

非相对论量子色动力学(NRQCD)有效理论

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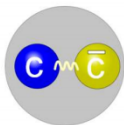


Table 1: Quarkonium energy scales

	$c\bar{c}$	$b\bar{b}$	$t\bar{t}$
M	1.5 GeV	4.7 GeV	180 GeV
Mv	0.9 GeV	1.5 GeV	16 GeV
Mv^2	0.5 GeV	0.5 GeV	1.5 GeV

Charmonia: $v^2/c^2 \sim 0.3$

Bottomonia: $v^2/c^2 \sim 0.1$

Introduction to the NRQCD factorization approach to heavy quarkonium

#1

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42 citations

1 NRQCD的基本概念

- NRQCD的能量标度
- NRQCD的拉矢量
- 速度标度率
- 四费米子算符

2 重夸克偶素的产生和湮灭

- 重夸克偶素到轻强子的衰变
- 重夸克偶素的电磁湮灭

3 赝标量重夸克偶素衰变到双光子

4 讨论

- M (表征重夸克的产生和湮灭),
- Mv (v 在静止框架下, 根据 $\lambda = \frac{\hbar}{Mv}$, 则 $\frac{1}{Mv}$ 表征重夸克偶素的大小),
- Mv^2 (表征重夸克偶素相邻径向激发态或角动量激发态的能级差),

$$M \gg Mv \gg Mv^2$$

描述带有重夸克的拉矢量可写为:

$$\mathcal{L}_{QCD} = \mathcal{L}_{light} + \bar{\psi}(i\gamma^\mu D_\mu - M)\psi,$$

NRQCD可通过QCD的拉矢量做两步修改得到:

1.截断

2.FWT变换

$$\mathcal{L}_{NRQCD} = \mathcal{L}_{light} + \mathcal{L}_{heavy} + \delta\mathcal{L},$$

$$\mathcal{L}_{light} = -\frac{1}{2} \text{Tr} G_{\mu\nu} G^{\mu\nu} + \Sigma \bar{q} i \not{D} q,$$

$$\psi^\dagger \gamma^0 (D_0 - m) \psi$$

$$\psi \rightarrow e^{-i\frac{\vec{p} \cdot \vec{r}}{2M}} \psi, \quad \psi^\dagger \rightarrow \psi^\dagger e^{i\frac{\vec{p} \cdot \vec{r}}{2M}} = \psi^\dagger e^{i\vec{p} \cdot \vec{r}}$$

$$\psi^\dagger e^{-i\frac{\vec{p} \cdot \vec{r}}{2M}} (i\vec{D} - \vec{D} \cdot \vec{\tau} - m) e^{-i\frac{\vec{p} \cdot \vec{r}}{2M}} \psi$$

$$= \psi^\dagger (i\vec{D} e^{-i\frac{\vec{p} \cdot \vec{r}}{2M}} i\vec{D} e^{i\frac{\vec{p} \cdot \vec{r}}{2M}} - i\vec{D} e^{-i\frac{\vec{p} \cdot \vec{r}}{2M}} \vec{D} e^{i\frac{\vec{p} \cdot \vec{r}}{2M}} - i\vec{D} e^{-i\frac{\vec{p} \cdot \vec{r}}{2M}} m e^{i\frac{\vec{p} \cdot \vec{r}}{2M}}) \psi$$

$$= \psi^\dagger (i\vec{D} - i\vec{D} e^{-i\frac{\vec{p} \cdot \vec{r}}{2M}} \vec{D} e^{i\frac{\vec{p} \cdot \vec{r}}{2M}} - i\vec{D} m e^{-i\frac{\vec{p} \cdot \vec{r}}{2M}}) \psi \quad (\text{因为 } e^{i\vec{p} \cdot \vec{r}} e^{-i\vec{p} \cdot \vec{r}} = 1 + i\vec{p} \cdot \vec{r} + \frac{(\vec{p} \cdot \vec{r})^2}{2!} + \dots)$$

