 64$	405	6.9	1
	3.2	-0.0737	-0.0723	-0.058	$32^3 \times 64$	347	5.9	4
	3.2	-0.0733	-0.0727	-0.058	$32^3 \times 64$	345	5.8	1
	3.2	-0.0776	-0.0764	-0.05	$32^3 \times 64$	289	4.9	4
	3.2	-0.0805	-0.0795	-0.044	$32^3 \times 64$	235	4.0	12
	3.2	-0.0806	-0.0794	-0.033	$32^3 \times 64$	256	4.4	12
	3.2	-0.0686	-0.0674	-0.02	$32^3 \times 64$	440	8.1	4
	3.2	-0.0737	-0.0723	-0.025	$32^3 \times 64$	377	6.8	4
	3.2	-0.0776	-0.0764	-0.029	$32^3 \times 64$	317	5.6	4
	3.2	-0.077	-0.0643	-0.0297	$32^3 \times 64$	404	7.3	4
	3.2	-0.073	-0.0629	-0.0351	$32^3 \times 64$	435	7.8	4
	3.2	-0.077	-0.0669	-0.0391	$32^3 \times 64$	378	6.7	4
~0.089 fm	3.3	-0.0486	-0.0474	-0.048	$32^3 \times 64$	407	6.1	1
	3.3	-0.0537	-0.0523	-0.038	$32^3 \times 64$	341	5.1	2
	3.3	-0.0535	-0.0525	-0.038	$32^3 \times 64$	340	5.0	2
	3.3	-0.0576	-0.0564	-0.03	$32^3 \times 64$	269	4.0	12
	3.3	-0.0576	-0.0564	-0.019	$32^3 \times 64$	281	4.2	12
	3.3	-0.0606	-0.0594	-0.024	$48^3 \times 64$	197	4.3	20
~0.077 fm	3.4	-0.034	-0.033	-0.0335	$32^3 \times 64$	403	5.0	4
	3.4	-0.0385	-0.0375	-0.0245	$32^3 \times 64$	318	4.0	4
	3.4	-0.0423	-0.0417	-0.0165	$48^3 \times 64$	219	4.1	4
~0.064 fm	3.5	-0.0218	-0.0212	-0.0215	$32^3 \times 64$	420	4.4	4
	3.5	-0.0254	-0.0246	-0.0145	$48^3 \times 64$	341	5.4	4
	3.5	-0.0268	-0.0262	-0.0115	$48^3 \times 64$	307	4.8	8
	3.5	-0.0269	-0.0261	-0.0031	$48^3 \times 64$	306	4.9	8
	3.5	-0.0285	-0.0275	-0.0085	$48^3 \times 64$	262	4.1	8
	3.5	-0.0302	-0.0294	-0.0049	$64^3 \times 96$	197	4.1	4

BMWc, Science 347(2015)1452

QED corrections:

$6/g^2$	e	am_u	am_d	am_s	$L^3 \times T$	m_π[MeV]	$m_\pi L$	$\times 1000$ trajectories
3.2	1.00	-0.0819	-0.0752	-0.0352	$32^3 \times 64$	373	6.6	4
3.2	$\sqrt{4\pi/137}$	-0.07788	-0.07722	-0.05022	$32^3 \times 64$	290	4.9	4
3.2	1.00	-0.0859	-0.0792	-0.0522	$32^3 \times 64$	290	4.9	4
3.2	1.41	-0.0943	-0.0812	-0.0542	$32^3 \times 64$	290	4.9	4
3.2	0.71	-0.0815	-0.0781	-0.0511	$32^3 \times 64$	290	4.9	4
3.2	1.00	-0.0889	-0.0822	-0.0462	$32^3 \times 64$	236	4.0	4
3.2	1.00	-0.0859	-0.0792	-0.0522	$24^3 \times 48$	292	3.7	5
3.2	1.00	-0.0859	-0.0792	-0.0522	$48^3 \times 96$	290	7.3	4
3.2	1.00	-0.0859	-0.0792	-0.0522	$80^3 \times 64$	289	12.2	1
3.3	1.00	-0.063	-0.0555	-0.0405	$48^3 \times 96$	335	7.4	4
3.3	1.00	-0.0666	-0.0592	-0.0329	$48^3 \times 96$	270	6.0	4
3.5	1.00	-0.034	-0.02575	-0.02575	$32^3 \times 64$	411	4.3	4
3.5	1.00	-0.0359	-0.0277	-0.0173	$48^3 \times 96$	362	5.7	4
3.5	1.00	-0.0389	-0.0307	-0.0111	$48^3 \times 96$	283	4.5	4

BMWc, Science 347(2015)1452

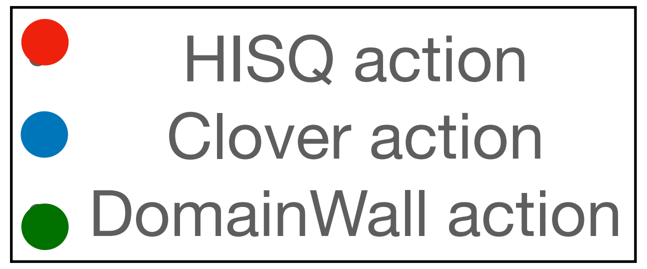


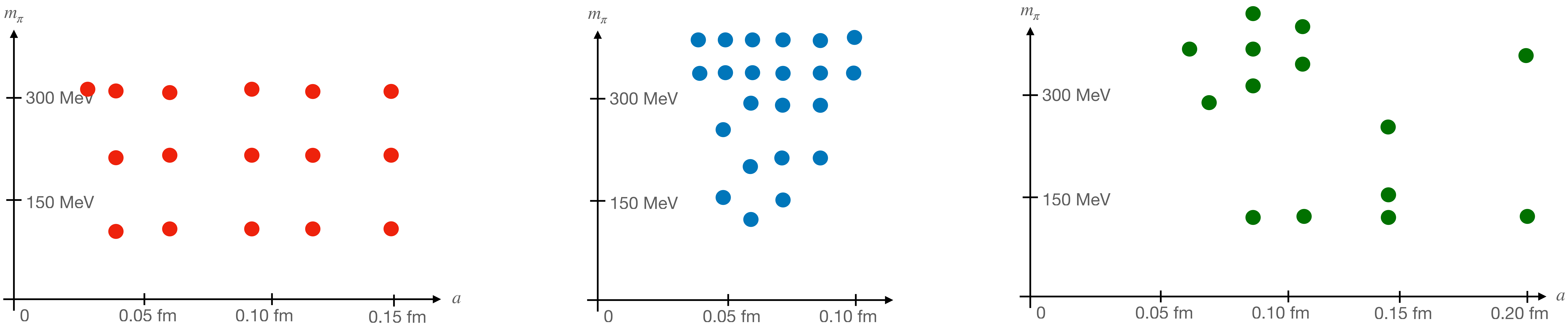
- With the simulation on the ensembles with $e \sim \mathcal{O}(1)$, they obtain the QED correction of the neutron-proton mass difference to be ~-1.00(7)(14) MeV.
- It suggests that the neutron can be lighter than proton when $\alpha \geq 2\alpha_{\text{phys}}$.

Configurations：

Lattice QCD FLAG criteria

- Three different m_π with the lightest m_π smaller than 200 MeV, or one $m_\pi \sim$ 135 MeV and another $m_\pi <$ 200 MeV;
- Three lattice spacings a, with two of $a < 0.1$ fm, and $(a_{max}/a_{min})^2 \geq 2$;
- $(m_{\pi,\mathrm{min}}/M_{\pi,\mathrm{phys}})^2 \exp\{4 - m_{\pi,\mathrm{min}}L\} < 2$, or at least three volumes.





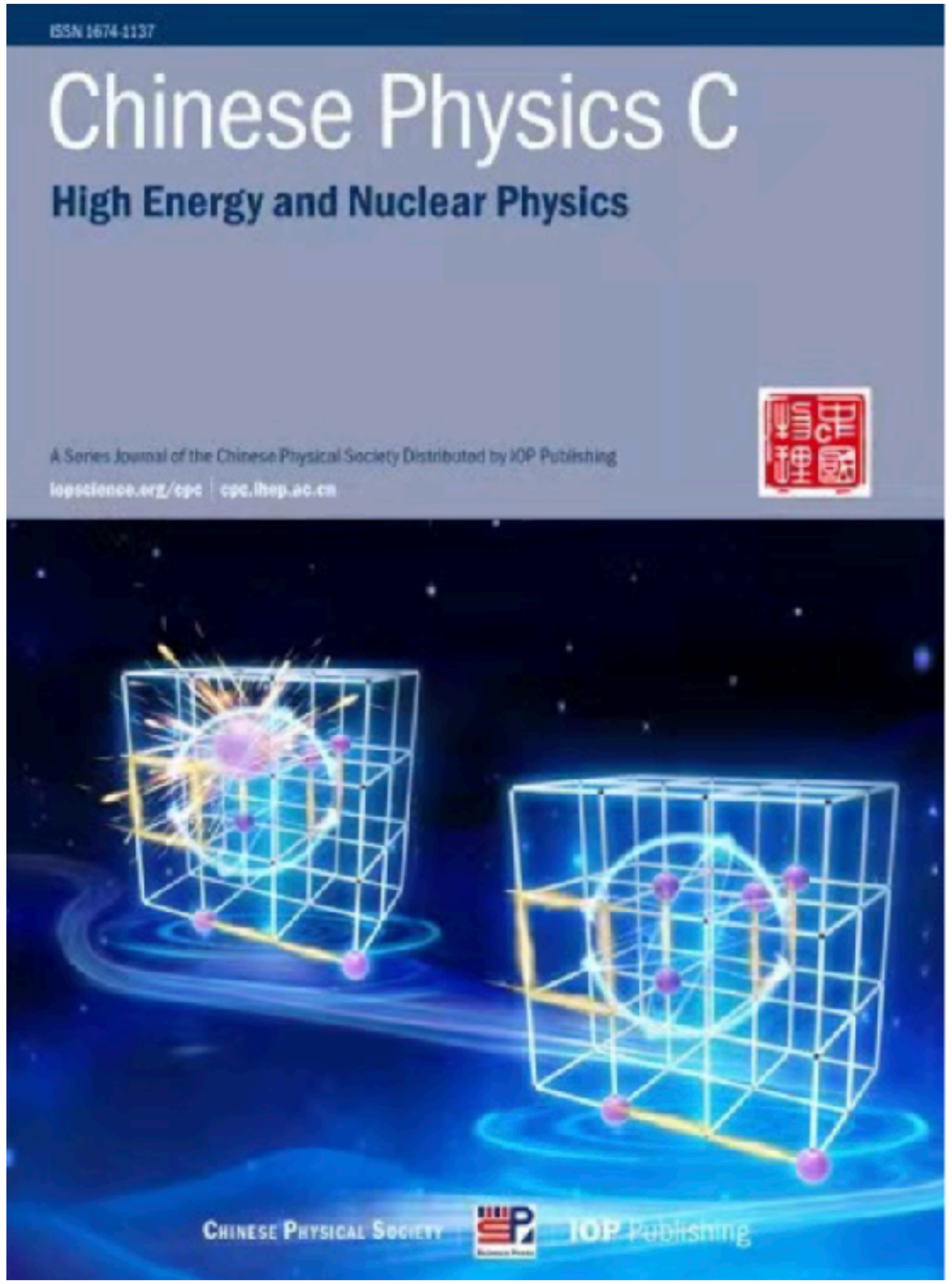
MILC collaboration

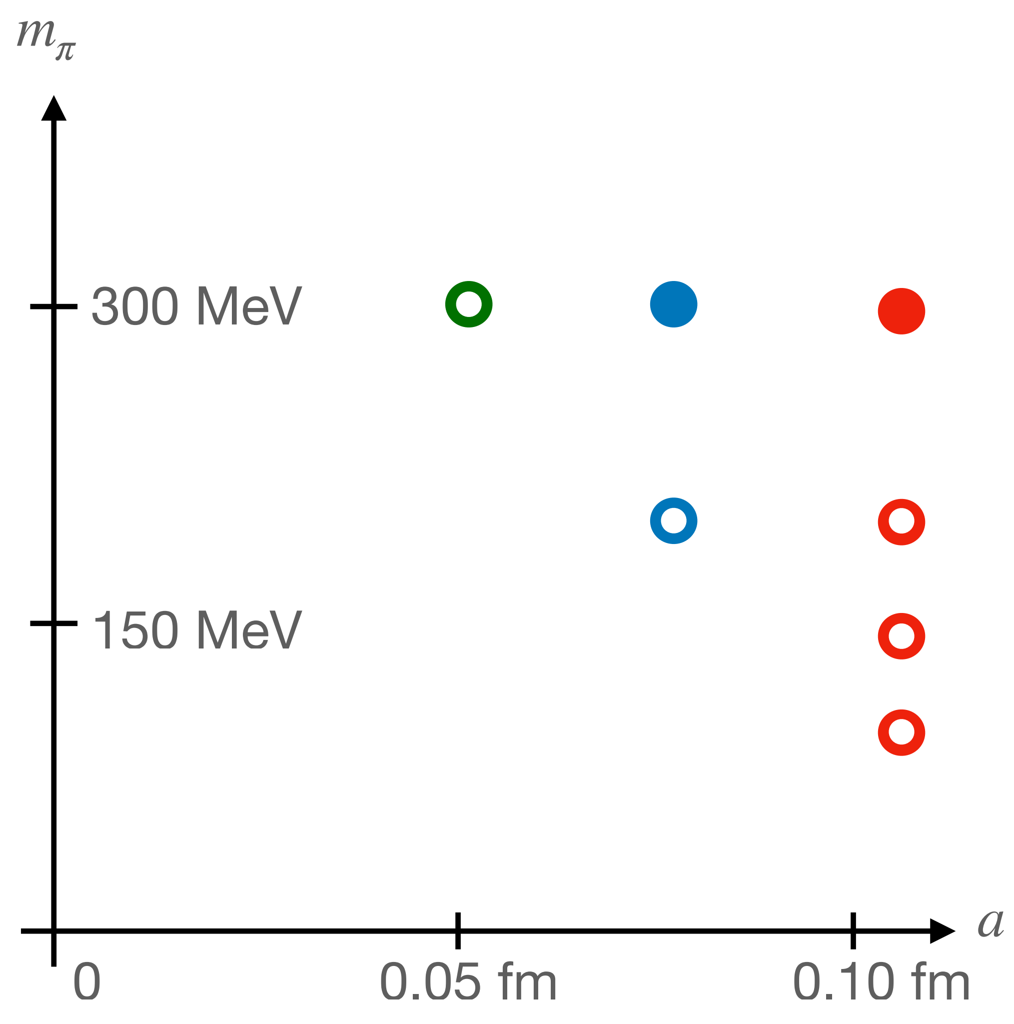
CLS collaboration

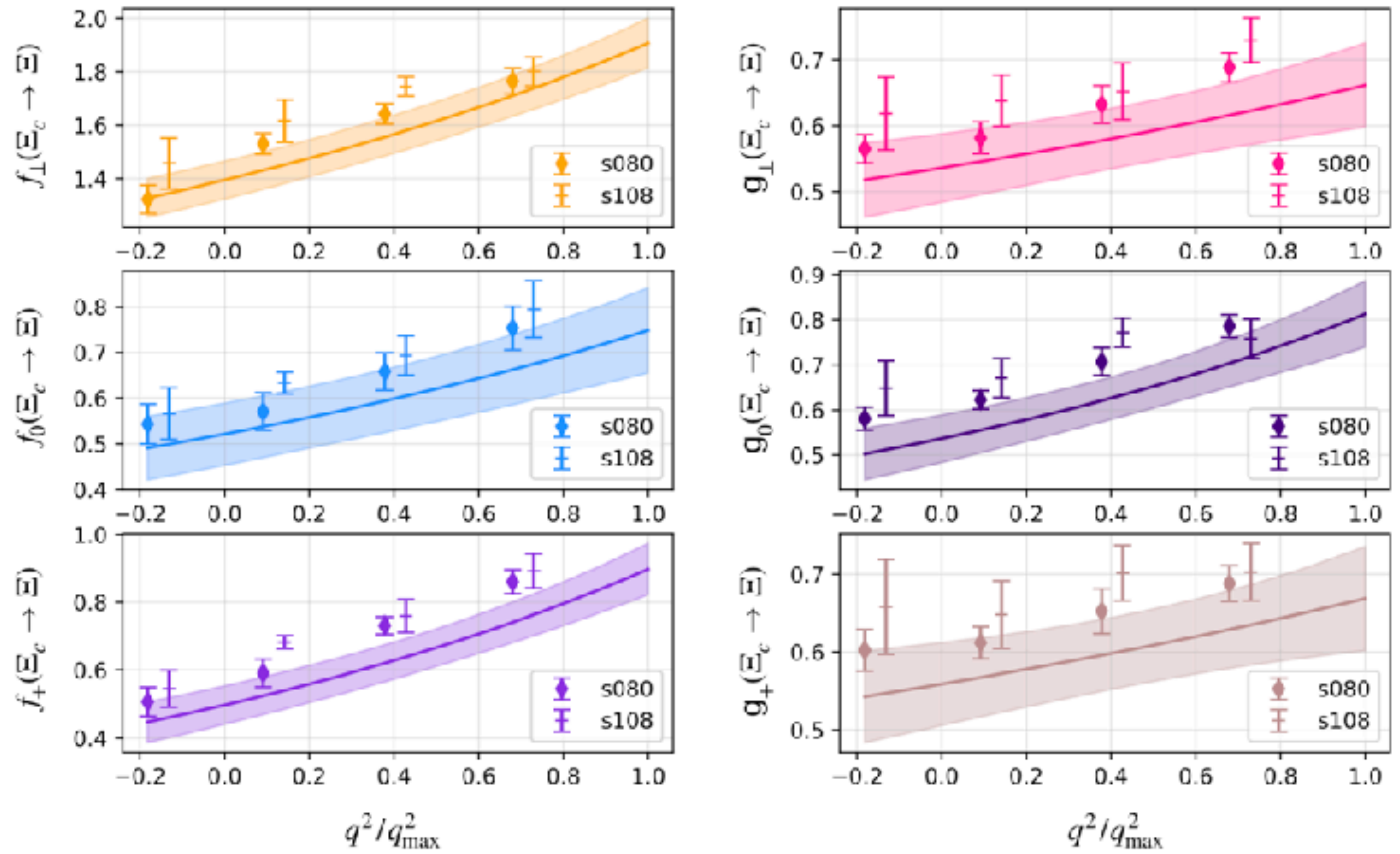
RBC/UKQCD collaboration

Configurations in China：

Form factor







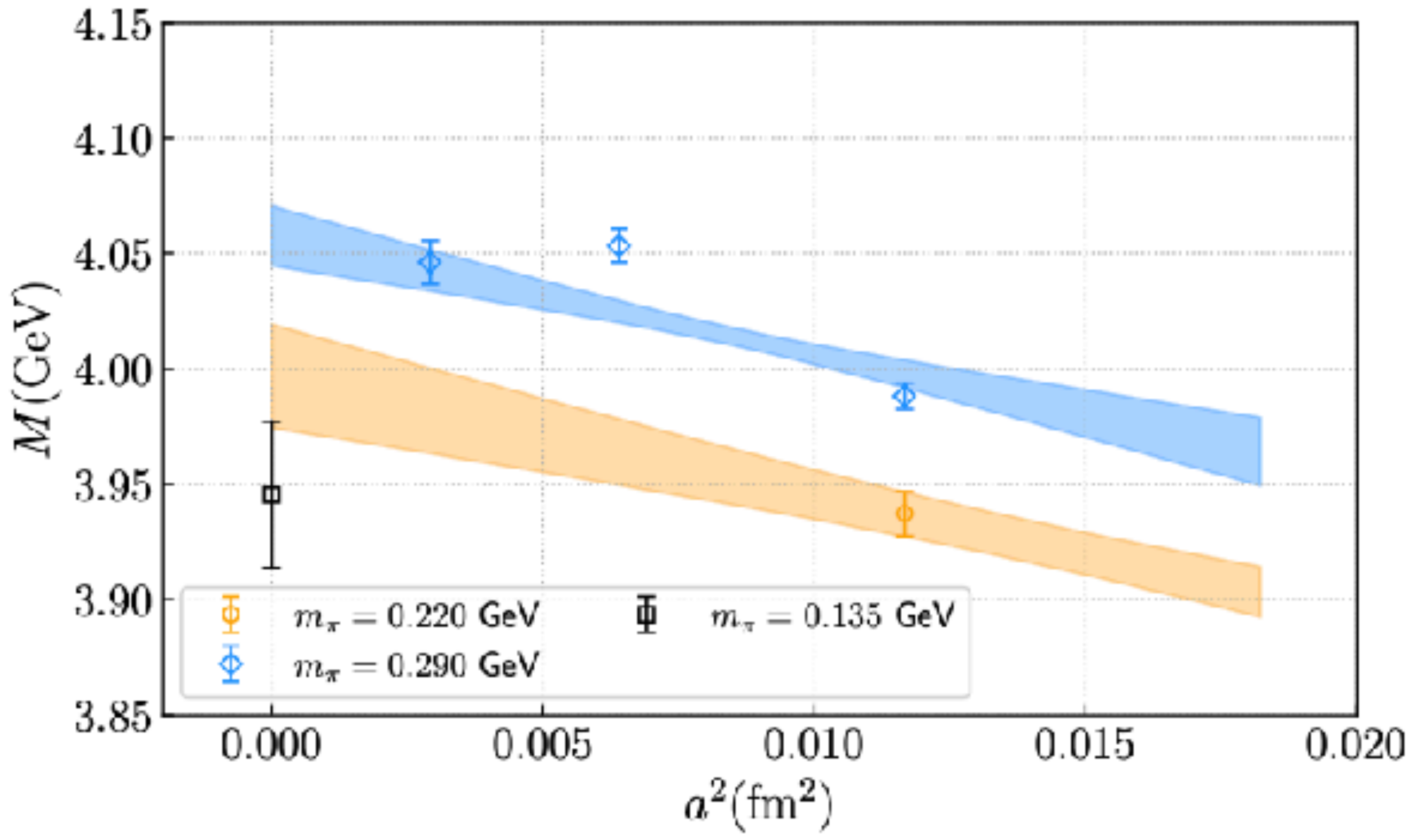
- Our clover ensembles with multiple lattice spacings， a=0.11/0.08fm;
- First lattice QCD calculation on $\Xi_c \to \Xi$ form factor；
- Predict $|V_{cs}| = 0.834(127)_{exp}(74)_{th}$ with the Belle experiment.
- It is essential to improve the ensembles sets to control the systematic uncertainty from lattice spacing and pion mass.

ITP+SJTU+NNU

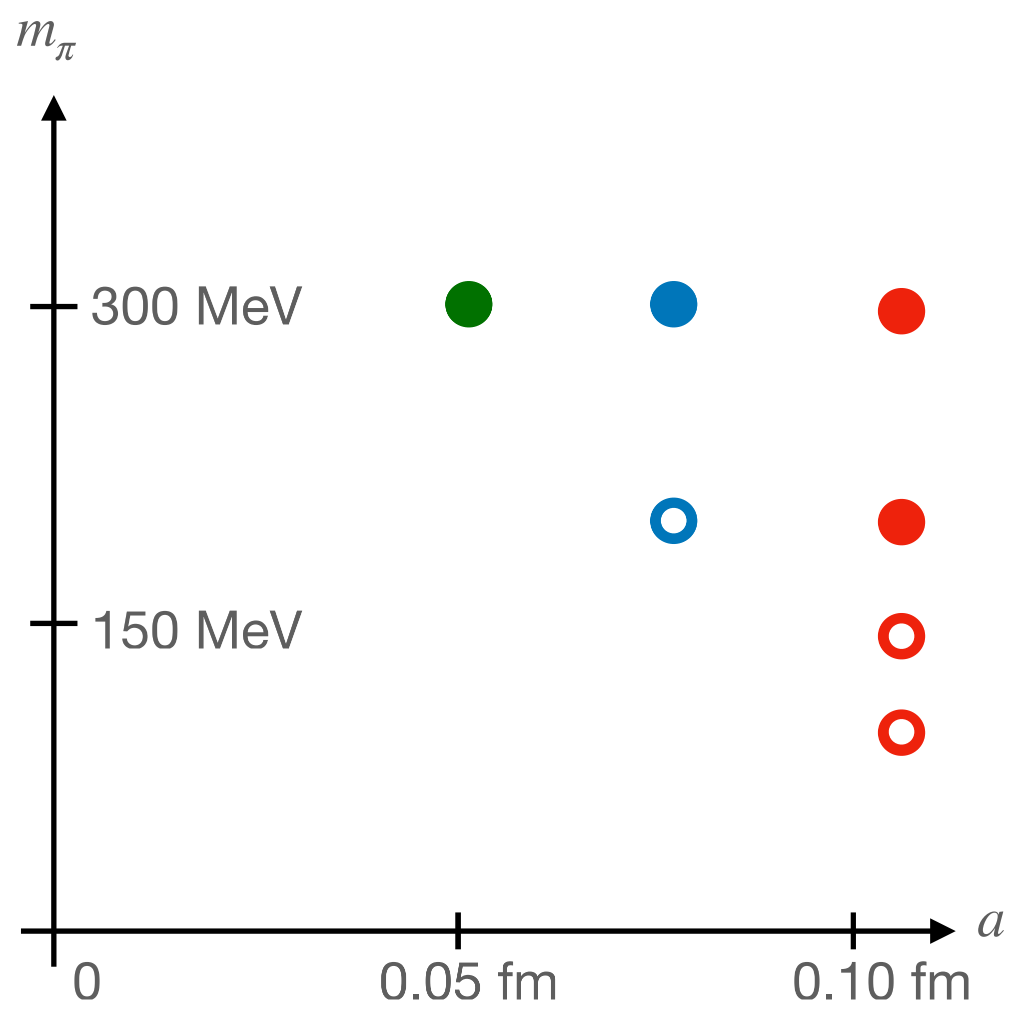
Q.-A. Zhang et al., CPC 46(2022)011002, arXiv:2103.07064

