重味物理中轻子普适性破坏的研究

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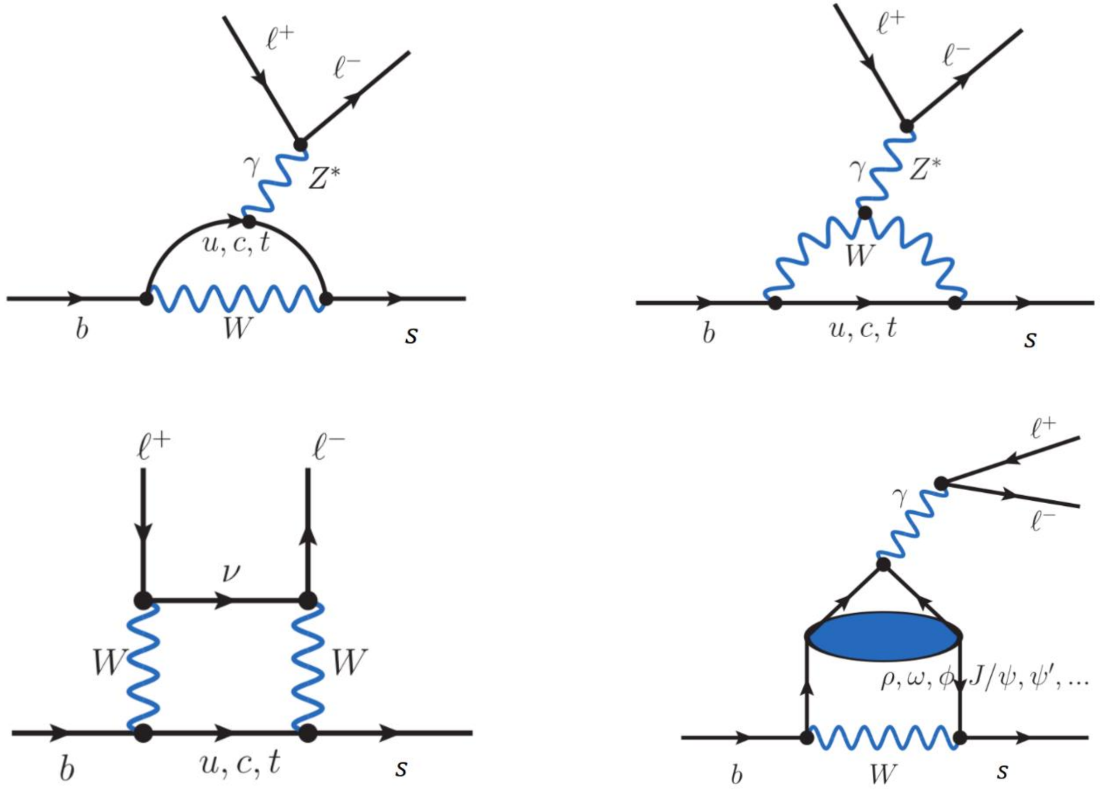
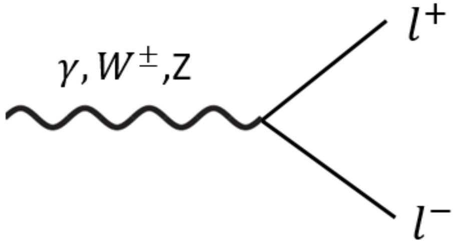
1.研究动机

Motivation

重味物理作为间接寻找新物理的重要领域，最近在B介子半轻衰变中出现了很多反常，尤其是$b \to s\ell^+\ell^-$味道改变中性流过程可能有轻子味普适性破缺迹象，引起了很多物理学家的关注 。本文对$b \to s\ell^+\ell^-$诱导的$B \to K^{(*)}\ell^+\ell^-(\ell = e,\mu,\tau)$过程可能存在的新物理效应进行了模型无关研究。



$B \to K^{(*)}\ell^+\ell^-$过程$R_{K^{(*)}}^{\mu/e}$反常



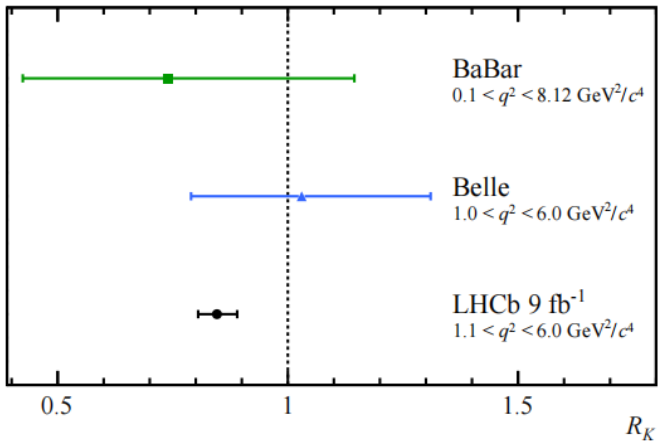
$$R_{K^{(*)}}^{\mu/e} = \frac{\mathcal{B}(B \to K^{(*)}\mu^+\mu^-)}{\mathcal{B}(B \to K^{(*)}e^+e^-)},$$

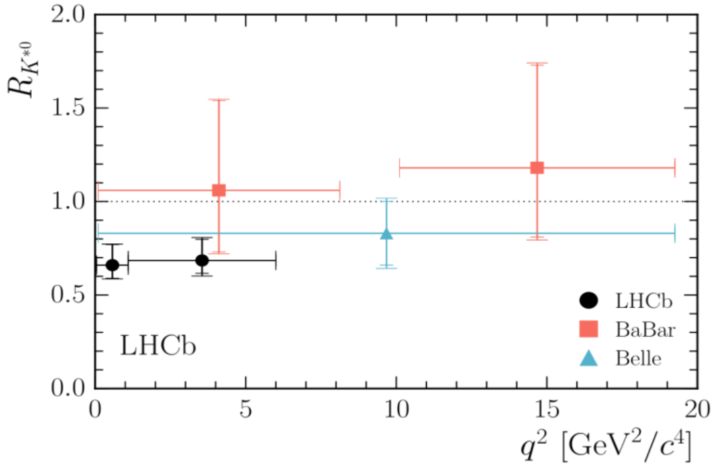
标准模型理论值：$R_{K^{(*)}}^{\mu/e} \simeq 1.0004$，LHCb实验组2021年测得的最新值：

$$R_K^{\exp} = 0.846_{-0.039-0.012}^{+0.042+0.013}, \qquad 1.1\mathrm{GeV}^2 \le q^2 \le 6\mathrm{GeV}^2,$$

$$R_{K^*}^{\exp} = \begin{cases} 0.660_{-0.07}^{+0.11} \pm 0.03, & 0.045\mathrm{GeV}^2 \le q^2 \le 1.1\mathrm{GeV}^2, \\ 0.685_{-0.07}^{+0.11} \pm 0.05, & 1.1\mathrm{GeV}^2 \le q^2 \le 6\mathrm{GeV}^2, \end{cases}$$

实验值与标准模型理论值存在2.5σ，2.1σ，2.5σ的偏差，轻子的普适性受到破坏，其中可能存在**新物理**效应。





LHCb Collaboration, Nature Phys., 2022, 18(3): 277-282.

LHCb Collaboration, JHEP, 2017, 2017(8): 55.

实验上，欧洲核子研究中心的 **LHCb**和另外两个探测器 **ATLAS** 和 **CMS**，日本**Belle-II**实验正在从事相关的研究。

理论方面，寻找新物理一般可以通过两种方法：

- 利用模型无关的有效场论方法进行计算，将所有可能贡献的算符用有效哈密顿量表示。然后对实验数据做**全局拟合**(Global fits)，得出这些算符的Wilson系数最优值。
  - 一维拟合(1D fits)　　Descotes-Genon S, Hofer L, Matias J, et al. JHEP, 2016, 6: 92.
  - 二维拟合(2D fits)　　Algueró M, Capdevila B, Descotes-Genon S, et al. EPJC, 2022, 82(4): 326.
  - 六维拟合(6D fits)　　Algueró M, Capdevila B, Descotes-Genon S, et al. EPJC, 2022, 82(4): 326.
- 通过尝试构建**新物理模型**使其产生的数据与实验数据相同，从而确定新物理。
  - LQ模型(Leptoquarks)　　Bečirević D, Košnik N, Sumensari O, et al. JHEP, 2016,11: 035.
  - 矢量玻色子(Z')模型　　King S F. JHEP 2017, 08 : 019.

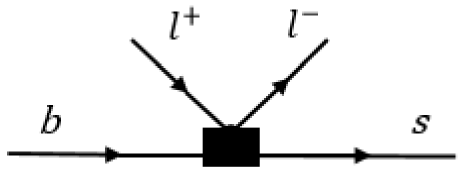
2.解析计算

Calculation

$\mathbf{b \to s\ell^+\ell^-}$过程的低能有效哈密顿量

$$\mathcal{H}^{\text{eff}} = -\frac{4G_{\text{F}}}{\sqrt{2}} V_{tb}V_{ts}^* \frac{\alpha_e}{4\pi}\left(\sum_i C_i\, O_i + \sum_j C_j' O_j'\right)$$

$$O_7^{(\prime)} = \frac{m_b}{e}\left[\bar{s}\sigma^{\mu\nu}P_{R(L)}b\right]F_{\mu\nu},$$



$$O_9^{(\prime)} = \left[\bar{s}\gamma^{\mu}P_{L(R)}b\right]\left[\bar{\ell}\gamma_{\mu}\ell\right], \qquad O_{10}^{(\prime)} = \left[\bar{s}\gamma^{\mu}P_{L(R)}b\right]\left[\bar{\ell}\gamma_{\mu}\gamma_5\ell\right],$$

$$O_S^{(\prime)} = \left[\bar{s}P_{R(L)}b\right]\left[\bar{\ell}\ell\right], \qquad O_P^{(\prime)} = \left[\bar{s}P_{R(L)}b\right]\left[\bar{\ell}\gamma_5\ell\right],$$

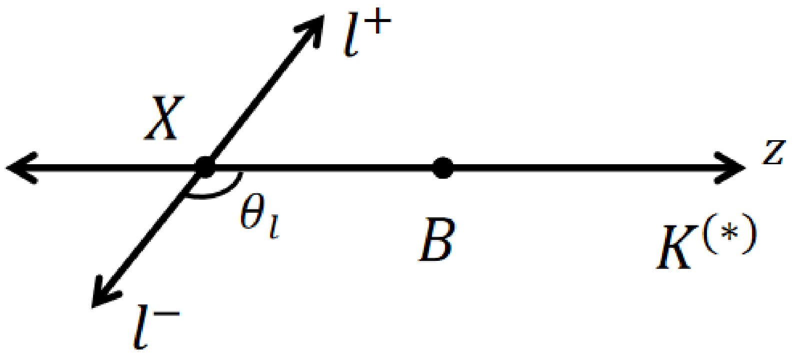
$$O_T = \left[\bar{s}\sigma^{\mu\nu}b\right]\left[\bar{\ell}\sigma_{\mu\nu}\ell\right], \qquad O_{T5} = -\frac{\mathrm{i}}{2}\epsilon_{\mu\nu\alpha\beta}\left[\bar{s}\sigma^{\mu\nu}b\right]\left[\bar{\ell}\sigma^{\alpha\beta}\ell\right].$$

