$B \to D^*$ form factors beyond leading power

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Motivations

- Understanding strong interaction dynamics in heavy hadron system
  - Factorization property
  - Higher order (α_s) impact
  - Renormalization and asymptotic properties of the (higher-twist) B-meson DAs
  - Interplay of different QCD techniques
- Understanding the general properties of power expansion in EFTs
- Extracting $|V_{cb}|$ exclusively (inclusive vs. exclusive)
- Testing $R(D^*)$, ΔA_{FB} $\cdots$

State of art of $B \to D^{(*)}$ FF

- Heavy-quark expansion [Isgur/Wise 89' 90']
  - Power corrections at $1/m_Q$ and $1/m_Q^2$ [Luke 90', Neubert 92', Ball 92', Mannel 94' ; Falk/Neubert 92' 93']
  - QCD corrections at $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ [Gounaris 83' 84', Shifman 88'; Neubert 92', Czarnecki 96', 97']
  - QCDSR for subleading I-W func [Ligeti/Nir/Neubert 92' 92' 93']
  - QED correction at $\mathcal{O}(\alpha)$ [Sirlin 82']
- LCSR with B-meson LCDAs [Faller *et al.* 09']
  - LCSR at $\mathcal{O}(\alpha_s)$ [Wang *et al.* 16']
  - LCSR with higher-twist contributions [Gubernari *et al.* 19']
  - LCSR NLL + 3 NLP [Gao *et al.*21']
- Parametrization [BGL 97'; CLN 98']
- Lattice QCD [FNAL/MILC04' 08' 13' 14' 15' 21'; HPQCD15' 17' 17' 20' 21'; UKQCD 19'; JLQCD ?]
- Experiment [Belle 16' 17' 19' 20'; BarBar 08' 09' 10'; LHCb 20']

Leading power factorization formulae

- Factorization formulae for semileptonic decay at leading power

$$F_i^{B\to D^*}(n\cdot p) = C_i^{(\mathrm{A0})}(n\cdot p)\,\xi_a(n\cdot p) + \int d\tau\, C_i^{(\mathrm{B1})}(\tau, n\cdot p)\,\Xi_a(\tau, n\cdot p)\,, (a = \parallel, \perp)$$

- Hard matching coefficients $C^{(A0)}$
  - One-loop perturbative calculations [Bauer/Fleming/Pirjol/Stewart 01'; Beneke/Kiyo/Yang 04']
  - Two-loop perturbative calculations [Bonciani/Ferroglia 08'; Asatrian/Greub/Pecjak 08'; Beneke/Huber/Li 09'; Bell 09'; Bell/Beneke/Huber/Li 11']
- Hard matching coefficients $C^{(B1)}$
  - One-loop perturbative calculations [Becher/Hill 04'; Hill/Becher/Lee/Neubert 04'; Beneke/Yang 06']
- Hard-collinear matching coefficients $J^{(A0)}_{\parallel(\perp),-(+)}, J^{(B1)}_{\parallel(\perp),+}$
  - This work (power counting: $m_c \sim \mathcal{O}(\sqrt{m_b \Lambda_{\mathrm{QCD}}})$)
- B-meson LCDAs [KKQT 01'; Braun,Ji,Manashov 17'; Beneke/Braun/Ji/Wei 18']
- NLL resummation for C, J, ϕ_B

Subleading Power Corrections

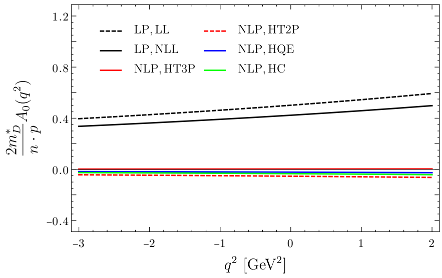
- Higher-twist B-meson LCDAs including both 2-& 3-particle (up to twist-6)
- HQET representation of QCD b-quark field

$$\bar{q}\,\gamma_\mu\, b \to \bar{q}\,\gamma_\mu\, h_v + \frac{1}{2m_b}\,\bar{q}\,\gamma_\mu\, i\vec{\not{D}} h_v + \mathcal{O}(\frac{1}{m_b^2}) + \cdots$$

