

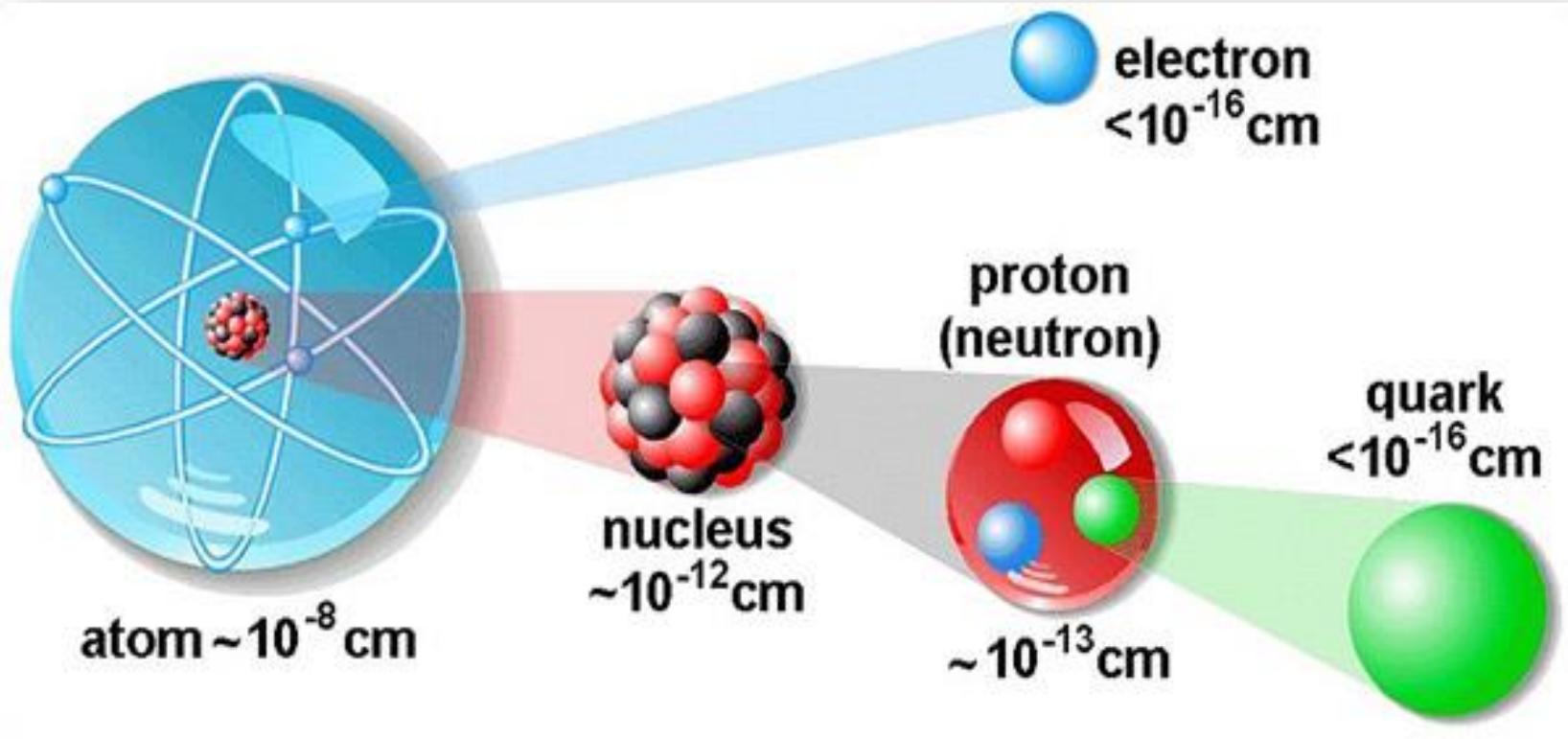
# Dynamical Models & the fluid nature of the QGP

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第10届华大QCD讲席班

线上, 2022年10月30–11月4日



# Landscape of nuclear physics

degrees of freedom

**quarks  
& gluons**



Energy  
(MeV)

940  
neutron mass

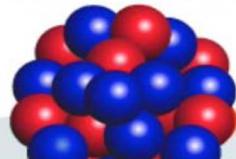
constituent quarks

**hadrons**



140  
pion mass

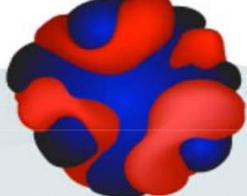
baryons, mesons



8  
proton separation  
energy in lead

protons, neutrons

**nuclei**



1.32  
vibrational  
state in tin

nucleonic densities  
and currents

0.043  
rotational  
state in uranium

collective coordinates

# Landscape of nuclear physics

degrees of freedom

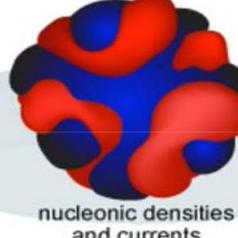
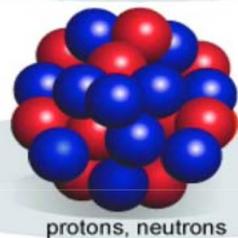
quarks  
& gluons



hadrons



nuclei



Energy  
(MeV)

940  
neutron mass

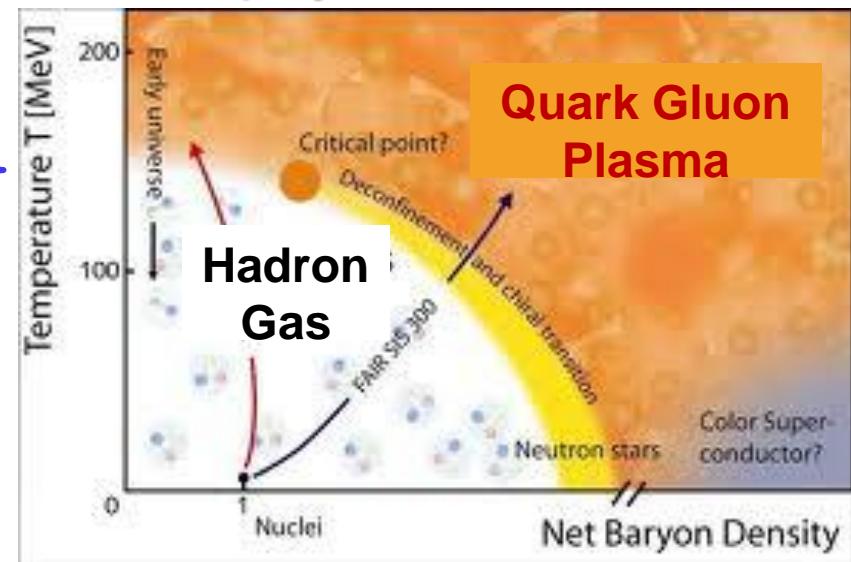
140  
pion mass

8  
proton separation  
energy in lead

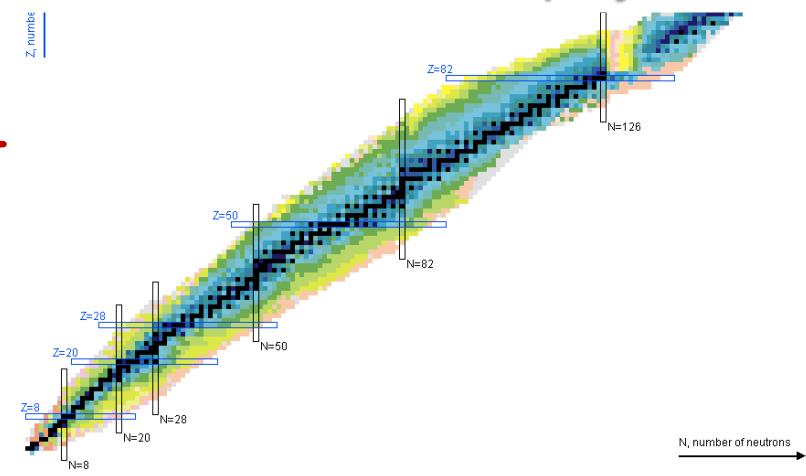
1.32  
vibrational  
state in tin

0.043  
rotational  
state in uranium

-intermediate and high energy  
nuclear physics



-nuclear structure physics



# degrees of freedom

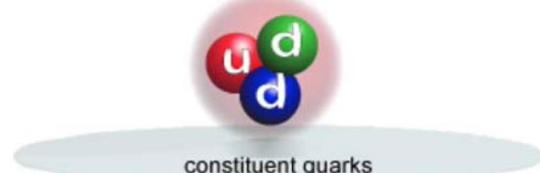
quarks  
& gluons



Energy  
(MeV)

940

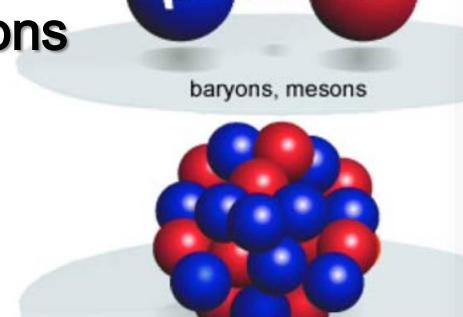
neutron mass



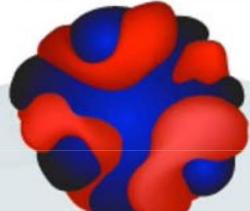
hadrons



140



nuclei



atom  $\sim 10^{-8}$  cm

0.043

rotational state in uranium



Quark and Gluons: confined in proton and neutrons through strong forces described by QCD

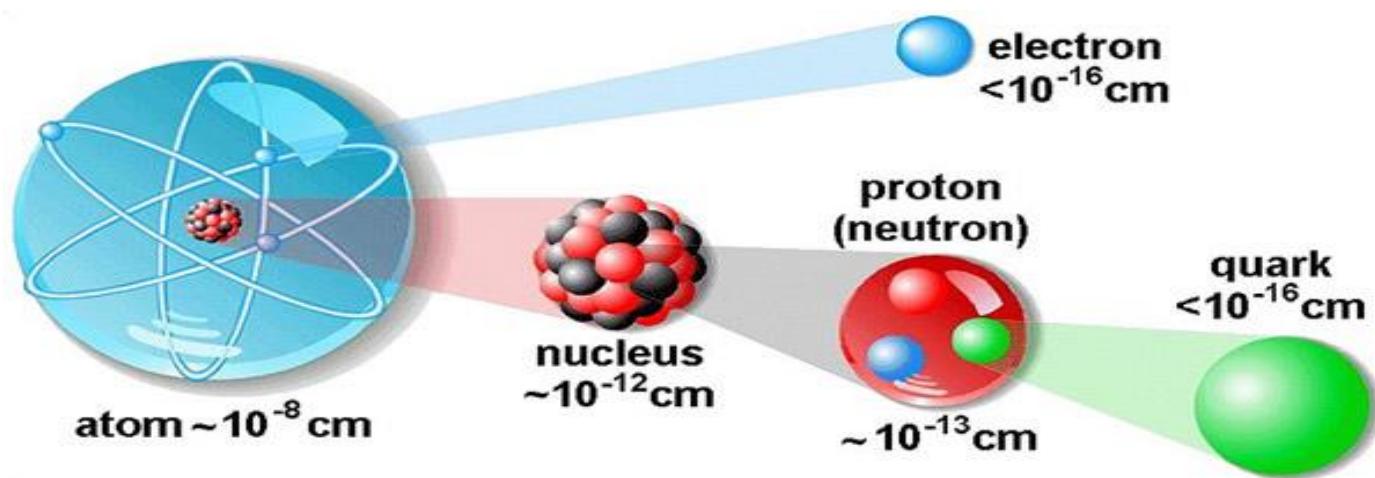
electron  
 $<10^{-16}$  cm

proton  
(neutron)

quark  
 $<10^{-16}$  cm

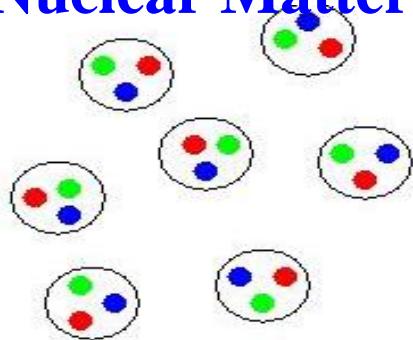
nucleus  
 $\sim 10^{-12}$  cm

$\sim 10^{-13}$  cm



**Quark and Gluons:** confined in proton and neutrons through strong forces described by QCD

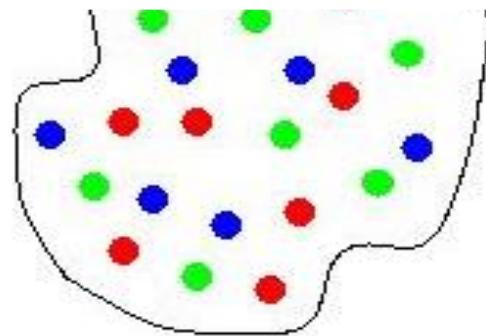
### Nuclear Matter



Phase Transition

$$T_c \sim 2 \times 10^{12} \text{ K}$$

### Quark Gluon Plasma

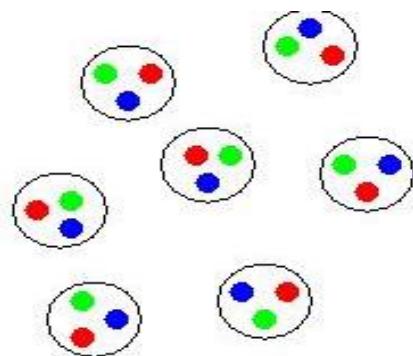


Confinement

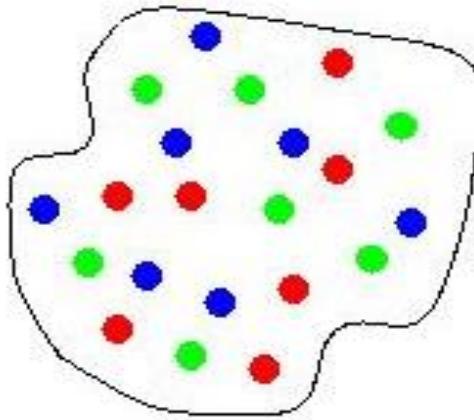
Deconfinement

**QGP (quark gluon plasma):** a deconfinement phase of the QCD matter

## Nuclear Matter



## Quark Gluon Plasma



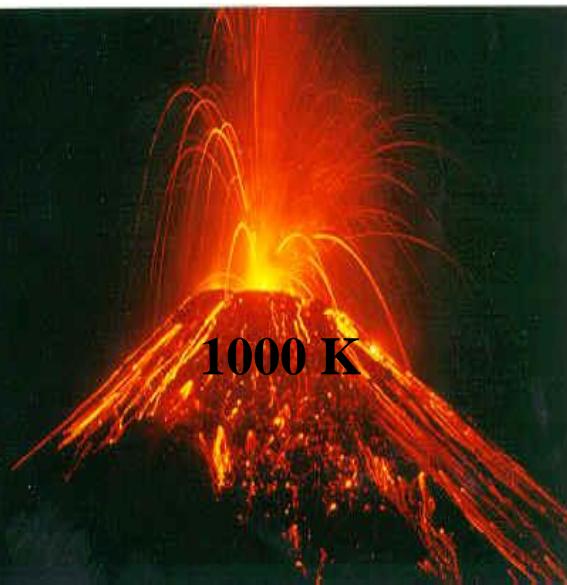
Phase Transition

$$T_c \sim 2 \times 10^{12} \text{ K}$$

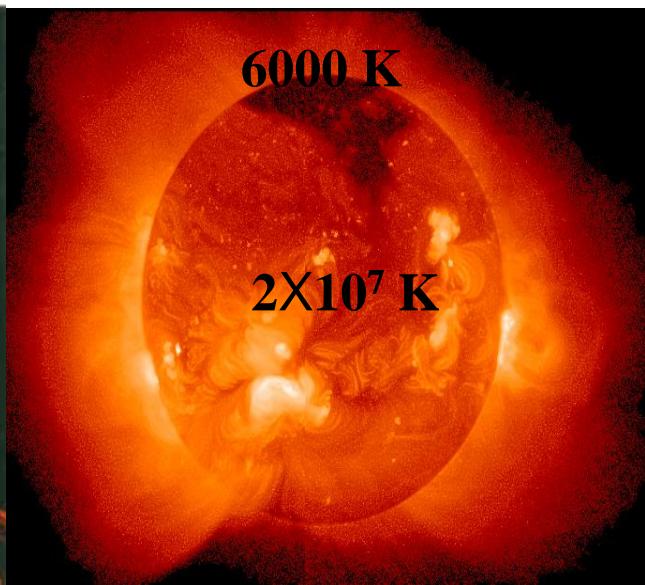
Confinement

Deconfinement

**QGP (quark gluon plasma):** a deconfinement phase of the QCD matter

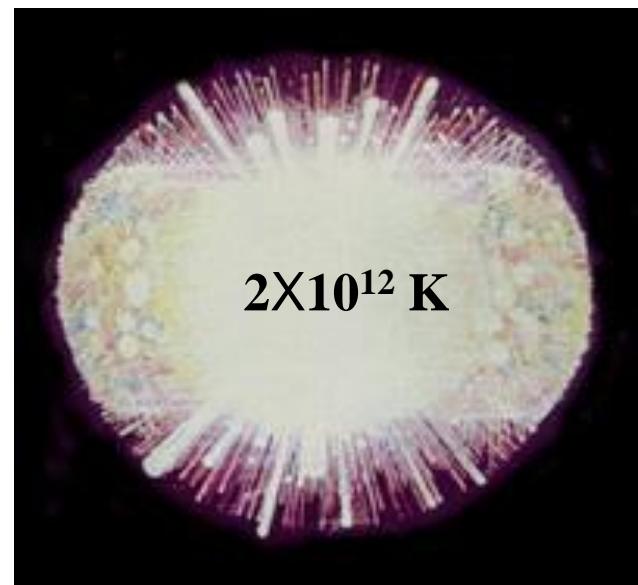


1000 K

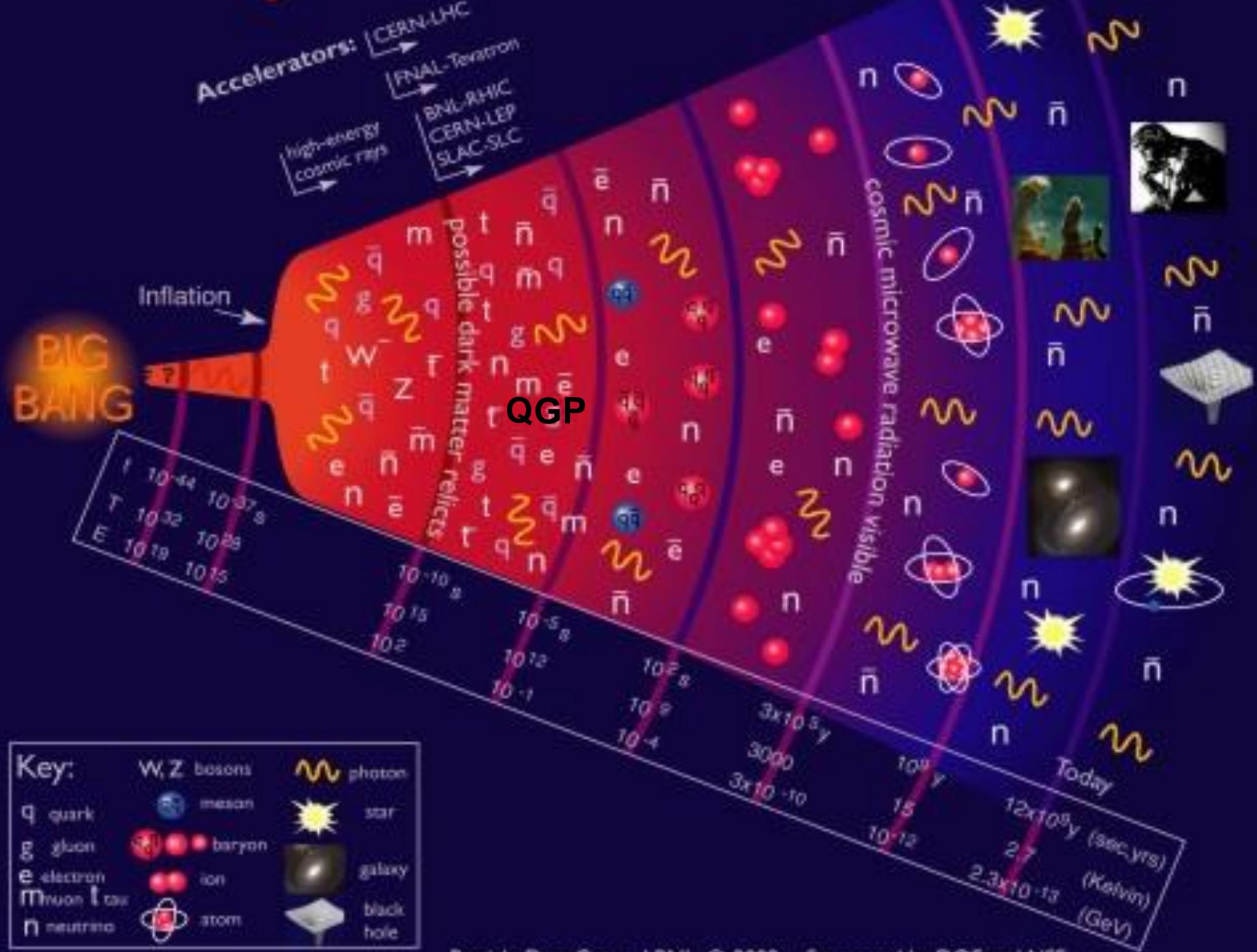


6000 K

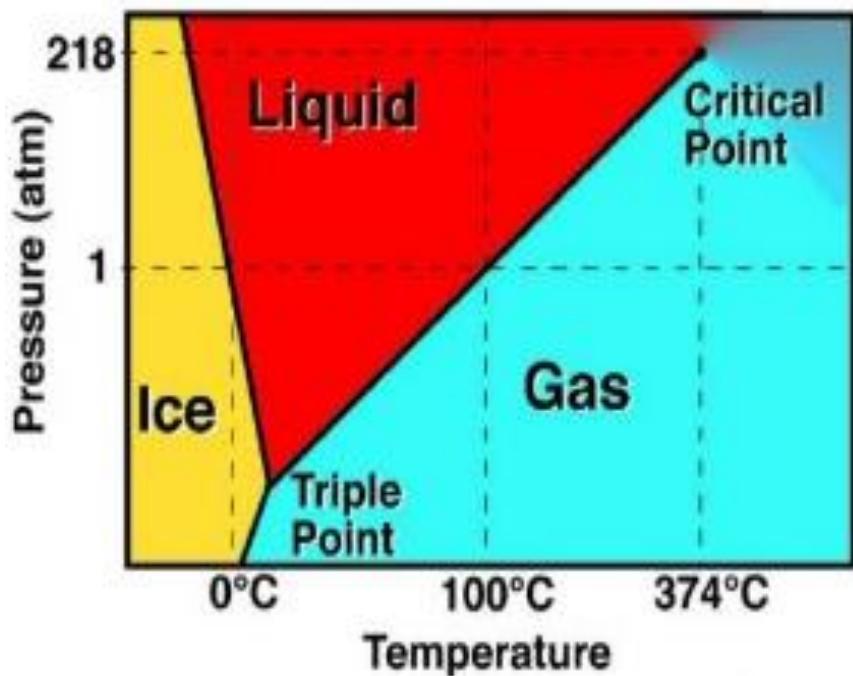
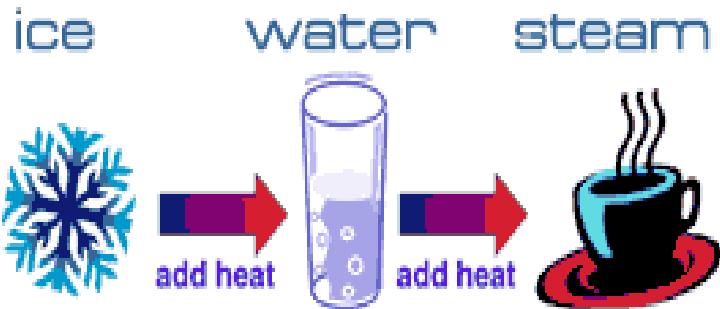
$2 \times 10^7 \text{ K}$



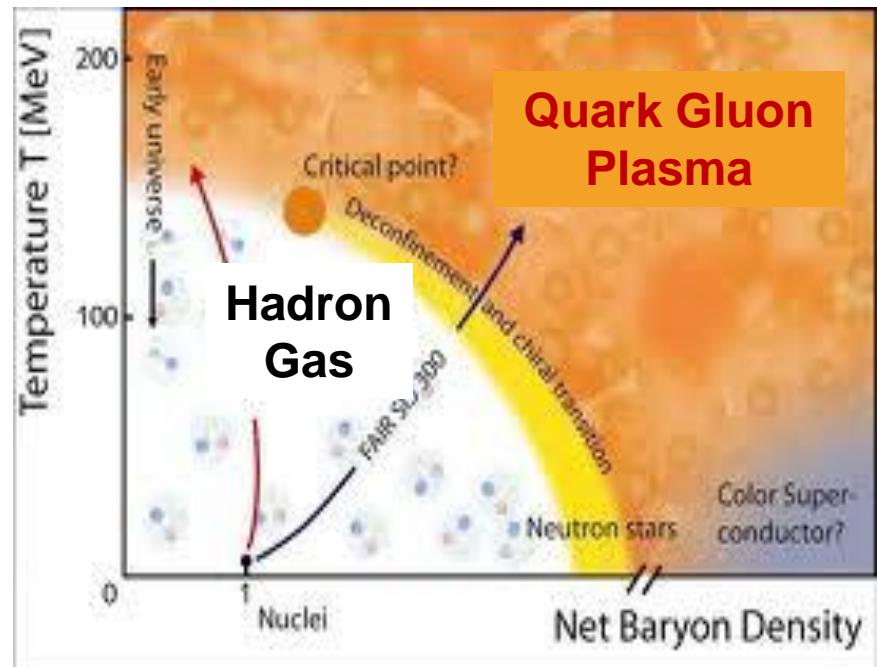
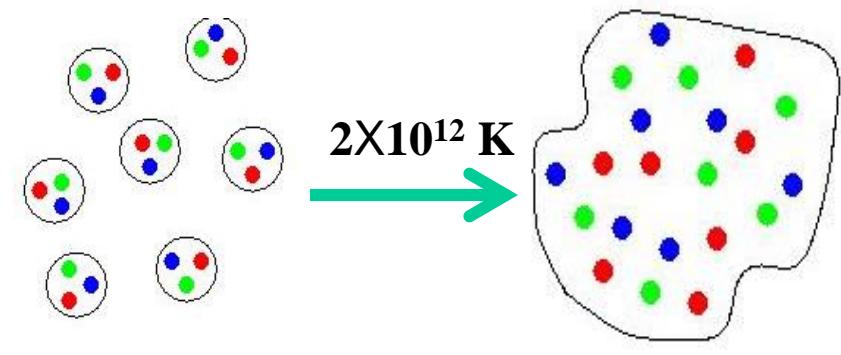
$2 \times 10^{12} \text{ K}$