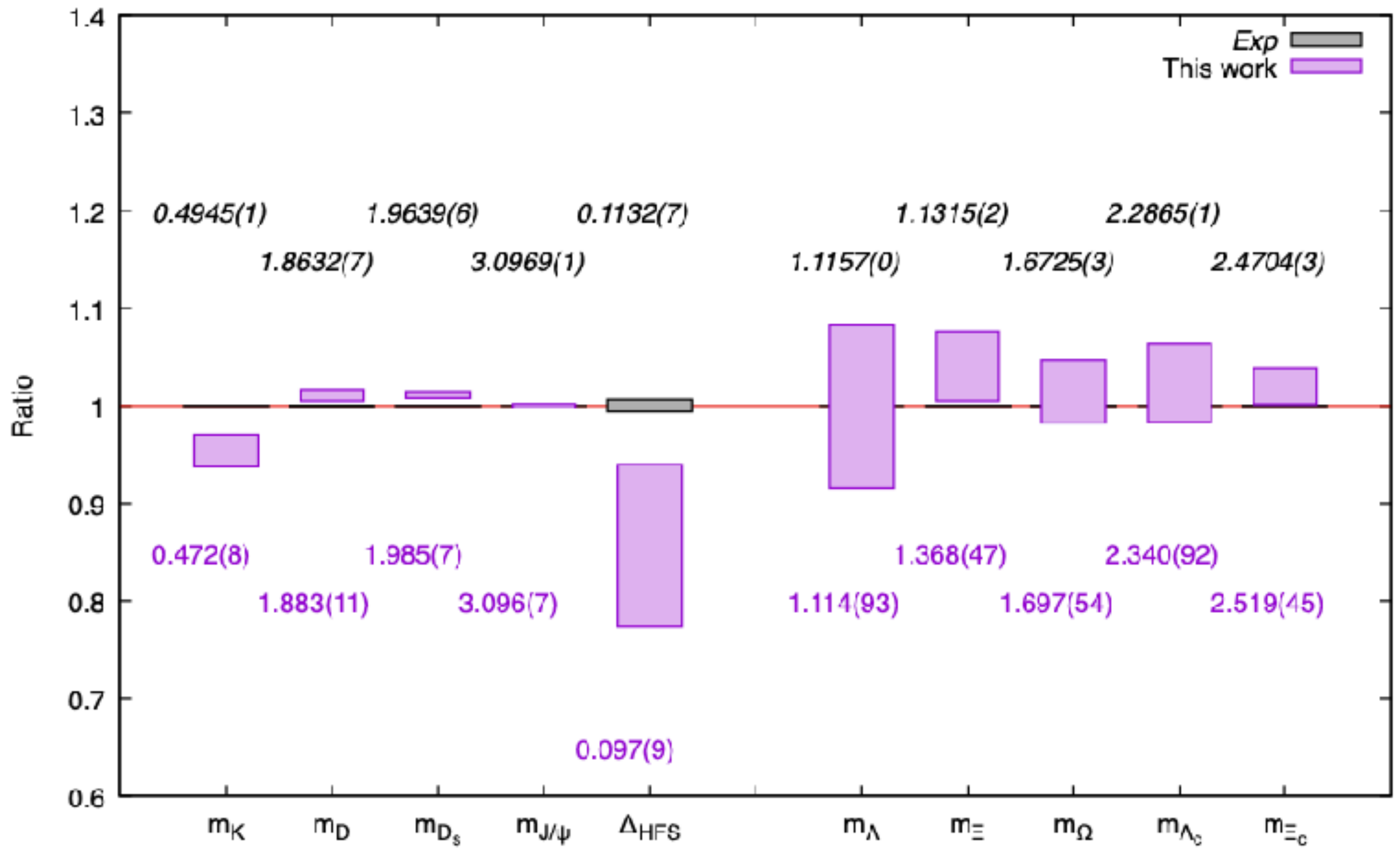
Configurations in China：

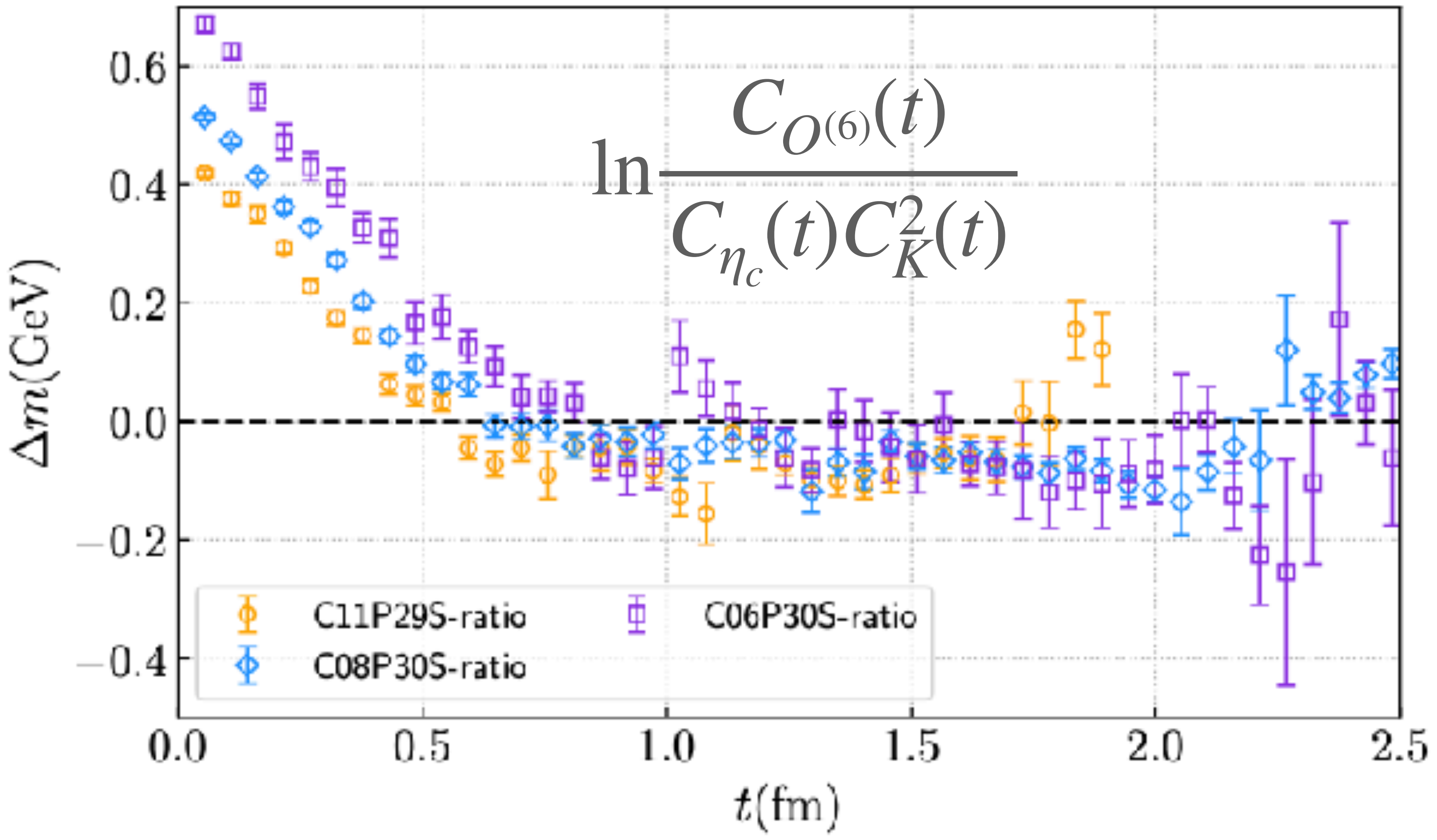
Hadron spectrum



(a) 0^{-+}







- Our clover ensembles with 3 lattice spacings，a=0.11/0.08/0.06 fm and two pion masses;
- First lattice QCD calculation on hidden charm hexaquark spectrum；

ITP+SJTU+IMP

- The ground state mass (3.95(3) for 0^{-+} channel) is much lower than the $\Xi_c - \bar{\Xi}_c$ threshold (4.938 GeV), but close to that of $\eta_c K^+ K^-$.
- The "real" hexaquark state can be hidden in the excited states of $\eta_c K^+ K^-$.

H. Liu, J. He et al., arXiv:2207.00183

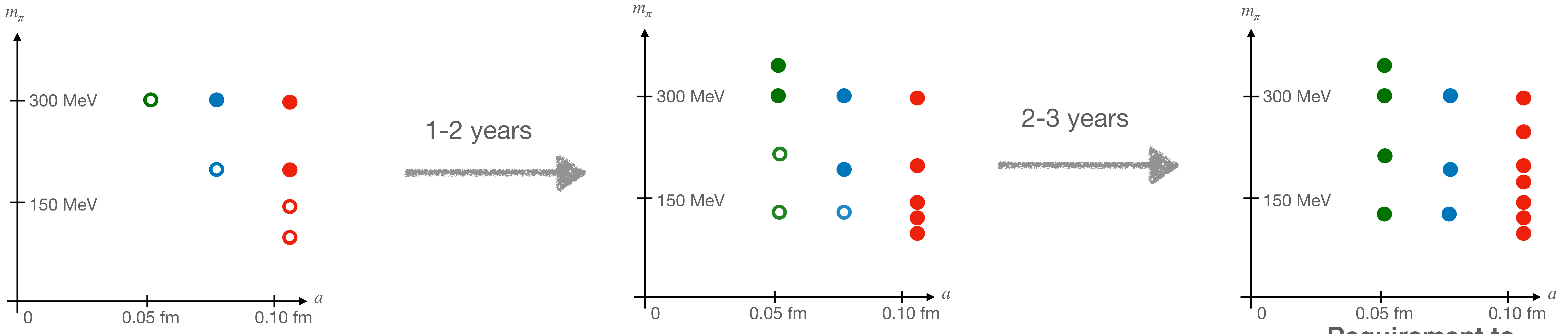
Configurations in China：

Towards the FLAG criteria

- Generate the configurations using the domestic super computers is the foundation of any high-precision lattice QCD study.
- FLAG criteria is the current status-of-the-arts in the lattice community.
- Major contributors：P. Sun, L. Liu, YBY, W. Sun…

- Three different m_π with the lightest m_π smaller than 200 MeV, or one $m_\pi \sim$ 135 MeV and another $m_\pi <$ 200 MeV;
- Three lattice spacings a, with two of $a < 0.1$ fm, and $(a_{max}/a_{min})^2 \geq 2$;
- $(m_{\pi,\min}/M_{\pi,\text{phys}})^2 \exp\{4 - m_{\pi,\min}L\} < 2$, or at least three volumes.