振幅--螺旋度振幅方法

$$\mathcal{M}^{\lambda_1,\lambda_2} = -\frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* \frac{\alpha_e}{4\pi} \sum_{i=L,R}\sum_{\lambda}\left[\eta_\lambda H_{VA,\lambda}^{i,s_p,s_k} L_{\lambda_1,\lambda_2,\lambda}^{VA,i} + H_{SP}^{i,s_p,s_k} L_{\lambda_1,\lambda_2}^{SP,i} + \eta_\lambda\eta_\lambda' H_{T,\lambda,\lambda'}^{i,s_p,s_k} L_{\lambda_1,\lambda_2,\lambda,\lambda'}^{T,i}\right]$$

$\mathbf{B \to K^{(*)}\ell^+\ell^-}$衰变过程的运动学约定

$$B(p,s_p) \to K^{(*)}(k,s_k)\ell^+(q_1)\ell^-(q_2)$$



- **在B介子静止系中**：

$p^\mu = (m_B, 0,0,0)$,　$k^\mu = (E_{K^{(*)}}, 0,0, |\boldsymbol{p}_{K^{(*)}}|)$,

$q^\mu = (q_0, 0,0, -|\boldsymbol{q}|)$,

K^*介子的极化矢量：

$\epsilon^\mu(0) = -\frac{1}{m_{K^*}}(-E_{K^*}, 0,0, -|\boldsymbol{p}_{K^*}|)$;

$\epsilon^\mu(t) = -\frac{1}{m_{K^*}}(|\boldsymbol{p}_{K^{(*)}}|, 0,0, E_{K^*})$;

$\varepsilon^\mu(\pm) = \frac{1}{\sqrt{2}}(0, \pm 1, \mathrm{i}, 0)$。

虚规范玻色子的极化矢量：

$\epsilon^\mu(0) = \frac{1}{\sqrt{q^2}}(|\boldsymbol{q}|, 0,0, -q_0)$;

$\epsilon^\mu(t) = \frac{1}{\sqrt{q^2}}(q_0, 0,0, -|\boldsymbol{q}|)$;　$\varepsilon^\mu(\pm) = \frac{1}{\sqrt{2}}(0, \mp 1, \mathrm{i}, 0)$。

- **在双轻子系统的静止系中**：

$q^\mu = \left(\sqrt{q^2}, 0,0,0\right)$,

$q_2^\mu\big|_{2\ell} = (E_\ell, |\boldsymbol{q}_{2\ell}|\sin\theta_\ell, 0, |\boldsymbol{q}_{2\ell}|\cos\theta_\ell)$,

$q_1^\mu\big|_{2\ell} = (E_\ell, -|\boldsymbol{q}_{2\ell}|\sin\theta_\ell, 0, -|\boldsymbol{q}_{2\ell}|\cos\theta_\ell)$。

虚规范玻色子的极化矢量：

$\epsilon^\mu(0) = (0,0,0,-1)$;

$\epsilon^\mu(t) = (1,0,0,0)$;

$\varepsilon^\mu(\pm) = \frac{1}{\sqrt{2}}(0, \mp 1, \mathrm{i}, 0)$。

强子螺旋度振幅定义----$B \to K^*\ell^+\ell^-$

$$H^{L(R)}_{SP,\lambda_{K^*}} = \langle K^*(k,s_k)|[(C_S^+ \mp C_P^+)\bar{s}b + (C_S^- \mp C_P^-)\bar{s}\gamma_5 b]|\bar{B}(p,s_p)\rangle,$$

$$H^{L(R)}_{VA,\lambda,\lambda_{K^*}} = \epsilon^{\mu*}(\lambda)\langle K^*(k,s_k)|(C_9^+ \mp C_{10}^+)\bar{s}\gamma_\mu b + (C_9^- \mp C_{10}^-)\bar{s}\gamma_\mu\gamma_5 b|\bar{B}(p,s_p)\rangle$$

$$+\langle K^*(k,s_k)|C_7^+\bar{s}\sigma_{\mu\nu}q^\nu b + C_7^-\bar{s}\sigma_{\mu\nu}q^\nu\gamma_5 b|\bar{B}(p,s_p)\rangle,$$

$$H^{L(R)}_{T,\lambda,\lambda',\lambda_{K^*}} = \mathrm{i}\epsilon^{\mu*}(\lambda)\epsilon^{\nu*}(\lambda')\langle K^*(k,s_k)|(C_T \mp C_{T5})\bar{s}\sigma_{\mu\nu}b|\bar{B}(p,s_p)\rangle,$$

$$\mathcal{C}_\mathrm{S}^\pm = \mathcal{C}_\mathrm{S} \pm \mathcal{C}_{\mathrm{S}'}\,,\qquad \mathcal{C}_\mathrm{P}^\pm = \mathcal{C}_\mathrm{P} \pm \mathcal{C}_{\mathrm{P}'}\,,$$

$$\mathcal{C}_7^\pm = \frac{-2m_b}{q^2}(\mathcal{C}_7 + \mathcal{C}_7^\mathrm{NP} \pm \mathcal{C}_{7'})\,,\qquad \mathcal{C}_9^\pm = \pm\,\mathcal{C}_9 \pm \mathcal{C}_9^\mathrm{NP} + \mathcal{C}_{9'}\,,$$

$$\mathcal{C}_{10}^\pm = \pm\,\mathcal{C}_{10} \pm \mathcal{C}_{10}^\mathrm{NP} + \mathcal{C}_{10'}\,.$$

强子跃迁矩阵元----$B \to K^* \ell^+ \ell^-$

$$\langle K^*(k,s_k)|\bar{s}\gamma_\mu b|\bar{B}(p,s_p)\rangle = -\mathrm{i}\varepsilon_{\mu\nu\rho\sigma}\epsilon^{\mu*}p^\rho k^\sigma \frac{2V(q^2)}{m_B + m_{K^*}},$$

$$\langle K^*(k,s_k)|\bar{s}\gamma_\mu\gamma_5 b|\bar{B}(p,s_p)\rangle = 2m_{K^*}\frac{\epsilon^*\cdot q}{q^2}q_\mu(-\mathrm{i})A_0(q^2) - \mathrm{i}(m_B + m_{K^*})\left(\epsilon^*_\mu - \frac{\epsilon^*\cdot q}{q^2}q_\mu\right)A_1(q^2)$$

$$+\frac{\epsilon^*\cdot q}{m_B + m_{K^*}}\left[(p+k)_\mu - \frac{m_B^2 - m_{K^*}^2}{q^2}q_\mu\right]A_2(q^2),$$

$$\langle K^*(k,s_k)|\bar{s}b|\bar{B}(p,s_p)\rangle = 0,$$

$$\langle K^*(k,s_k)|\bar{s}\gamma_5 b|\bar{B}(p,s_p)\rangle = -\mathrm{i}\epsilon^*\cdot q\frac{2m_{K^*}}{m_b + m_s}A_0(q^2)$$

$$\langle K^*(k,s_k)|\bar{s}\sigma_{\mu\nu} b|\bar{B}(p,s_p)\rangle = \varepsilon_{\mu\nu\rho\sigma}[\mathrm{i}\epsilon^{\rho*}(p+k)^\sigma T_1(q^2) + \frac{m_B^2 - m_{K^*}^2}{q^2}\epsilon^{\rho*}\cdot q^\sigma(-\mathrm{i}T_1(q^2) + T_2(q^2))$$

$$+2\frac{\epsilon^*\cdot q}{q^2}p^\rho k^\sigma\left(-\mathrm{i}T_1(q^2) + T_2(q^2) + \frac{q^2}{m_B^2 - m_{K^*}^2}T_3(q^2)\right)],$$

$$\langle K^*(k,s_k)|\bar{s}\sigma_{\mu\nu}q^\nu b|\bar{B}(p,s_p)\rangle = -2\mathrm{i}\varepsilon_{\mu\nu\rho\sigma}\epsilon^{\nu*}p^\rho k^\sigma T_1(q^2),$$

$$\langle K^*(k,s_k)|\bar{s}\sigma_{\mu\nu}q^\nu\gamma_5 b|\bar{B}(p,s_p)\rangle = [(m_B^2 - m_{K^*}^2)\epsilon^{\mu*} - \epsilon^*\cdot q(p+k)_\mu]T_2(q^2)$$

$$+\epsilon^*\cdot q\cdot\left[q_\mu - \frac{q^2}{m_B^2 - m_{K^*}^2}(p+k)_\mu\right]T_3(q^2),$$

Bharucha, A, Straub D M, Zwicky R. JHEP, 2016, 08: 098.
Sakaki Y, Tanaka M, Tayduganov A, et al. Phys.Rev.D, 2013, 88 (9): 094012.