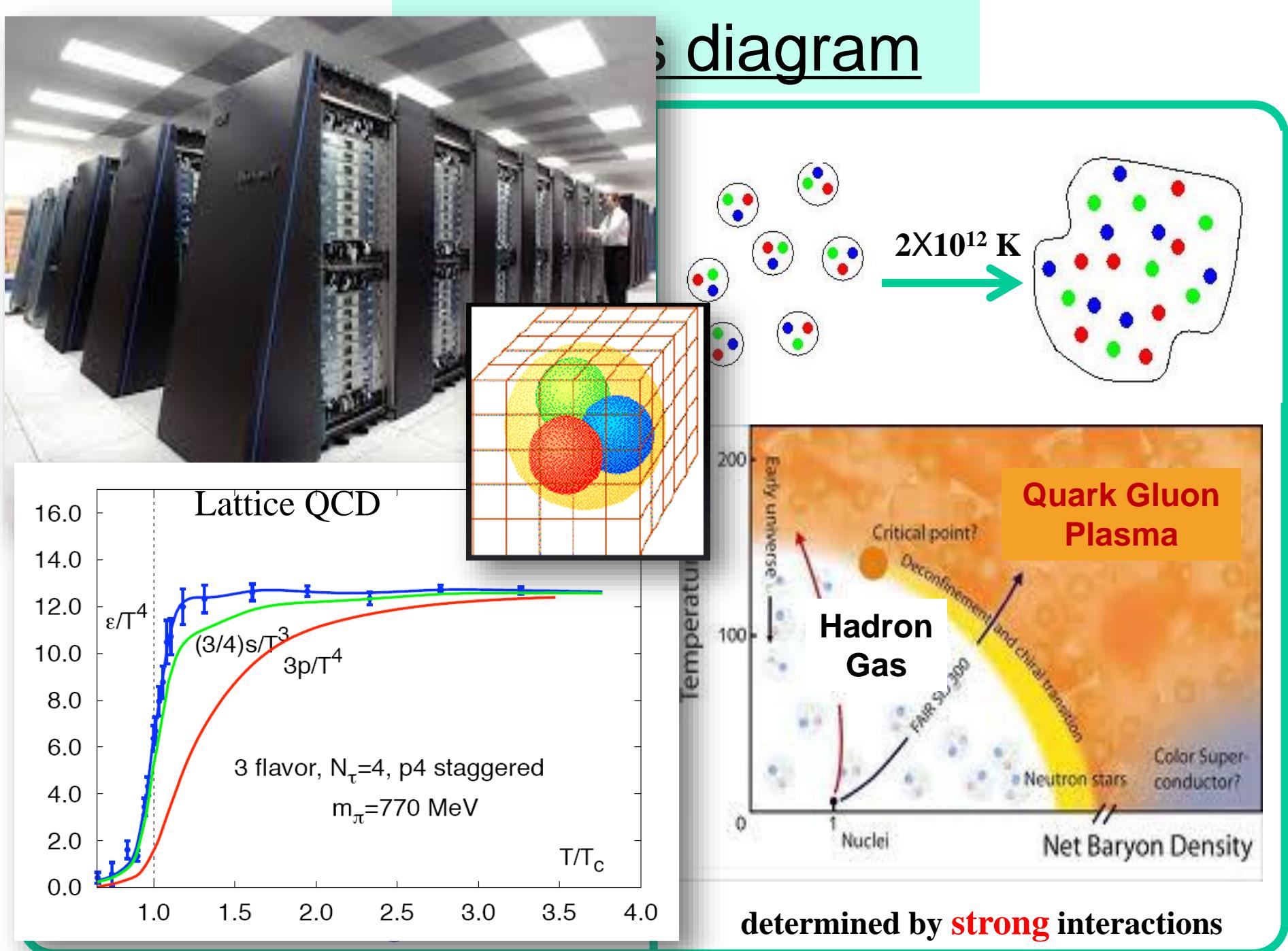
# Phases diagram



determined by **electromagnetic** interactions



determined by **strong** interactions



# QCD Phase transition

$$\varepsilon = g \frac{\pi^2}{30} T^4$$

Energy density for “g” massless d.o.f.

$$\varepsilon = 3 \cdot \frac{\pi^2}{30} T^4$$

Hadronic Matter:  
3  $\pi$  with spin=0

$$\varepsilon = 37 \cdot \frac{\pi^2}{30} T^4$$

Quark Gluon Plasma:

# QCD Phase transition

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$$\varepsilon = 37 \cdot \frac{\pi^2}{30} T^4$$

Quark Gluon Plasma:

$$\varepsilon = g \frac{\pi^2}{30} T^4$$

$$= \left\{ 2 \cdot 8_g + \frac{7}{8} \cdot 2_s \cdot 2_a \cdot 2_f \cdot 3_c \right\} \frac{\pi^2}{30} T^4$$

**8 gluons, 2 spins;  
2 quark flavors, anti-quarks,  
2 spins, 3 colors**

$$= 37 \cdot \frac{\pi^2}{30} T^4$$

**d.o.f : 3  $\rightarrow$  37 (!)**

$$\varepsilon = g \frac{\pi^2}{30} T^4$$

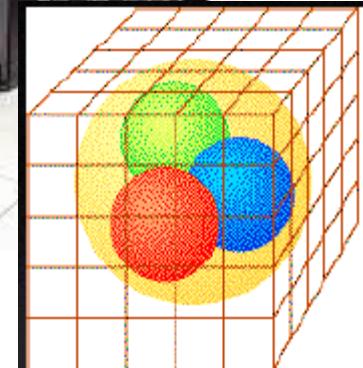
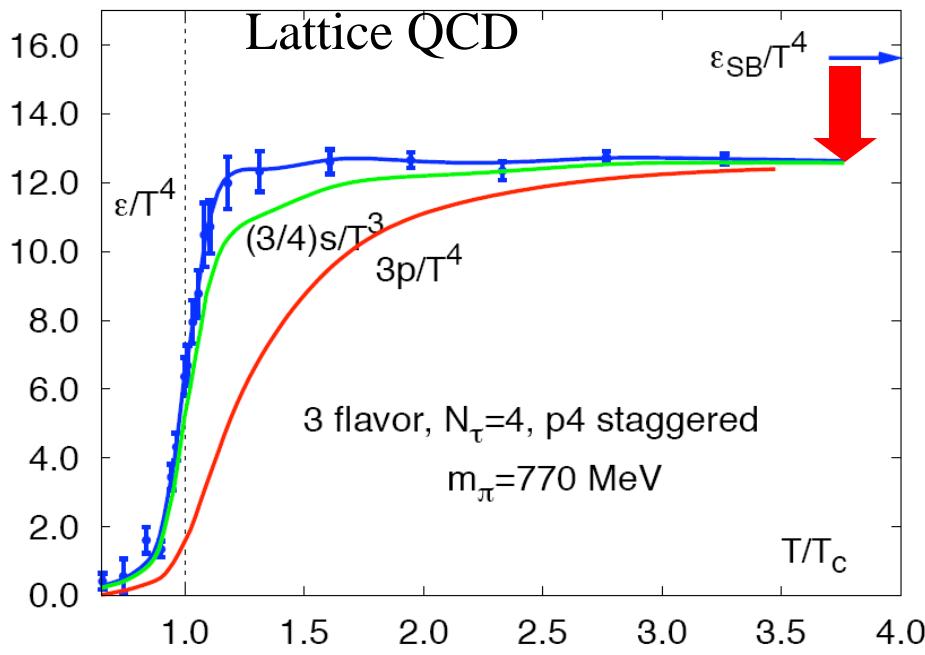
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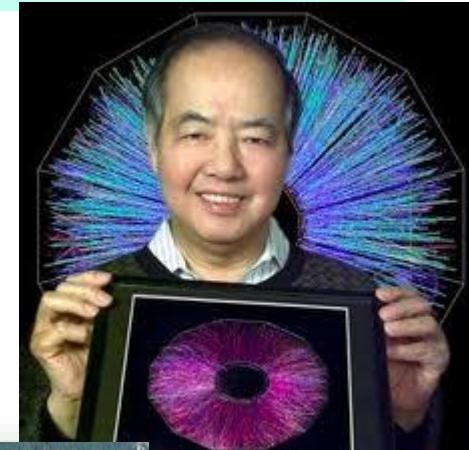
Quark Gluon Plasma:  
8 gluons;  
2 quark flavors, antiquarks, 2 spins, 3 colors



# A brief history of relativistic heavy ion physics

1974: Workshop on “GeV/nucleon collisions of heavy ions”

We should investigate.... phenomena by distributing energy of high nucleon density of a relatively large volume”  
---T.D.Lee



1984: SPS starts, (end 2003)

1986: AGS starts, (end 2000)

2000: RHIC starts

2010: LHC starts

Future: FAIR & NICA



# A brief history of relativistic heavy ion physics

1974: Workshop on “GeV/nucleon collisions of heavy ions”

We should investigate.... phenomena by distributing energy of high nucleon density of a relatively large volume”  
---T.D.Lee



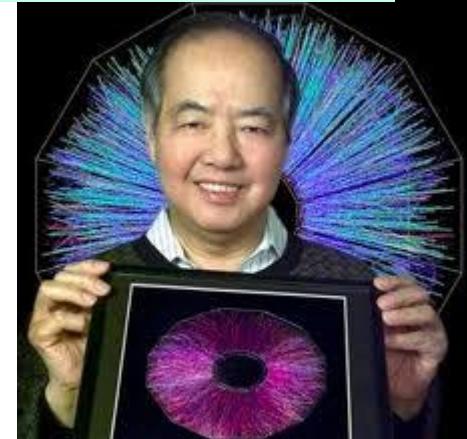
LHC, CERN



# A brief history of relativistic heavy ion physics

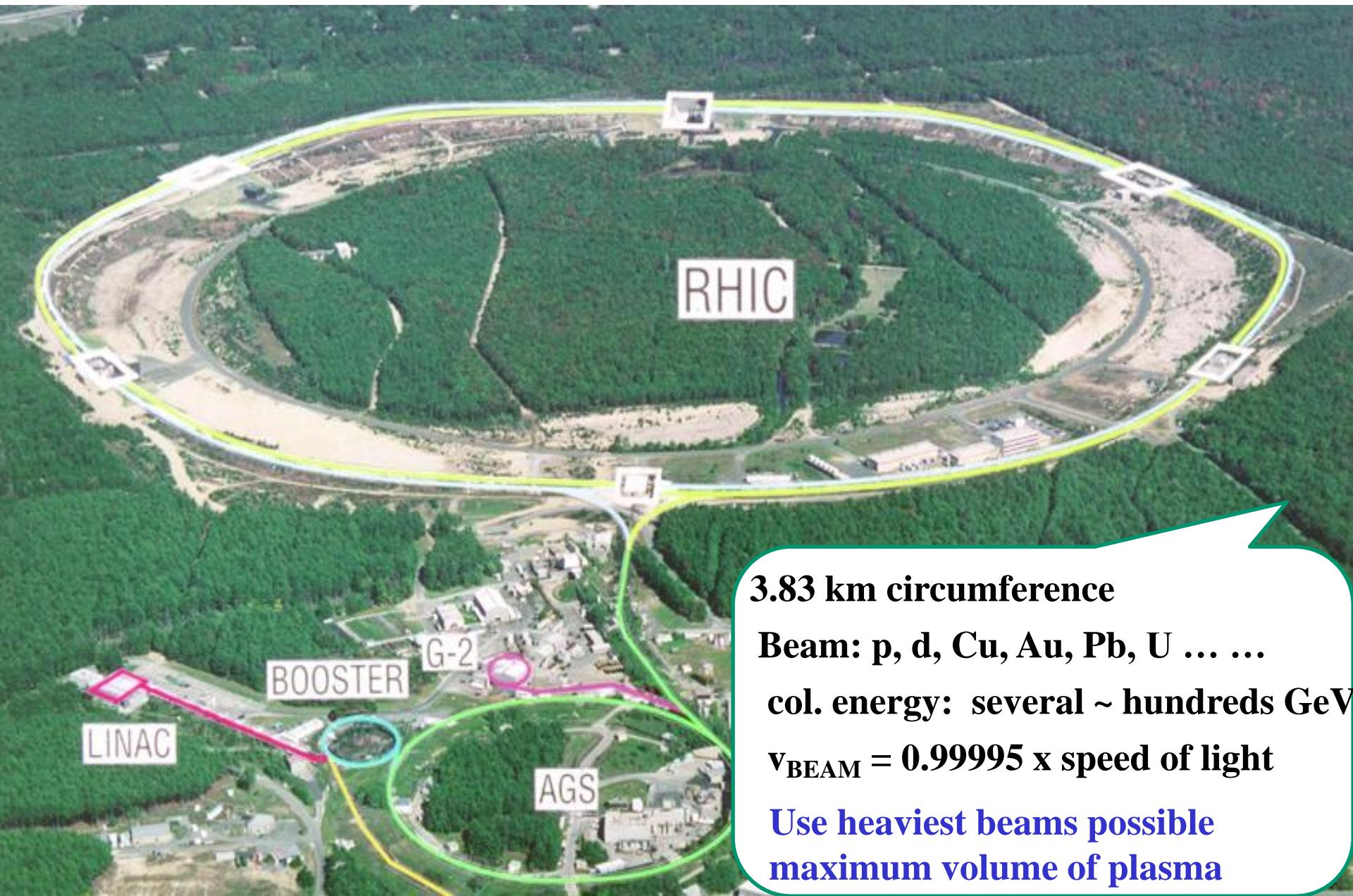
1974: Workshop on “GeV/nucleon collisions of heavy ions”

We should investigate.... phenomena by distributing energy of high nucleon density of a relatively large volume”  
---T.D.Lee



核子重如牛，对撞生新态

# Relativistic Heavy Ion Collider



**3.83 km circumference**

**Beam: p, d, Cu, Au, Pb, U ... ...**

**col. energy: several ~ hundreds GeV**

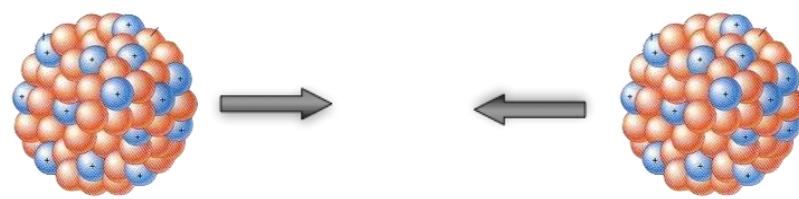
**$v_{BEAM} = 0.99995 \times \text{speed of light}$**

**Use heaviest beams possible  
maximum volume of plasma**

- RHIC = Relativistic Heavy Ion Collider
- Located at Brookhaven National Laboratory



RHIC



$$v_{\text{BEAM}} = 0.99995 \times \text{speed of light}$$



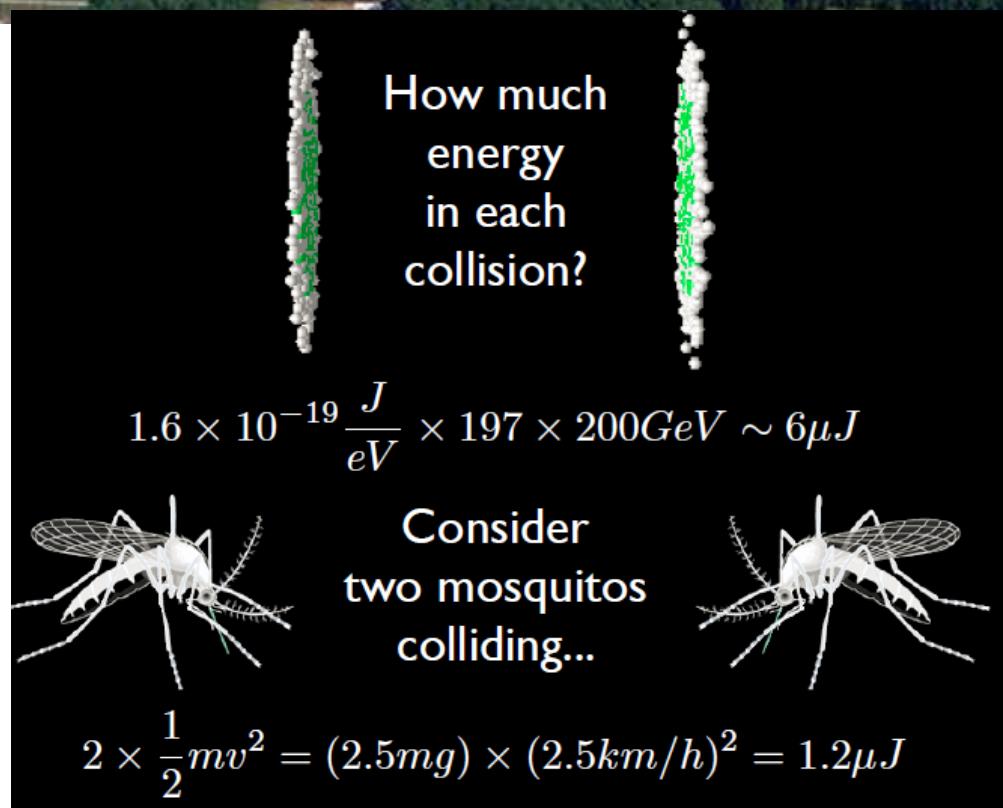
# RHIC



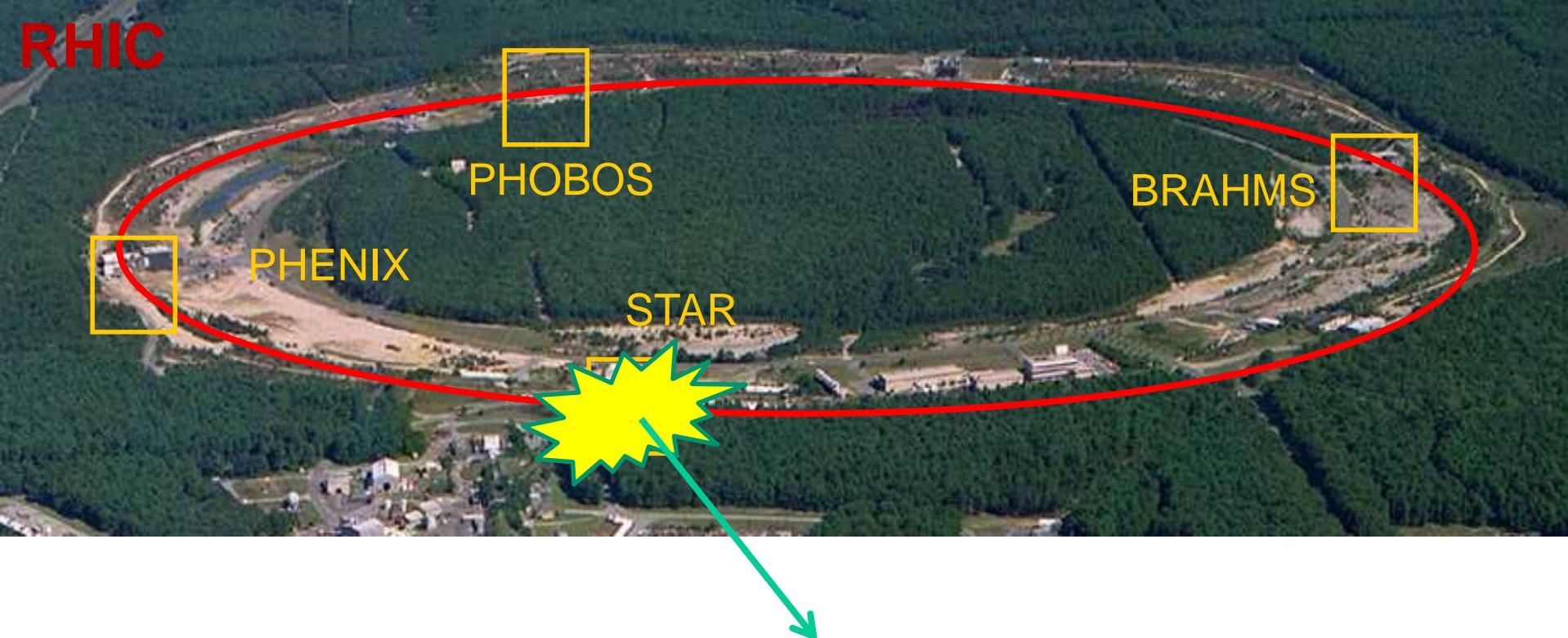
$$v_{\text{BEAM}} = 0.99995 \times \text{speed of light}$$



Energy deposition in super tiny volume



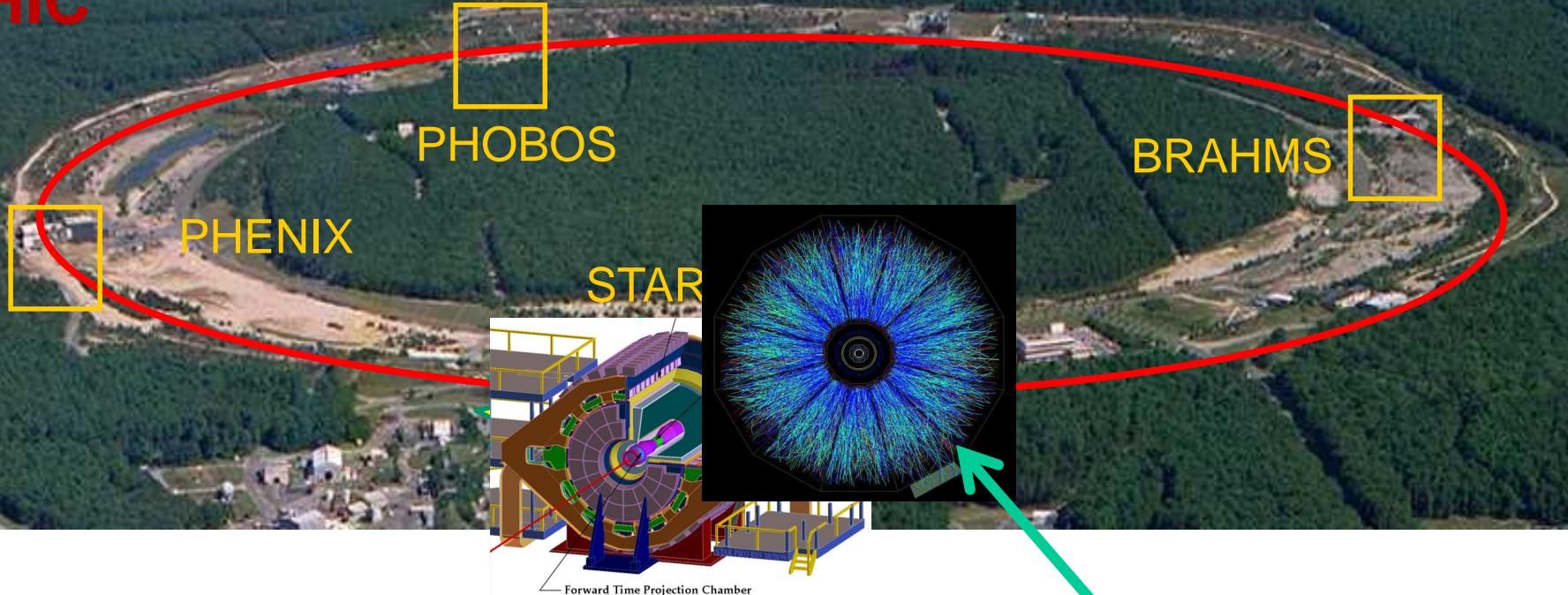
RHIC



little bang: the different stage for a relativistic heavy ion collisions



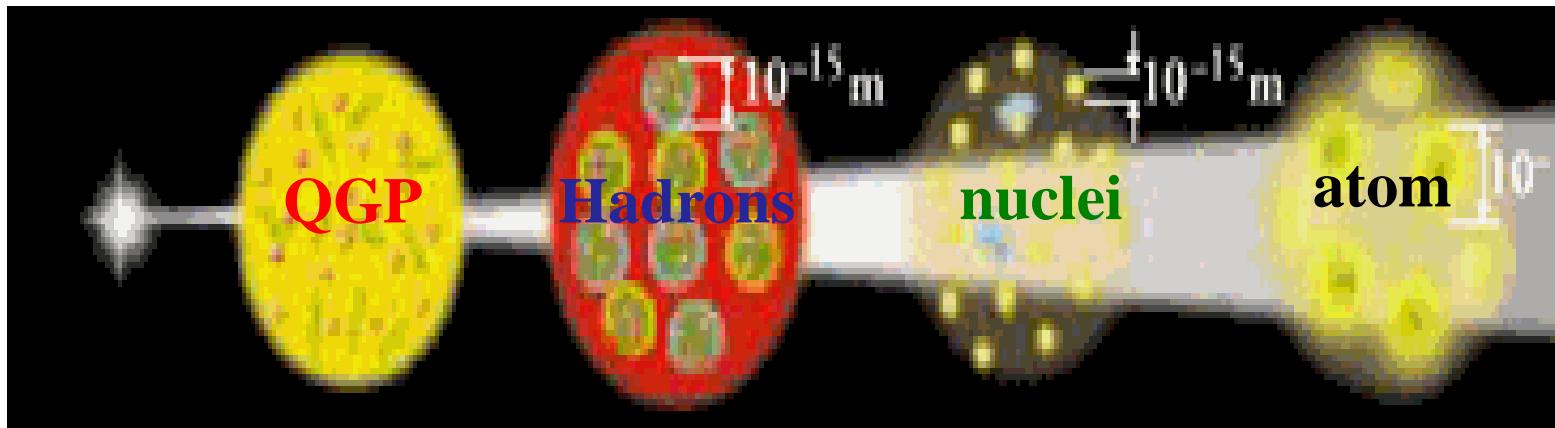
RHIC



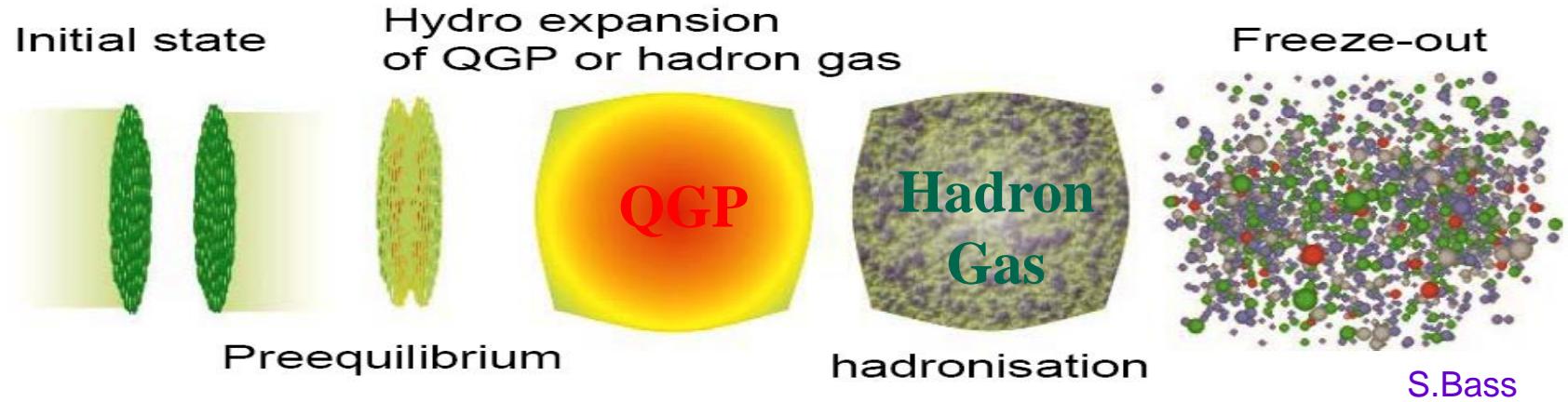
little bang: the different stage for a relativistic heavy ion collisions



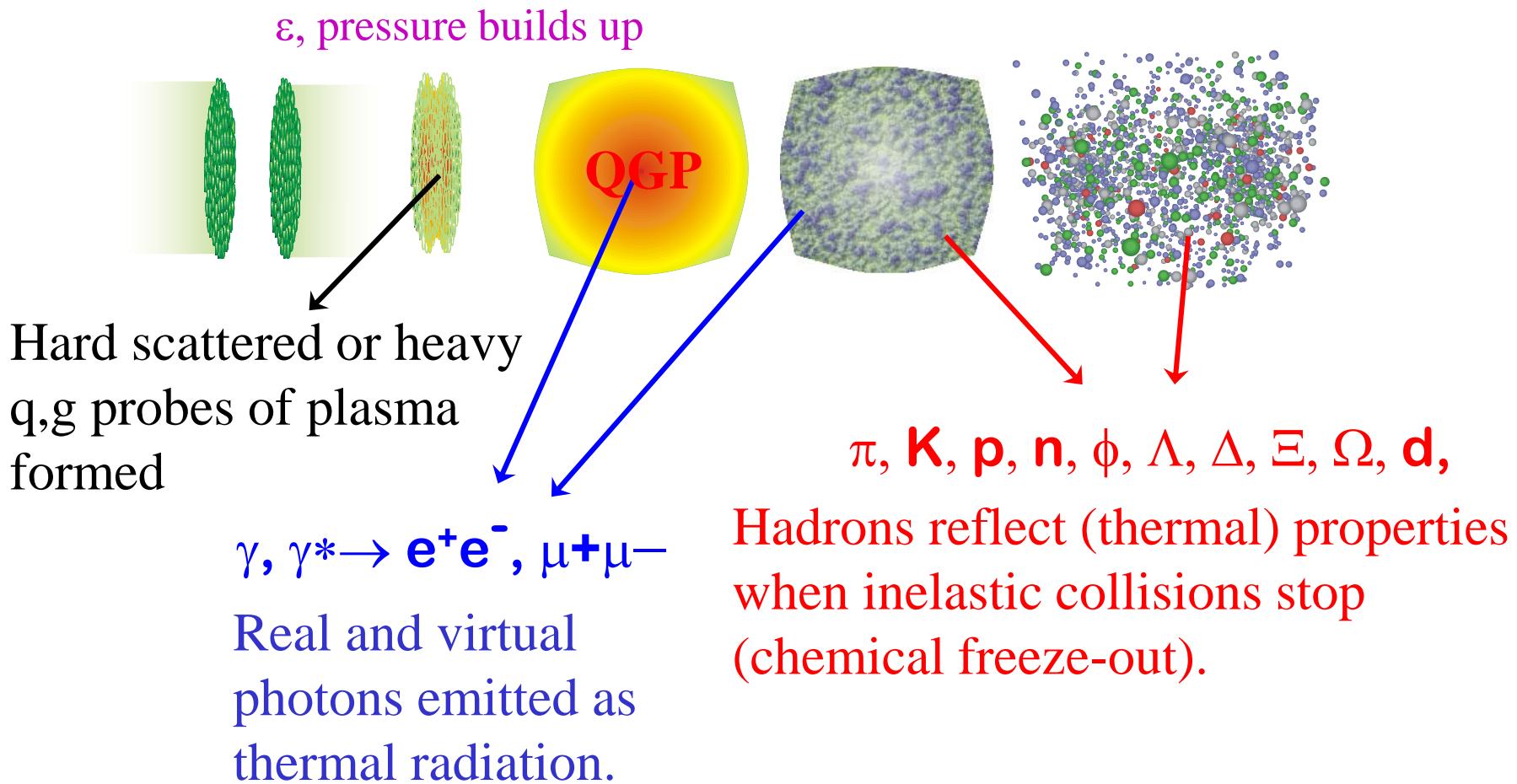
## big bang: the very early history of the universe



## little bang: the different stage for a relativistic heavy ion collisions



# Different stages for heavy ion collisions



# What State of Matter?



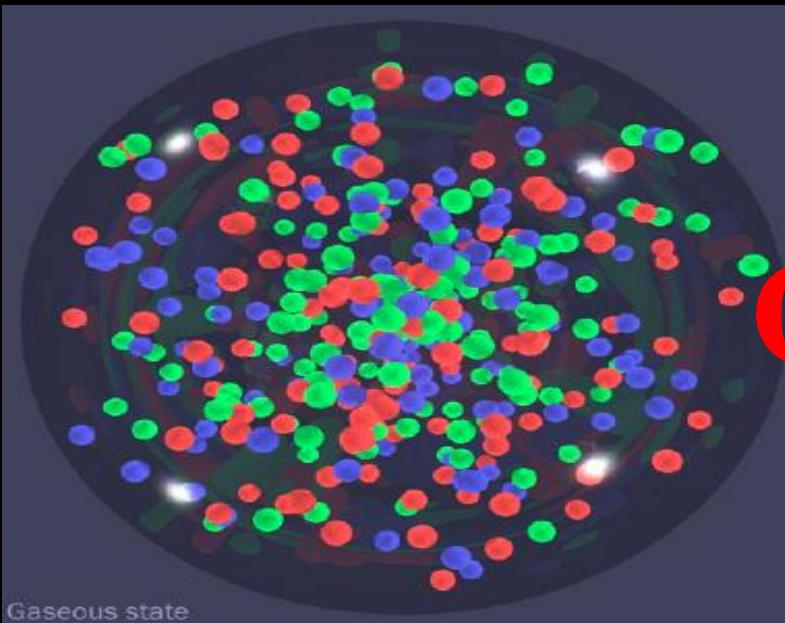
QGP



Does it act  
like an ideal gas?

