Strong Phase Measurements at BESIII

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On behalf of BESIII Collaboration

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BESIII实验上粲强子、QCD及新物理研讨会

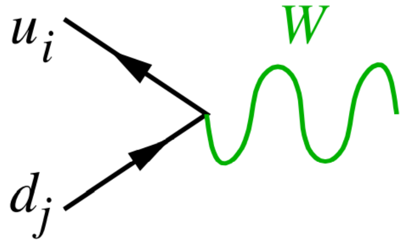
Outline

- **Introduction**
  - CKM Angle γ/φ_3 to strong phase
- **BESIII Experiment**
  - BEPCII & BESIII
  - Quantum correlated $D^0\overline{D}^0$ sample at BESIII
- **Strong Phase Measurement**
  - $D^0 \to K^-\pi^+,\ K^-\pi^+\pi^+\pi^-,\ K^-\pi^+\pi^0$
  - $D^0 \to K_S^0 h^+h^-$
  - Impact of Strong Phase Input
  - Status of Strong Phase Measurements
- **Summary**

CKM Matrix

- **Flavor-changing charged currents**
  - mass eigenstates ≠ weak interaction eigenstates

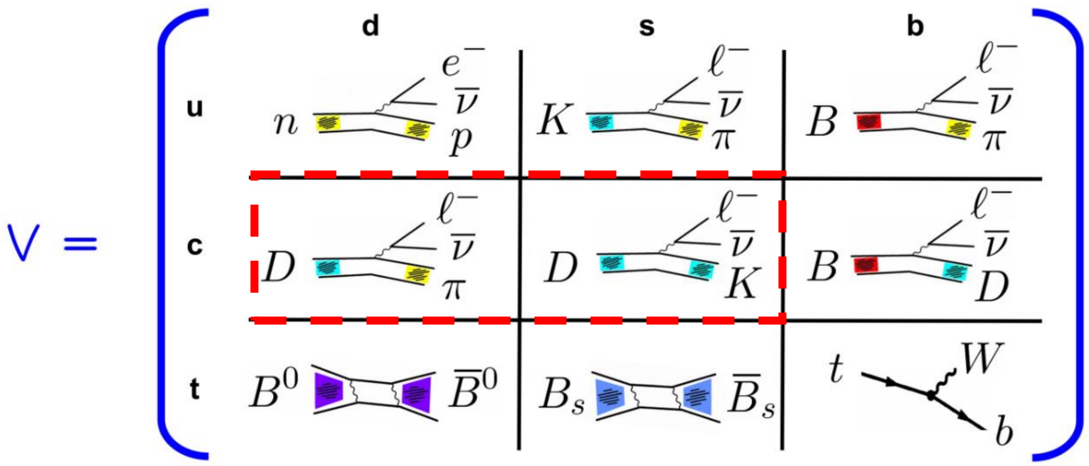
$$J^+_\mu = \bar{u}_i \gamma_\mu (1-\gamma_5) V_{ij} d_j$$

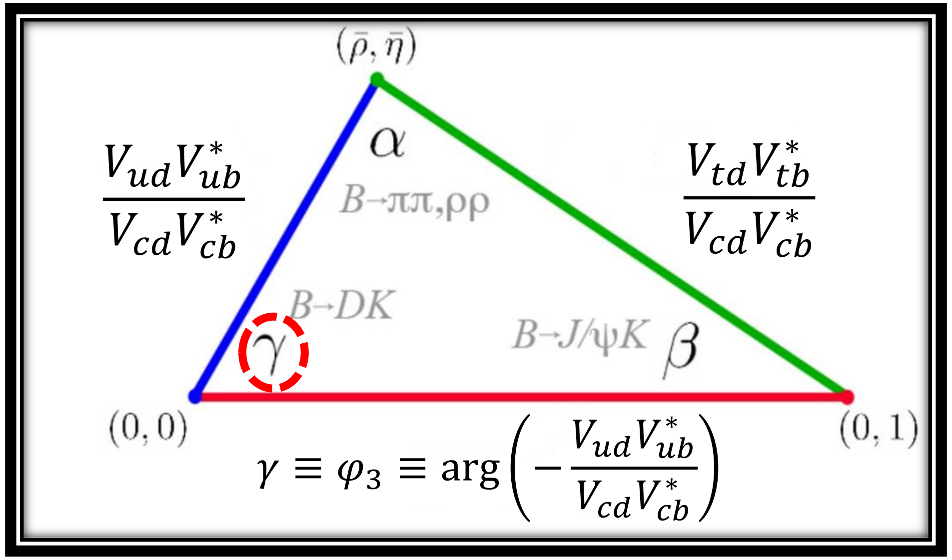


- **CKM Matrix (**3x3 unitary complex matrix**)**
  - Fundamental parameters of the Standard Model (SM)
  - 3 mixing angles + 1 complex phase ⇒ source of CPV in SM
  - Precise measurement ⇒ Test SM and search for new physics (NP)

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \simeq \begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\bar{\rho}-i\bar{\eta}) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2 & 1 \end{bmatrix} + O(\lambda^4)$$

Measurement of CKM





- **unitarity triangle**
  - Related to B meson decays:

$$\underset{\sim\lambda^3}{V_{ud}V_{ub}^*}+\underset{\sim\lambda^3}{V_{cd}V_{cb}^*}+\underset{\sim\lambda^3}{V_{td}V_{tb}^*}=0$$

  - Measurement of γ/ϕ_3 involve neutral D meson decays

- Indirect (CKMfitter Spring 21)

$$\gamma=(65.5^{+1.1}_{-2.7})^\circ$$

- Direct (HFLAV & PDG 2022)

$$\gamma=(65.9^{+3.3}_{-3.5})^\circ$$

- Good agreement but need be improved

Measurement of γ/φ_3

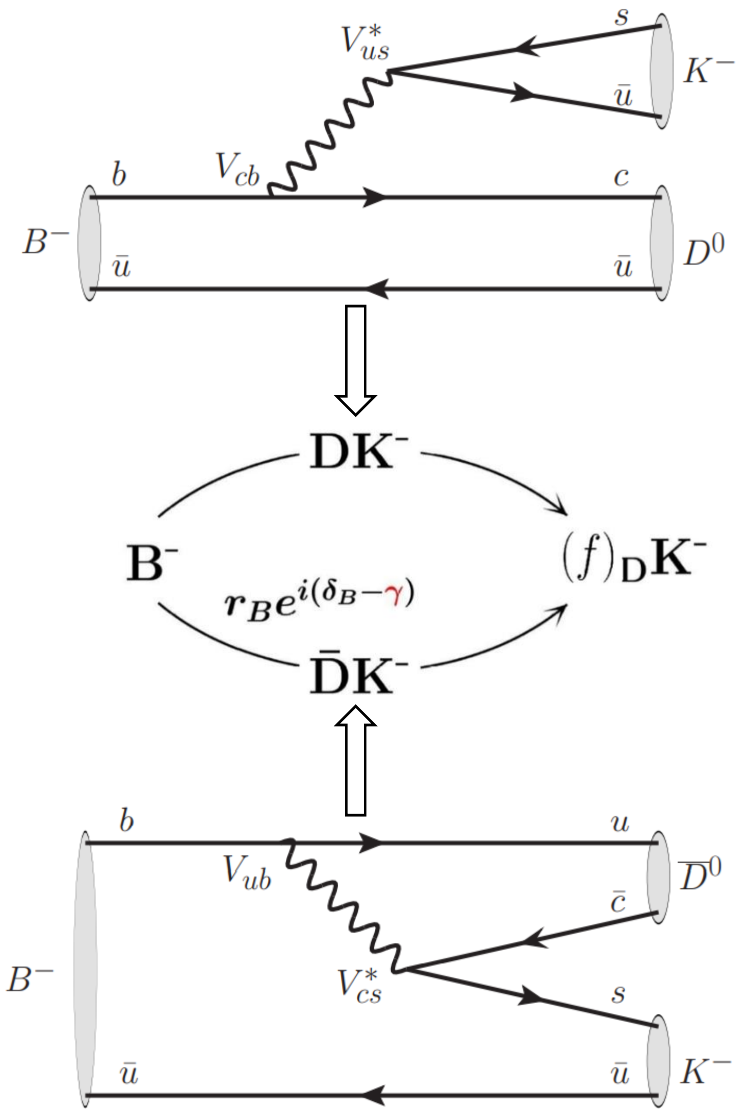
- **Measurement of γ/φ_3**
  - Measured entirely in tree-level transitions in the interference of $b\to c$ and $b\to u$ diagrams

$$\Gamma(B^-\to[f]_D K^-)\propto 1+r_B^2 r_D^2+2R_f r_D r_B\cos(\delta_B-\gamma-\delta_D)$$

  - Inputs from $D^0\to f$ and $\overline{D}^0\to f$ decays are needed

$$(r_D^f)^2=\int|\bar{A}_f|^2\mathrm{d}\Phi/\int|A_f|^2\mathrm{d}\Phi,\qquad R_f e^{-i\delta_D^f}=\frac{\int A_f^*\bar{A}_f\mathrm{d}\Phi}{\sqrt{\int|A_f|^2\mathrm{d}\Phi\int|\bar{A}_f|^2\mathrm{d}\Phi}}$$

- **Different D decay models:**
  - ADS: CF and DCS decays (e.g. $K\pi, K\pi\pi^0$) $\leftarrow R_f,\delta_D^f$
  - GLW: (Quasi-)CP eigenstates (e.g. $KK,\pi^+\pi^-\pi^0$) $\leftarrow F_+$
  - GGSZ: Multi-body Self-conjugate decay (e.g. $K_S^0\pi^+\pi^-$) $\leftarrow c_i, s_i$

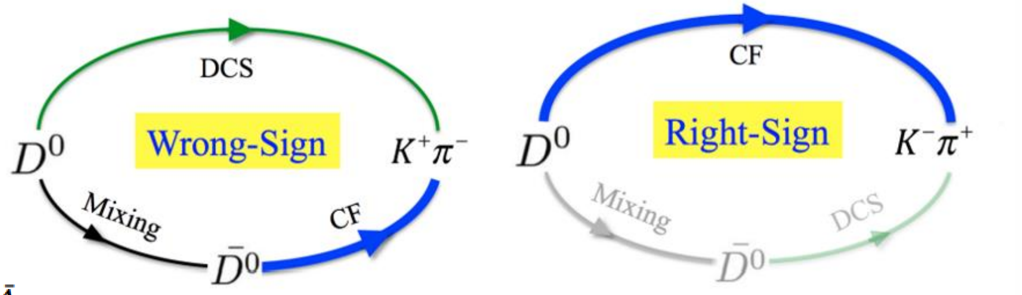


Other Application of Strong Phase

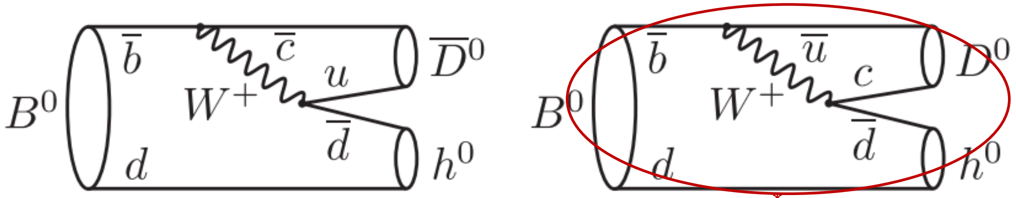
- $\boldsymbol{D^0}$- $\boldsymbol{\bar{D}^0}$ **mixing (Time dependent analysis)**

$$\frac{d\Gamma(D^0(t)\to f)}{d\Phi_f} \propto |A_f|^2 e^{-\Gamma t}[\frac{1+|\lambda_f|^2}{2}\cosh(y\Gamma t)+\frac{1-|\lambda_f|^2}{2}\cos(x\Gamma t)$$
$$-\mathrm{Re}(\lambda_f)\sinh(y\Gamma t)+\mathrm{Im}(\lambda_f)\sin(x\Gamma t)],$$

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}$$



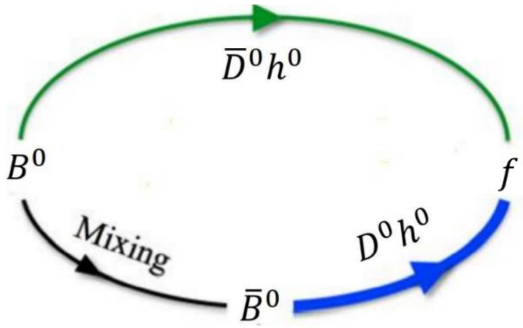
- $\boldsymbol{\beta}$ **measurement in** $\boldsymbol{B^0 \to \bar{D}^0 h^0}$ $\boldsymbol{(\beta \equiv \varphi_2 \equiv arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right))}$
  - $b \to c\bar{u}d$ decays $B^0 \to \bar{D}^0 h^0$, have no penguin contributions, and provide theoretically clean $\sin(2\beta)$ measurements.
  - $\cos(2\beta)$ can be determined with multibody final states (e.g. $D^0 \to K_S^0\pi^+\pi^-$)



Neglect $|\frac{V_{ub}^*V_{cd}}{V_{cb}V_{ud}^*}|$~0.02

$$\frac{q}{p} = \frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*} = e^{-2i\beta+O(\lambda^4)} \qquad \mathrm{Im}(\lambda_f) = \left|\frac{A(\bar{B}^0\to f)}{A(B^0\to f)}\right|\sin(\delta_f - 2\beta)$$

$$B^0 \to \bar{D}^0 h^0 \to f: \delta_f = \arg\left(\eta_{CP}^{h^0}(-1)^L\right) + \delta_D$$