○ Input parameters determined
● Enough configurations obtained



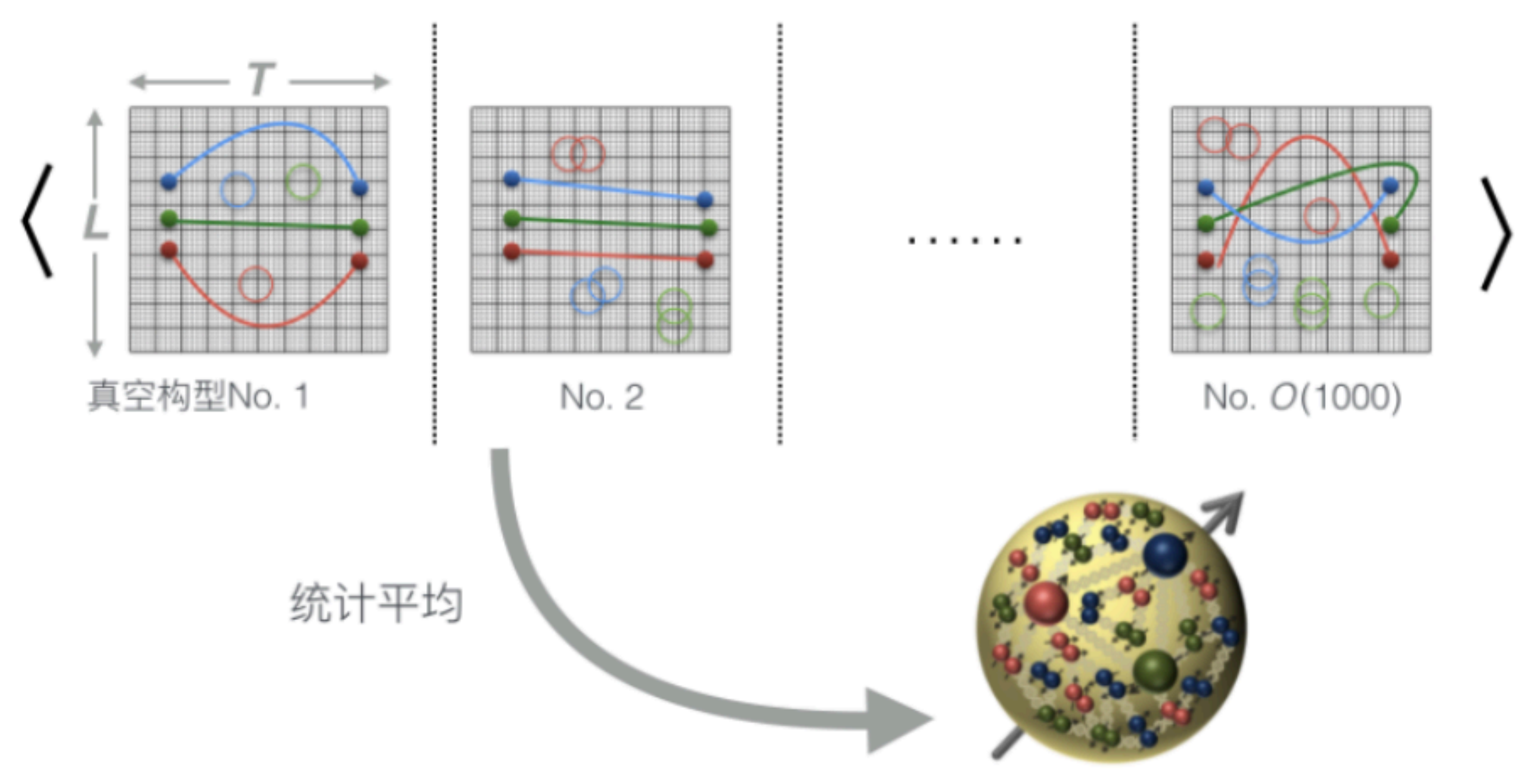
Current status

Present FLAG "Green star" requirement

Requirement to control all the systematic uncertainties

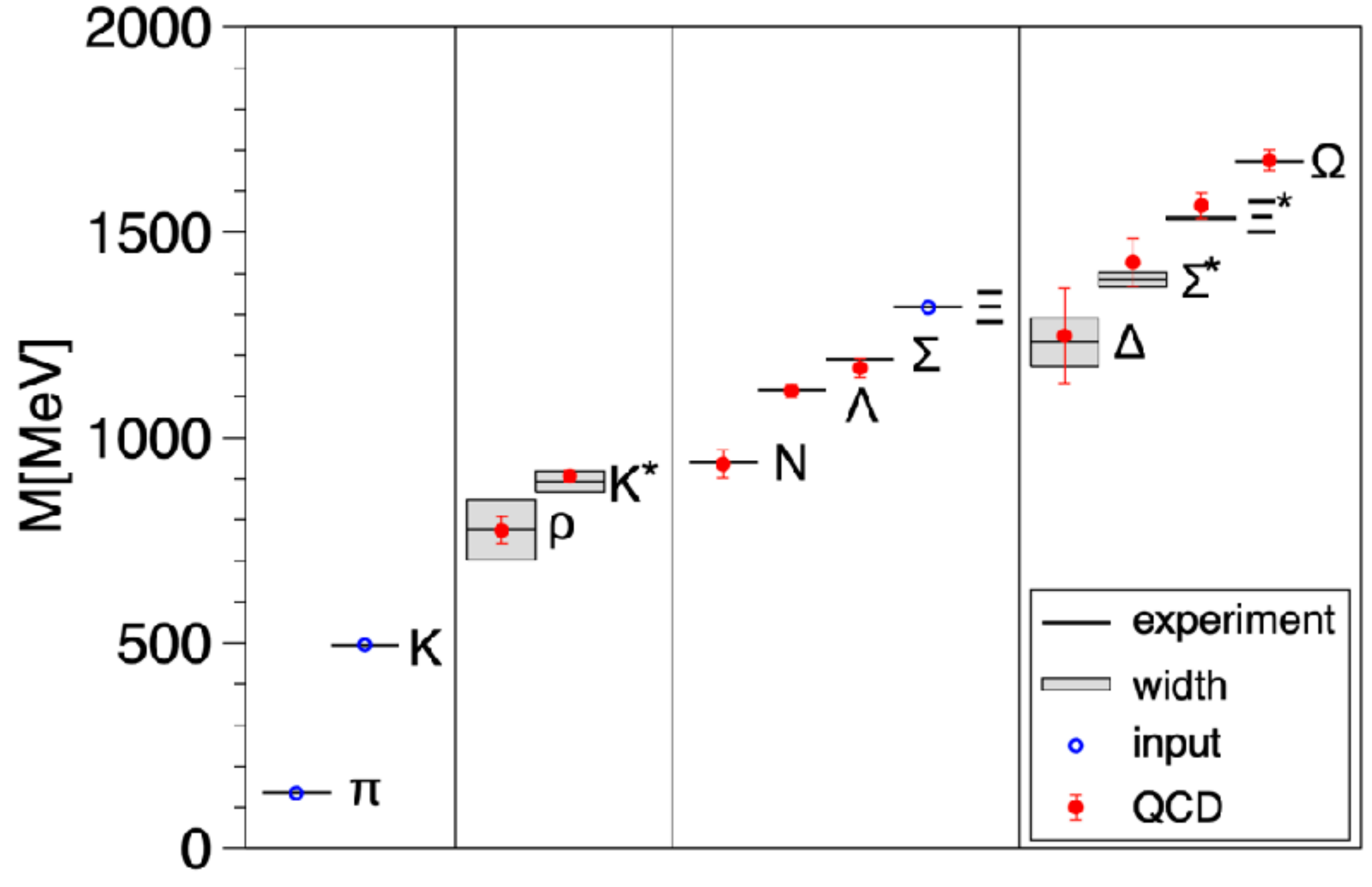
Outline

Basic idea of LQCD



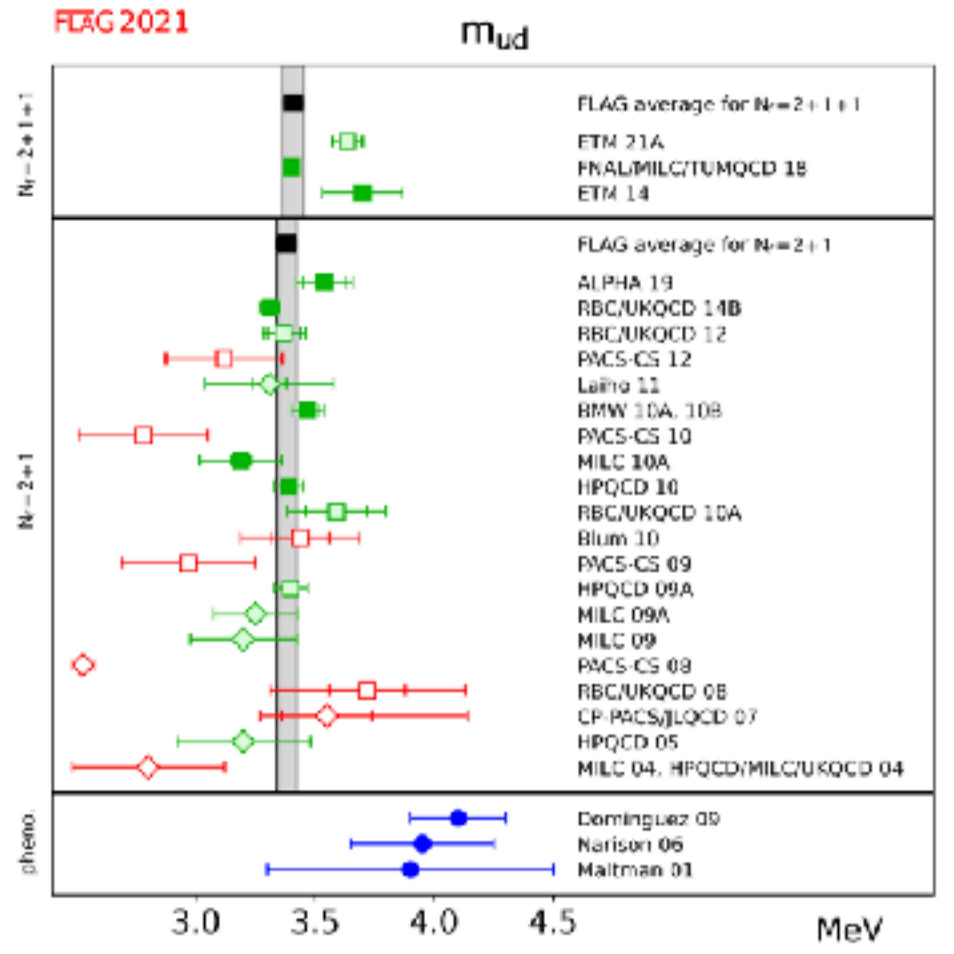
Hadron spectrum

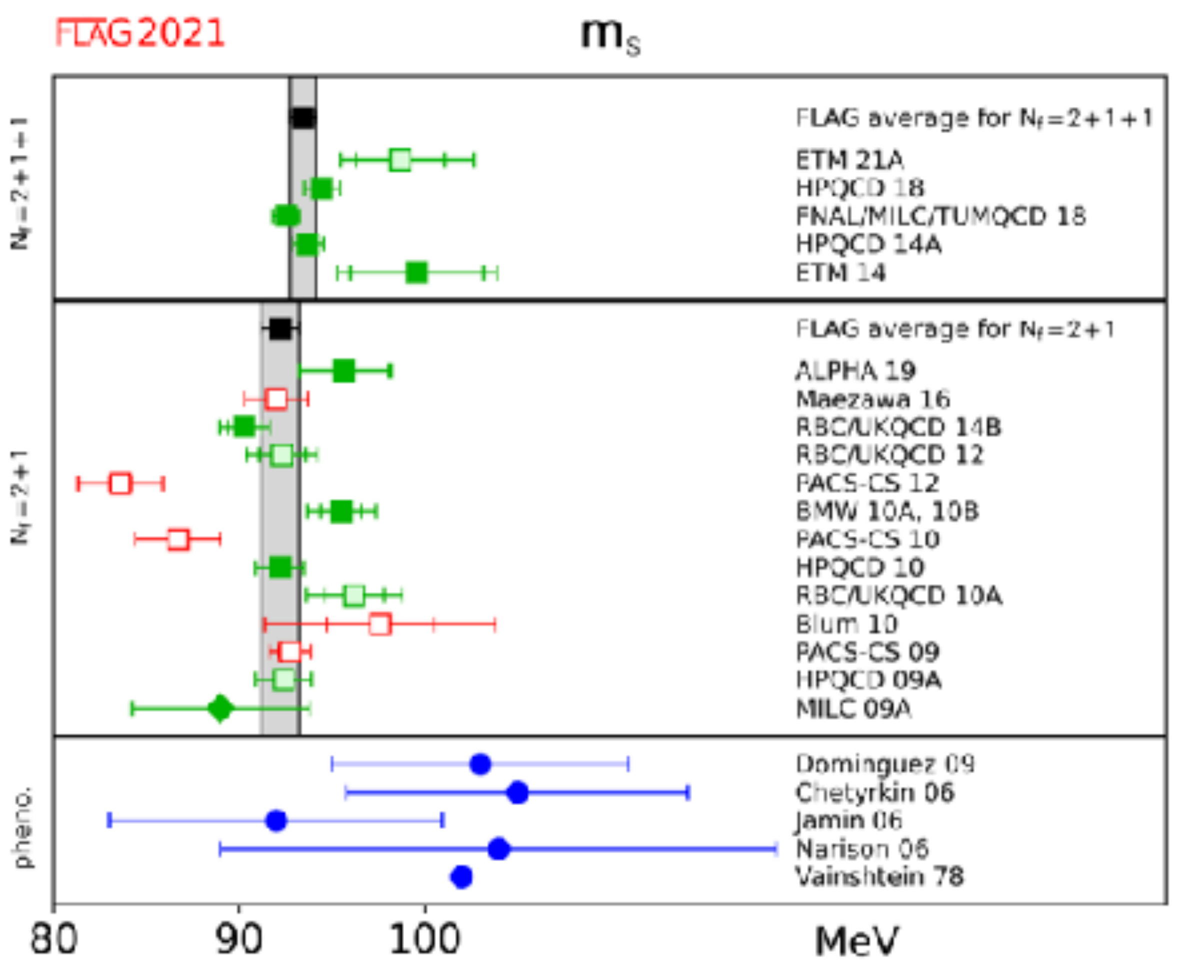
BMWc, Science 322(2008)1224



Quark mass determination

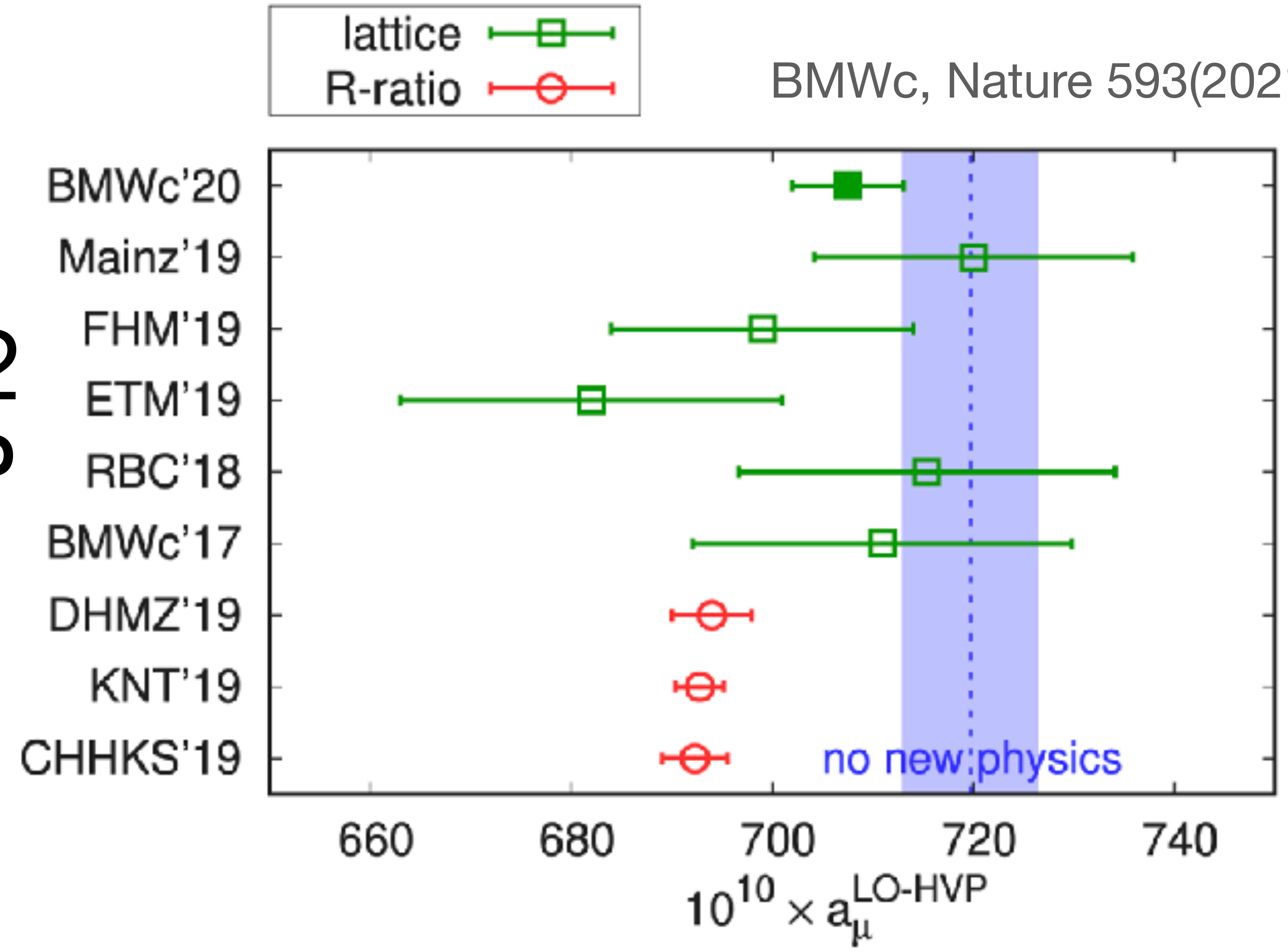
FLAG, EPJC80(2020)113



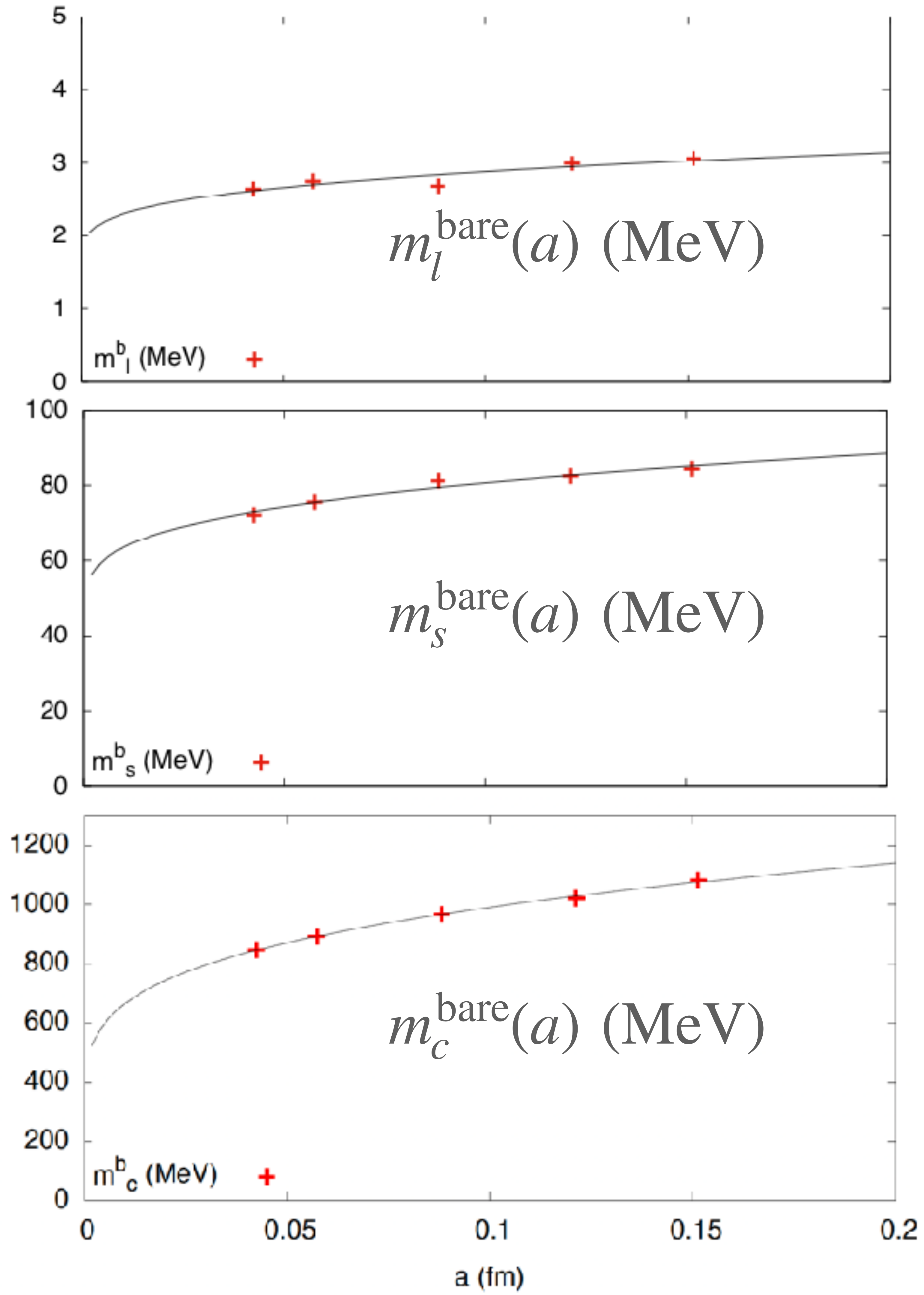


Muon g-2 from HVP

BMWc, Nature 593(2021)51



The quark masses



A. Bazavov, et,al., MILC, PRD87(2013)054505

Bare values

- The bare quark mass depends on 1/a logarithmically.
- The widely used $\overline{\mathrm{MS}}$ scheme can not be apply to the bare quark mass, as the dimension change 2ϵ doesn't exist on the lattice.
- Non-trivial renormalization is needed to convert the bare quark masses to MS-bar.

Lattice regularization

Feynman rules

- The Feynman rule under **the lattice regularization** can be extracted in the weak coupling limit.
- It approaches to the continuum form in the $a \to 0$ limit.
- But the Feynman rule of the multi-gluon vertex is very complicated, especially for the improved discretized actions.
- For example, the 4-gluon vertex of the simplest Wilson gauge actions

$$S_G = \frac{1}{2g_0^2} \sum_{x,\mu\nu} \mathrm{Re}\left[1 - \mathrm{Tr}[\mathscr{P}_{\mu\nu}(x)]\right] = \frac{1}{2}\mathrm{Tr}\int \mathrm{d}^4x F_{\mu\nu}F_{\mu\nu} + \mathcal{O}(a^2)$$

With $\mathscr{P}_{\mu\nu}(x) = U_\mu(x)U_\nu(x+a\hat{\mu})U_\mu^\dagger(x+a\hat{\nu})U_\nu^\dagger(x)$:

$$(S_G)_{A^4} = -\frac{1}{4!}\int_{BZ} \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{d^4r}{(2\pi)^4}\frac{d^4s}{(2\pi)^4} \times \sum_{\mu\nu\lambda\rho}\sum_{ABCD} \Gamma^{ABCD}_{\mu\nu\lambda\rho}(k,q,r,s)\tilde{A}^A_\mu(k)\tilde{A}^B_\nu(q)\tilde{A}^C_\lambda(r)\tilde{A}^D_\rho(s)\ .$$

$$\begin{aligned}
&\Gamma^{ABCD}_{\mu\nu\lambda\rho}(k,q,r,s) = \\
&-g_0^2\Big[\sum_E f_{ABE}f_{CDE}\Big\{\delta_{\mu\lambda}\delta_{\nu\rho}[\cos\frac{1}{2}a(q-s)_\mu \cos\frac{1}{2}a(k-r)_\nu - \frac{a^4}{12}\tilde{k}_\nu\tilde{q}_\mu\tilde{r}_\nu\tilde{s}_\mu] \\
&\quad - \delta_{\mu\rho}\delta_{\nu\lambda}[\cos\frac{1}{2}a(q-r)_\mu \cos\frac{1}{2}a(k-s)_\nu - \frac{a^4}{12}\tilde{k}_\nu\tilde{q}_\mu\tilde{r}_\mu\tilde{s}_\nu] \\
&\quad + \frac{1}{6}\delta_{\nu\lambda}\delta_{\nu\rho}a^2(\widetilde{s-r})_\mu\tilde{k}_\nu\cos(\frac{1}{2}aq_\mu) \\
&\quad - \frac{1}{6}\delta_{\mu\lambda}\delta_{\mu\rho}a^2(\widetilde{s-r})_\nu\tilde{q}_\mu\cos(\frac{1}{2}ak_\nu) \\
&\quad + \frac{1}{6}\delta_{\mu\nu}\delta_{\mu\rho}a^2(\widetilde{q-k})_\lambda\tilde{r}_\rho\cos(\frac{1}{2}as_\lambda) \\
&\quad - \frac{1}{6}\delta_{\mu\nu}\delta_{\mu\lambda}a^2(\widetilde{q-k})_\rho\tilde{s}_\lambda\cos(\frac{1}{2}ar_\rho) \\
&\quad + \frac{1}{12}\delta_{\mu\nu}\delta_{\mu\lambda}\delta_{\mu\rho}a^2\sum_\sigma(\widetilde{q-k})_\sigma(\widetilde{s-r})_\sigma\Big\} \\
&\quad + (B\leftrightarrow C, \nu\leftrightarrow\lambda, q\leftrightarrow r) + (B\leftrightarrow D, \nu\leftrightarrow\rho, q\leftrightarrow s)\Big] \\
&\quad + \frac{g_0^2}{12}a^4\Big\{\frac{2}{3}(\delta_{AB}\delta_{CD} + \delta_{AC}\delta_{BD} + \delta_{AD}\delta_{BC}) \\
&\quad + \sum_E(d_{ABE}d_{CDE} + d_{ACE}d_{BDE} + d_{ADE}d_{BCE})\Big\} \\
&\quad \times \Big\{\delta_{\mu\nu}\delta_{\mu\lambda}\delta_{\mu\rho}\sum_\sigma\tilde{k}_\sigma\,\tilde{q}_\sigma\tilde{r}_\sigma\tilde{s}_\sigma - \delta_{\mu\nu}\delta_{\mu\lambda}\tilde{k}_\rho\tilde{q}_\rho\tilde{r}_\rho\tilde{s}_\mu \\
&\quad - \delta_{\mu\nu}\delta_{\mu\rho}\tilde{k}_\lambda\tilde{q}_\lambda\tilde{s}_\lambda\tilde{r}_\mu - \delta_{\mu\lambda}\delta_{\mu\rho}\tilde{k}_\nu\tilde{r}_\nu\tilde{s}_\nu\tilde{q}_\mu - \delta_{\nu\lambda}\delta_{\nu\rho}\tilde{q}_\mu\tilde{r}_\mu\tilde{s}_\mu\tilde{k}_\nu \\
&\quad + \delta_{\mu\nu}\delta_{\lambda\rho}\tilde{k}_\lambda\tilde{q}_\lambda\tilde{r}_\mu\tilde{s}_\mu + \delta_{\mu\lambda}\delta_{\nu\rho}\tilde{k}_\nu\tilde{r}_\nu\tilde{q}_\mu\tilde{s}_\mu + \delta_{\mu\rho}\delta_{\nu\lambda}\tilde{k}_\nu\tilde{s}_\nu\tilde{q}_\mu\tilde{r}_\mu\Big\}
\end{aligned}$$

H. Rothe, 《Lattice Gauge Theories, An Introduction》, Eq.15.53

Lattice regularization

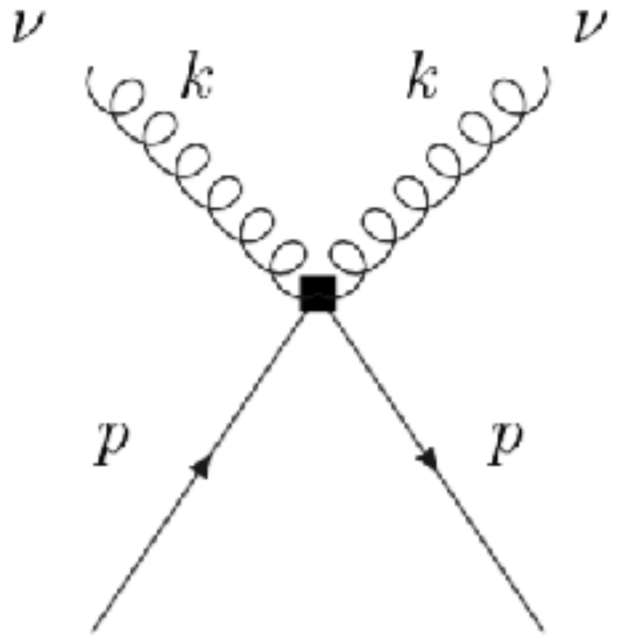
Additional vertex

Taking the simplest Wilson fermion as example:

$$\mathcal{S}_q^W(m) = \sum_{x,y} \bar{\psi}(x) D_w(m;x,y)\psi(y),\ D_w(m;x,y) = \frac{1}{2a}\sum_{\mu}\left((1+\gamma_\mu)U_\mu(x)\delta_{y,x+a\hat{\mu}} + (1-\gamma_\mu)U_\mu^\dagger(x-a\hat{\mu})\delta_{y,x-a\hat{\mu}}\right) - (m+\frac{4}{a})\delta_{y,x}$$

where $U(x) \equiv e^{ig_0\int_x^{x+a\hat{\mu}} dyA_\mu(y)}$:

There is a g-g-q-q vertex at $\mathcal{O}(a)$: $\cdot g_0^2 \cdot$



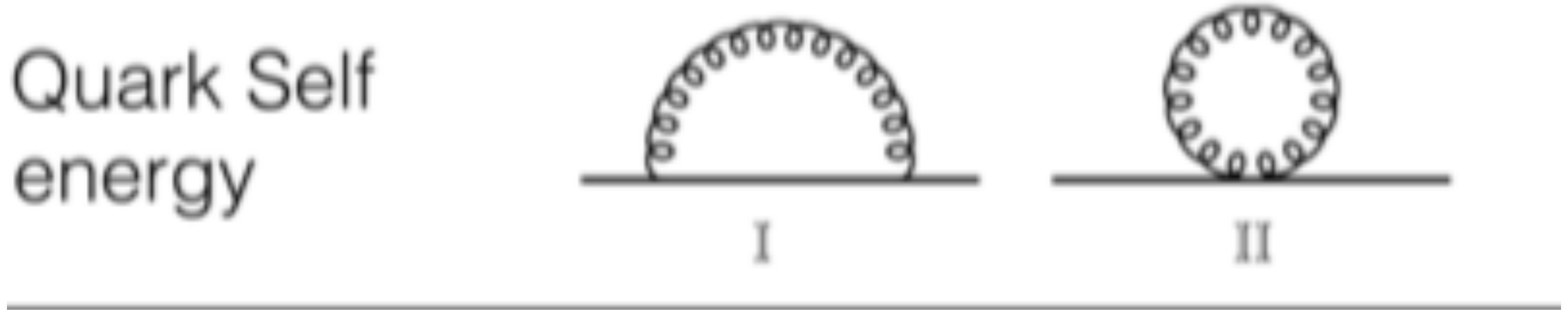
$-\frac{a}{2}\, C_F\, \mathrm{i}\gamma_\mu \sin ap_\nu$

Such a vertex is $\mathcal{O}(a)$ at tree level, but it can introduce $\mathcal{O}(\alpha_s)$ correction at quantum level!

Lattice regularization

Loop correction

Taking the quark self energy as an example：



The result can be quite different finite $\mathcal{O}(\alpha_s)$ corrections with different discretization：

$$Z_Q^{RI}(p^2) = 1 + \frac{g^2 C_F}{16\pi^2}[(1-\xi)\log(a^2p^2) + B_Q + 4.79\xi] + O(a^2p^2) + O(g^4)$$

Gauge actions

Fermion actions

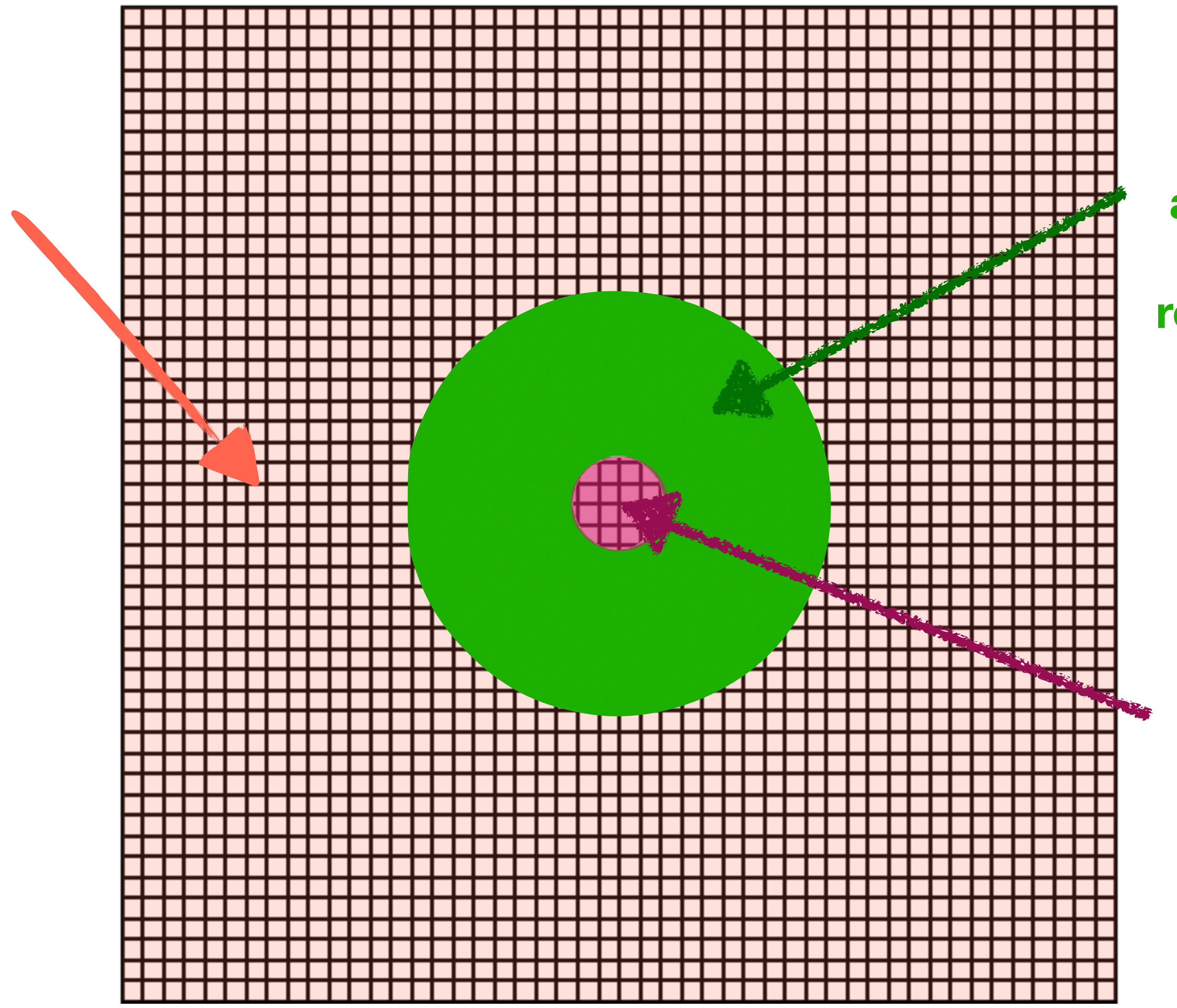
	B_Q		
	Wilson	Iwasaki	Iwasaki^{HYP}
Wilson	11.85	3.32	-4.22
overlap	-21.50	-13.58	-7.56

The light quark masses

Renormalization

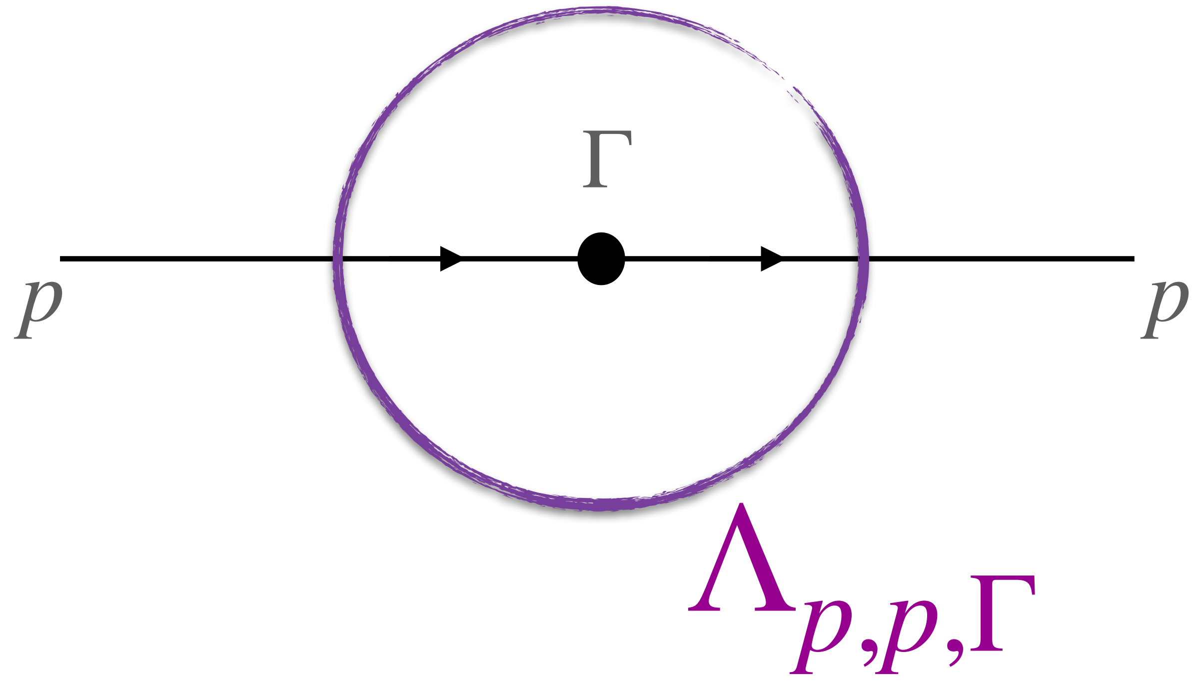
$$m_q^{\overline{\mathrm{MS}}}(\mu) = \frac{Z_m^{\mathrm{MOM,Lat}}(Q, 1/a)}{Z_m^{\mathrm{MOM,Dim}}(Q, \mu, \epsilon)} Z_m^{\overline{\mathrm{MS}},\mathrm{Dim}}(\epsilon) m_q^{\mathrm{Lat}}(1/a) + \mathcal{O}(a^m, \alpha_s^n)$$

- The RI/MOM renormalization targets to cancel the $\alpha_s \log(a)$ divergences using the off-shell quark matrix element;
- **Up to the $\mathcal{O}(a^2p^2)$ correction which can be eliminated by the $a^2p^2 \to 0$ extrapolation.**



G. Martinelli, et.al, NPB445(1995)81, arXiv: hep-lat/9411010

The regularization-independent momentum subtraction scheme



First of all, we introduce a perturbative calculable scale $Q^2 = -p^2$,

- Then we can calculate the quark propagator $S(p) = \sum_x e^{-i(p\cdot x)} \langle \psi(x)\bar{\psi}(0)\rangle$,

- And also vertex function $\Lambda(p,p,\Gamma) = S^{-1}(p) \sum_{x,y} e^{-i\, p\cdot(x-y)} \langle \psi(x)\bar{\psi}(0)\Gamma\psi(0)\bar{\psi}(y)\rangle S^{-1}(p)$ under the Landau gauge.

- Eventually we can define the RI/MOM renormalization condition as the following: $\dfrac{12 Z_q^{\mathrm{RI}}(Q)}{\mathrm{Tr}[\Gamma^\dagger \Lambda(p,p,\Gamma)]}\bigg|_{p^2=-Q^2} = \dfrac{C_0}{m_q^n} + Z_S^{\mathrm{MOM}}(Q) + \mathcal{O}(m_q)$。

- It can be applied to any regularization scheme.