Does it flow,  
like a (compressible) liquid?

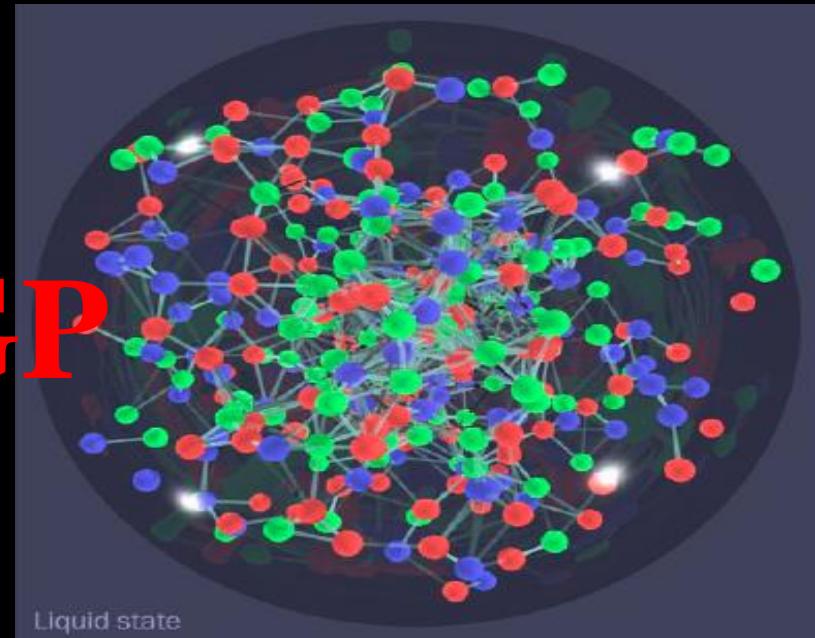
# What State of Matter?



Gaseous state

**Gas:** particles only know about each other when they bump

Does it act like an ideal gas?



Liquid state

**Liquid:** particles exert forces on one another all the time, flows in a coordinated fashion

Does it flow, like a (compressible) liquid?

Movie

# QGP-the most perfect fluid in the world

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Disappearing  
Superconductivity Reappears  
-- in 2-D

---

Electron Pairs Precede High-  
Temperature  
Superconductivity

---

World's biggest computing  
grid launched

---

First Beam for Large Hadron  
Collider

---

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## RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

April 18, 2005

TAMPA, FL -- The four detector groups conducting research at the [Relativistic Heavy Ion Collider](#) (RHIC) -- a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory -- say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In [peer-reviewed papers](#) summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

"Once again, the physics research sponsored by the Department of Energy is producing historic results," said Secretary of Energy Samuel Bodman, a trained chemical engineer. "The DOE is the principal federal funder of basic research in the physical sciences, including nuclear and high-energy physics. With today's announcement we see that investment paying off."

"The truly stunning finding at RHIC that the new state of matter created in the collisions of gold ions is more like a liquid than a gas gives us a profound insight into the earliest moments of the universe," said Dr. Raymond L. Orbach, Director of the DOE Office of Science.

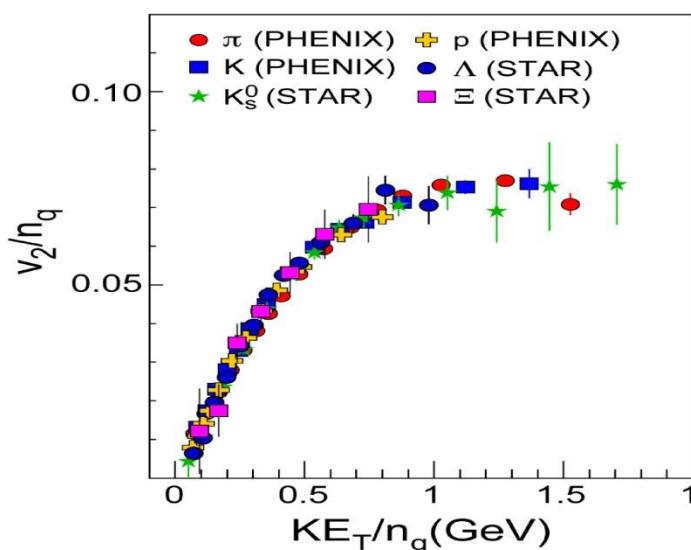
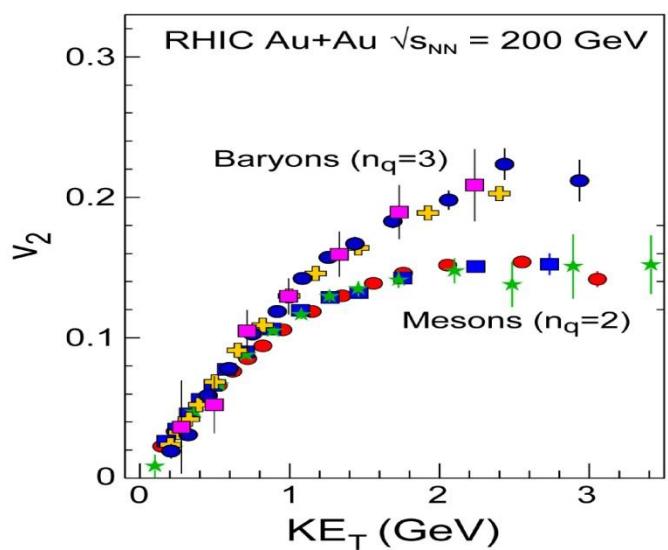
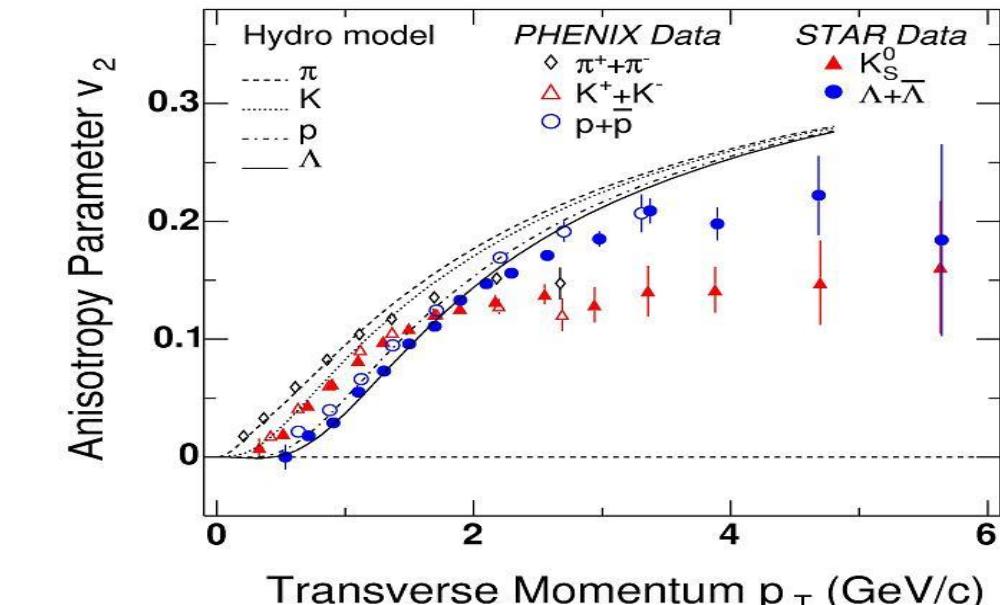
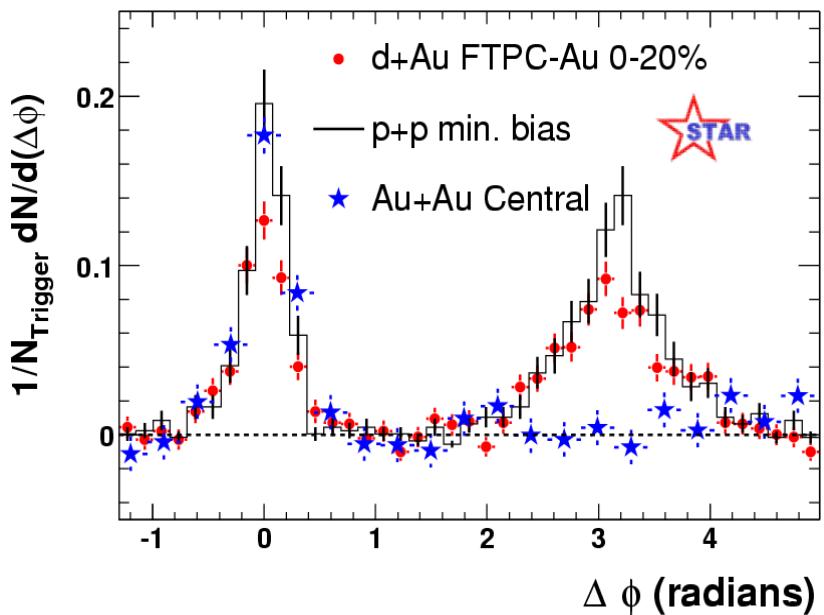
Also of great interest to many following progress at RHIC is the emerging connection between the collider's results and calculations using the methods of string theory, an approach that attempts to explain



Secretary of Energy  
Samuel Bodman

# The QGP was discovered

RHIC (2000-- )



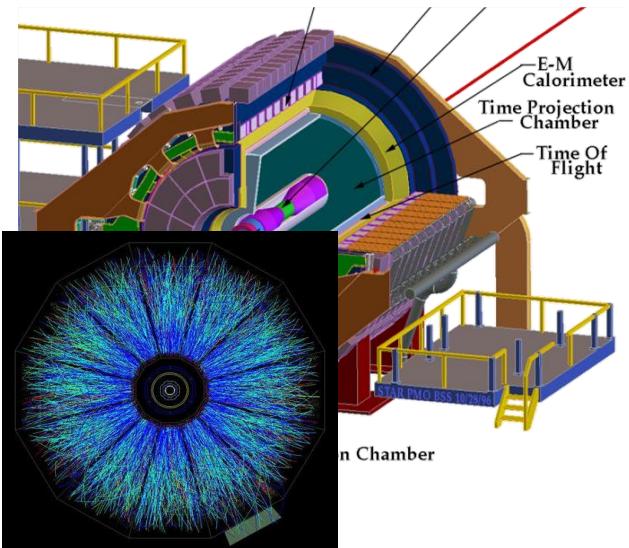
# Theoretical tools for QGP evolution

Life time  $\sim 10^{-23}$  s

size  $\sim 10^{-14}$  m



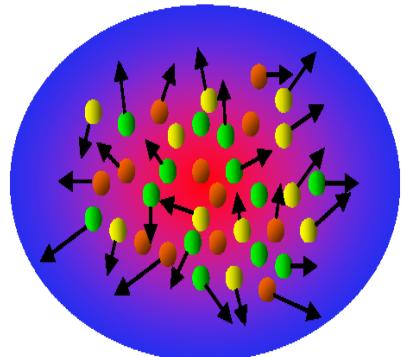
Numerical simulation



## Dynamical Model

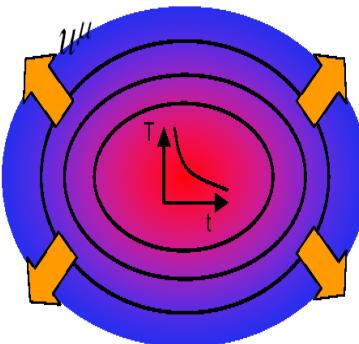
### Boltzmann approach

microscopic view



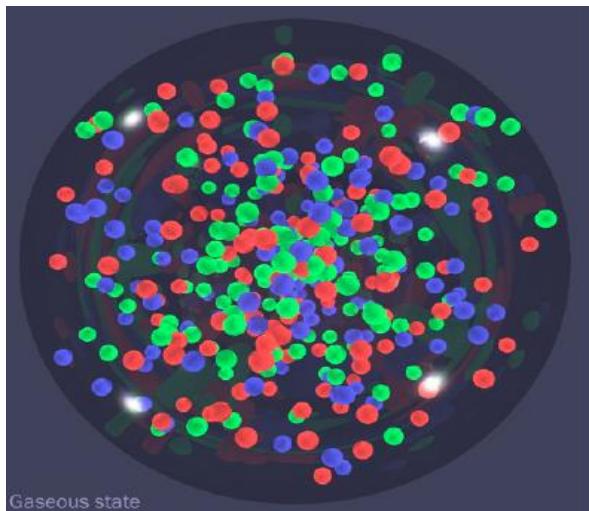
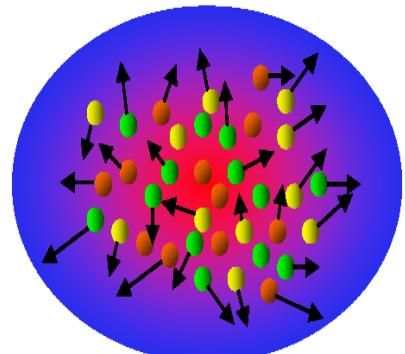
### Hydrodynamics

macroscopic view



# Boltzmann approach

microscopic view

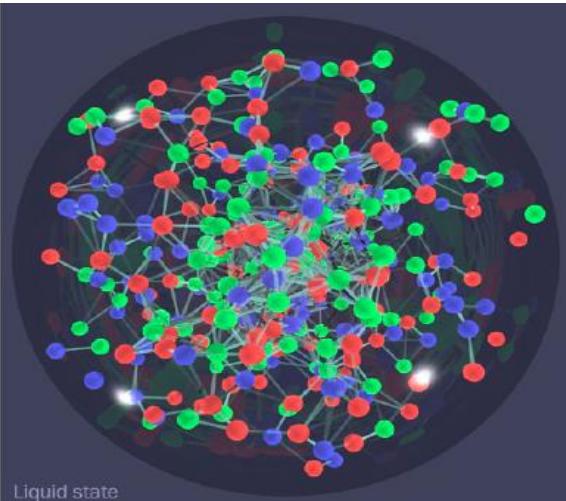
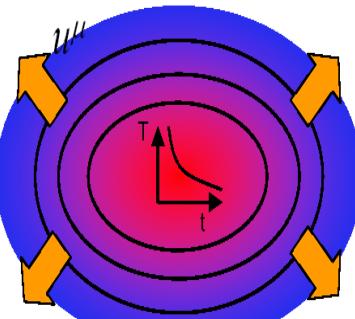


Gaseous state

Gas: particles only know about each other when they bump

# Hydrodynamics

macroscopic view



Liquid state

Liquid: particles exert forces on one another all the time, flows in a coordinated fashion

# Hydrodynamics

ideal hydro

$$\partial_\mu S^\mu = 0$$

Local equilibrium system

$$e(x) \ p(x) \ n(x) \ u^\mu(x)$$

viscous hydro

$$\partial_\mu S^\mu \geq 0$$

Near equilibrium system

$$e(x) \ p(x) \ n(x) \ u^\mu(x)$$
  
$$\pi^{\mu\nu}(x) \quad \Pi(x)$$

Initial state



Hydro expansion  
of QGP or hadron gas

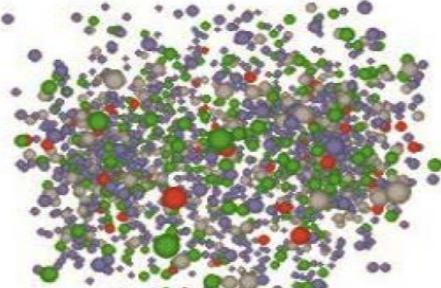


hydro

Preequilibrium

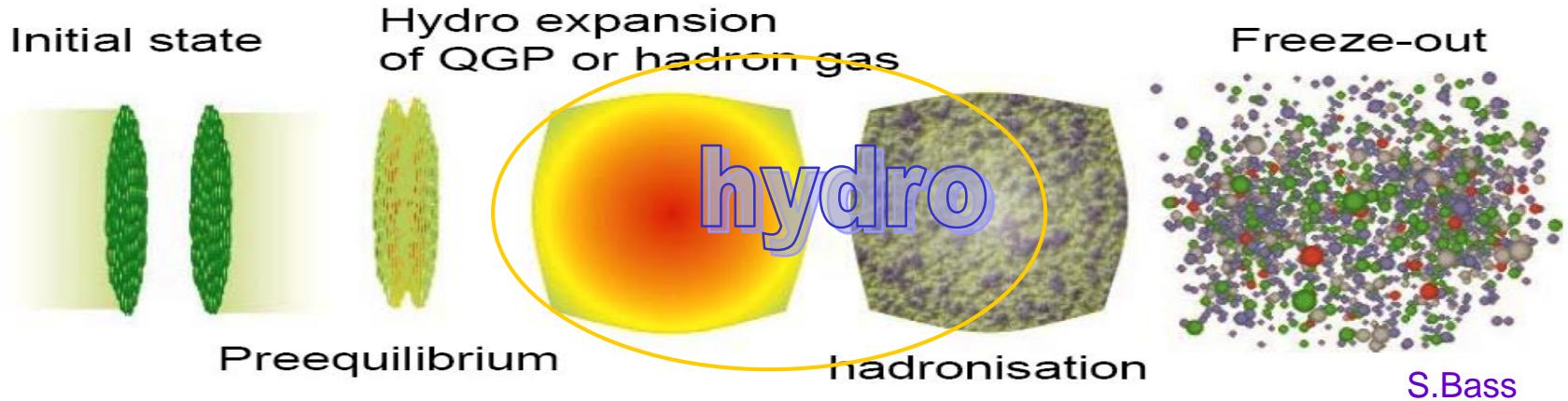
hadronisation

Freeze-out



S.Bass

# hydrodynamics



Hydrodynamics:

-A macroscopic tool to describe the expansion of QGP or hadronic matter

Conservation laws

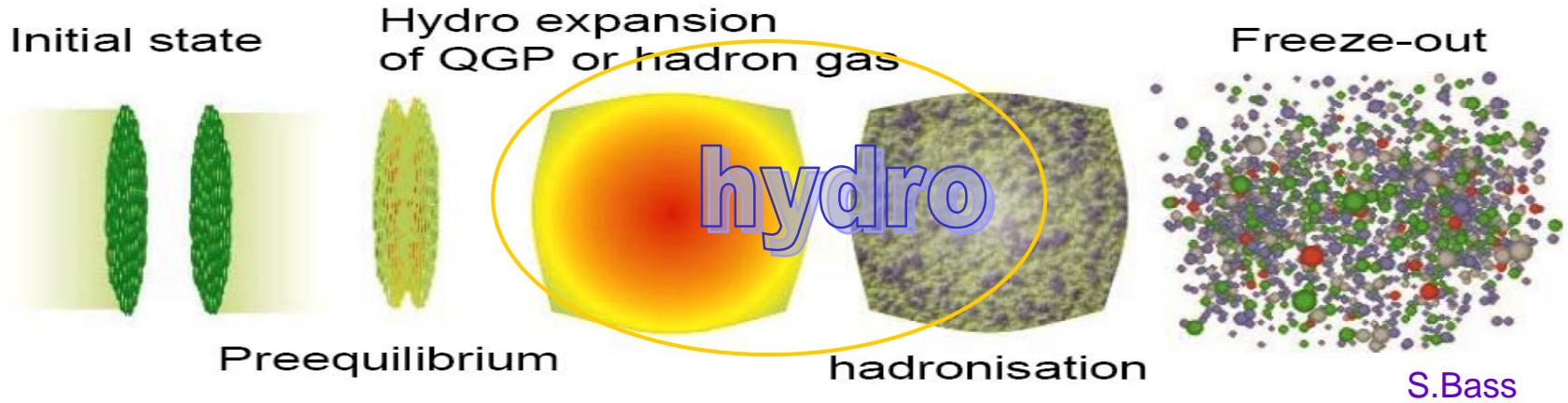
$$\partial_\mu N^\mu(x) = 0$$

$$\partial_\mu T^{\mu\nu}(x) = 0$$

5 equ. 14 independent variables

- reduce # of independent variables (**ideal hydro**)
- or provide more equations? (**viscous hydro**)

# Ideal hydrodynamics



## Hydrodynamics:

-A macroscopic tool to describe the expansion of QGP or hadronic matter

## Conservation laws

$$\partial_\mu N^\mu(x) = 0$$

$$\partial_\mu T^{\mu\nu}(x) = 0$$

5 equ. 14 independent variables

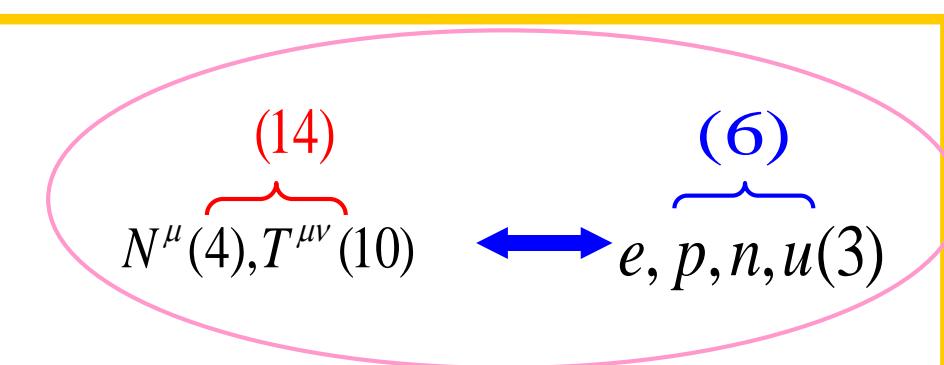
- reduce # of independent variables (**ideal hydro**)
- or provide more equations? (**viscous hydro**)

## Ideal hydrodynamics:

$$N^\mu = n u^\mu$$

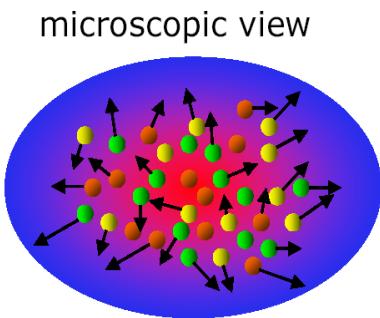
$$T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

Input: “EOS”  $\varepsilon = \varepsilon(p, n)$

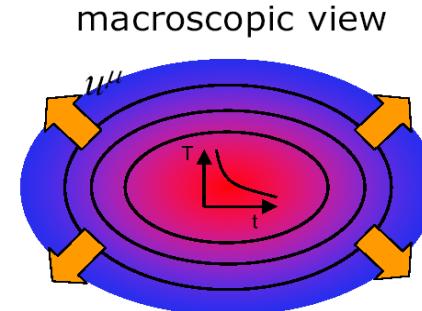


# A brief comment on ideal hydro

The basic assumption of ideal hydrodynamics : local equilibrium



$$l_{m.f.p.} \ll L$$



Particle scattering rate

>>

flow expansion rate

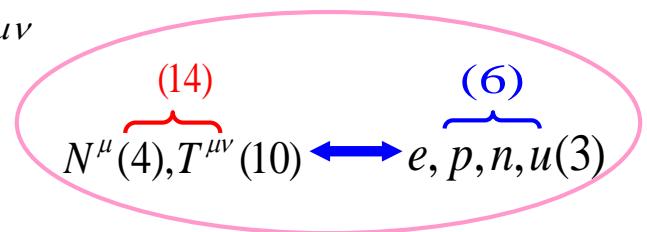
-entropy is conserved  $\partial_\mu S^\mu = 0$

-the independent variables is reduced  $T^{\mu\nu}$ ,  $N^\mu$

$$f_{eq}(x, p) = \frac{1}{e^{[p.u(x)+\mu(x)]/T(x)} \pm 1}$$

$$T^{\mu\nu}(x) = \int \frac{d^3 p}{E} p^\mu p^\nu (f_{eq} + \bar{f}_{eq}) = (\varepsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

$$N^\mu(x) = \int \frac{d^3 p}{E} p^\mu (f_{eq} + \bar{f}_{eq}) = n u^\mu$$



# Ideal hydro vs viscous hydro

The basic assumption of ideal hydrodynamics : local equilibrium

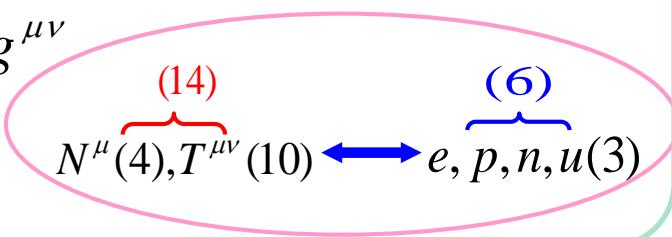
-entropy is conserved  $\partial_\mu S^\mu = 0$