$$= \psi^\dagger (i\vec{D} - i\vec{D} (1 - i\frac{\vec{p} \cdot \vec{r}}{2M} + \frac{1}{2} (\frac{\vec{p} \cdot \vec{r}}{2M})^2) \vec{D} (1 - i\frac{\vec{p} \cdot \vec{r}}{2M} + \frac{1}{2} (\frac{\vec{p} \cdot \vec{r}}{2M})^2) + \frac{1}{2} (\frac{\vec{p} \cdot \vec{r}}{2M})^2 - i\vec{D} m (1 - i\frac{\vec{p} \cdot \vec{r}}{2M} + \frac{1}{2} (\frac{\vec{p} \cdot \vec{r}}{2M})^2) + \frac{1}{2} (\frac{\vec{p} \cdot \vec{r}}{2M})^2 + \frac{1}{2} (\frac{\vec{p} \cdot \vec{r}}{2M})^2) \psi$$

① 先计算不含导数的结果

② 再计算导数项

$$\textcircled{1} \psi^\dagger (i\vec{D} - i\vec{D} \vec{D} \cdot \vec{\tau} - i\vec{D} (-\frac{\vec{p} \cdot \vec{r}}{2M}) \vec{D} - i\vec{D} \vec{D} \cdot \vec{\tau} (-\frac{\vec{p} \cdot \vec{r}}{2M}) - i\vec{D} m + i\vec{D} \vec{D} \cdot \vec{\tau} + \frac{(\vec{p} \cdot \vec{r})^2}{2M}) \psi$$

$$= \psi^\dagger (i\vec{D} - m - \frac{\vec{D} \cdot \vec{D}}{2M}) \psi$$

$$= \psi^\dagger \begin{pmatrix} iD_0 & 0 \\ 0 & iD_0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \psi$$

$$+ \begin{pmatrix} \frac{\vec{D}^2}{2M} & 0 \\ 0 & -\frac{\vec{D}^2}{2M} \end{pmatrix} + \begin{pmatrix} \frac{\vec{D} \cdot \vec{D}}{2M} & 0 \\ 0 & -\frac{\vec{D} \cdot \vec{D}}{2M} \end{pmatrix} \psi$$

$$\begin{aligned} \vec{r} \cdot \vec{D}^2 &= \vec{r} \cdot \vec{D} \vec{D} \cdot \vec{D} \\ &= \frac{1}{2} (\vec{r} \cdot \vec{D}) \vec{D} \cdot \vec{D} + \frac{1}{2} \vec{D} \cdot \vec{D} (\vec{r} \cdot \vec{D}) \\ &= \vec{D} \cdot \vec{D} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{aligned}$$

$$\text{取 Dirac 表象} \\ \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$$

$$\begin{pmatrix} \psi \\ \chi \end{pmatrix}^\dagger \begin{pmatrix} -m + iD_0 + \frac{\vec{D}^2}{2M} & 0 \\ 0 & m + iD_0 - \frac{\vec{D}^2}{2M} \end{pmatrix} \begin{pmatrix} \psi \\ \chi \end{pmatrix}$$

$$\psi^\dagger (-m + iD_0 + \frac{\vec{D}^2}{2M}) \psi \quad \psi \rightarrow e^{-i\vec{p} \cdot \vec{r}} \psi$$

$$\Rightarrow \psi^\dagger e^{i\vec{p} \cdot \vec{r}} (-m + iD_0 + \frac{\vec{D}^2}{2M}) e^{-i\vec{p} \cdot \vec{r}} \psi$$

$$= \psi^\dagger e^{i\vec{p} \cdot \vec{r}} e^{-i\vec{p} \cdot \vec{r}} (-m + iD_0 + \vec{p} \cdot \vec{p} + \frac{\vec{D}^2}{2M}) \psi$$

$$= \psi^\dagger (iD_0 + \frac{\vec{D}^2}{2M}) \psi$$

下同同理。

$$\mathcal{L}_{\text{heavy}} = \psi^\dagger (iD_0 + \frac{\vec{D}^2}{2M}) \psi + \chi^\dagger (iD_0 - \frac{\vec{D}^2}{2M}) \chi,$$

$$\begin{aligned} \delta\mathcal{L} = & \frac{c_1}{8M^3}\psi^\dagger(\mathbf{D}^2)^2\psi + \frac{c_2}{8M^2}\psi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\psi \\ & + \frac{c_3}{8M^2}\psi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\psi + \frac{c_4}{2M}\psi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\psi \\ & + \text{charge conjugate terms,} \end{aligned}$$