RI/MOM scheme

Perturbative calculation

- The RI/MOM renormalization constant of the quark mass under the lattice regularization is:

$$Z_m^{\mathrm{MOM,Lat}}(Q,1/a,\xi) = (Z_S^{\mathrm{MOM,Lat}}(Q,1/a,\xi))^{-1} = \langle q|\mathcal{O}|q\rangle^{\mathrm{Lat}} = 1 + \frac{\alpha_s C_F}{4\pi}[-3\log(a^2Q^2) - \xi + b_S] + \mathcal{O}(\alpha_s^2, a^2Q^2);$$

- The RI/MOM and $\overline{\mathrm{MS}}$ renormalization constants under the dimensional regularization are:

$$Z_m^{\mathrm{MOM,Dim}}(Q,\mu,\epsilon,\xi) = \langle q|\mathcal{O}|q\rangle^{\mathrm{Dim}} = 1 + \frac{\alpha_s C_F}{4\pi}[\frac{3}{\tilde{\epsilon}} - 3\log(\frac{Q^2}{\mu^2}) - \xi + 5] + \mathcal{O}(\alpha_s^2)$$

$$Z_m^{\overline{\mathrm{MS}},\mathrm{Dim}}(Q,\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\frac{3}{\tilde{\epsilon}} + \mathcal{O}(\alpha_s^2)$$

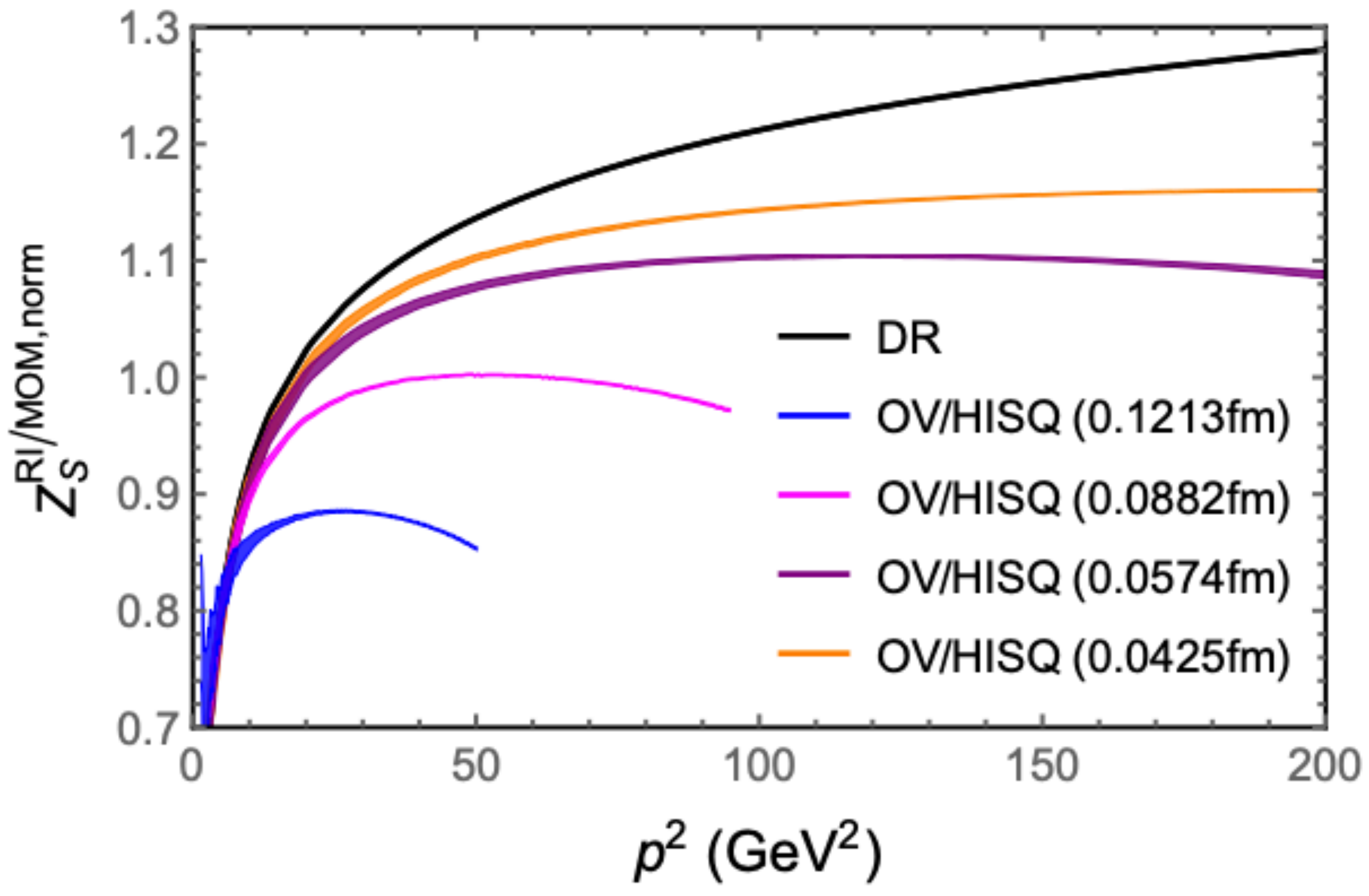
- Thus the renormalized quark mass under the $\overline{\mathrm{MS}}$ scheme can be defined by:

$$m_q^{\overline{\mathrm{MS}}}(\mu) = \frac{Z_m^{\mathrm{MOM,Lat}}(Q,1/a,\xi)}{Z_m^{\mathrm{MOM,Dim}}(Q,\mu,\epsilon,\xi)} Z_m^{\overline{\mathrm{MS}},\mathrm{Dim}}(\epsilon) m_q^{\mathrm{Lat}}(1/a) + \mathcal{O}(a^{2m}Q^{2m}, \alpha_s^n)$$

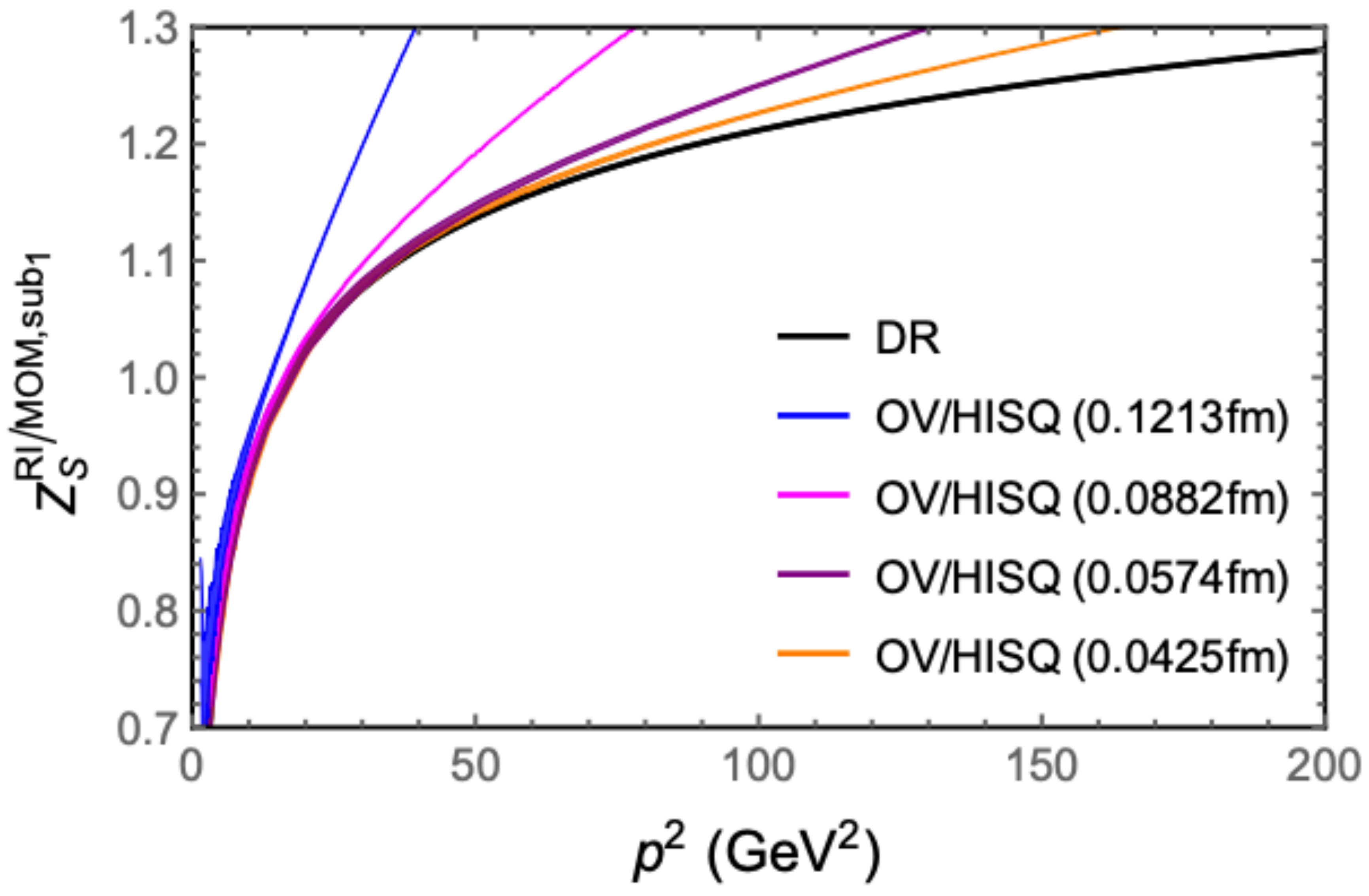
RI/MOM scheme

Discretization errors

$$Z_m^{\mathrm{MOM,Lat}}(Q,a,0) = (Z_S^{\mathrm{MOM,Lat}}(Q,a,0))^{-1}$$



Original lattice results

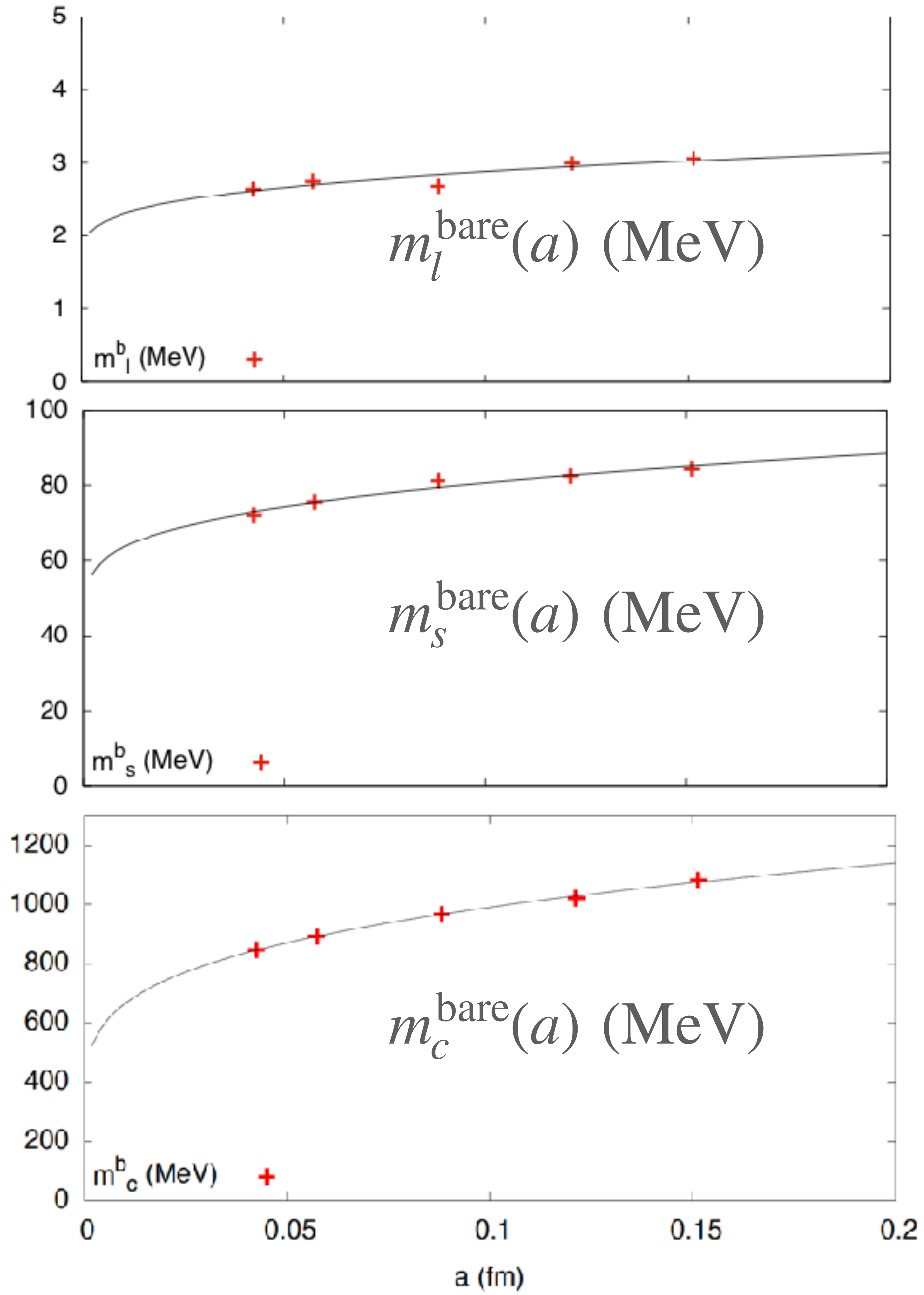


$\mathcal{O}(a^2p^2)$ term subtracted

F. He, et.al, χQCD, arXiv: 2204.09246

- The discretization is sizable at $a \sim 0.1$ fm;
- Becomes much smaller after the $\mathcal{O}(a^2p^2)$ correction is removed;
- The higher order $a^{2n}p^{2n}$ correction can also be removed in the practical calculation.

The light quark masses

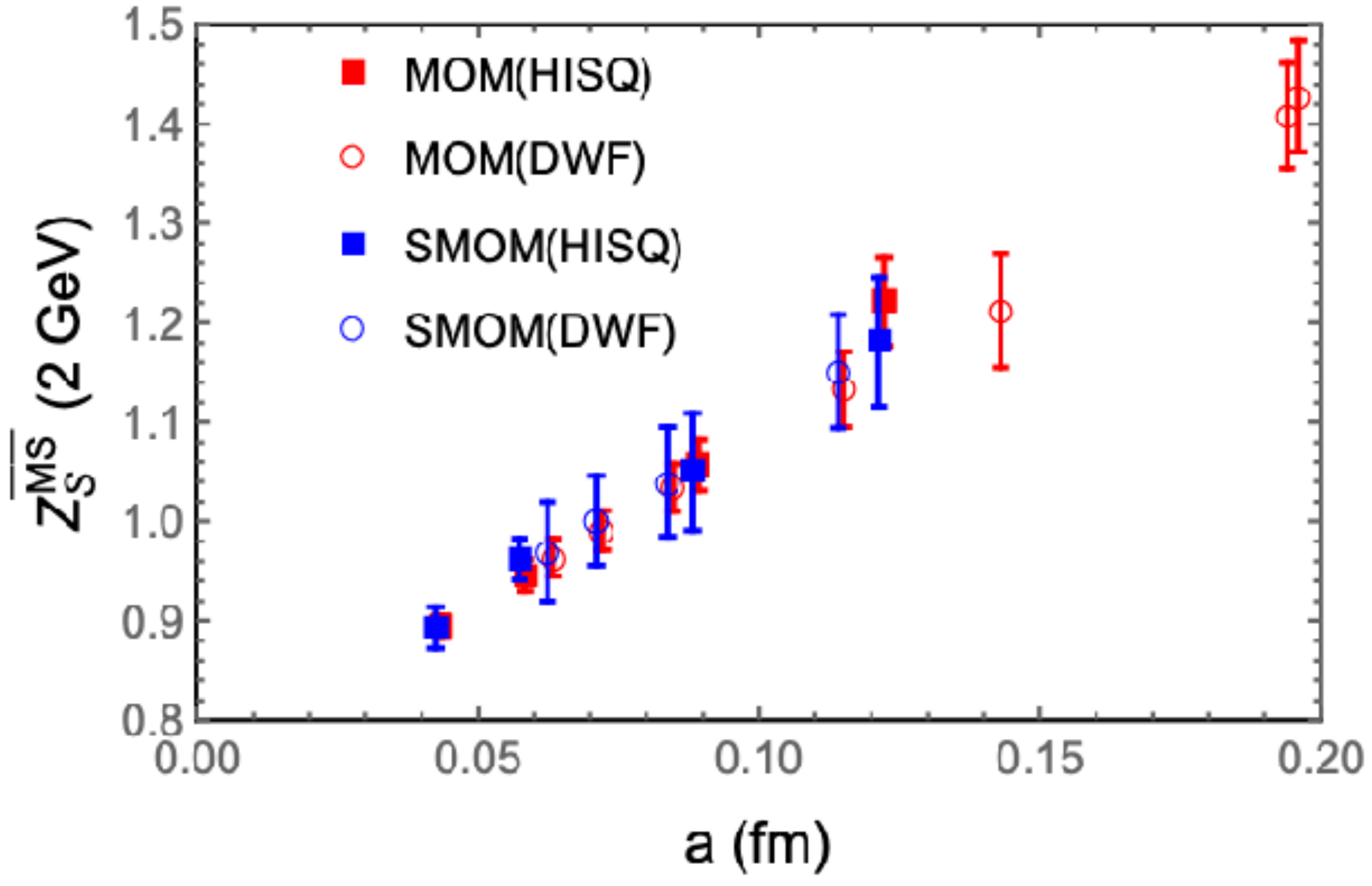


A. Bazavov, et,al., MILC, PRD87(2013)054505

Renormalization

$$m_q^{\overline{\text{MS}}}(\mu) = Z_m^{\overline{\text{MS}},\text{Lat}}(\mu,1/a)\, m_q^{\text{Lat}}(1/a) + \mathcal{O}(a^{2m}Q^{2m}, \alpha_s^n)$$

$$Z_m^{\overline{\text{MS}}}(\mu) = (Z_S^{\overline{\text{MS}}}(\mu))^{-1} = \frac{Z_m^{\text{MOM,Lat}}(Q,1/a,\xi)}{Z_m^{\text{MOM,Dim}}(Q,\mu,\epsilon,\xi)} Z_m^{\overline{\text{MS}},\text{Dim}}(\epsilon)\,|_{a^2Q^2\to 0} + \mathcal{O}(\alpha_s^4)$$

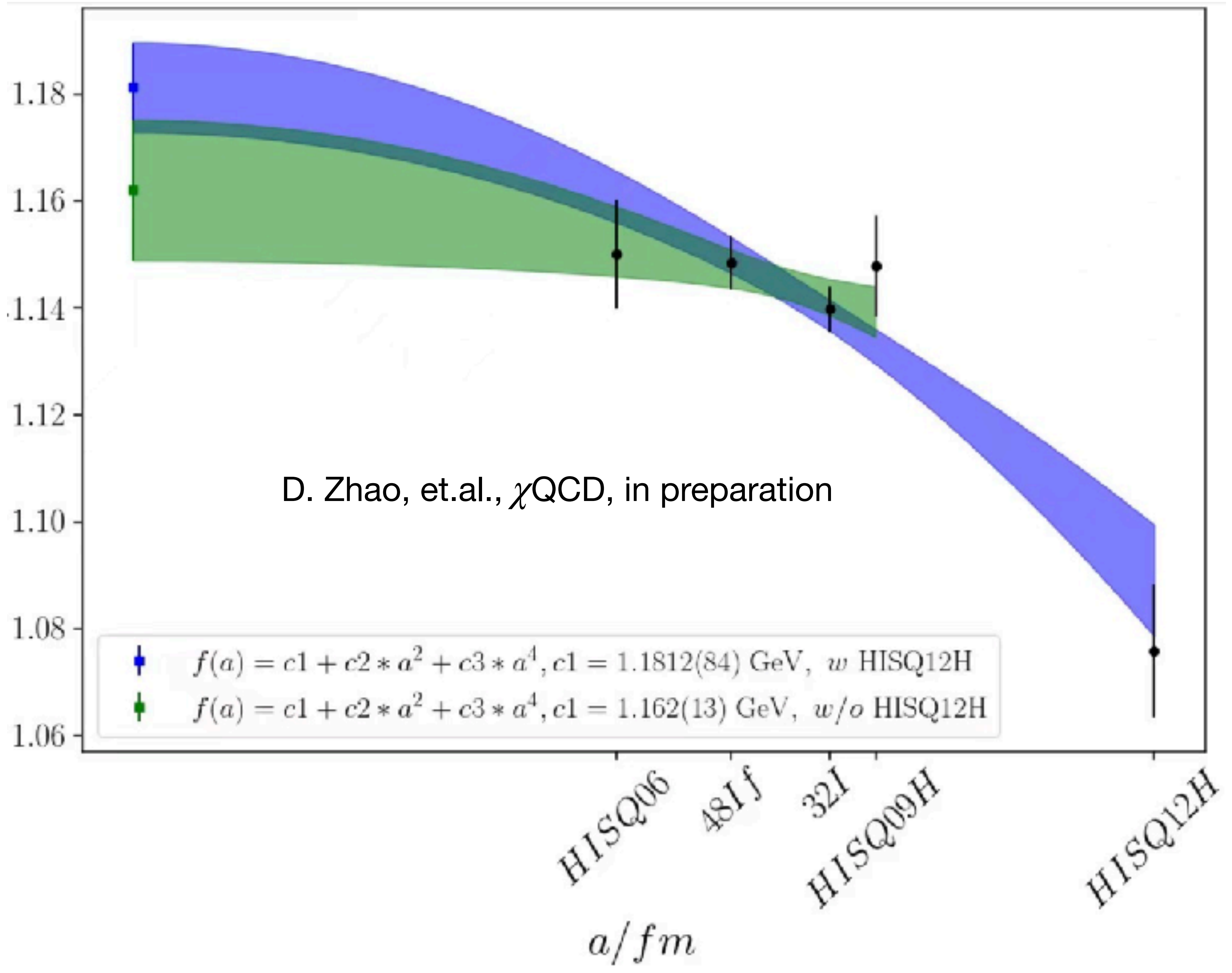


F. He, et.al, χQCD, arXiv: 2204.09246

- The scalar renormalization constant $Z_S^{\overline{\text{MS}}}(\mu)$ shares the similar lattice spacing dependence as the bare quark mass $m_q^{\text{Lat}}(1/a)$;
- The renormalized quark mass $m_q^{\overline{\text{MS}}}(\mu) = \dfrac{m_q^{\text{Lat}}(1/a)}{Z_S^{\overline{\text{MS}},\text{Lat}}(\mu,1/a)}$ should be free of $1/a$.

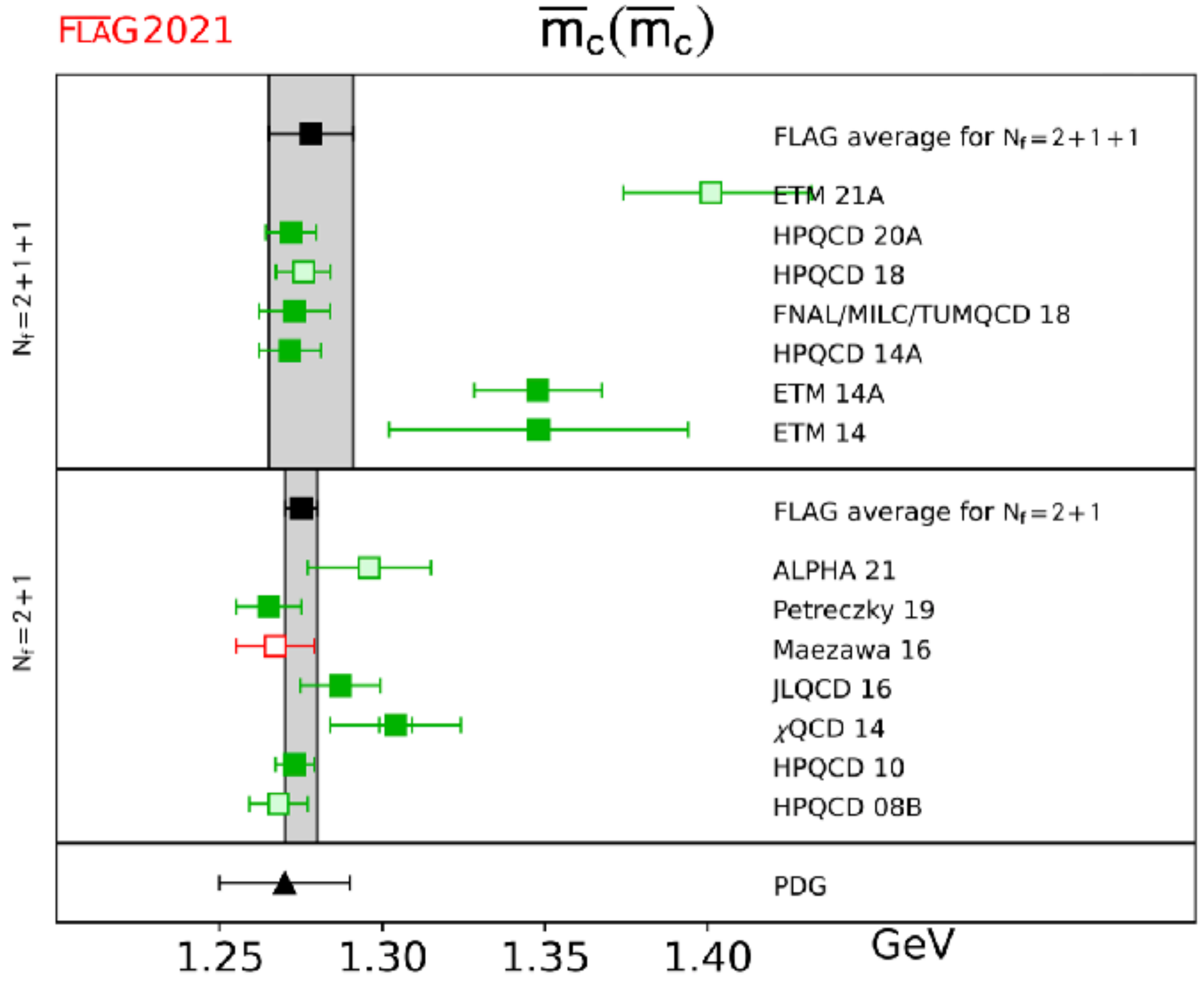
The charm quark masses

From lattice QCD



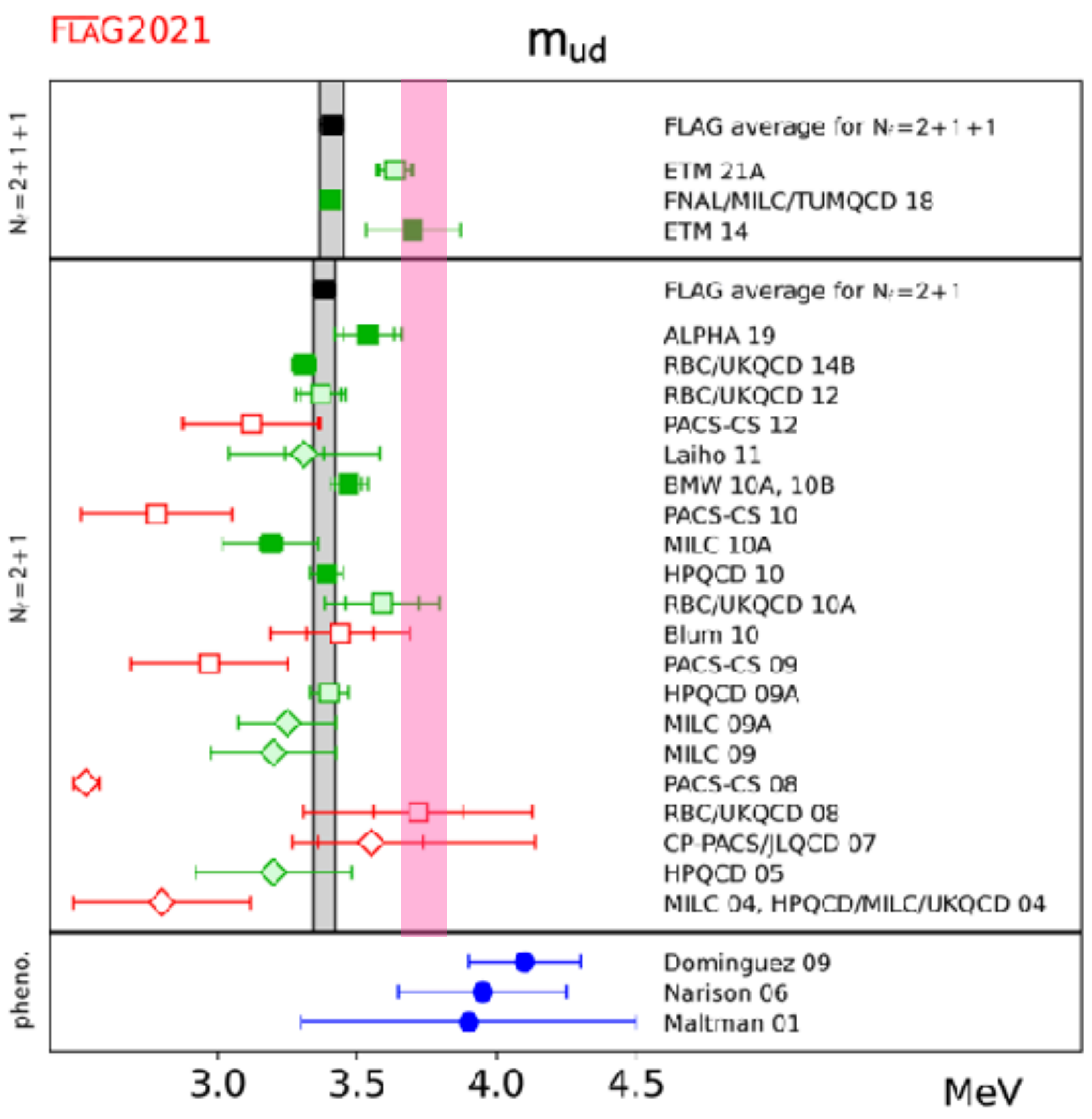
- Using $m_{J/\psi} = 3097$ MeV as input;
- $m_c(2\text{ GeV}) = 1.16(1)$ GeVcorresponds to $m_c(m_c) = 1.332(9)$ GeV.

