

# Lattice QCD study on charmonia radiative decay

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BESIII 实验上粲强子、QCD 及新物理研讨会  
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- Introduction
  - Experiments(BESIII)
  - Theories
- Charmonium radiative decay
  - $\eta_c \rightarrow 2\gamma$
  - $\chi_{c0} \rightarrow 2\gamma$
  - $J/\psi \rightarrow 3\gamma$
  - $J/\psi \rightarrow \gamma\nu\bar{\nu}$
- Conclusion and outlook

## Why charmonium decay ?

- Test the interplay of pert and non-pert QCD
  - intermediate energy scale
- More possibilities for the search of new physics
  - invisible decay of charmonium
- BESIII experiment

For more charm physics on lattice, see [Liu Zhaofeng's talk 14:00,08-22](#)

- Results

	$\text{Br} \times 10^5$	Note
CLEO	$1.4_{-0.5}^{+0.7} \pm 0.3$	PRL 101,101801(2008)
BESIII	$2.7 \pm 0.8 \pm 0.6$	PRD 87,032003(2013)
World average	$1.9_{-0.6}^{+0.7}$	PDG-aver
Global fit	$1.61 \pm 0.12$	PDG-fit

- Updated: 5.15(35) keV  $\Rightarrow$  5.43(+0.41-0.38) keV

- Datas before 1995 unused
- $J/\psi \rightarrow \gamma\eta_c$  unused
- 97 datas, 29 parameters,  $\chi^2 = 86$

H-P.Wang and C-Z.Yuan, "New puzzle in charmonium decays"  
CPC46,071001(2022)

# Experiments: $J/\psi \rightarrow 3\gamma$

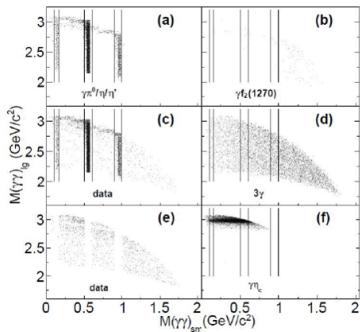
- Results:

Exp	$\mathcal{B} \times 10^5$	$\delta\mathcal{B} \times 10^5$	Refs
Crystal Ball	$< 5.5$		PRL 44,712(1980)
CLEO	1.2	$0.3 \pm 0.2$	PRL 101,101801(2008)
BESIII	1.13	$0.18 \pm 0.2$	PRD 87,032003(2013)

- Difficulties:

- Multi-intermediate states
- Poor knowledge of matrix element

- Opportunities: BESIII with  $10^{10} J/\psi$



BESIII  $\sim 3 \times 10^7 J/\psi$

# Experiments: $J/\psi \rightarrow \gamma + X$

- BSM predict CP-odd pseudoscalar Higgs boson  $A_0$ , which can be coupled with SM particle
- Dark matter from collider

CLEO, PRD. **81**, 091101 (2010)

$$< 4.3 \times 10^{-6}$$

BESIII, PRD. **101**, 112005 (2020)

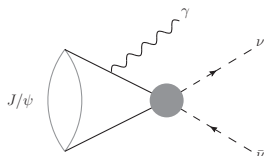
$$< 7.0 \times 10^{-7}$$

- SM background:  $J/\psi \rightarrow \gamma + \nu\bar{\nu}$

- Perturbative calculation

D.-N. Gao, Phys. Rev. D. **90**, 077501 (2014)

- No lattice study, YM et al, in progress



- $\eta_c \rightarrow 2\gamma$

	$\Gamma(\text{keV})$	Note
NRQCD	9.9 ~ 10.6	PRL <b>119</b> ,252001(2017)
DSE	6.32 ~ 6.39	PRD <b>95</b> ,016010(2017)
Lattice	6.04(68)	CLQCD(2020)
PDG-fit	<b>5.15(35)</b>	<b>PDG</b>
Lattice	<b>6.57(15)(8)</b>	YM et al,2109.09381

- $J/\psi \rightarrow 3\gamma$

- (i) NRQCD+model inputs

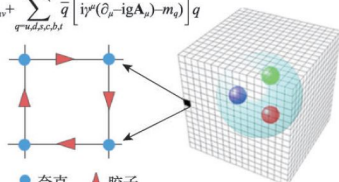
F. Feng, Y. Jia and W-L. Sang, PRD 87,051501(2013)