Table 3: Estimates of the magnitudes of NRQCD operators for matrix elements between heavy-quarkonium states.

Operator	Estimate
ψ	$(Mv)^{3/2}$
χ	$(Mv)^{3/2}$
D_0 (acting on ψ or χ)	Mv^2
$\mathbf{D}$	Mv
$g\mathbf{E}$	M^2v^3
$g\mathbf{B}$	M^2v^4
gA_0 (in Coulomb gauge)	Mv^2
$g\mathbf{A}$ (in Coulomb gauge)	Mv^3

由场的归一化关系 $\int d^3x \psi^\dagger(x) \psi(x) = 1$ 可得, $\psi^2 \sim \int \frac{1}{d^3x}$, 又因为 $\lambda = \frac{\hbar}{Mv}$, 得到尺度大小 $\sim \frac{1}{p}$,

即 $\int d^3x \sim \frac{1}{p^3}$, 因此 $\psi^\dagger(x) \psi(x) \sim p^3$, 则 $\psi \sim p^{3/2} \sim (Mv)^{3/2}$

$$\delta\mathcal{L}_{4\text{-fermion}} = \sum \frac{\text{Im}(f_n(\Lambda))}{M^{d_n-4}} \mathcal{O}_n(\Lambda)$$

$\mathcal{O}_n$ 是所有可能的四费米子算符, d_n 是算符 $\mathcal{O}_n$ 的量纲, M^{d_n-4} 的引入保证 f_n 是无量纲的。

下面只给出S波和P波低阶(按照速度标度率)的几个算符的形式。对于S波, 最低阶算符的量纲为六, 四费米相互作用项为:

$$\begin{aligned} (\delta\mathcal{L}_{4\text{-fermion}})_{d=6} = & \frac{f_1(^1S_0)}{M^2} \mathcal{O}_1(^1S_0) + \frac{f_1(^3S_1)}{M^2} \mathcal{O}_1(^3S_1) \\ & + \frac{f_8(^1S_0)}{M^2} \mathcal{O}_8(^1S_0) + \frac{f_8(^3S_1)}{M^2} \mathcal{O}_8(^3S_1) \end{aligned}$$

其中四费米算符用夸克场和反夸克场显式写出为:

$$\mathcal{O}_1(^1S_0) = \psi^+ \chi \chi^+ \psi.$$

$$\mathcal{O}_1(^3S_1) = \psi^+ \boldsymbol{\sigma} \chi \cdot \chi^+ \boldsymbol{\sigma} \psi.$$

$$\mathcal{O}_8(^1S_0) = \psi^+ \boldsymbol{T}^\alpha \chi \chi^+ \boldsymbol{T}^\alpha \psi.$$

$$\mathcal{O}_8(^3S_1) = \psi^+ \boldsymbol{\sigma} \boldsymbol{T}^\alpha \chi \cdot \chi^+ \boldsymbol{\sigma} \boldsymbol{T}^\alpha \psi.$$

$$\begin{aligned}
 \Gamma(H \rightarrow LH) &= 2\text{Im}\langle H | \delta\mathcal{L}_{4\text{-fermion}} | H \rangle \\
 &= \sum \frac{2\text{Im}(f_n(\Lambda))}{M^{d_n-4}} \langle H | \mathcal{O}_n(\Lambda) | H \rangle
 \end{aligned}$$