形状因子误差分析----$B \to K^*\ell^+\ell^-$

$B \to K^*\ell^+\ell^-$衰变过程中，形状因子采用光锥求和规则（Light-cone sum rules）的结果作为输入。对$B \to K^*\ell^+\ell^-$衰变过程的形状因子A_0，A_1，A_2，V，T_1，T_2，T_3的表达式可以统一为以下形式：

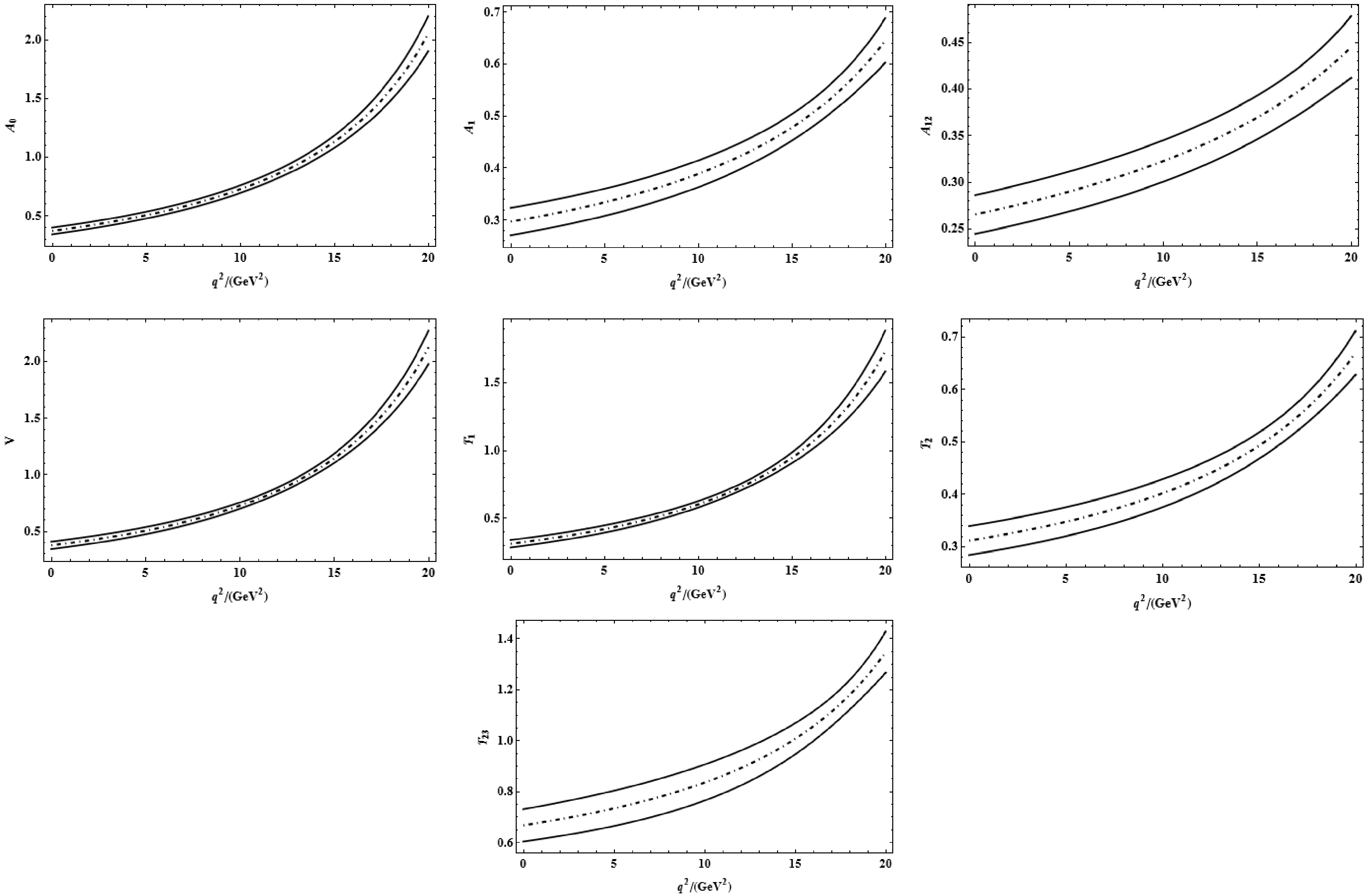
$$F_i(q^2) = P_i(q^2)\sum_k a_k^i\,[z(q^2) - z(0)]^k,$$

其中，$P_i(q^2) = \left(1 - q^2/\left(m_{pole}^f\right)^2\right)^{-1}$，$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$，$t_\pm = (m_B \pm m_{K^*})^2$，$t_0 = t_+\left(1 - \sqrt{1 - t_-/t_+}\right)$。

- 误差计算公式如下：

$$\sigma^2(F_i) = \sum_{k,l,i,j} \frac{\partial F_i(q^2)}{\partial a_k^i} cov\left(a_k^i, a_l^j\right) \frac{\partial F_i(q^2)}{\partial a_l^j}$$

Bharucha, A, Straub D M, Zwicky R. JHEP, 2016, 08: 098.



A0
A1
A12
V
T1
T2
T23
q^2/(GeV^2)

强子螺旋度振幅结果----$B \to K^*\ell^+\ell^-$

$$H_0^{\mathrm{SPL(R)}}(q^2) = -\mathcal{C}_{\mathrm{P}}^{\mathrm{L(R)}}\frac{\sqrt{Q_-Q_+}}{m_b+m_s}iA_0(q^2)\,,$$

$$H_{\pm,\pm}^{\mathrm{VAL(R)}}(q^2) = \mathcal{C}_{\mathrm{A}}^{\mathrm{L(R)}}m_+iA_1(q^2) \pm \mathcal{C}_{\mathrm{V}}^{\mathrm{L(R)}}\frac{\sqrt{Q_-Q_+}}{m_+}iV(q^2) + i\mathcal{C}_7^-m_+m_-T_2(q^2) \mp \mathcal{C}_7^+\sqrt{Q_-Q_+}iT_1(q^2)\,,$$

$$H_{0,t}^{\mathrm{VAL(R)}}(q^2) = \mathcal{C}_{\mathrm{A}}^{\mathrm{L(R)}}\sqrt{\frac{Q_-Q_+}{q^2}}iA_0(q^2),$$

$$H_{0,0}^{\mathrm{VAL(R)}}(q^2) = \frac{\mathcal{C}_{\mathrm{A}}^{\mathrm{L(R)}}}{2m_{K^*}\sqrt{q^2}}\left[m_+(m_+m_- - q^2)iA_1(q^2) - \frac{Q_+Q_-}{m_+}iA_2(q^2)\right]$$

$$+ i\mathcal{C}_7^-\frac{\sqrt{q^2}}{2m_+m_-M_{K^*}}[-Q_-Q_+T_3(q^2) + m_+m_-(m_B^2+3m_{K^*}^2-q^2)T_2(q^2)]\,,$$

$$H_{\pm,\pm,t}^{\mathrm{TL(R)}}(q^2) = \mp\frac{\mathcal{C}_{\mathrm{T}}^{\mathrm{L(R)}}}{\sqrt{q^2}}\sqrt{Q_+Q_-}iT_1(q^2)\,,$$

$$H_{\pm,\pm,0}^{\mathrm{TL(R)}}(q^2) = \pm\frac{\mathcal{C}_{\mathrm{T}}^{\mathrm{L(R)}}}{\sqrt{q^2}}m_+m_-T_2(q^2)\,,$$

$$H_{0,+,-}^{\mathrm{TL(R)}}(q^2) = \frac{\mathcal{C}_{\mathrm{T}}^{\mathrm{L(R)}}}{2m_{K^*}}\left[(m_B^2+3m_{K^*}^2-q^2)T_2(q^2) - \frac{Q_+Q_-}{m_+m_-}T_3(q^2)\right]\,,$$

轻子螺旋度振幅定义----$B \to K^{(*)}\ell^+\ell^-$

$$
\begin{aligned}
L^{\mathrm{S}}_{\lambda_{\ell_1},\lambda_{\ell_2}} &= \langle \ell_1\ell_2 | \bar{\ell}_1\ell_2 | 0\rangle = \bar{u}_{\ell_1}(\vec{p}_{\ell_1},\lambda_{\ell_1}) v_{\ell_2}(-\vec{p}_{\ell_1},\lambda_{\ell_2}) \,, \\
L^{\mathrm{P}}_{\lambda_{\ell_1},\lambda_{\ell_2}} &= \langle \ell_1\ell_2 | \bar{\ell}_1\gamma_5\ell_2 | 0\rangle = \bar{u}_{\ell_1}(\vec{p}_{\ell_1},\lambda_{\ell_1}) \gamma_5 v_{\ell_2}(-\vec{p}_{\ell_1},\lambda_{\ell_2}) \,, \\
L^{\mathrm{V}}_{\lambda_{\ell_1},\lambda_{\ell_2},\lambda_W} &= \bar{\epsilon}^{\mu}(\lambda_W) \langle \ell_1\ell_2 | \bar{\ell}_1\gamma_\mu\ell_2 | 0\rangle = \bar{\epsilon}^{\mu}(\lambda_W) \bar{u}_{\ell_1}(\vec{p}_{\ell_1},\lambda_{\ell_1}) \gamma_\mu v_{\ell_2}(-\vec{p}_{\ell_1},\lambda_{\ell_2}) \,, \\
L^{\mathrm{A}}_{\lambda_{\ell_1},\lambda_{\ell_2},\lambda_W} &= \bar{\epsilon}^{\mu}(\lambda_W) \langle \ell_1\ell_2 | \bar{\ell}_1\gamma_\mu\gamma_5\ell_2 | 0\rangle = \bar{\epsilon}^{\mu}(\lambda_W) \bar{u}_{\ell_1}(\vec{p}_{\ell_1},\lambda_{\ell_1}) \gamma_\mu\gamma_5 v_{\ell_2}(-\vec{p}_{\ell_1},\lambda_{\ell_2}) \,, \\
L^{\mathrm{T0}}_{\lambda_{\ell_1},\lambda_{\ell_2},\lambda_{W_1},\lambda_{W_2}} &= -i\bar{\epsilon}^{\mu}(\lambda_{W_1})\bar{\epsilon}^{\nu}(\lambda_{W_2}) \langle \ell_1\ell_2 | \bar{\ell}_1\sigma_{\mu\nu}\ell_2 | 0\rangle \\
&= -i\bar{\epsilon}^{\mu}(\lambda_{W_1})\bar{\epsilon}^{\nu}(\lambda_{W_2}) \bar{u}_{\ell_1}(\vec{p}_{\ell_1},\lambda_{\ell_1}) \sigma_{\mu\nu} v_{\ell_2}(-\vec{p}_{\ell_1},\lambda_{\ell_2}) \,, \\
L^{\mathrm{T5}}_{\lambda_{\ell_1},\lambda_{\ell_2},\lambda_{W_1},\lambda_{W_2}} &= -i\bar{\epsilon}^{\mu}(\lambda_{W_1})\bar{\epsilon}^{\nu}(\lambda_{W_2}) \langle \ell_1\ell_2 | \bar{\ell}_1\sigma_{\mu\nu}\gamma_5\ell_2 | 0\rangle \\
&= -i\bar{\epsilon}^{\mu}(\lambda_{W_1})\bar{\epsilon}^{\nu}(\lambda_{W_2}) \bar{u}_{\ell_1}(\vec{p}_{\ell_1},\lambda_{\ell_1}) \sigma_{\mu\nu}\gamma_5 v_{\ell_2}(-\vec{p}_{\ell_1},\lambda_{\ell_2})
\end{aligned}
$$

$$
u_{\ell 1}(\lambda) = \begin{pmatrix} \sqrt{E_\ell + m_\ell}\chi^u_\lambda \\ 2\lambda\sqrt{E_\ell - m_\ell}\chi^u_\lambda \end{pmatrix}, \qquad \chi^u_{+\frac{1}{2}} = \begin{pmatrix} cos\frac{\theta_\ell}{2} \\ sin\frac{\theta_\ell}{2} \end{pmatrix}, \qquad \chi^u_{-\frac{1}{2}} = \begin{pmatrix} -sin\frac{\theta_\ell}{2} \\ cos\frac{\theta_\ell}{2} \end{pmatrix},
$$

$$
v_{\ell 2}(\lambda) = \begin{pmatrix} \sqrt{E_\ell - m_\ell}\chi^v_{-\lambda} \\ -2\lambda\sqrt{E_\ell + m_\ell}\chi^v_{-\lambda} \end{pmatrix}, \qquad \chi^v_{+\frac{1}{2}} = \begin{pmatrix} sin\frac{\theta_\ell}{2} \\ -cos\frac{\theta_\ell}{2} \end{pmatrix}, \qquad \chi^v_{-\frac{1}{2}} = \begin{pmatrix} cos\frac{\theta_\ell}{2} \\ sin\frac{\theta_\ell}{2} \end{pmatrix}
$$