- (ii) Lattice QCD

YM,C.Liu, K-L.Zhang Phys.Rev.D.**102**,054506(2020)

Precision test for QCD  $\Leftarrow$  High precision lattice simulation  $\Leftarrow$   
 new lattice method+systematic analysis

- K.G.Wilson, PRD 10, 2445(1974)
- Idea: put QCD on 4-d lattice
  - Quark field  $\rightarrow$  site
  - Gauge field  $\rightarrow$  link

$$\mathcal{L} = -\frac{1}{4} \mathbf{F}^{\mu\nu} \mathbf{F}_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q} \left[ i \gamma^\mu (\partial_\mu - i g \mathbf{A}_\mu) - m_q \right] q$$


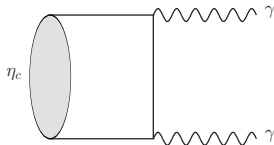
• 夸克    ▲ 胶子

格点量子色动力学在中国 [J]. 现代物理知识, 2020, 32(01): 36-44.

- “格点场论既是世界观 (非微扰的定义) 又是方法论 (非微扰的计算)”
  - 刘川, 《格点量子色动力学导论》
  - **世界观**  $\Rightarrow$  Non-perturbative definition of QCD,  
natural ultraviolet and infrared truncation
  - **方法论**  $\Rightarrow$  Non-perturbative calculation of QCD,  
Monte-Carlo simulation



$$\eta_c \rightarrow 2\gamma$$



- Amplitude:

$$\begin{aligned}\mathcal{M} &= e^2 \epsilon_\mu(p) \epsilon_\nu(p') H_{\mu\nu}(p, q) \\ H_{\mu\nu}(p, q) &= \int d^4x e^{-ipx} \mathcal{H}_{\mu\nu}(x, q), \quad \mathcal{H}_{\mu\nu} = \langle 0 | \text{Tr}[J_\mu(x) J_\nu(0)] | \eta_c(q) \rangle\end{aligned}$$

- Form factor:

$$H_{\mu\nu}(p, q) = \epsilon_{\mu\nu\alpha\beta} p_\alpha q_\beta F(p^2)$$

- Decay width:

$$\Gamma_{\eta_c \gamma\gamma} = \alpha_{\text{em}}^2 \frac{\pi}{4} m_{\eta_c}^3 |F_{\eta_c \gamma\gamma}|^2, \quad F_{\eta_c \gamma\gamma} = F(0)$$

- Off-shell form factors by projecting discrete momentum  $\vec{p} = \frac{2\pi\vec{n}}{L}$

$$F(p^2) \xrightarrow{\text{Cont.Limit}} F(0), \quad |\vec{p}| = \frac{m_{\eta_c}}{2}$$

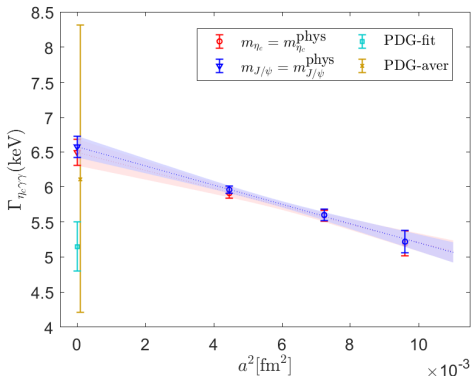
Y.Chen *et al*(CLQCD), CPC44,083108(2020)

- Additional computation costs and extrapolation error.
  - Lattice discretization effect: only two lattice spacings used
  - Excited-state contamination: without considering the excited-state effect on the lattice
- Scalar function method: calculate on-shell form factor directly

$$F_{\eta_c\gamma\gamma}(\Delta t) = -\frac{1}{2m_{\eta_c}} \int dt e^{m_{\eta_c} t/2} \int d^3\vec{x} \frac{j_1(|\vec{p}|, |\vec{x}|)}{|\vec{p}| |\vec{x}|} \epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}_{\mu\nu}(x, \Delta t)$$