Measurement of Strong Phase Parameters

- **Quantum correlated (QC) $D^0\bar{D}^0$ decay at $\psi(3770)$**

$$e^+e^- \to \psi(3770) \to D^0\bar{D}^0 \to fg$$

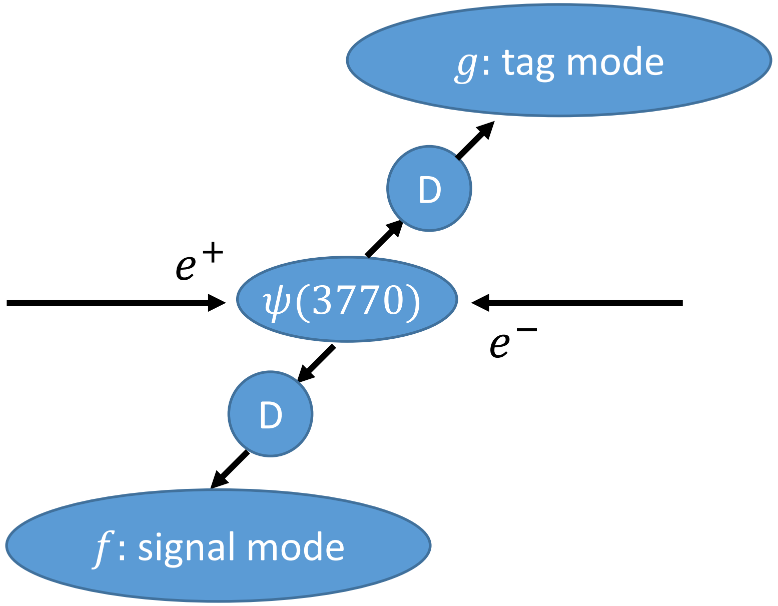
$$\Big\Downarrow \quad C_{\psi(3770)} = -1$$

$$|\psi(3770)\rangle \to \frac{1}{\sqrt{2}}\left(|D^0\rangle|\bar{D}^0\rangle - |\bar{D}^0\rangle|D^0\rangle\right)$$

$$\Big\Downarrow \quad \text{No CPV}$$

$$\boxed{\Gamma(f|g)} \propto \left(1+\frac{y^2-x^2}{2}\right)\left[\left(r_D^f\right)^2+\left(r_D^g\right)^2-2r_D^f r_D^g \boxed{R_f} R_g \cos\left(\boxed{\delta_D^f}-\delta_D^g\right)\right]$$
$$+\frac{y^2+x^2}{2}\left[1+\left(r_D^f r_D^g\right)^2-2r_D^f r_D^g R_f R_g \cos\left(\delta_D^f+\delta_D^g\right)\right]$$

$\propto$ Number of events   coherence factor   strong phase difference



- **Strong phase parameters of $D^0 \to f$ decay can be measured in quantum correlated $D^0\bar{D}^0$ data**

BEPCII & BESIII

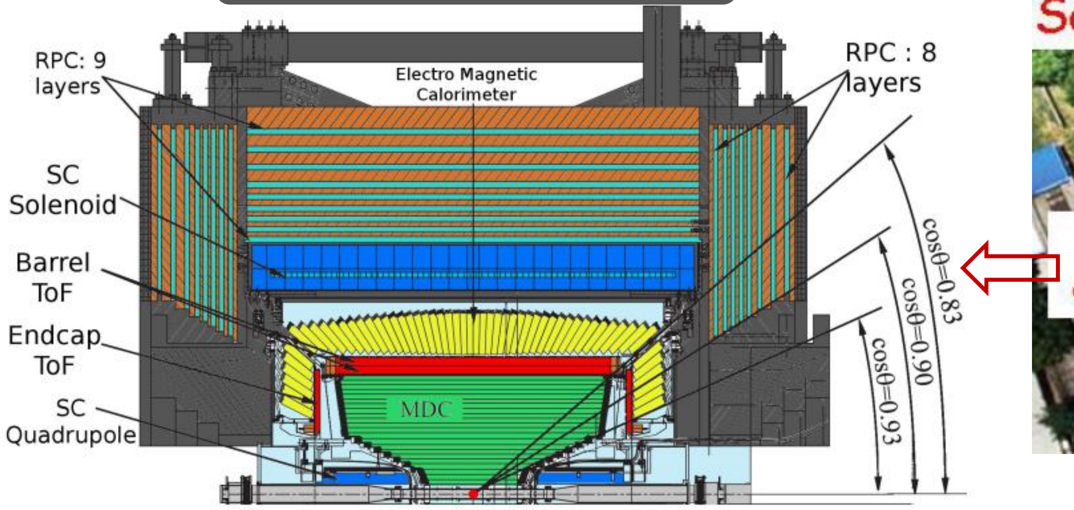
MDC: small cell & Gas: $He/C_3H_8(60/40)$, 43 layers
$\sigma_p/p = 0.5\%@1GeV$,
$\sigma_{dE/dx} = 6\%$

TOF: Barrel: $\sigma_T = 100ps$
endcap: $\sigma_T = 110ps$
(60ps for endcap after upgraded to MRPC in 2015)

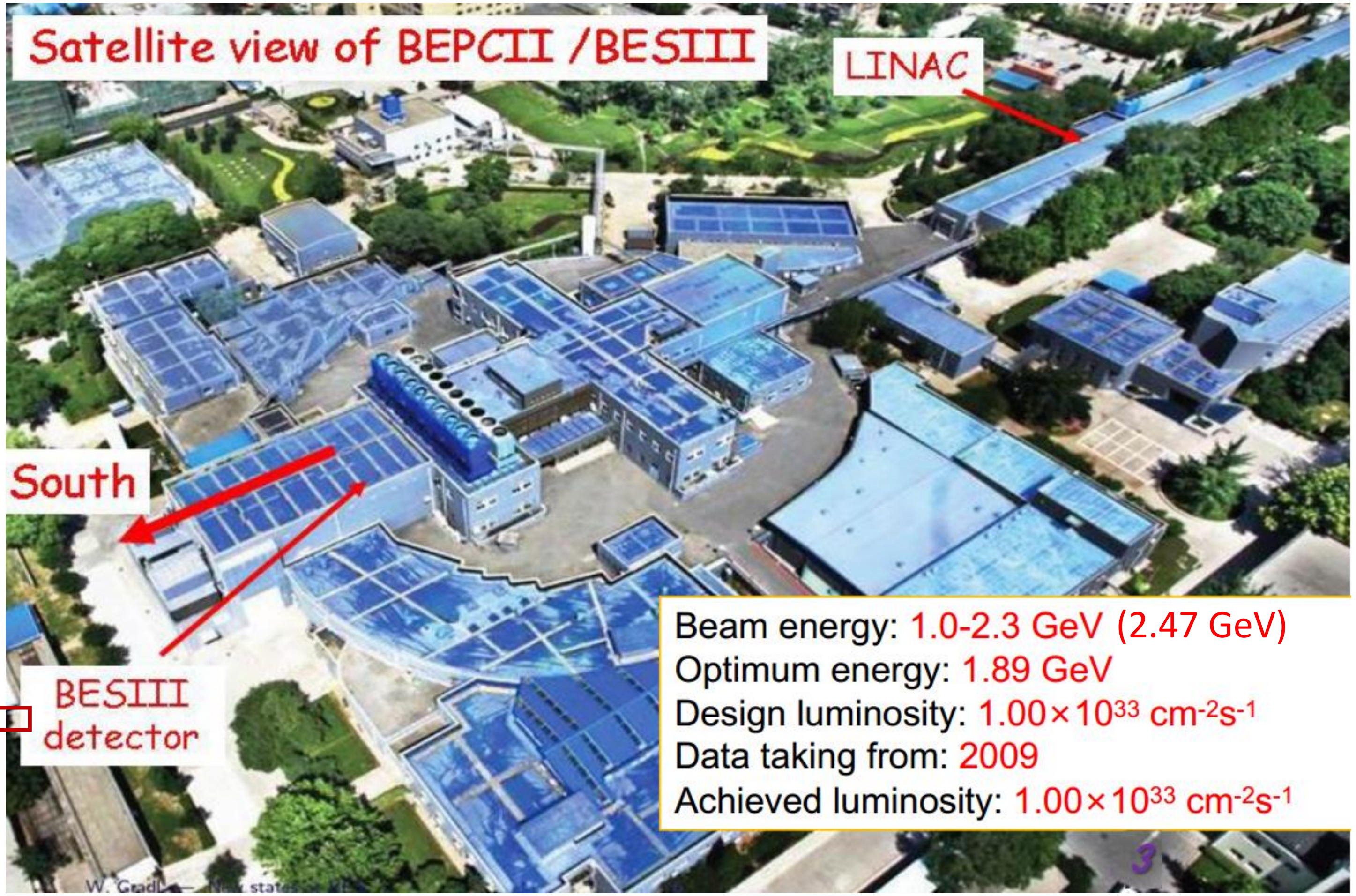
EMC: CsI crystal. 28cm
$\Delta E/E = 2.5\%@1GeV$,
$\sigma_z = 0.6cm/\sqrt{E}$

MUC: 9layers RPC
(8 layers in Endcap)
$\sigma_{R\phi} = 1.4\sim1.7cm$

Magnet: 1T Super conducting



[Nucl. Instr. Meth. A614, 345(2010)]

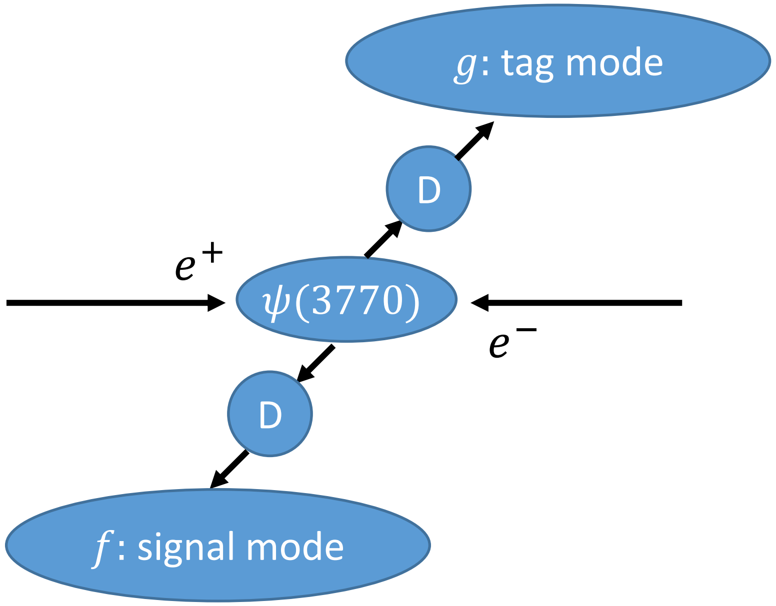


Quantum Correlated $D^0\bar{D}^0$ Data @ BESIII

- **Quantum correlated $D^0\bar{D}^0$ produced at BESIII**

$$e^+e^- \to \psi(3770) \to D^0\bar{D}^0$$

  - $2.93\mathrm{fb}^{-1}$@ $\mathrm{E_{cm}} = 3.773$ GeV (~3.6x CLEO's)
  - ~10.5M $D^0\bar{D}^0$ pairs produced



- **Analysis method in pair production:**
  - Single Tag(ST): reconstruct one of $D\bar{D}$
  - Double Tag(DT): reconstruct both of $D\bar{D}$

- **Typical Tag modes:**

Flavor	$K^\pm\pi^\mp, K^\pm\pi^\mp\pi^0, K^\pm\pi^\mp\pi^\pm\pi^\mp, K^\pm e^\mp\nu_e$
CP even	$K^+K^-, \pi^+\pi^-, K_S^0\pi^0\pi^0, K_L^0\pi^0, \pi^+\pi^-\pi^0$
CP odd	$K_S^0\pi^0, K_S^0\eta^{(\prime)}, K_S^0\omega, K_L^0\pi^0\pi^0$
Mixed CP	$K_S^0\pi^+\pi^-, K_L^0\pi^+\pi^-$

$D^0 \to K^-\pi^+, K^-\pi^+\pi^0$ and $K^-\pi^+\pi^+\pi^-$

- **Measurement Parameters:** R_f, δ_D^f

$$R_f e^{-i\delta_D^f} = \frac{\int A_f^* \bar{A}_f d\Phi}{\sqrt{\int |A_f|^2 d\Phi \int |\bar{A}_f|^2 d\Phi}}$$

- **PHSP average analysis with 2.93 fb^{-1} data @ $E_{cm} = 3.773$ GeV**

- **Improved precision compared to CLEO-c results**

  - BESIII [PLB 734 227 (2014)]

    $\cos\delta_{K\pi} = 1.02 \pm 0.11 \pm 0.06 \pm 0.01$

    (*Will be updated : more tag modes and more fit configurations)

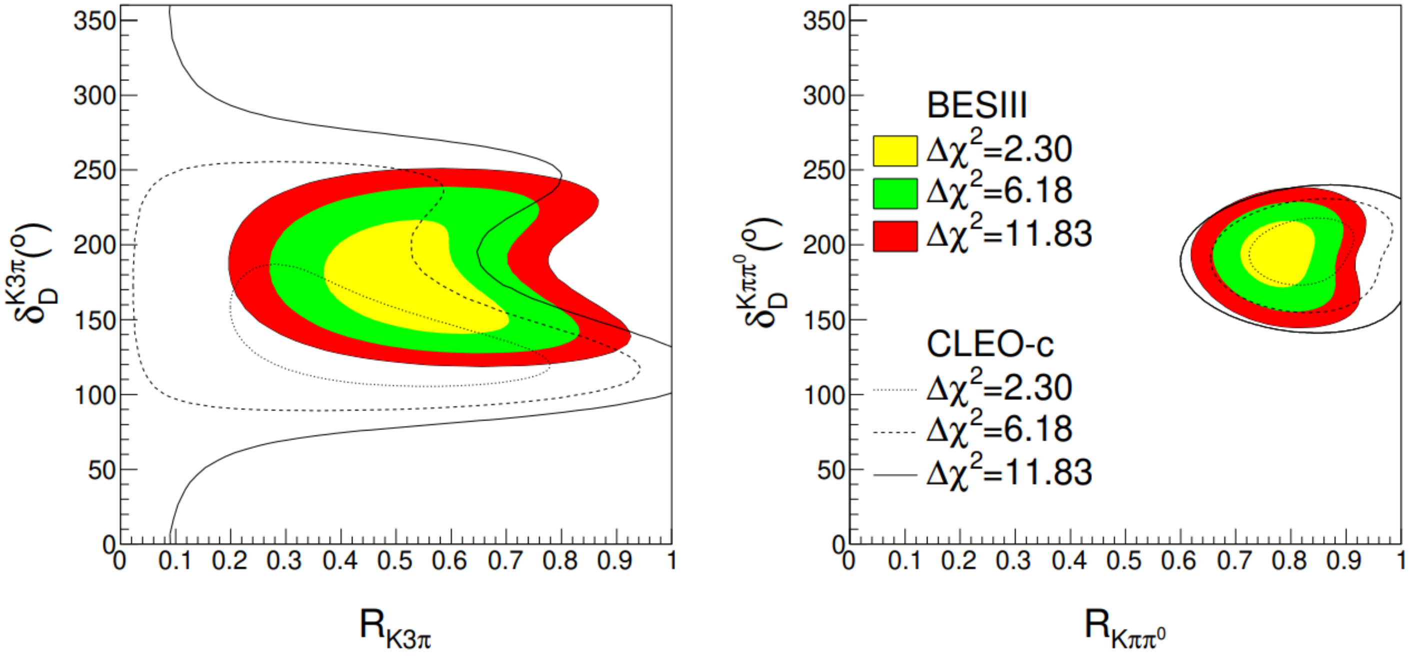
  - CLEOc [PRD 86 112001 (2012)]

    $$\cos\delta_{K\pi} = 0.81^{+0.22+0.07}_{-0.18-0.05}$$

  - HFLAV 2022:

$$\delta_{K\pi} = 11.7^{+3.6}_{-3.8} \Rightarrow \cos\delta_{K\pi} = 0.979^{+0.011}_{-0.015}$$