- Hard-collinar charm quark propagator

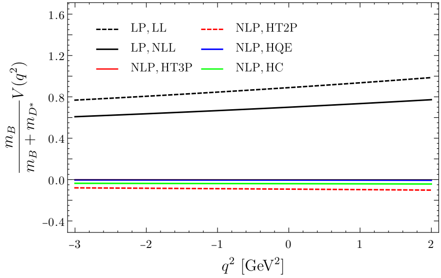
$$\frac{\not{p} - \not{k} + m_c}{(p-k)^2 - m_c^2} \to \underbrace{\frac{1}{\bar{n}\cdot\hat{p}}\frac{\not{\bar{n}}}{2}}_{\text{LP}} + \underbrace{\frac{1}{n\cdot p\bar{n}\cdot\hat{p}}\left[\bar{n}\cdot p\frac{\not{n}}{2} - \not{k} + \frac{n\cdot k\bar{n}\cdot p}{\bar{n}\cdot\hat{p}}\frac{\not{\bar{n}}}{2}\right]}_{\text{NLP}} + \underbrace{\frac{m_c}{n\cdot p}\frac{1}{\bar{n}\cdot\hat{p}}\left[1 + \frac{n\cdot k\bar{n}\cdot p}{n\cdot p\bar{n}\cdot\hat{p}}\right]}_{m_c\ \text{NLP}}$$

Predictions for form factors



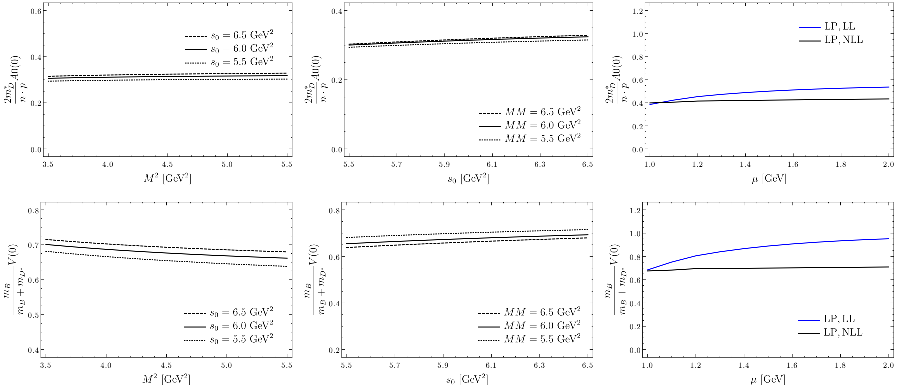
At $q^2 = 0$

- NLL $\sim$ -15.6%
- HT3P $\sim$ 0.4%
- HT2P $\sim$ -10.8%
- HQE $\sim$ -4.5%
- HC $\sim$ -6.9%



- NLL $\sim$ -21.3%
- HT3P $\sim$ -0.3%
- HT2P $\sim$ -10.4%
- HQE $\sim$ -0.5%
- HC $\sim$ -4.5%

Dependence on the M^2, s_0 and μ



BGL parametrization

- Parametrization

$$g(z) = \frac{1}{P_{1-}(z)\phi_g(z)} \sum_{n=0}^{\infty} a_n z^n ,$$

$$f(z) = \frac{1}{P_{1+}(z)\phi_f(z)} \sum_{n=0}^{\infty} b_n z^n ,$$

$$\mathcal{F}_1(z) = \frac{1}{P_{1+}(z)\phi_{\mathcal{F}_1}(z)} \sum_{n=0}^{\infty} c_n z^n ,$$

$$\mathcal{F}_2(z) = \frac{1}{P_{0-}(z)\phi_{\mathcal{F}_2}(z)} \sum_{n=0}^{\infty} d_n z^n$$

$$P(z) = \prod_{P=1}^{n} \frac{z - z_P}{1 - z z_p}, \qquad z_P = \frac{\sqrt{t_+ - m_P^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - m_P^2} + \sqrt{t_+ - t_0}}$$

