



Phenomenological studies on neutral B -meson decays into $J/\psi f_1$ and $\eta c f_1$

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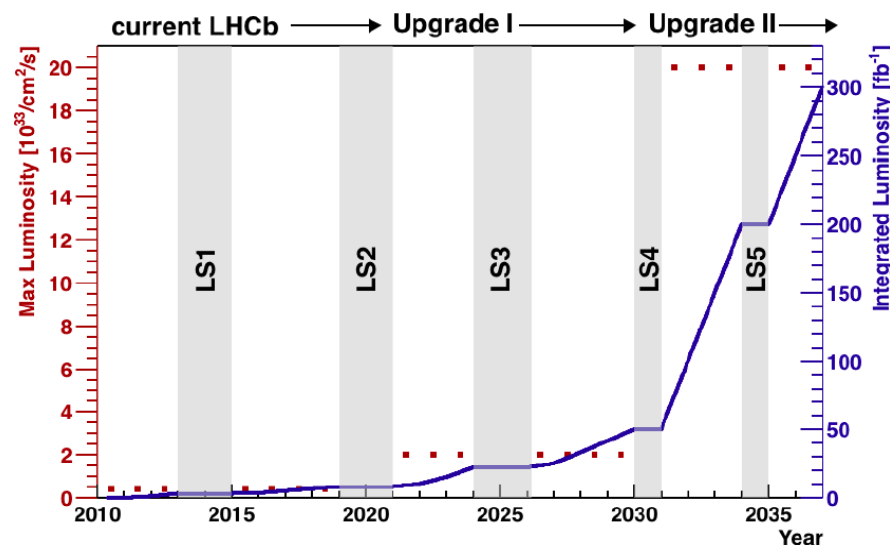
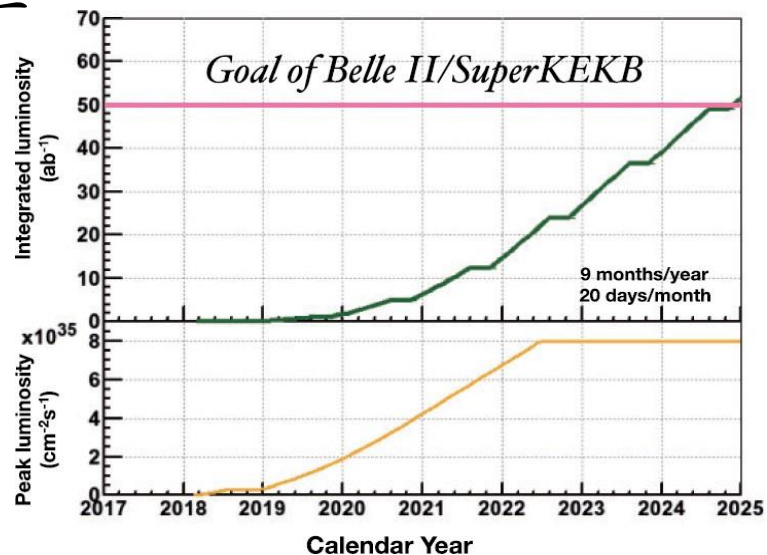
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概述

- ① 研究动机
- ② 理论框架
- ③ 数值结果
- ④ 总结展望

研究动机

1.1 动机一



- 随着探测器的不断升级，未来Belle-II探测器的亮度将会达到 $50ab^{-1}$, LHCb Upgradell 探测器的亮度将会达到 $300fb^{-1}$ 。
- 探测技术也在不断地提高，探测器的测量精度将会大大提升，并且能够收集更多关于 B 介子的数据。

研究动机

$$\phi_s = -2\beta_s = -2 \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{tb}V_{cb}^*}\right) = -0.036 \pm 0.002 (\text{rad}). \text{ (SM)} \quad B_s^0 \rightarrow J/\psi \phi \text{ or } B_s^0 \rightarrow J/\psi f_0$$

QCD求和规则认为轴矢量介子 f_1 和矢量介子 ϕ 的动力学行为相似;

$B_s^0 \rightarrow (J/\psi, \eta_c) f_1$ 的衰变中, 在夸克层次上也是由 $b \rightarrow c\bar{c}s$ 跃迁所控制,

类似于 $B_s^0 \rightarrow J/\psi \phi$ 衰变道, 对于混合相角 ϕ_s 的提取起到一定约束作用。



$$\begin{cases} B_s^0 \rightarrow J/\psi f_1(1420) \\ B_s^0 \rightarrow \eta_c f_1(1420) \end{cases}$$

研究动机

LHCb通过 $B \rightarrow J/\psi(2\pi^+2\pi^-)_{f_1(1285)}$ 衰变道得到了 $B \rightarrow J/\psi f_1(1285)$ 衰变的分支比和混合角：