上式中, $|H\rangle$ 是指重夸克偶素, 已经包含了所有的Fock态展开, 而 $|LH\rangle$ 是指一切可能的末态轻强子。 $\text{Im}(f_n(\Lambda))$ 是短距离系数, 可通过完整QCD和NRQCD衰变宽度匹配得到。长程矩阵元 $\langle H | \mathcal{O}_n(\Lambda) | H \rangle$ 与具体过程无关, 可以通过格点或者势模型计算。

对于S波重夸克偶素的衰变, 例如 J/ψ , 主要贡献来自S波算符。因子化公式为:

$$\Gamma(J/\psi \rightarrow LH) = \frac{2\text{Im}f_1(^3S_1)}{m^2} \langle J/\psi | \mathcal{O}_1(^3S_1) | J/\psi \rangle$$

量纲为六的电磁相互作用的四费米算符为：

$$(\delta\mathcal{L}_{4-fermion}^{EM})_{d=6} = \frac{f_1(^1S_0)}{M^2}\psi^+\chi|0\rangle\langle 0|\chi^+\psi + \frac{f_1(^3S_1)}{M^2}\psi^+\boldsymbol{\sigma}\chi|0\rangle\langle 0|\chi^+\boldsymbol{\sigma}\psi \quad (2.17)$$

其中八重态算符如 $\psi^+\mathbf{T}^a\chi|0\rangle\langle 0|\chi^+\mathbf{T}^a\psi$ 插入真空态后为0，因为重夸克偶素是色单态，真空态也是色单态，所以电磁衰变的有效拉氏量只包含色单态的算符。

量纲为八的电磁相互作用的四费米算符为

$$\begin{aligned} (\delta\mathcal{L}_{4-fermion}^{EM})_{d=8} = & \frac{f_{EM}(^3P_0)}{M^4}\frac{1}{3}\psi^+(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})\cdot\boldsymbol{\sigma}\chi|0\rangle\langle 0|\chi^+(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})\cdot\boldsymbol{\sigma}\psi \\ & + \frac{f_{EM}(^3P_2)}{M^4}\psi^+(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\sigma^j))\chi|0\rangle\langle 0|\chi^+(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\sigma^j))\psi \\ & + \frac{g_{EM}(^1S_0)}{M^4}\frac{1}{2}[\psi^+\chi|0\rangle\langle 0|\chi^+(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\psi + H.C.] \\ & + \frac{g_{EM}(^3S_1)}{M^4}\frac{1}{2}[\psi^+\boldsymbol{\sigma}\chi|0\rangle\langle 0|\chi^+\boldsymbol{\sigma}(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\psi + H.C.] \\ & + \dots \end{aligned}$$

重夸克偶素H到电磁末态的衰变类似到轻强子的衰变，也可以因子化为：

$$\Gamma(H \rightarrow EM) = \sum \frac{2\text{Im}(f_{EM,n}(\Lambda))}{M^{d_n-4}} \langle H | \psi^+ \mathcal{K}'_n \chi | 0 \rangle \langle 0 | \chi^+ \mathcal{K}_n \psi | H \rangle$$

其中 $\mathcal{K}'_n$, $\mathcal{K}_n$ 是由单位色矩阵，自旋矩阵以及协变微商 $\mathbf{D}$ 的多项式和其他场产生

$$\mathcal{A}(Q\bar{Q} \rightarrow Q\bar{Q})|_{\text{pert.}QCD} = \sum_n \frac{2\text{Im}(f_n(\Lambda))}{M^{d_n-4}} \langle Q\bar{Q} | \mathcal{O}_n(\Lambda) | Q\bar{Q} \rangle |_{\text{pert.}NRQCD}$$

$\mathcal{O}(\alpha_s v^2)$ correction to pseudoscalar quarkonium decay to two photons

#1

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Abstract

We investigate the $\mathcal{O}(\alpha_s v^2)$ correction to the process of pseudoscalar quarkonium decay to two photons in nonrelativistic QCD (NRQCD) factorization framework. The short-distance coefficient associated with the relative-order v^2 NRQCD matrix element is determined to next-to-leading order in α_s through the perturbative matching procedure. Some technical subtleties encountered in calculating the $\mathcal{O}(\alpha_s)$ QCD amplitude are thoroughly addressed.

