The electromagnetic form factors of hyperons in a modified VMD model

杨永亮

青岛大学

Y. Yang, D. Y. Chen and Z. Lu, Phys. Rev. D 100, 073007 (2019)
J. Wan, Y. Yang and Z. Lu, Eur. Phys. J. Plus 136, 949 (2021)

BESIII实验上粲强子、QCD及新物理研讨会 2022年8月21~24日 兰州

outline

- Overview the VMD model and EMFFs in time-like region
- Numerical results of Λ and discussions
- The results of EMFFs for Λ_c
- Summary

Overview the VMD model

- The electromagnetic form factors (EMFFs) G_E and G_M of hadrons are important physical quantities that encode the information of internal structure of hadrons
- A reasonable theoretical approach to understand the nucleon EMFFs in the spacelike region is the vector meson dominance (VMD) model
- The time-like and space-like EMFFs of the proton and neutron have been extensively studied
- The EMFFs of hyperons (e.g.,$\Lambda, \Lambda_c, \Sigma$) in the time-like region have also been investigated

The observed EMFFs can be expressed in terms of the Dirac form factor $F_1(Q^2)$ and Pauli form factor $F_2(Q^2)$ as

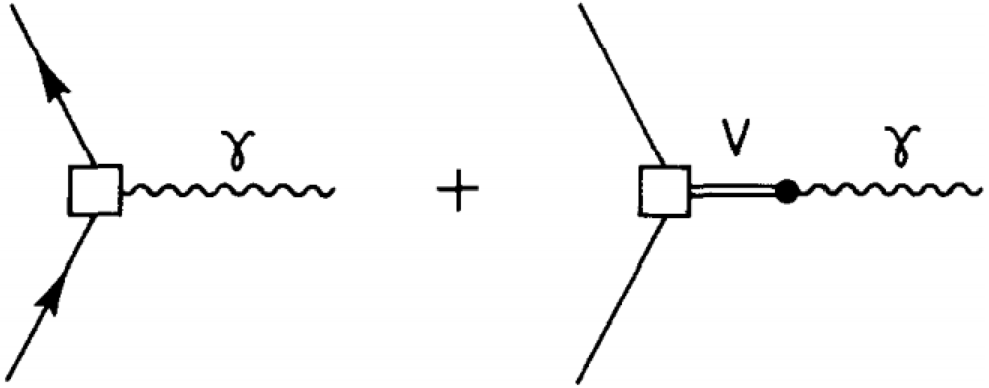
$$G_M = F_1 + F_2, \qquad G_E = F_1 - \tau F_2$$

$$J^\mu = \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_\Lambda} F_2(Q^2)$$

In VMD model, form factors can be decomposed as isoscalar and isovector components

$$F_i = F_i^S + F_i^V$$

Diagrams describing the interaction of nucleons with the EM field



the square box: intrinsic form factor

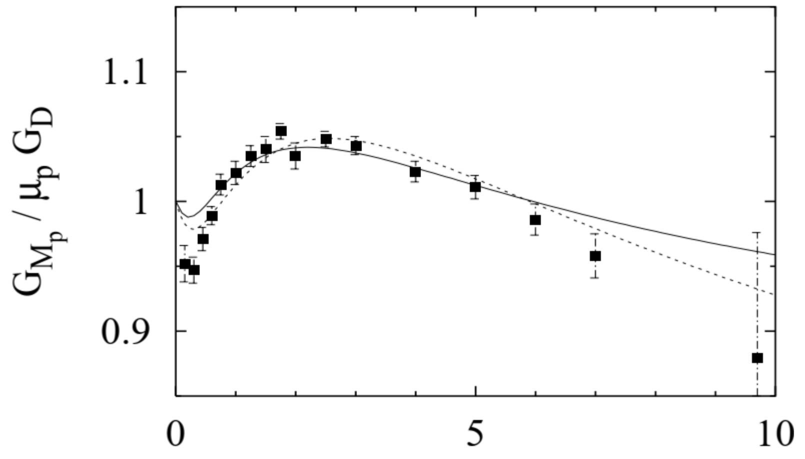
V : vector mesons (ρ, ω, ϕ)

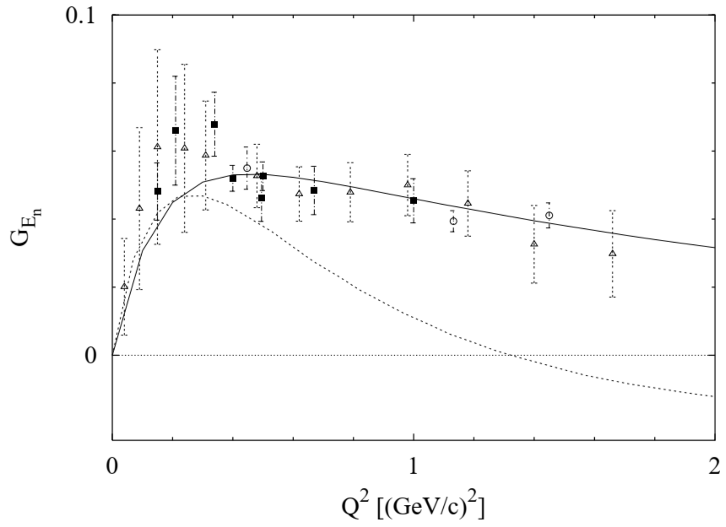
F. Iachello, A. D. Jackson, and A. Lande, Phys.Lett. 43B, 191 (1973).

In the VMD models, the form factors as the product of an intrinsic structure and the meson cloud

- intrinsic structure: is a characteristic of valence quark structure, is consistent with pQCD, described by the form factor $g(Q^2)$

- meson cloud : the interaction between the bare nucleon and the photon in the framework of the vector meson dominance





R. Bijker and F. Iachello, Phys. Rev. C 69, 068201 (2004).

Λ hyperon in spacelike region

- is a isospin singlet (Isospin is zero)

$$F_i^V \equiv 0$$

- The Dirac and Pauli form factors of Λ hyperon are normalized at Q^2 as

$$F_1(0) = 0, \qquad F_2(0) = G_M(0) = \mu_\Lambda.$$

$$\mu_\Lambda = -0.613\hat{\mu}_N = -0.723\hat{\mu}_\Lambda$$

- For large values of Q^2, the development of pQCD has put constraints to the asymptotic behavior of form factors

$$F_1 \sim 1/Q^4 \text{ and } F_2 \sim 1/Q^6$$

Λ spacelike form factor in a modified VMD

These constraints lead to the parametrized forms of the isoscalar Dirac and Pauli form factors in the VMD model as follows

$$F_1^S(Q^2) = \frac{g(Q^2)}{3}\Sigma_{i=1}^N\left[-\beta_{\omega_i} - \beta_{\phi_i} + \beta_{\omega_i}\frac{m_{\omega_i}^2}{m_{\omega_i}^2 + Q^2} + \beta_{\phi_i}\frac{m_{\phi_i}^2}{m_{\phi_i}^2 + Q^2}\right]$$