[R. Aaij et al. [LHCb Collaboration], Phys. Rev. D, 2013, 112(9):091802.]

$$\begin{cases} \mathcal{B}(B_d^0 \rightarrow J/\psi f_1(1285))_{\text{Exp}} = 8.37_{-2.09}^{+2.11} \times 10^{-6} \\ \mathcal{B}(B_s^0 \rightarrow J/\psi f_1(1285))_{\text{Exp}} = 7.14_{-1.41}^{+1.36} \times 10^{-5}, \end{cases} \quad \varphi^{\text{Exp}} = \pm \left(24.0_{-2.7}^{+3.2} \right)^\circ.$$

理论上从 $B_s^0 \rightarrow J/\psi f_1(1285)$ 衰变道抽取出的混合角值为： $\varphi^{\text{Theo}} = \left(15.0_{-1.5}^{+1.5} \right)^\circ$.

[Liu X, Xiao Z J. Phys. Rev. D, 2014, 89(9):097503.]

研究动机

1.2 动机二

发现 p -波介子,这些粒子的分类和内部结构等基本问题还存在诸多争议, 轴矢量介子 K_{1A} 和 K_{1B} 的混合复杂, 实验和理论上对其混合角仍不十分确定。

$n^{2s+1}\ell_J$	J^{PC}	$I = 1$ $u\bar{d}, \bar{u}d, \frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$	$I = \frac{1}{2}$ $u\bar{s}, d\bar{s}; \bar{d}s, -\bar{u}s$	$I = 0$ f'	$I = 0$ f	θ_{quad} [°]	θ_{lin} [°]
1^1S_0	0^{-+}	π	K	η	$\eta'(958)$	-11.5	-24.6
1^3S_1	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	38.7	36.0
1^1P_1	1^{+-}	$b_1(1235)$	$K_{1B}^\dagger$	$h_1(1380)$	$h_1(1170)$		
1^3P_0	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
1^3P_1	1^{++}	$a_1(1260)$	$K_{1A}^\dagger$	$f_1(1420)$	$f_1(1285)$		
1^3P_2	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	29.6	28.0

研究动机

$f_1(1285)$ 和 $f_1(1420)$ 介子混合:
$$\begin{pmatrix} f_1(1285) \\ f_1(1420) \end{pmatrix} = \begin{pmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{pmatrix} \begin{pmatrix} f_n \\ f_s \end{pmatrix}$$

$$\begin{pmatrix} f_1(1285) \\ f_1(1420) \end{pmatrix} = \begin{pmatrix} \cos \theta_{f_1} & \sin \theta_{f_1} \\ -\sin \theta_{f_1} & \cos \theta_{f_1} \end{pmatrix} \begin{pmatrix} f_1 \\ f_8 \end{pmatrix}$$

混合角关系式: $\varphi = \theta_i - \theta_{f_1}$;

$$\tan^2 \theta_{f_1} = \frac{4m_{K_{1A}}^2 - m_{a_1}^2 - 3m_{f_1(1420)}^2}{-4m_{K_{1A}}^2 + m_{a_1}^2 + 3m_{f_1(1285)}^2}, \quad m_{K_{1A}}^2 = m_{K_1(1400)}^2 \cos^2 \theta_{K_1} + m_{K_1(1270)}^2 \sin^2 \theta_{K_1}$$

$K_1(1270)$ 和 $K_1(1400)$ 混合形式:
$$\begin{pmatrix} K_1(1270) \\ K_1(1400) \end{pmatrix} = \begin{pmatrix} \sin \theta_{K_1} & \cos \theta_{K_1} \\ \cos \theta_{K_1} & -\sin \theta_{K_1} \end{pmatrix} \begin{pmatrix} K_{1A} \\ K_{1B} \end{pmatrix}$$

理论框架

PQCD因子化方法：

基于 k_T 因子化定理，消除端点发散，所有拓扑结构费曼图均可微扰计算。

$$\frac{1}{l^2(p_b^2 - m_B^2)} \sim -\frac{1}{m_B^4 x_1 x_3 (1 - x_3)} \rightarrow -\frac{1}{((1 - x_3)m_B^2 + k_{3T}^2)(x_1 x_3 m_B^2 + (k_{1T} - k_{3T})^2)}$$