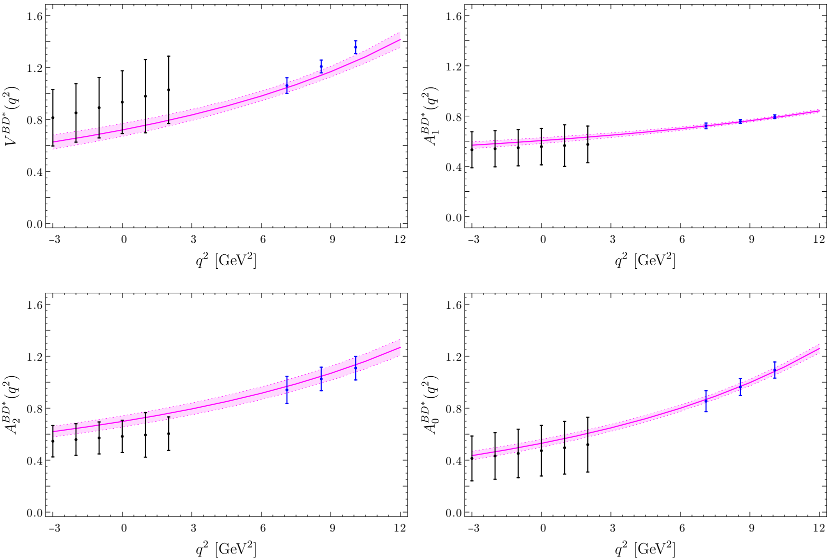
- Constrains

$$\mathcal{F}_1(z=0) = (m_B - m_{D^*}) f(z=0) \qquad \mathcal{F}_1(z=z_{max}) = \frac{1+r}{(1-r)m_B^2(1+\omega)} \mathcal{F}_2(z=z_{max})$$

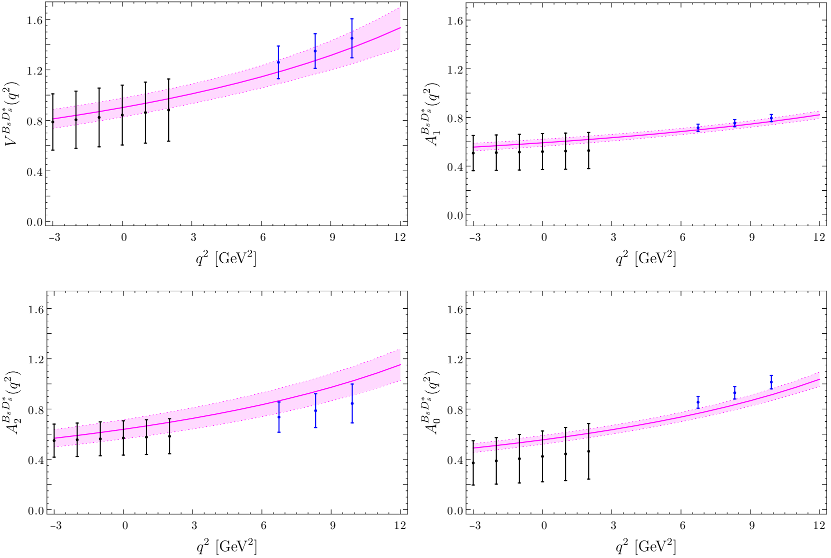
- (Weak) Unitary bound

$$\sum_0^{\infty} a_n^2 < 1 \qquad \sum_0^{\infty} b_n^2 + c_n^2 < 1 \qquad \sum_0^{\infty} d_n^2 < 1$$

Joint fit with FNAL/MILC 21'



Joint fit with HPQCD 21' $(B_s \to D_s^* \ell \nu)$



Results from joint fit ($B \to D^*\ell\nu$)

q^2 [GeV^2]	-3.0	-2.0	-1.0	0.0	1.0	2.0
A_0	0.413(172)	0.432(179)	0.452(187)	0.473(194)	0.495(203)	0.520(211)
A_1	0.532(144)	0.540(144)	0.549(145)	0.557(145)	0.566(164)	0.575(146)
A_2	0.546(121)	0.558(122)	0.571(123)	0.583(125)	0.594(171)	0.604(129)
V	0.813(218)	0.851(225)	0.891(233)	0.933(241)	0.979(282)	1.028(259)