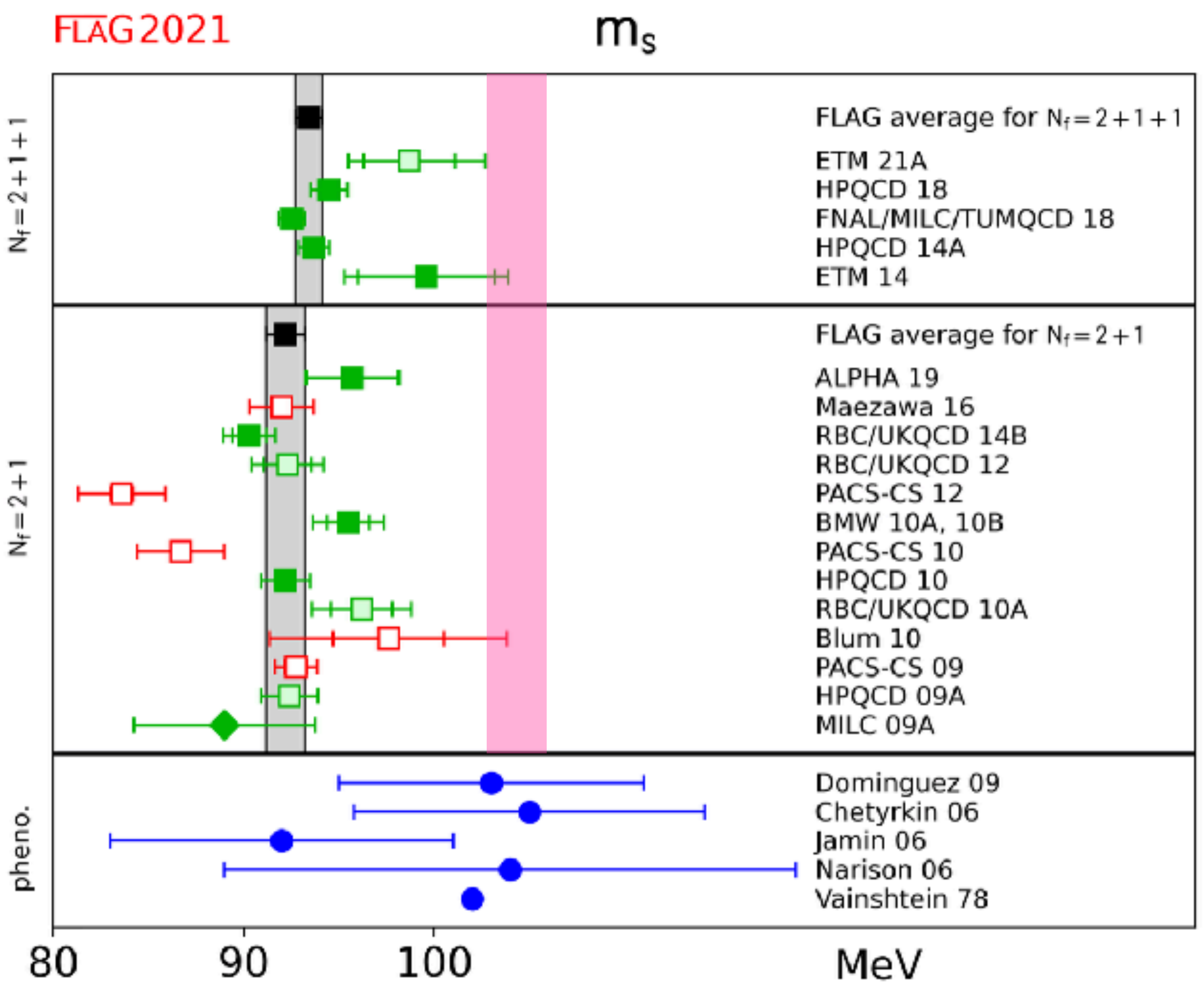
- 4% higher than the current FLAG average of $m_c(m_c)$, corresponds to 6% for $m_c(2\text{ GeV})$.



Light quark masses

From lattice QCD





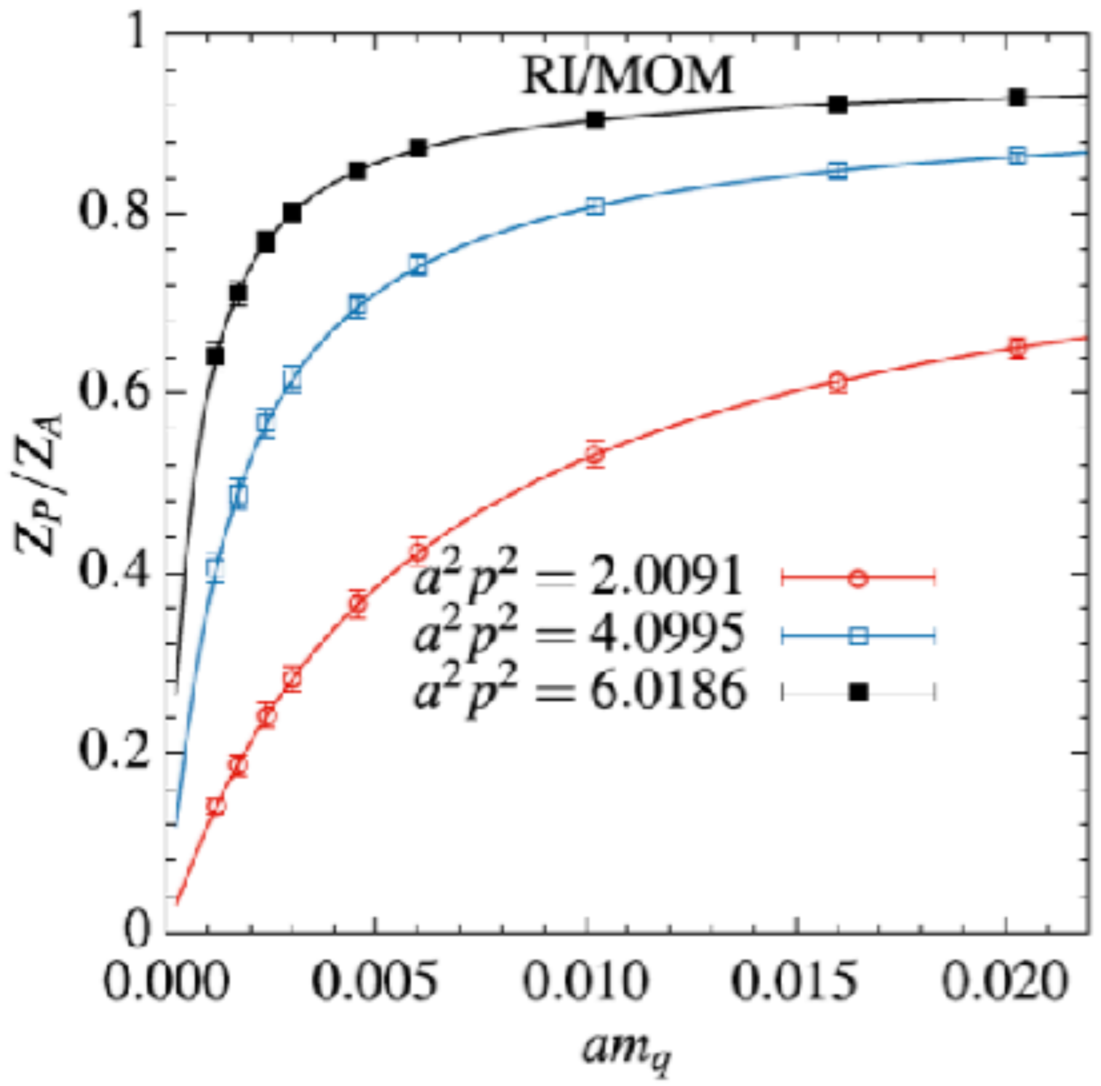
- χQCD prediction of the light and strange quark masses are also higher than the current FLAG averages which uses the SMOM scheme;
- while consistent with that from ETM using the RI/MOM scheme.

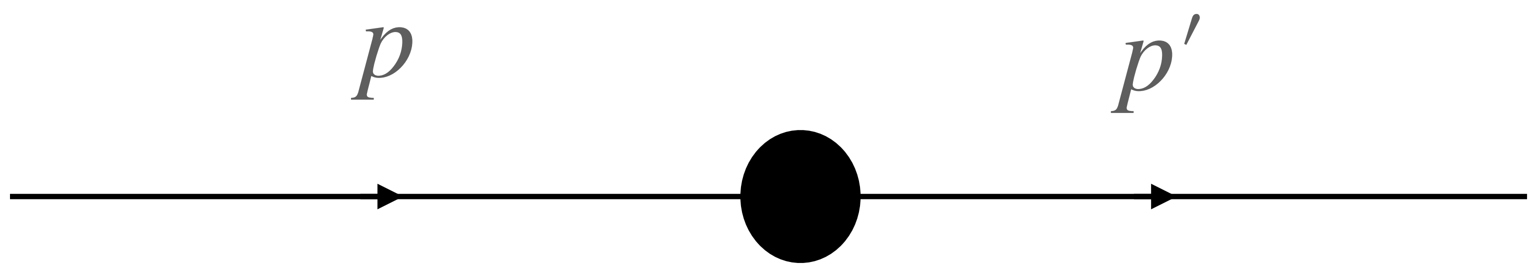
FLAG, EPJC80(2020)113

Preliminary result from χQCD

A. T. Lytle, et.al., HPQCD, PRD98(2018)014513

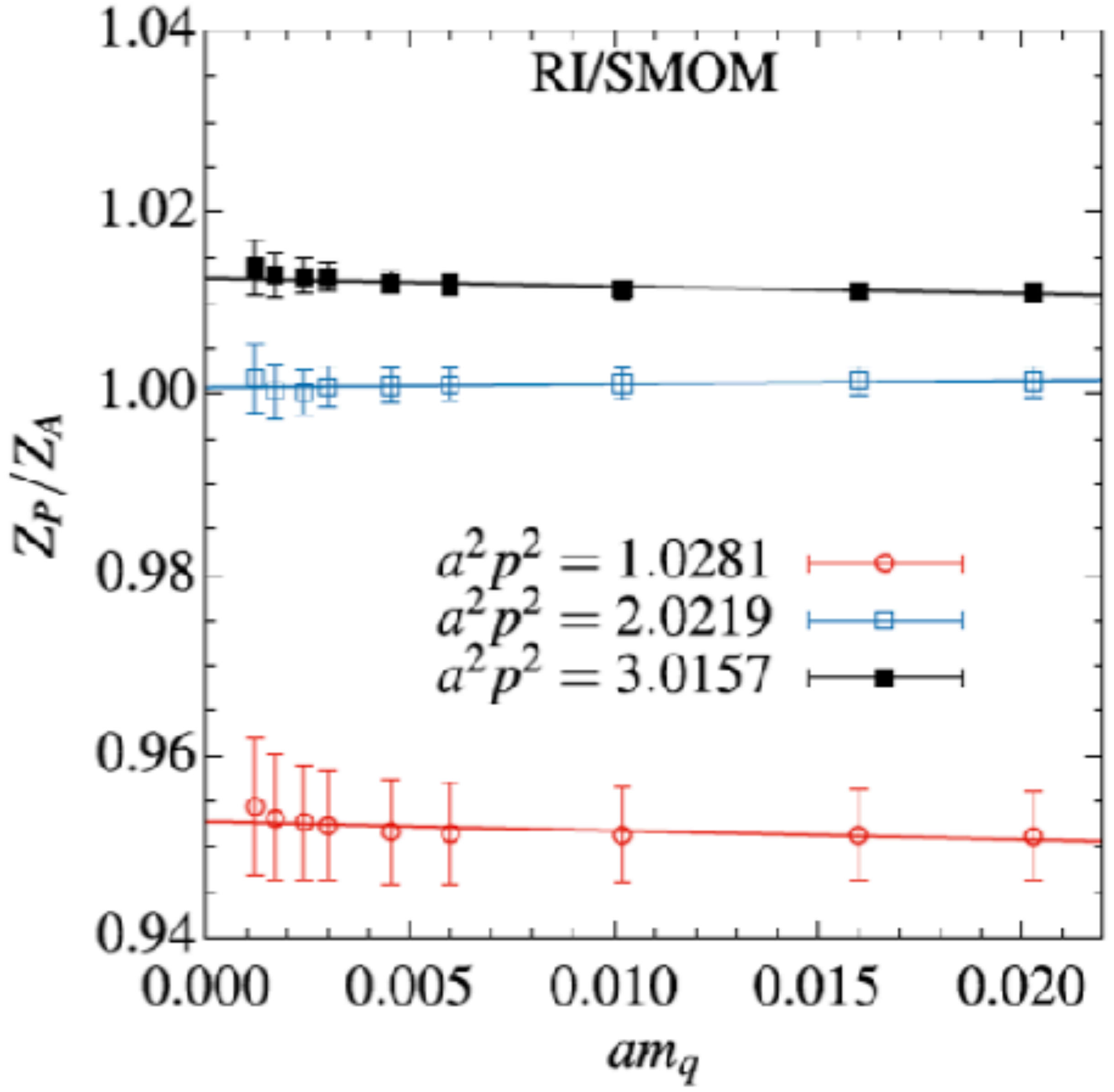
MOM v.s. SMOM



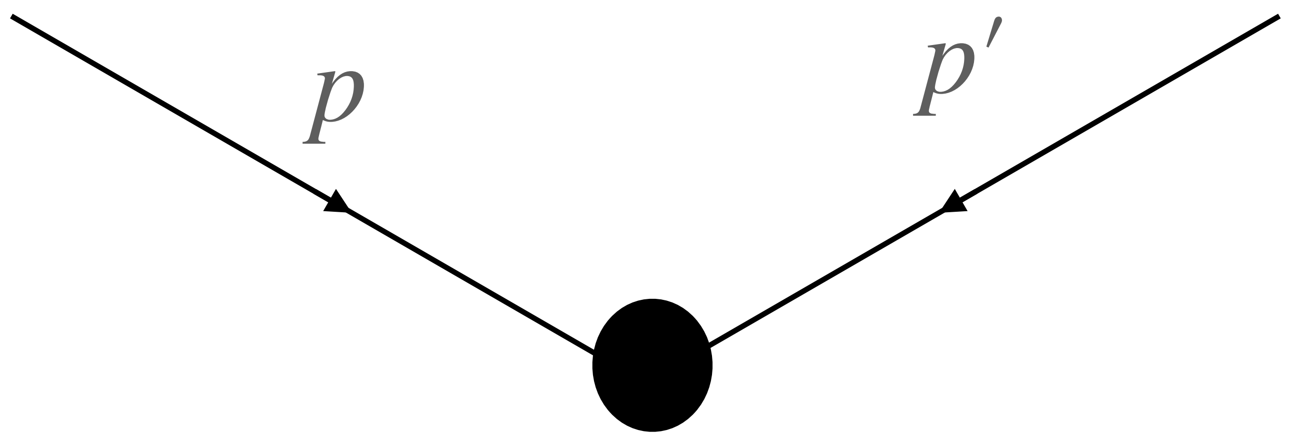


$$(p'-p)^2 = 0$$

- **MOM scheme**: The zero momentum transfer at the current can introduce additional mixing/non-perturbative effect;



- **SMOM scheme**: Requiring the momentum transfer to be non-zero can avoid such a problem.

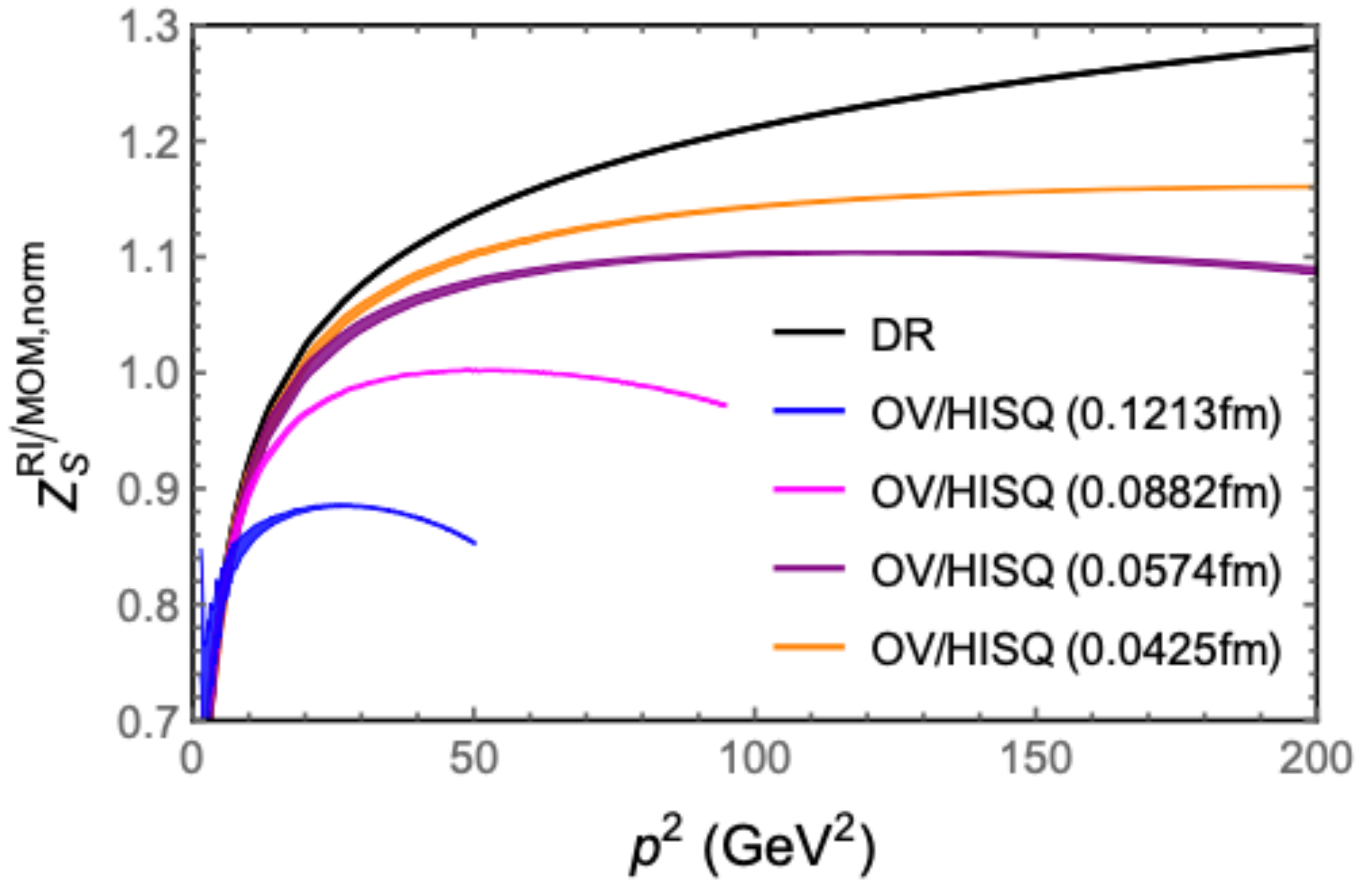


$$(p'-p)^2 = p^2$$

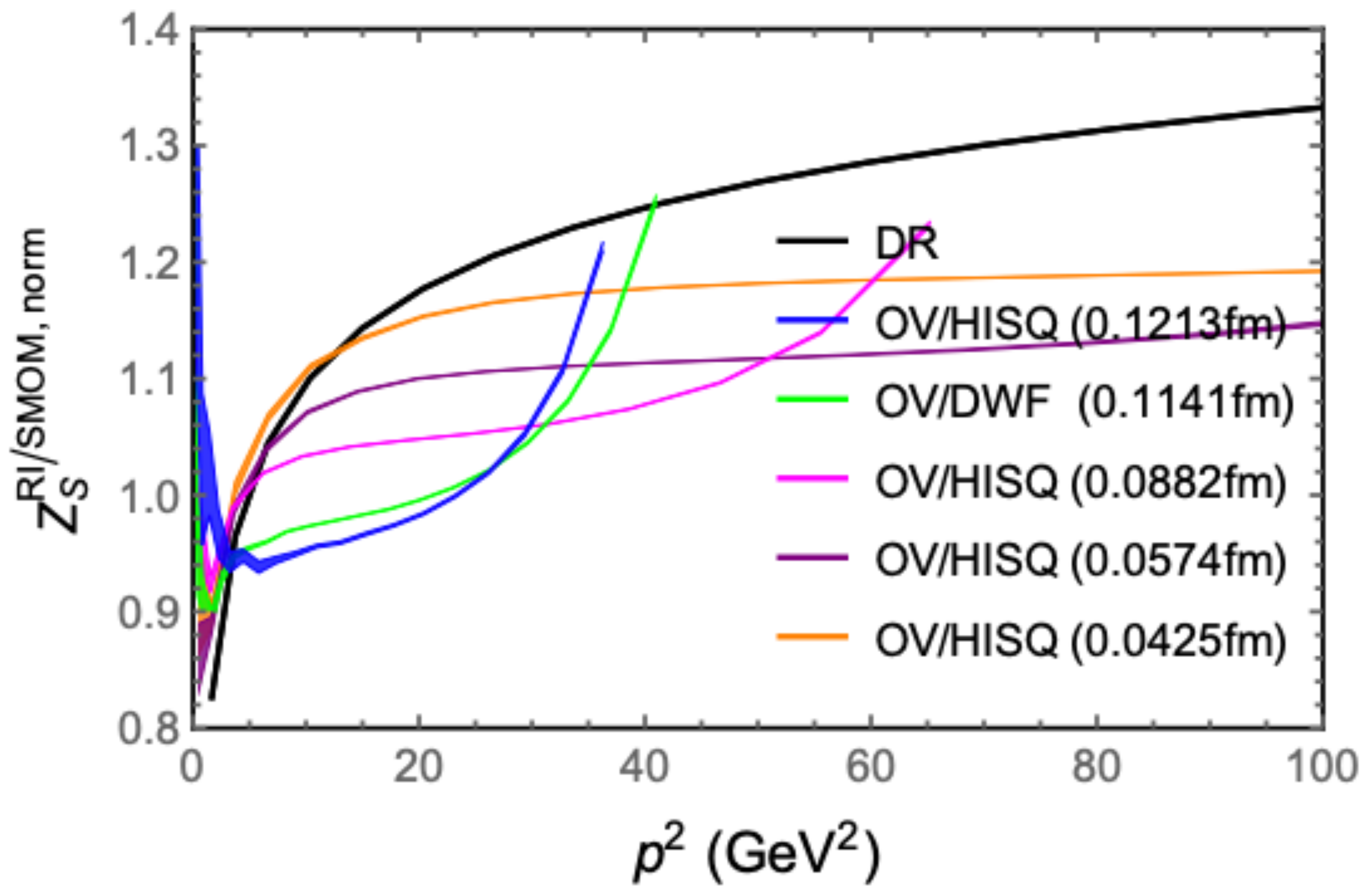
Y. Aoki, et.al, PRD78(2009)054520, 0712.1061
C. Sturm. et.al, PRD90(2009)014501, 0901.2599