-the independent variables is reduced  $T^{\mu\nu}, N^\mu$

$$f_{eq}(x, p) = \frac{1}{e^{[p.u(x)+\mu(x)]/T(x)} \pm 1}$$

$$T^{\mu\nu}(x) = \int \frac{d^3 p}{E} p^\mu p^\nu (f_{eq} + \bar{f}_{eq}) = (\varepsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

$$N^\mu(x) = \int \frac{d^3 p}{E} p^\mu (f_{eq} + \bar{f}_{eq}) = n u^\mu$$



The basic assumption of viscous hydro : near equilibrium

$$f(x, p) = f_{eq}(x, p) + \delta f(x, p)$$

$$T^{\mu\nu}(x) = \int \frac{d^3 p}{E} p^\mu p^\nu (f + \bar{f}) = T_{eq}^{\mu\nu} + \delta T^{\mu\nu}$$

Dissipative flow

$$N^\mu(x) = \int \frac{d^3 p}{E} p^\mu (f - \bar{f}) = N_{eq}^\mu + \delta N^\mu$$

# Viscous hydrodynamics--theory

Tensor decomposition in frame of  $u^\mu$

$$N^\mu = n u^\mu + V^\mu$$

$$(\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu)$$

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - p g^{\mu\nu}$$

$$+ (W^\mu u^\nu + W^\nu u^\mu) - \Pi \Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$S^\mu = s u^\mu + \phi^\mu$$

$$V^\mu u_\mu = 0 \quad W^\mu u_\mu = 0 \quad W^\mu = \frac{e+P}{n} V^\mu + q^\mu$$

$$\pi^{\mu\nu} u_\nu = 0 \quad \pi^\mu_\mu = 0 \quad \phi^\mu u_\mu = 0$$

## Viscous terms

$V^\mu$  charge flow

$W^\mu$  energy flow

$\Pi$  bulk pressure

$\pi^{\mu\nu}$  shear pressure tensor

$\phi^\mu$  entropy flow

$q^\mu$  heat flow

$$N^\mu(4) \rightarrow n, V^\mu(3)$$

$$T^{\mu\nu}(10) \rightarrow (p + \Pi), \varepsilon, W^\mu(3), \pi^{\mu\nu}(5)$$

# Viscous hydrodynamics--theory

Choice of frame:

Eckart frame:  $u_E^\mu = \frac{N^\mu}{\sqrt{N \cdot N}}$     $V^\mu = 0,$     $q^\mu = W^\mu$

Landau frame:  $u_L^\mu = \frac{T_v^\mu u_L^\nu}{\sqrt{u_L^\alpha T_\alpha^\beta T_{\beta\gamma} u_L^\gamma}}$     $W^\mu = 0,$     $q^\mu = -\frac{e+p}{n} V^\mu$

-RHIC & LHC, low net baryon density – choose Landau frame:  $W^\mu = 0,$

-The previous 3 independent variables in  $W^\mu(3)$  are then replaced by  $u^\mu(3)$

General conservation laws

$$\partial_\mu T^{\mu\nu}(x) = 0$$

$$\partial_\mu N^\mu(x) = 0$$

more equ. for these viscous terms

-Phenomenologically:  $\partial_\mu S^\mu \geq 0$

-From kinetic theory  $p^\mu d_\mu f(\vec{x}, t, \vec{p}) = C(x)$

# I-S formulism: phenomenological approach

-Thermodynamics:  $s = \beta p - \alpha n + \beta e \quad Ts = p - \mu n + e, \quad \alpha = \mu/T, \beta = 1/T$

-Relativistic, equilibrium:  $S^\mu = p\beta^\mu - \alpha N_{eq}^\mu + \beta_\nu T_{eq}^{\mu\nu} \quad \alpha = \mu/T, \beta^\mu = u^\mu/T$

-Relativistic, off equilibrium:  $S^\mu = p\beta^\mu - \alpha(N_{eq}^\mu + \delta N^\mu) + \beta_\nu(T_{eq}^{\mu\nu} + \delta T^{\mu\nu}) + Q^\mu(\delta N^\mu, \delta T^{\mu\nu})$

$T\partial_\mu S^\mu \geq 0$

2<sup>nd</sup> order and higher  
order corrections

# I-S formulism: phenomenological approach

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 $T\partial_\mu S^\mu \geq 0$   
2<sup>nd</sup> order and higher order corrections

$$T\partial_\mu S^\mu = \Pi X - q^\mu X_\mu + \pi^{\mu\nu} X_{\mu\nu} + T\partial_\mu Q^\mu \geq 0$$

$$X \equiv -\theta = -\nabla \cdot \mathbf{u}, \quad X^\mu \equiv -\frac{nT}{e+p} \nabla^\mu \left( \frac{\mu}{T} \right), \quad X^{\mu\nu} = \nabla^{<\mu} \mathbf{u}^{\nu>} \equiv \sigma^{\mu\nu}$$

$$Q_\mu = 0$$

$$\begin{aligned} \Pi &= -\zeta\theta = \zeta X \\ q^\mu &= \kappa \left( \nabla^\mu T - T \dot{u}^\mu \right) = \kappa X^\mu \\ \pi^{\mu\nu} &= 2\eta\sigma^{\mu\nu} = 2\eta X^{\mu\nu} \end{aligned}$$

# I-S formulism: phenomenological approach

- Thermodynamics:  $s = \beta p - \alpha n + \beta e \quad Ts = p - \mu n + e, \quad \alpha = \mu/T, \beta = 1/T$
- Relativistic, equilibrium:  $S^\mu = p\beta^\mu - \alpha N_{eq}^\mu + \beta_\nu T_{eq}^{\mu\nu} \quad \alpha = \mu/T, \beta^\mu = u^\mu/T$
- Relativistic, off equilibrium:  $S^\mu = p\beta^\mu - \alpha(N_{eq}^\mu + \delta N^\mu) + \beta_\nu(T_{eq}^{\mu\nu} + \delta T^{\mu\nu}) + Q^\mu(\delta N^\mu, \delta T^{\mu\nu})$

-a simple case for second order theory (without heat flow & bulk pressure)

$$T\partial_\mu S^\mu = \Pi X - q^\mu X_\mu + \pi^{\mu\nu} X_{\mu\nu} + T\partial_\mu Q^\mu \geq 0$$

$$Q^\mu = -\left(\beta_2 \pi^{\lambda\nu} \pi_{\lambda\nu}\right) \frac{u^\mu}{2T}, \quad (\Pi = 0, \quad q^\mu = 0)$$

A. Muronga 00-04    W. Israel, J. Stewart 79

$$T\partial_\mu S^\mu = \pi^{\alpha\beta} [\sigma_{\alpha\beta} - \beta_2 \Delta^{\mu\alpha} \Delta^{\nu\beta} D\pi_{\mu\nu} + \frac{T}{2} \partial_\lambda \left( \frac{\beta_2}{T} u^\lambda \right) \pi_{\alpha\beta} - 2\pi^{\alpha(\mu} \omega_{\alpha}^{\nu)}]$$

$$T\partial_\mu S^\mu = \frac{\pi_{\alpha\beta} \pi^{\alpha\beta}}{2\eta} \geq 0$$

$$\Delta^{\mu\alpha} \Delta^{\nu\beta} D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi} \left[ \pi^{\mu\nu} - 2\eta \sigma^{\mu\nu} + \pi^{\mu\nu} \eta T \partial_\lambda \left( \frac{\tau_\pi}{2\eta T} u^\lambda \right) \right] - 2\pi^{\alpha(\mu} \omega_{\alpha}^{\nu)}$$

# I-S formulism: phenomenological approach

- Thermodynamics:  $s = \beta p - \alpha n + \beta e \quad Ts = p - \mu n + e, \quad \alpha = \mu/T, \beta = 1/T$
- Relativistic, equilibrium:  $S^\mu = p\beta^\mu - \alpha N_{eq}^\mu + \beta_\nu T_{eq}^{\mu\nu} \quad \alpha = \mu/T, \beta^\mu = u^\mu/T$
- Relativistic, off equilibrium:  $S^\mu = p\beta^\mu - \alpha(N_{eq}^\mu + \delta N^\mu) + \beta_\nu(T_{eq}^{\mu\nu} + \delta T^{\mu\nu}) + Q^\mu(\delta N^\mu, \delta T^{\mu\nu})$

## -full second order theory

A. Muronga 00-04   W. Israel, J. Stewart 79

$$Q^\mu = -(\beta_0 \Pi^2 - \beta_1 q^\mu q_\mu + \beta_2 \pi_{\mu\nu} \pi^{\mu\nu}) \frac{u^\mu}{2T} - \frac{\alpha_0 \Pi q^\mu}{T} + \frac{\alpha_1 \pi^{\mu\nu} q_\nu}{T},$$

$$\dot{\Pi} = -\frac{1}{\tau_\Pi} \left[ \Pi + \zeta \theta - l_{\Pi q} \nabla_\mu q^\mu + \Pi \zeta T \partial_\mu \left( \frac{\tau_\Pi u^\mu}{2\zeta T} \right) \right],$$

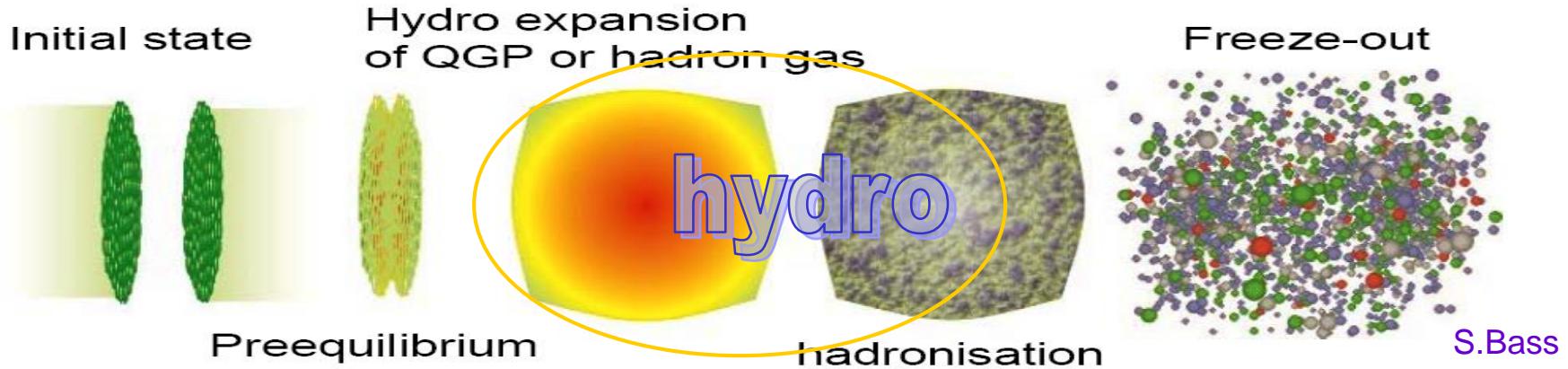
$$\Delta_\nu^\mu \dot{q}^\nu = -\frac{1}{\tau_q} \left[ q_\mu + \lambda \frac{nT^2}{e+p} \nabla^\mu \frac{\nu}{T} + l_{q\pi} \nabla_\nu \pi^{\mu\nu} + l_{q\Pi} \nabla^\mu \Pi - \lambda T^2 q^\mu \partial_\mu \left( \frac{\tau_q u^\mu}{2\lambda T^2} \right) \right],$$

$$\Delta^{\mu\alpha} \Delta^{\nu\beta} \dot{\pi}_{\alpha\beta} = -\frac{1}{\tau_\pi} \left[ \pi^{\mu\nu} - 2\eta \nabla^{\langle\mu} u^{\nu\rangle} - l_{\pi q} \nabla^{\langle\mu} q^{\nu\rangle} + \pi_{\mu\nu} \eta T \partial_\alpha \left( \frac{\tau_\pi u^\alpha}{2\eta T} \right) \right],$$

$T \partial_\mu S^\mu \geq 0$

$$\tau_\Pi = \zeta \beta_0, \quad \tau_q = \lambda T \beta_1, \quad \tau_\pi = 2\eta \beta_2 \quad \tau_0 = \zeta \alpha_0, \quad \tau_1 = \lambda T \alpha_1, \quad \tau_2 = 2\eta \alpha_1$$

# Viscous hydrodynamics



Conservation laws:

$$\partial_\mu T^{\mu\nu}(x) = 0, \quad \partial_\mu N_i^\mu(x) = 0,$$

2<sup>nd</sup> order I-S equ:

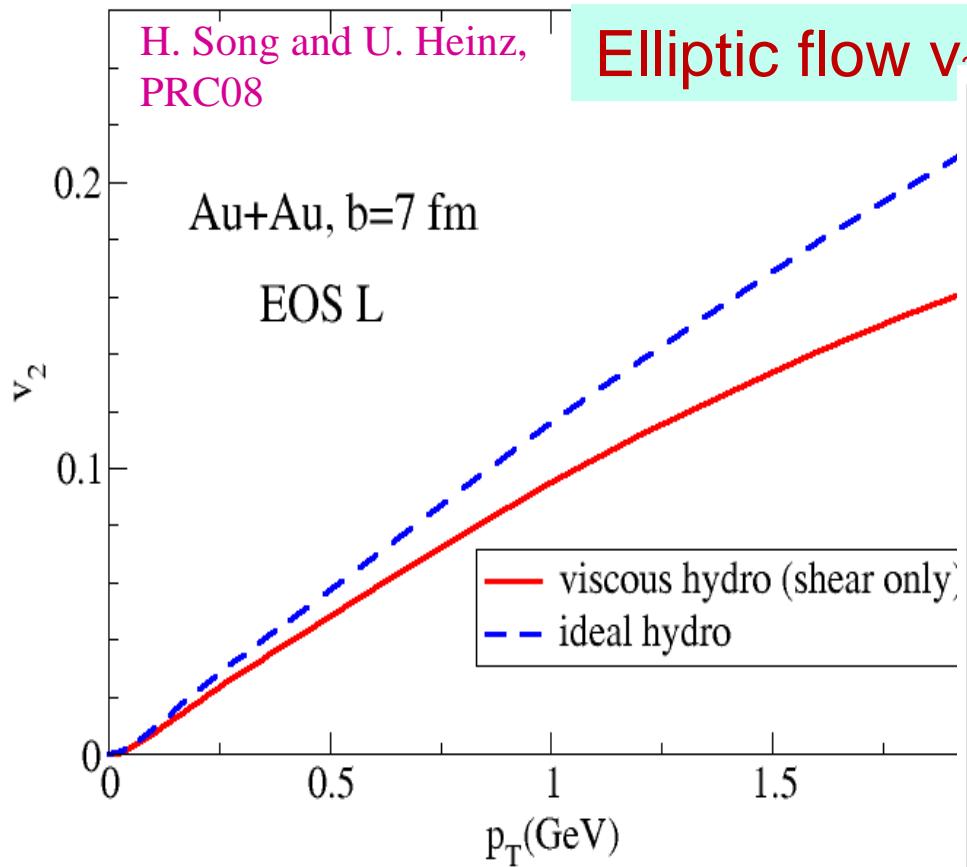
$$\dot{\Pi} = -\frac{1}{\tau_\Pi} \left[ \Pi + \zeta \theta - l_{\Pi q} \nabla_\mu q^\mu + \Pi \zeta T \partial_\mu \left( \frac{\tau_\Pi u^\mu}{2\zeta T} \right) \right],$$

$$\Delta_\nu^\mu \dot{q}^\nu = -\frac{1}{\tau_q} \left[ q_\mu + \lambda \frac{nT^2}{e+p} \nabla^\mu \frac{\nu}{T} + l_{q\pi} \nabla_\nu \pi^{\mu\nu} + l_{q\Pi} \nabla^\mu \Pi - \lambda T^2 q^\mu \partial_\mu \left( \frac{\tau_q u^\mu}{2\lambda T^2} \right) \right],$$

$$\Delta^{\mu\alpha} \Delta^{\nu\beta} \dot{\pi}_{\alpha\beta} = -\frac{1}{\tau_\pi} \left[ \pi^{\mu\nu} - 2\eta \nabla^{\langle\mu} u^{\nu\rangle} - l_{\pi q} \nabla^{\langle\mu} q^{\nu\rangle} + \pi_{\mu\nu} \eta T \partial_\alpha \left( \frac{\tau_\pi u^\alpha}{2\eta T} \right) \right],$$

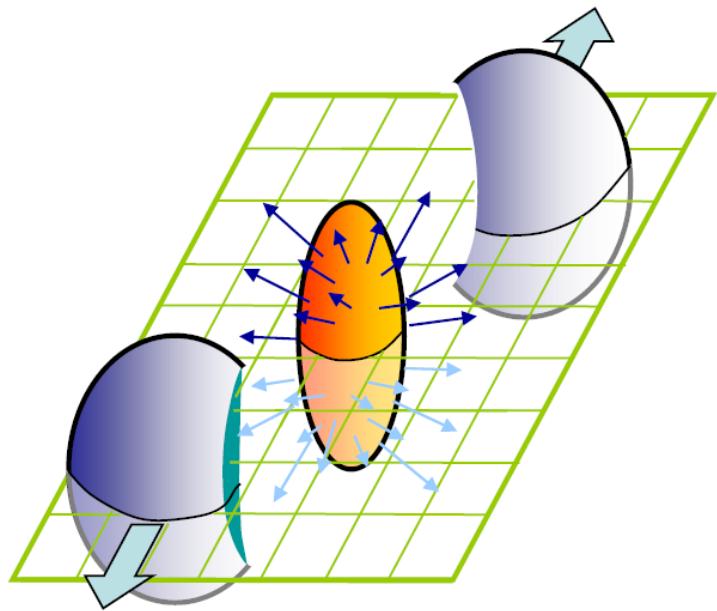
Input: “EOS”  $\varepsilon = \varepsilon(p)$  initial and final conditions

# Viscous hydro: Shear viscosity $\eta$ & elliptic flow $v_2$



Elliptic flow  $v_2$

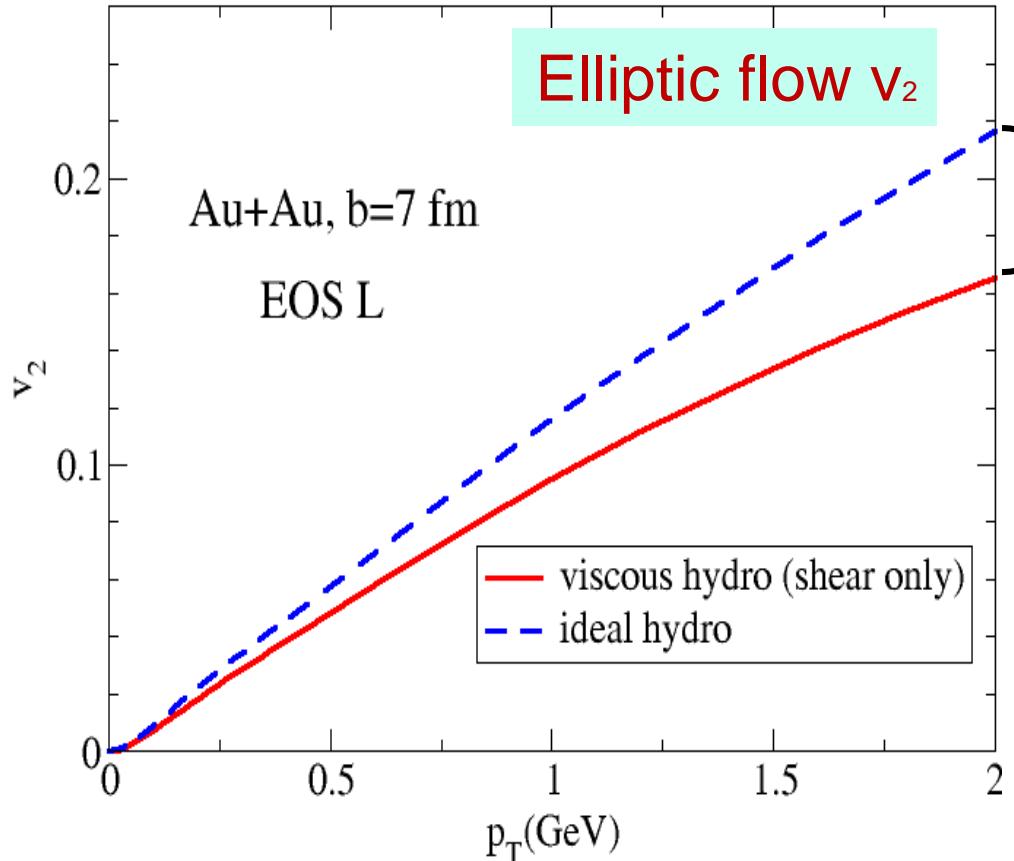
Elliptic flow



$$E \frac{dN}{d^3 p} = \frac{dN}{dp_T dp_T d\phi}$$

$$= \frac{1}{2\pi} \frac{dN}{dp_T dp_T} [1 + 2v_2 \cos(2\phi) + \dots]$$

# Viscous hydro: Shear viscosity $\eta$ & elliptic flow $V_2$



20-25%  $v_2$  suppression     $\frac{\eta}{s} = \frac{1}{4\pi}$

H. Song and U. Heinz, PLB08

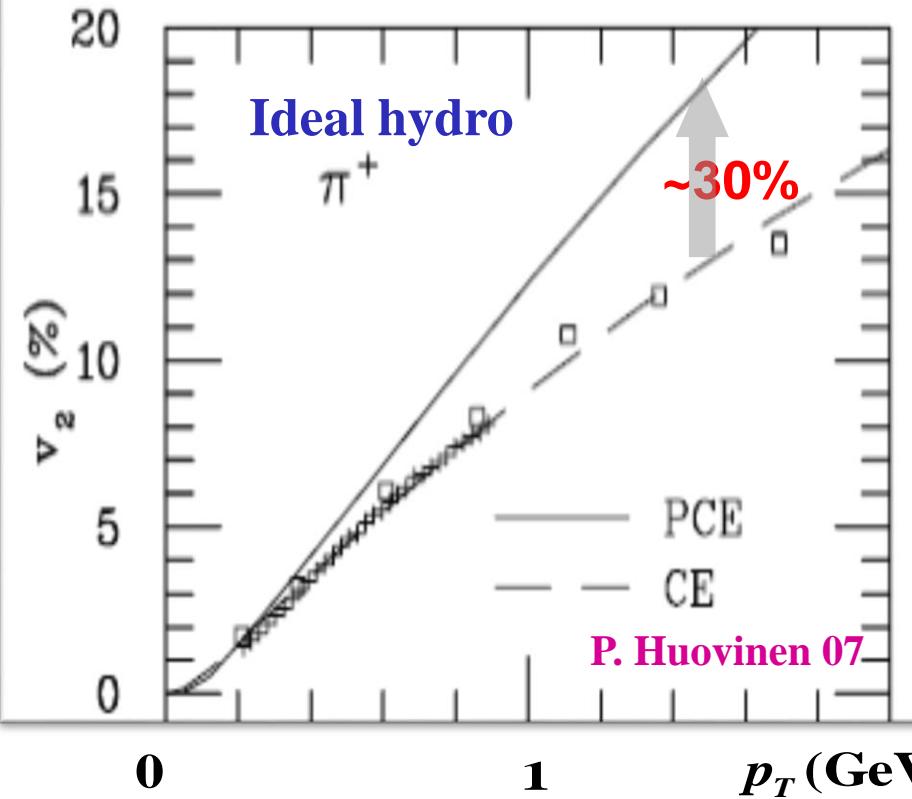
H. Song and U. Heinz, PRC08

-Elliptic flow is sensitive to the QGP shear viscosity, minimal value of  $\eta/s$  lead to 20-30%  $V_2$  suppression

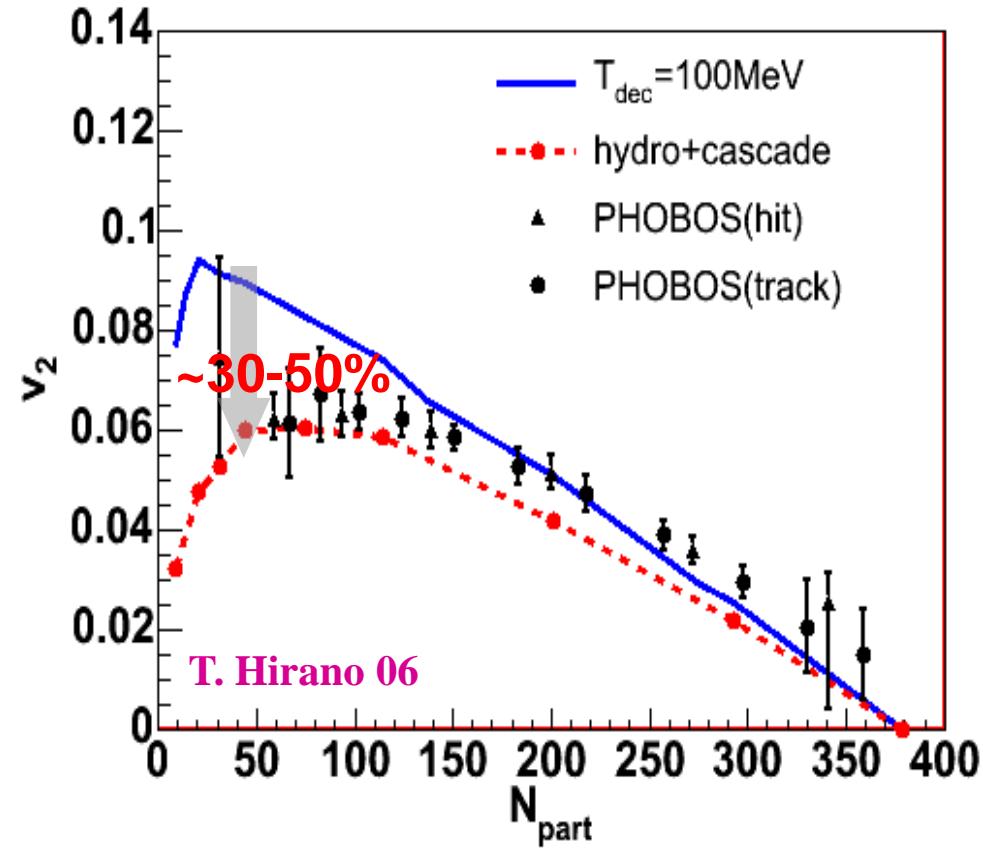
- $V_2$  can be used to extract the QGP shear viscosity

# Effect from Hadronic evolution

## Partially Chemical equil.



## hadronic dissipative effects

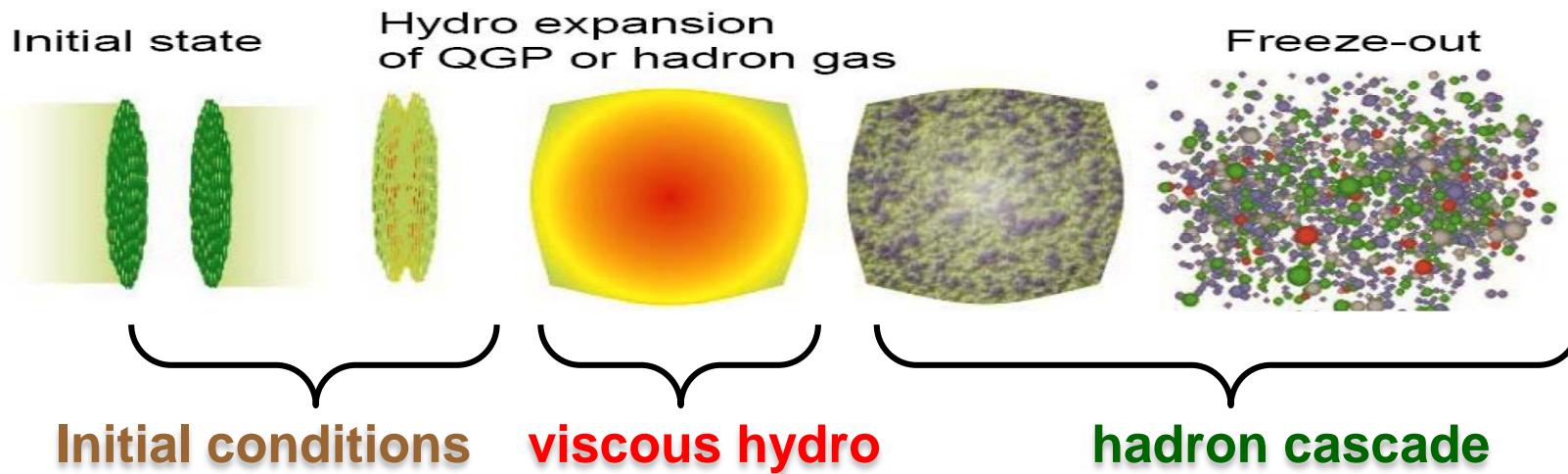


-These two HRG effects are not included in early viscous hydro calculations

→ **viscous hydro + hadron cascade (URQMD) hybrid approach**

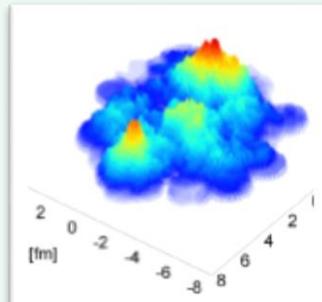
URQMD includes the **partially chemical equilibrium** nature & **hadronic dissipative** effects