[JHEP 05, 164 (2021)]

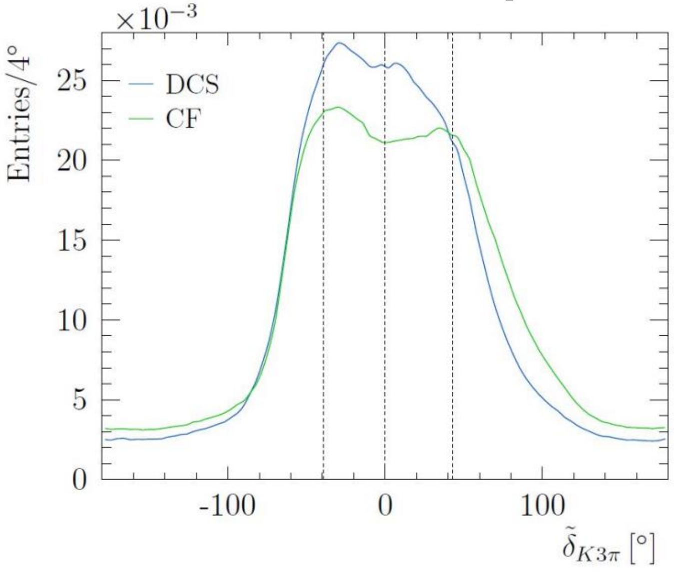


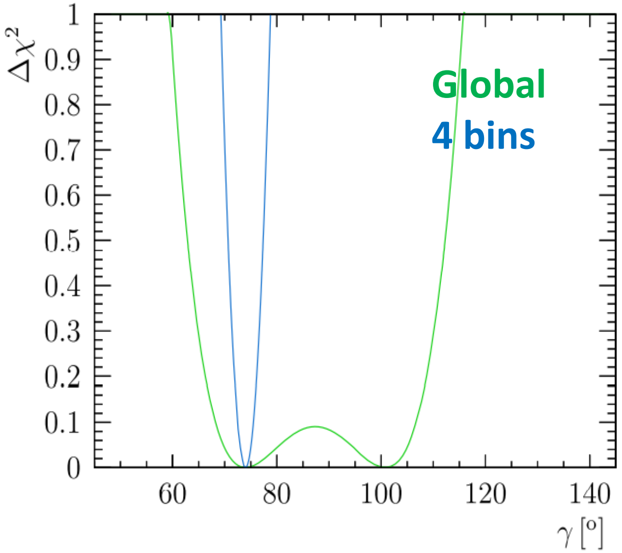
$D^0 \to K^-\pi^+\pi^+\pi^-$ (Binned)

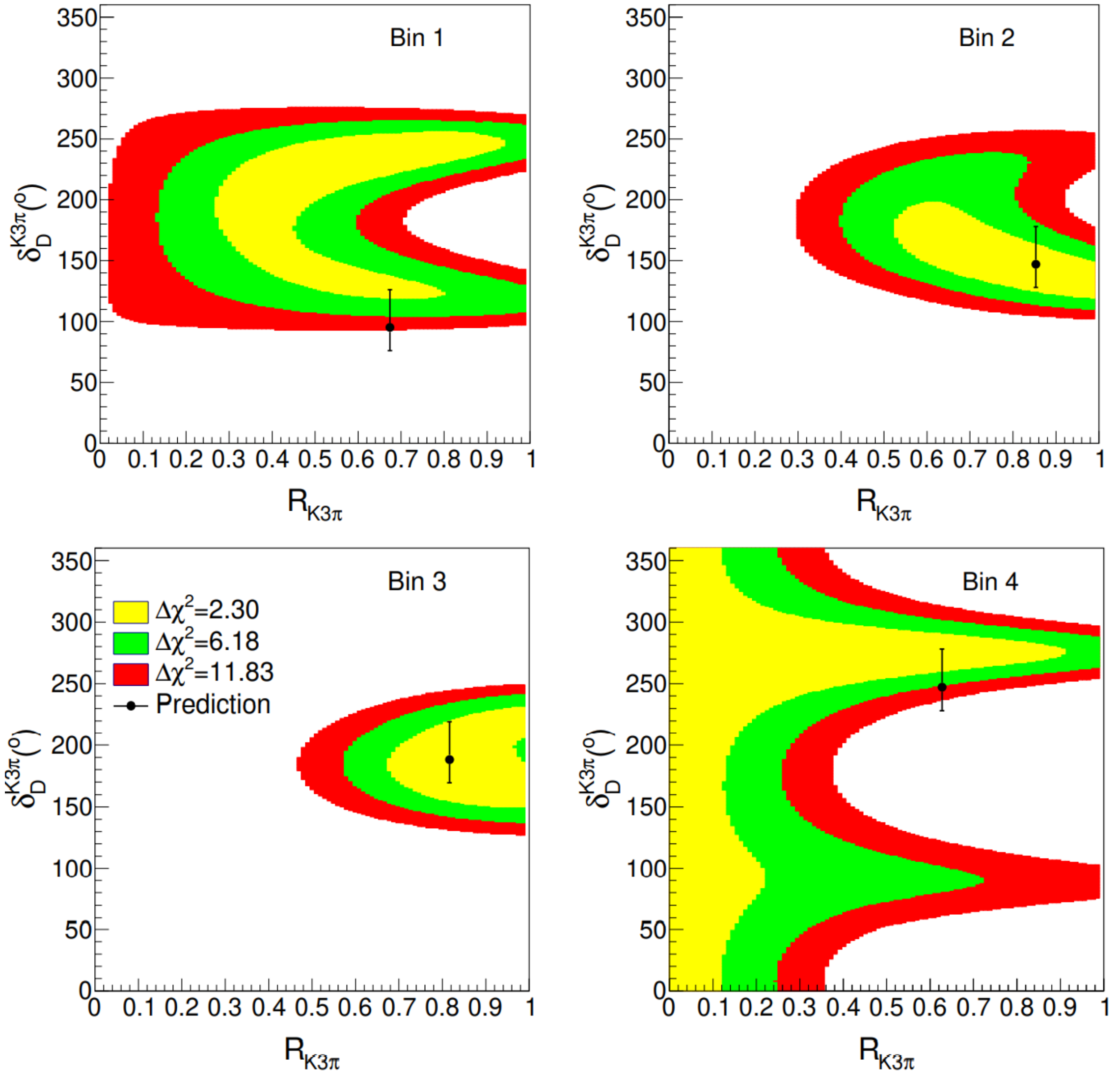
- Sensitivity on γ can be improved with binned measurement in $D^0 \to K^-\pi^+\pi^+\pi^-$
- Binned analysis with $2.93\ \mathrm{fb}^{-1}$ data @ $E_{cm} = 3.773$ GeV
- 4-bin binning scheme from [PLB 802, 135188 (2020)]

[JHEP 05, 164 (2021)]

[PLB 802, 135188 (2020)]







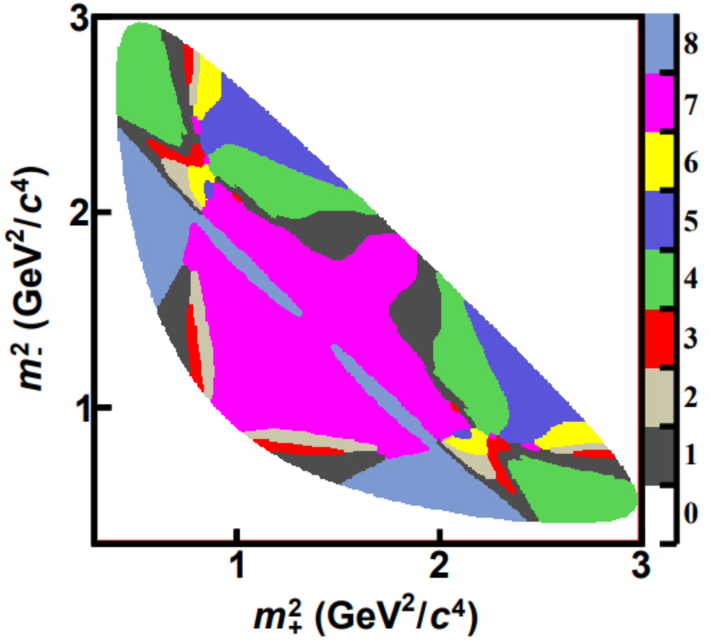
$D^0 \to K_S^0 h^+ h^-$

- **Measurement Parameters:** binned strong parameters $\boldsymbol{c_i, s_i}$

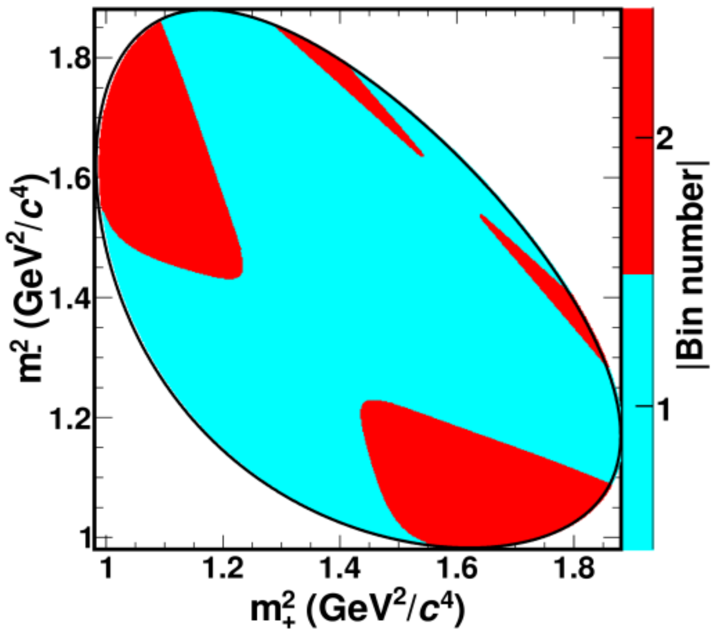
$$c_i = \frac{1}{\sqrt{K_i K_{-i}}} \int_i |A_f||\bar{A}_f| \cos[\delta_D]\, dm_+^2 dm_-^2 \xRightarrow{\cos\to\sin} s_i \qquad K_i = \int_i |A_f|^2 dm_+^2 dm_-^2$$

- **Binned analysis with 2.93 $\mathbf{fb^{-1}}$ data @ $E_{cm} = 3.773$ GeV**

Binning scheme of $K_S^0\pi^+\pi^-$
[PRD 82, 112006 (2010)]



Binning scheme of $K_S^0K^+K^-$
[PRD 78, 034023 (2008)]



$D^0 \to K_S^0 h^+ h^-$

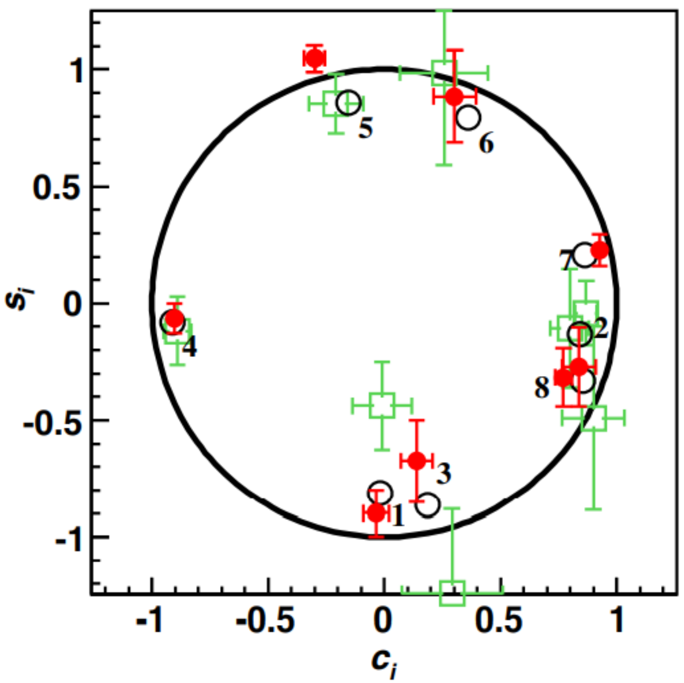
- Consistent with CLEOc's result
- Improved precision compared to CLEO-c results

BESIII

CLEO-c
[PRD 82, 112006 (2010)]