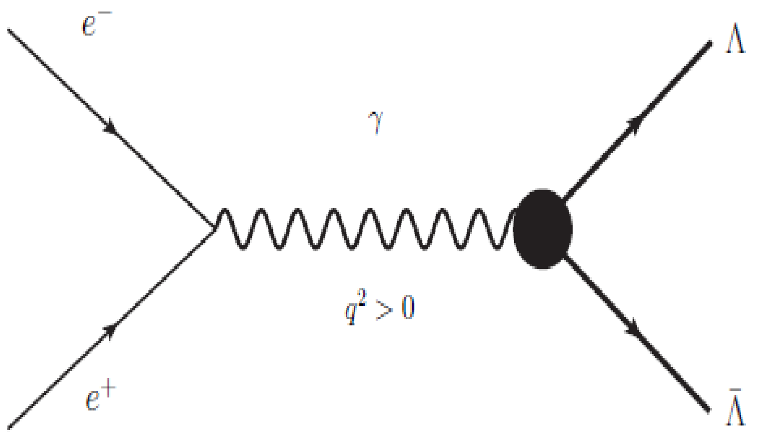
$$F_2^S(Q^2) = \frac{g(Q^2)}{3}\Sigma_{i=1}^N\left[(\mu_\Lambda - \alpha_{\phi_i})\frac{m_{\omega_i}^2}{m_{\omega_i}^2 + Q^2} + \alpha_{\phi_i}\frac{m_{\phi_i}^2}{m_{\phi_i}^2 + Q^2}\right]$$

$I^G(J^{PC}) = 0^-(1^{--})$

- ω_i $(i = 1,2,3)$ denotes the vector meson states $\underline{\omega(782), \omega(1420)\ and\ \omega(1650)}$
- ϕ_i $(i = 1,2,3)$ denotes the vector meson states $\underline{\phi(1020), \phi(1680)\ and\ \phi(2170)}$
- $g(Q^2) = \frac{1}{(1+\gamma\, Q^2)^2}$
- β, α as the products of a $V\gamma$ coupling constant and VBB coupling constant

To timelike region

- the hyperons are unstable and hyperon targets are unfeasible
- the reaction $e^+e^- \to \bar{\Lambda}\Lambda$ can offer a unique opportunity to study the electromagnetic property of Λ hyperon
- the cross sections have been measured by BABAR and BESIII Collaborations



Under one-photon exchange approximation

The cross section $e^+e^- \to \bar{\Lambda}\Lambda$ is govern by G_E and G_M

$$\sigma(s) = \frac{4\pi\alpha^2\beta}{3s} C_\Lambda \left[|G_M(s)|^2 + \frac{1}{2\tau} |G_E(s)|^2 \right]$$

G_E and G_M are the EMFFs in the timelike region

$$|G_{\rm eff}(s)| = \sqrt{\frac{2\tau|G_M(s)|^2 + |G_E(s)|^2}{1+2\tau}}$$

$$= \sqrt{\frac{\sigma_{e^+e^-\to\bar{\Lambda}\Lambda}(s)}{\frac{4\pi\alpha^2\beta}{3s} C_\Lambda [1+\frac{1}{2\tau}]}}.$$

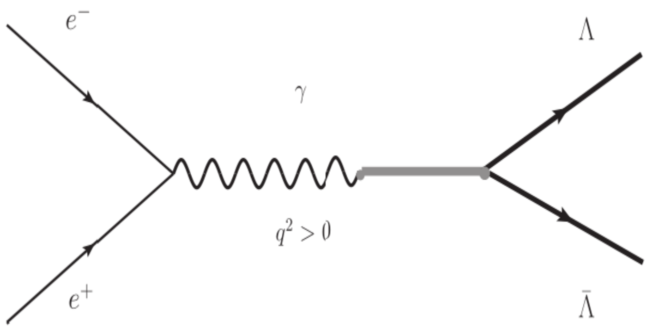
can be extracted from experimental measurements on the cross section

$|G_E/G_M|$

Data from BABAR and BESIII

Phys. Rev. D 76, 092006 (2007)
Phys. Rev. D 95, 052003 (2017)
Phys. Rev. D 97, 032013 (2018)
Phys. Rev. Lett. 123, 122003 (2019)

EMFFs in time-like region



- Λ hyperon is neutral → the near threshold enhancement effect independent on the Coulomb final-state interactions

$$G_E/G_M = |G_E/G_M| e^{i\Delta\Phi}$$

- relative phase angle $\Delta\Phi$ → G_E and G_M in the timelike region are complex
- the form factors in the spacelike region are real.
- the parameters are considered to be unified in both regions

these parameters should be real

the analytical continuation to the timelike region based on the following relation

$$Q^2 = -q^2 = q^2 e^{i\pi}.$$

E. Tomasi-Gustafsson, F. Lacroix, C. Duterte, and G. I. Gakh, Eur. Phys. J. A 24, 419 (2005)

- the intrinsic structure

$$g(q^2) = \frac{1}{(1-\gamma q^2)^2}. \quad \gamma > 1/(4m_\Lambda^2)$$

retain the analytical continuation

- the meson cloud part (included the width of vector meson)

$$\beta_{\omega_i}\frac{m_{\omega_i}^2}{m_{\omega_i}^2+Q^2} \to \beta_{\omega_i}\frac{m_{\omega_i}^2}{m_{\omega_i}^2-q^2-im_{\omega_i}\Gamma_{\omega_i}},$$

$$\beta_{\phi_i}\frac{m_{\phi_i}^2}{m_{\phi_i}^2+Q^2} \to \beta_{\phi_i}\frac{m_{\phi_i}^2}{m_{\phi_i}^2-q^2-im_{\phi_i}\Gamma_{\phi_i}},$$

are crucial for reproducing the relative phase angle

Numerical results of Λ and discussions

Fit the effective form factor in the region 2.2324 GeV $<\sqrt{s}<$ 3.08 GeV and the electromagnetic form factor ratio

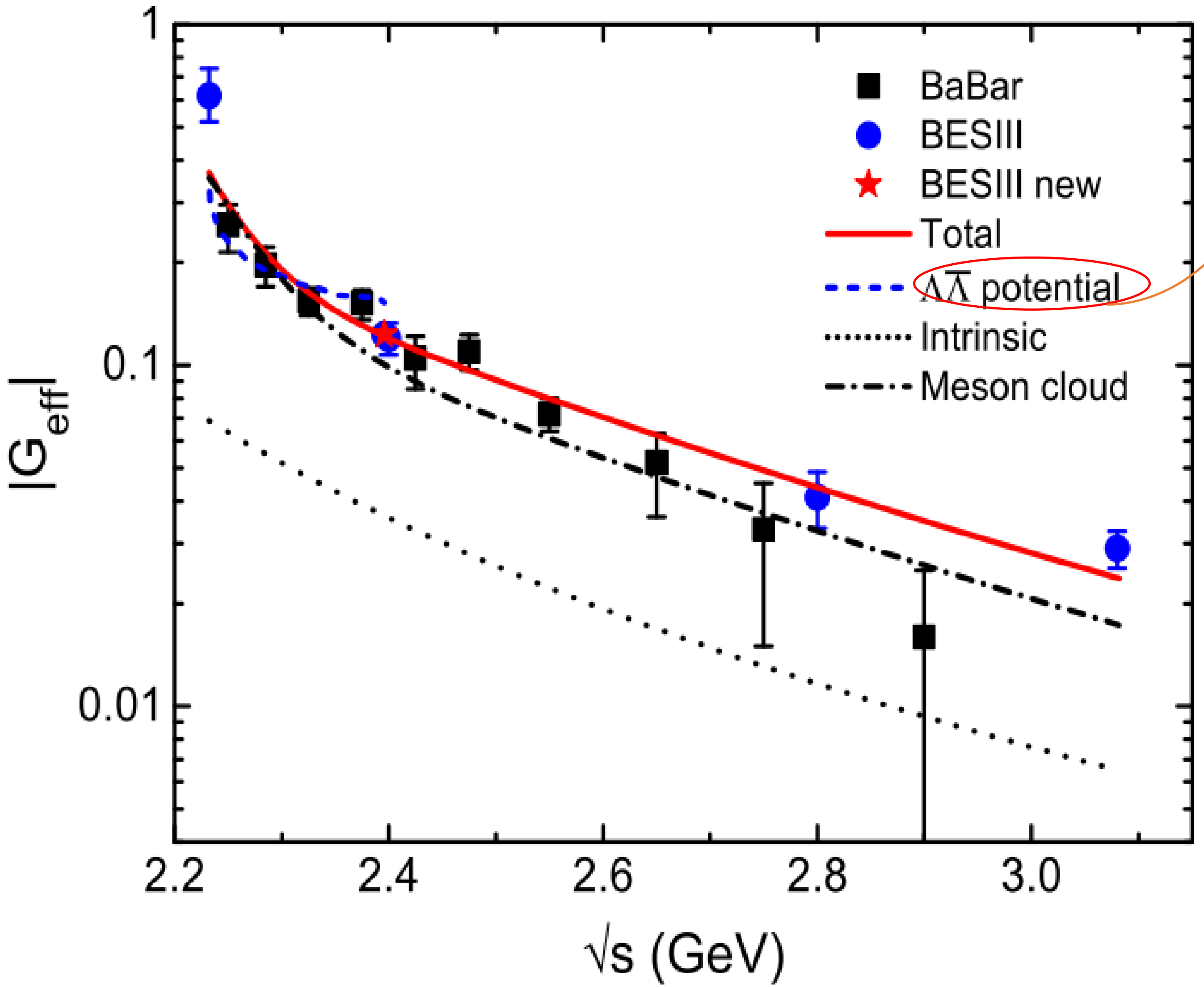
TABLE I. The masses and widths of the involved vector mesons in unit of MeV [55–58].