# VISHNU & iEBE-VISHNU hybrid approach



**VISHNU:** H. Song, S. Bass, U. Heinz, PRC2011

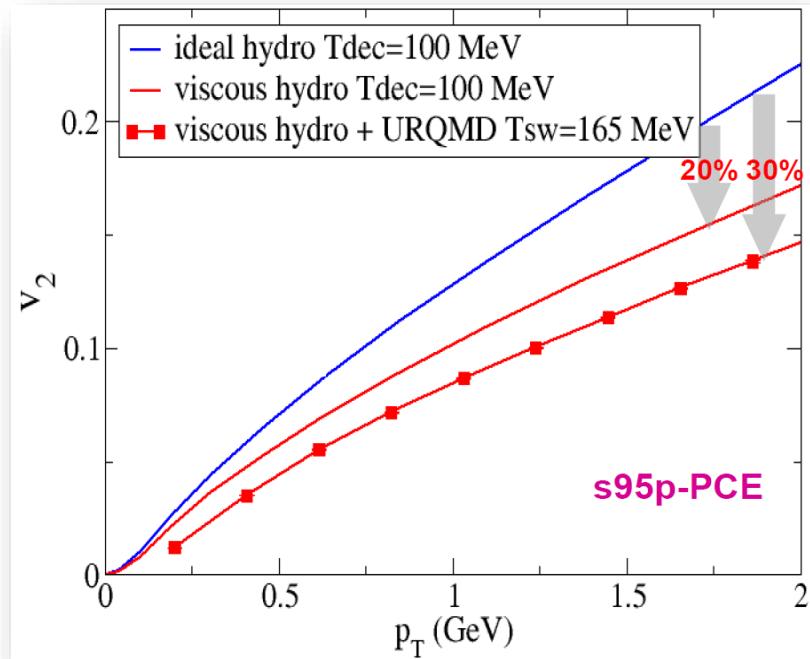
- initial conditons
- Viscous hydro (VISH2+1)
- Hadron Cascade
- EoS: (s95p-PCE, etc)



**iEBE- VISHNU:**

- Event-by-Event VISHNU

Shen, Qiu, Song et al CPC2016



# Initialization Models

-fluctuations of nucleon positions:

**MC-Glauber:**

**MC-KLN:**

} T. Hirano & Y. Nara, Phys. Rev. C 79 064904 (2009)

-fluctuations of color charges or gluon number

**IP-Glasma:** Schenke, Tribedy & Venugopalan PRC, PRL2012

-early flow & fluctuations from dynamical models:

**URQMD initialization:** H.Petersen & M. Bleicher, Phys. Rev. C81, 044906,2010

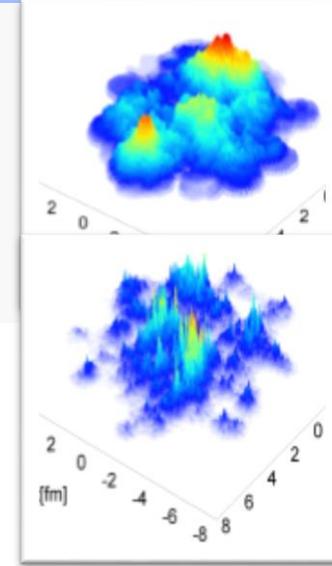
**AMPT initialization:** L. Pang, Q.Wang & X.Wang, PRC 2012

H. Xu Z.Li and H. Song PRC2016

**EPOS/NEXUS initialization:** K. Werner et al., Phys. Rev. C83:044915,2011;  
H. J. Drescher et, al., Phys.Rev. C65 , 054902 (2002)

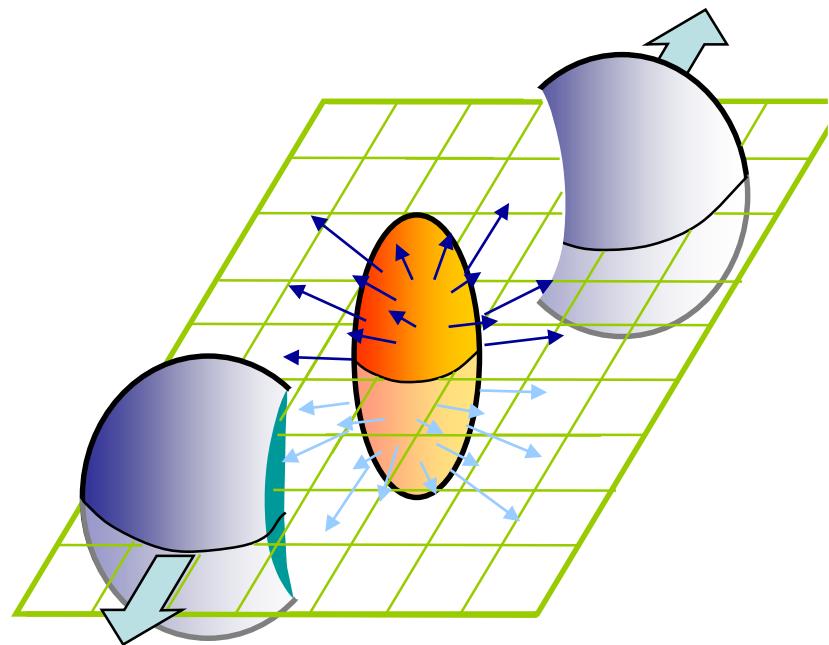
-Parameterized initial condition:

**Trento initialization:** S. Moreland, J. E. Bernhard and S.A. Bass, Phys. Rev. C92no.1,  
011901 (2015)

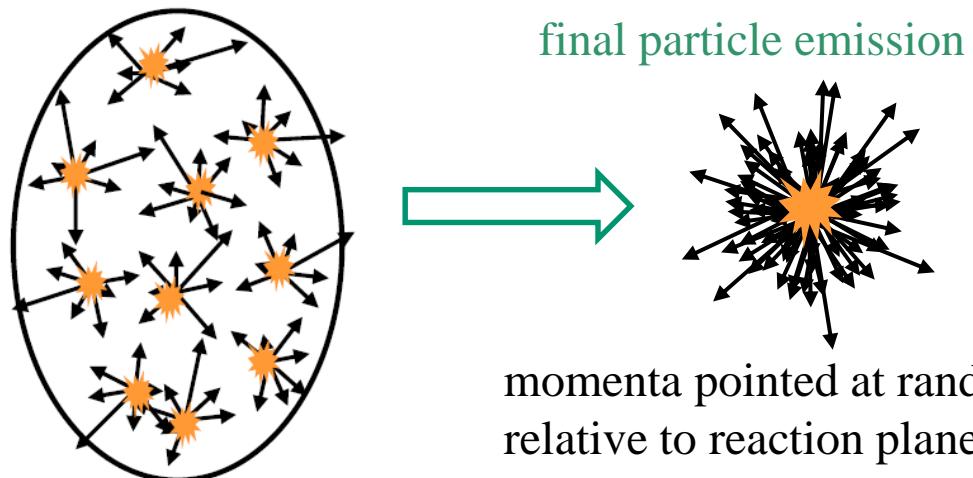


# Collective Flow

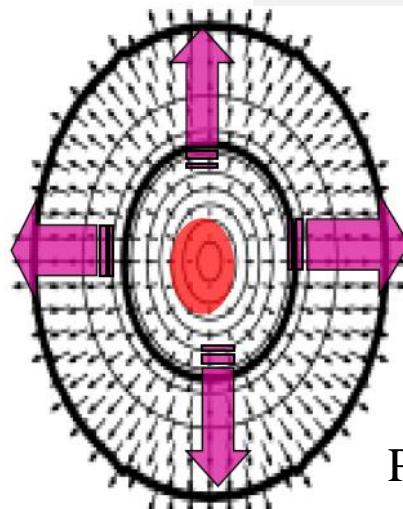
# Collective expansion



## Superposition of independent p+p:



## Evolution as a bulk system

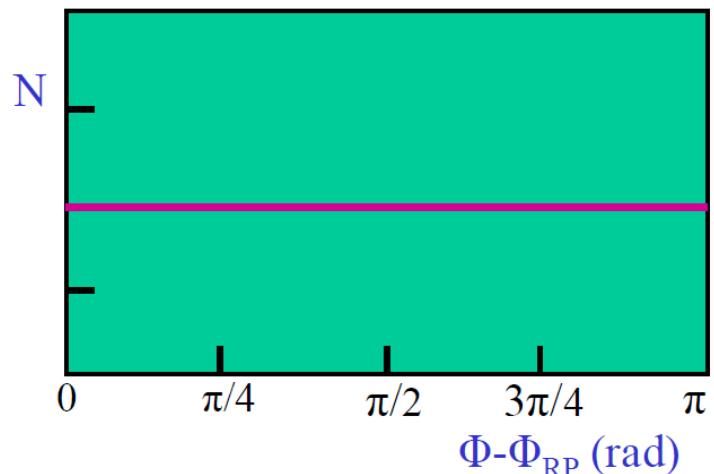
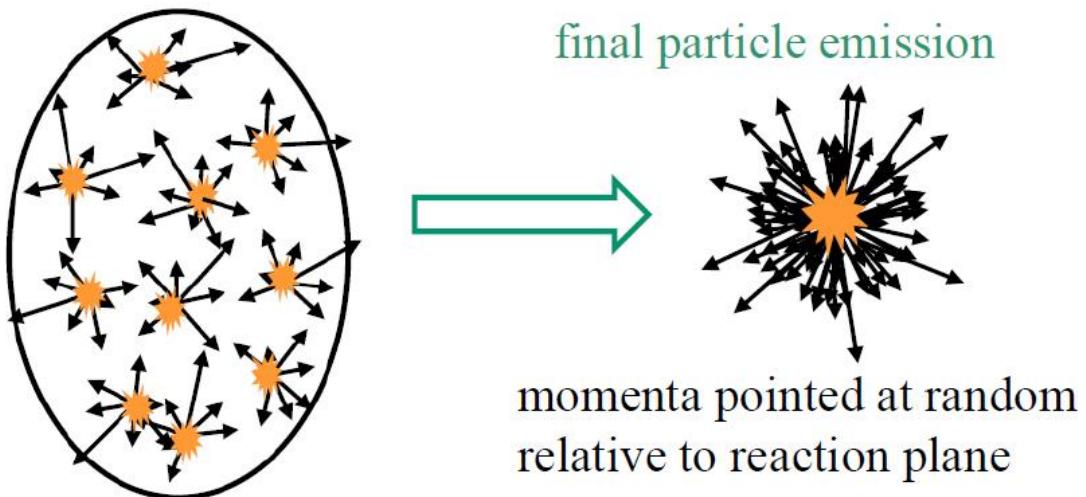


final particle emission

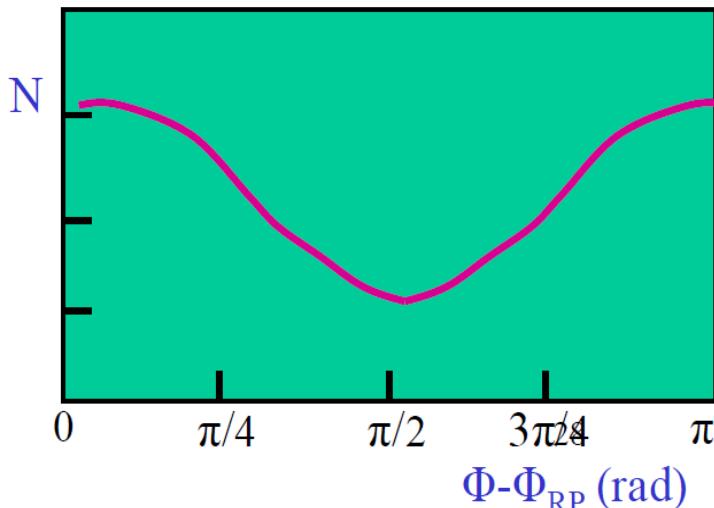
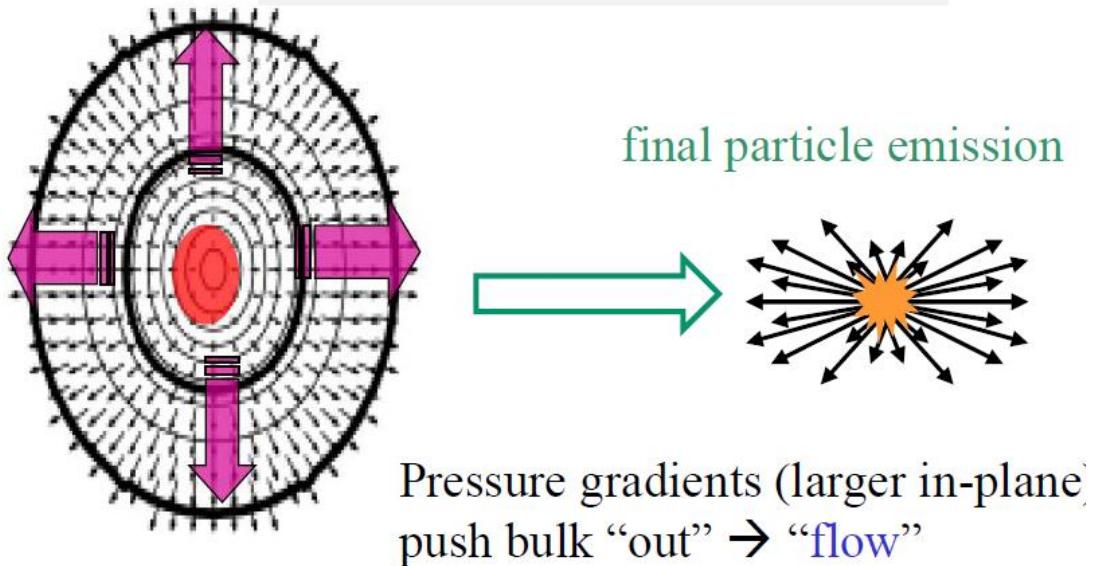
Pressure gradients (larger in-plane)  
push bulk “out” → “flow”

# Azimuthal distributions

## Superposition of independent p+p:

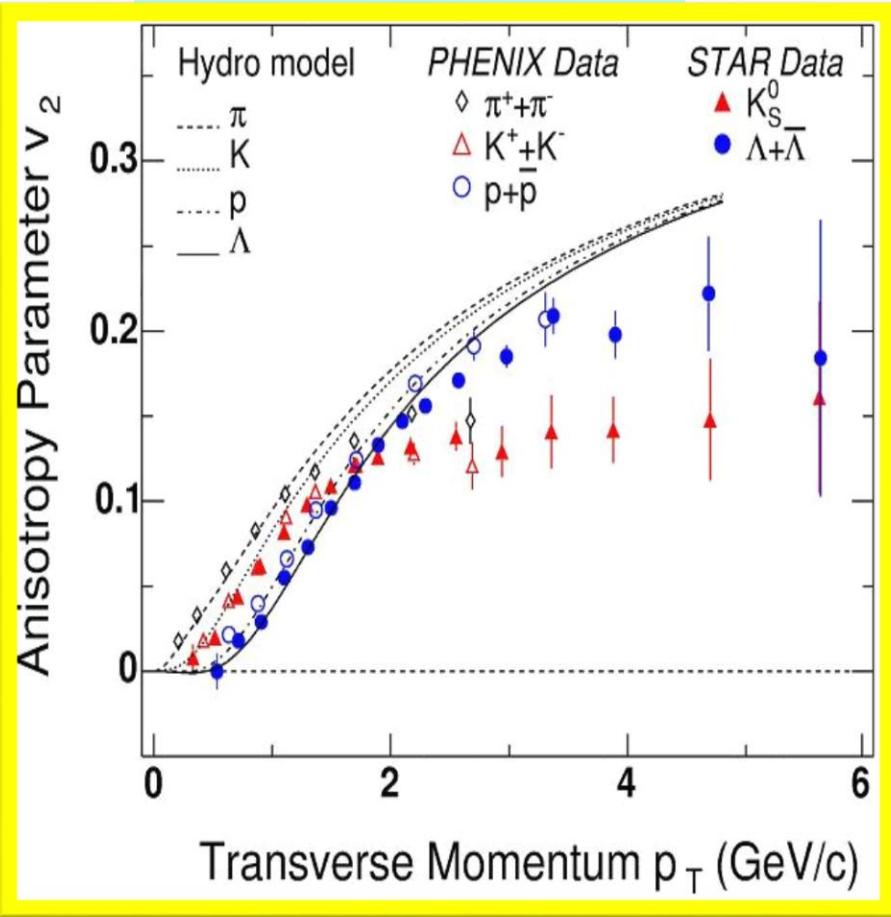


## Evolution as a bulk system

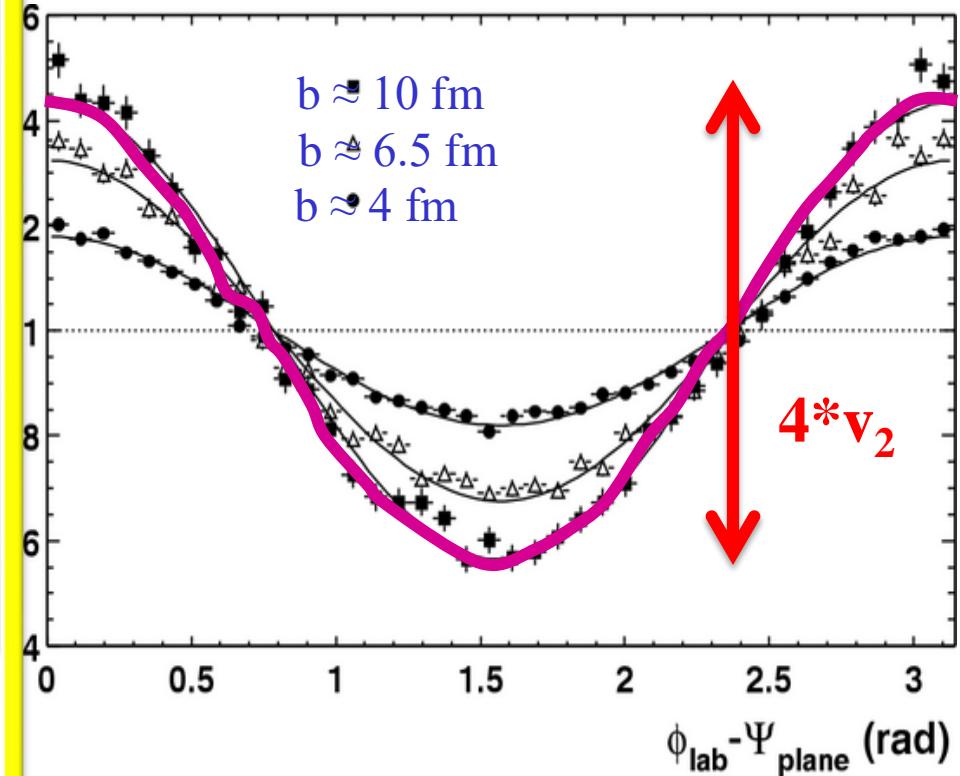


# Azimuthal distributions

## Elliptic Flow

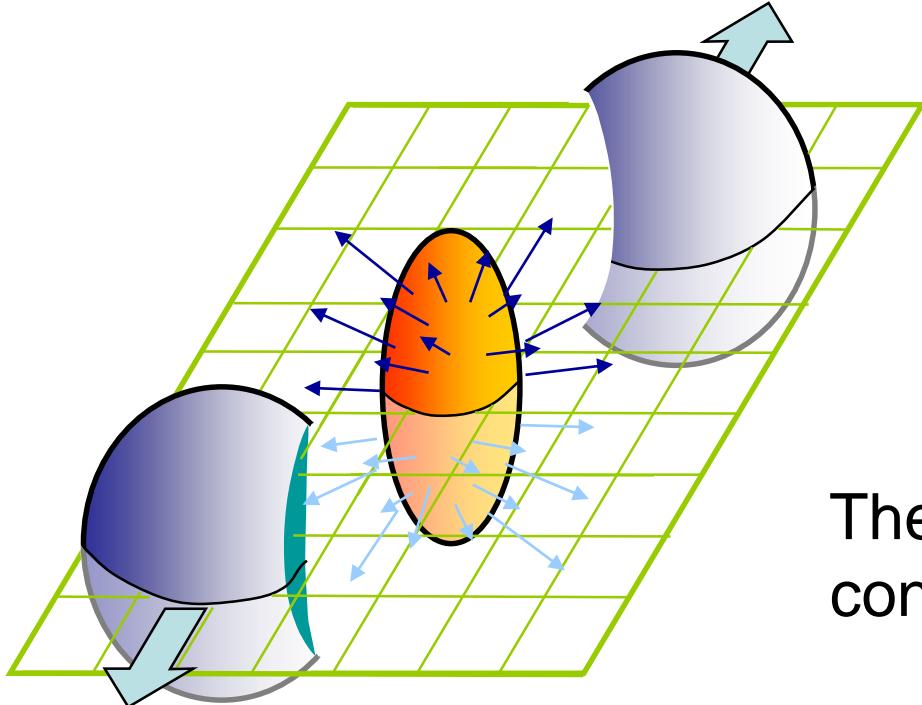


STAR, PRL90 032301 (2003)



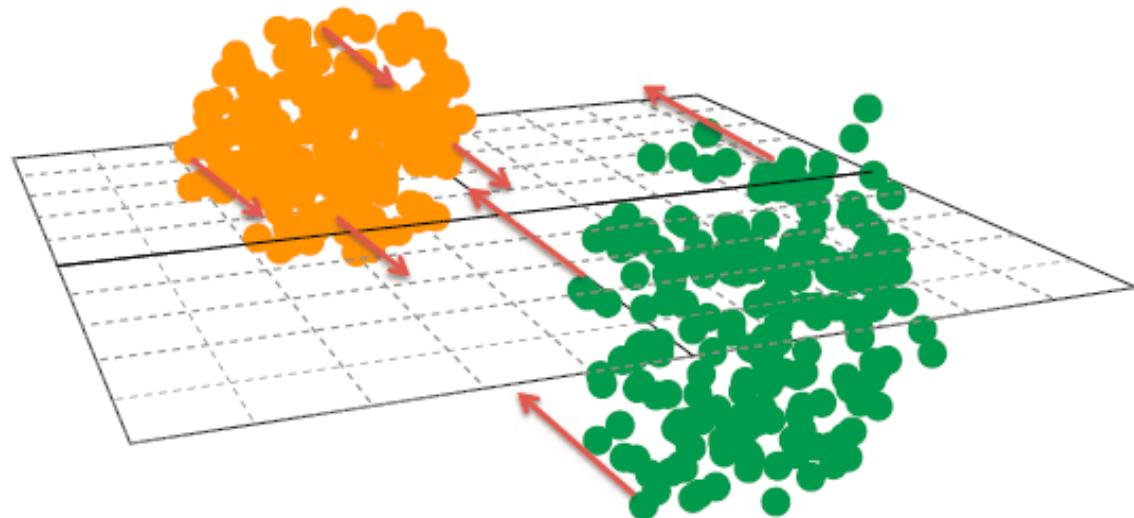
$$E \frac{dN}{d^3p} = \frac{dN}{dyp_T dp_T d\varphi} = \frac{1}{2\pi} \frac{dN}{dyp_T dp_T} [1 + 2v_2(p_T, b)\cos(2\varphi) + \dots]$$

Instead of two smooth colliding nuclei

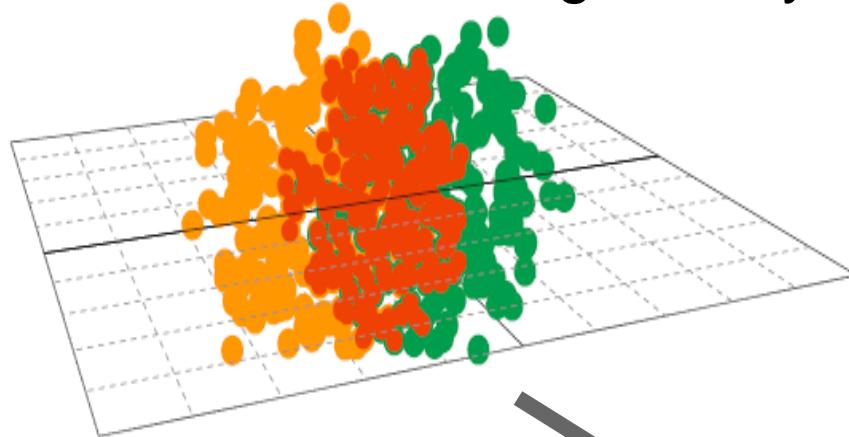


### Initial stage fluctuations

The position of initial nucleons constantly fluctuate

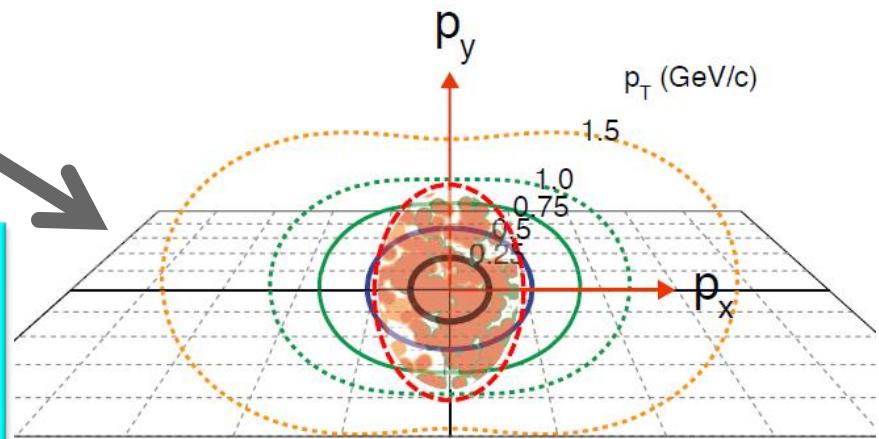
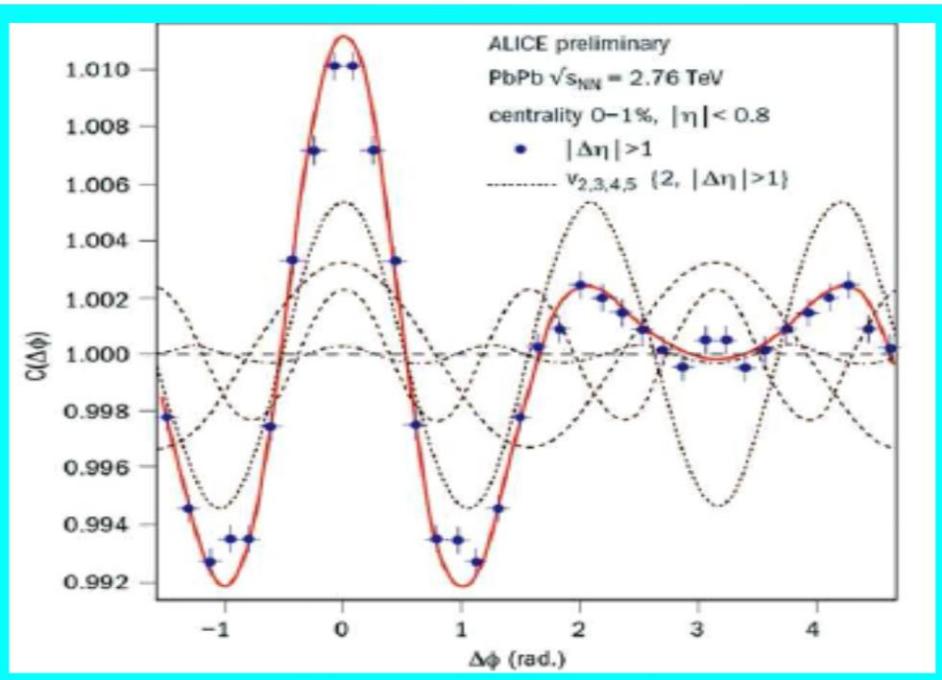