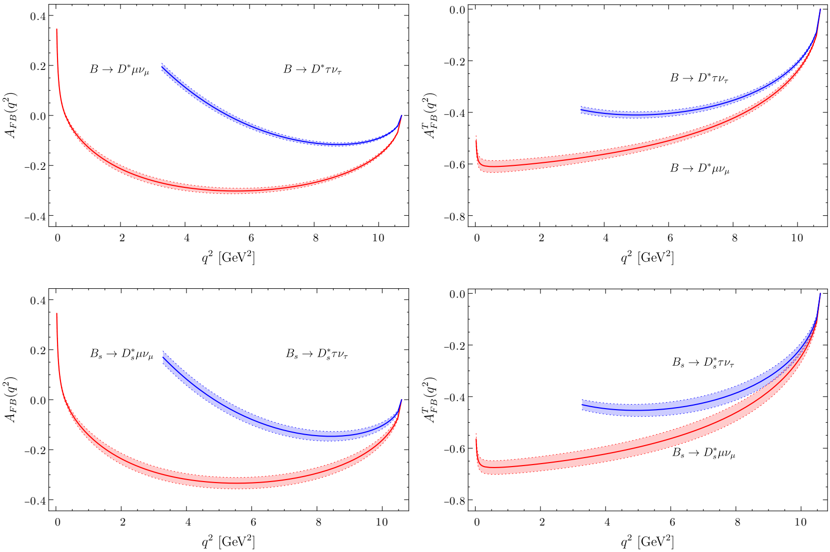
		correlation matrix					
	value	a_0	a_1	b_0	b_1	c_1	d_1
a_0	0.0305(12)	1	-0.4673	0.0904	-0.0409	0.0395	-0.1570
a_1	$-0.0632(366)$		1	-0.0476	0.2351	0.1468	0.1986
b_0	0.0124(2)			1	-0.2547	-0.2545	-0.3396
b_1	0.0014(87)				1	0.5990	0.6753
c_1	0.0061(18)					1	0.8699
d_1	$-0.4411(678)$						1

Results from joint fit $(B_s \to D_s^* \ell \nu)$

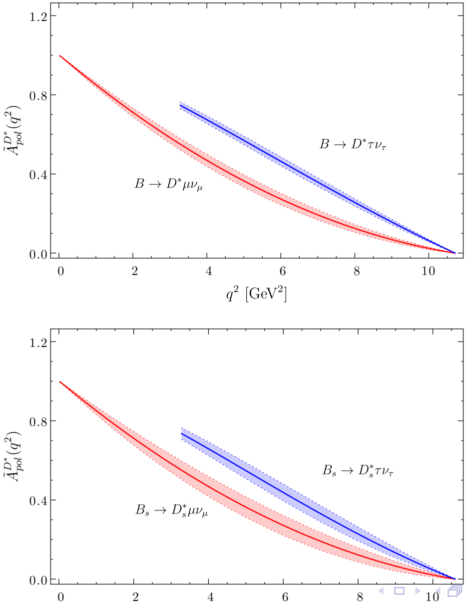
q^2 [GeV^2]	-3.0	-2.0	-1.0	0.0	1.0	2.0
A_0	0.372(176)	0.388(185)	0.405(193)	0.424(202)	0.443(211)	0.464(221)
A_1	0.507(145)	0.511(145)	0.515(147)	0.519(148)	0.523(148)	0.527(149)
A_2	0.549(131)	0.556(133)	0.563(135)	0.570(136)	0.577(138)	0.584(139)
V	0.787(223)	0.805(227)	0.823(233)	0.842(237)	0.861(242)	0.882(246)

		correlation matrix					
	value	a_0	a_1	b_0	b_1	c_1	d_1
a_0	0.0076(7)	1	−0.5469	0.0053	0.0067	−0.0040	−0.0008
a_1	0.0149(139)		1	0.0009	0.0027	0.0051	0.0191
b_0	0.0034(1)			1	−0.1207	−0.3475	0.0527
b_1	0.0059(29)				1	0.0437	−0.0651
c_1	−0.0003(7)					1	0.5235
d_1	−0.0244(262)						1