SMOM scheme

Discretization errors

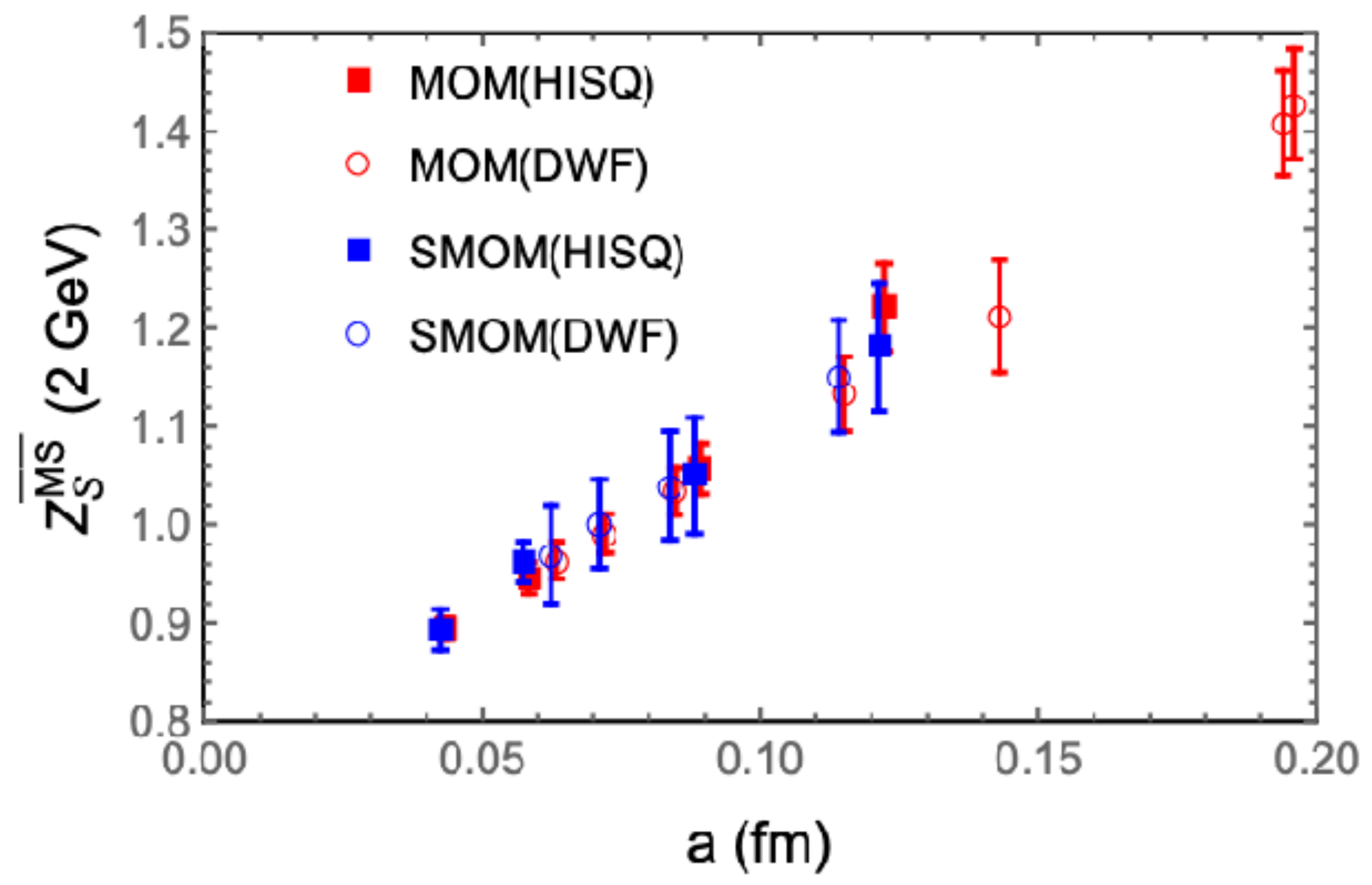


$Z_S^{\rm MOM}$



$Z_S^{\rm SMOM}$

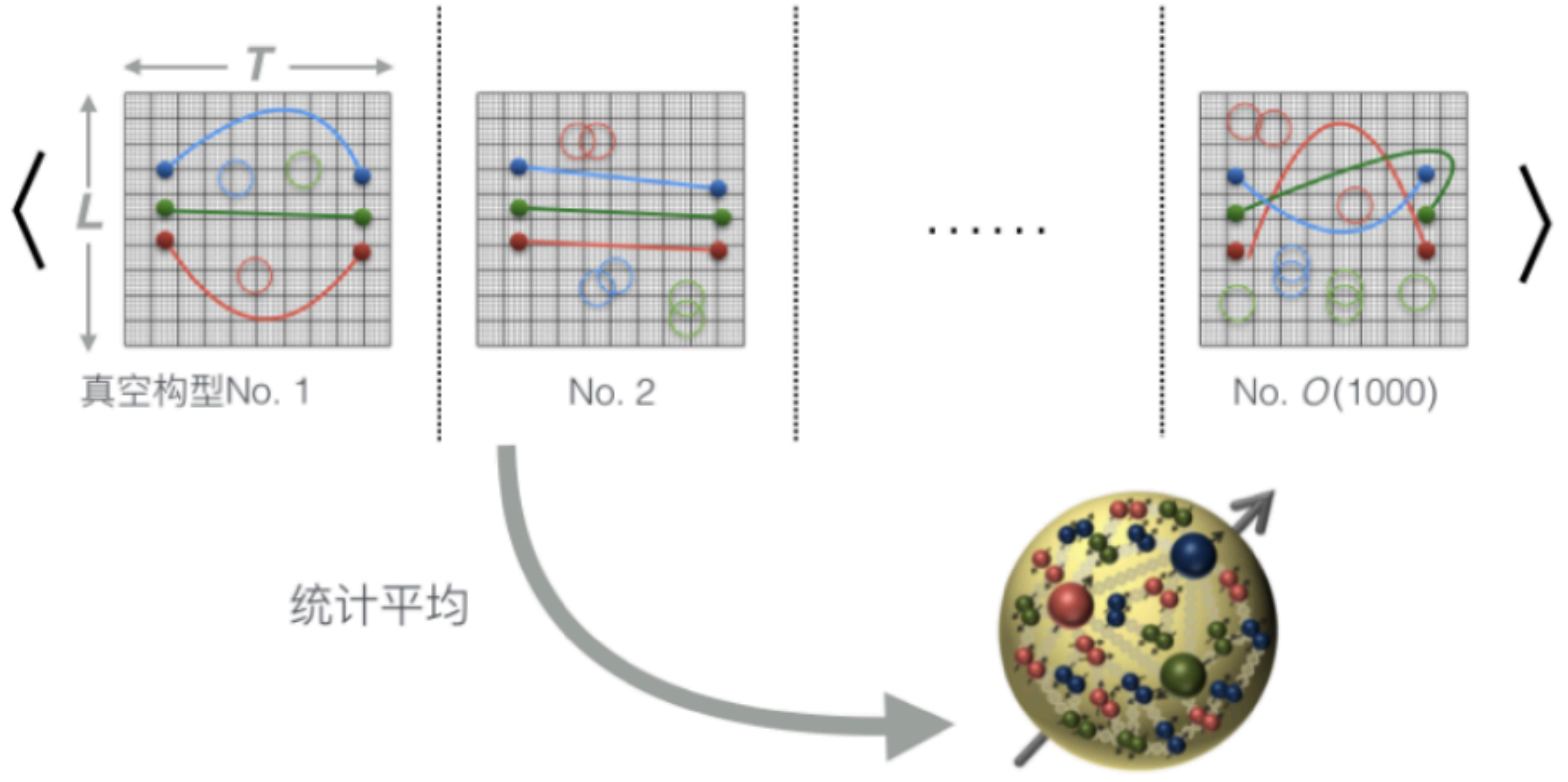
- $Z_S^{\rm SMOM}$ has much larger discretization error comparing to $Z_S^{\rm MOM}$, and require very careful treatment.
- Eventually both the MOM and SMOM should provide consistent $\overline{\rm MS}$ result, as we verified here.



F. He, et.al, χQCD, arXiv: 2204.09246

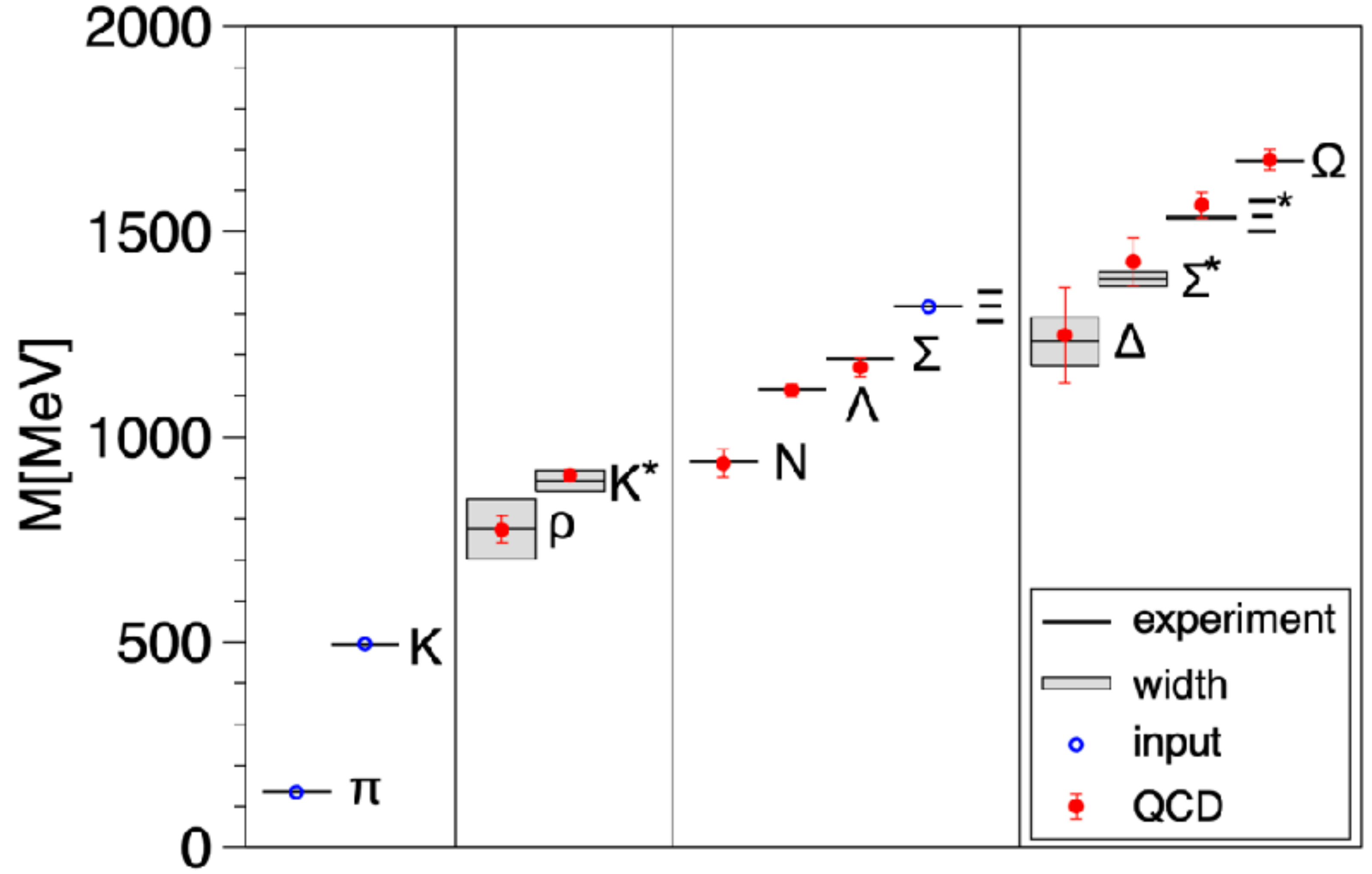
Outline

Basic idea of LQCD



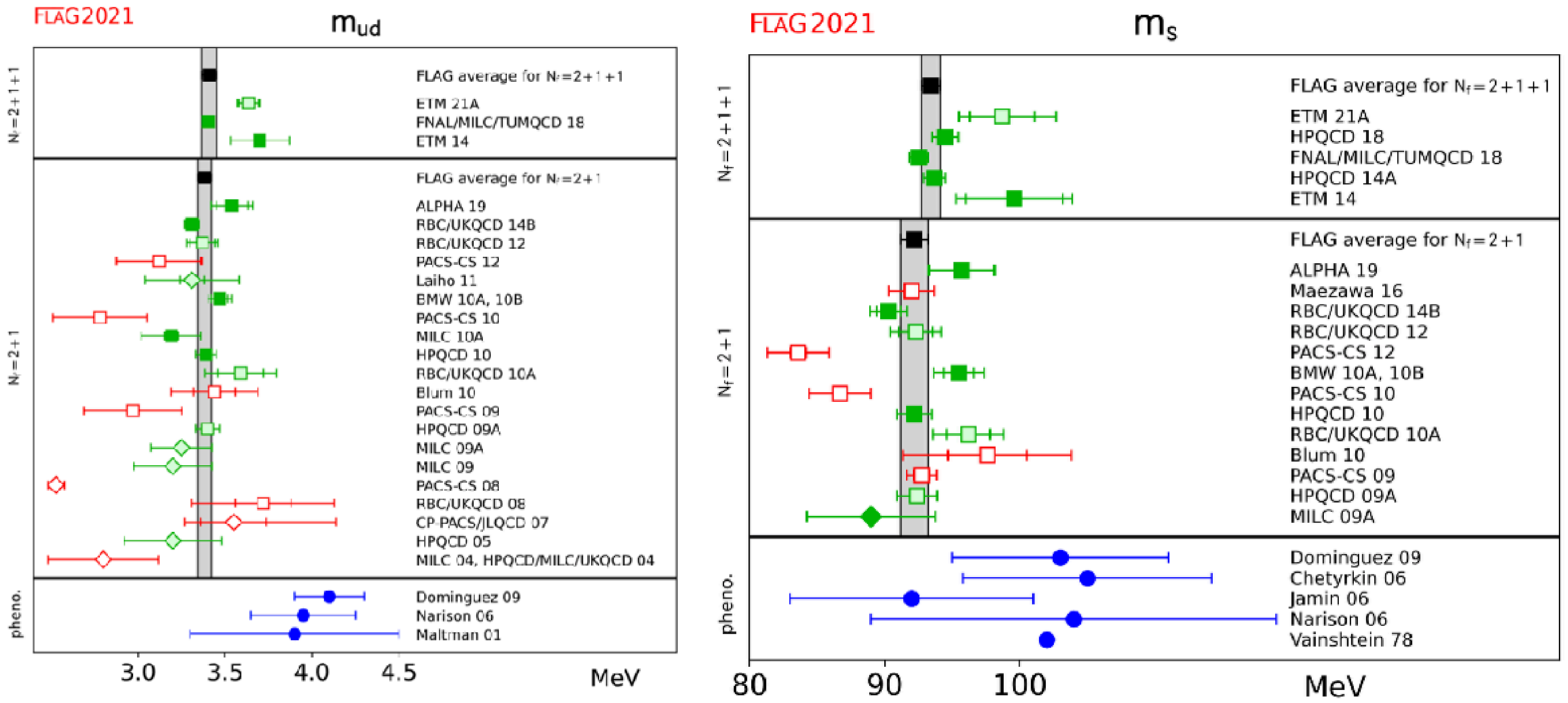
Hadron spectrum

BMWc, Science 322(2008)1224



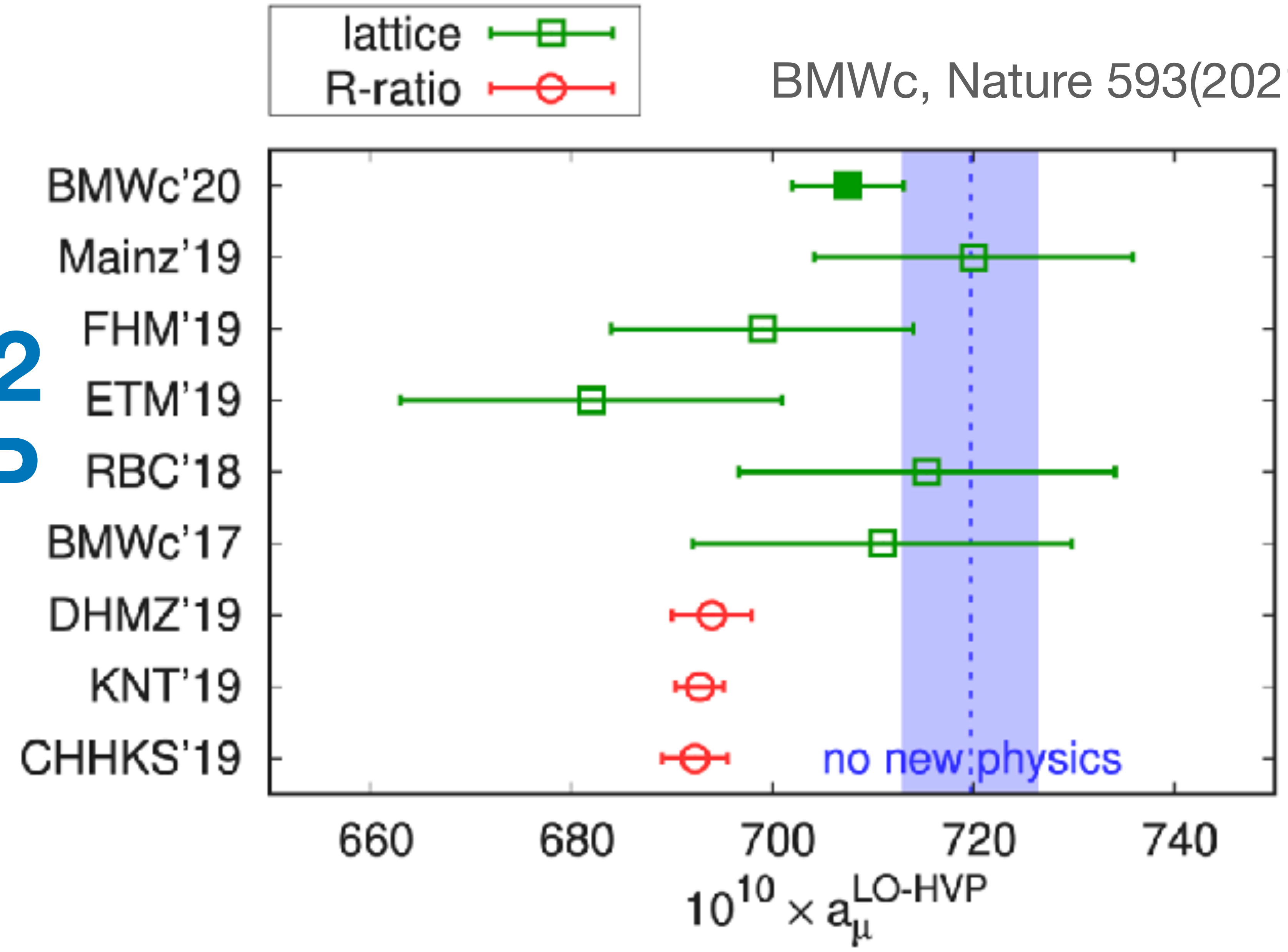
Quark mass determination

FLAG, EPJC80(2020)113



Muon g-2 from HVP

BMWc, Nature 593(2021)51



Anomalous magnetic moment

of electron

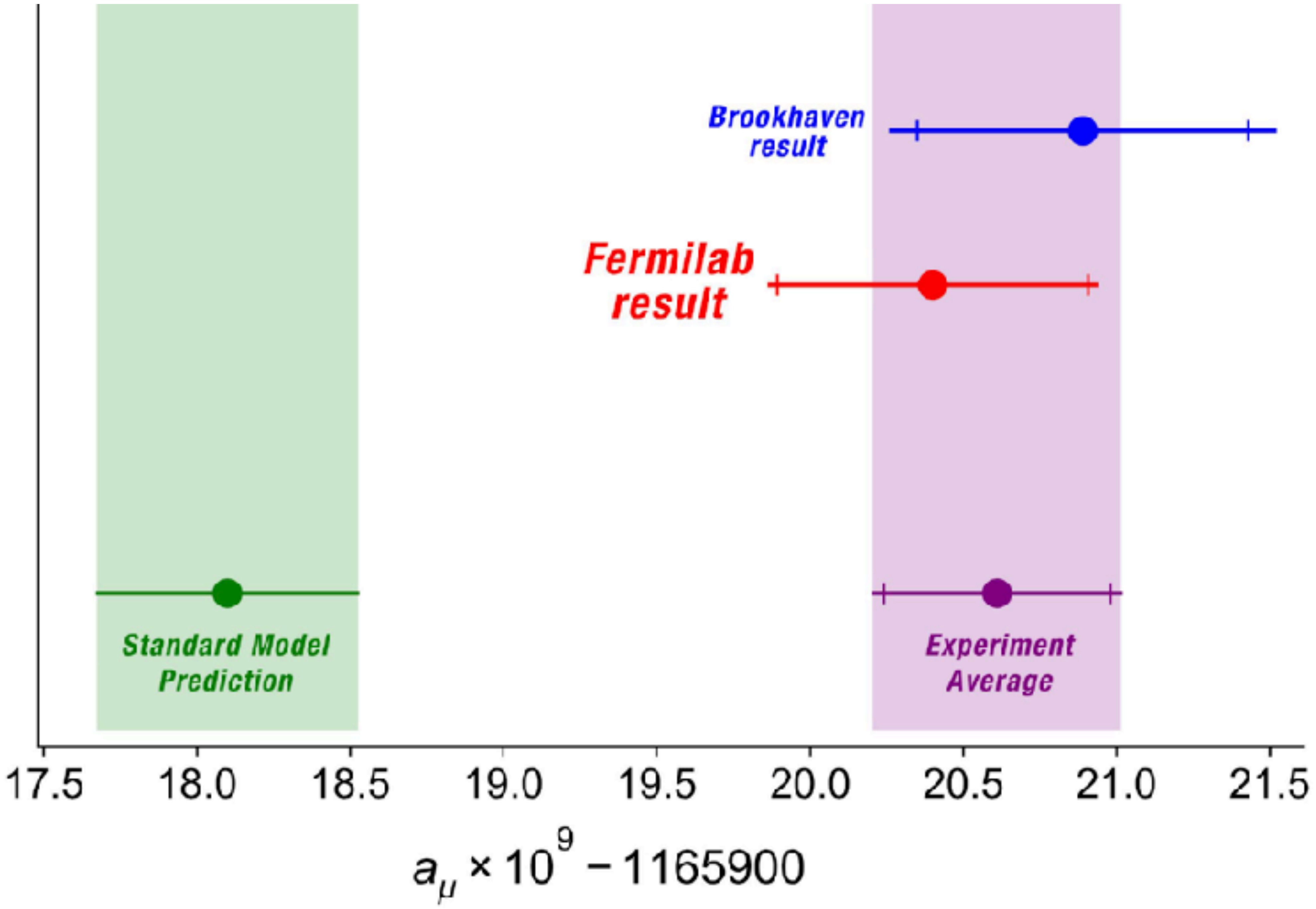
- The magnetic moment g of the fermion should be 2 based on the non-realistic of the Dirac equation: $\frac{(H-eA_0)^2}{2m}\psi = (\frac{m}{2} + \frac{\overrightarrow{P} - e\overrightarrow{A})^2}{2m} - 2\frac{e}{2m}\overrightarrow{B}\cdot\overrightarrow{S} + \dots)\psi$;

- The quantum effect introduces a correction: $g = 2 + \frac{\alpha_R}{\pi} + \mathcal{O}(\alpha_R^2) = 2.00232 + \mathcal{O}(10^{-5})$.

- Standard model prediction: $a^{\text{th}} = \frac{g-2}{2} = 1159652181.6(2) \times 10^{-12}$;
- Experiment measurement: $a^{\text{ex}} = \frac{g-2}{2} = 1159652181.3(2) \times 10^{-12}$.

Anomalous magnetic moment of muon

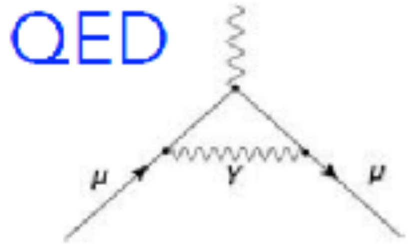
$$g_\mu = 2(1 + a_\mu) = 2(1 + \frac{e^2}{2\pi} + \mathcal{O}(e^4)) \sim 2 * 1.00116......$$



Experimental result of muon g-2

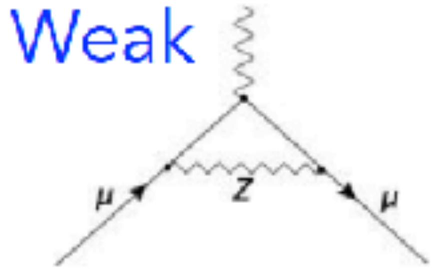
$$a_\mu(\mathrm{SM}) = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{Weak}) + a_\mu(\mathrm{Hadronic})$$

QED



$+\ldots$ $116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

Weak

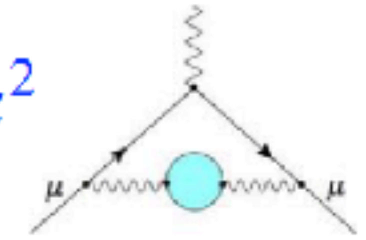


$+\ldots$ $153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic...

...Vacuum Polarization (HVP)

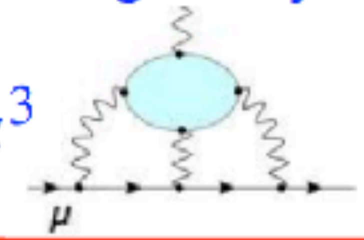
α^2



$+\ldots$ $6845\,(40) \times 10^{-11}$ [0.6%] 0.37 ppm

...Light-by-Light (HLbL)

α^3



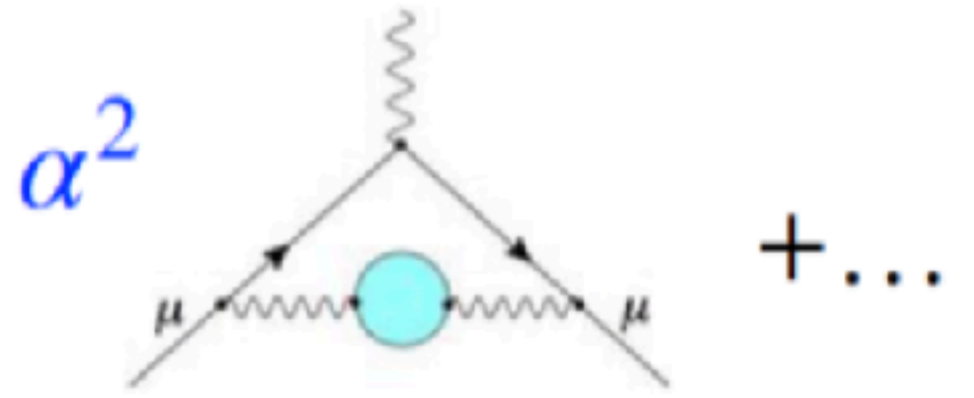
$+\ldots$ $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

muon g-2 from standard model prediction

Leading order HVP contribution to g-2

Hadronic…

…Vacuum Polarization (HVP)



$$a_\mu^{\mathrm{LO-HVP}} = 4\alpha^2 \int_0^\infty \frac{\mathrm{d}q^2}{m_\mu^2} f(\frac{q^2}{m_\mu^2})(\Pi(q^2) - \Pi(0))$$

$$\Pi^{\mu\nu}(q) = \int \mathrm{d}^4 x e^{iqx} \langle j_\mu(x) j_\nu(0) \rangle = \Pi(q^2)(q^2\delta_{\mu\nu} - q_\mu q_\nu), \quad j_\mu = \sum_{f=u,d,s,c\ldots} Q_f \bar{\psi}_f \gamma_\mu \psi_f$$

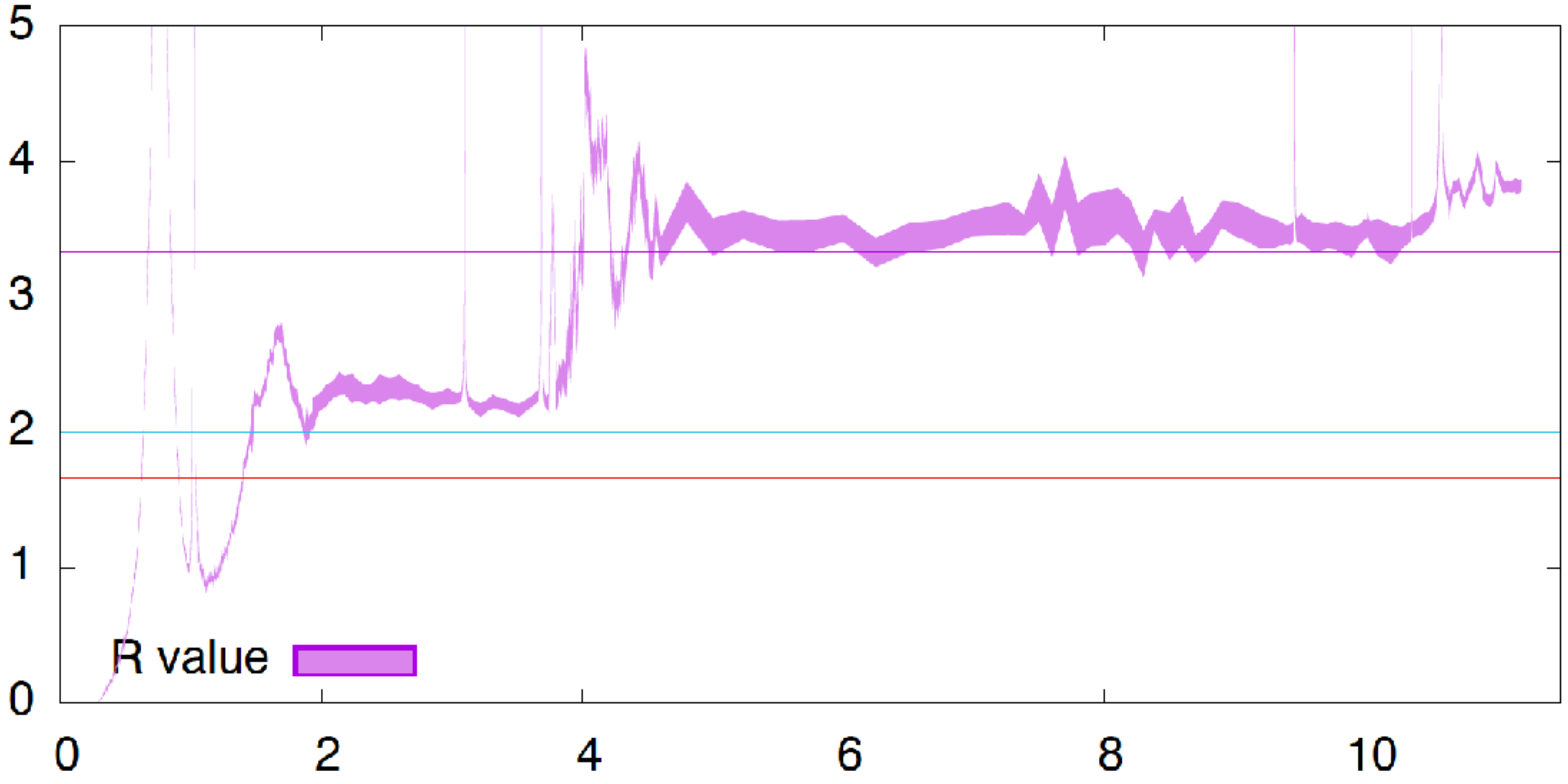
- LO-HVP contribution is crucial to understand the muon g-2 tension.
- Standard model prediction from the R-ratio suggests $\bar{a} \equiv a_\mu^{\mathrm{LO-HVP}} \times 10^{10} = 692.8(2.4)$;

A. Keshavarzi, D. Nomura, and T. Teubner, PRD101(2020)014029

- The present experiment requires for no new physics is $\bar{a} = 718(4)$.
- This tension is suppressed by m_e^2/m_μ^2 in the electron g-2 and then comparable with the other uncertainties.