# Results

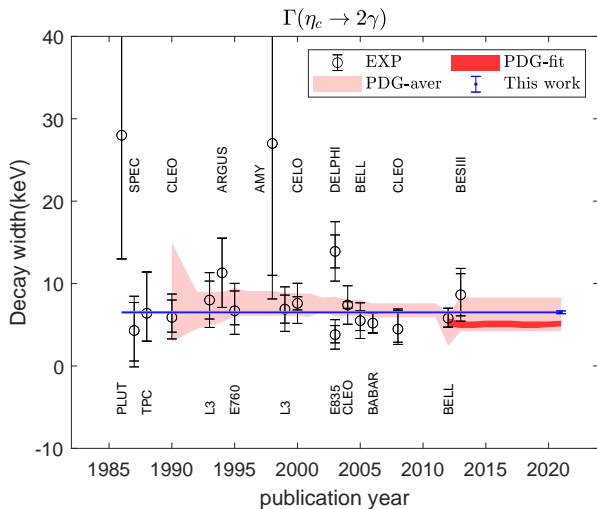
- NRQCD(NLO)  $\sim 6.2$   
K.-T.Chao *et al*, PRD**56**,368(1997)
- NRQCD(NNLO)  $\sim 10$   
F.Feng *et al*, PRL**119**,252002(2017)
- DSE  $\sim 6.4$   
J.Chen *et al*, PRD**95**,016010(2017)
- CLQCD  $\sim 6.0(7)$   
Y.Chen *et al*, CPC**44**,083108(2020)



$$\Gamma(\eta_c \rightarrow 2\gamma) = \begin{cases} 6.57(15)(8) \text{ keV} & \text{YM } et al, 2109.09381 \\ 5.15(35) \text{ keV} & \text{PDG-fit} \\ 6.11_{-1.9}^{+2.2} \text{ keV} & \text{PDG-aver} \end{cases}$$

- HPQCD-preliminary,  $\Gamma_{\eta_c \rightarrow 2\gamma} = 6.78(13) \text{ keV}$ , Lattice 2022

# Lattice & Experiments



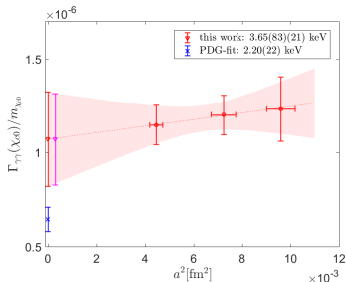
$$\chi_{c0} \rightarrow 2\gamma$$

- Overviews

Method	$\Gamma(\text{keV})$	$\delta\Gamma(\text{keV})$	Refs
CLEO	2.36	$0.35 \pm 0.22$	PRD 78,091501(2008)
BESIII	2.03	$0.08 \pm 0.14$	PRD 96,092007(2013)
Dudek	2.41	$0.58 \pm 0.86$	PRL 97,172001(2006)
CLQCD	3.72	$0.76 \pm 1.0$	CPC 44,083108(2020)

- Notes:

- (a) Continuous extrapolation
- (b) Improved lattice method
- (c) Twm Ens  $\rightarrow$  parity violation  
 $\Rightarrow$  GEVP  $\rightarrow$  statistical error



- Our result:  $\Gamma_{\gamma\gamma}(\chi_{c0}) = 3.65(83)_{\text{stat}}(21)_{\text{lat.syst}}(66)_{\text{syst}} \text{ keV}$

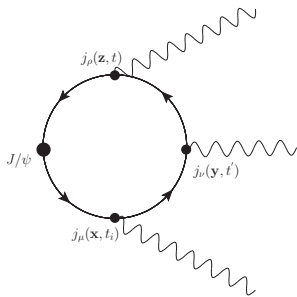
Z.-H.Zou, Y.M, C.Liu, CPC 46,053102(2022)