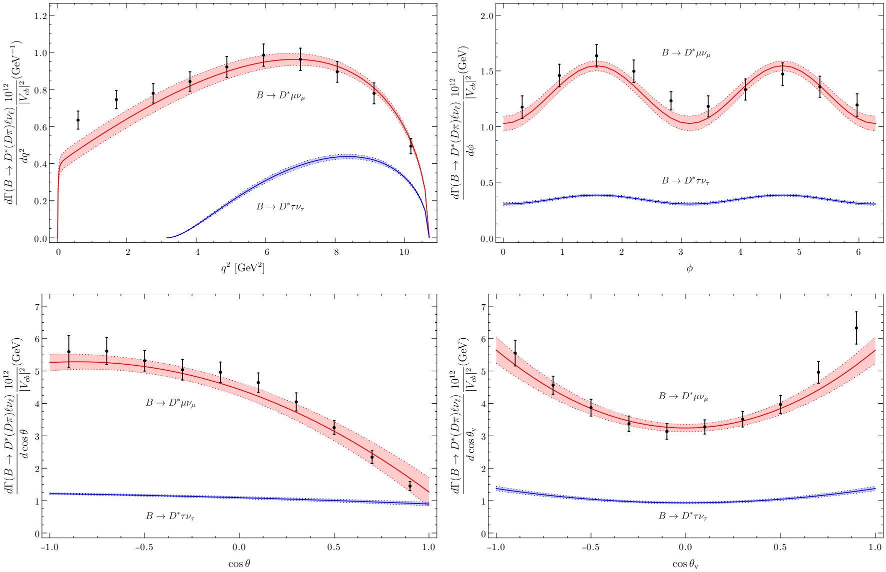
Forward-backward asymmetry



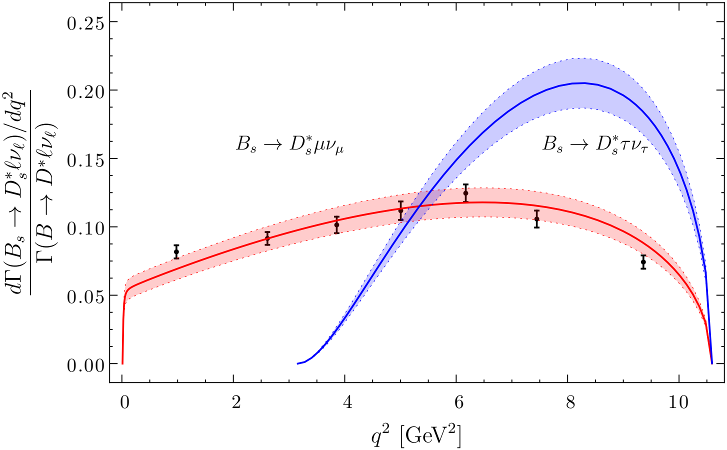
D^* polarization asymmetry



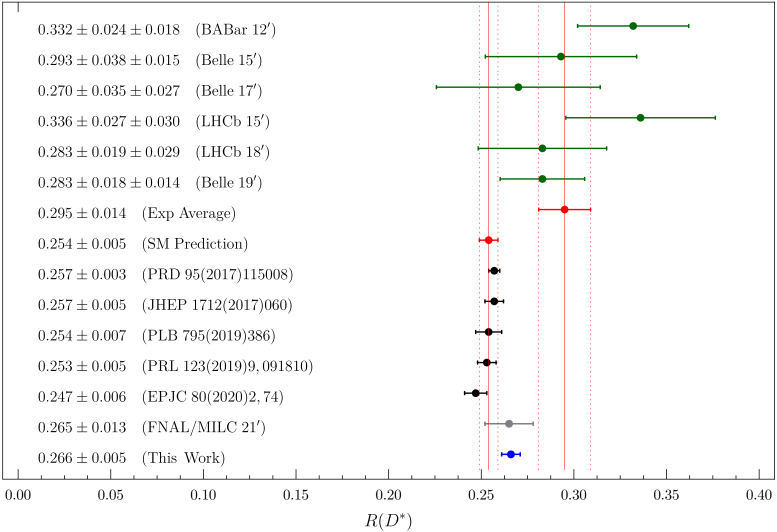
Differential decay rates and EXP from BELLE 17'