State	Mass	Width	State	Mass	Width
$\omega(782)$ [55]	782	8.1	$\phi(1020)$ [56]	1019	4.2
$\omega(1420)$ [57]	1418	104	$\phi(1680)$ [57]	1674	165
$\omega(1650)$ [57]	1679	121	$\phi(2170)$ [58]	2171	128

TABLE II. The parameters obtained from the combined fit.

Parameter	Value	Parameter	Value	Parameter	Value
$\beta_{\omega(782)}$	1.248	$\beta_{\omega(1420)}$	0.712	$\beta_{\omega(1650)}$	1.0129
$\beta_{\phi(1020)}$	−1.902	$\beta_{\phi(1680)}$	−0.581	$\beta_{\phi(2170)}$	−0.584
$\alpha_{\phi(1020)}$	−2.224	$\alpha_{\phi(1680)}$	2.748	$\alpha_{\phi(2170)}$	0.615

- 16 experimental data and 10 free parameters.
- Intrinsic parameter γ is 0.336 GeV^{-2}
- The pole $q = 1.725$ GeV



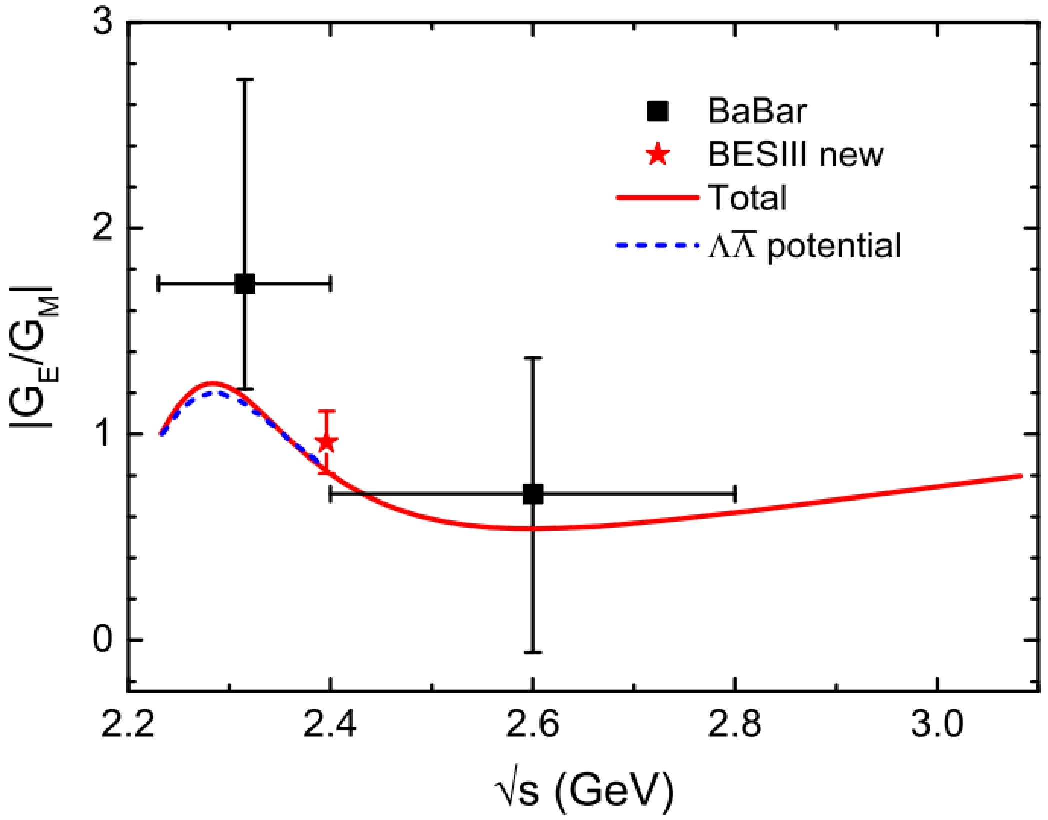
Y. Yang, D.Y. Chen, Z. Lu, Phys. Rev. D **100**(7), 073007 (2019)

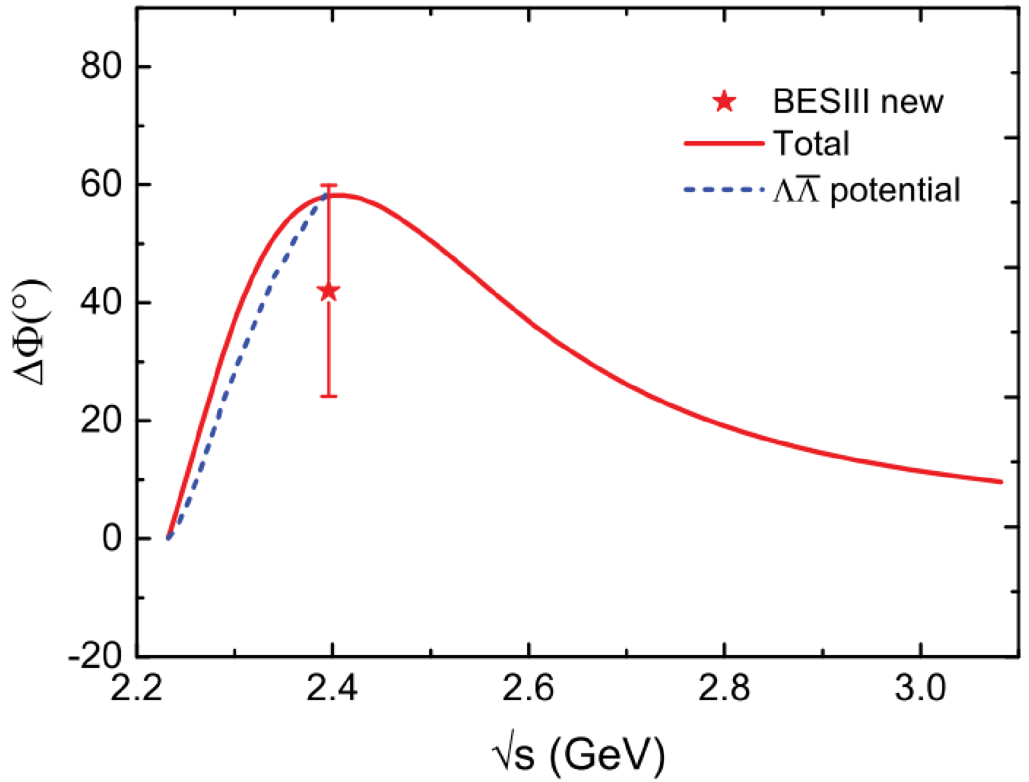
J. Haidenbauer and U. G. Meißner, Phys. Lett. B761, 456 (2016)