- Decay width

$$\Gamma_{3\gamma} = \frac{m_{J/\psi}}{1536\pi^3} \int_0^1 dx \int_{1-x}^1 dy \mathcal{T}(x, y)$$

$$\mathcal{T} = \frac{1}{3} \sum_{\mu\nu\rho\alpha} |\mathcal{M}_{\mu\nu\rho\alpha}|^2$$

$$x \equiv 1 - 2q_2 \cdot q_3/m^2, y \equiv 1 - 2q_1 \cdot q_2/m^2$$



- Traditional:**  $\mathcal{M}_{\mu\nu\rho\alpha}(q_1, q_2, q_3)$

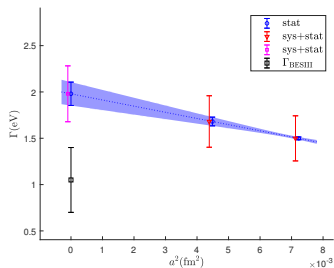
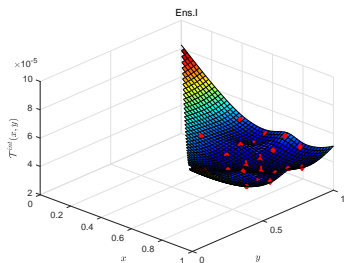
$$\mathcal{T}(x, y, Q_1^2, Q_2^2, Q_3^2) = \mathcal{T}(x, y) + \text{const} \times \sum_i Q_i^2$$

- Scalar function method**

YM et al, in progress

$$\mathcal{M}^{\mu\nu\rho\alpha}(q_1, q_2, q_3) = \sum_{[123]} \sum_{i=1}^3 F_i(s_1, s_2, s_3) \Pi_i^{[\mu\nu\rho]\alpha}(q_1, q_2, q_3)$$

# Traditional calculation



YM, C.Liu, K-L.Zhang Phys.Rev.D.102,054506(2020)

- Naive continuum extrapolation;
- The intermediate contribution  $J/\psi \rightarrow \gamma \eta_c \rightarrow 3\gamma$ ;
- Excited-state contamination;
- Systematical error by the region without data covered;

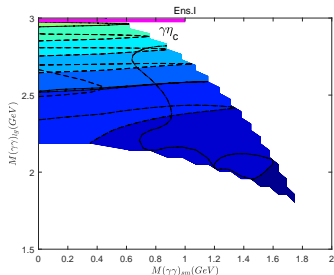
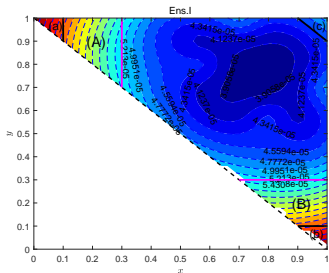
# Dalitz analysis

- Dalitz variables:

$$\frac{M(\gamma\gamma)_{lg/sm}}{m_{J/\psi}} = \frac{\max}{\min} \left\{ \sqrt{1-x}, \sqrt{1-y}, \sqrt{x+y-1} \right\}$$

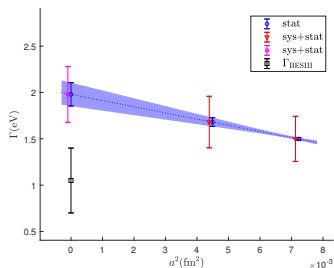
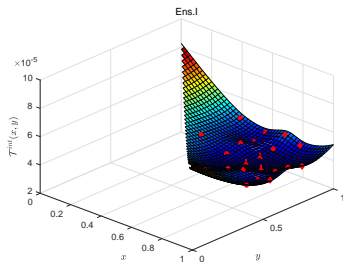
- Dalitz plot is the direct observable for the experiments.
- Bands in Dalitz plot indicate the intermediate two-body states.

- Dalitz plot





# Toward a systematic study



YM,C.Liu, K-L.Zhang Phys.Rev.D.102,054506(2020)

- Naive continuum extrapolation;  Three lattice spacings
- The intermediate contribution  $J/\psi \rightarrow \gamma\eta_c \rightarrow 3\gamma$ ;
- Excited-state contamination;
- Systematical error by the region without data covered;  Scalar function method

## • Summary

- Lattice calculation on  $\eta_c/\chi_{c0} \rightarrow 2\gamma$  with new method
- An explorative calculation on  $J/\psi \rightarrow 3\gamma$  by traditional way

## • Outlook

- Systematic study on  $J/\psi \rightarrow \gamma\nu\bar{\nu}$  and  $J/\psi \rightarrow 3\gamma$
- QCD precision test + new physics in charmonium physics

BESIII,etc & Lattice QCD & Phenomenological

特别致谢： BESIII 项目组及实验同行