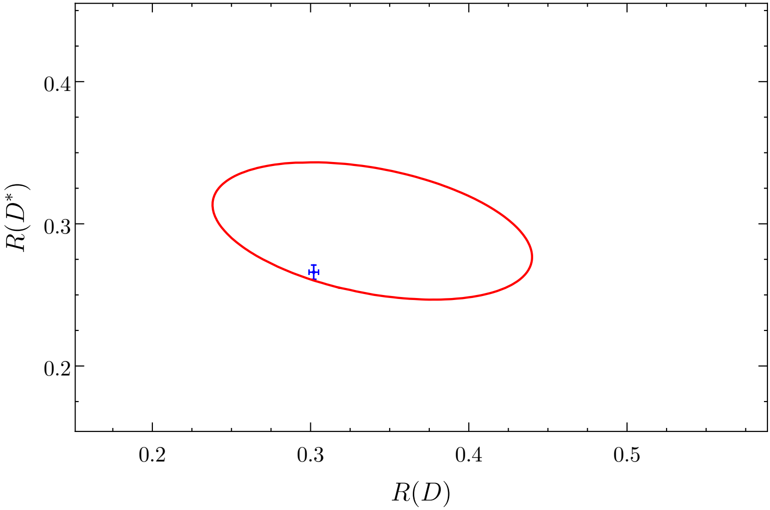
Normalized Differential decay rates and exp from LHCb 20'



Lepton flavor universality $R(D^*)$



$R(D) - R(D^*)$ & HFLAV averaged (Babar 12',15', Belle 19') 3σ



Extracting $|V_{cb}|$

- BaBar 07'

$$|V_{cb}| = \left(44.2^{+1.1}_{-1.0}\big|_{\rm th}\ {}^{+1.2}_{-1.2}\big|_{\rm exp}\right) \times 10^{-3}$$

- Belle 20'

$$|V_{cb}| = \left(40.1^{+1.0}_{-0.9}\big|_{\rm th}\ {}^{+0.3}_{-0.3}\big|_{\rm exp}\right) \times 10^{-3}$$

- LHCb 20' $(B_s \to D_s^* \ell \nu)$

$$|V_{cb}| = \left(43.5^{+1.6}_{-1.4}\big|_{\rm th}\ {}^{+2.4}_{-2.5}\big|_{\rm exp}\right) \times 10^{-3}$$

Summary and outlook

- Leading power
  - Factorization formulae
  - NLO calculations for hard-collinear matching Coe
- Subleading power
  - Two-& Three-particle higher-twist
  - HQET Rep. of b-quark field
  - Hard-collinear charm-quark propagator
- Joint fit LCSR at small q^2 and Lattice at large q^2
- Observales
  - $d\Gamma/d[q^2, \cos\theta_D, \cos\theta_\ell, \phi]$, A_{FB}, A^T_{FB}, $A^{D^*}_{pol}$, $R(D^*)$
  - Extracting $|V_{cb}|$

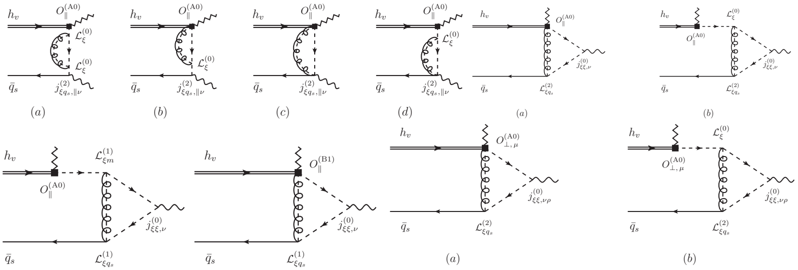
Outlook:

- Theoretical correlation between diff FF, q^2, processes
- Joint fit with Belle17' (40-Bin)