轻子螺旋度振幅结果----$B \to K^{(*)}\ell^+\ell^-$

$$L^{\rm S}_{\pm1/2,\pm1/2} = \pm\sqrt{q^2}\beta_\ell\,, \qquad L^{\rm P}_{\pm1/2,\pm1/2} = -\sqrt{q^2}\,,$$

$$L^{\rm V}_{1/2,1/2,\pm} = L^{\rm V}_{-1/2,-1/2,\mp} = \mp\sqrt{2}m_\ell\sin\theta_\ell\,, \qquad L^{\rm V}_{\pm1/2,\pm1/2,0} = \mp 2m_\ell\cos\theta_\ell\,,$$

$$L^{\rm V}_{\pm1/2,\mp1/2,\pm} = \pm\frac{\sqrt{q^2}}{\sqrt{2}}(1-\cos\theta_\ell)\,, \qquad L^{\rm V}_{\pm1/2,\mp1/2,0} = \sqrt{q^2}\sin\theta_\ell\,,$$

$$L^{\rm V}_{\pm1/2,\mp1/2,\mp} = \pm\frac{\sqrt{q^2}}{\sqrt{2}}(1+\cos\theta_\ell)\,, \qquad L^{\rm A}_{\pm1/2,\pm1/2,t} = -2m_\ell\,,$$

$$L^{\rm A}_{\pm1/2,\mp1/2,\pm} = \frac{\sqrt{q^2}}{\sqrt{2}}(1-\cos\theta_\ell)\beta_\ell\,, \qquad L^{\rm A}_{\pm1/2,\mp1/2,0} = \pm\sqrt{q^2}\sin\theta_\ell\beta_\ell\,,$$

$$L^{\rm A}_{\pm1/2,\mp1/2,\mp} = \frac{\sqrt{q^2}}{\sqrt{2}}(1+\cos\theta_\ell)\beta_\ell\,.$$

$$L^{\rm T0,1/2}_{1/2,t,\pm} = -L^{\rm T0,-1/2}_{-1/2,t,\pm} = \mp L^{\rm T5,1/2}_{1/2,\pm,0} = \pm L^{\rm T5,-1/2}_{-1/2,\pm,0} = \mp\frac{\sqrt{q^2}\sin\theta_\ell}{\sqrt{2}}\,,$$

$$L^{\rm T0,1/2}_{1/2,\pm,0} = L^{\rm T0,-1/2}_{-1/2,\pm,0} = \mp L^{\rm T5,1/2}_{1/2,t,\pm} = \mp L^{\rm T5,-1/2}_{-1/2,t,\pm} = -\frac{\beta_\ell\sqrt{q^2}\sin\theta_\ell}{\sqrt{2}}\,,$$

$$L^{\rm T0,\pm1/2}_{\mp1/2,t,\pm} = \mp L^{\rm T5,\pm1/2}_{\mp1/2,\pm,0} = \pm\sqrt{2}(1-\cos\theta_\ell)m_\ell\,,$$

$$L^{\rm T0,\pm1/2}_{\mp1/2,t,\mp} = \pm L^{\rm T5,\pm1/2}_{\mp1/2,\mp,0} = \pm\sqrt{2}(1+\cos\theta_\ell)m_\ell\,,$$

$$L^{\rm T0,\pm1/2}_{\pm1/2,+,-} = -L^{\rm T5,\pm1/2}_{\pm1/2,t,0} = -\sqrt{q^2}\beta_\ell\cos\theta_\ell\,,$$

$$L^{\rm T0,\pm1/2}_{\pm1/2,t,0} = L^{\rm T5,\mp1/2}_{\mp1/2,+,-} = \mp\sqrt{q^2}\cos\theta_\ell\,,$$

$$L^{\rm T0,\pm1/2}_{\mp1/2,t,0} = L^{\rm T5,\pm1/2}_{\mp1/2,-,+} = 2\sin\theta_\ell m_\ell\,.$$

强子螺旋度振幅----$B \to K\ell^+\ell^-$

$$\langle K(k)|\bar{s}\gamma^\mu b|\bar{B}(p)\rangle = f_+(q^2)\left[(p_B+p_K)^\mu - \frac{m_B^2-m_K^2}{q^2}q^\mu\right]$$
$$+f_0(q^2)\frac{m_B^2-m_K^2}{q^2}q^\mu,$$

$$\langle K(k)|\bar{s}\gamma^\mu\gamma_5 b|\bar{B}(p)\rangle = 0,$$

$$\langle K(k)|\bar{s}b|\bar{B}(p)\rangle = f_0(q^2)\frac{m_B^2-m_K^2}{m_b-m_s},$$

$$\langle K(k)|\bar{s}\gamma_5 b|\bar{B}(p)\rangle = 0,$$

$$\langle K(k)|\bar{s}\sigma^{\mu\nu}b|\bar{B}(p)\rangle = -\mathrm{i}(p^\mu k^\nu - p^\nu k^\mu)\frac{2f_T(q^2)}{m_B+m_K},$$

$$\langle K(k)|\bar{s}\sigma^{\mu\nu}\gamma_5 b|\bar{B}(p)\rangle = -\varepsilon^{\mu\nu\rho\sigma}p_\rho k_\sigma\frac{2f_T(q^2)}{m_B+m_K}$$

$$\langle K(k)|\bar{s}\sigma^{\mu\nu}q_\nu b|\bar{B}(p)\rangle = -\mathrm{i}(p^\mu k^\nu - p^\nu k^\mu)\frac{2f_T(q^2)}{m_B+m_K}q_\nu,$$

$$\langle K(k)|\bar{s}\sigma^{\mu\nu}q_\nu\gamma_5 b|\bar{B}(p)\rangle = -\varepsilon^{\mu\nu\rho\sigma}p_\rho k_\sigma\frac{2f_T(q^2)}{m_B+m_K}q_\nu。$$

$$H_{SP}^{L(R)} = C_S^{L(R)}\frac{m_B^2-m_K^2}{m_b-m_s}f_0(q^2),$$

$$H_{VA,0}^{L(R)} = C_V^{L(R)}\sqrt{\frac{s_+s_-}{q^2}}f_+(q^2) - C_7^+\frac{\sqrt{q^2s_+s_-}}{m_B+m_K}f_T(q^2),$$

$$H_{VA,t}^{L(R)} = C_V^{L(R)}\frac{m_B^2-m_K^2}{\sqrt{q^2}}f_0(q^2),$$

$$H_{T,t,0}^{L(R)} = C_T^{L(R)}\frac{\sqrt{q^2s_+s_-}}{m_B+m_K}f_T(q^2)。$$

Sakaki Y, Tanaka M, Tayduganov A, et al. Phys.Rev.D, 2013, 88 (9): 094012.

形状因子误差分析----$B \to K\ell^+\ell^-$

$B \to K\ell^+\ell^-$衰变过程中有三个形状因子f_0、f_+、f_T，采用格点量子色动力学（Lattice Quantum chromodynamics, Lattice QCD）的结果，具体表达式为

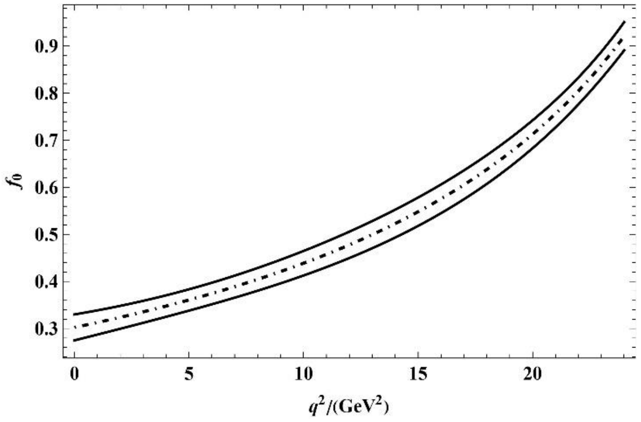
$$f_0(q^2) = \sum_k^3 a_k^0\, z(q^2)^k,$$

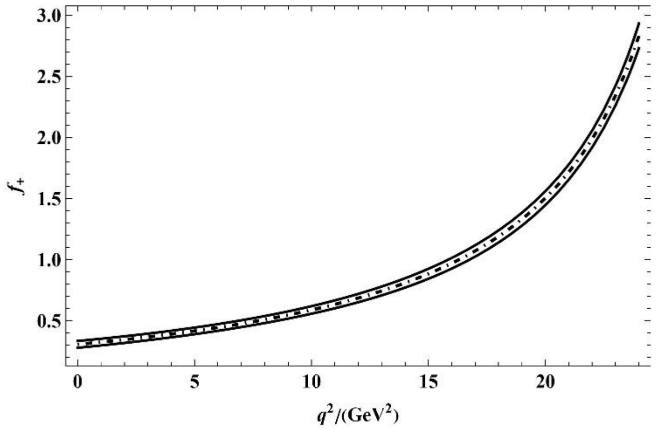
$$f_+(q^2) = \left(1 - q^2/\left(m_{pole}^f\right)^2\right)^{-1} \sum_k^2 a_k^+ \left[z(q^2)^k - (-1)^{k-3}\frac{k}{3} z(q^2)^3\right],$$

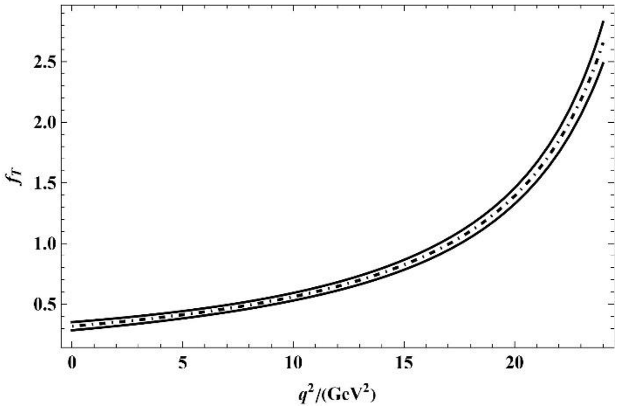
$$f_T(q^2) = \left(1 - q^2/\left(m_{pole}^f\right)^2\right)^{-1} \sum_k^2 a_k^T \left[z(q^2)^k - (-1)^{k-3}\frac{k}{3} z(q^2)^3\right]。$$

HPQCD Collaboration. Phys.Rev.D, 2013, 88 (5): 054509.