$$\Gamma[\eta_Q \rightarrow \gamma\gamma] = \frac{F(1S_0)}{m^2} |\langle 0 | \chi^\dagger \psi | \eta_Q \rangle|^2 + \frac{G(1S_0)}{m^4} \text{Re} \left\{ \langle \eta_Q | \psi^\dagger \chi | 0 \rangle \langle 0 | \chi^\dagger (-\frac{i}{2} \overleftrightarrow{\mathbf{D}})^2 \psi | \eta_Q \rangle \right\}$$

标记 η_Q 和末态两光子的能动量分别为 P 、 k_1 、 k_2 ，光子的极化矢量为 ϵ_1 、 ϵ_2 ，则在 $\eta_Q \rightarrow \gamma\gamma$ 的衰变振幅中，唯一可能的张量结构是 $\mathcal{M} \sim \epsilon^{\mu\nu k_1 P} \epsilon_{1\mu}^* \epsilon_{2\nu}^*$ ，在 η_Q 的静止系，可利用下面关系：

$$\hat{\mathbf{k}}_1 \cdot \epsilon_1^* \times \epsilon_2^* = \frac{2}{M_{\eta_Q}^2} \epsilon^{\mu\nu\alpha\beta} P_\mu k_{1\nu} \epsilon_{1\alpha}^* \epsilon_{2\beta}^*.$$

其中 $\hat{\mathbf{k}}_1 = \frac{\mathbf{k}_1}{|\mathbf{k}_1|}$ ，是 $\mathbf{k}_1$ 方向上的单位矢量。因此在 η_Q 的静止系，衰变振幅正比于动力学不变量 $\hat{\mathbf{k}}_1 \cdot \epsilon_1^* \times \epsilon_2^*$ 。

在振幅层次上进行NRQCD因子化和匹配，因此在NRQCD框架下，只需保留连接真空和自旋单态 η_c 的算符矩阵元，衰变振幅有如下的因子化公式：

$$\mathcal{M}[\eta_Q \rightarrow \gamma\gamma] = \hat{\mathbf{k}}_1 \cdot \epsilon_1^* \times \epsilon_2^* \left[c_0 \langle 0 | \chi^\dagger \psi | \eta_Q \rangle + \frac{c_2}{m^2} \langle 0 | \chi^\dagger (-\frac{i}{2} \overleftrightarrow{\mathbf{D}})^2 \psi | \eta_Q \rangle + \mathcal{O}(v^4) \right]$$

利用 $\sum_{\text{Pol}} |\hat{\mathbf{k}}_1 \cdot \boldsymbol{\varepsilon}_1^* \times \boldsymbol{\varepsilon}_2^*|^2 = 2$, 对末态光子极化矢量求和, 乘以两体末态相空间, 得到 $\eta_Q \rightarrow \gamma\gamma$ 的衰变宽度:

$$\begin{aligned}\Gamma[\eta_Q \rightarrow \gamma\gamma] &= \frac{1}{2!} \int \frac{d^3 k_1}{(2\pi)^3 2k_1^0} \frac{d^3 k_2}{(2\pi)^3 2k_2^0} (2\pi)^4 \delta^{(4)}(P - k_1 - k_2) \sum |\mathcal{A}[\eta_Q \rightarrow \gamma\gamma]|^2 \\ &= \frac{1}{8\pi} \left| c_0 \langle 0 | \chi^\dagger \psi | \eta_Q \rangle + \frac{c_2}{m^2} \langle 0 | \chi^\dagger (-\frac{i}{2} \overleftrightarrow{\mathbf{D}})^2 \psi | \eta_Q \rangle + \dots \right|^2,\end{aligned}$$

$$F(^1S_0) = \frac{m^2}{8\pi} |c_0|^2,$$

$$G(^1S_0) = \frac{m^2}{4\pi} \text{Re}[c_0 c_2^*].$$

$$\langle 0 | \chi^\dagger \psi | Q\bar{Q}(^1S_0) \rangle = \sqrt{2N_c} (1 + O(\alpha_s)),$$

$$\langle 0 | \chi^\dagger (-\frac{i}{2} \overleftrightarrow{\mathbf{D}})^2 \psi | Q\bar{Q}(^1S_0) \rangle = \sqrt{2N_c} \mathbf{q}^2 (1 + O(\alpha_s)),$$