PQCD因子化方法的中心思想是将QCD参与的过程中可微扰部分分离出来用微扰论处理，而非微扰部分用普适的强子波函数表示。

理论框架

1. B 介子波函数: $\Phi_B = \frac{i}{\sqrt{2N_C}} \{(\not{P} + m_B) \gamma_5 \phi_B(x, k_T)\}_{\alpha\beta}$

2. 矢量介子 J/ψ 和赝标介子 η_c 的波函数:

$$\begin{aligned}\Phi_{J/\psi}^L(x) &= \frac{1}{\sqrt{2N_C}} \{m_{J/\psi} \not{\epsilon}_L \phi_{J/\psi}^L(x) + \not{\epsilon}_L \not{P} \phi_{J/\psi}^t(x)\}_{\alpha\beta} \\ \Phi_{J/\psi}^T(x) &= \frac{1}{\sqrt{2N_C}} \{m_{J/\psi} \not{\epsilon}_T \phi_{J/\psi}^V(x) + \not{\epsilon}_T \not{P} \phi_{J/\psi}^T(x)\}_{\alpha\beta}\end{aligned}$$

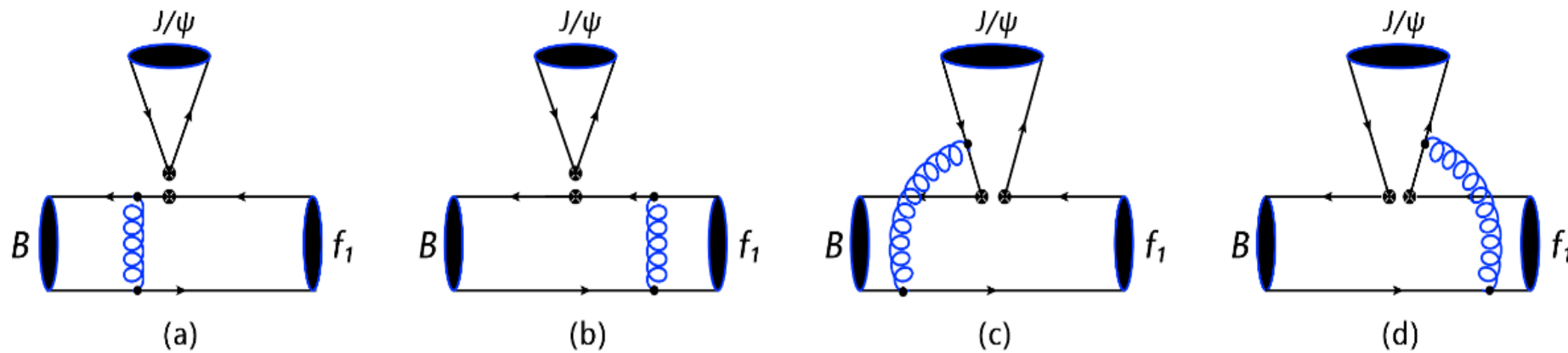
$$\Phi_{\eta_c} = \frac{i}{\sqrt{2N_C}} \gamma_5 \{ \not{P} \phi_{\eta_c}^V(x) + m_{\eta_c} \phi_{\eta_c}^S(x) \}_{\alpha\beta}$$

3. 轴矢量介子夸克-味态 $f_q(q=n,s)$ 的波函数:

$$\begin{aligned}\Phi_{f_q}^L &= \frac{1}{\sqrt{2N_C}} \gamma_5 \{m_{f_q} \not{\epsilon}_L \phi_{f_q}(x) + \not{\epsilon}_L \not{P} \phi_{f_q}^t(x) + m_{f_q} \phi_{f_q}^S(x)\}_{\alpha\beta} \\ \Phi_{f_q}^T &= \frac{1}{\sqrt{2N_C}} \gamma_5 \{m_{f_q} \not{\epsilon}_T \phi_{f_q}^V(x) + \not{\epsilon}_T \not{P} \phi_{f_q}^T(x) + m_{f_q} i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^\mu \not{\epsilon}_T n^\rho v^\sigma \phi_{f_q}^a(x)\}_{\alpha\beta}\end{aligned}$$