1. After including the resonances of ω and ϕ mesons, the fit demonstrates that our model can accurately describe the effective form factor

2. Intrinsic contribution is much smaller than the meson cloud

3. the new BESIII data (a star) is also consistent with our fit curve.





- The maximum of ratio~1.25 at $\simeq$ 2.28 GeV, which is consistent with the estimation of $\bar{\Lambda}\Lambda$ potential model

- The numerical result of relative phase is consistent with the new experimental measurement from BESIII

The spin polarized observables appearing in the reaction

$$A_y = \frac{-2m_\Lambda\sqrt{s}\sin(2\theta)\mathrm{Im}(G_M G_E^*)}{D_c - D_s\sin^2(\theta)},$$

$$A_{xz} = \frac{2m_\Lambda\sqrt{s}\sin(2\theta)\mathrm{Re}(G_M G_E^*)}{D_c - D_s\sin^2(\theta)},$$

$$A_{xx} = \frac{[D_c - D_s]\sin^2(\theta)}{D_c - D_s\sin^2(\theta)},$$

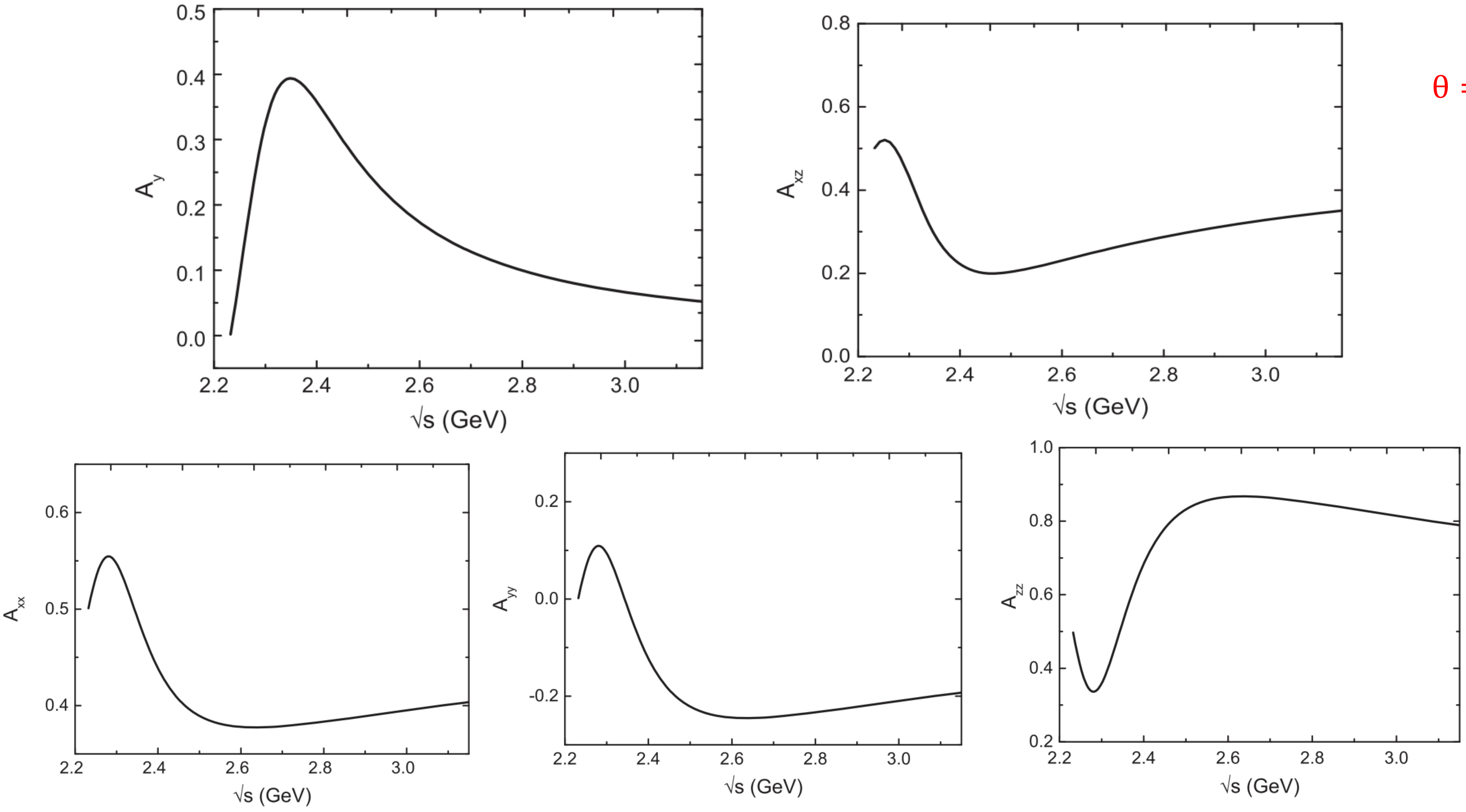
$$A_{yy} = \frac{-D_s\sin^2(\theta)}{D_c - D_s\sin^2(\theta)},$$

$$A_{zz} = \frac{[D_s\sin^2(\theta) + D_c\cos^2(\theta)]}{D_c - D_s\sin^2(\theta)},$$

$$\begin{cases} D_c = 2s|G_M|^2, \\ D_s = s|G_M|^2 - 4m_\Lambda^2|G_E|^2 \end{cases}$$

A_y and A_{xz}, depend on the relative phase

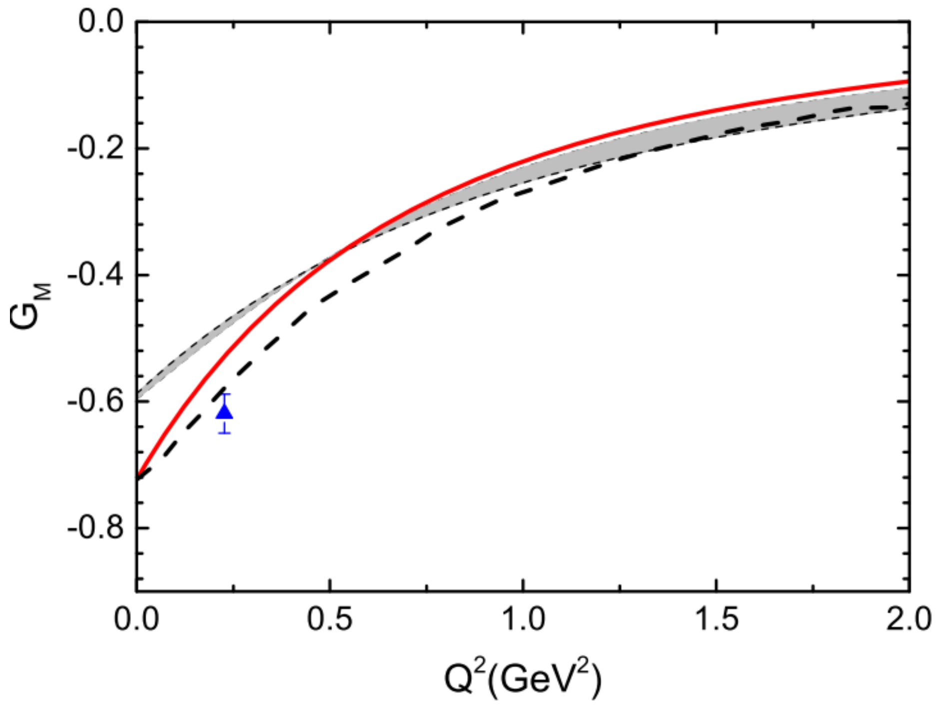
$$\mathrm{Im}(G_M G_E^*) \sim \sin(\Delta\Phi).$$