Thank You

Back-up

Diagrammatic representations



Angular distribution

$$
\begin{aligned}
\frac{d^4\Gamma(\bar{B}\to D^*(\to D\pi)\ell^-\bar{\nu}_\ell)}{dq^2\, d\cos\theta\, d\phi\, d\cos\theta_V} &= \mathcal{N}_\pi |\vec{p}_{D^*}| \left(1-\frac{m_\ell^2}{q^2}\right)^2 \Big\{ I_{1s}^\pi \sin^2\theta_V + I_{1c}^\pi \cos^2\theta_V \\
&+ \left(I_{2s}^\pi \sin^2\theta_V + I_{2c}^\pi \cos^2\theta_V\right)\cos 2\theta \\
&+ I_3^\pi \sin^2\theta_V \sin^2\theta \cos 2\phi + I_4^\pi \sin 2\theta_V \sin 2\theta \cos\phi \\
&+ I_5^\pi \sin 2\theta_V \sin\theta \cos\phi + \left(I_{6s}^\pi \sin^2\theta_V + I_{6c}^\pi \cos^2\theta_V\right)\cos\theta \\
&+ I_7^\pi \sin 2\theta_V \sin\theta \sin\phi \Big\} ,
\end{aligned}
$$

with $\mathcal{N}_\pi = \frac{3G_F^2|V_{cb}|^2\mathcal{B}(D^*\to D\pi)}{128(2\pi)^4 m_B^2}$, and coefficients of the angular terms are related to the helicity amplitudes

$$
\begin{aligned}
I_{1s}^\pi &= \frac{1}{2}(H_+^2+H_-^2)(m_\ell^2+3q^2) , & I_{1c}^\pi &= 2(2m_\ell^2 H_t^2 + H_0^2(m_\ell^2+q^2)) , \\
I_{2s}^\pi &= \frac{1}{2}(H_+^2+H_-^2)(q^2-m_\ell^2) , & I_{2c}^\pi &= 2H_0^2(m_\ell^2-q^2) , \\
I_3^\pi &= 2H_+H_-(m_\ell^2-q^2) , & I_4^\pi &= H_0(H_++H_-)(m_\ell^2-q^2) , \\
I_5^\pi &= -2(H_++H_-)H_t m_\ell^2 - 2H_0(H_+-H_-)q^2 , & & \\
I_{6s}^\pi &= 2(H_+^2-H_-^2)q^2 , & I_{6c}^\pi &= -8H_0H_t m_\ell^2 , \\
I_7^\pi &= 0 & &
\end{aligned}
$$

Sample point from parameter space

Parameter	value/interval	unit	prior	source/comments
μ	1.5 $\pm$ 0.5	GeV	uniform @ 100%	-
μ_h	$[m_b/2, 2m_b]$	GeV	$\ln(\mu_h)$uniform @ 100%	-
M^2	4.5 $\pm$ 1	GeV^2	gaussian @ 68%	Wang *et. al.* 17'
s_0	6.0 $\pm$ 0.5	GeV^2	uniform @ 100%	Wang *et. al.* 17'
λ_B	0.35 $\pm$ 0.15	GeV	uniform @ 100%	Shen *et. al.* 20'
$\{\hat{\sigma}_1, \hat{\sigma}_2\}$	$[\{-0.7, -6\}, \{0.7, 6\}]$	-	uniform @ 100%	Shen *et. al.* 20'
λ_E^2/λ_H^2	0.5 $\pm$ 0.1	-	uniform @ 100%	Beneke *et. al.* 18'
$2\lambda_E^2 + \lambda_H^2$	0.25 $\pm$ 0.15	GeV	uniform @ 100%	Beneke *et. al.* 18'
f_B	0.19$\pm$ 0.0013	GeV	gaussian @ 68%	FLAG 21'
$\{f_D, f_D^T\}$	$\{[0.2102, 0.245], [0.1858, 0.2181]\}$	GeV	gaussian @ 68%	Pullin/Zwicky 21'
$m_b(m_b)$	$[4.158, 4.215]$	GeV	gaussian @ 68%	Beneke *et. al.* 14'
$m_c(m_c)$	$[1.268, 1.308]$	GeV	gaussian @ 68%	Dehnadi *et. al.* 15'
m_b(Pole)	4.8 $\pm$0.1	GeV	gaussian @ 68%	PDG 20'