其中, 本文所采用的波函数的分布振幅和 Gegenbauer 矩是由 QCD 求和规则得到。

理论框架

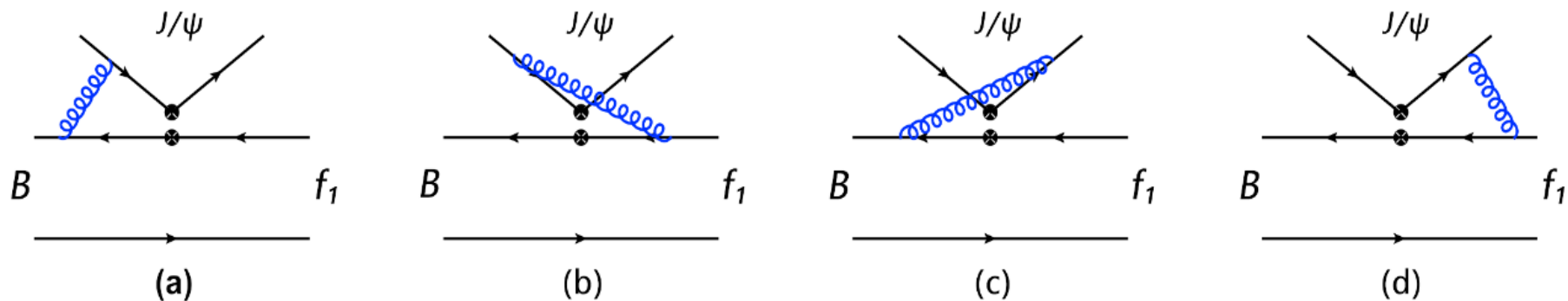


可因子化发射图

不可因子化发射图

$$F_{J/\psi}^L = 8\pi C_F f_{J/\psi} m_B^4 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1, b_1) \left\{ \left[\sqrt{1-r_2^2} \left(r_3 \left(\sqrt{1-r_2^2} (2x_3 - 1) \right) \right. \right. \right. \\ \times \phi_{f_q}^s(x_3) - \phi_{f_q}^t(x_3) \left(2x_3(r^2 - 1) + r^2 + 1 \right) \left. \right) + \phi_{f_q}(x_3) \left((r^2 - 1)x_3 - 1 \right) \left. \right] \\ \times h_{fe}(x_1, x_3, b_1, b_3) E_{fe}(t_a) - \left[2r_3(1 - r^2) \phi_{f_q}^s(x_3) \right] h_{fe}(x_3, x_1, b_3, b_1) E_{fe}(t_b) \left. \right\},$$

理论框架



次领头阶顶角修正费曼图

$$\begin{aligned}
 a_2 \rightarrow \tilde{a}_2^h &= a_2 + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_2 \left(-18 + 12 \ln \frac{m_b}{\mu} + f_I^h \right), & a_7 \rightarrow \tilde{a}_7^h &= a_7 + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_8 \left(6 - 12 \ln \frac{m_b}{\mu} - f_I^h \right), \\
 a_3 \rightarrow \tilde{a}_3^h &= a_3 + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_4 \left(-18 + 12 \ln \frac{m_b}{\mu} + f_I^h \right), & a_9 \rightarrow \tilde{a}_9^h &= a_9 + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_{10} \left(-18 + 12 \ln \frac{m_b}{\mu} + f_I^h \right), \\
 a_5 \rightarrow \tilde{a}_5^h &= a_5 + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_6 \left(6 - 12 \ln \frac{m_b}{\mu} - f_I^h \right), & &
 \end{aligned}$$

其中, 函数 $f_I^0 = f_I + g_I(1-r_2^2)$, $f^\pm = f_I$.

[Cheng H Y, Keum Y Y, Yang K C. Phys. Rev. D, 2002, 65(9):094023.]

数值结果

3.1 分支比

混合角 $\varphi^{\text{Theo}} \sim 15.0^\circ$:

f_q Gegenbauer

$$\mathcal{B}(B_d^0 \rightarrow J/\psi f_1(1285)) = 3.05^{+0.73+0.22+1.51+0.42+1.01+0.02+0.09+0.01+0.04}_{-0.56-0.20-1.08-0.40-0.77-0.02-0.12-0.00-0.04} \left[3.05^{+2.02}_{-1.51} \right] \times 10^{-5}$$

$$\mathcal{B}(B_d^0 \rightarrow J/\psi f_1(1420)) = 1.99^{+0.47+0.14+0.99+0.28+0.64+0.01+0.06+0.00+0.41}_{-0.37-0.13-0.71-0.26-0.49-0.01-0.08-0.00-0.37} \left[1.99^{+1.37}_{-1.05} \right] \times 10^{-6}$$

$$\mathcal{B}(B_s^0 \rightarrow J/\psi f_1(1285)) = 0.89^{+0.25+0.07+0.08+0.13+0.32+0.01+0.04+0.00+0.19}_{-0.18-0.06-0.06-0.12-0.24-0.00-0.04-0.00-0.16} \left[0.89^{+0.48}_{-0.37} \right] \times 10^{-4}$$

$$\mathcal{B}(B_s^0 \rightarrow J/\psi f_1(1420)) = 1.14^{+0.32+0.08+0.10+0.17+0.40+0.02+0.05+0.00+0.02}_{-0.23-0.07-0.08-0.15-0.30-0.01-0.05-0.00-0.01} \left[1.14^{+0.56}_{-0.42} \right] \times 10^{-3}$$