$\sqrt{2N_c}$ 因子来自 NRQCD 中归一化态 $Q\bar{Q}(^1S_0)$ 的自旋耦合和色因子。



在完整QCD中, 非相对论极限下的 $Q\bar{Q}(^1S_0) \rightarrow \gamma(k_1, \varepsilon_1) + \gamma(k_2, \varepsilon_2)$ 的衰变振幅为:

$$\bar{v}(\bar{p})T u(p) = \text{Tr}[u(p)\bar{v}(\bar{p})T].$$

其中 T 是除去四分量旋量 u, v 外的旋量矩阵和色矩阵。

$$\begin{aligned}\Pi_1^{(1)}(p, \bar{p}) &= \sum_{s_1, s_2} u(p, s_1) \bar{v}(\bar{p}, s_2) \langle \frac{1}{2}, s_1; \frac{1}{2}, s_2 | 00 \rangle \otimes \frac{\mathbf{1}_c}{\sqrt{N_c}} \\ &= \frac{1}{8\sqrt{2}E^2(E+m)} (\not{p} + m)(\not{p} + 2E) \gamma_5 (\not{\bar{p}} - m) \otimes \frac{\mathbf{1}_c}{\sqrt{N_c}}\end{aligned}$$

其中 $\mathbf{1}_c$ 是SU(3)群基础表示的单位矩阵, $\frac{1}{\sqrt{N_c}}$ 是色因子。

$$\mathcal{M}(Q\bar{Q}(^1S_0) \rightarrow \gamma\gamma) |_{FullQCD} = \text{Tr} \left\{ \Pi_1^{(1)}(p, \bar{p}, \lambda) T \right\},$$

$$T^{(0)} = -ie^2 e_Q^2 \left[\not{\varepsilon}_2^* \frac{\not{p} - \not{k}_1 + m}{-2p \cdot k_1} \not{\varepsilon}_1^* + \not{\varepsilon}_1^* \frac{-\not{p} + \not{k}_1 + m}{-2\bar{p} \cdot k_1} \not{\varepsilon}_2^* \right] \otimes \mathbf{1}_c$$

$$\begin{aligned}
\mathcal{M}(Q\bar{Q}(^1S_0) \rightarrow \gamma\gamma) |_{q_4} &= (e^2 e_Q^2) \sqrt{N_c} \frac{m}{\sqrt{2}(E^4 - (k_1 \cdot q)^2)} \epsilon^{\mu\nu k_1 P} \epsilon_{1\mu}^* \epsilon_{2\nu}^* \\
&= (e^2 e_Q^2) \hat{\mathbf{k}}_1 \cdot \boldsymbol{\epsilon}_1^* \times \boldsymbol{\epsilon}_2^* \sqrt{2N_c} \frac{mE^2}{(E^4 - (k_1 \cdot q)^2)}
\end{aligned}$$

$$\mathcal{M}[\eta_Q \rightarrow \gamma\gamma] = \hat{\mathbf{k}}_1 \cdot \boldsymbol{\epsilon}_1^* \times \boldsymbol{\epsilon}_2^* \left[c_0 \langle 0 | \chi^\dagger \psi | \eta_Q \rangle + \frac{c_2}{m^2} \langle 0 | \chi^\dagger (-\frac{i}{2} \overleftrightarrow{\mathbf{D}})^2 \psi | \eta_Q \rangle + \mathcal{O}(v^4) \right]$$