# QGP with fluctuating density

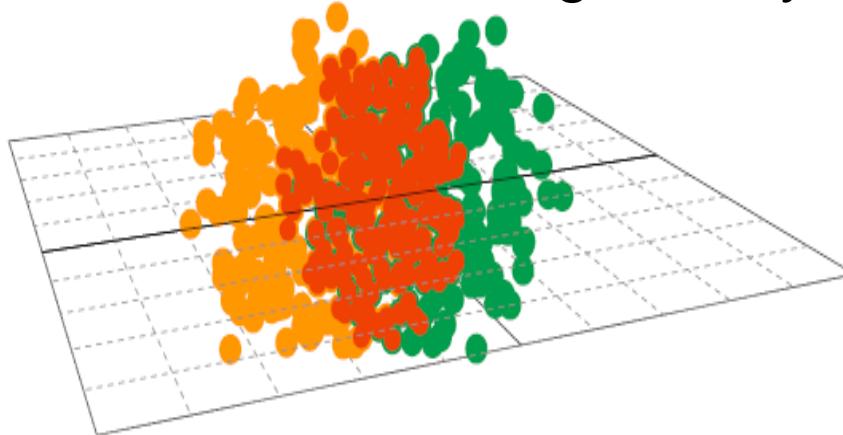


Azimuthal distribution in p-space

→ measured flow:  $v_n$

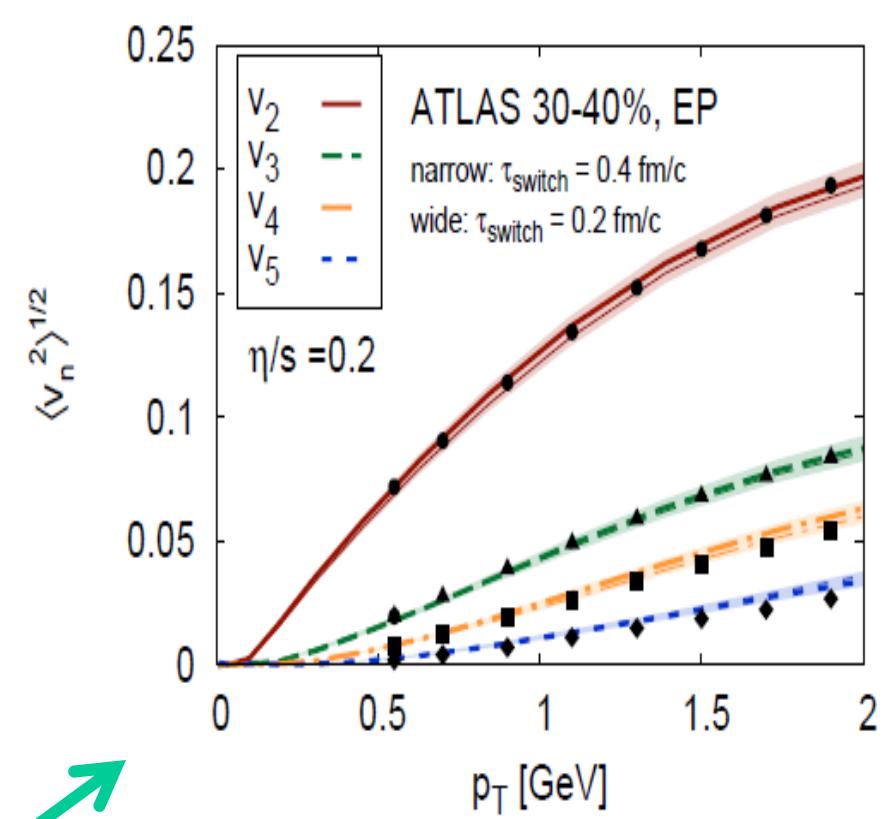
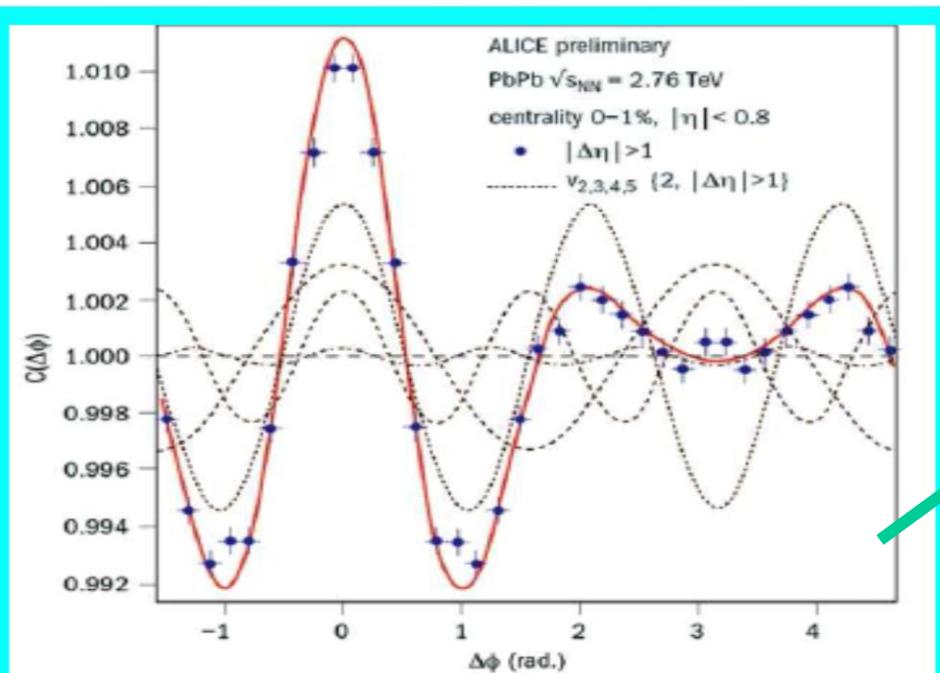


# QGP with fluctuating density



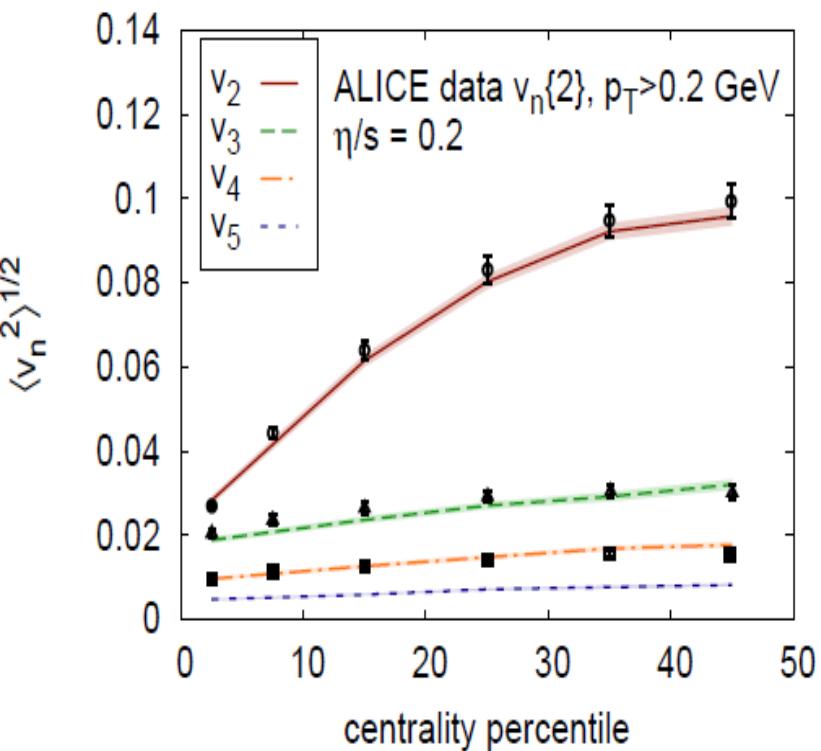
## Elliptic Flow & higher order flow harmonics

→ measured flow:  $v_n$



$N(\phi) \propto 1 + 2v_1 \cos(\phi) + 2v_2 \cos(2\phi)$   
 $+ 2v_3 \cos(3\phi) + \dots$

# The Success of Hydrodynamics



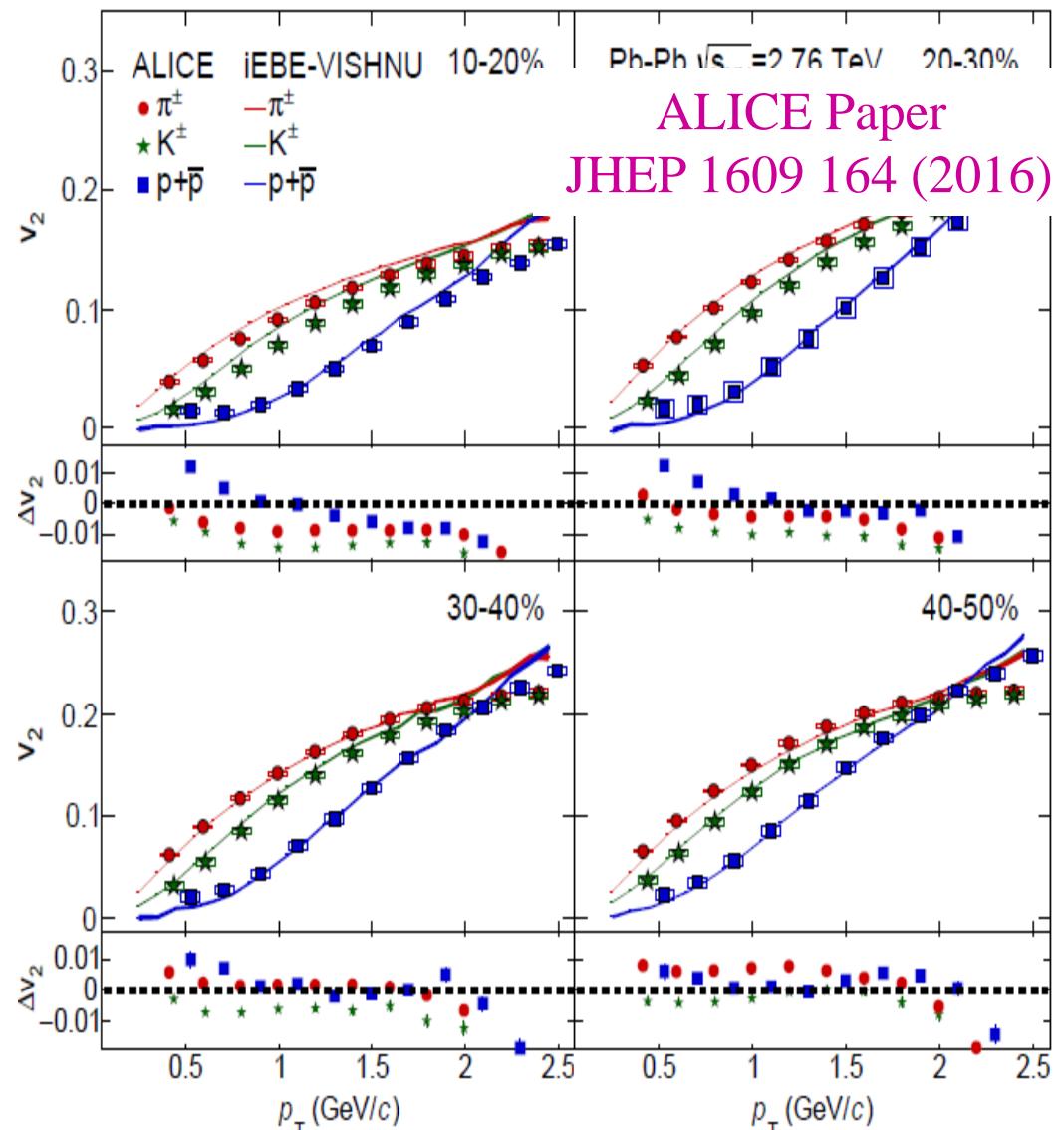
-Hydro + IP-Glasma

Gale, et. Al, PRL2013

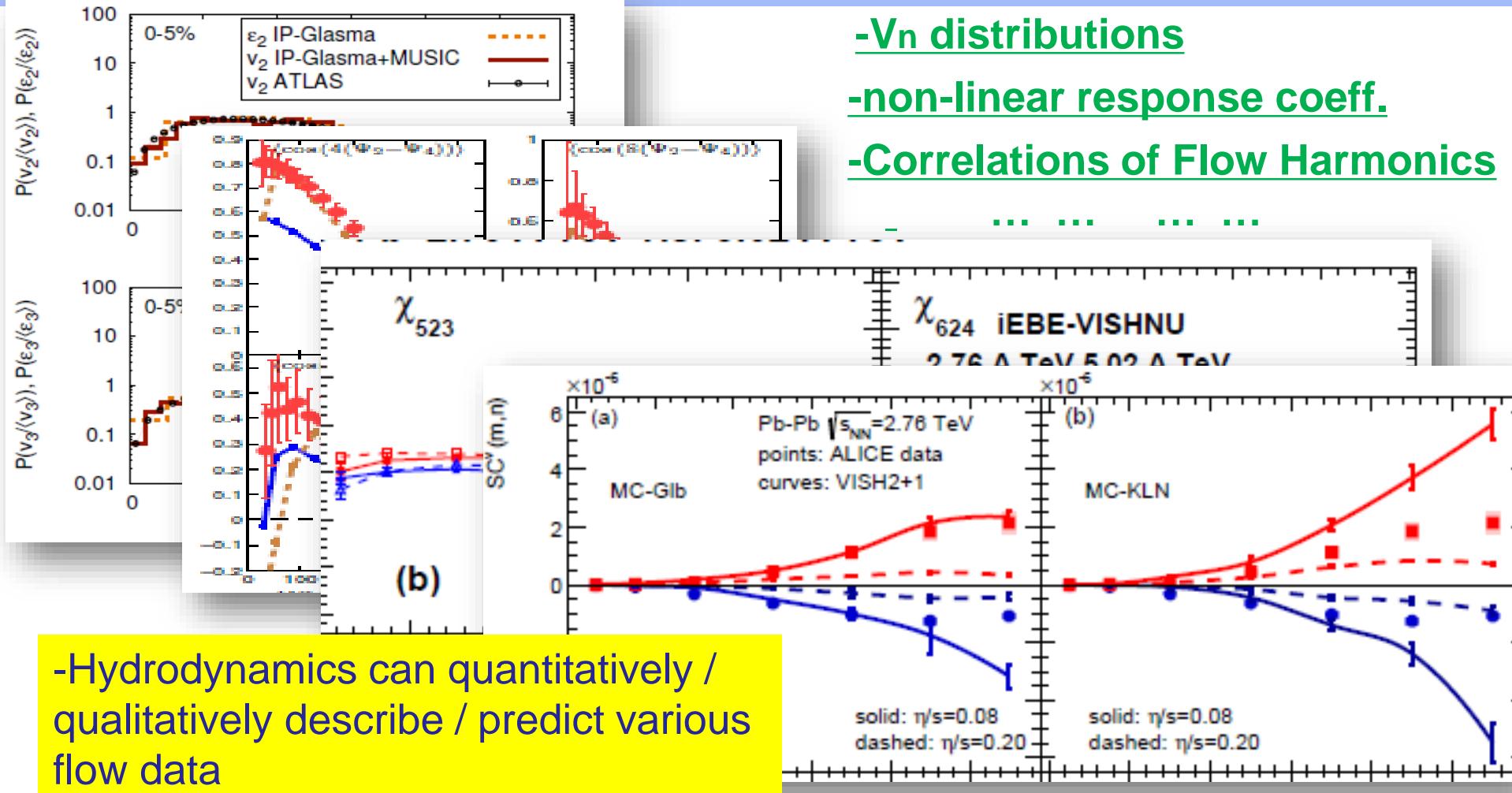
-iEBE-VISHNU + AMPT

Xu, Li, H. S\*, PRC 2016

-hydrodynamics nice describe of integrated and differential  $V_n$  of all charged and identified hadrons



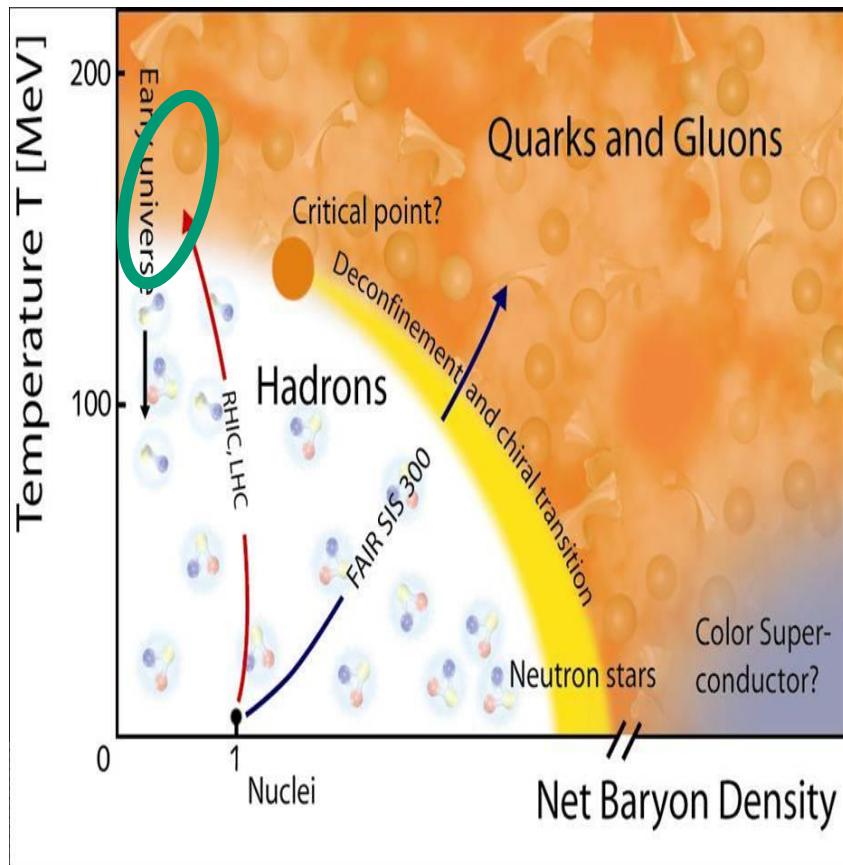
# Various Flow Predictions from Hydrodynamics



H. Xu, Z. Li and H. S\*, Phys. Rev. C93, no. 6, 064905 (2016); W. Zhao, H. Xu and H. S\*, Eur. Phys. J. C 77, no. 9, 645 (2017); X. Zhu, Y. Zhou, H. Xu and H. S\*, Phys. Rev. C95, no. 4, 044902 (2017); W. Zhao, L. Zhu, H. Zheng, C. M. Ko and H. S\*, Phys. Rev. C 98, no. 5, 054905 (2018); Li, Zhao, Zhou, H.S\*, in preparation (2020) ... ... ... ...

# Flow & QGP viscosity

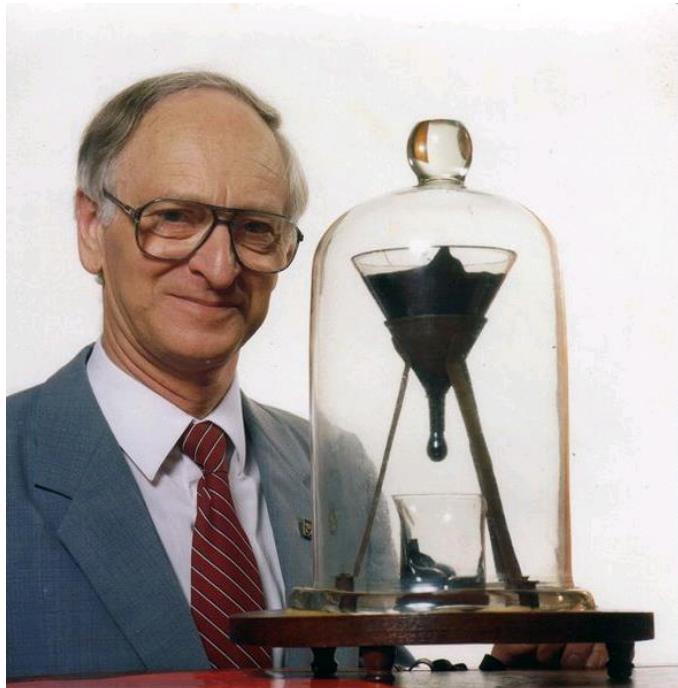
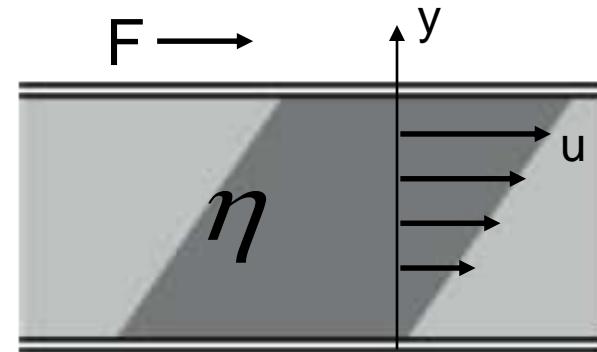
@ top RHIC and LHC energies



# Shear Viscosity

-classical definition:

$$\frac{F}{A} = \eta \frac{du}{dy}$$



## A super viscous liquid - Pitch

Pitch has viscosity approximately 230 billion times that of water.

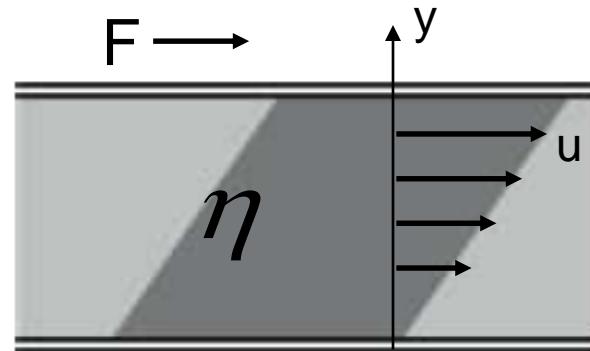
Longest running experiment (1927-present)  
8 drops so far, none ever seen fall!

[http://en.wikipedia.org/wiki/Pitch\\_drop\\_experiment](http://en.wikipedia.org/wiki/Pitch_drop_experiment)

# Lowest bound of specific shear viscosity

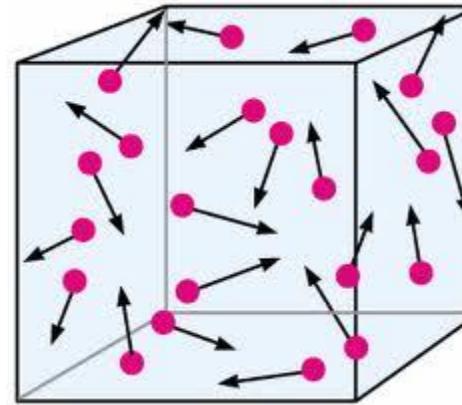
-classical definition:

$$\frac{F}{A} = \eta \frac{du}{dy}$$



-kinetic theory:

$$\eta \sim mn\bar{v}l_{mfp}$$



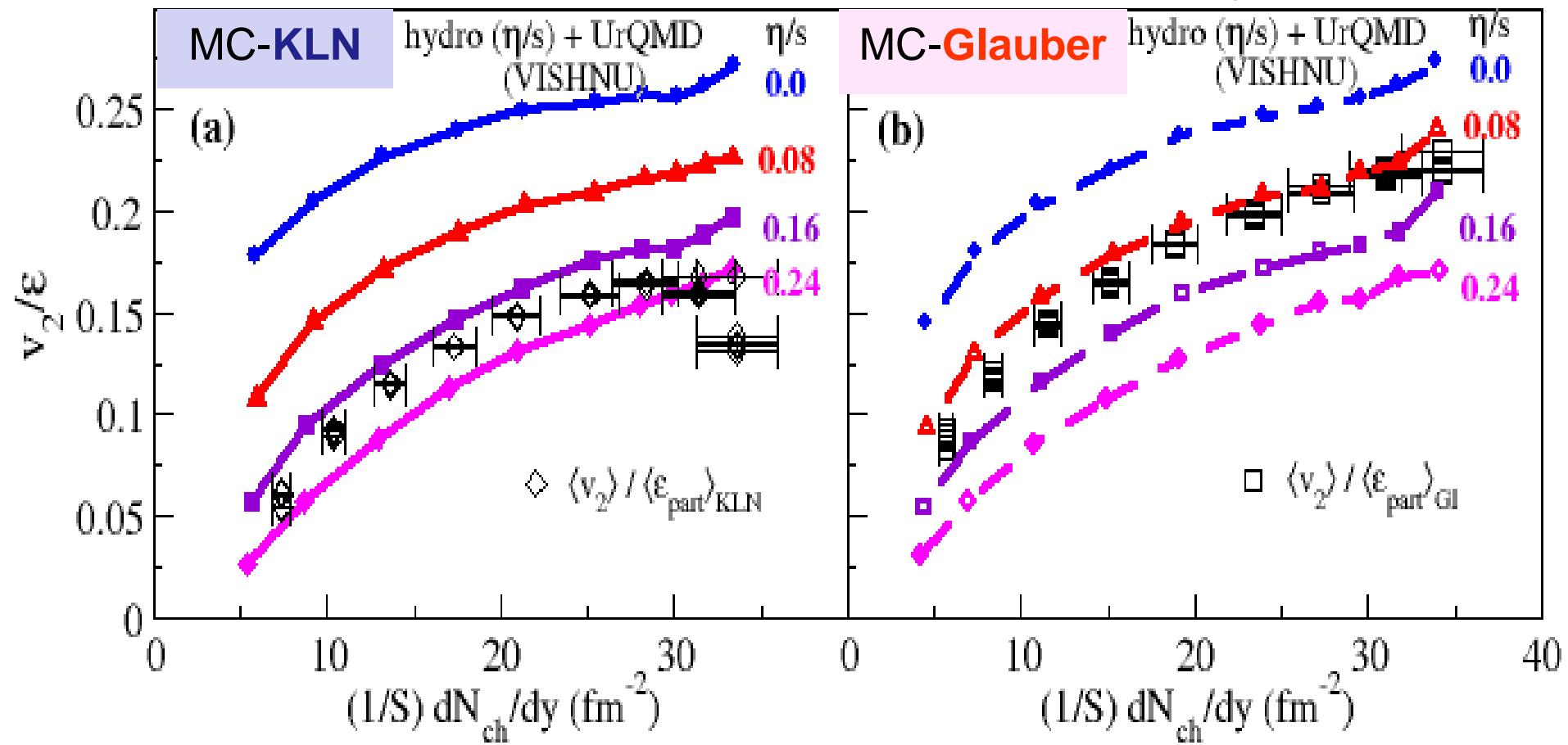
$$\frac{\eta}{s} \sim \frac{1}{k_B} \bar{v} m l_{mfp} \sim \frac{1}{k_B} \left( \frac{1}{2} m \bar{v}^2 \right) \left( \frac{l_{mfp}}{\bar{v}} \right) \sim \frac{e \tau}{k_B} \quad (s \sim k_B n)$$