混合角 $\varphi^{\text{Exp}} \sim 24.0^\circ$:

$$\mathcal{B}(B_d^0 \rightarrow J/\psi f_1(1285)) = 2.73^{+0.65+0.19+1.35+0.38+0.90+0.02+0.08+0.01+0.11}_{-0.50-0.18-0.97-0.36-0.69-0.02-0.11-0.00-0.14} \left[2.73^{+1.80}_{-1.36} \right] \times 10^{-5}$$

$$\mathcal{B}(B_d^0 \rightarrow J/\psi f_1(1420)) = 4.92^{+1.16+0.34+2.43+0.68+1.58+0.03+0.14+0.00+1.29}_{-0.91-0.34-1.75-0.65-1.22-0.04-0.20-0.01-1.00} \left[4.92^{+3.47}_{-2.64} \right] \times 10^{-6}$$

$$\mathcal{B}(B_s^0 \rightarrow J/\psi f_1(1285)) = 2.21^{+0.61+0.15+0.18+0.31+0.79+0.02+0.09+0.00+0.58}_{-0.46-0.15-0.17-0.30-0.60-0.02-0.10-0.00-0.45} \left[2.21^{+1.22}_{-0.96} \right] \times 10^{-4}$$

$$\mathcal{B}(B_s^0 \rightarrow J/\psi f_1(1420)) = 1.02^{+0.28+0.08+0.08+0.15+0.36+0.01+0.04+0.00+0.04}_{-0.21-0.07-0.08-0.14-0.27-0.01-0.04-0.00-0.05} \left[1.02^{+0.50}_{-0.39} \right] \times 10^{-3}$$

数值结果

混合角 $\varphi^{\text{Theo}} \sim 15.0^\circ$:

$$\mathcal{B}(B_d^0 \rightarrow \eta_c f_1(1285)) = 7.64^{+2.42+1.92+3.77+0.92+4.08+0.10+0.58+0.01+0.10}_{-1.78-1.71-2.71-0.86-3.03-0.10-0.66-0.02-0.11} \left[7.64^{+6.45}_{-4.88} \right] \times 10^{-6}$$

$$\mathcal{B}(B_d^0 \rightarrow \eta_c f_1(1420)) = 5.16^{+1.64+1.31+2.56+0.63+2.77+0.07+0.40+0.02+1.06}_{-1.20-1.16-1.83-0.58-2.04-0.06-0.44-0.01-0.96} \left[5.16^{+4.51}_{-3.61} \right] \times 10^{-7}$$

$$\mathcal{B}(B_s^0 \rightarrow \eta_c f_1(1285)) = 2.15^{+0.81+0.54+0.17+0.26+1.28+0.04+0.18+0.00+0.44}_{-0.58-0.48-0.17-0.25-0.92-0.04-0.18-0.00-0.40} \left[2.15^{+1.71}_{-1.30} \right] \times 10^{-5}$$

$$\mathcal{B}(B_s^0 \rightarrow \eta_c f_1(1420)) = 2.84^{+1.07+0.71+0.22+0.35+1.69+0.05+0.24+0.00+0.03}_{-0.77-0.64-0.22-0.33-1.22-0.06-0.25-0.00-0.05} \left[2.84^{+2.18}_{-1.65} \right] \times 10^{-4}$$

混合角 $\varphi^{\text{Exp}} \sim 24.0^\circ$:

$$\mathcal{B}(B_d^0 \rightarrow \eta_c f_1(1285)) = 6.83^{+2.16+1.72+3.38+0.83+3.66+0.09+0.52+0.02+0.28}_{-1.59-1.53-2.42-0.77-2.71-0.09-0.58-0.01-0.35} \left[6.83^{+5.79}_{-4.37} \right] \times 10^{-6}$$

$$\mathcal{B}(B_d^0 \rightarrow \eta_c f_1(1420)) = 1.28^{+0.40+0.32+0.63+0.15+0.68+0.01+0.09+0.00+0.33}_{-0.30-0.29-0.46-0.15-0.51-0.02-0.11-0.01-0.26} \left[1.28^{+1.12}_{-0.87} \right] \times 10^{-6}$$

$$\mathcal{B}(B_s^0 \rightarrow \eta_c f_1(1285)) = 5.31^{+2.00+1.13+0.42+0.65+3.16+0.10+0.45+0.00+1.39}_{-1.43-1.19-0.41-0.62-2.28-0.11-0.46-0.01-1.08} \left[5.31^{+4.24}_{-3.26} \right] \times 10^{-5}$$

$$\mathcal{B}(B_s^0 \rightarrow \eta_c f_1(1420)) = 2.54^{+0.96+0.64+0.20+0.31+1.51+0.05+0.22+0.00+0.10}_{-0.69-0.57-0.20-0.30-1.09-0.05-0.22-0.00-0.13} \left[2.54^{+1.95}_{-1.48} \right] \times 10^{-4}$$