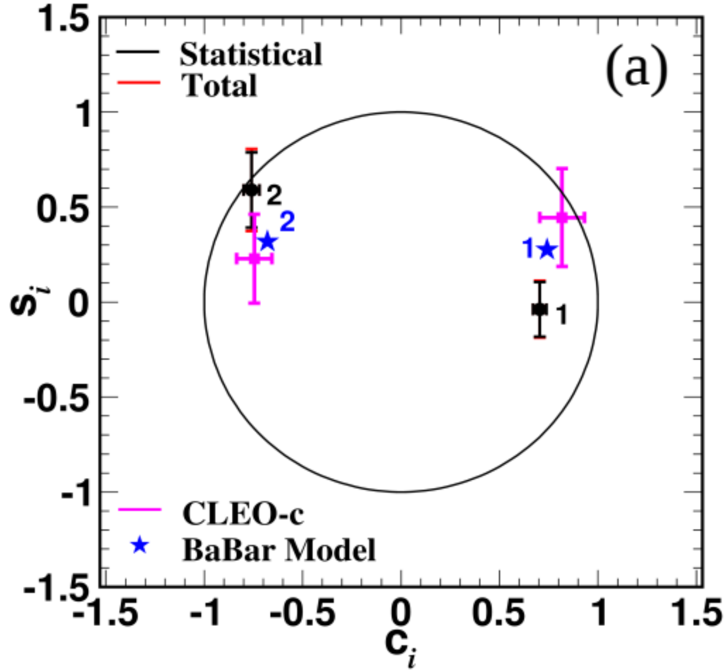
Model
[PRD 98, no.11, 112012 (2018)]

$D^0 \to K_S^0 \pi^+ \pi^-$



[PRL 124, 241802 (2020); PRD 101, 112002 (2020)]

$D^0 \to K_S^0 K^+ K^-$



[PRD 102, 052008 (2020)]

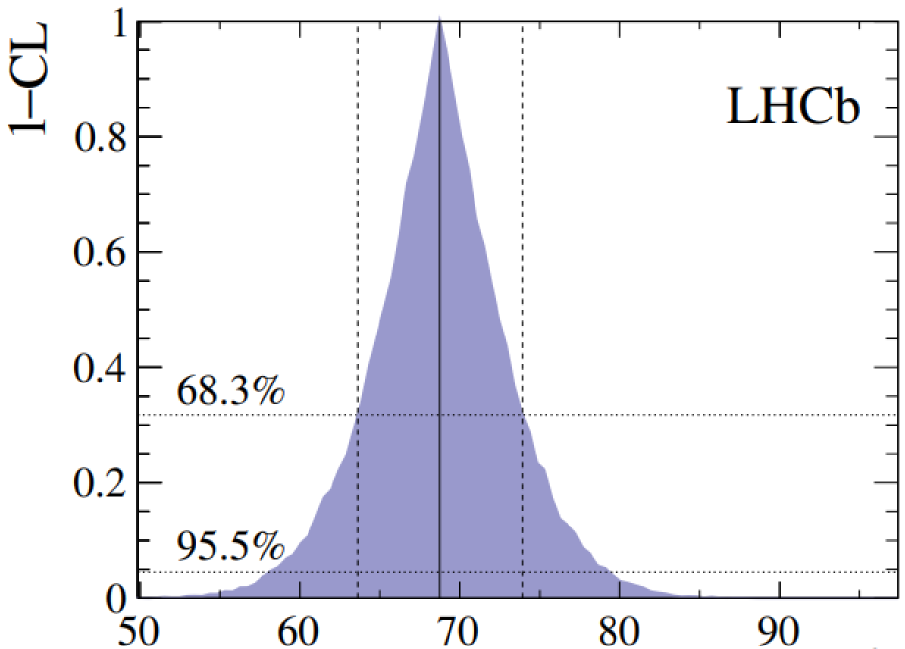
BESIII

CLEO-c
[PRD 82, 112006 (2010)]

Model
[PRD 78, 034023 (2008)]

Impact on γ/φ_3

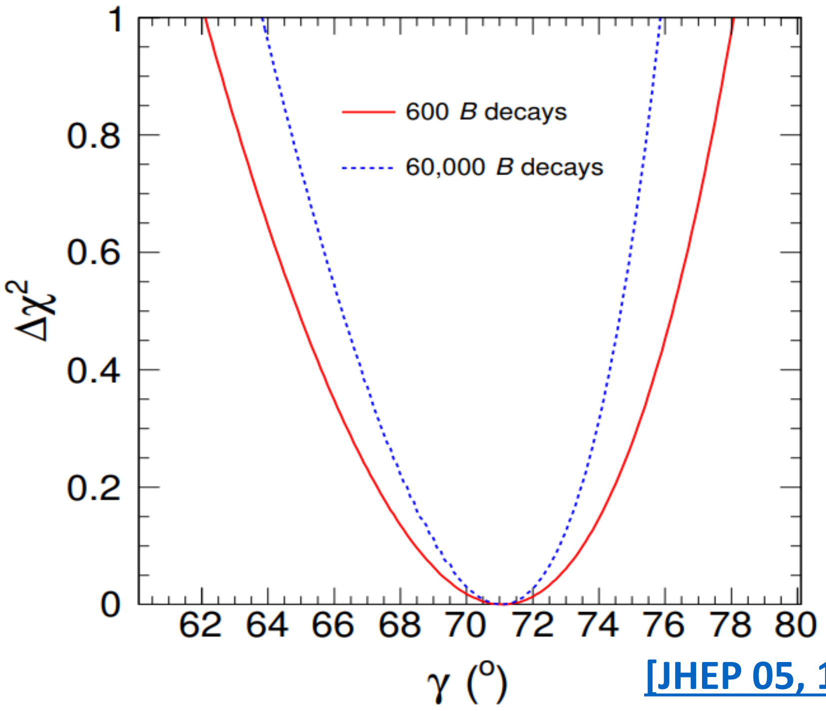
$D^0 \to K_S^0 h^+ h^-$



[JHEP 02 (2021) 169]

- Total uncertainty 5° @ LHCb (run1+run2)
- Leading to the best single γ measurement
- 1° from strong phase input

$D^0 \to K3\pi$ (Binning)

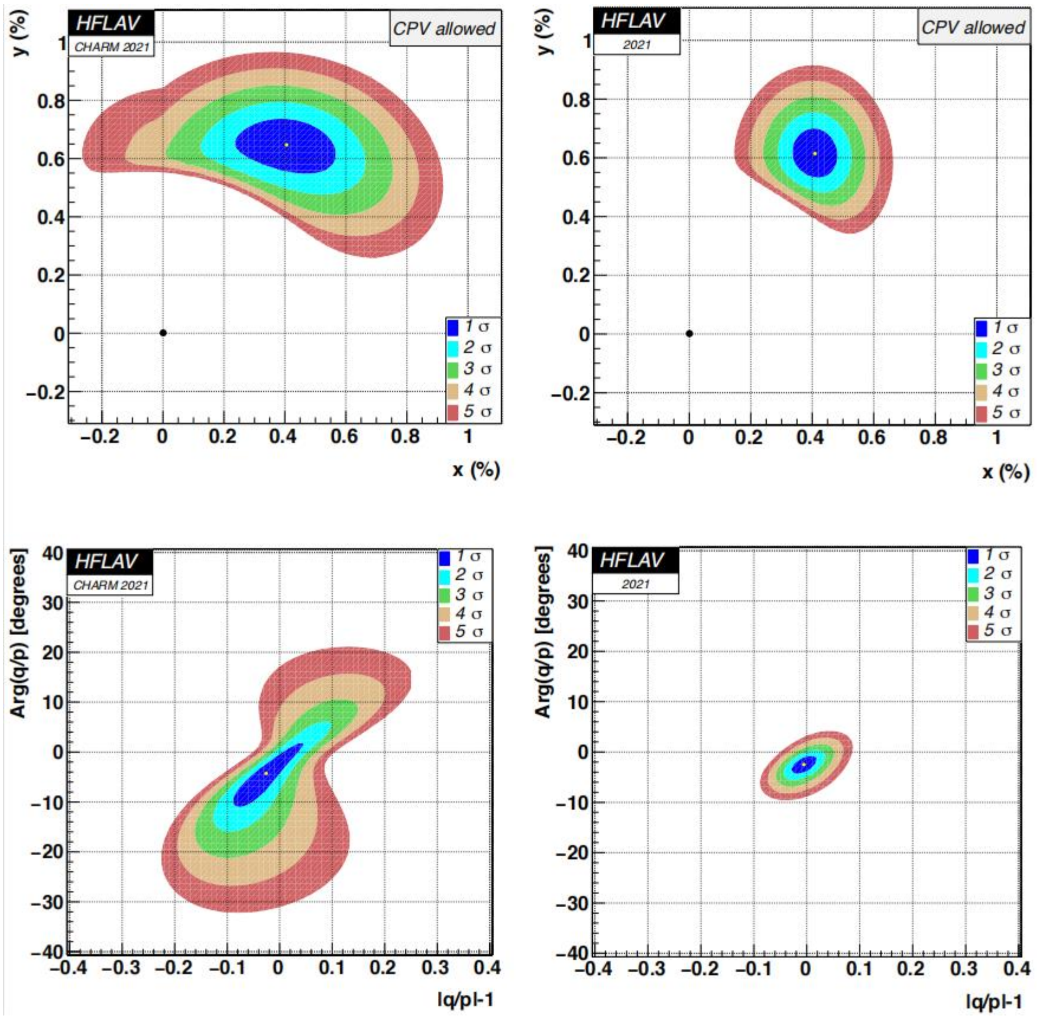


[JHEP 05, 164 (2021)]

- Estimated uncertainty $\left({}^{+7}_{-9}\right)^\circ$ @ LHCb (run1+run2)
- Second leading contribution to γ
- $\left({}^{+5}_{-7}\right)^\circ$ from strong phase input

Impact on D^0- $\bar{D}^0$ mixing

without $D^0 \to K_S^0\pi^+\pi^-$ include $D^0 \to K_S^0\pi^+\pi^-$



D^0- $\bar{D}^0$ mixing with $D^0 \to K_S^0\pi^+\pi^-$

[PRL 127, 111901 (2021)]

Source	x_{CP}	y_{CP}	Δx	Δy
Reconstruction and selection	0.199	0.757	0.009	0.044
Secondary charm decays	0.208	0.154	0.001	0.002
Detection asymmetry	0.000	0.001	0.004	0.102
Mass-fit model	0.045	0.361	0.003	0.009
Total systematic uncertainty	0.291	0.852	0.010	0.110
Strong phase inputs	0.23	0.66	0.02	0.04
Detection asymmetry inputs	0.00	0.00	0.04	0.08
Statistical (w/o inputs)	0.40	1.00	0.18	0.35
Total statistical uncertainty	0.46	1.20	0.18	0.36

Status of Strong Phase Measurement @ BESIII

Decay channels required in [LHCb-PUB-2016-025]

Decay mode	Quantity	Status （2.93 fb^{-1}）
$K_S^0\pi^+\pi^-$	c_i, s_i	Finished
$K_S^0K^+K^-$	c_i, s_i	Finished
$K^-\pi^+\pi^+\pi^-$	R, δ	Finished
$K^+K^-\pi^+\pi^-$	F_+ or c_i, s_i	On going
$\pi^+\pi^-\pi^+\pi^-$	F_+ or c_i, s_i	On going
$K^-\pi^+\pi^0$	R, δ	Finished
$K_S^0K^\pm\pi^\mp$	R, δ	…

Decay mode	Quantity	Status （2.93 fb^{-1}）
$\pi^+\pi^-\pi^0$	F_+	On going
$K_S^0\pi^+\pi^-\pi^0$	F_+ or c_i, s_i	On going
$K^+K^-\pi^0$	F_+	On going
$K^-\pi^+$	δ	Finished (Update)

- The precision will be improved with a factor ~2 compared with CLEOc's measurement

Status of Strong Phase Measurement @ BESIII

➢ Expected γ/φ_3 precision of LHCb[1] and Belle II[2] experiment

[1]. arXiv: 1808.08865
[2]. PTEP 2019, 123C01 (2019)

Runs	Collected / Expected integrated luminosity	Year attained	γ/ϕ_3 sensitivity
LHCb Run-1 [7, 8 TeV]	3 fb^{-1}	2012	8°
LHCb Run-2 [13 TeV]	6 fb^{-1}	2018	4°
Belle II Run	50 ab^{-1}	2025	1.5°
LHCb upgrade I [14 TeV]	50 fb^{-1}	2030	< 1°
LHCb upgrade II [14 TeV]	300 fb^{-1}	(>)2035	< 0.4°

➢ γ/φ_3 uncertainty from strong phase input

2.93fb^{-1}@ $\text{E}_{\text{cm}} = 3.773$ GeV @ BESIII
$D^0 \to K_S^0 h^+ h^- \sim 1°$

$\Longrightarrow$

20fb^{-1}@ $\text{E}_{\text{cm}} = 3.773$ GeV @ BESIII
$D^0 \to K_S^0 h^+ h^- \sim 0.4°$

[Chinese Phys. C 44, 040001 (2020)]

Summary

- BESIII provides unique quantum correlated $D^0\overline{D}^0$ data to measure the strong phase parameters in D decays as inputs to LHCb and Belle II for CKM angle γ measurement and search of indirect CPV in charm mixing

- Using 2.93 fb^{-1} e^+e^- collision data taken @ 3.773 GeV with BESIII detector, strong phase parameters of five channels are reported
  - Global parameters are measured for $K^-\pi^+, K^-\pi^+\pi^0, K^-\pi^+\pi^+\pi^-$
  - Binned parameters are measured for $K^-\pi^+\pi^+\pi^-, K_S^0\pi^+\pi^-, K_S^0K^+K^-$

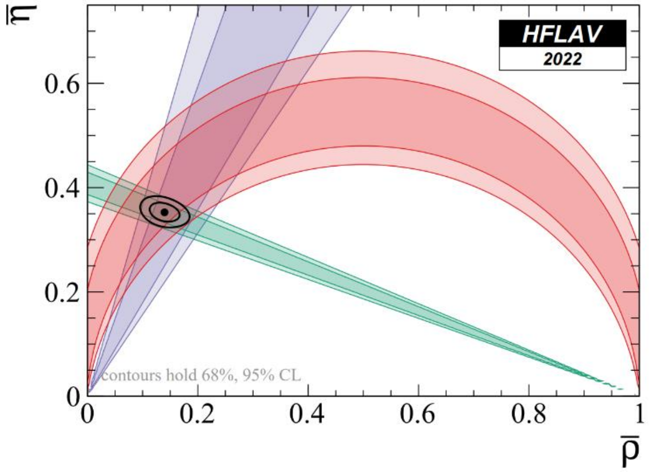
- $20\ \mathrm{fb}^{-1}$ $\psi(3770)$ data will be collected in the near future @ BESIII
  - More decays (e.g. $\pi^+\pi^-\pi^+\pi^-, K^+K^-\pi^+\pi^-$, $K_S^0\pi^+\pi^-\pi^0$ …)
  - Higher precision (e.g. uncertainty on γ $\sim 1^\circ \rightarrow \sim 0.4^\circ$ for $K_S^0h^+h^-$)

Thank you!