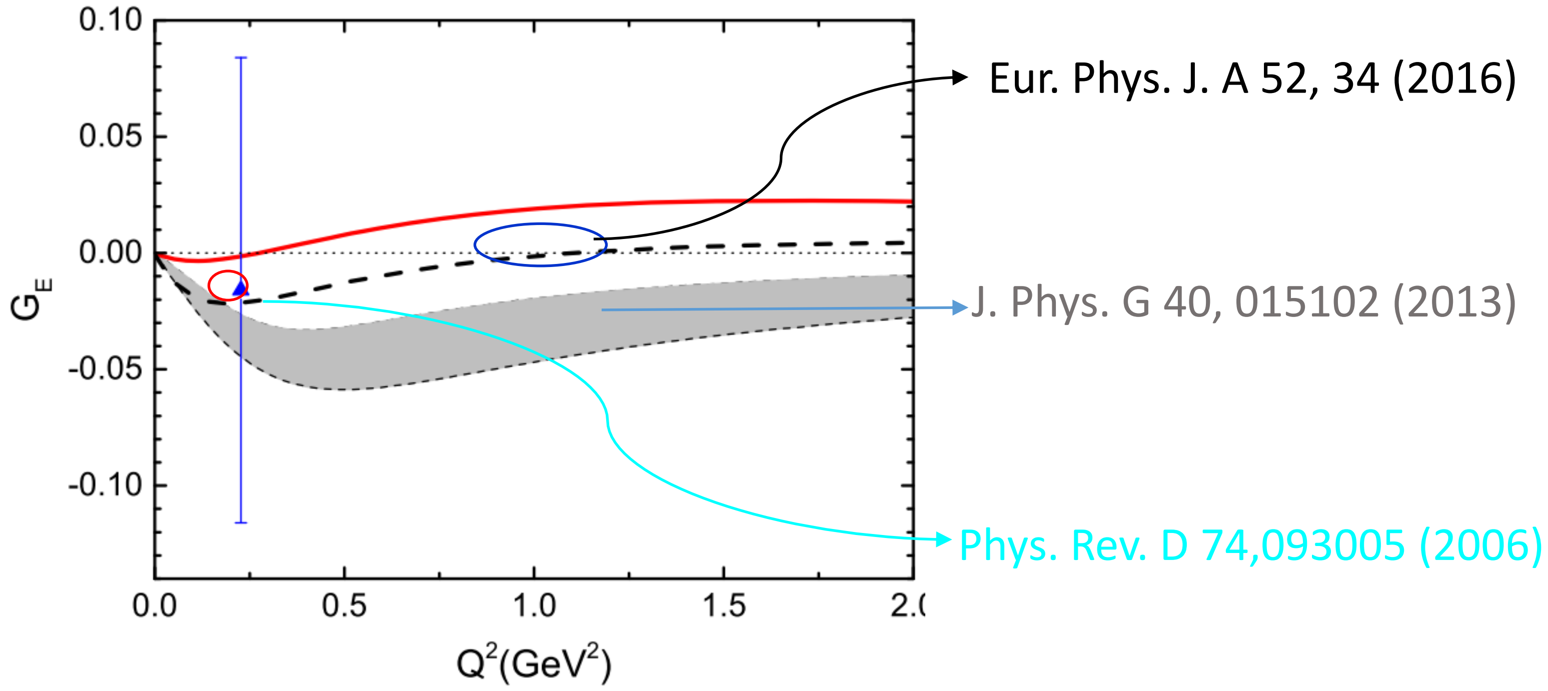


A_y
0.5
0.4
0.3
0.2
0.1
0.0
2.2
2.4
2.6
2.8
3.0
√s (GeV)
A_xz
0.8
0.6
0.4
0.2
0.0
2.2
2.4
2.6
2.8
3.0
√s (GeV)
A_xx
0.6
0.5
0.4
2.2
2.4
2.6
2.8
3.0
√s (GeV)
A_yy
0.2
0.0
-0.2
2.2
2.4
2.6
2.8
3.0
√s (GeV)
A_zz
1.0
0.8
0.6
0.4
0.2
2.2
2.4
2.6
2.8
3.0
√s (GeV)

$\theta = 45°$

Form factors in spacelike region





the magnetic and charge radius

$$\langle r^2\rangle_M = -6\frac{1}{\mu_\Lambda}\frac{dG_M}{dQ^2}\bigg|_{Q^2=0}, \quad \langle r^2\rangle_E = -6\frac{dG_E}{dQ^2}\bigg|_{Q^2=0}.$$

$$r_M = \sqrt{\langle r^2\rangle_M} = 0.42\,\text{fm}, \quad r_E = \sqrt{\langle r^2\rangle_E} = 0.11\,\text{fm}.$$

The results of EMFFs for Λ_c

- As charmed hyperon, the contribution of charmed mesons J/ψ ($c\bar{c}$) and their excitations are considered

$I^G(J^{PC}) = 0^-(1^{--})$

$$\psi(1S), \psi(2S), \psi(3770), \psi(4040), \psi(4160) \text{ and } \psi(4415).$$

- only investigate the charmed meson decay

$$\left|\frac{m^2_{\omega(\phi)}}{m^2_{\omega(\phi)} - q^2}\right| \ll \left|\frac{m^2_{J/\psi}}{m^2_{J/\psi} - q^2}\right|$$

- compare them to ω and ϕ in light of the magnitude of the mass

$$\omega(782) \to \psi(1S), \phi(1020) \to \psi(2S)$$

- is an isospin singlet

$$F_i^V \equiv 0,$$

- At $Q^2 = 0$, the EMFFs of c hyperon

$$G_E(0) = 1, \qquad G_M(0) = \mu_{\Lambda_c}$$
$$\mu_{\Lambda_c} = 1.039\ \hat{\mu}_{\Lambda_c} \text{ using } \hat{\mu}_{\Lambda_c} = \frac{M_N}{M_c}\hat{\mu}_N$$

the parameterized forms in spacelike region

$$F_1^S(Q^2) = \frac{g(Q^2)}{3}\Sigma_{i=1}^N\left[1 - \beta_{\Omega_i} - \beta_{\Phi_i} + \beta_{\Omega_i}\frac{m_{\Omega_i}^2}{m_{\Omega_i}^2 + Q^2}\right.$$
$$\left. + \beta_{\Phi_i}\frac{m_{\Phi_i}^2}{m_{\Phi_i}^2 + Q^2}\right]$$
$$F_2^S(Q^2) = \frac{g(Q^2)}{3}\Sigma_{i=1}^N\left[(\mu_{\Lambda_c} - 1 - \alpha_{\Phi_i})\frac{m_{\Omega_i}^2}{m_{\Omega_i}^2 + Q^2}\right.$$
$$\left. + \alpha_{\Phi_i}\frac{m_{\Phi_i}^2}{m_{\Phi_i}^2 + Q^2}\right]$$

- Ω_i $(i = 1,2,3)$ denotes the vector meson states $\psi(1S), \psi(3770), \psi(4160)$
- Φ_i $(i = 1,2,3)$ represents the vector meson states $\psi(2S), \psi(4040), \psi(4415)$.