通过误差计算公式得到







微分衰变宽度

$$\frac{d\Gamma\left(B\to K^{(*)}l^+l^-\right)}{dq^2}=\frac{1}{2m_B}\sum_{\lambda}|\mathcal{M}|^2\,d\Phi,\qquad d\Phi=\frac{\sqrt{Q_+Q_-}}{256\pi^3m_B^2}\sqrt{1-\frac{4m_\ell^2}{q^2}}\,dq^2d\cos\theta$$

物理可观测量

(1) $\mathcal{B}\left(B\to K^{(*)}l^+l^-\right)=\tau_B\int_{4m_\ell^2}^{m_{\mathrm{B}}^2-m_{K^{(*)}}^2}dq^2\,\frac{d\Gamma}{dq^2},$

(2) $A_{FB}(q^2)=\frac{\int_0^1 d\cos\theta\left(d^2\Gamma/dq^2d\cos\theta\right)-\int_{-1}^0 d\cos\theta\left(d^2\Gamma/dq^2d\cos\theta\right)}{d\Gamma/dq^2},$

(3) $F_L(q^2)=\frac{\int_{-1}^{+1} d\cos\theta(2-5\cos^2\theta)\left(d^2\Gamma/dq^2d\cos\theta\right)}{\int_{-1}^{+1} d\cos\theta(d^2\Gamma/dq^2d\cos\theta)},$

(4) $P_L^{K^*}(q^2)=\frac{d\Gamma^{\lambda_{K^*}=0}/dq^2}{d\Gamma/dq^2},$

(5) $P_T^{K^*}(q^2)=\frac{d\Gamma^{\lambda_{K^*}=0}/dq^2-d\Gamma^{\lambda_{K^*}=-1}/dq^2}{d\Gamma/dq^2}.$

输入参数

Parameter	Value
G_F	$1.166378 \times 10^{-5}\ \mathrm{GeV}^{-2}$
m_B	5.27963 GeV
m_{K^*}	0.89495 GeV
m_e	0.511 MeV
m_μ	105.66 MeV
m_τ	1.7769 GeV
m_b	4.2 GeV
m_s	0.095 GeV
τ_B	2.49 ps
$V_{tb}V_{ts}^*$	0.0401
形状因子	LCSR、Lattice QCD

Wilson系数：

C1 = -0.294， C2 = 1.017， C3 = -0.0059， C4 = -0.087， C5 = 0.0004，

C6 = 0.0011，C7 = -0.324，C8 = -0.176， C9 = 4.114， C10 = -4.193。

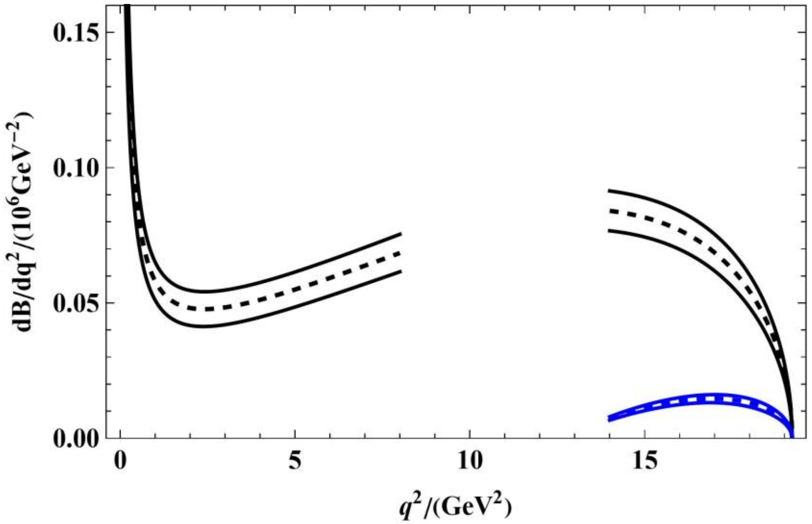
Hu Q Y, Li X Q, Yang Y D. EPJC, 2017, 77 (4): 228.

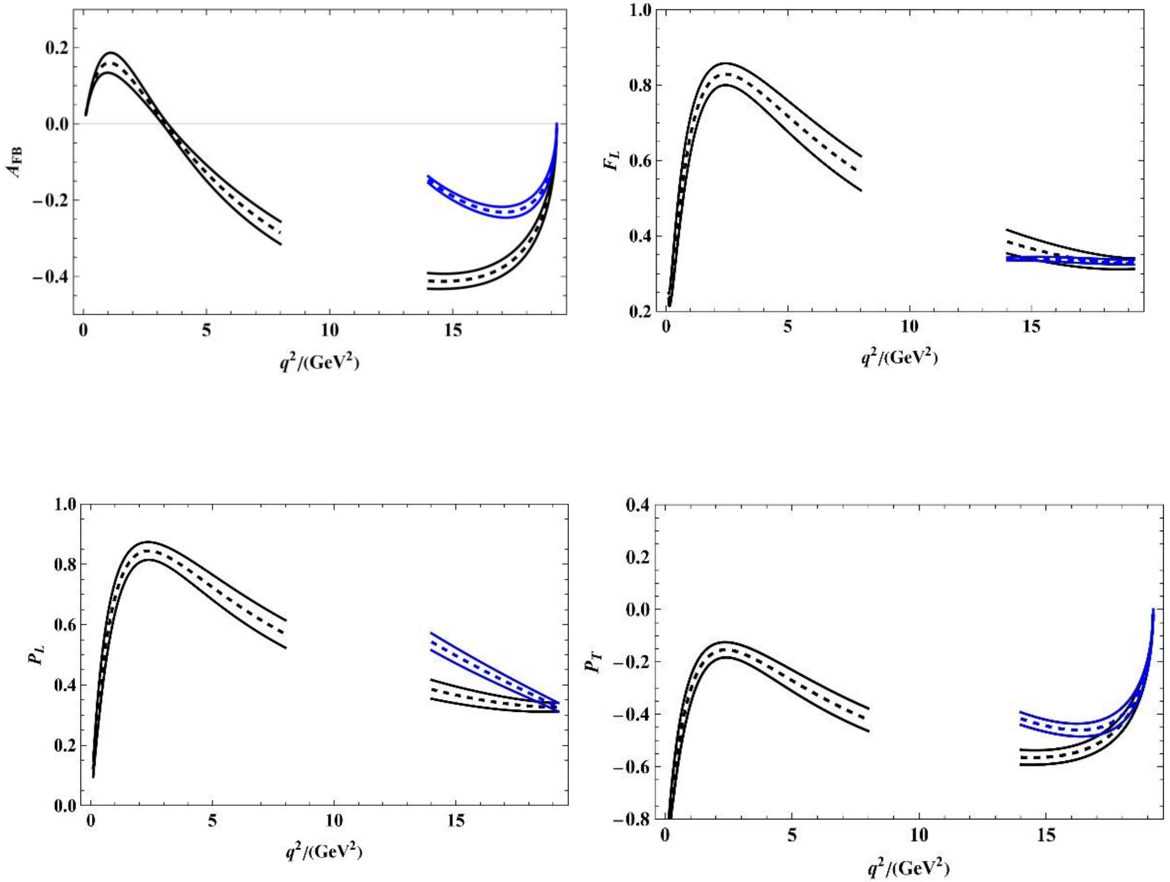
（$C_{7,9}^{\mathrm{eff}}$的表达式参考Du D, El-Khadra A X, Gottlieb S, et al. Physical Review D, 2016, 93(3): 034005.）

3.结果分析

Analysis

标准模型计算结果----$B \to K^*\ell^+\ell^-$





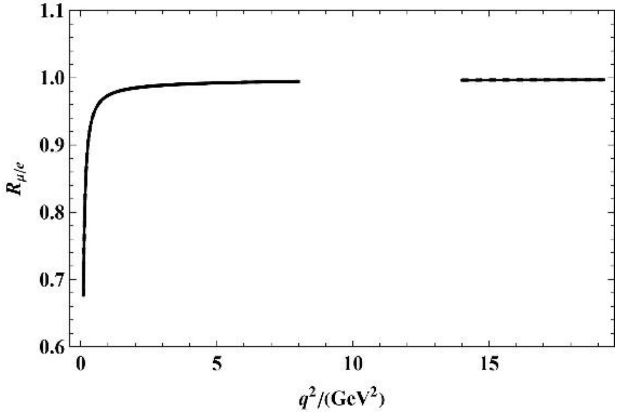
$$\mathcal{B}(B \to K^*\mu^+\mu^-)\Big|_{[0.045,1.1]}^{\mathrm{SM}} = (1.04 \pm 0.14) \times 10^{-7},$$

$$\mathcal{B}(B \to K^*\mu^+\mu^-)\Big|_{[1.1,6]}^{\mathrm{SM}} = (2.53 \pm 0.32) \times 10^{-7},$$

$$\mathcal{B}(B \to K^*\mu^+\mu^-)\Big|_{[14,(m_B-m_{K^*})^2]}^{\mathrm{SM}} = (3.43 \pm 0.33) \times 10^{-7},$$

$$\mathcal{B}(B \to K^*\tau^+\tau^-)\Big|_{[14,(m_B-m_{K^*})^2]}^{\mathrm{SM}} = (0.62 \pm 0.06) \times 10^{-7}。$$

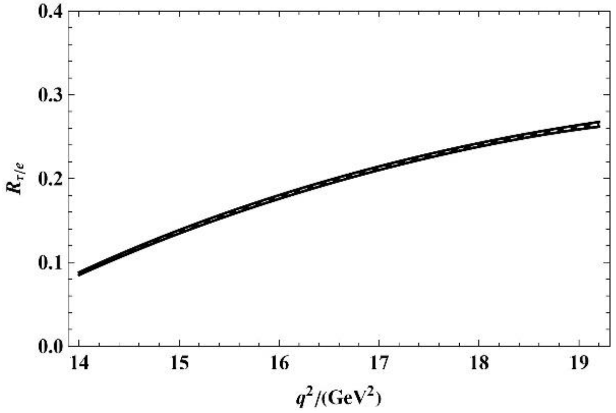
标准模型计算结果----$B \to K^{*}\ell^{+}\ell^{-}$



$$R_{K^*}^{\mu/e}\Big|_{[0.045,1.1]}^{\mathrm{SM}} = \frac{\mathcal{B}(B \to K^*\mu^+\mu^-)|_{[0.045,1.1]}^{\mathrm{SM}}}{\mathcal{B}(B \to K^*e^+e^-)|_{[0.045,1.1]}^{\mathrm{SM}}} = 0.8030 \pm 0.0087,$$