BACK-UP

Measurement of γ/φ_3

➢ CKM Matrix

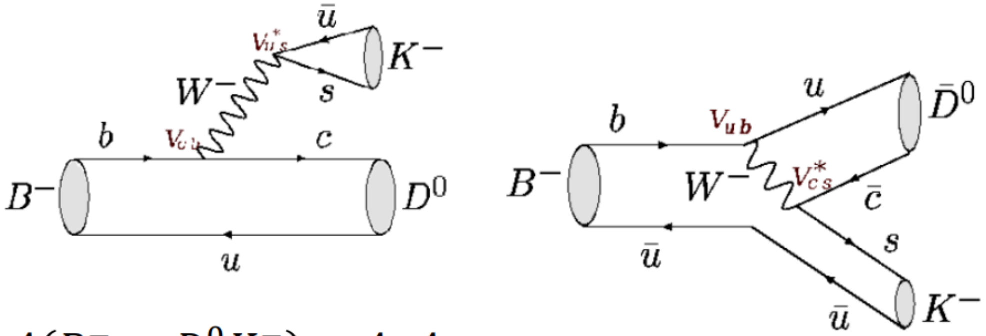


$$\alpha \equiv \varphi_1 \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) = (85.2^{+4.8}_{-4.3})^\circ$$

$$\beta \equiv \varphi_2 \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) = (22.2 \pm 0.7)^\circ$$

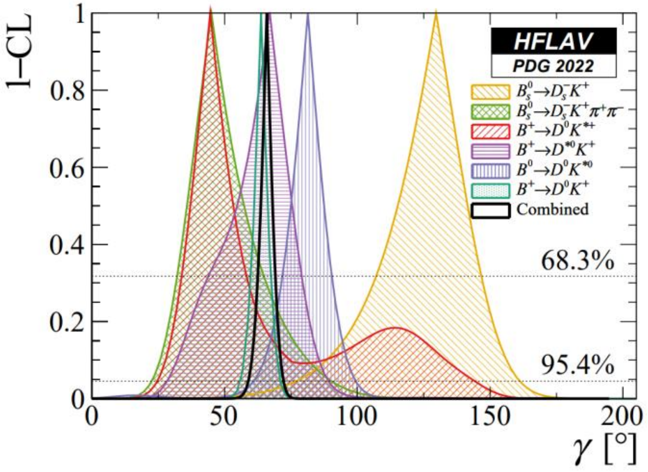
$$\gamma \equiv \varphi_3 \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = (65.9^{+3.3}_{-3.5})^\circ$$

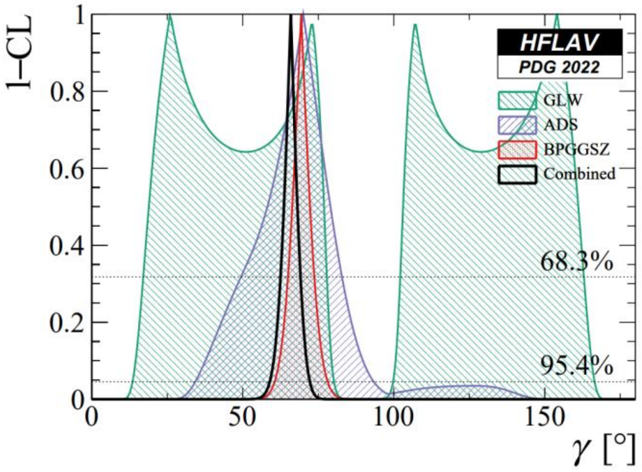
➢ Measurement of γ/φ_3



$$A(B^- \to D^0K^-) = A_B A_D$$

$$A(B^- \to \bar{D}^0K^-) = A_B r_B e^{i(\delta_B-\gamma)} A_{\bar{D}}$$





$D^0 \to K^-\pi^+$

[JHEP 05, 164 (2021)]

ST mode	$n_{S\pm}$	$\varepsilon_{S\pm}(\%)$
K^+K^-	$56\,156 \pm 261$	62.99 ± 0.26
$\pi^+\pi^-$	$20\,222 \pm 187$	65.58 ± 0.26
$K_S^0\pi^0\pi^0$	$25\,156 \pm 235$	16.46 ± 0.07
$\pi^0\pi^0$	7610 ± 156	42.77 ± 0.21
$\rho\pi^0$	$41\,117 \pm 354$	36.22 ± 0.21
$K_S^0\pi^0$	$72\,710 \pm 291$	41.95 ± 0.21
$K_S^0\eta$	$10\,046 \pm 121$	35.12 ± 0.20
$K_S^0\omega$	$31\,422 \pm 215$	17.88 ± 0.10
DT mode	$n_{K\pi,S\pm}$	$\varepsilon_{K\pi,S\pm}(\%)$
$K\pi,\ K^+K^-$	1671 ± 41	42.33 ± 0.21
$K\pi,\ \pi^+\pi^-$	610 ± 25	44.02 ± 0.21
$K\pi,\ K_S^0\pi^0\pi^0$	806 ± 29	12.86 ± 0.13
$K\pi,\ \pi^0\pi^0$	213 ± 14	30.42 ± 0.18
$K\pi,\ \rho\pi^0$	1240 ± 35	25.48 ± 0.16
$K\pi,\ K_S^0\pi^0$	1689 ± 41	29.06 ± 0.17
$K\pi,\ K_S^0\eta$	230 ± 15	24.84 ± 0.16
$K\pi,\ K_S^0\omega$	747 ± 27	12.60 ± 0.06

$$\mathcal{A}_{K\pi} \equiv \frac{\mathcal{B}(D_- \to K^-\pi^+) - \mathcal{B}(D_+ \to K^-\pi^+)}{\mathcal{B}(D_- \to K^-\pi^+) + \mathcal{B}(D_+ \to K^-\pi^+)},$$

$$\mathcal{A}_{K\pi} = \frac{-2r_D^{K\pi}\cos\delta_D^{K\pi} + y}{1 + (r_D^{K\pi})^2}.$$

$D^0 \to K^-\pi^+\pi^0$ and $K^-\pi^+\pi^+\pi^-$

[JHEP 05, 164 (2021)]

- **Measurement of γ (ADS) $\leftarrow$ R_f, δ_D^f** $\left(r_D^f\right)^2 = \int \left|\bar{A}_f\right|^2 d\Phi / \int \left|A_f\right|^2 d\Phi$ $R_f e^{-i\delta_D^f} = \dfrac{\int A_f^* \bar{A}_f d\Phi}{\sqrt{\int \left|A_f\right|^2 d\Phi \int \left|\bar{A}_f\right|^2 d\Phi}}$
- **Global analysis and binned analysis for $D^0 \to K^-\pi^+\pi^+\pi^-$**

  *Binning scheme (N=4)
  [PLB 802, 135188 (2020)]
- **Observables**
  - $\rho = \dfrac{DT\ Yield\ with\ QC}{DT\ Yield\ without\ QC}$ (*D^0- $\bar{D}^0$ mixing is ignored for simplicity, but considered in the analysis)
    - ✓ CP tag: $\rho^{\mathrm{f}}_{\mathrm{CP}\pm} = 1 \mp \dfrac{2r_D^f R_f}{1+\left(r_D^f\right)^2} \cos\left(\delta_D^f\right)$, $\Delta^f_{CP} = \pm(\rho^{\mathrm{f}}_{\mathrm{CP}\pm} - 1)$ (e.g. f vs CP + $\bar{f}$ vs CP)
    - ✓ Like-sign tag (same charge of Kaon in tag side and signal side):

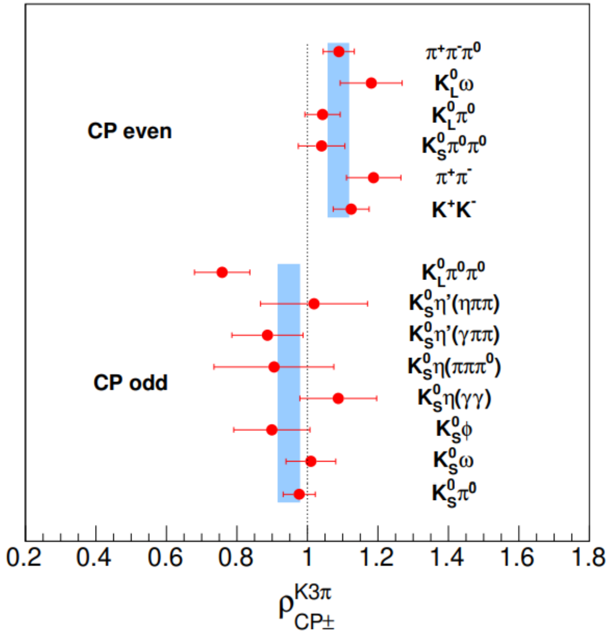
      $\rho^{\mathrm{f}}_{\mathrm{LS}} = 1 - R_f^2$ (e.g. f vs f + $\bar{f}$ vs $\bar{f}$)

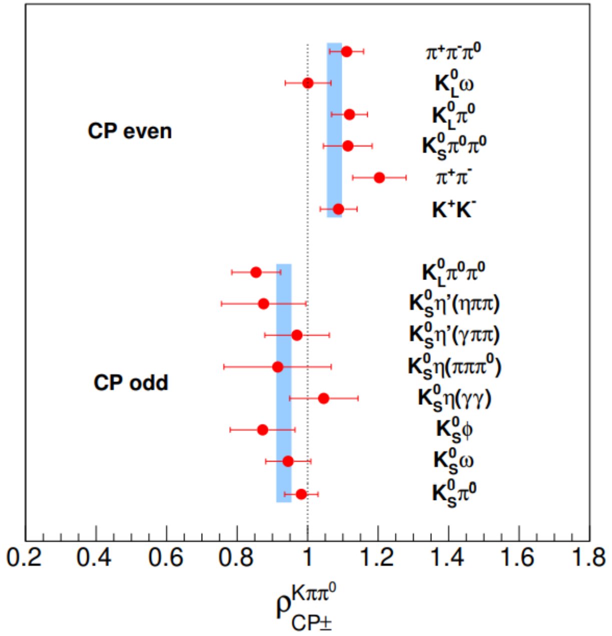
      $\rho^{\mathrm{f}}_{\mathrm{T,LS}} = 1 - \dfrac{2r_D^f r_D^T}{\left(r_D^f\right)^2 + \left(r_D^T\right)^2} R_f R_T \cos(\delta_D^T - \delta_D^f)$ (e.g. $K^-\pi^+$ vs f + $K^+\pi^-$ vs $\bar{f}$)
  - Y_i (DT yield for $K_S^0\pi^+\pi^-$ tag): $\propto [K_i + \left(r_D^f\right)^2 K_{-i} - 2r_D^f R_f \sqrt{K_i K_{-i}}(c_i \cos\left(\delta_D^f\right) - s_i \sin(\delta_D^f))]$
- **Extraction of DT yields**
  - Full reconstruction: Fit to M_{bc}^{sig}
  - Partial reconstruction: Fit to M_{miss}^2

$D^0 \to K^-\pi^+\pi^0$ and $K^-\pi^+\pi^+\pi^-$

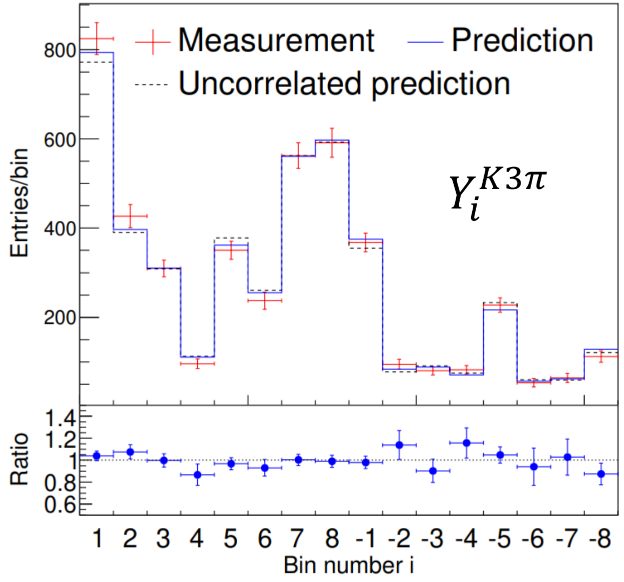
[JHEP 05, 164 (2021)]

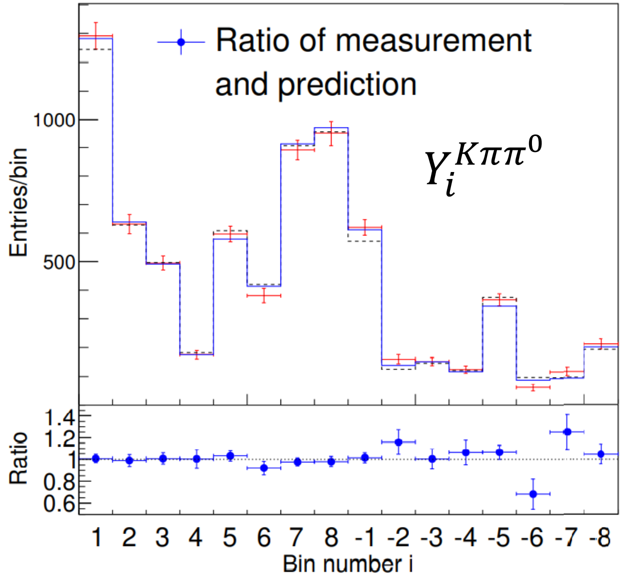
➢ Observed values of ρ & Y_i





Observable	Value	Observable	Value
$\Delta^{K3\pi}_{CP}$	$0.070 \pm 0.011 \pm 0.012$	$\Delta^{K\pi\pi^0}_{CP}$	$0.078 \pm 0.007 \pm 0.012$
$\rho^{K3\pi}_{LS}$	$0.740 \pm 0.157 \pm 0.161$	$\rho^{K\pi\pi^0}_{LS}$	$0.440 \pm 0.095 \pm 0.014$
$\rho^{K3\pi}_{K\pi,LS}$	$0.570 \pm 0.109 \pm 0.069$	$\rho^{K\pi\pi^0}_{K\pi,LS}$	$0.213 \pm 0.062 \pm 0.004$
$\rho^{K3\pi}_{K\pi\pi^0,LS}$	$0.715 \pm 0.094 \pm 0.089$		