State	Mass	Width	State	Mass	Width
$\psi(1S)$	3097	0.093	$\psi(2S)$	3686	0.294
$\psi(3770)$	3773	27.2	$\psi(4040)$	4039	80
$\psi(4160)$	4191	70	$\psi(4415)$	4421	62

4.58 GeV $<\sqrt{s}<$ 5.39 GeV

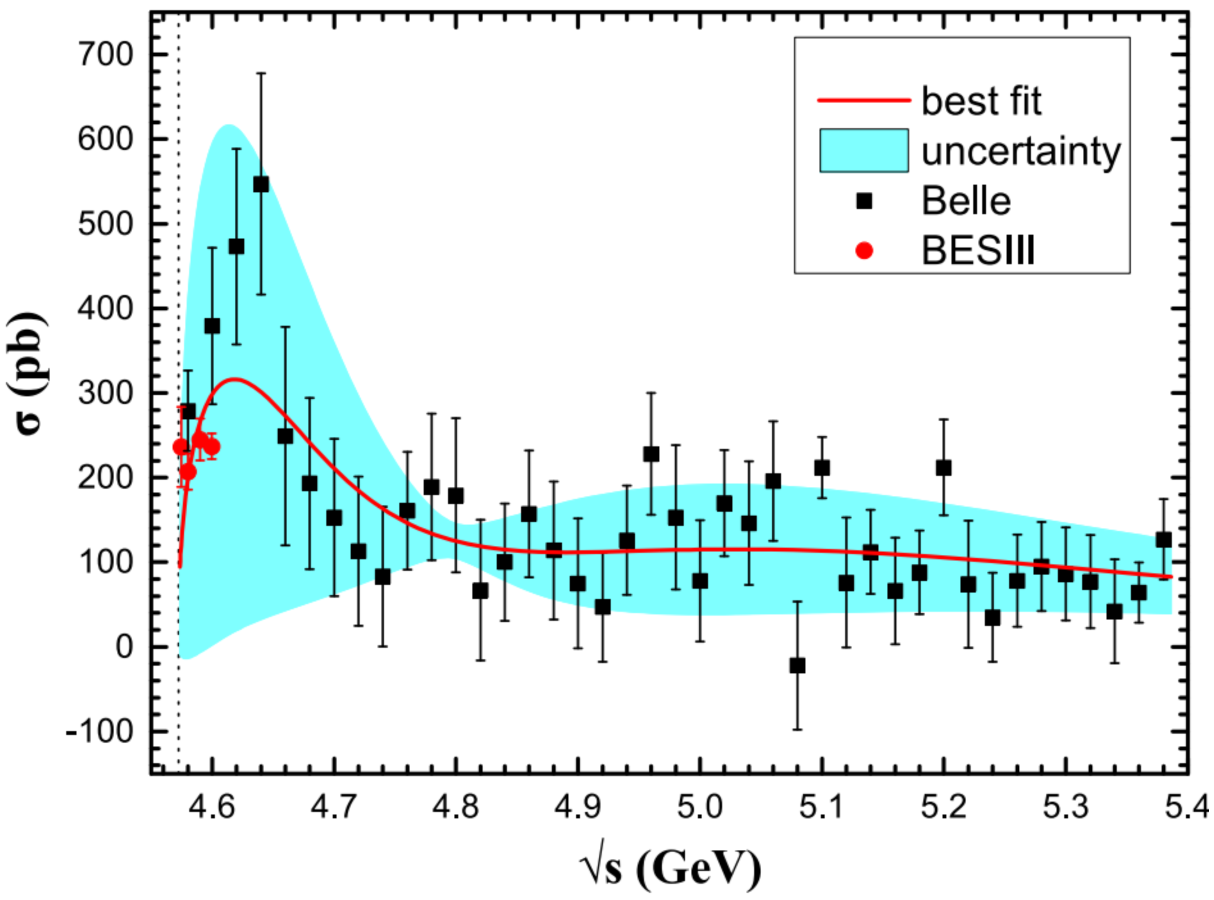
$$\Delta\chi^2 = 1.436$$

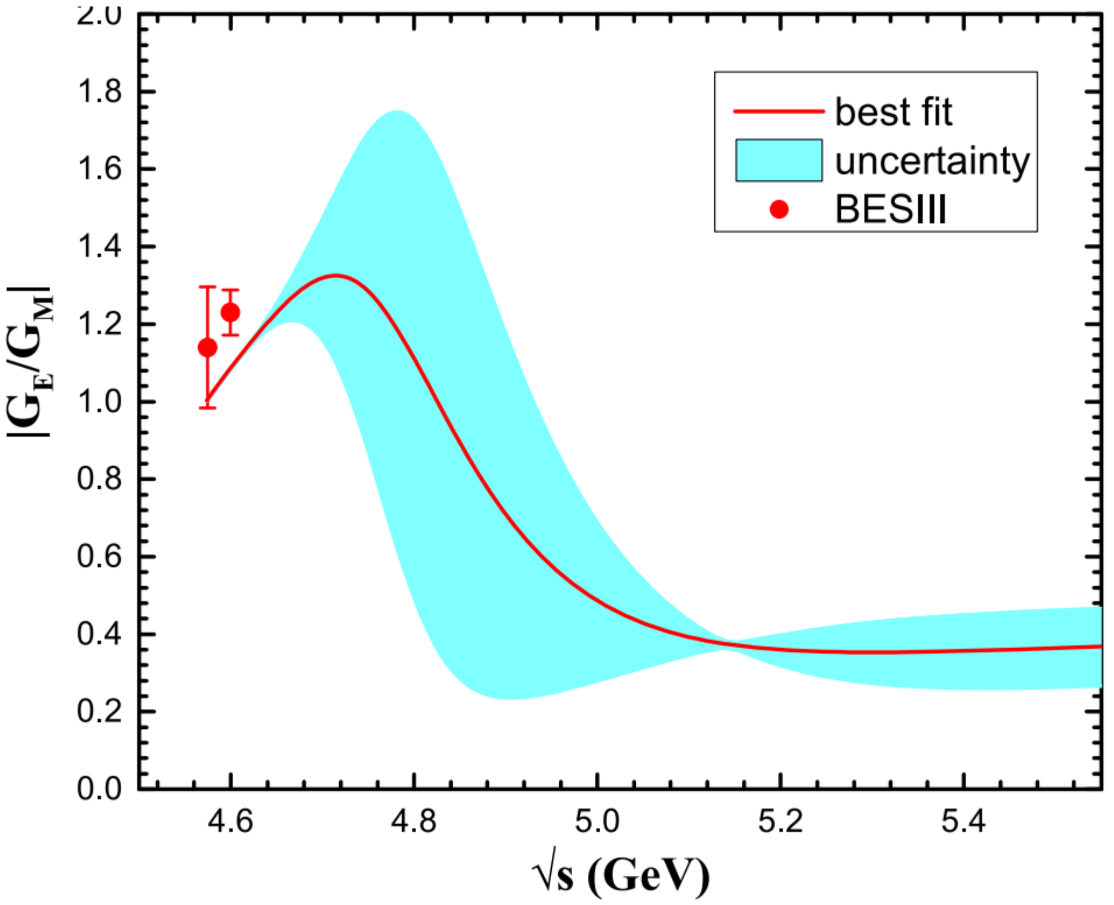
Table 2 The values and the errors of the parameters obtained from the fit