数值结果

根据窄宽度近似：

$$\mathcal{B}\left(B_d^0 \rightarrow J/\psi\left(\eta\pi^+\pi^-\right)_{f_1(1285)}\right) \equiv \mathcal{B}\left(B_d^0 \rightarrow J/\psi f_1(1285)\right) \cdot \mathcal{B}\left(f_1(1285) \rightarrow \eta\pi^+\pi^-\right) \approx \begin{cases} 1.07^{+0.84}_{-0.70} \times 10^{-5} \\ 0.96^{+0.75}_{-0.63} \times 10^{-5} \end{cases}$$

$$\mathcal{B}\left(B_d^0 \rightarrow \eta_c\left(\eta\pi^+\pi^-\right)_{f_1(1285)}\right) \equiv \mathcal{B}\left(B_d^0 \rightarrow \eta_c f_1(1285)\right) \cdot \mathcal{B}\left(f_1(1285) \rightarrow \eta\pi^+\pi^-\right) \approx \begin{cases} 2.67^{+2.54}_{-2.06} \times 10^{-6} \\ 2.39^{+2.27}_{-1.84} \times 10^{-6} \end{cases}$$

$$\mathcal{B}\left(B_s^0 \rightarrow J/\psi\left(K_S^0 K^+\pi^-\right)_{f_1(1420)}\right) \equiv \mathcal{B}\left(B_s^0 \rightarrow J/\psi f_1(1420)\right) \cdot \mathcal{B}\left(f_1(1420) \rightarrow K_S^0 K^+\pi^-\right) \approx \begin{cases} 0.73^{+0.36}_{-0.28} \times 10^{-3} \\ 0.65^{+0.33}_{-0.25} \times 10^{-3} \end{cases}$$

$$\mathcal{B}\left(B_s^0 \rightarrow \eta_c\left(K_S^0 K^+\pi^-\right)_{f_1(1420)}\right) \equiv \mathcal{B}\left(B_s^0 \rightarrow \eta_c f_1(1420)\right) \cdot \mathcal{B}\left(f_1(1420) \rightarrow K_S^0 K^+\pi^-\right) \approx \begin{cases} 1.82^{+1.40}_{-1.07} \times 10^{-4} \\ 1.63^{+1.26}_{-0.96} \times 10^{-4} \end{cases}$$

数量级较大，有望在LHCb和Belle-II实验中被探测到

数值结果

3.2 分支比比值

从实验的角度，分支比的比值通常比单个分支比测得的精度要高，定义了多组分支比比值：

$$R_d^{J/\psi} \equiv \frac{\mathcal{B}(B_d^0 \rightarrow J/\psi f_1(1285))}{\mathcal{B}(B_d^0 \rightarrow J/\psi f_1(1420))} \Big|_{\text{PQCD}} = \frac{\Phi_{d,J/\psi}^{f_1(1285)}}{\Phi_{d,J/\psi}^{f_1(1420)}} \cdot \cot^2 \varphi \approx \begin{cases} 15.33^{+0.15}_{-0.14} \\ 5.55^{+0.06}_{-0.04} \end{cases}$$

$$R_s^{J/\psi} \equiv \frac{\mathcal{B}(B_s^0 \rightarrow J/\psi f_1(1420))}{\mathcal{B}(B_s^0 \rightarrow J/\psi f_1(1285))} \Big|_{\text{PQCD}} = \frac{\Phi_{s,J/\psi}^{f_1(1420)}}{\Phi_{s,J/\psi}^{f_1(1285)}} \cdot \cot^2 \varphi \approx \begin{cases} 12.81^{+0.17}_{-0.18} \\ 4.62^{+0.08}_{-0.03} \end{cases}$$

$$R_d^{\eta_c} \equiv \frac{\mathcal{B}(B_d^0 \rightarrow \eta_c f_1(1285))}{\mathcal{B}(B_d^0 \rightarrow \eta_c f_1(1420))} \Big|_{\text{PQCD}} = \frac{\Phi_{d,\eta_c}^{f_1(1285)}}{\Phi_{d,\eta_c}^{f_1(1420)}} \cdot \cot^2 \varphi \approx \begin{cases} 14.81^{+3.12}_{-2.36} \\ 5.34^{+1.02}_{-0.92} \end{cases}$$

$$R_s^{\eta_c} \equiv \frac{\mathcal{B}(B_s^0 \rightarrow \eta_c f_1(1420))}{\mathcal{B}(B_s^0 \rightarrow \eta_c f_1(1285))} \Big|_{\text{PQCD}} = \frac{\Phi_{s,\eta_c}^{f_1(1420)}}{\Phi_{s,\eta_c}^{f_1(1285)}} \cdot \cot^2 \varphi \approx \begin{cases} 13.21^{+2.73}_{-2.13} \\ 4.78^{+0.91}_{-0.84} \end{cases}$$