$D^0 \to K^-\pi^+\pi^0$ and $K^-\pi^+\pi^+\pi^-$

[JHEP 05, 164 (2021)]

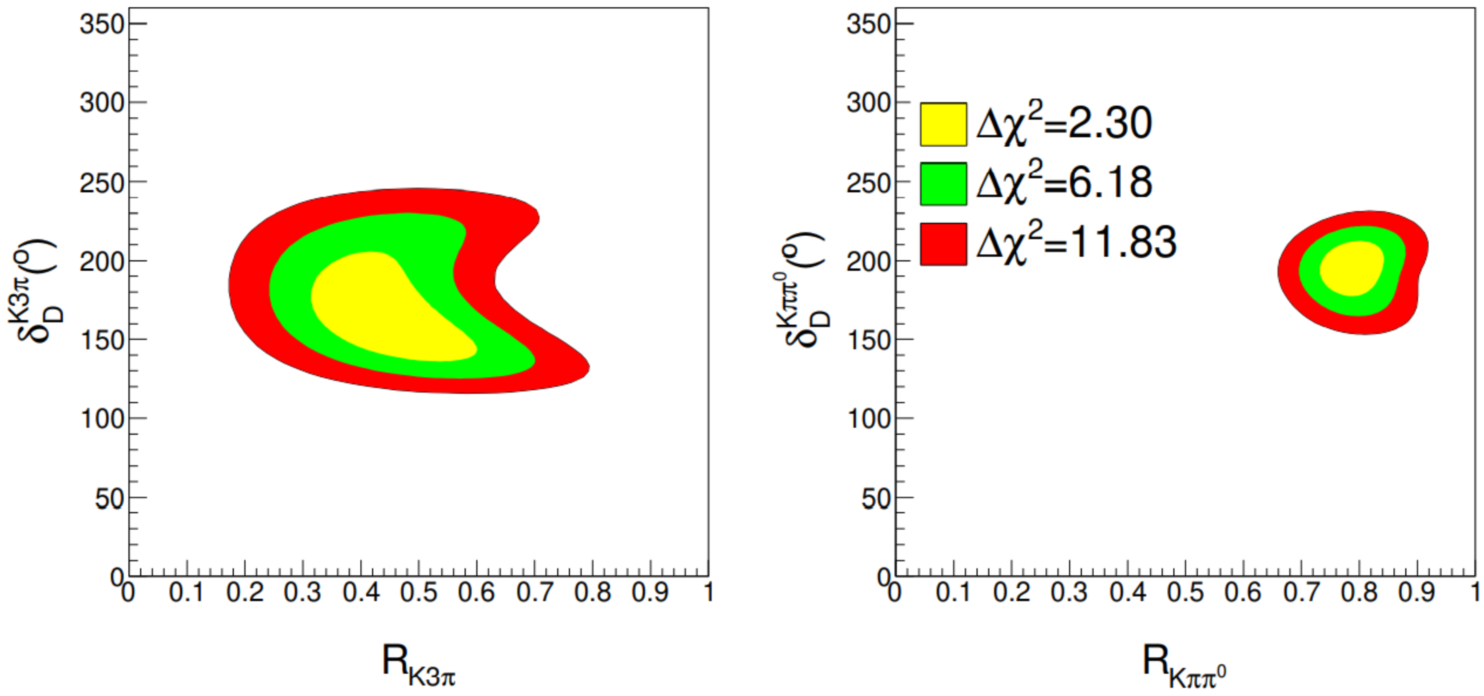
Systematics	$R_{K3\pi}$	$\delta_D^{K3\pi}$	$R_{K\pi\pi^0}$	$\delta_D^{K\pi\pi^0}$
Size of CP-tagged $D \to K^-\pi^+$ samples	0.04	7.0°	0.02	6.9°
K/π tracking and identification	0.02	3.8°	< 0.01	2.3°
π^0 reconstruction	< 0.01	$< 0.1^\circ$	< 0.01	$< 0.1^\circ$
Impact of resonance modelling on efficiency	< 0.01	2.5°	< 0.01	0.4°
Size of Monte Carlo samples	0.01	1.5°	< 0.01	1.3°
$D \to K_S^0 K^-\pi^+$ background	0.05	1.0°	0.01	4.6°
Fit method for signal yields	0.02	3.4°	< 0.01	1.1°
c_i, s_i	${}^{+0.01}_{-0.00}$	3.0°	< 0.01	$\left({}^{+0.6}_{-0.7}\right)^\circ$
K_i	0.01	$\left({}^{+6.7}_{-6.1}\right)^\circ$	0.01	$\left({}^{+3.1}_{-4.4}\right)^\circ$
$\mathcal{B}(D^0 \to S)$, with $S = K^-\pi^+\pi^+\pi^-$ and $K^-\pi^+\pi^0$	0.01	$\left({}^{+1.7}_{-1.5}\right)^\circ$	0.01	$\left({}^{+3.4}_{-2.2}\right)^\circ$
$\mathcal{B}(D^0 \to \bar{S})/\mathcal{B}(D^0 \to S)$	${}^{+0.02}_{-0.01}$	2.7°	< 0.01	0.2°
$\mathcal{B}(D^0 \to K^-\pi)$	0.01	$\left({}^{+0.8}_{-1.2}\right)^\circ$	< 0.01	$\left({}^{+0.9}_{-0.7}\right)^\circ$
$r_D^{K\pi}$	< 0.01	$\left({}^{+0.2}_{-0.1}\right)^\circ$	< 0.01	0.2°
$\delta_D^{K\pi}$	< 0.01	$< 0.1^\circ$	< 0.01	$< 0.1^\circ$
x, y	< 0.01	$\left({}^{+1.0}_{-1.1}\right)^\circ$	< 0.01	0.5°
$F^+_{\pi\pi\pi^0}$	< 0.01	$\left({}^{+0.3}_{-0.4}\right)^\circ$	< 0.01	0.1°
Statistical	${}^{+0.08}_{-0.09}$	$\left({}^{+29.3}_{-18.7}\right)^\circ$	0.04	$\left({}^{+10.6}_{-12.6}\right)^\circ$

$D^0 \to K^-\pi^+\pi^0$ and $K^-\pi^+\pi^+\pi^-$

[JHEP 05, 164 (2021)]

Combination of global R & δ_D of BESIII, CLEO-c and LHCb

Parameter	BESIII and CLEO-c	BESIII, CLEO-c and LHCb
$R_{K3\pi}$	$0.49^{+0.11}_{-0.10}$	$0.44^{+0.10}_{-0.09}$
$\delta_D^{K3\pi}$	$(154^{+22}_{-14})^\circ$	$(161^{+28}_{-18})^\circ$
$r_D^{K3\pi}$ $(\times 10^{-2})$	5.46 ± 0.08	5.50 ± 0.07
$R_{K\pi\pi^0}$	0.79 ± 0.04	0.79 ± 0.04
$\delta_D^{K\pi\pi^0}$	$(196\pm11)^\circ$	$(196\pm11)^\circ$
$r_D^{K\pi\pi^0}$ $(\times 10^{-2})$	4.41 ± 0.11	4.41 ± 0.11

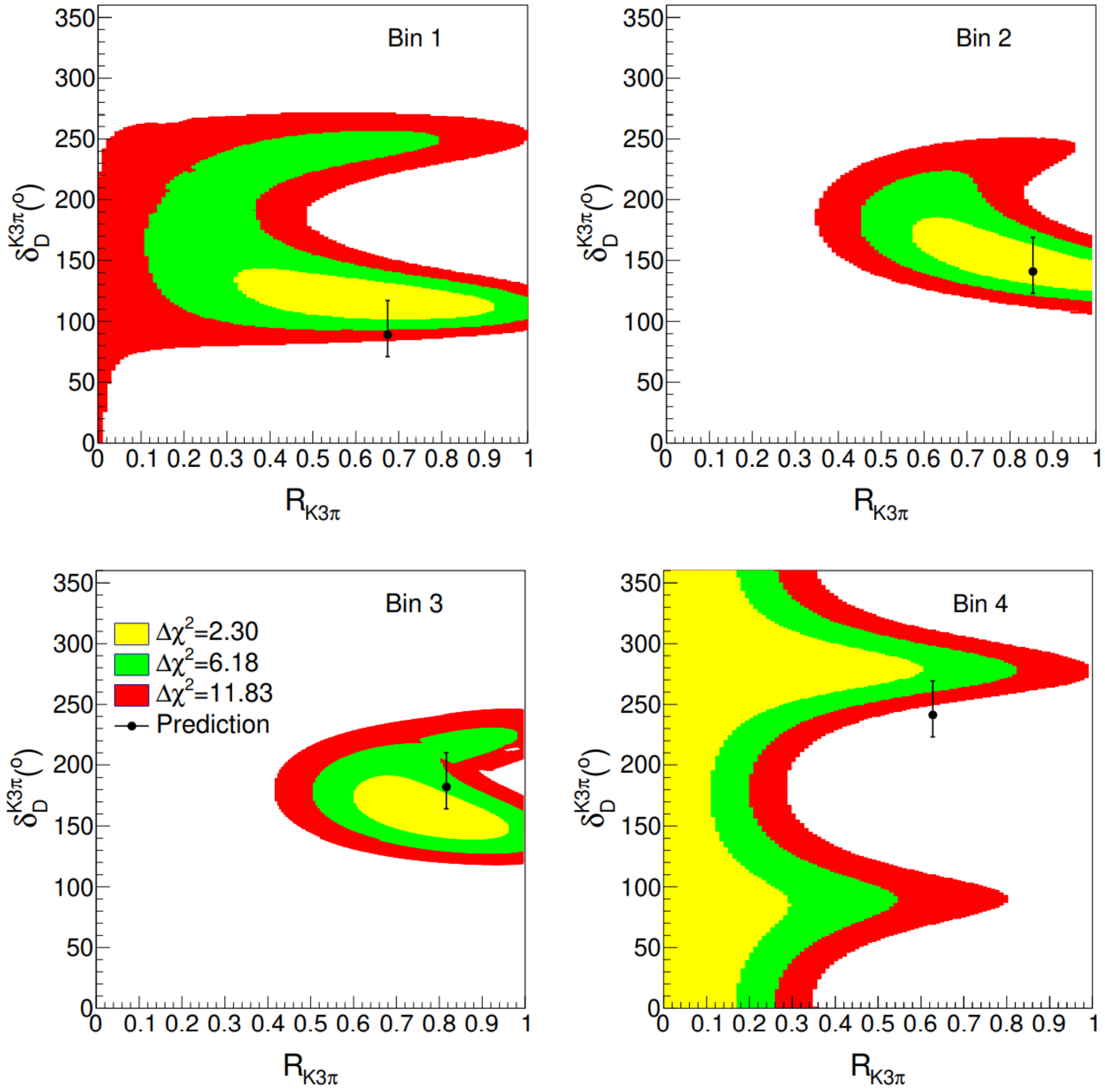


$D^0 \to K^-\pi^+\pi^0$ and $K^-\pi^+\pi^+\pi^-$

[JHEP 05, 164 (2021)]

Combination of binned $R_{K3\pi}$ & $\delta_D^{K3\pi}$ of BESIII and CLEO-c

Parameter	Bin 1	Bin 2	Bin 3	Bin 4
$R_{K3\pi}$	$0.66^{+0.18}_{-0.21}$	$0.85^{+0.14}_{-0.21}$	$0.78^{+0.12}_{-0.12}$	$0.25^{+0.16}_{-0.25}$
$\delta_D^{K3\pi}$	$(117^{+14}_{-9})^\circ$	$(145^{+23}_{-14})^\circ$	$(160^{+19}_{-20})^\circ$	$(288^{+15}_{-29})^\circ$
$r_D^{K3\pi}$ $(\times 10^{-2})$	5.43 ± 0.10	5.78 ± 0.11	5.76 ± 0.10	5.06 ± 0.12
$R_{K\pi\pi^0}$		0.80 ± 0.04		
$\delta_D^{K\pi\pi^0}$		$(203\pm11)^\circ$		
$r_D^{K\pi\pi^0}$ $(\times 10^{-2})$		4.49 ± 0.11		



$D^0 \to K_S^0\pi^+\pi^-$

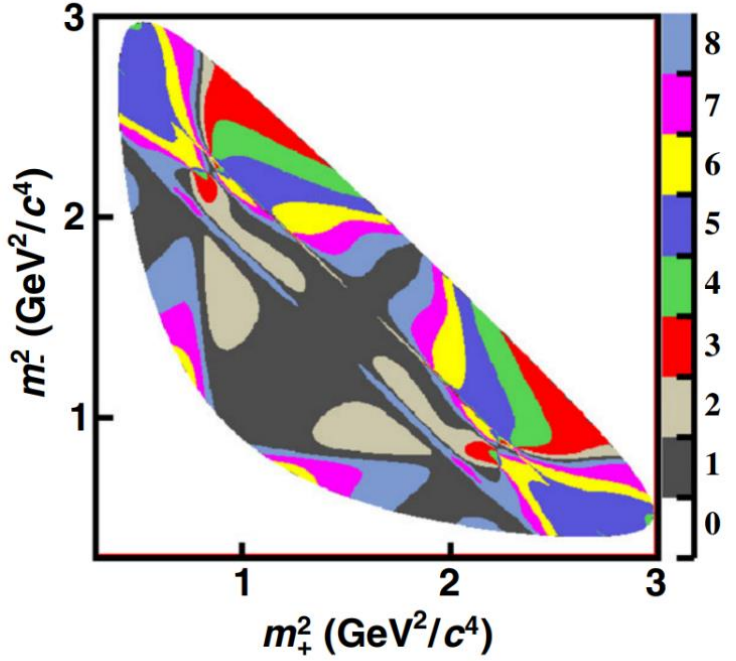
[PRL 124, 241802 (2020); PRD 101, 112002 (2020)]