Parameter	Value	Parameter	Value	Parameter	Value
$\beta_{\psi(1S)}$	7.9636 ± 0.3507	$\beta_{\psi(3770)}$	-2.1338 ± 0.1746	$\beta_{\psi(4160)}$	-1.2008 ± 0.1092
$\beta_{\psi(2S)}$	-2.1857 ± 0.1839	$\beta_{\psi(4040)}$	-1.6917 ± 0.1529	$\beta_{\psi(4415)}$	-1.0592 ± 0.0597
$\alpha_{\psi(2S)}$	-4.5310 ± 0.1107	$\alpha_{\psi(4040)}$	23.2260 ± 0.7772	$\alpha_{\psi(4415)}$	0.6651 ± 0.0934

$$\gamma = 0.0899 \pm 0.0017\ \mathrm{GeV}^{-2}$$

The pole q = 3.335 GeV

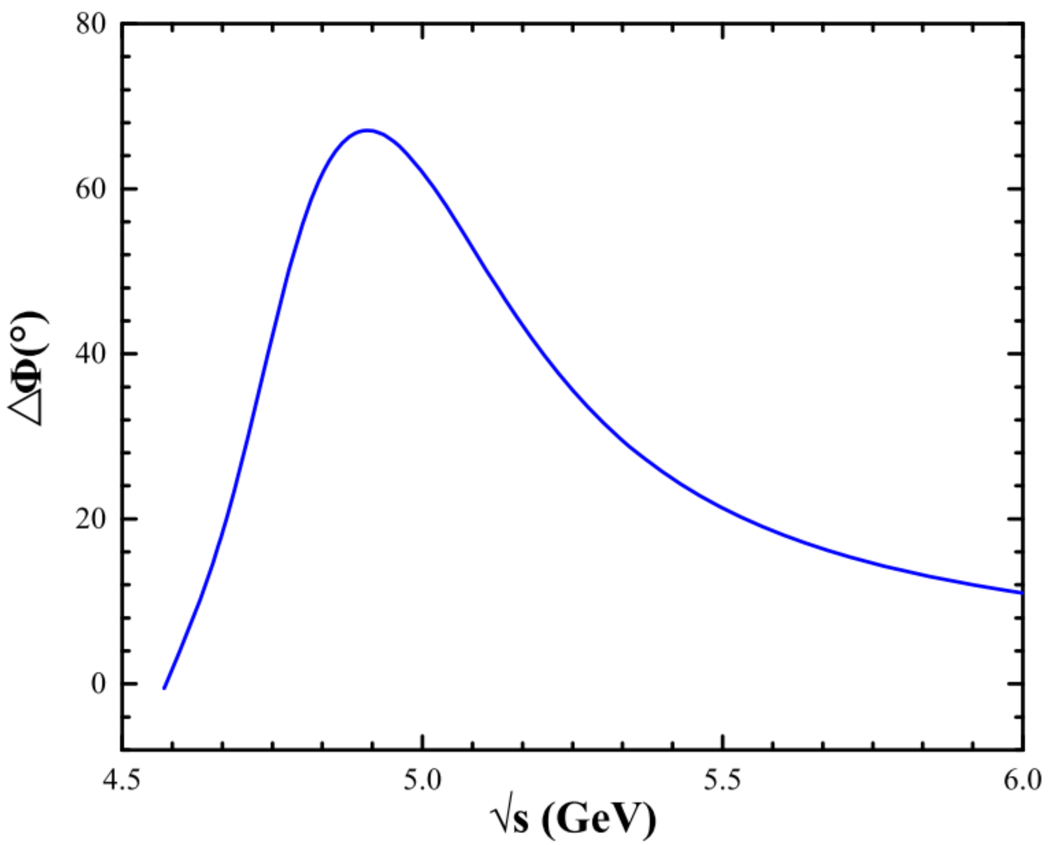


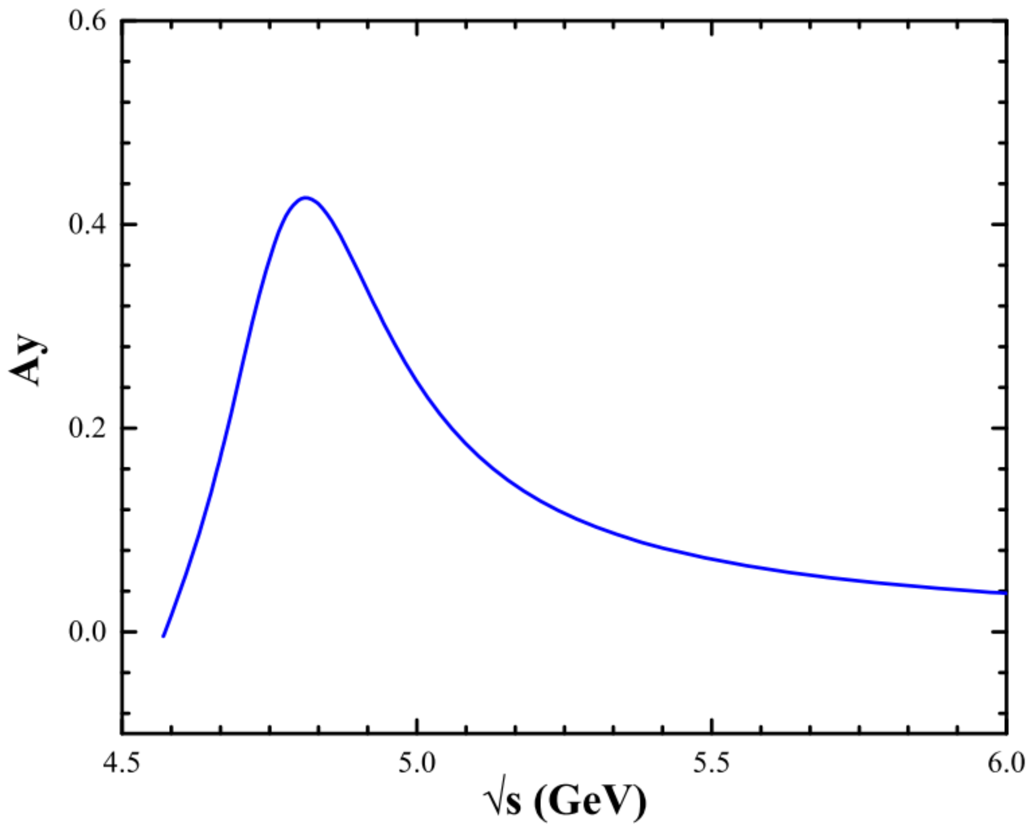


J. Wan, Y. Yang and Z. Lu, Eur. Phys. J. Plus 136, 949 (2021)

- We also provide the theoretical band corresponding to the uncertainty of parameters obtained from the errors of the data
- qualitatively predict the behavior of ratio