uncertainty principle:  $\rightarrow$

$$\frac{\eta}{s} \geq \frac{h}{k_B}$$

# Extracting QGP viscosity-early attempt

H. Song,et.al, PRL2011



$$1 \times (1/4\pi) \leq (\eta/s)_{QGP} \leq 2.5 \times (1/4\pi)$$

# Extract QGP properties from bulk observ.

-massive data evaluation

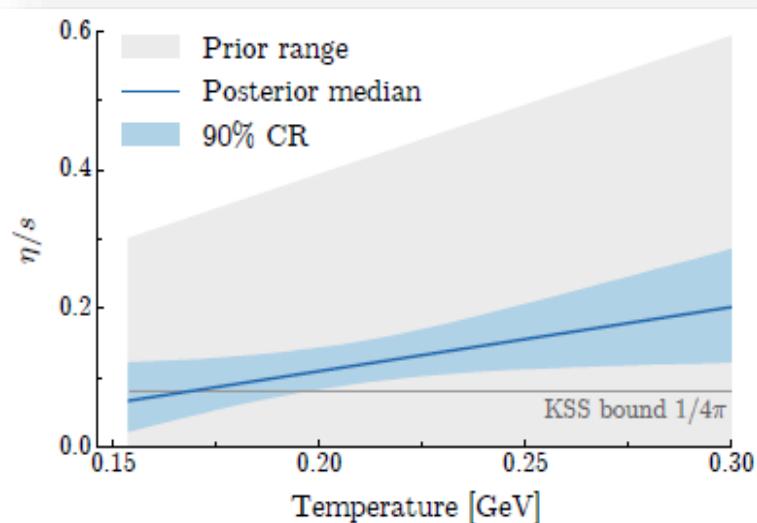
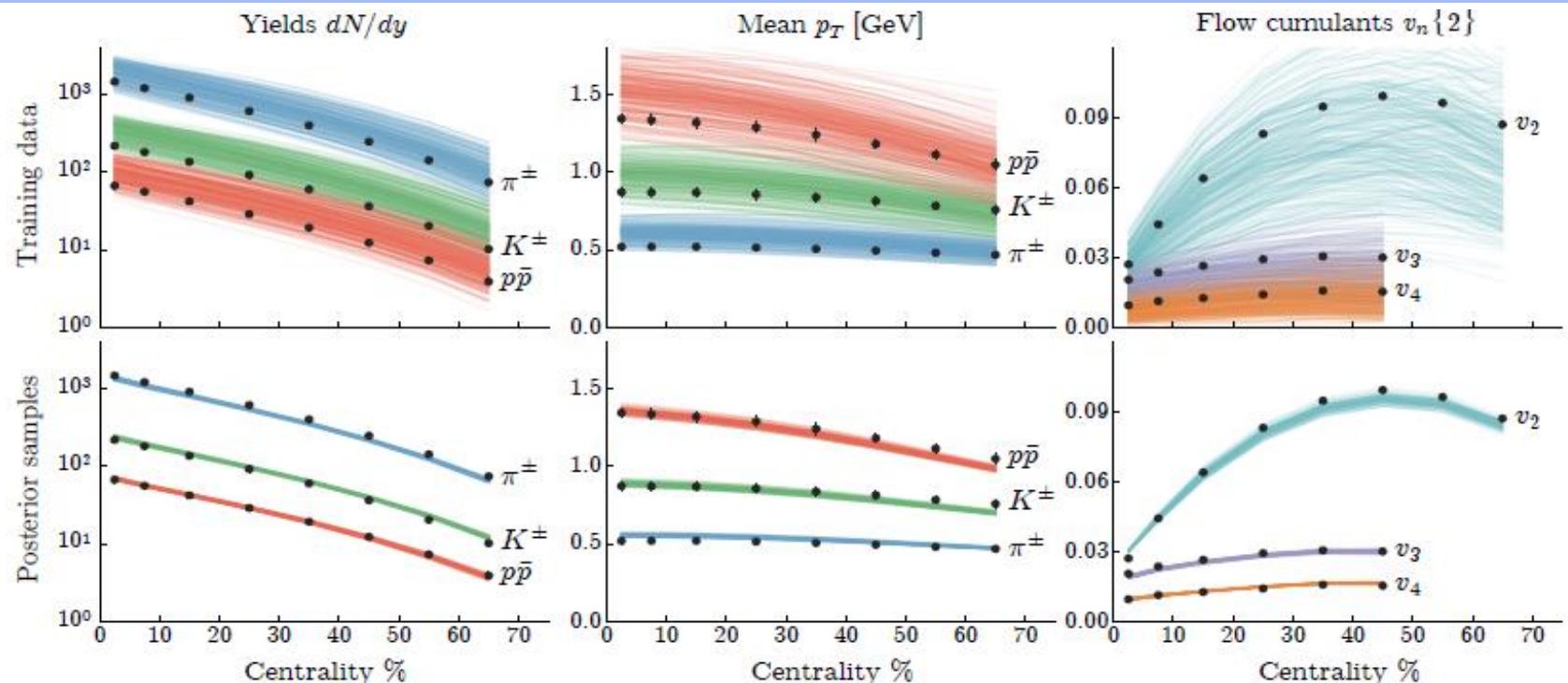
## Exp Observables

- particle yields
  - spectra
  - elliptic flow
  - triangular flow & higher order flow harmonics
  - event by event  $v_n$  distributions
  - higher-order event plane correlations
- .... .... .... ....

## Hydro model & its Inputs:

- Initial conditions
  - EoS
  - shear viscosity
  - bulk viscosity
  - Heat conductivity
  - relaxation times
  - freeze-out/switching cond.
- .... .... .... ....

# An quantitatively extraction of the QGP viscosity

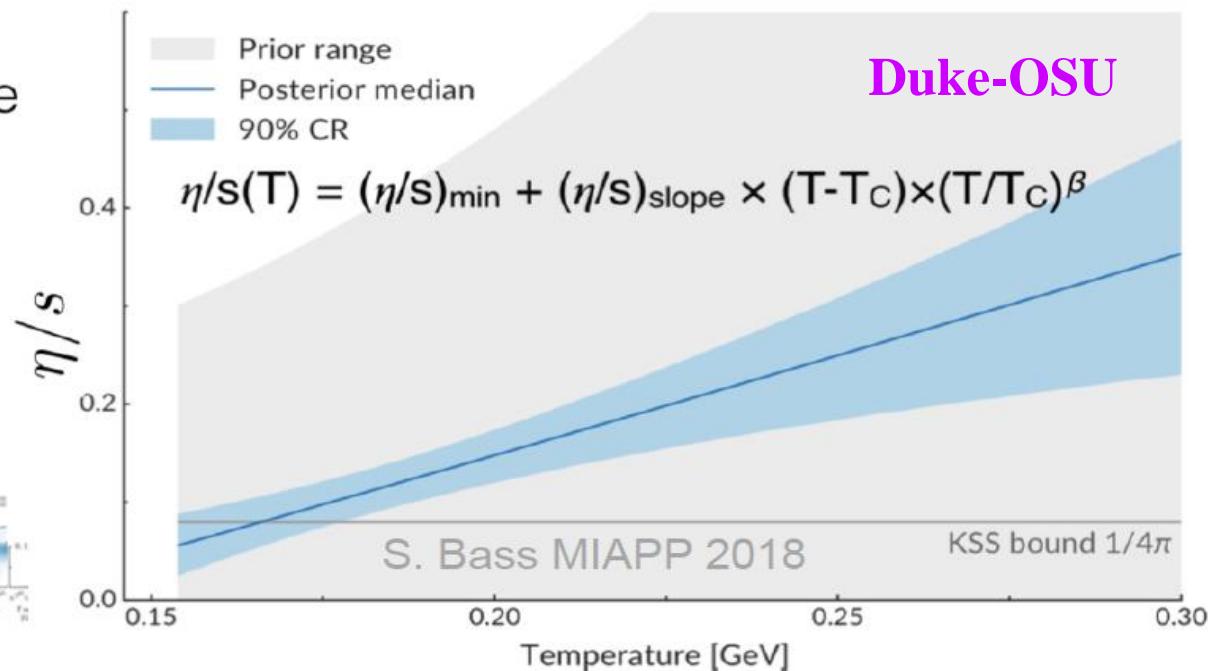
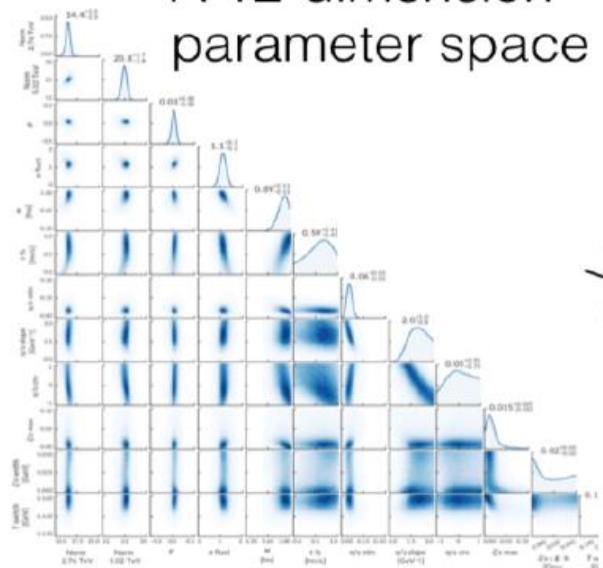


-An quantitatively extraction of the QGP viscosity with iEBE-VISHNU and the massive data evaluation  
- $\eta/s(T)$  is very close to the KSS bound of  $1/4\pi$

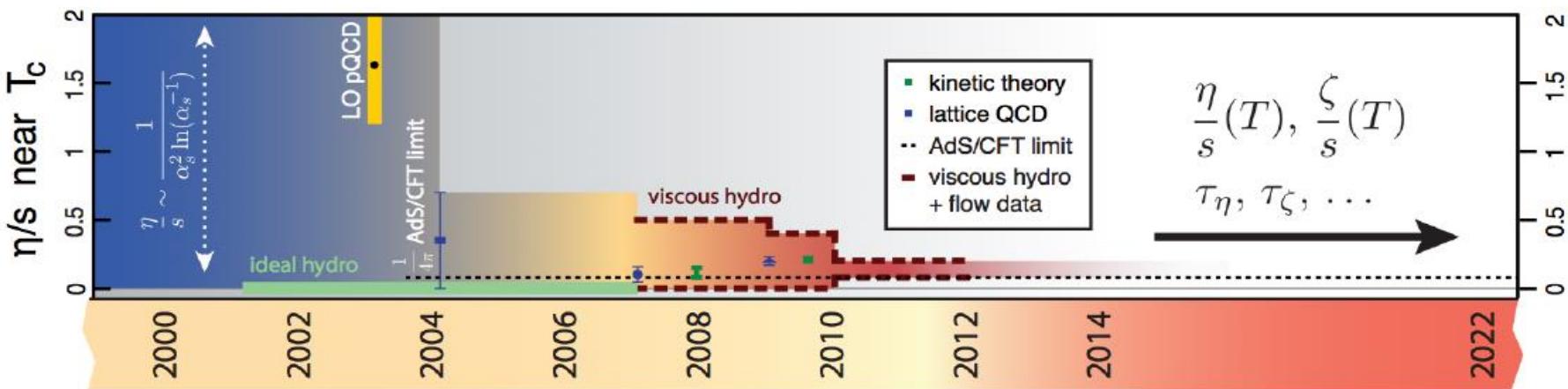
J. Bernhard, S. Moreland, S.A. Bass,  
J. Liu, U. Heinz, PRC 2015

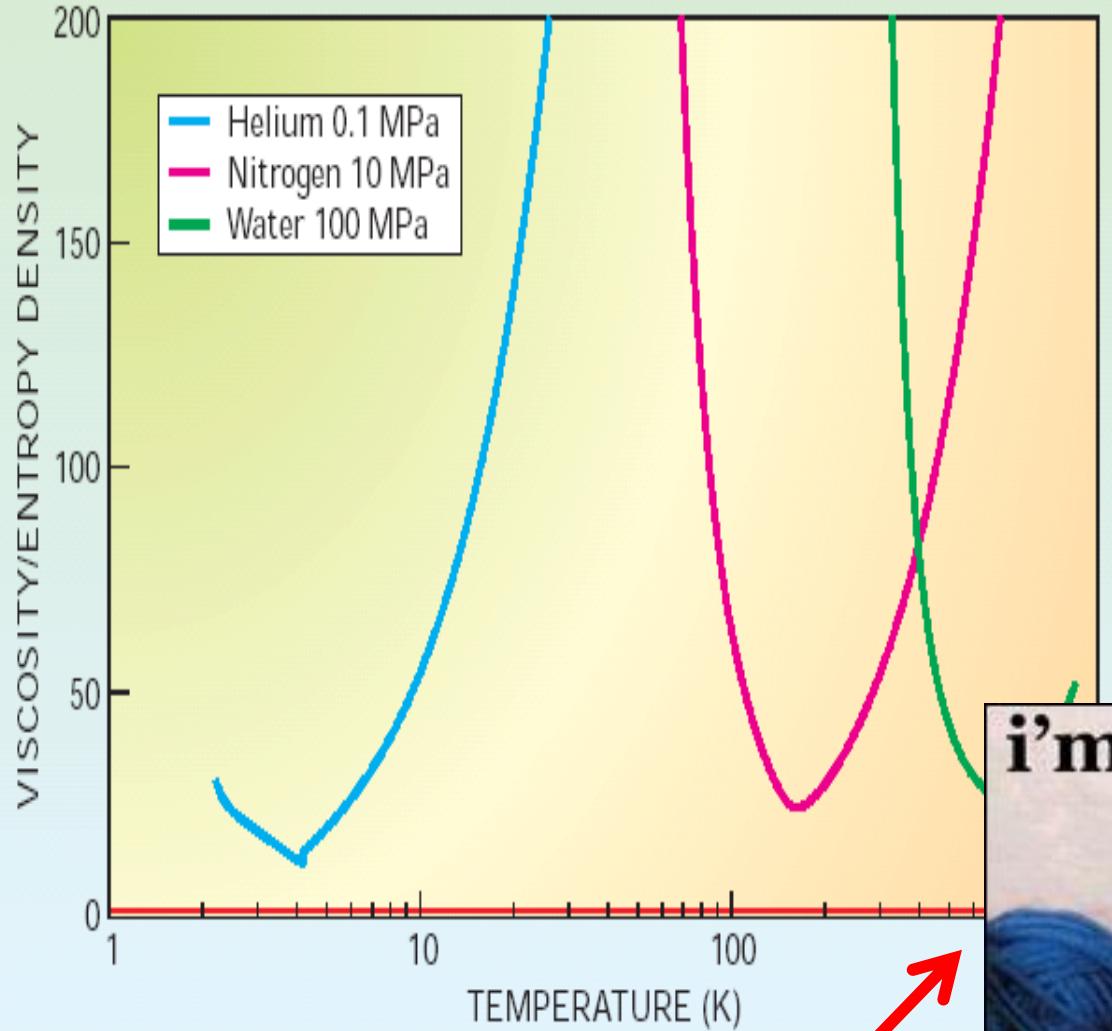
# Extracting QGP viscosity with massive data evaluation

A 12-dimension parameter space



## Extracted QGP viscosity with ever increasing precision





AdS/CFT

$$\frac{\eta}{s} \geq \frac{h}{k_B}$$

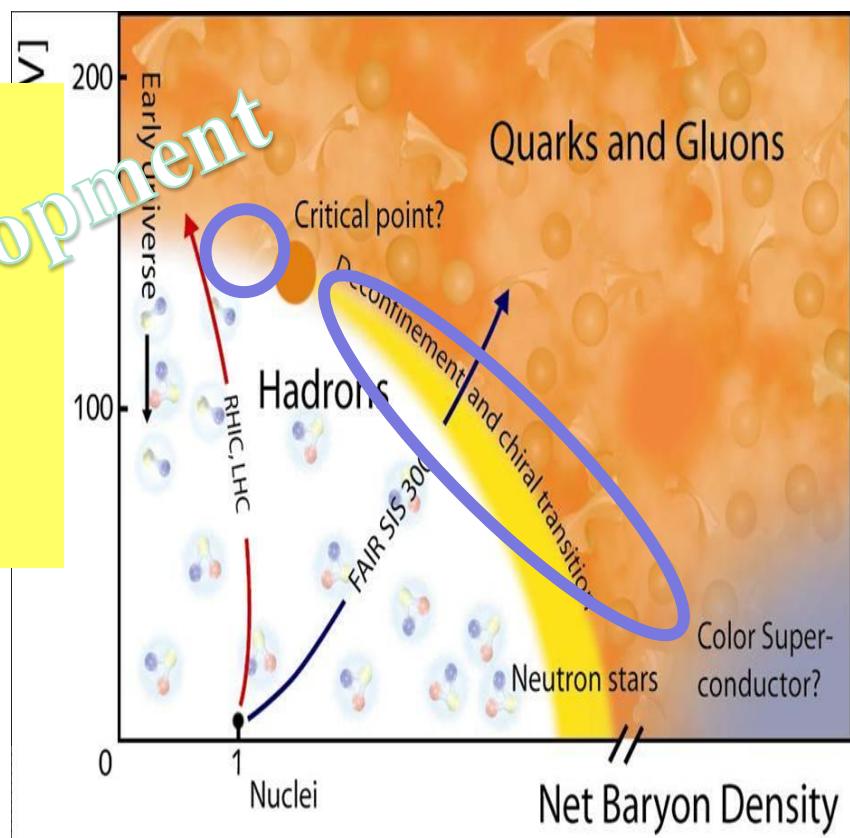


QGP specific shear viscosity  
Extracted from exp data

# Flow & viscosity at RHIC-BES

## Hybrid model for RHIC BES

- proper initial condition  
(**dynamical initial cond.,...**)
- 3+1-d hydro  
(**effects from heat conductivity ...**)
- hadronic afterburner
- EoS with T & P



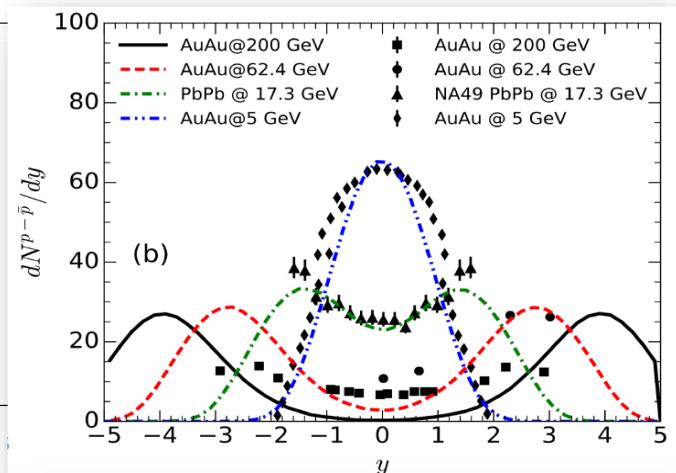
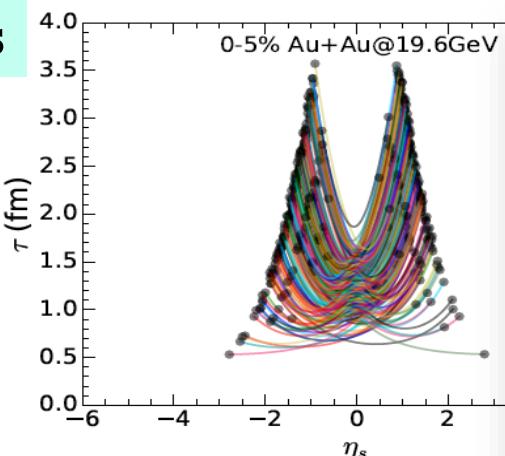
# Recent model development for RHIC BES

## Dynamical initial conditions

$$\partial_\mu T^{\mu\nu} = J_{\text{source}}^\nu$$

$$\partial_\mu J^\mu = \rho_{\text{source}}.$$

C. Shen and B. Schenke, Phys. Rev. C97 (2018) 024907



## Net baryon diffusion

$$\begin{aligned} \Delta^{\mu\nu} D q_\nu &= -\frac{1}{\tau_q} \left( q^\mu - \kappa_B \nabla^\mu \frac{\mu_B}{T} \right) - \frac{\delta_{qq}}{\tau_q} q^\mu \theta - \frac{\lambda_{qq}}{\tau_q} q_\nu \sigma^{\mu\nu} \\ &+ \frac{l_{q\pi}}{\tau_q} \Delta^{\mu\nu} \partial_\lambda \pi^\lambda{}_\nu - \frac{\lambda_{q\pi}}{\tau_q} \pi^{\mu\nu} \nabla_\nu \frac{\mu_B}{T}, \quad (13) \end{aligned}$$

$$\begin{aligned} \Delta_{\alpha\beta}^{\mu\nu} D \pi^{\alpha\beta} &= -\frac{1}{\tau_\pi} (\pi^{\mu\nu} - 2\eta \sigma^{\mu\nu}) \\ &- \frac{\delta_{\pi\pi}}{\tau_\pi} \pi^{\mu\nu} \theta - \frac{\tau_{\pi\pi}}{\tau_\pi} \pi^\lambda \langle \sigma^\nu \rangle_\lambda + \frac{\phi_7}{\tau_\pi} \pi^{\langle\mu}{}_\alpha \pi^{\nu\rangle\alpha} \\ &+ \frac{l_{\pi q}}{\tau_\pi} \nabla^{\langle\mu} q^{\nu\rangle} + \frac{\lambda_{\pi q}}{\tau_\pi} q^{\langle\mu} \nabla^{\nu\rangle} \frac{\mu_B}{T}. \end{aligned}$$

G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke and C. Shen, Phys. Rev. C98, 034916 (2018); M. Li and C. Shen, Phys. Rev. C98, 064908 (2018)

Net baryon diffusion transports more baryon numbers to the mid-rapidity region / extracting heat conductivity  
In the future

# Recent development of hybrid model for RHIC BES

## Dynamical initial conditions

$$\partial_\mu T^{\mu\nu} = J_{\text{source}}^\nu$$

$$\partial_\mu J^\mu = \rho_{\text{source}}.$$

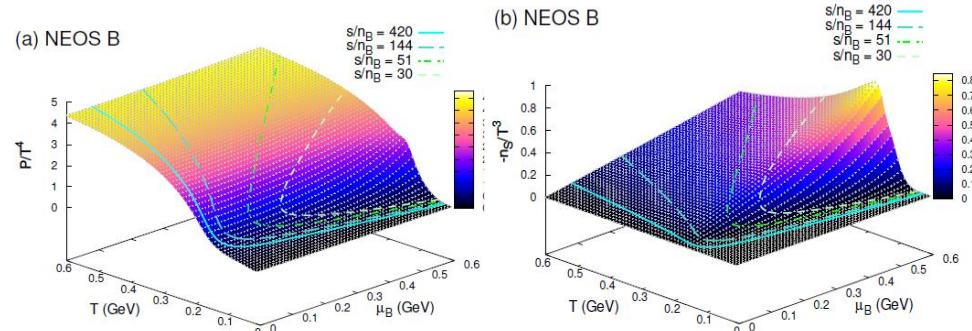
C. Shen and B. Schenke, Phys. Rev. C97 (2018) 024907

## Net baryon diffusion

$$\begin{aligned} \Delta^{\mu\nu} D q_\nu &= -\frac{1}{\tau_q} \left( q^\mu - \kappa_B \nabla^\mu \frac{\mu_B}{T} \right) - \frac{\delta_{qq}}{\tau_q} q^\mu \theta - \frac{\lambda_{qq}}{\tau_q} q_\nu \sigma^{\mu\nu} \\ &\quad + \frac{l_{q\pi}}{\tau_q} \Delta^{\mu\nu} \partial_\lambda \pi^\lambda{}_\nu - \frac{\lambda_{q\pi}}{\tau_q} \pi^{\mu\nu} \nabla_\nu \frac{\mu_B}{T}, \end{aligned} \quad (13)$$

$$\begin{aligned} \Delta_{\alpha\beta}^{\mu\nu} D \pi^{\alpha\beta} &= -\frac{1}{\tau_\pi} (\pi^{\mu\nu} - 2\eta \sigma^{\mu\nu}) \\ &\quad - \frac{\delta_{\pi\pi}}{\tau_\pi} \pi^{\mu\nu} \theta - \frac{\tau_{\pi\pi}}{\tau_\pi} \pi^\lambda \langle \sigma^\nu \rangle_\lambda + \frac{\phi_7}{\tau_\pi} \pi^{\langle\mu}{}_\alpha \pi^{\nu\rangle\alpha} \\ &\quad + \frac{l_{\pi q}}{\tau_\pi} \nabla^{\langle\mu} q^{\nu\rangle} + \frac{\lambda_{\pi q}}{\tau_\pi} q^{\langle\mu} \nabla^{\nu\rangle} \frac{\mu_B}{T}. \end{aligned} \quad (14)$$

## EoS with finite $T$ & $\mu$

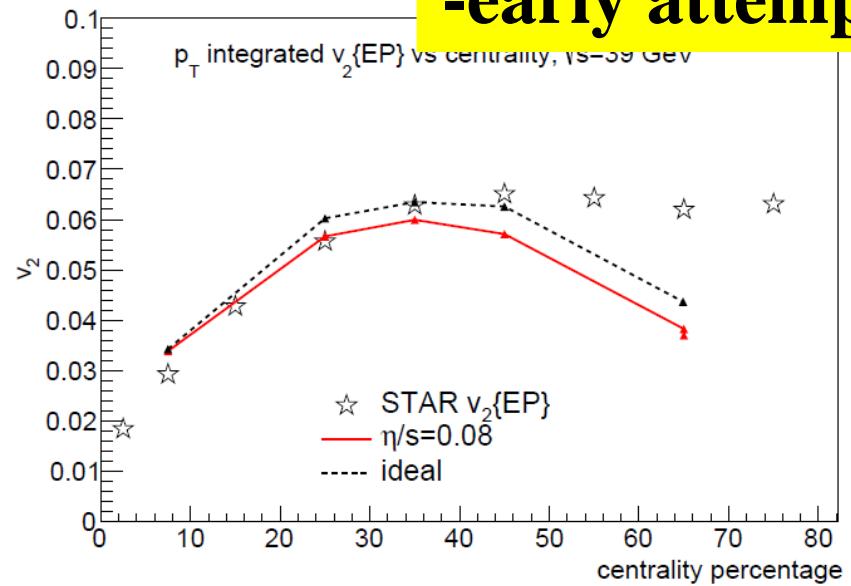
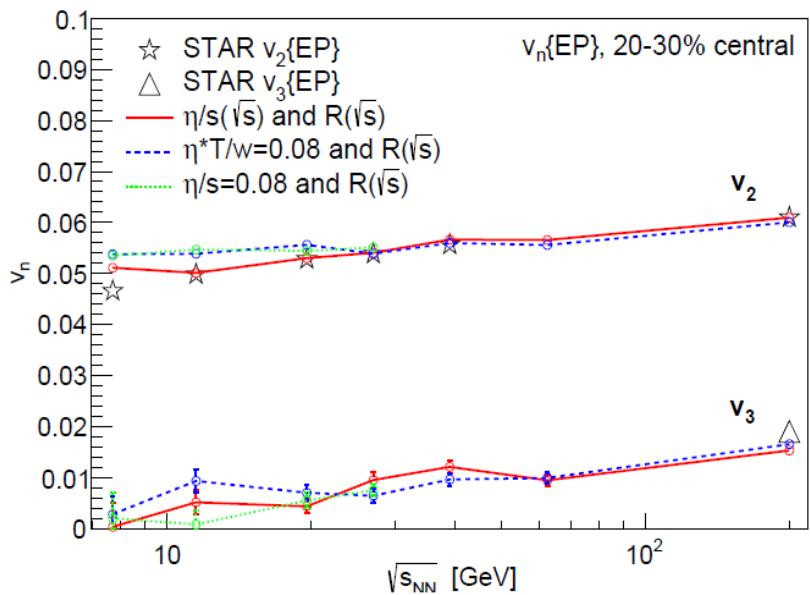


A. Monnai, B. Schenke and C. Shen, arXiv:1902.05095 [nucl-th].

G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke and C. Shen, Phys. Rev. C98, 034916 (2018); M. Li and C. Shen, Phys. Rev. C98, 064908 (2018)