$$\mathcal{M}[Q\bar{Q}(^1S_0) \rightarrow \gamma\gamma] |_{FullQCD} = \hat{\mathbf{k}}_1 \cdot \boldsymbol{\epsilon}_1^* \times \boldsymbol{\epsilon}_2^* \left[\mathcal{A}_0 + \frac{\mathbf{q}^2}{m^2} \mathcal{A}_2 + \mathcal{O}\left(\frac{\mathbf{q}^4}{m^4}\right) \right]$$

$$\mathcal{A}_0^{(0)} = \sqrt{2N_c} \frac{4\pi e_Q^2 \alpha}{m},$$

$$\mathcal{A}_2^{(0)} \Big|_{q_4 \text{ scheme}} = -\sqrt{2N_c} \frac{8\pi e_Q^2 \alpha}{3m},$$

$$\mathbb{A}_{\text{NRQCD}}^{(0)} = \sqrt{2N_c} \left[c_0^{(0)} + c_2^{(0)} v^2 + \dots \right].$$

$$c_0^{(0)} = \frac{4\pi e_Q^2 \alpha}{m},$$

$$c_2^{(0)} \Big|_{q_4 \text{ scheme}} = -\frac{8\pi e_Q^2 \alpha}{3m},$$

■ NRQCD的成功和挑战

A.理论值与实验值符合较好

B.相对于色单态而言，能很好解释其微分截面。

a.对于极化问题存在争议

b.矩阵元的选取

NRQCD框架下的 χ_{cJ} 衰变宽度的理论值与实验值的对比。

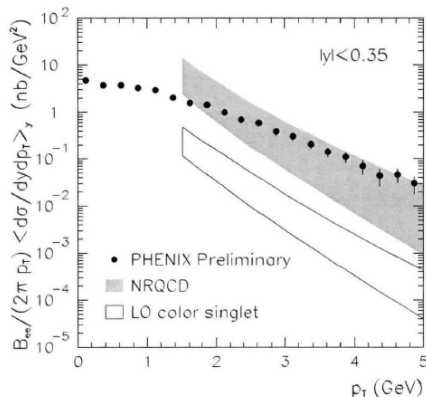
分支比	领头阶	次领头阶	PDG
$\frac{\Gamma(\chi_{c0} \rightarrow \gamma\gamma)}{\Gamma(\chi_{c2} \rightarrow \gamma\gamma)}$	3.75	5.43	4.43 ± 0.99
$\frac{\Gamma(\chi_{c0} \rightarrow LH) - \Gamma(\chi_{c1} \rightarrow LH)}{\Gamma(\chi_{c0} \rightarrow \gamma\gamma)}$	1300	2781	3600 ± 700
$\frac{\Gamma(\chi_{c2} \rightarrow LH) - \Gamma(\chi_{c1} \rightarrow LH)}{\Gamma(\chi_{c0} \rightarrow \gamma\gamma)}$	347	383	410 ± 100
$\frac{\Gamma(\chi_{c0} \rightarrow LH) - \Gamma(\chi_{c1} \rightarrow LH)}{\Gamma(\chi_{c2} \rightarrow LH) - \Gamma(\chi_{c1} \rightarrow LH)}$	3.75	7.63	8.9 ± 0.1
$\frac{\Gamma(\chi_{c0} \rightarrow LH) - \Gamma(\chi_{c2} \rightarrow LH)}{\Gamma(\chi_{c2} \rightarrow LH) - \Gamma(\chi_{c1} \rightarrow LH)}$	2.75	6.63	7.9 ± 1.5

N. Brambilla et al. [Quarkonium Working Group], *Heavy quarkonium physics*, Report No. CERN-2005- 005.

K. Nakamura et al. [Particle Data Group], *Review of particle physics*, Jour. Phys. G **37**, 075021 (2010).

H. S. Chung, S. Kim, J. Lee, and C. Yu, *Polarization of prompt J/ψ in proton-proton collisions at RHIC*, Phys. Rev. D **81**, 014020 (2010).