- **Measurement of γ (GGSZ)** ← binned parameters c_i, s_i

$$K_i = \int_i |A_f|^2 \mathrm{d}m_+^2 \mathrm{d}m_-^2 \qquad c_i = \frac{1}{\sqrt{K_i K_{-i}}} \int_i |A_f||\bar{A}_f| \cos[\Delta\delta_D]\, \mathrm{d}m_+^2 \mathrm{d}m_-^2 \xRightarrow{\cos\to\sin} s_i$$

- **Divide phase space (DP) into symmetrical bins**

*Binning scheme (N=8)
[Phys. Rev. D 82, 112006 (2010)]



- **Observables (DT yields)**
  - Flavor tag: $\propto K_i/f_i$ (f_i: correct factor for hadronic flavor tag modes)
  - CP tag: $\propto [K_i + K_{-i} - (2F_+ - 1)2c_i\sqrt{K_i K_{-i}}]$
  - $K_S^0\pi^+\pi^-$ tag (binned): $\propto [K_i K_{-j} + K_{-i}K_j - 2\sqrt{K_i K_{-i} K_j K_{-j}}(c_i c_j + s_i s_j)]$
  - $K_L^0\pi^+\pi^-$ tag (binned): $\propto [K_i K'_{-j} + K_{-i}K'_j + 2\sqrt{K_i K_{-i} K'_j K'_{-j}}(c_i c'_j + s_i s'_j)]$

- **Extraction of DT yields:**
  - Full reconstruction: Fit to M_{bc}^{sig}
  - Partial reconstruction: Fit to U_{miss} for ν and M_{miss}^2 for other

$$M_{bc} = \sqrt{(\sqrt{s}/2)^2 - |\vec{p}_D|^2}$$

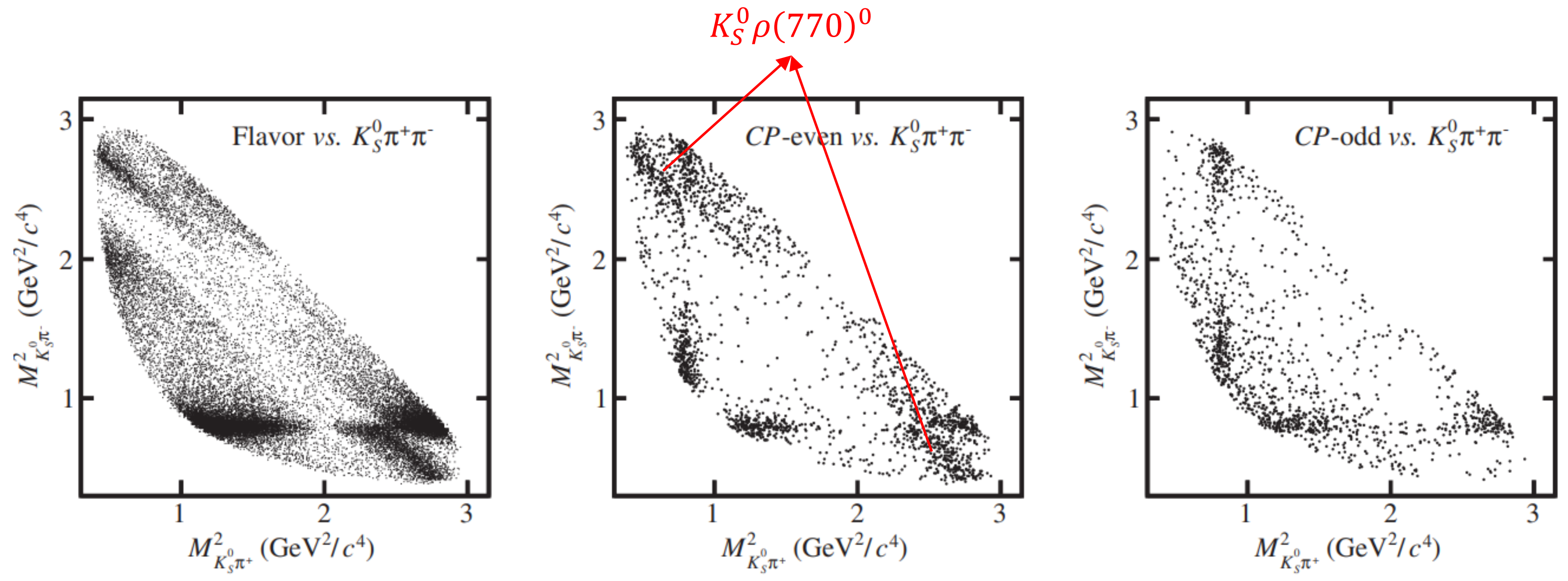
$$U_{miss} = E_{miss} - |\vec{p}_{miss}|$$

$$M_{miss}^2 = E_{miss}^2 - |\vec{p}_{miss}|^2$$

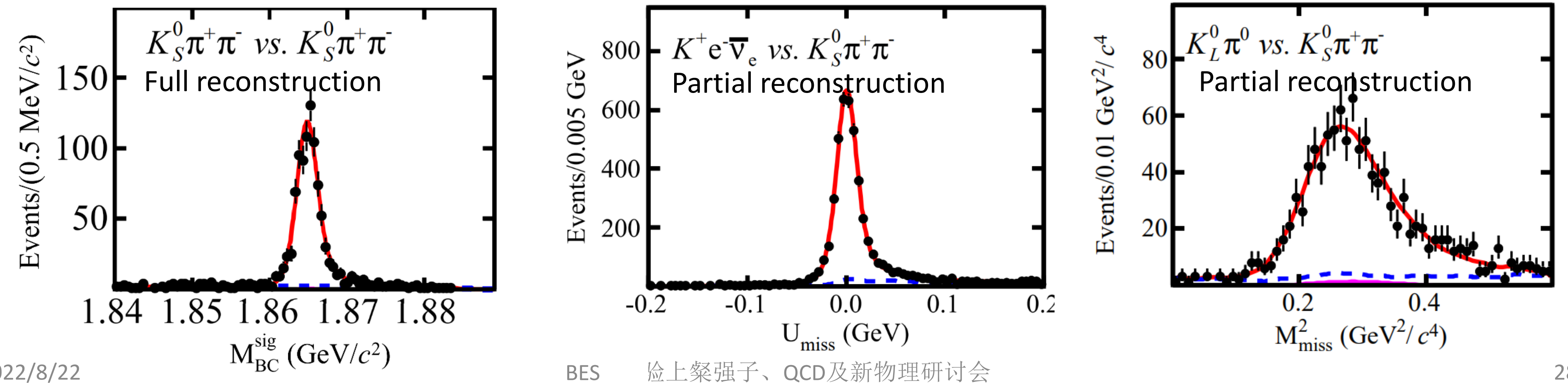
$D^0 \to K_S^0\pi^+\pi^-$

[PRL 124, 241802 (2020); PRD 101, 112002 (2020)]

- Dalitz Plot



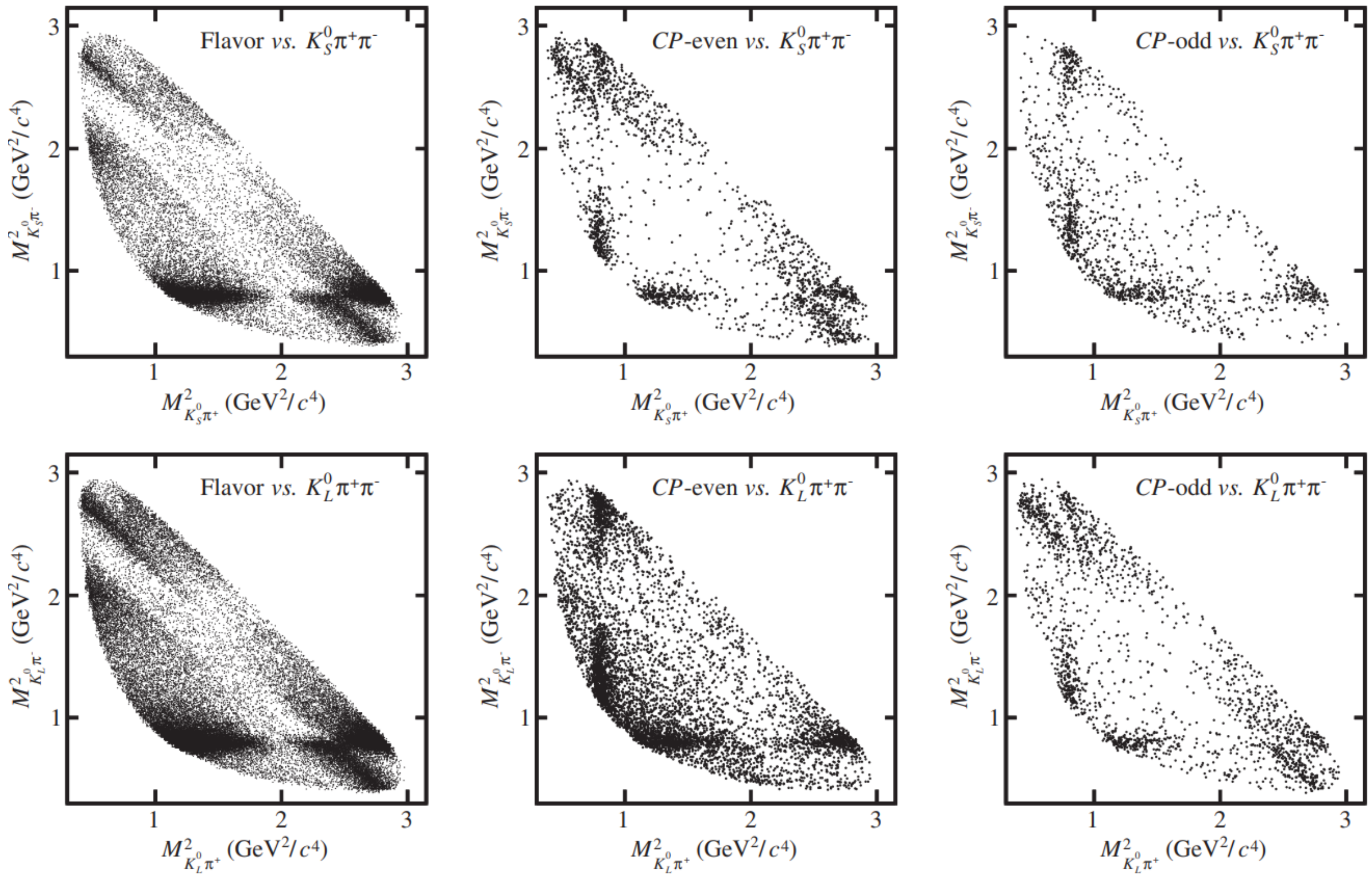
- Fit Plot



$D^0 \to K^0_{S/L}\pi^+\pi^-$

[PRL 124, 241802 (2020)] [PRD 101, 112002 (2020)]

Mode	$N_{\rm ST}$	$N^{\rm DT}_{K^0_S\pi^+\pi^-}$	$N^{\rm DT}_{K^0_L\pi^+\pi^-}$
Flavor tags			
$K^+\pi^-$	549373 ± 756	4740 ± 71	9511 ± 115
$K^+\pi^-\pi^0$	1076436 ± 1406	5695 ± 78	11906 ± 132
$K^+\pi^-\pi^-\pi^+$	712034 ± 1705	8899 ± 95	19225 ± 176
$K^+e^-\bar{\nu}_e$	458989 ± 5724	4123 ± 75	
CP-even tags			
K^+K^-	57050 ± 231	443 ± 22	1289 ± 41
$\pi^+\pi^-$	20498 ± 263	184 ± 14	531 ± 28
$K^0_S\pi^0\pi^0$	22865 ± 438	198 ± 16	612 ± 35
$\pi^+\pi^-\pi^0$	107293 ± 716	790 ± 31	2571 ± 74
$K^0_L\pi^0$	103787 ± 7337	913 ± 41	
CP-odd tags			
$K^0_S\pi^0$	66116 ± 324	643 ± 26	861 ± 46
$K^0_S\eta_{\gamma\gamma}$	9260 ± 119	89 ± 10	105 ± 15
$K^0_S\eta_{\pi^+\pi^-\pi^0}$	2878 ± 81	23 ± 5	40 ± 9
$K^0_S\omega$	24978 ± 448	245 ± 17	321 ± 25
$K^0_S\eta'_{\pi^+\pi^-\eta}$	3208 ± 88	24 ± 6	38 ± 8
$K^0_S\eta'_{\gamma\pi^+\pi^-}$	9301 ± 139	81 ± 10	120 ± 14
$K^0_L\pi^0\pi^0$	50531 ± 6128	620 ± 32	
Mixed CP tags			
$K^0_S\pi^+\pi^-$	188912 ± 756	899 ± 31	3438 ± 72
$K^0_S\pi^+\pi^-_{\rm miss}$		224 ± 17	
$K^0_S(\pi^0\pi^0_{\rm miss})\pi^+\pi^-$		710 ± 34	



$D^0 \to K^0_{S/L}\pi^+\pi^-$

[PRL 124, 241802 (2020)] [PRD 101, 112002 (2020)]

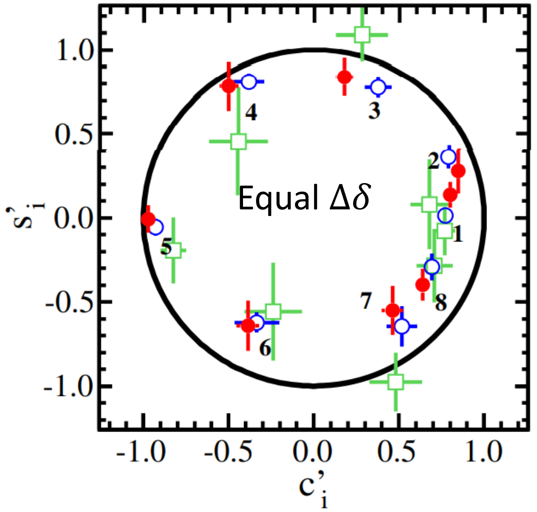
Likelihood function in fit:

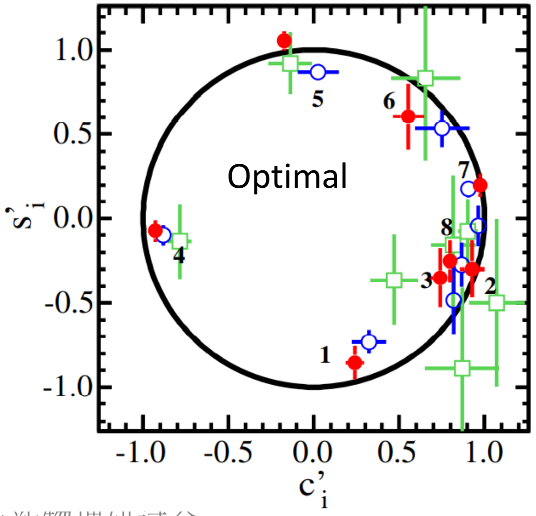
$$
\begin{aligned}
-2\log\mathcal{L} = & -2\sum_{i=1}^{8}\ln P(N_i^{\mathrm{obs}}, \langle N_i^{\mathrm{exp}}\rangle)_{CP,K_S^0\pi^+\pi^-} \\
& -2\sum_{i=1}^{8}\ln P(N_i^{\mathrm{obs}}, \langle N_i^{\mathrm{exp}}\rangle)_{CP,K_L^0\pi^+\pi^-} \\
& -2\sum_{n=1}^{72}\ln P(N_n^{\mathrm{obs}}, \langle N_n^{\mathrm{exp}}\rangle)_{K_S^0\pi^+\pi^-,K_S^0\pi^+\pi^-} \\
& -2\sum_{n=1}^{128}\ln P(N_n^{\mathrm{obs}\prime}, \langle N_n^{\mathrm{exp}}\rangle)_{K_L^0\pi^+\pi^-,K_S^0\pi^+\pi^-} + \chi^2
\end{aligned}
$$

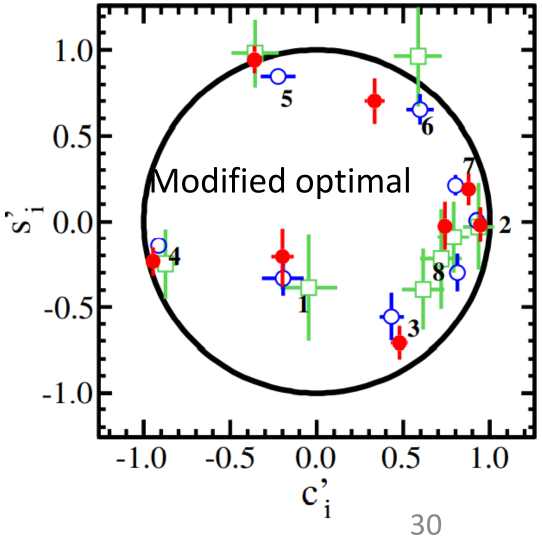
$$
\chi^2 = \sum_i \left(\frac{c_i' - c_i - \boxed{\Delta c_i}}{\boxed{\delta\Delta c_i}}\right)^2 + \sum_i \left(\frac{s_i' - s_i - \boxed{\Delta s_i}}{\boxed{\delta\Delta s_i}}\right)^2
$$

[Phys. Rev. D 81, 112002 (2010); Phys. Rev. Lett. 95, 121802 (2005)]

- c_i' and s_i' for $D^0 \to K_L^0\pi^+\pi^-$
- A factor of 2.8 (2.2) more precise for c_i' and s_i' than previous measurements







$D^0 \to K^0_{S/L}\pi^+\pi^-$

[PRL 124, 241802 (2020)] [PRD 101, 112002 (2020)]

- Systematic uncertainty for equal $\Delta\delta$ binning scheme

Uncertainty	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
K_i and K_i'	0.004	0.013	0.005	0.007	0.005	0.014	0.006	0.007
ST yields	0.007	0.007	0.013	0.008	0.004	0.014	0.019	0.011
MC statistics	0.001	0.003	0.003	0.003	0.001	0.004	0.004	0.003
DT peaking-background subtraction	0.002	0.003	0.002	0.007	0.005	0.007	0.003	0.002
DT yields	0.001	0.002	0.002	0.001	0.001	0.002	0.003	0.002
Momentum resolution	0.002	0.003	0.012	0.011	0.010	0.010	0.011	0.009
$D^0\bar{D}^0$ mixing	0.001	0.000	0.002	0.001	0.000	0.002	0.002	0.001
Total systematic	0.009	0.016	0.019	0.017	0.013	0.024	0.023	0.017
Statistical plus $K^0_L\pi^+\pi^-$ model	0.020	0.035	0.047	0.053	0.019	0.062	0.057	0.036
$K^0_L\pi^+\pi^-$ model alone	0.011	0.009	0.027	0.030	0.007	0.034	0.033	0.017
Total	0.022	0.039	0.051	0.055	0.023	0.066	0.061	0.039

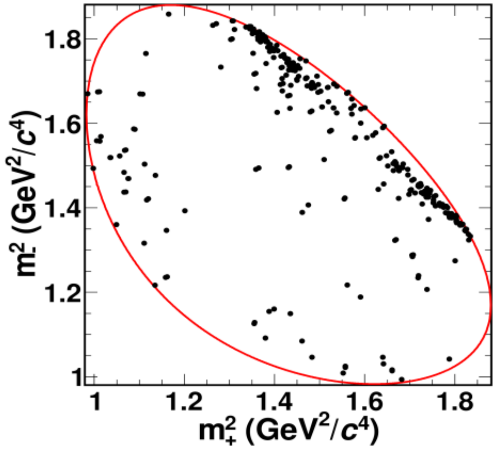
Uncertainty	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
K_i and K_i'	0.004	0.006	0.012	0.005	0.003	0.018	0.022	0.008
ST yields	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.001
MC statistics	0.007	0.011	0.009	0.010	0.005	0.009	0.011	0.006
DT peaking-background subtraction	0.007	0.005	0.007	0.018	0.005	0.009	0.011	0.004
DT yields	0.005	0.005	0.003	0.004	0.003	0.004	0.005	0.003
Momentum resolution	0.012	0.005	0.011	0.001	0.003	0.022	0.006	0.025
$D^0\bar{D}^0$ mixing	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000
Total systematic	0.017	0.015	0.020	0.022	0.009	0.031	0.028	0.027
Statistical plus $K^0_L\pi^+\pi^-$ model	0.076	0.134	0.112	0.143	0.081	0.147	0.143	0.091
$K^0_L\pi^+\pi^-$ model alone	0.017	0.029	0.022	0.018	0.012	0.017	0.036	0.028
Total	0.078	0.135	0.114	0.144	0.081	0.150	0.146	0.095

$D^0 \to K^0_{S/L} K^+ K^-$

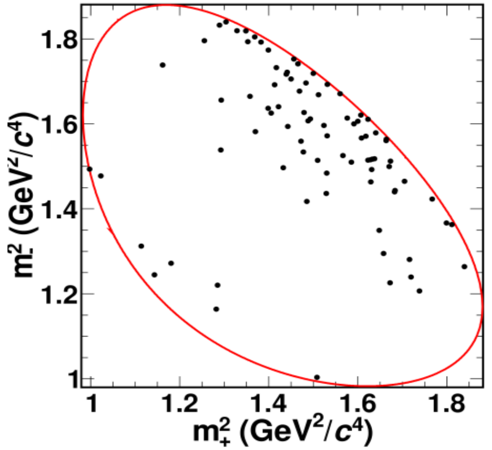
[PRD 102, 052008 (2020)]

Mode	ST N_{ST}	ST $\epsilon_{\mathrm{ST}}(\%)$	DT $N_{\mathrm{DT}}^{K^0_S K^+ K^-}$	DT $N_{\mathrm{DT}}^{K^0_L K^+ K^-}$	DT $\epsilon_{\mathrm{DT}}^{K^0_S K^+ K^-}(\%)$	DT $\epsilon_{\mathrm{DT}}^{K^0_L K^+ K^-}(\%)$
Flavor-tags						
$K^-\pi^+$	524307 ± 742	63.31 ± 0.06	323	743	12.43 ± 0.07	15.85 ± 0.08
$K^-\pi^+\pi^0$	995683 ± 1117	31.70 ± 0.03	596	1769	5.86 ± 0.05	7.94 ± 0.06
$K^-e^+\nu_e$	752387 ± 12795		263		3.23 ± 0.04	
CP-even tags						
K^+K^-	53481 ± 247	61.02 ± 0.11	42	112	12.07 ± 0.07	15.52 ± 0.08
$\pi^+\pi^-$	19339 ± 163	64.52 ± 0.11	10	31	12.16 ± 0.07	15.70 ± 0.08
$K^0_S\pi^0\pi^0$	19882 ± 233	14.86 ± 0.08	7	45	2.49 ± 0.04	3.79 ± 0.04
$\pi^+\pi^-\pi^0$	99981 ± 618	37.65 ± 0.11	51	254	6.79 ± 0.06	9.54 ± 0.07
$K^0_L\pi^0$	209445 ± 14796		90		8.88 ± 0.06	
$K^0_L\eta(\gamma\gamma)$	40009 ± 2543		19		6.60 ± 0.06	
$K^0_L\omega$	207376 ± 11498		44		3.42 ± 0.04	
$K^0_L\eta'(\pi^+\pi^-\eta)$	33683 ± 1909		7		3.23 ± 0.04	
CP-odd tags						
$K^0_S\pi^0$	65072 ± 281	36.92 ± 0.11	39	89	6.75 ± 0.06	9.33 ± 0.07
$K^0_S\eta(\gamma\gamma)$	9524 ± 134	32.94 ± 0.11	9	10	6.05 ± 0.05	9.05 ± 0.06
$K^0_S\omega$	19262 ± 157	12.14 ± 0.07	16	27	2.20 ± 0.03	3.42 ± 0.04
$K^0_S\eta'(\pi^+\pi^-\eta)$	3301 ± 62	12.46 ± 0.07	2	5	2.20 ± 0.03	3.46 ± 0.04
Mixed CP tags						
$K^0_S\pi^+\pi^-$			78	265	6.35 ± 0.05	8.32 ± 0.06
$K^0_L\pi^+\pi^-$			282		9.56 ± 0.07	
$K^0_S K^+K^-$	12949 ± 119	18.35 ± 0.09	4	19	2.99 ± 0.04	3.40 ± 0.04

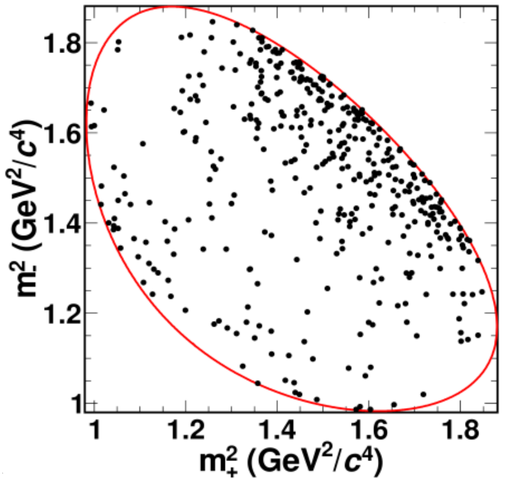
CP-even vs. $K^0_S K^+ K^-$



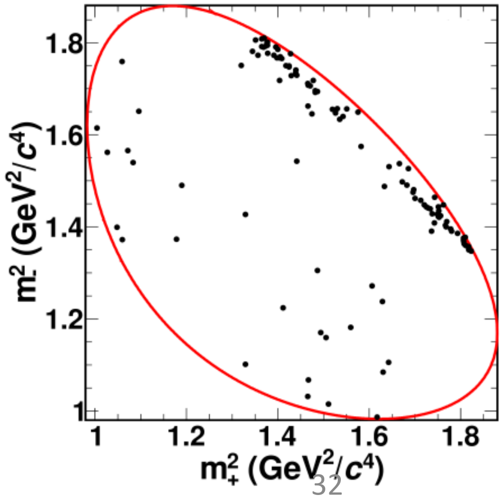
CP-odd vs. $K^0_S K^+ K^-$



CP-even vs. $K^0_L K^+ K^-$



CP-odd vs. $K^0_L K^+ K^-$



$D^0 \to K^0_{S/L} K^+ K^-$

[PRD 102, 052008 (2020)]

Likelihood function in fit:

$$
\begin{aligned}
-2\ln\mathcal{L} = &-2\sum_i \ln P\left(N_i^{\pm}, \langle N_i^{\pm}\rangle\right)_{K^0_S K^+K^-,\ CP} \\
&-2\sum_i \ln P\left(N_i'^{\pm}, \langle N_i'^{\pm}\rangle\right)_{K^0_L K^+K^-,\ CP} \\
&-2\sum_{i,j} \ln P\left(N_{ij}, \langle N_{ij}\rangle\right)_{K^0_S K^+K^-,\ K^0_S K^+K^-} \\
&-2\sum_{i,j} \ln P\left(N'_{ij}, \langle N'_{ij}\rangle\right)_{K^0_S K^+K^-,\ K^0_L K^+K^-} \\
&-2\sum_{i,j} \ln P\left(N_{ij}, \langle N_{ij}\rangle\right)_{K^0_S K^+K^-, K^0_S \pi^+\pi^-} \\
&-2\sum_{i,j} \ln P\left(N'_{ij}, \langle N'_{ij}\rangle\right)_{K^0_S K^+K^-,\ K^0_L \pi^+\pi^-} \\
&-2\sum_{i,j} \ln P\left(N'_{ij}, \langle N'_{ij}\rangle\right)_{K^0_L K^+K^-,\ K^0_S \pi^+\pi^-} \\
&+\chi^2 . \qquad\qquad (28)
\end{aligned}
$$

- c_i' and s_i' for $D^0 \to K^0_L K^+ K^-$