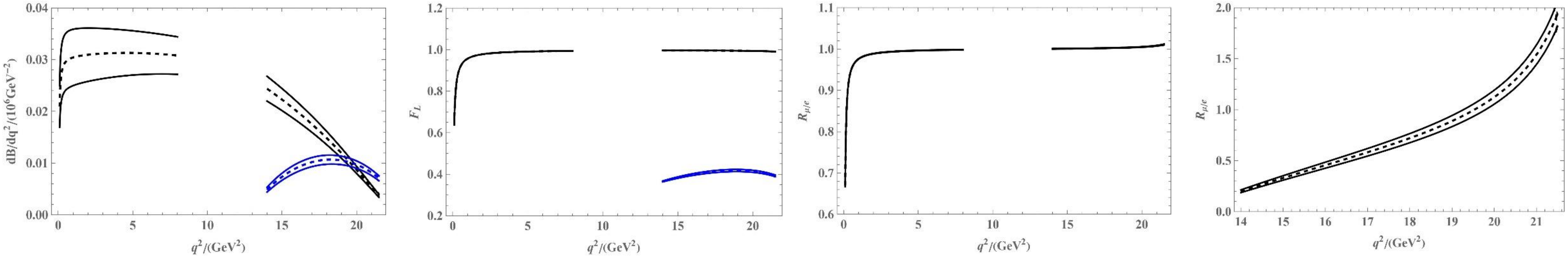
$$R_{K^*}^{\mu/e}\Big|_{[1.1,6]}^{\mathrm{SM}} = \frac{\mathcal{B}(B \to K^*\mu^+\mu^-)|_{[1.1,6]}^{\mathrm{SM}}}{\mathcal{B}(B \to K^*e^+e^-)|_{[1.1,6]}^{\mathrm{SM}}} = 0.9887 \pm 0.0005,$$

$$R_{K^*}^{\mu/e}\Big|_{[14,(m_{\mathrm{B}}-m_{K^*})^2]}^{\mathrm{SM}} = \frac{\mathcal{B}(B \to K^*\mu^+\mu^-)|_{[14,(m_{\mathrm{B}}-m_{K^*})^2]}^{\mathrm{SM}}}{\mathcal{B}(B \to K^*e^+e^-)|_{[14,(m_{\mathrm{B}}-m_{K^*})^2]}^{\mathrm{SM}}} = 0.9967 \pm 0,$$



$$R_{K^*}^{\tau/e}\Big|_{[14,(m_{\mathrm{B}}-m_{K^*})^2]}^{\mathrm{SM}} = \frac{\mathcal{B}(B \to K^*\tau^+\tau^-)|_{[14,(m_{\mathrm{B}}-m_{K^*})^2]}^{\mathrm{SM}}}{\mathcal{B}(B \to K^*e^+e^-)|_{[14,(m_{\mathrm{B}}-m_{K^*})^2]}^{\mathrm{SM}}} = 0.1794 \pm 0.0030。$$

标准模型计算结果----$B \to K\ell^+\ell^-$



$$\mathcal{B}(B \to K\mu^+\mu^-)\Big|_{[1.1,6]}^{\mathrm{SM}} = (1.52 \pm 0.23) \times 10^{-7},$$

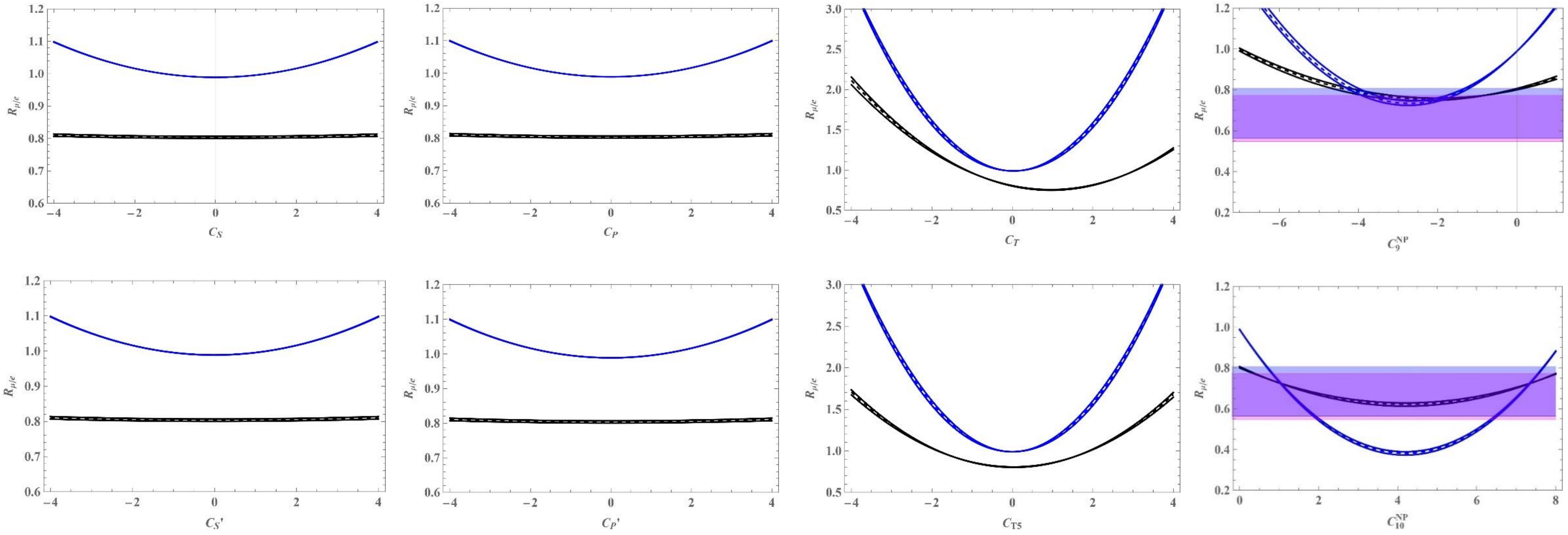
$$\mathcal{B}(B \to K\tau^+\tau^-)\Big|_{[14,(m_B-m_K)^2]}^{\mathrm{SM}} = (0.68 \pm 0.06) \times 10^{-7}。$$

$$R_K^{\mu/e}\Big|_{[1.1,6]}^{\mathrm{SM}} = \frac{\mathcal{B}(B \to K\mu^+\mu^-)|_{[1.1,6]}^{\mathrm{SM}}}{\mathcal{B}(B \to Ke^+e^-)|_{[1.1,6]}^{\mathrm{SM}}} = 0.9927 \pm 0.0002,$$

$$R_K^{\tau/e}\Big|_{[14,(m_{\mathrm{B}}-m_K)^2]}^{\mathrm{SM}} = \frac{\mathcal{B}(B \to K\tau^+\tau^-)|_{[14,(m_{\mathrm{B}}-m_K)^2]}^{\mathrm{SM}}}{\mathcal{B}(B \to Ke^+e^-)|_{[14,(m_{\mathrm{B}}-m_K)^2]}^{\mathrm{SM}}} = 0.6080 \pm 0.0043。$$

新物理对$B \to K^*\mu^+\mu^-$中$R_{K^*}^{\mu/e}$的影响----一维假设

[0.045，1.1]
[1.1，6]

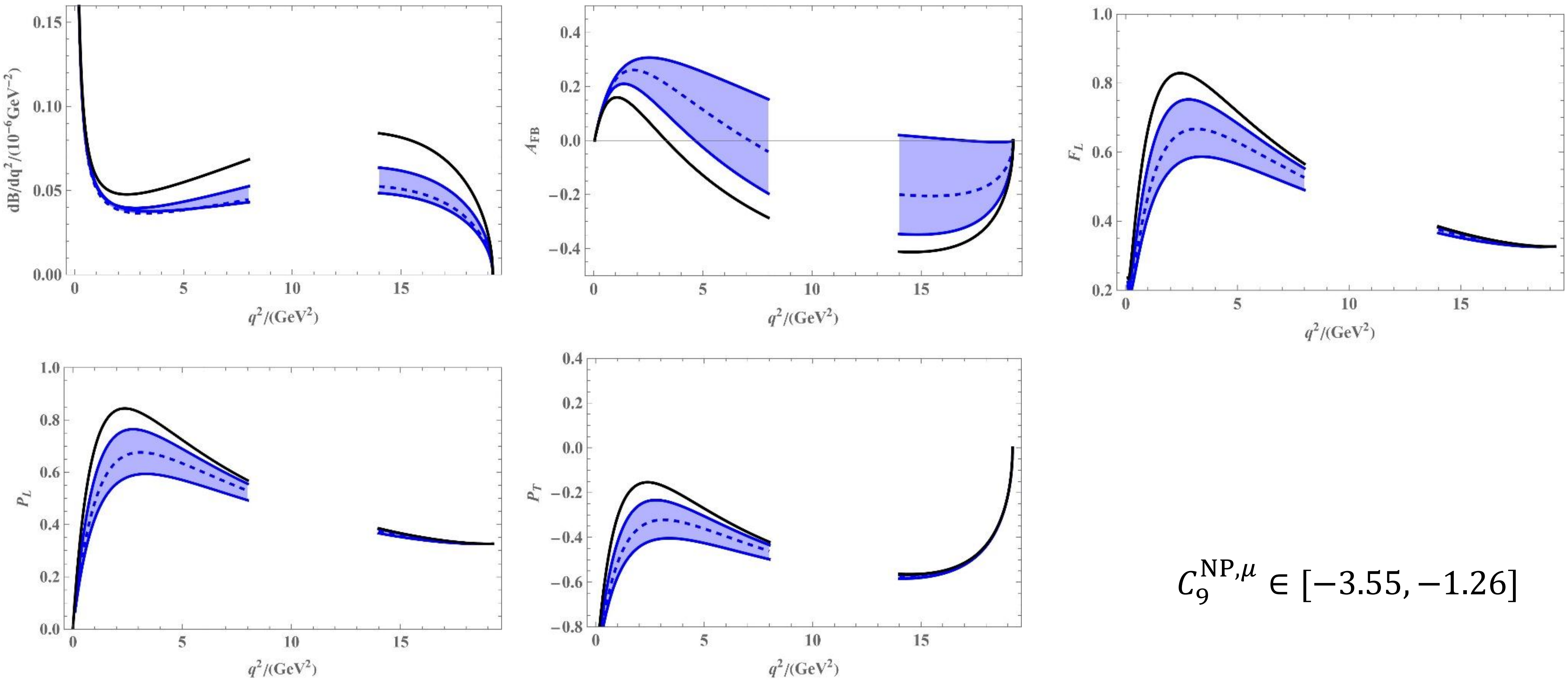


$q^2 = [0.045\text{GeV}^2, 1.1\text{GeV}^2]$： $C_9^{\text{NP},\mu} \in [-3.55, -0.78]$，$C_{10}^{\text{NP},\mu} \in [0.34, 8.00]$；