Correlations matrix

Correlations matrix for form factors (V, A_1, A_2, A_0) at $q^2 = (-3, -2, -1, 0, 1, 2)$

```
MatrixForm[corrLCSR]
```

//MatrixForm=

1.	0.998938	0.998866	0.997446	0.998331	0.997954	0.990822	0.988151	0.986522	0.982996	0.98096	0.977467	0.84987	0.855364	0.869068	0.87443	0.884833	0.890709	0.848905	0.840935	0.842532	0.835455	0.832098	0.820511
0.	1.	0.999299	0.997965	0.998975	0.998722	0.989885	0.989313	0.987077	0.98365	0.981752	0.978401	0.845858	0.853293	0.866516	0.872158	0.882818	0.888983	0.84477	0.838531	0.839503	0.832517	0.829211	0.817694
0.	0.	1.	0.99815	0.999255	0.999105	0.989614	0.988425	0.987572	0.98364	0.981842	0.978596	0.844002	0.850928	0.865653	0.870759	0.881544	0.887893	0.842645	0.83587	0.838289	0.830718	0.827469	0.816066
0.	0.	0.	1.	0.998152	0.99809	0.988208	0.987105	0.985745	0.985456	0.980756	0.977602	0.843322	0.850312	0.864327	0.873499	0.881038	0.887446	0.841802	0.835163	0.836945	0.833034	0.826637	0.815371
0.	0.	0.	0.	1.	0.999539	0.989296	0.988347	0.987115	0.983959	0.982936	0.979437	0.841799	0.849048	0.863294	0.869474	0.881191	0.887231	0.840337	0.833805	0.835674	0.829102	0.826688	0.814828
0.	0.	0.	0.	0.	1.	0.988784	0.987959	0.986831	0.983767	0.982357	0.979772	0.841211	0.848625	0.862964	0.869293	0.880467	0.887553	0.839907	0.83354	0.835509	0.82909	0.8261	0.815271
0.	0.	0.	0.	0.	0.	1.	0.998902	0.998616	0.996829	0.996887	0.995437	0.878121	0.88503	0.897638	0.903136	0.914415	0.921066	0.872539	0.867257	0.8687	0.862971	0.861741	0.852541
0.	0.	0.	0.	0.	0.	0.	1.	0.999249	0.997708	0.998094	0.996971	0.875775	0.884681	0.89674	0.902513	0.914114	0.921104	0.869566	0.866212	0.86708	0.861574	0.860603	0.851699
0.	0.	0.	0.	0.	0.	0.	0.	1.	0.998119	0.998808	0.99799	0.875768	0.884274	0.897732	0.90298	0.914781	0.922016	0.868755	0.865113	0.867447	0.86151	0.860817	0.852264
0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.998064	0.997561	0.876779	0.885461	0.89813	0.907381	0.916072	0.923436	0.869007	0.865771	0.867525	0.86535	0.86181	0.853665
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.999407	0.876604	0.885662	0.898451	0.904761	0.917652	0.92473	0.868234	0.865406	0.867295	0.862684	0.863337	0.854912
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.877696	0.887061	0.899826	0.906289	0.918751	0.926949	0.868735	0.866416	0.86846	0.864205	0.864605	0.857542
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.994706	0.997846	0.994153	0.993772	0.990103	0.995445	0.99052	0.996033	0.993497	0.995551	0.993113
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.994931	0.991972	0.992515	0.989737	0.989749	0.995648	0.991309	0.988829	0.991005	0.988529
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.996617	0.997729	0.995498	0.992604	0.988884	0.9951	0.991739	0.993909	0.991357
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.996533	0.994941	0.98877	0.985327	0.990678	0.993457	0.990125	0.987578
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.998397	0.987919	0.984772	0.990008	0.987746	0.990982	0.987561
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.984146	0.981357	0.986539	0.98451	0.987053	0.98654
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.994001	0.998558	0.99543	0.996325	0.993205
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.99398	0.991167	0.992423	0.989704
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.996566	0.998077	0.995744
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.996285	0.994375
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.997685
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.

Updated results will appear soon...