数值结果

$$R_{d, f_1(1285)}^{s, f_1(1420)}[J/\psi] \equiv \frac{\mathcal{B}(B_s^0 \rightarrow J/\psi f_1(1420))}{\mathcal{B}(B_d^0 \rightarrow J/\psi f_1(1285))} \Big|_{\text{PQCD}} = \frac{\tau_{B_s^0}}{\tau_{B_d^0}} \cdot \left(\frac{m_{B_s^0}}{m_{B_d^0}} \right)^7 \frac{\Phi_{s, J/\psi}^{f_1(1420)}}{\Phi_{d, J/\psi}^{f_1(1285)}} \cdot \frac{|\mathcal{A}(B_s^0 \rightarrow J/\psi f_s)/m_{B_s^0}^2|^2}{|\mathcal{A}(B_d^0 \rightarrow J/\psi f_n)/m_{B_d^0}^2|^2} \approx 37.47_{-10.50}^{+16.22}$$

$$R_{d, f_1(1420)}^{s, f_1(1285)}[J/\psi] \equiv \frac{\mathcal{B}(B_s^0 \rightarrow J/\psi f_1(1285))}{\mathcal{B}(B_d^0 \rightarrow J/\psi f_1(1420))} \Big|_{\text{PQCD}} = \frac{\tau_{B_s^0}}{\tau_{B_d^0}} \cdot \left(\frac{m_{B_s^0}}{m_{B_d^0}} \right)^7 \frac{\Phi_{s, J/\psi}^{f_1(1285)}}{\Phi_{d, J/\psi}^{f_1(1420)}} \cdot \frac{|\mathcal{A}(B_s^0 \rightarrow J/\psi f_s)/m_{B_s^0}^2|^2}{|\mathcal{A}(B_d^0 \rightarrow J/\psi f_n)/m_{B_d^0}^2|^2} \approx 44.92_{-12.64}^{+19.47}$$

$$R_{d, f_1(1285)}^{s, f_1(1420)}[\eta_c] \equiv \frac{\mathcal{B}(B_s^0 \rightarrow \eta_c f_1(1420))}{\mathcal{B}(B_d^0 \rightarrow \eta_c f_1(1285))} \Big|_{\text{PQCD}} = \frac{\tau_{B_s^0}}{\tau_{B_d^0}} \cdot \left(\frac{m_{B_s^0}}{m_{B_d^0}} \right)^7 \cdot \frac{\Phi_{s, \eta_c}^{f_1(1420)}}{\Phi_{d, \eta_c}^{f_1(1285)}} \cdot \frac{|\mathcal{A}(B_s^0 \rightarrow \eta_c f_s)/m_{B_s^0}^2|^2}{|\mathcal{A}(B_d^0 \rightarrow \eta_c f_n)/m_{B_d^0}^2|^2} \approx 37.13_{-10.66}^{+16.19}$$

$$R_{d, f_1(1420)}^{s, f_1(1285)}[\eta_c] \equiv \frac{\mathcal{B}(B_s^0 \rightarrow \eta_c f_1(1285))}{\mathcal{B}(B_d^0 \rightarrow \eta_c f_1(1420))} \Big|_{\text{PQCD}} = \frac{\tau_{B_s^0}}{\tau_{B_d^0}} \cdot \left(\frac{m_{B_s^0}}{m_{B_d^0}} \right)^7 \cdot \frac{\Phi_{s, \eta_c}^{f_1(1285)}}{\Phi_{d, \eta_c}^{f_1(1420)}} \cdot \frac{|\mathcal{A}(B_s^0 \rightarrow \eta_c f_s)/m_{B_s^0}^2|^2}{|\mathcal{A}(B_d^0 \rightarrow \eta_c f_n)/m_{B_d^0}^2|^2} \approx 41.60_{-11.94}^{+18.14}$$