Leading order HVP contribution from R-ratio

$\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$



A. Keshavarzi, D. Nomura, and T. Teubner, PRD101(2020)014029

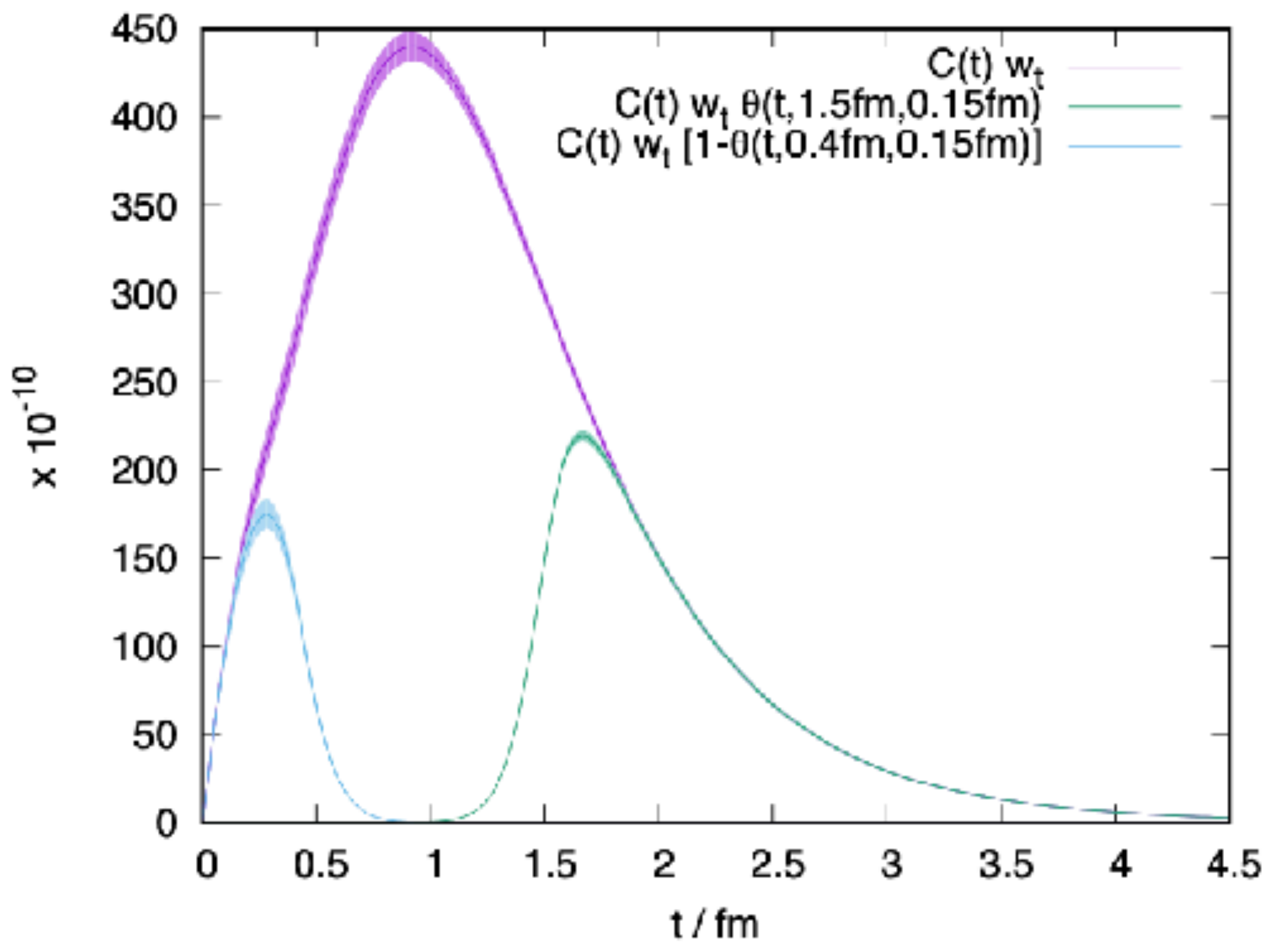
- The HVP contribution can be extracted from the famous R-ratio which was used to confirm $N_c = 3$:
- $a_\mu^{\mathrm{LO-HVP}} = \int \mathrm{d}t\, \omega(t) C(t);$

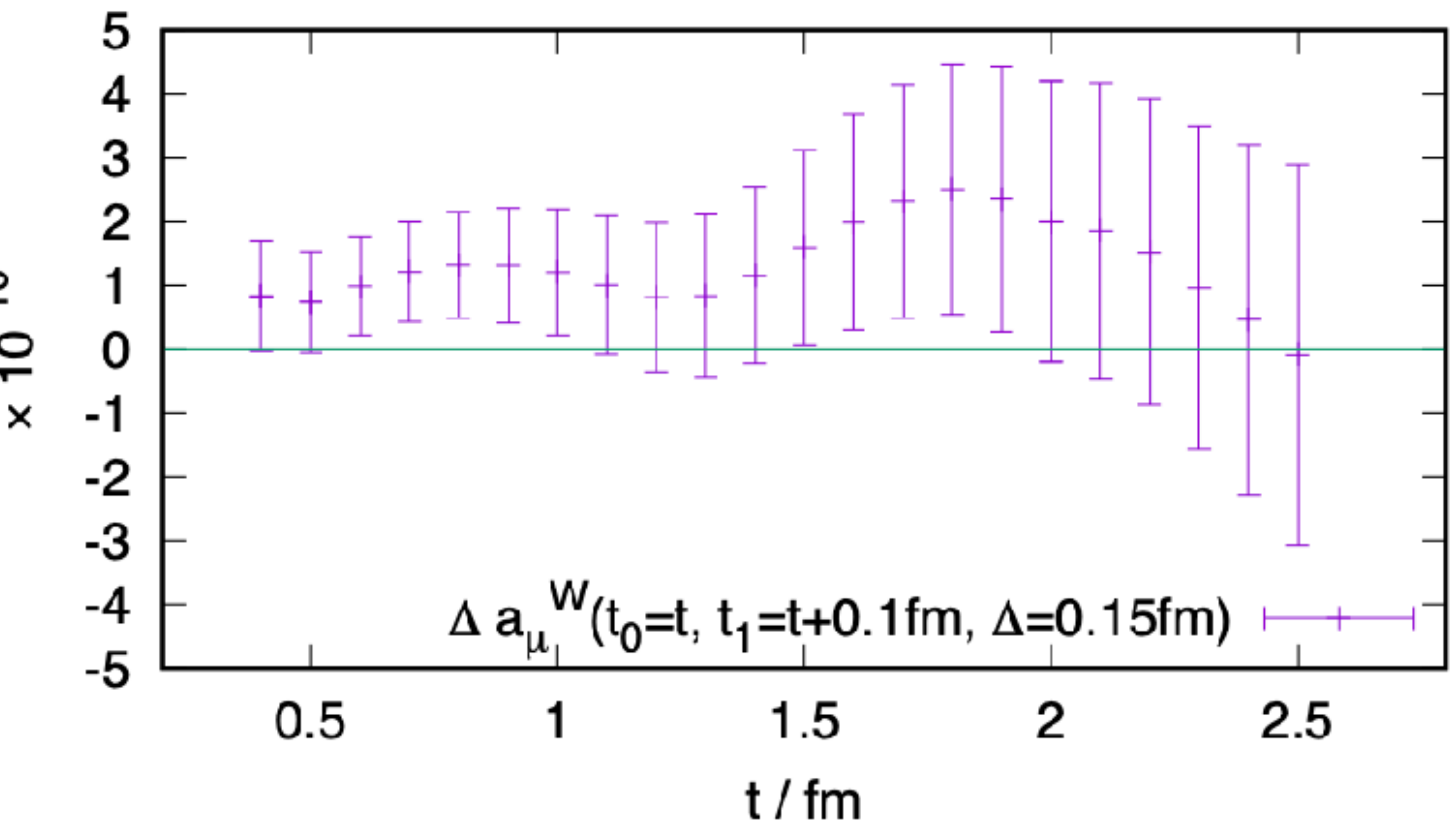
$$\omega(t) = 4\alpha^2 \int_0^\infty \frac{\mathrm{d}q^2}{m_\mu^2} f\left(\frac{q^2}{m_\mu^2}\right)\left[\frac{\cos(tq)-1}{q^2} + \frac{1}{2}t^2\right]$$

$$C(t) = 1/(12\pi^2)\int_0^\infty d(\sqrt{s})R(s)se^{-\sqrt{s}t}$$

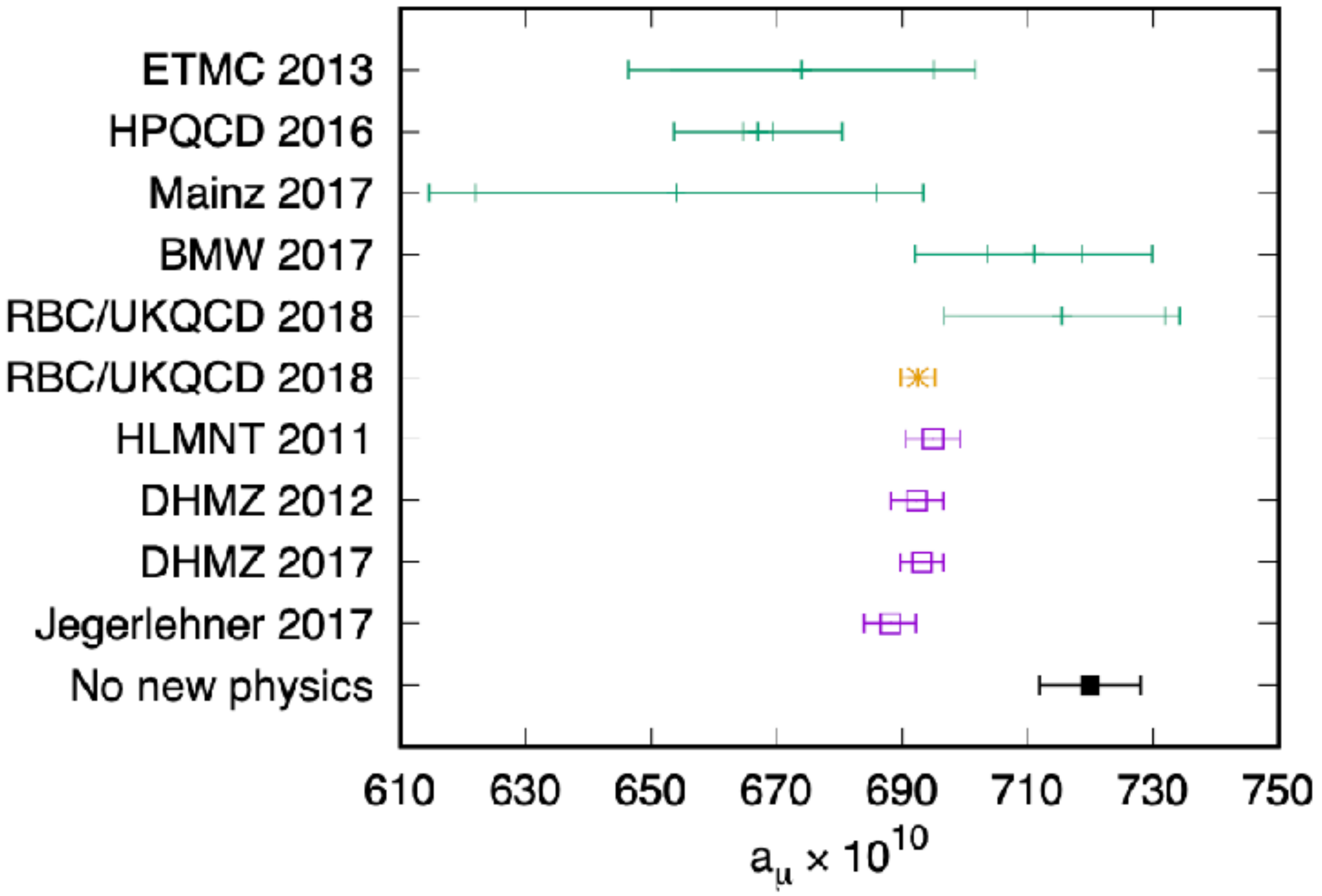
- $C(t)$ can also be calculated from LQCD directly.

Leading order HVP contribution from RBC/UKQCD





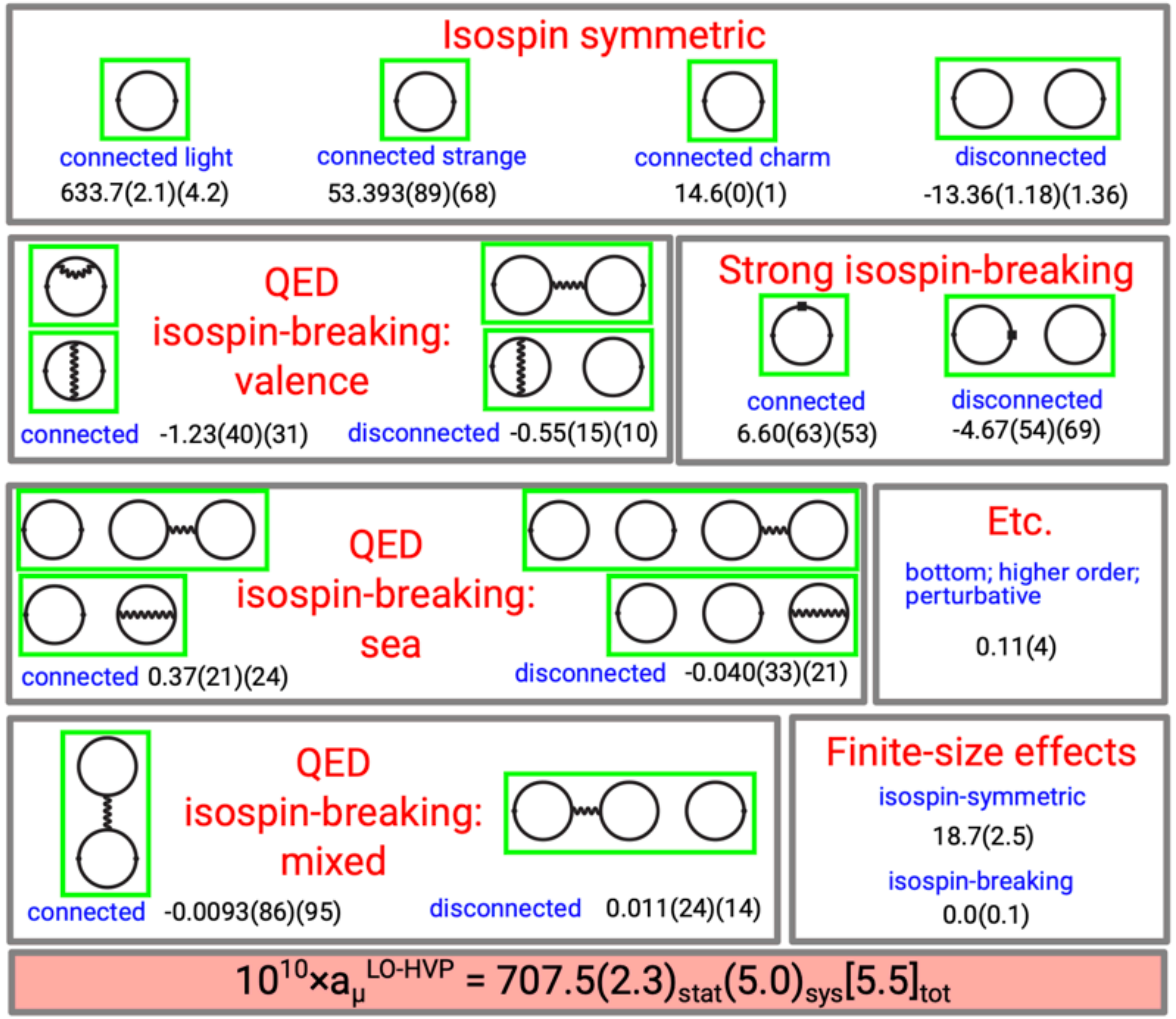
- RBC result of $C(t)$ is consistent with that from R-ratio in all the range,
- and has smaller uncertainty in the medium range of t.
- Thus they also present a LQCD-RR combined prediction 693(3), together with the pure LQCD prediction 715(18).



T. Blum, RBC, PRL121(2018)022003

Leading order HVP contribution

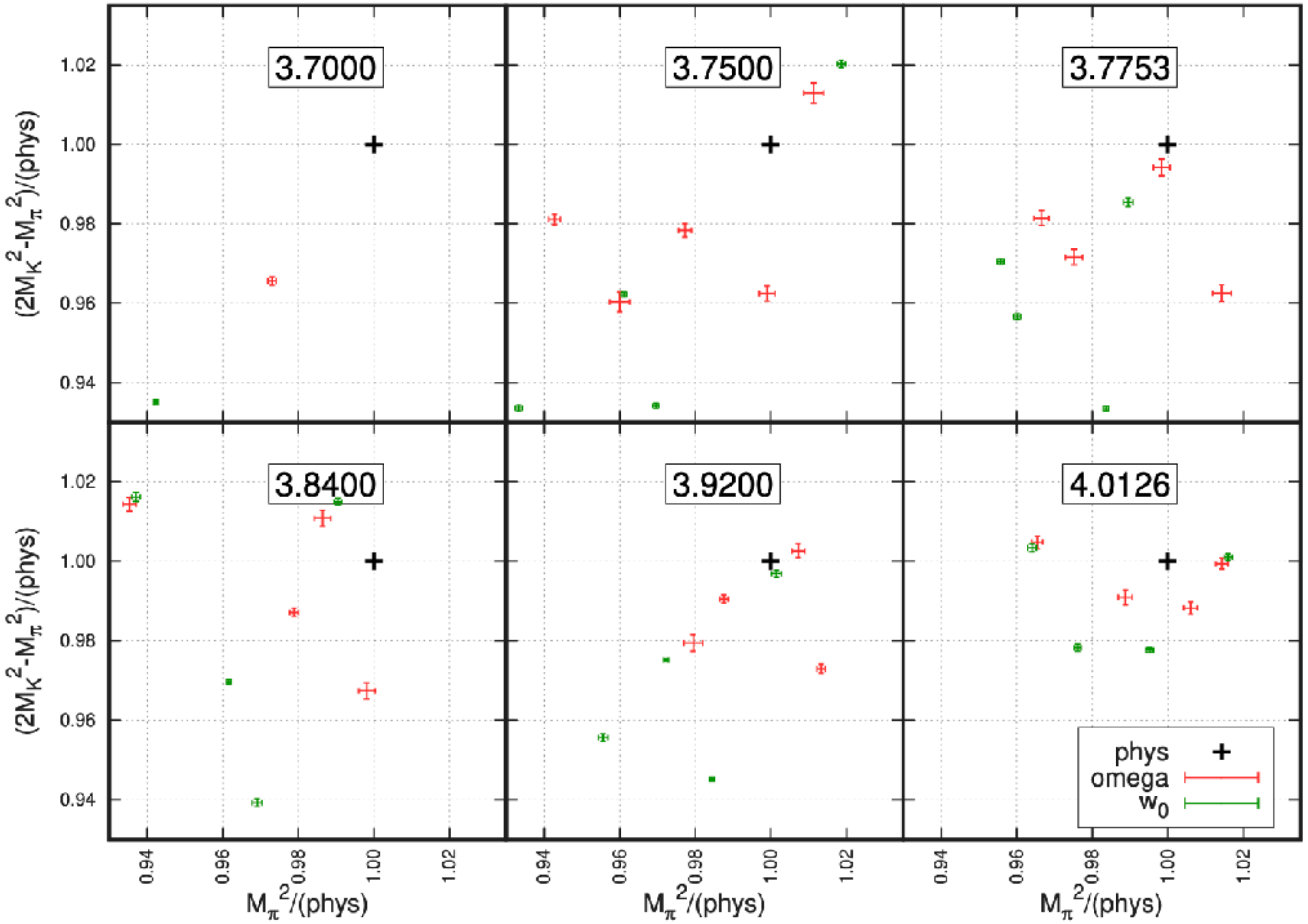
from BMWc



BMWc, Nature 593(2021)51

- $$C(t) = \frac{5}{9}C^{\text{con}}(t; m_l) + \frac{1}{9}C^{\text{con}}(t; m_s) + \frac{4}{9}C^{\text{con}}(t; m_c) + C^{\text{dis}}(t)$$
  $$+\alpha C^{\text{QED}}(t) + \Delta m C^{\text{SIB}}(t) + \mathcal{O}(\alpha^2, \alpha\Delta m, \Delta m^2);$$
- BMW HVP result suppress the uncertainty by a factor of 3 and reach the 1% precision;
- The charm and disconnected contributions cancel each other within the uncertainty;
- Connected and disconnected part of $\Delta m C^{\text{SIB}}(t)$ almost cancel each other;
- The QED contributions are negligible.