C. L. da Silva et al. [PHENIX Collaboration], Nucl. Phys. A **830**, 227 (2009).



： J/ψ 在质心系能量 $\sqrt{s} = 200 \text{ GeV}$ 下的 pp 对撞机上的微分截面。

Next-to-leading order QCD calculation of B_c to charmonium tensor form factors

#1

Wei Tao (Nanjing Normal U.), Zhen-Jun Xiao (Nanjing Normal U.), Ruilin Zhu (Nanjing Normal U.) (Apr 13, 2022)

e-Print: [2204.06385](#) [hep-ph]



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1 citation

Scrutinizing New Physics in Semi-leptonic $B_c \rightarrow J/\psi \pi \nu$ Decay

#2

Ru-Ying Tang (Beijing, Inst. High Energy Phys. and Beijing, GUCAS), Zhuo-Ran Huang (APCTP, Pohang), Cai-Dian Lü (Beijing, Inst. High Energy Phys. and Beijing, GUCAS), Ruilin Zhu (Nanjing Normal U.) (Apr 8, 2022)

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2 citations

Relativistic corrections to the form factors of B_c into S -wave Charmonium

#1

Ruilin Zhu (Nanjing Normal U.), Yan Ma (Nanjing Normal U.), Xin-Ling Han (Nanjing Normal U.), Zhen-Jun Xiao (Nanjing Normal U.) (Mar 10, 2017)

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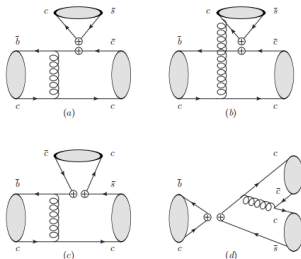


FIG. 2: Typical Feynman diagrams for $B_c^+ \rightarrow J/\psi + D_s^{(*)}$, where two “ $\oplus$ ” denote four-fermion weak interaction operators. There are four types of topologies: (a) factorizable diagrams; (b) non-factorizable diagrams; (c) color-suppressed diagrams; (d) annihilation diagrams.

Based on the NRQCD framework, we can calculate the amplitudes in Fig. 2 and numerical results indicate that the factorizable diagrams dominate the contribution of the decay widths of $B_c^+ \rightarrow J/\psi + D_s^{(*)}$, but colour-suppressed and annihilation topologies diagrams contribute less than 10 percent. The factorizable diagrams can be factorized into the form factor part and the $D_s^{(*)}$ decay constant part. Thus we can employ the results of NLO QCD and relativistic corrections to the form factors, and obtain more precise predictions.

TABLE II: Comparisons of the results of the decay ratios of $B_c \rightarrow J/\psi + D_s^{(*)}$ with data and other theoretical predictions.

$R_{D_s^+/\pi^+}$	$R_{D_s^{*+}/\pi^+}$	$R_{D_s^{*+}/D_s^+}$	$\Gamma_{\pm\pm}/\Gamma$	Refs.
3.8 ± 1.2	10.4 ± 3.5	$2.8^{+1.2}_{-0.9}$	0.38 ± 0.24	ATLAS [25]
2.90 ± 0.62	—	2.37 ± 0.57	0.52 ± 0.20	LHCb [24].
2.6	4.5	1.7	—	Potential model [32]
1.3	5.2	3.9	—	QCD SR [33]
2.0	5.7	2.9	—	RCQM [34]
2.2	—	—	—	BSW [35]
2.06 ± 0.86	—	3.01 ± 1.23	—	LFQM [17]
$3.45^{+0.49}_{-0.17}$	—	$2.54^{+0.07}_{-0.21}$	0.48 ± 0.04	PQCD [7]
—	—	—	0.410	RIQM [2]
$3.07^{+0.21+0.14}_{-0.38-0.13}$	$11.8^{+1.0+2.3}_{-1.4-0}$	$3.85^{+0.04+0.54}_{-0.02-0}$	$0.601^{+0.001+0.033}_{-0.001-0.040}$	NRQCD NLO+RC