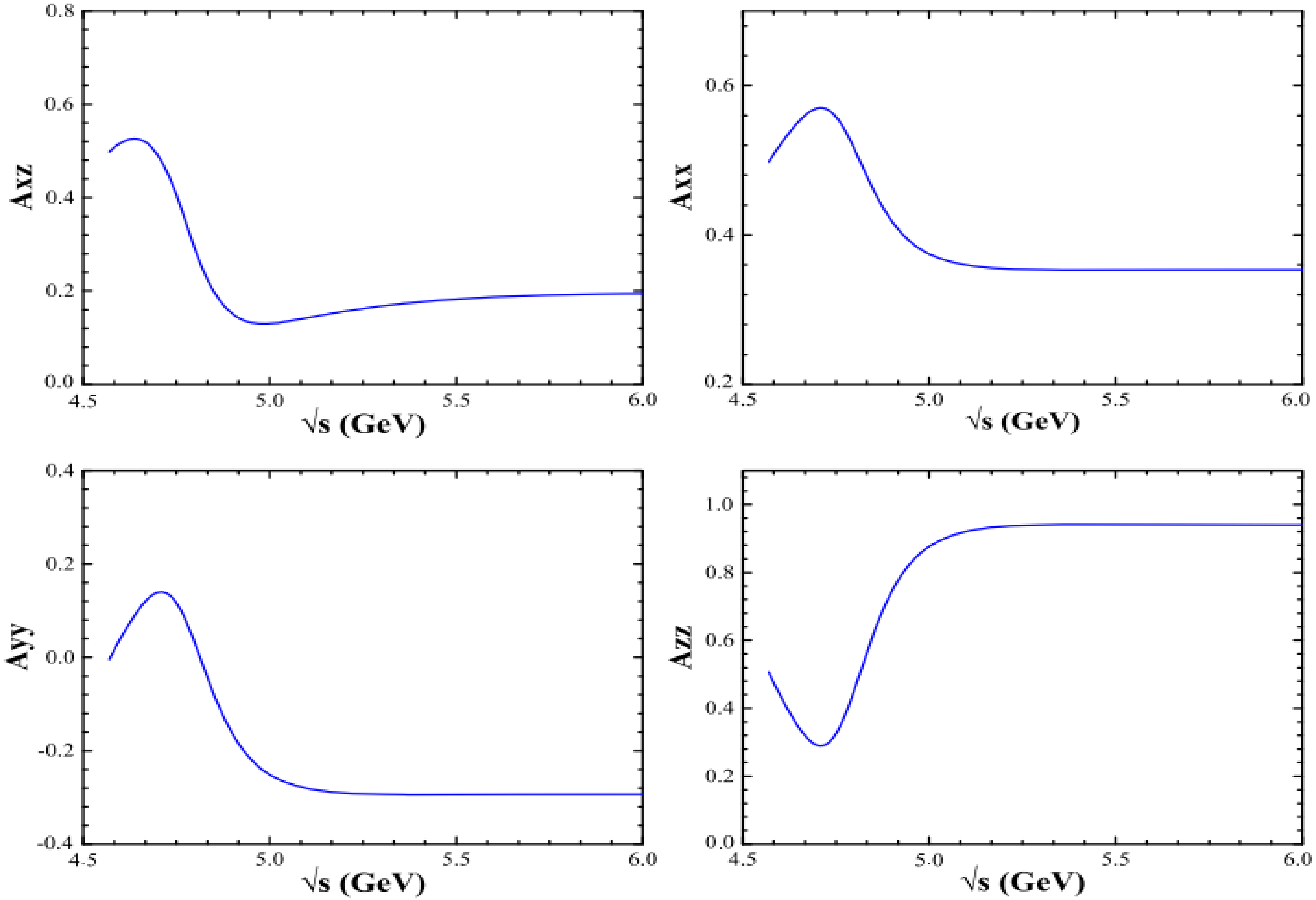
Prediction of the relative phase





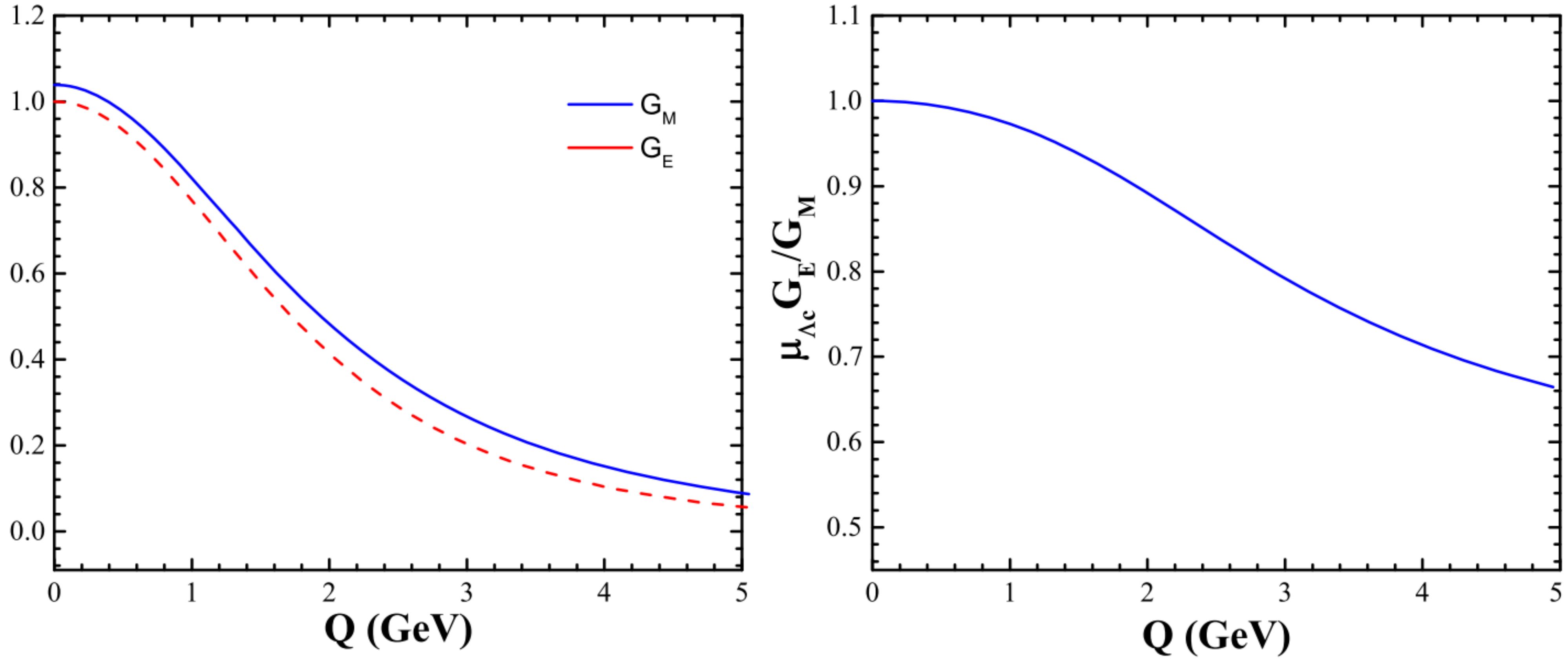
$\theta = 45°$

- One should note that $\Delta\Phi = 0$ at the threshold due to $G_E = G_M$ at $s = 4M_c^2$
- A_y is similar to that of the relative phase



The measurement of these quantities could be used to verify the validity of the model

Form factors in space-like region



- G_E and G_M are rather similar
- The ratio is similar to the case of the proton

Summary

- The inclusion of these mesons and their widths can naturally produce the complex structure of the time-like EMFFs

- The enhancement effect near the threshold of the hyperon pair can be described in the VMD model

- The vector resonances (with width) are crucial in depicting the experimental data

- The model can provide a description for the relative phase

Thanks！