Ensembles used by BMWc

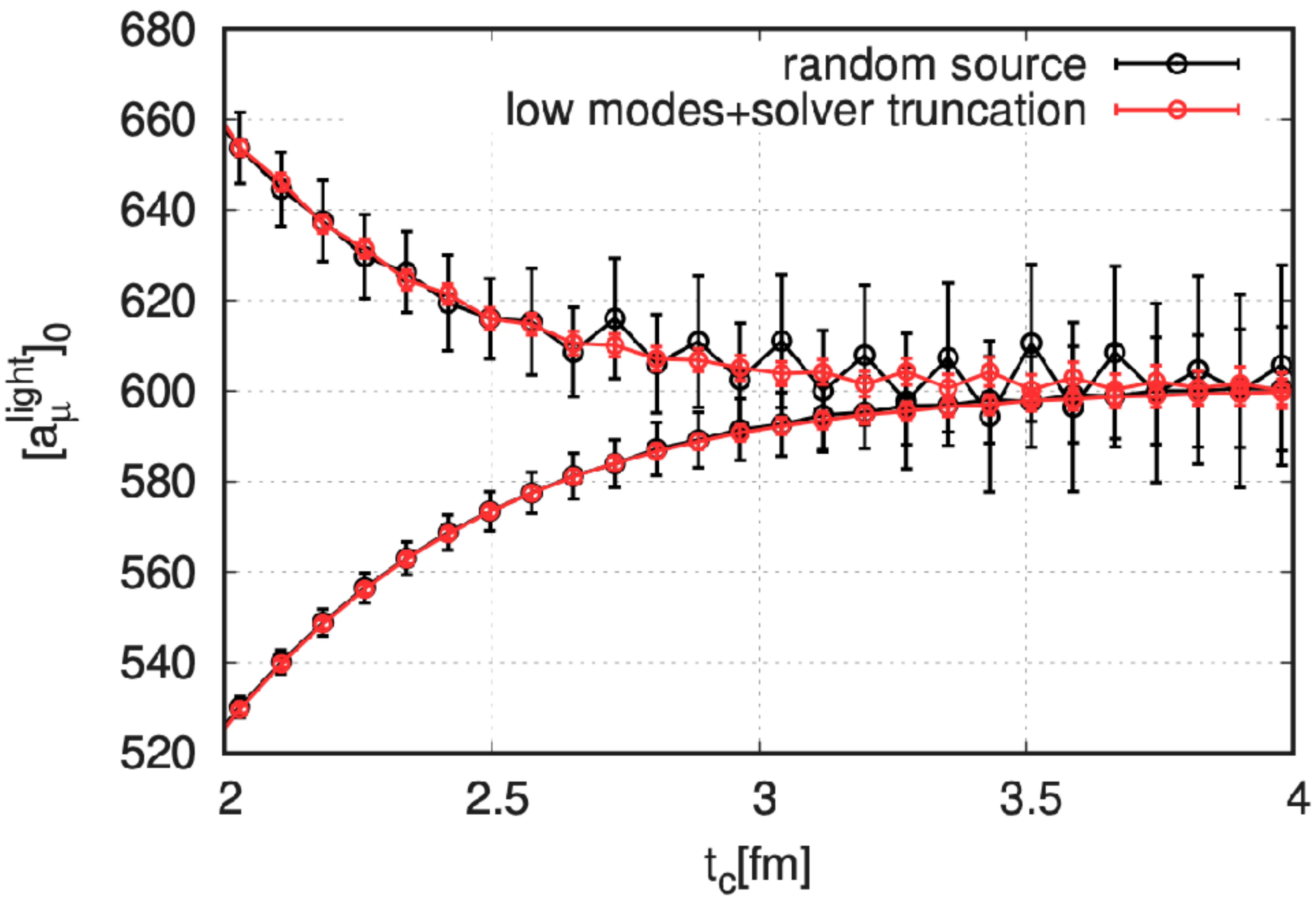


BMWc, Nature 593(2021)51

β	a[fm]	$L \times T$	m_s	m_s/m_l	#conf
3.7000	0.1315	48×64	0.057291	27.899	904
3.7500	0.1191	56×96	0.049593	28.038	315
			0.049593	26.939	516
			0.051617	29.183	504
			0.051617	28.038	522
			0.055666	28.038	215
3.7753	0.1116	56×84	0.047615	27.843	510
			0.048567	28.400	505
			0.046186	26.479	507
			0.049520	27.852	385
3.8400	0.0952	64×96	0.043194	28.500	510
			0.043194	30.205	190
			0.043194	30.205	436
			0.040750	28.007	1503
			0.039130	26.893	500
3.9200	0.0787	80×128	0.032440	27.679	506
			0.034240	27.502	512
			0.032000	26.512	1001
			0.032440	27.679	327
			0.033286	27.738	1450
			0.034240	27.502	500
4.0126	0.0640	96×144	0.026500	27.634	446
			0.026500	27.124	551
			0.026500	27.634	2248
			0.026500	27.124	1000
			0.027318	27.263	985
			0.027318	28.695	1750

Bounding method

for a_μ^{light}



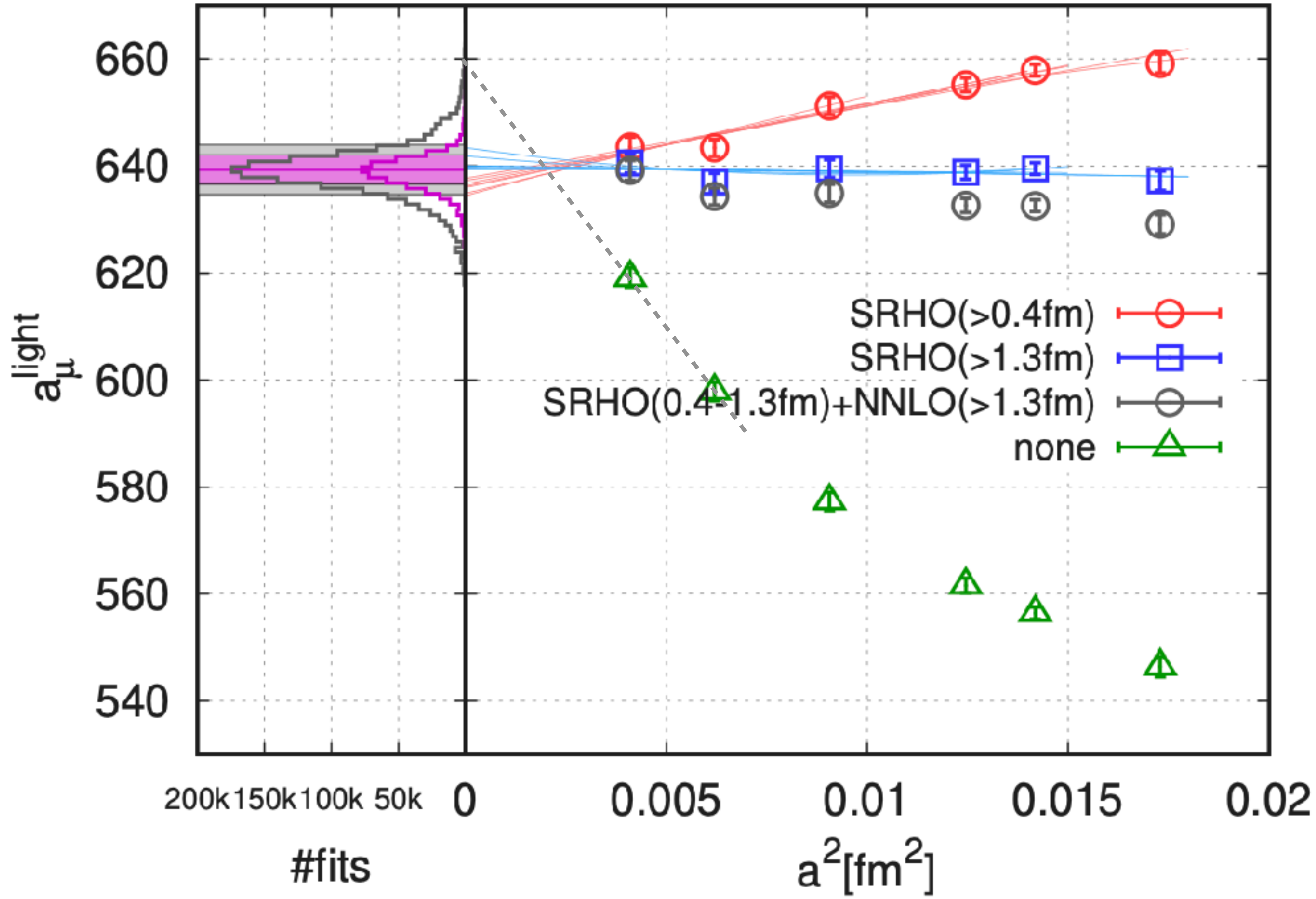
- The upper/lower band of the light quark contribution is defined by:

- $a_{\mu,\text{lower band}}^{\text{LO-HVP}} = \int_0^{t_c} \mathrm{d}t\, \omega(t) C(t)$
- $a_{\mu,\text{upper band}}^{\text{LO-HVP}} = \int_0^{t_c} \mathrm{d}t\, \omega(t) C(t) + \int_{t_c}^{\infty} \mathrm{d}t\, \omega(t) C(T_{cut}) e^{-2\sqrt{m_\pi^2 + (\frac{2\pi}{L})^2}}$

- One can take the value at t_c where $a_{\mu,\text{lower band}}^{\text{LO-HVP}} = a_{\mu,\text{upper band}}^{\text{LO-HVP}}$ to suppress the uncertainty.

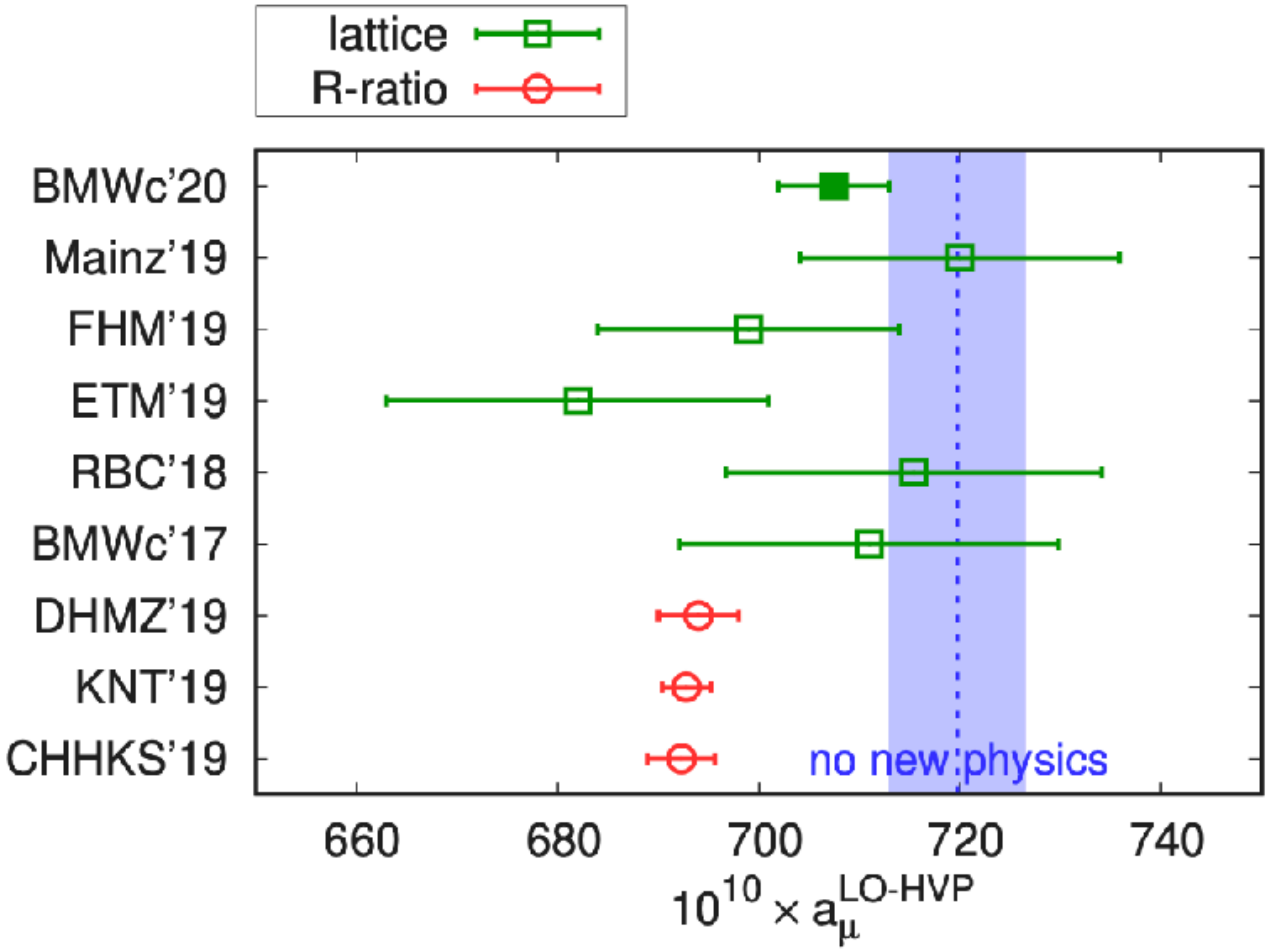
Lattice spacing dependence

of a_μ^{light}



- The original result is obviously non-linear and kinds of the analytical corrections are applied;
- The distribution of kinds of fits looks like a Gaussian one and the central value is around 640.
- But the continuum extrapolation based on the original data can be much higher…

Leading order HVP contribution from kinds of LQCD groups



BMWc, Nature 593(2021)51

- The present BMWc result is in the middle of the R-ratio and "no new physics requirement".
- $O(100)$ cheaper fermion action (comparing to the RBC/UKQCD setup) is used to obtain the results at smaller lattice spacing.
- The result requires further verification from the other groups.

χQCD efforts

nature

NEWS | 30 June 2022

Physicists spellbound by deepening mystery of muon particle's magnetism

Theoretical predictions move closer to experimental results, but questions remain about possible gaps in the standard model of particle physics.

……

Two preliminary results from this energy 'window' were posted on the arXiv preprint repository in April 2022: one by Christopher Aubin at Fordham University in New York City and his collaborators[4], and the other by Gen Wang at the University of Aix-Marseille in France[5]. Earlier this month, two more groups – one led by Hartmut Wittig at Johannes Gutenberg University in Mainz, Germany, the other by Silvano Simula at the National Institute for Nuclear Physics in Rome – announced their own window results at a muon conference in Los Angeles, California. Simula's group is writing a preprint, and Wittig's group submitted its preprint on 14 June[6]. All four calculations validated BMW's own window results, even though their lattice techniques vary. "Very different ways of approaching the problem are getting a very similar result," says Aubin.

……

Muon g - 2 with overlap valence fermion

G. Wang, et.al., χQCD, arXiv: 2204.01280

(χQCD Collaboration)

Gen Wang,[1,2,*] Terrence Draper,[2] Keh-Fei Liu,[2] and Yi-Bo Yang[3,4,5,6,†]

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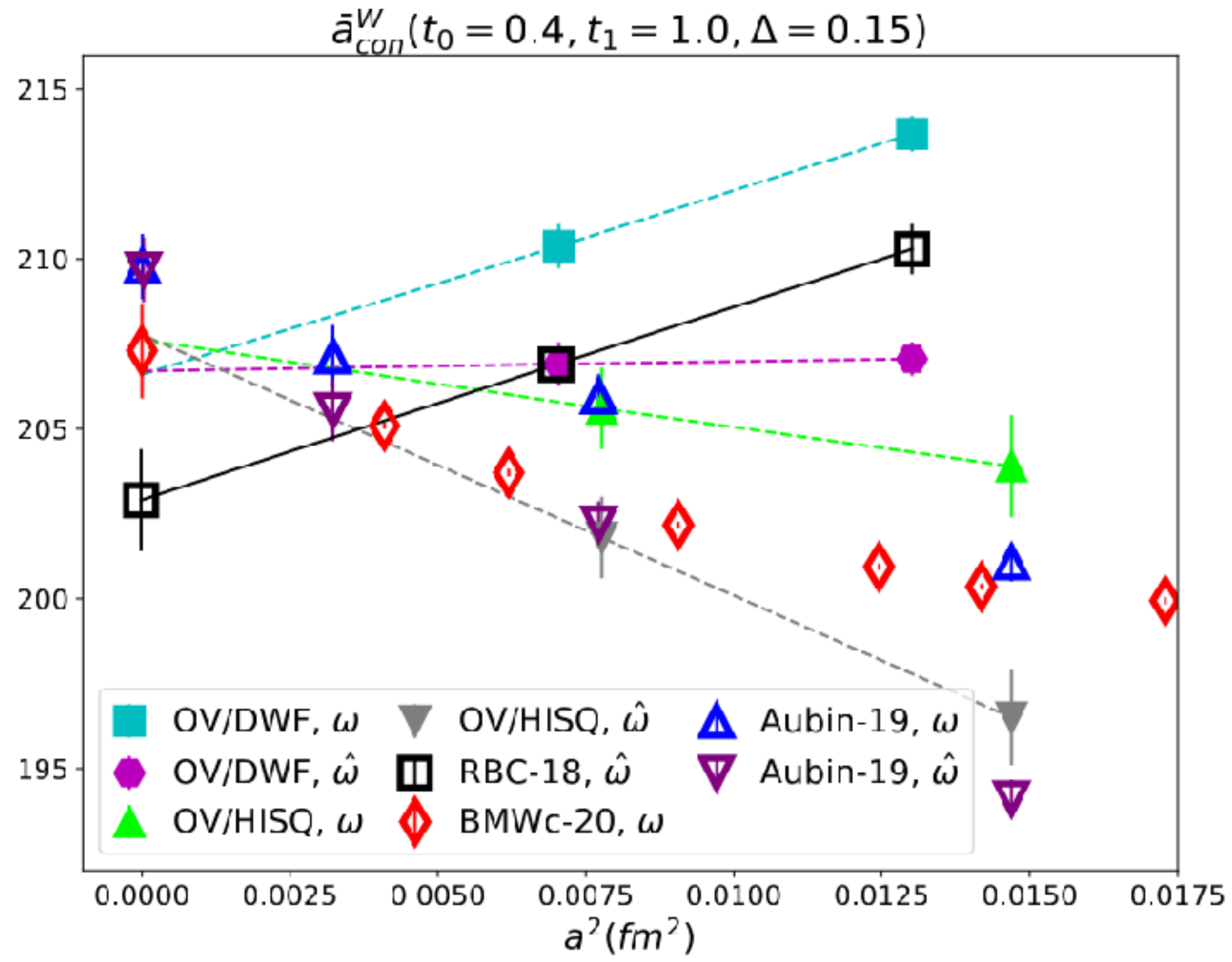
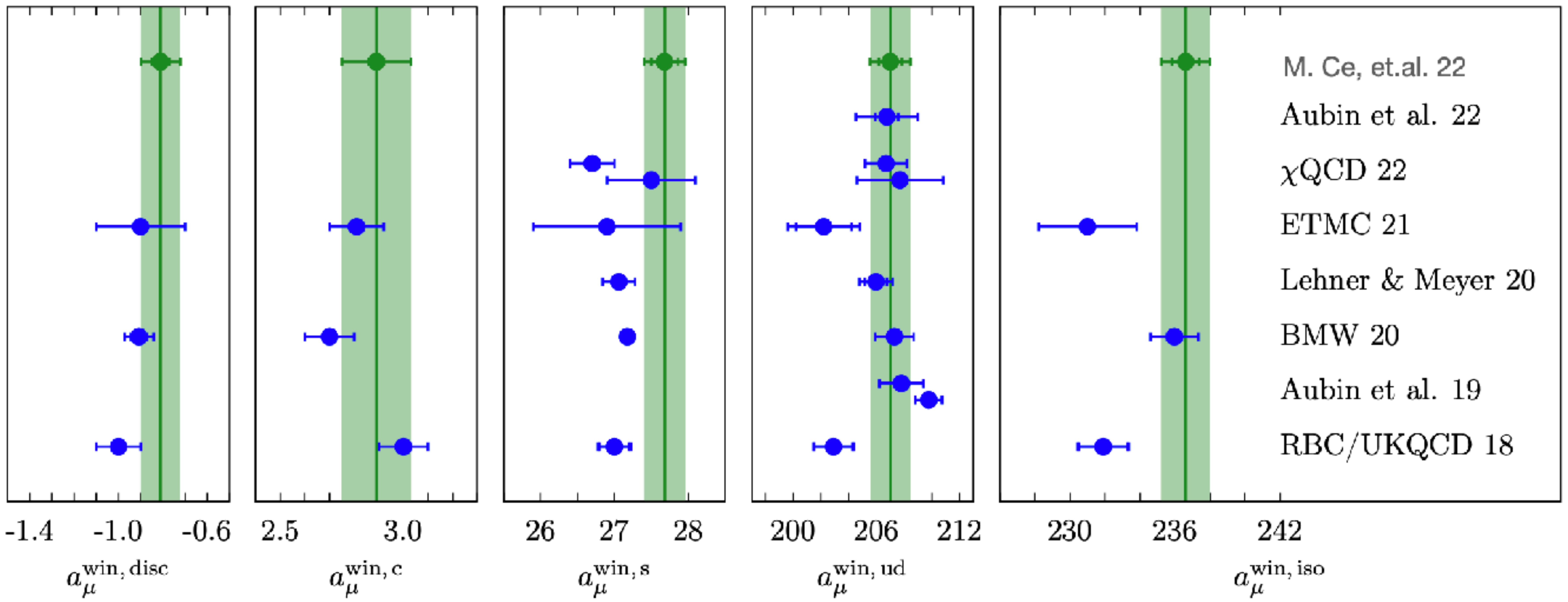
[5] *School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China*
[6] *International Centre for Theoretical Physics Asia-Pacific, Beijing/Hangzhou, China*



Leading order HVP contribution in the "window"

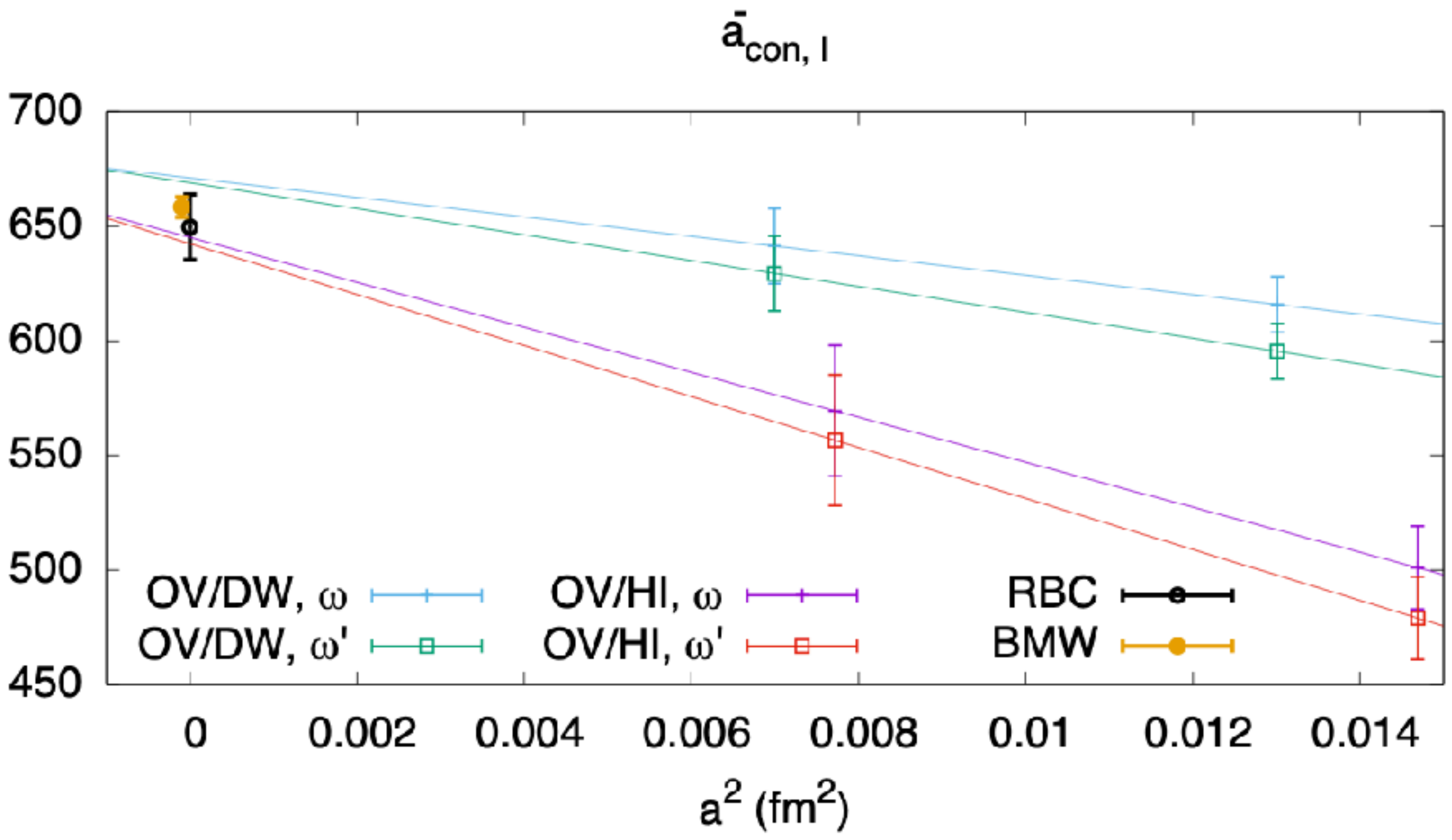


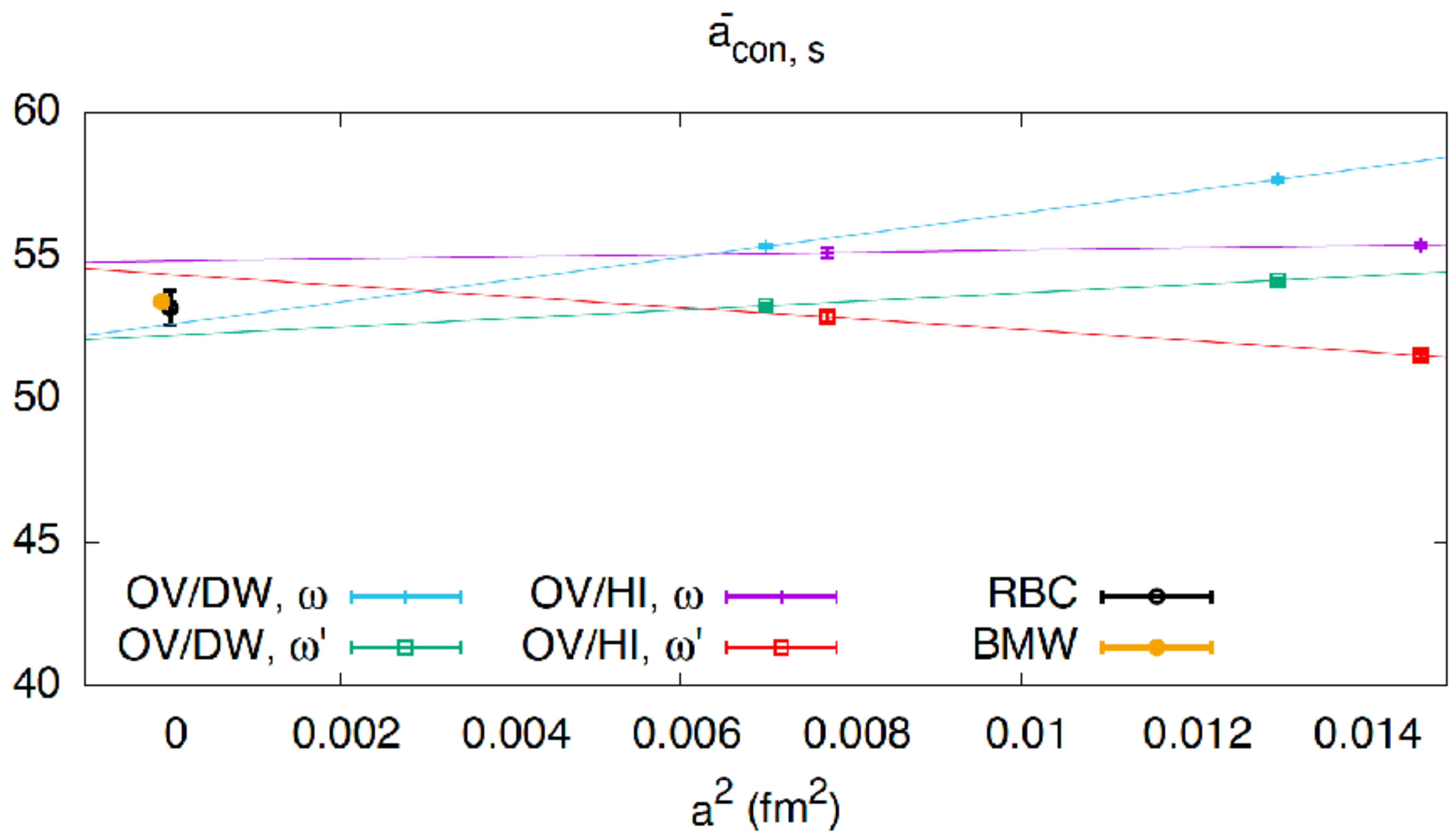
M. Ce, et.al., arXiv: 2206.06582

- The "window" value is much easier to be precise and accurate from LQCD;
- and then become a good way to verify the consistency between different LQCD groups;
- There are still some tensions which can be resolved within one year or so.

χQCD efforts

on the total contribution





G. Wang, et.al., χQCD, in preparation

Summary

- LQCD: using the samples from stochastic quantization to simulate the correlation function of path integral;
- State-of-the-arts ensembles are the foundation of the high accuracy LQCD prediction, and the efforts in China are in progress;
- Renormalization issue we found should be verified by the other collaborations, before high accuracy quark mass prediction can be made;
- LO-HVP contribution of muon g-2 is converging, while more efforts are needed.