$q^2 = [1.1\text{GeV}^2, 6\text{GeV}^2]$： $C_9^{\text{NP},\mu} \in [-4.17, -1.26]$，$C_{10}^{\text{NP},\mu} \in [0.68, 1.93] \cup [6.49, 7.70]$。

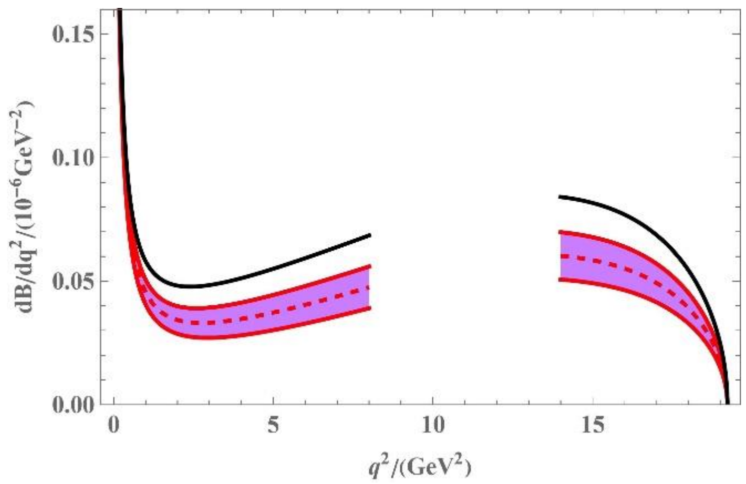
$\boldsymbol{C_9^{\text{NP},\mu} \in [-3.55, -1.26]}$，$\boldsymbol{C_{10}^{\text{NP},\mu} \in [0.68, 1.93] \cup [6.49, 7.70]}$

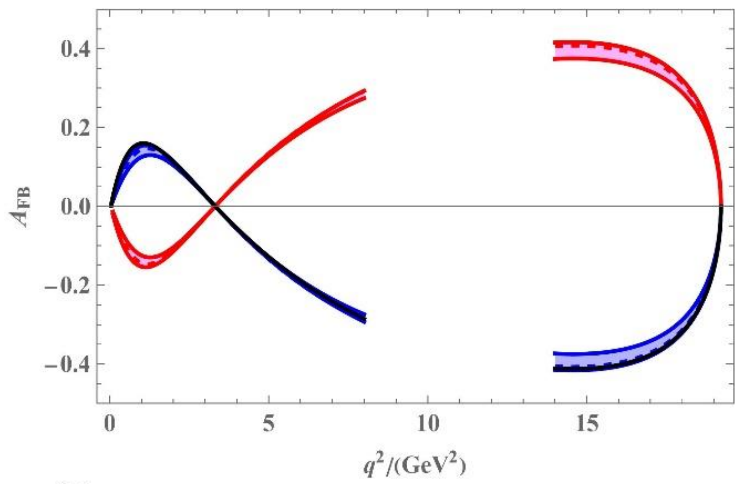
$C_9^{\mathrm{NP},\mu}$取特定值$B \to K^*\mu^+\mu^-$中其他可观测量结果

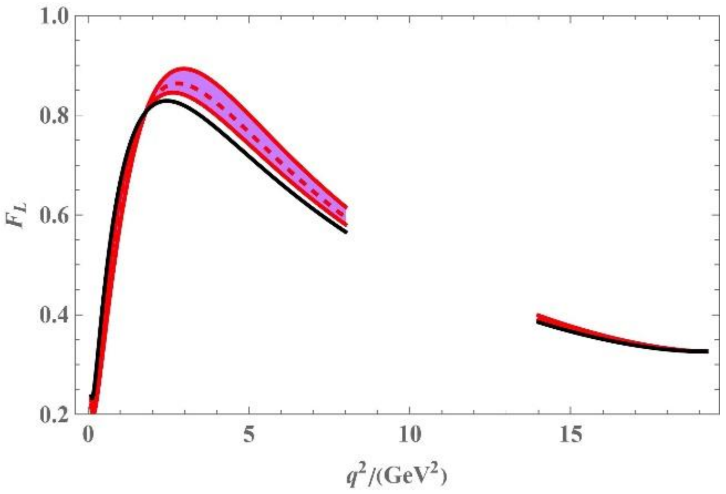


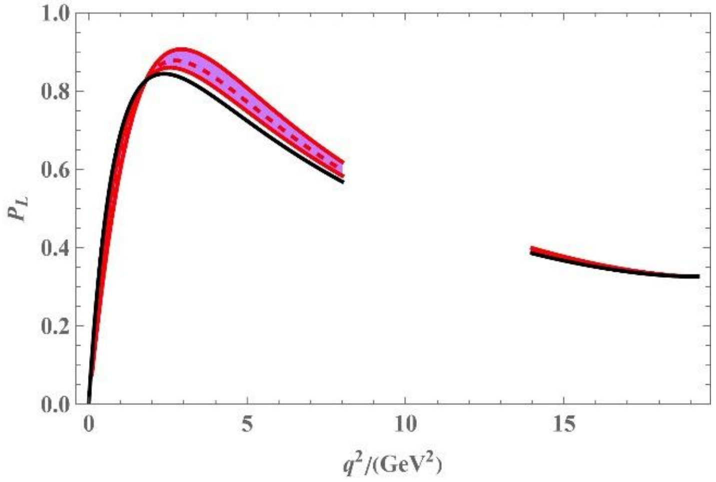
$$C_9^{\mathrm{NP},\mu} \in [-3.55, -1.26]$$

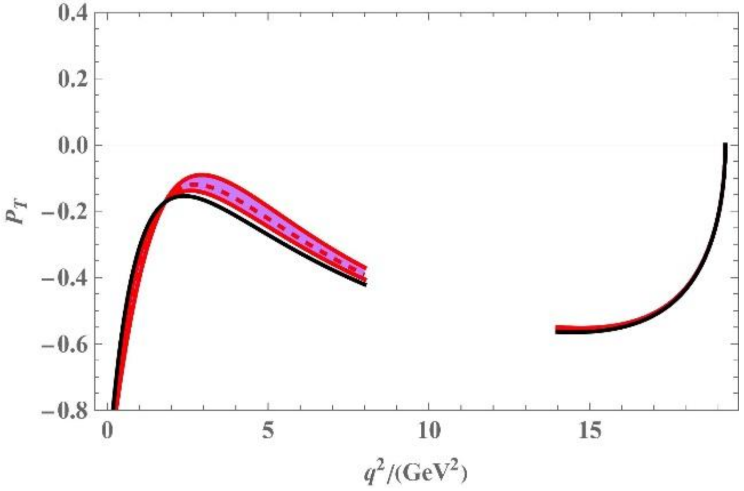
$C_{10}^{\mathrm{NP},\mu}$取特定值$B \to K^*\mu^+\mu^-$中其他可观测量结果





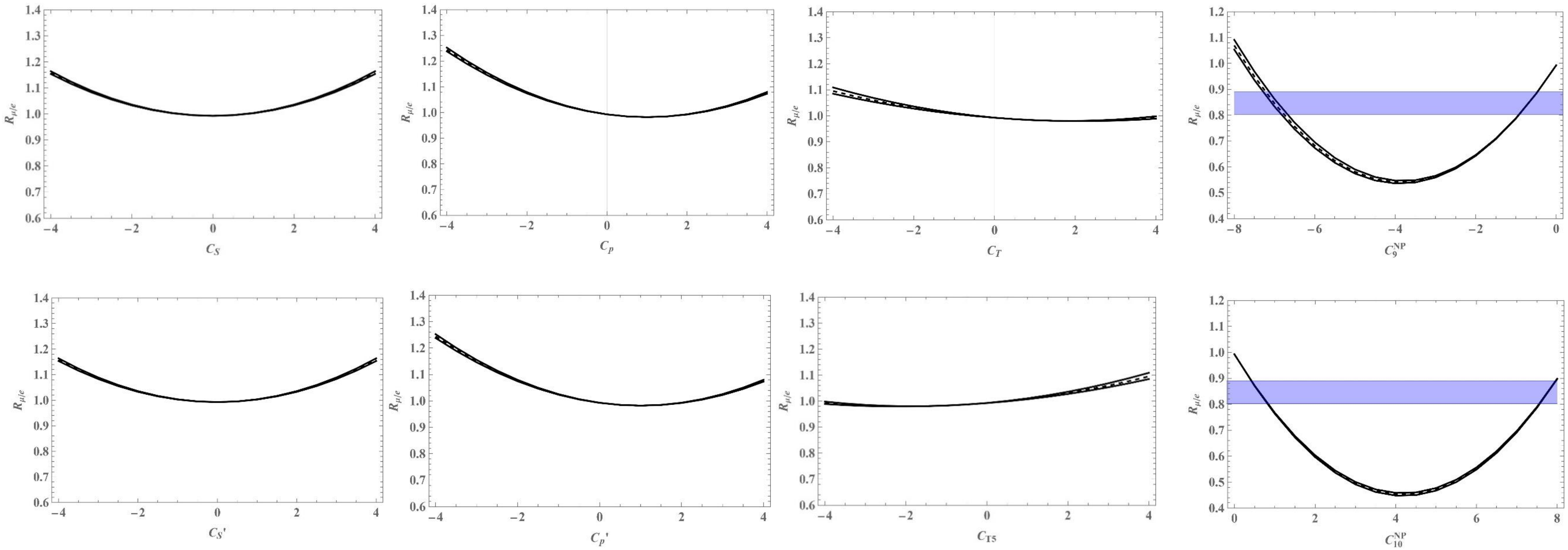






$C_{10}^{\mathrm{NP},\mu} \in [0.68, 1.93\,] \cup [6.49, 7.70]$

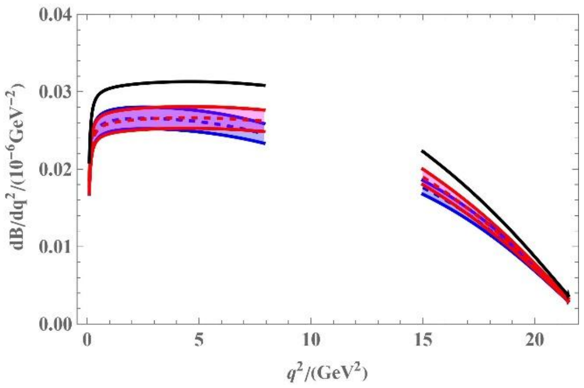
新物理对$B \to K\mu^+\mu^-$中$R_K^{\mu/e}$的影响