数值结果

实验数据【PDG2020】：

$$\mathcal{B}(B_s^0 \rightarrow J/\psi \phi) = 1.07^{+0.28+0.07+0.04+0.14+0.05+0.03+0.05+0.00}_{-0.22-0.08-0.05-0.13-0.06-0.04-0.07-0.00} \left[1.07^{+0.33}_{-0.29} \right] \times 10^{-3} \quad \left[1.08^{+0.08}_{-0.08} \right] \times 10^{-3}$$

$$\mathcal{B}(B_d^0 \rightarrow J/\psi \rho^0) = 2.98^{+0.67+0.21+0.10+0.35+0.08+0.14+0.09+0.00}_{-0.53-0.20-0.10-0.32-0.07-0.13-0.13-0.00} \left[2.98^{+0.81}_{-0.69} \right] \times 10^{-5} \quad \longleftrightarrow \quad \left[2.55^{+0.18}_{-0.16} \right] \times 10^{-5}$$

$$\mathcal{B}(B_s^0 \rightarrow \eta_c \phi) = 3.57^{+1.39+0.91+0.29+0.26+0.25+0.56+0.00}_{-0.96-0.80-0.27-0.23-0.23-0.50-0.00} \left[3.57^{+1.81}_{-1.41} \right] \times 10^{-4} \quad \left[5.00^{+0.90}_{-0.90} \right] \times 10^{-4}$$

$$\mathcal{B}(B_d^0 \rightarrow \eta_c \rho^0) = 7.92^{+2.63+1.99+0.57+0.81+0.52+1.15+0.01}_{-1.92-1.78-0.55-0.76-0.50-1.13-0.02} \left[7.92^{+3.67}_{-3.04} \right] \times 10^{-6}$$

$$R_d^{J/\psi} \left[f_1(1285)/\rho^0 \right] \equiv \frac{\mathcal{B}(B_d^0 \rightarrow J/\psi f_1(1285))}{\mathcal{B}(B_d^0 \rightarrow J/\psi \rho^0)} \bigg|_{\text{PQCD}} \approx \begin{cases} 1.02^{+0.55}_{-0.42} \\ 0.92^{+0.49}_{-0.37} \end{cases} \quad R_s^{\eta_c} \left[f_1(1420)/\phi \right] \equiv \frac{\mathcal{B}(B_s^0 \rightarrow \eta_c f_1(1420))}{\mathcal{B}(B_s^0 \rightarrow \eta_c \phi)} \bigg|_{\text{PQCD}} \approx \begin{cases} 0.80^{+0.08}_{-0.08} \\ 0.71^{+0.06}_{-0.06} \end{cases}$$

$$R_s^{J/\psi} \left[f_1(1420)/\phi \right] \equiv \frac{\mathcal{B}(B_s^0 \rightarrow J/\psi f_1(1420))}{\mathcal{B}(B_s^0 \rightarrow J/\psi \phi)} \bigg|_{\text{PQCD}} \approx \begin{cases} 1.07^{+0.34}_{-0.25} \\ 0.95^{+0.31}_{-0.23} \end{cases} \quad R_d^{\eta_c} \left[f_1(1285)/\rho^0 \right] \equiv \frac{\mathcal{B}(B_d^0 \rightarrow \eta_c f_1(1285))}{\mathcal{B}(B_d^0 \rightarrow \eta_c \rho^0)} \bigg|_{\text{PQCD}} \approx \begin{cases} 0.96^{+0.39}_{-0.31} \\ 0.86^{+0.35}_{-0.27} \end{cases}$$

总结展望

- 中性 B 介子衰变到 $J/\psi f_1$ 和 $\eta_c f_1$ 的**大分支比**易被实验探测到。在两个参考角度下得到的理论结果在误差范围内基本一致，这表明混合角仍然需要更多不同视角的约束。
- 定义了 $B_{d,s}^0 \rightarrow (J/\psi, \eta_c) f_1$ 衰变分支比之间多个有意义的**比值**，可用于精确**确定**夸克-味态 f_n 和 f_s 之间的**混合角** φ 和**研究**其中所包含的 $SU(3)_F$ **对称破缺效应**。
- 对于含有粲偶素过程的QCDF研究发现，粲偶素中重夸克对**色八重态**贡献很大，这应该对应于一些**幂次修正贡献**，接下来工作可以考虑这一部分贡献。



谢谢各位老师、同学的聆听！

理论框架

$B \rightarrow M_2 M_3$ 衰变振幅的因子化形式为：

$$A(B \rightarrow M_2 M_3) \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \\ \cdot \text{Tr} \left[C(t) \Phi_B(x_1, b_1) \Phi_{M_2}(x_2, b_2) \Phi_{M_3}(x_3, b_3) H(x_i, b_i, t) S_t(x_i) e^{-S(t)} \right].$$

- 第二行的第一项表示Wilson系数；第二到四项表示初末态介子的波函数，属于非微扰部分。
- 第二行的第五项表示可微扰计算的“硬核部分”；最后两项是喷注函数和Sudakov因子，二者一起确保了PQCD方法微扰计算的可靠性。

夸克-味混合方案下，其中非奇异和奇异夸克-味态：

$$\begin{cases} f_n = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \\ f_s = s\bar{s} \end{cases}$$