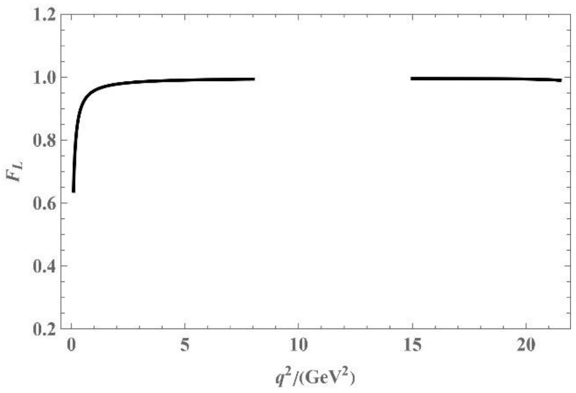


$q^2 = [1.1\text{GeV}^2, 6\text{GeV}^2]$： $C_9^{\text{NP},\mu} \in [-7.20, -6.74] \cup [-0.92, -0.69]$,

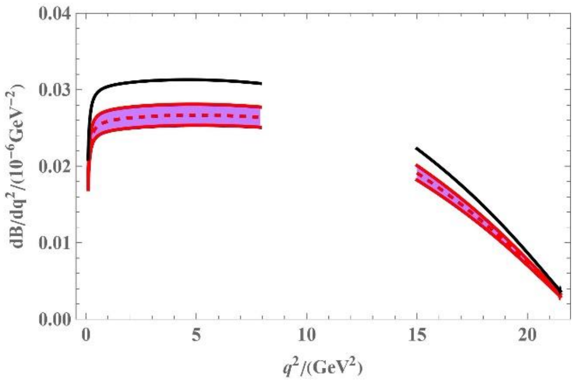
$C_{10}^{\text{NP},\mu} \in [0.42, 0.82] \cup [7.57, 7.77\,]$

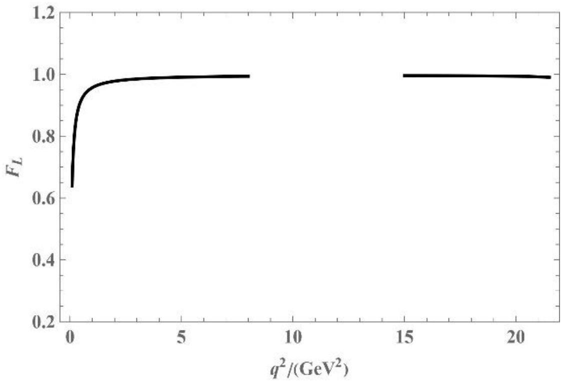
$C_{9}^{\mathrm{NP},\mu}$，$C_{10}^{\mathrm{NP},\mu}$取特定值$B \to K\mu^{+}\mu^{-}$中其他可观测量结果





$C_{9}^{\mathrm{NP},\mu} \in [-7.20, -6.74] \cup [-0.92, -0.69]$

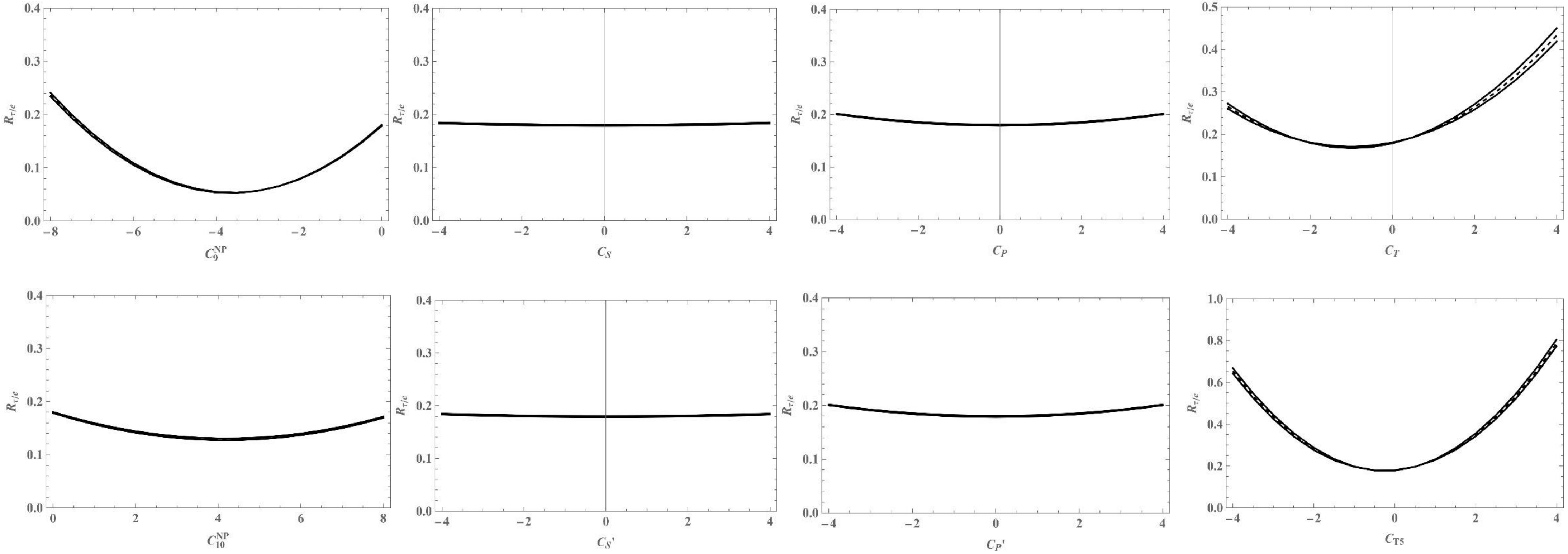




$C_{10}^{\mathrm{NP},\mu} \in [0.42, 0.82] \cup [7.57, 7.77\ ]$

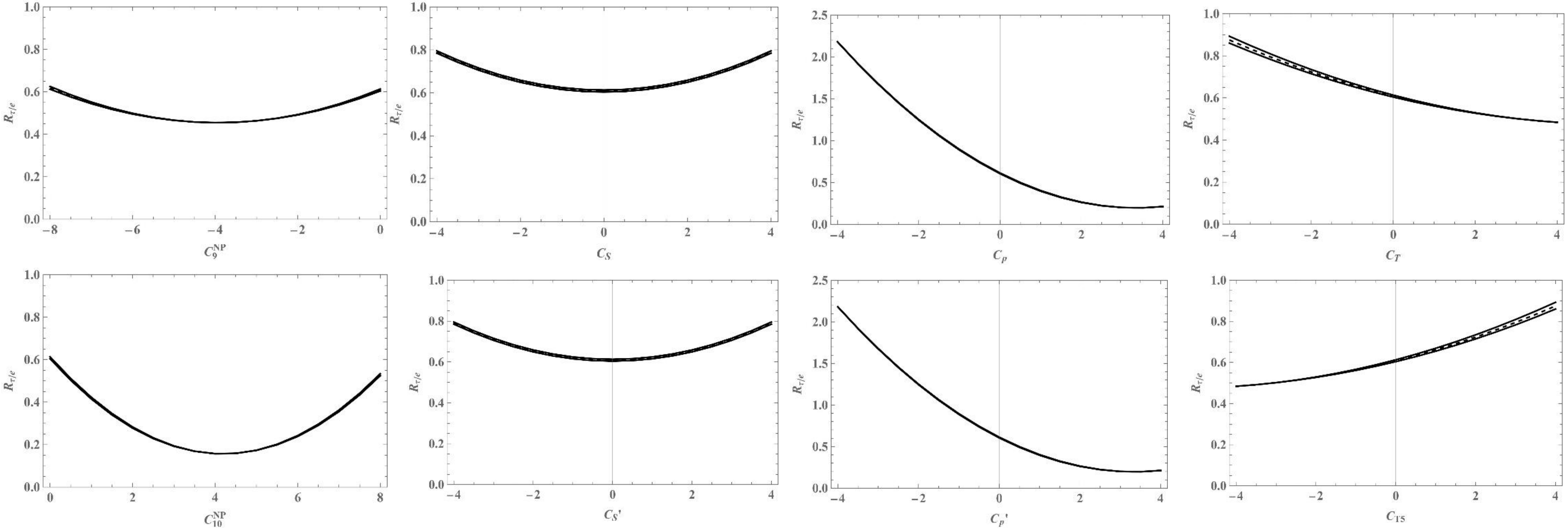
新物理对$B \to K^*\tau^+\tau^-$中$R_{K^*}^{\tau/e}$的影响

$q^2 = [14\text{GeV}^2, (m_\text{B} - m_{K^*})^2]$



新物理对$B \to K\tau^+\tau^-$中$R_K^{\tau/e}$的影响

$q^2 = [14\mathrm{GeV}^2, (m_\mathrm{B} - m_K)^2]$



4.总结展望

Summary

- 本文利用低能有效理论计算了$B \to K^{(*)}\ell^+\ell^-$过程的螺旋度振幅，得到了**标准模型**的结果。
- 假设新物理粒子不与第一代轻子发生耦合，基于**一维假设**详细讨论了$\boldsymbol{B \to K^*\mu^+\mu^-}$过程的新物理贡献，利用最新测量的$R_{K^*}^{\mu/e}$数值，得到$C_9^{\mathrm{NP},\mu} \in [-3.55,-1.26]$，$C_{10}^{\mathrm{NP},\mu} \in [0.68,1.93] \cup [6.49,7.70]$，并在此参数空间计算了其它物理观测量，如微分宽度，轻子前后不对称性，末态轻子和强子的极化等；利用同样的技术分析了$\boldsymbol{B \to K\mu^+\mu^-}$过程，得到$C_9^{\mathrm{NP},\mu} \in [-7.20,-6.74] \cup [-0.92,-0.69]$，$C_{10}^{\mathrm{NP},\mu} \in [0.42,0.82] \cup [7.57,7.77]$，发现$C_{10}^{\mathrm{NP},\mu}$的共同参数空间$[0.68,0.82] \cup [7.57,7.70]$。
- 讨论了**一维假设**下新物理算符对$\boldsymbol{B \to K^{(*)}\tau^+\tau^-}$的影响，供未来实验进行测量。

随着西欧大型强子对撞机和日本Belle II 数据的累计，本文的计算结果将有助于在重味物理上寻找新物理信号并辨别新物理模型，为寻找超出标准模型提供理论支持。

谢谢！