# Extracting $\eta/s(\sqrt{s})$ from RHIC BES (I)

-early attempt



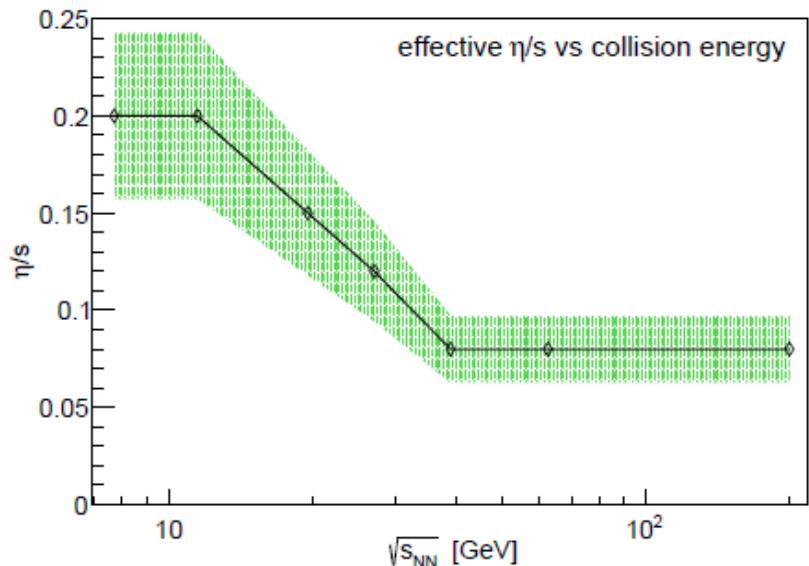
## Data

- RHIC BES Au+Au 7.7-200 A GeV

## Model

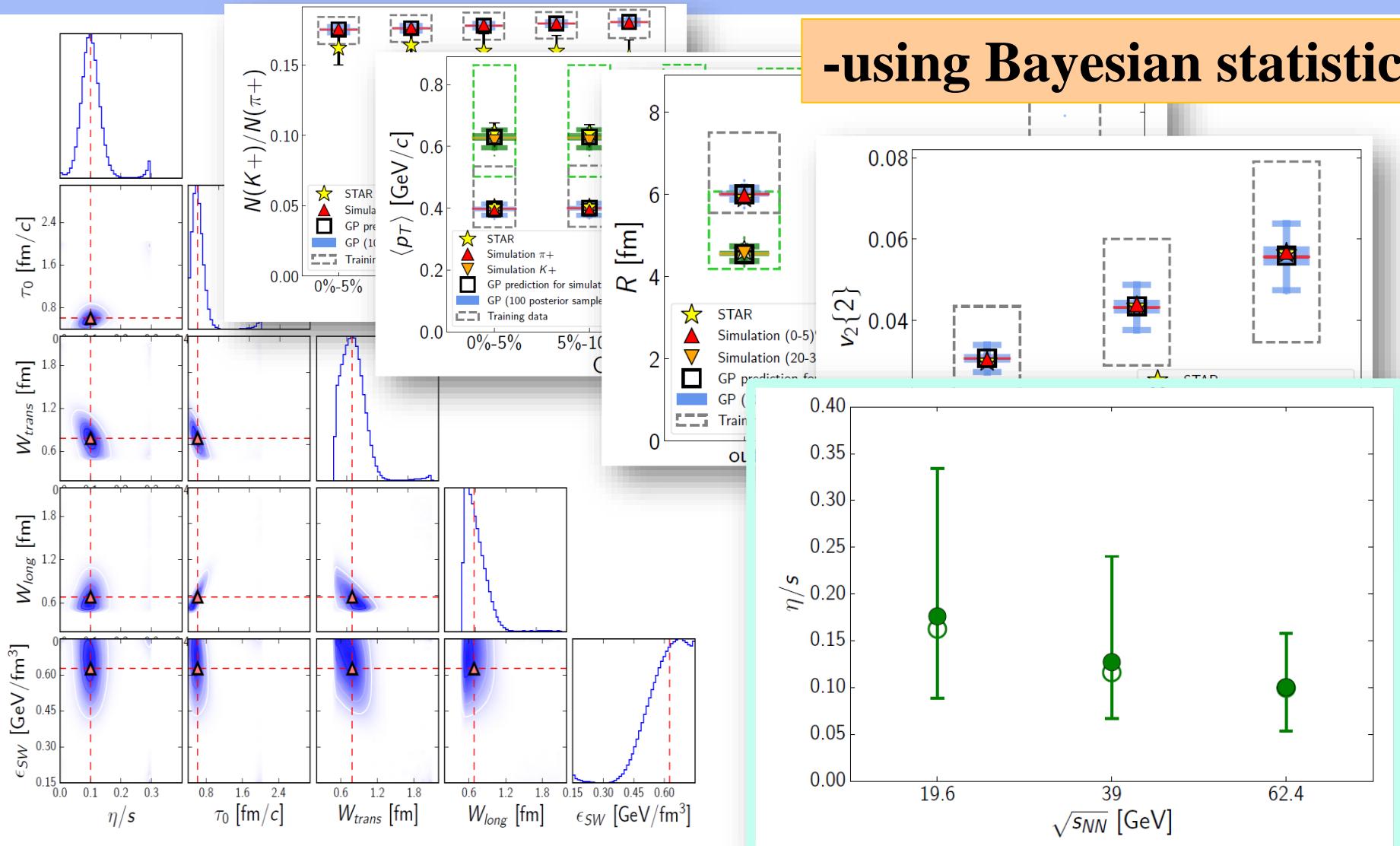
- 3+1d viscous hydro + UrQMD
- pre-equilibrium stage UrQMD
- EoS (Chiral Model with  $T, \mu$ )

I. A. Karpenko, P. Huovinen, H. Petersen and M. Bleicher, Phys. Rev. C91, no. 6, 064901 (2015)



# Extracting $\eta/s(\sqrt{s})$ from RHIC BES (II)

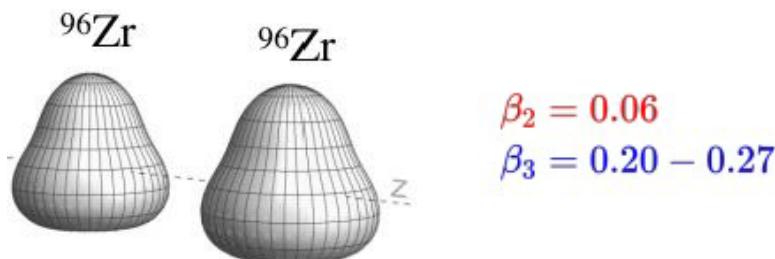
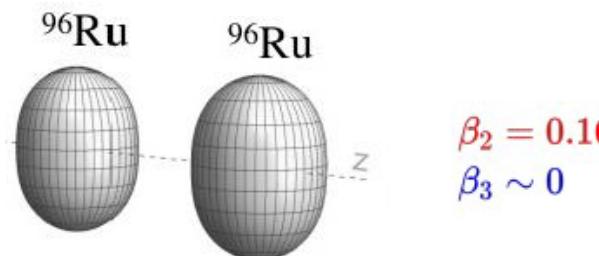
-using Bayesian statistics



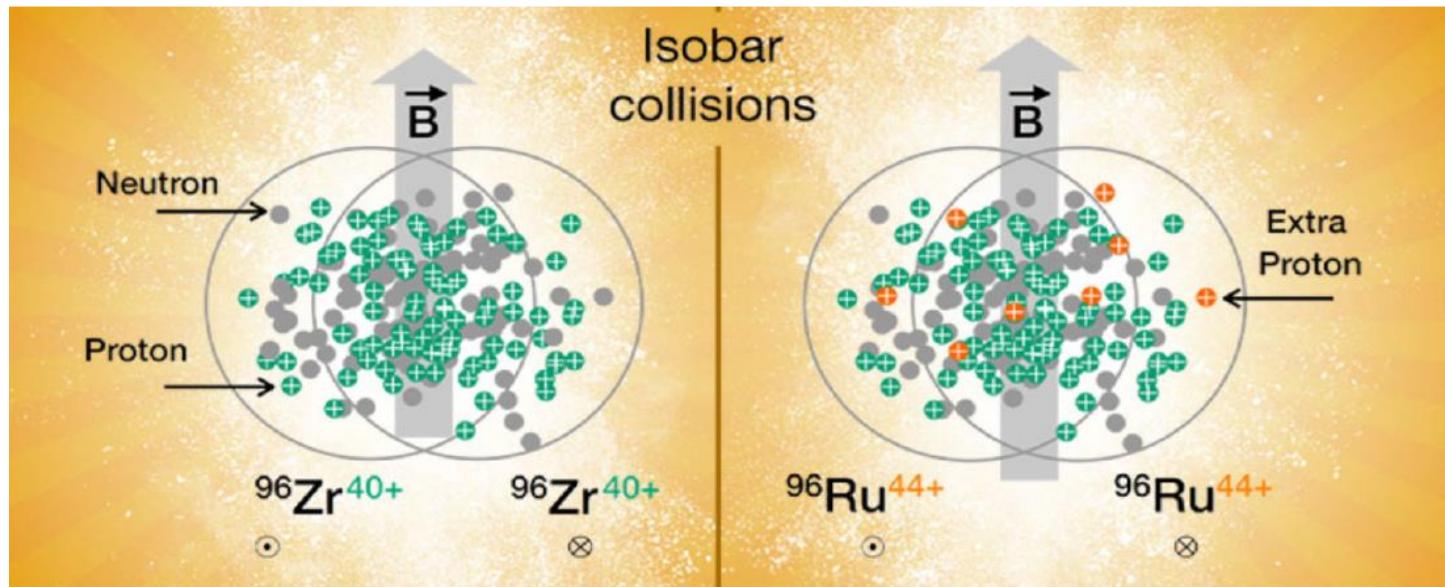
Future:  
 $\eta/s(T, \mu)$   $\zeta/s(T, \mu)$   $K/s(T, \mu)$

J. Auvinen, J. E. Bernhard, S. A. Bass and I. Karpenko, Phys. Rev. C97, no. 4, 044905 (2018)

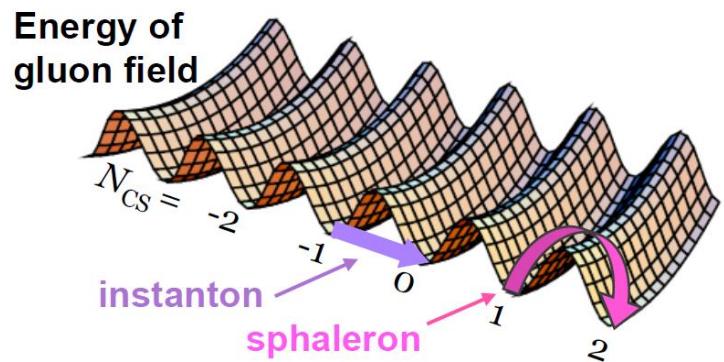
# Probe the Nuclear Deformation with high energy nucleus-nucleus collisions



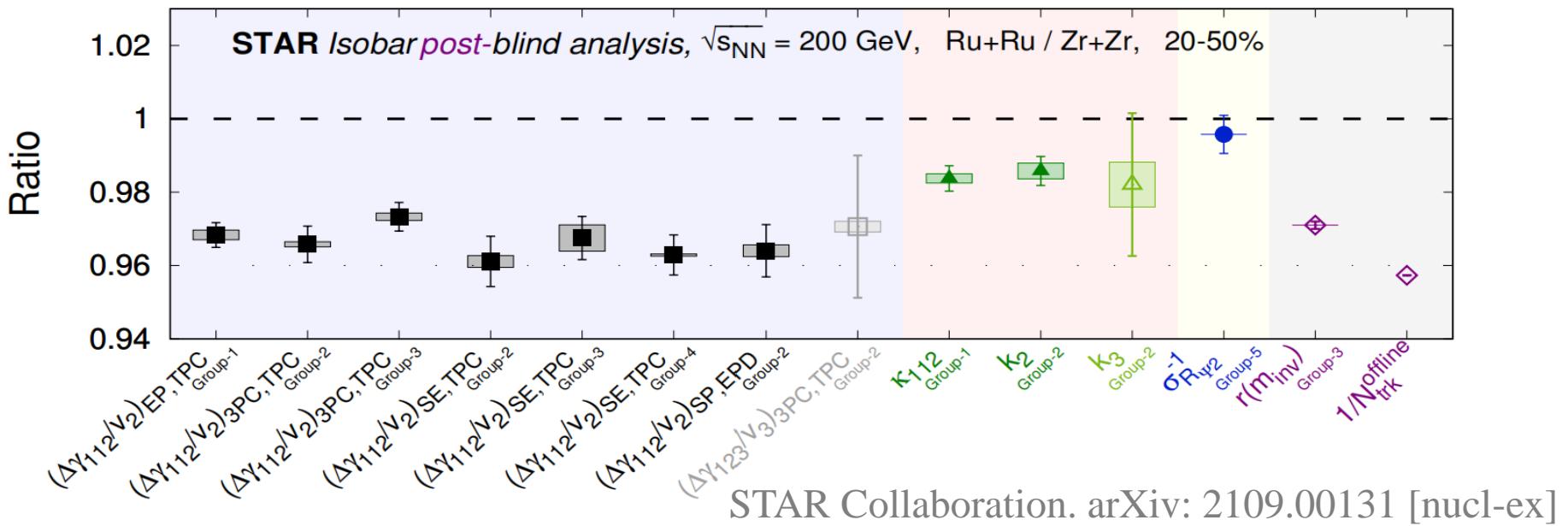
# $^{96}\text{Ru} + ^{96}\text{Ru}$ and $^{96}\text{Zr} + ^{96}\text{Zr}$ Collisions @ RHIC isobar run



- Obviously different early magnetic field for Ru+Ru and Zr+Zr collisions
- Aim to search the Chiral Magnetic Effect (CME) and probe nontrivial structure of the QCD vacuum



# Search CME with Isobar collisions



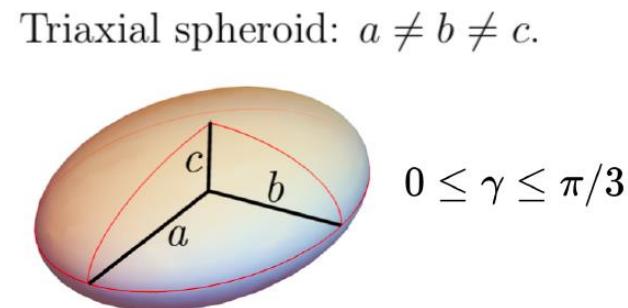
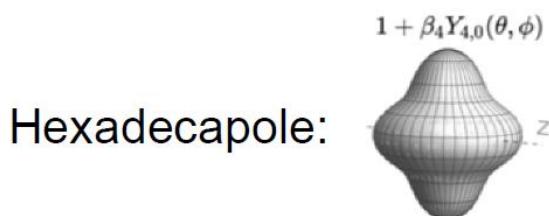
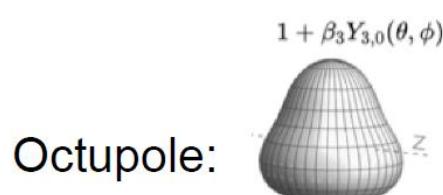
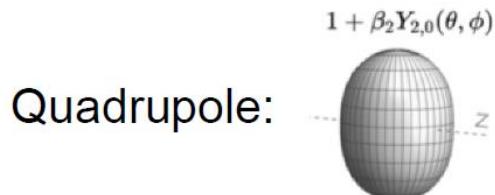
between the two isobar systems. Observed differences in the multiplicity and flow harmonics at the matching centrality indicate that the magnitude of the CME background is different between the two species. No CME signature that satisfies the predefined criteria has been observed in isobar collisions in this blind analysis.

-Observed differences in both multiplicity and v2 imply that **CME background are different for  $^{96}\text{Ru} + ^{96}\text{Ru}$  and  $^{96}\text{Zr} + ^{96}\text{Zr}$  Collisions** at matching centralities

# Nuclear Deformation

$$\rho(r, \theta, \phi) = \frac{\rho_0}{1 + e^{(r - R(\theta, \phi))/a_0}}$$

$$R(\theta, \phi) = R_0 \left( 1 + \beta_2 [\cos \gamma Y_{2,0} + \sin \gamma Y_{2,2}] + \beta_3 \sum_{m=-3}^3 \alpha_{3,m} Y_{3,m} + \beta_4 \sum_{m=-4}^4 \alpha_{4,m} Y_{4,m} \right)$$

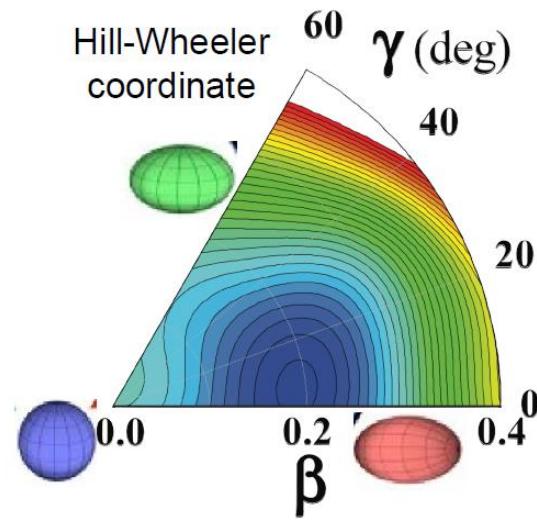
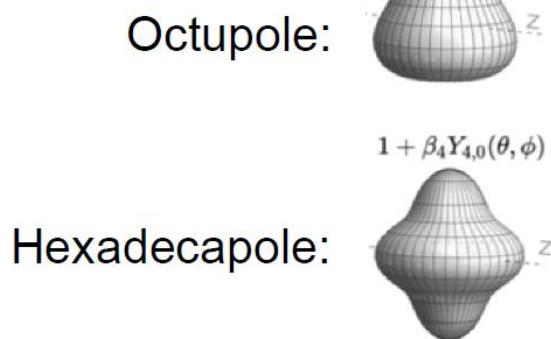
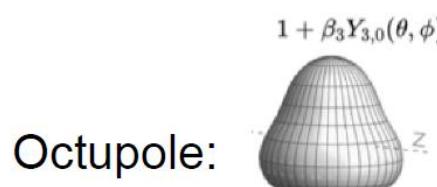
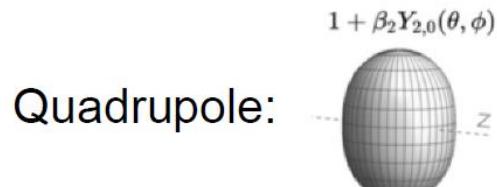


Prolate:  $a=b<c \rightarrow \beta_2, \gamma=0$   
 Oblate:  $a<b=c \rightarrow \beta_2, \gamma=\pi/3$  or  $-\beta_2, \gamma=0$

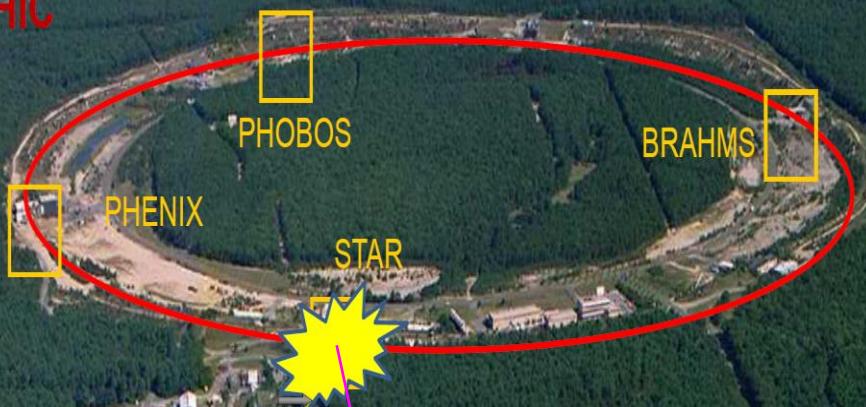
# Nuclear Deformation

$$\rho(r, \theta, \phi) = \frac{\rho_0}{1 + e^{(r - R(\theta, \phi))/a_0}}$$

$$R(\theta, \phi) = R_0 \left( 1 + \beta_2 [\cos \gamma Y_{2,0} + \sin \gamma Y_{2,2}] + \beta_3 \sum_{m=-3}^3 \alpha_{3,m} Y_{3,m} + \beta_4 \sum_{m=-4}^4 \alpha_{4,m} Y_{4,m} \right)$$

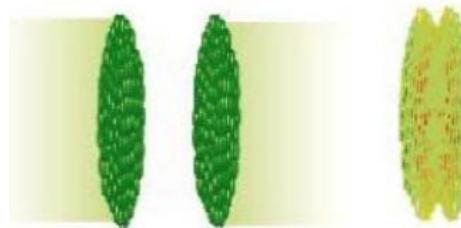
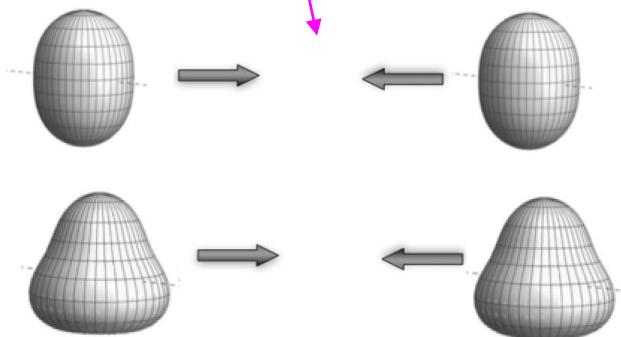


RHIC



Relativistic heavy ion collision can probe the nuclear deformation

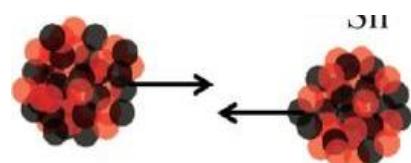
- Relativistic heavy collisions start from nuclei
- Collision time  $< 10^{-24}$  s directly probe the ground state of nuclei



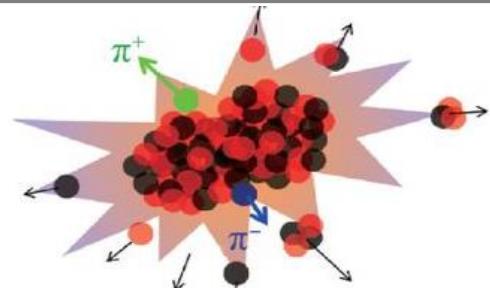
initial conditions:  
(with deformations)

Collision time  $< 10^{-24}$  s

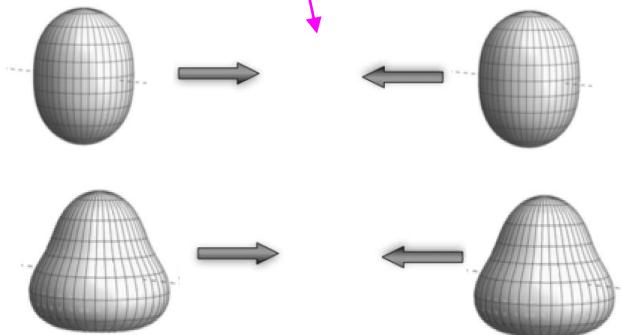
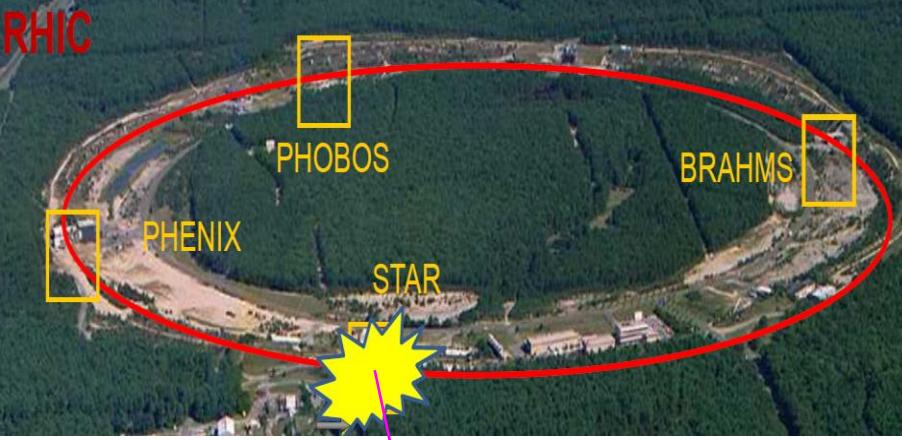
heavy ion collision at intermediate energies excites nuclei during the collision



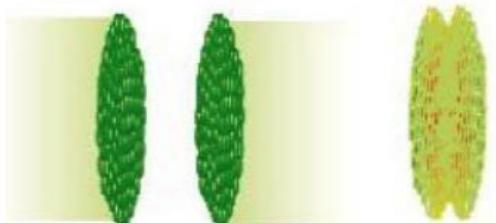
碰撞前



碰撞中



**initial conditions:  
(with deformations)**



**Initial conditions**

**viscous hydro**

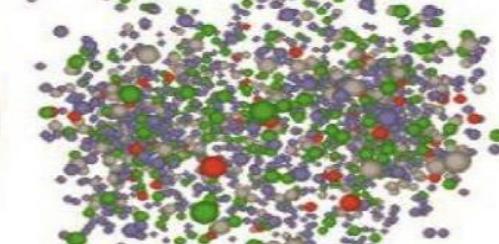
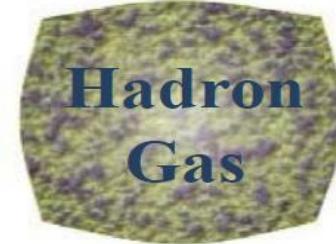


Relativistic heavy ion collision can  
probe the nuclear deformation

- Relativistic heavy collisions start from nuclei
- Collision time  $< 10^{-24}$  s directly probe the ground state of nuclei
- Well calibrated calculations to focus on the initial state effects from the succeeding evolution

**Well calibrated calculations**

**hadron cascade**



# Deformation of $^{96}\text{Ru}$ and $^{96}\text{Zr}$

PHYSICAL REVIEW C

VOLUME 42, NUMBER 3

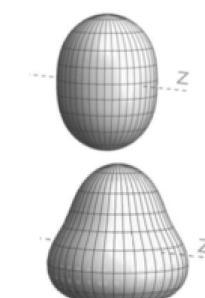
SEPTEMBER 1990

Strong octupole and dipole collectivity in  $^{96}\text{Zr}$ : Indication for  
octupole instability in the  $A = 100$  mass region

$^{96}\text{Zr}$  has very large octupole deformation from  $B(E3; 0_1^+ \rightarrow 3_1^-)$

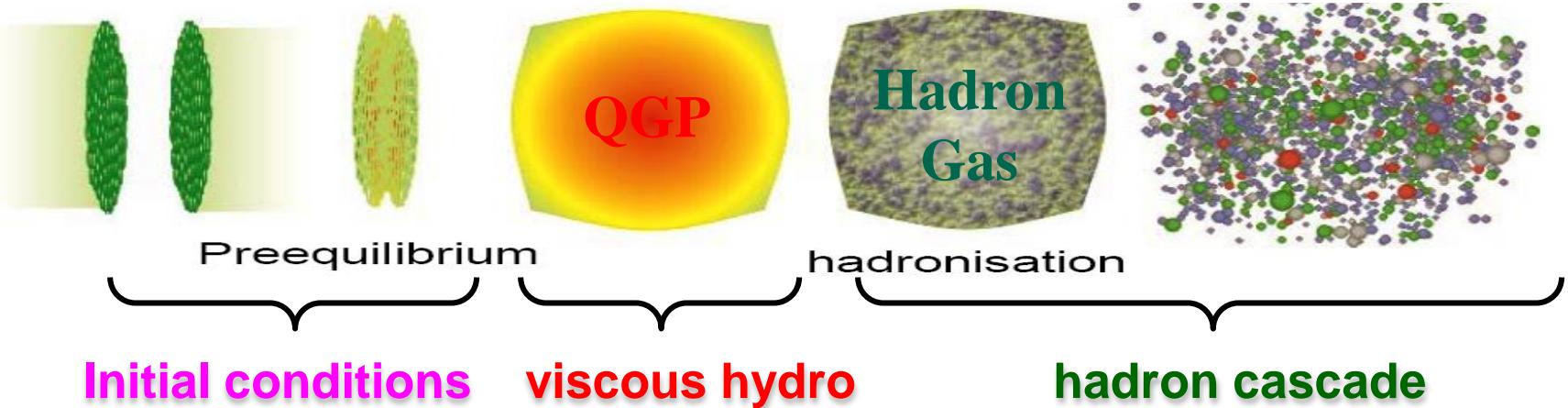
Conversion from  $B(\text{En})$  to  $\beta_n$  via:  $\beta_2 = \frac{4\pi}{3ZR_n^2} \sqrt{\frac{B(E2)\uparrow}{e^2}}$ ,  $\beta_3 = \frac{4\pi}{3ZR_0^3} \sqrt{\frac{B(E3)\uparrow}{e^2}}$

	$\beta_2$	$E_{2_1^+}$ (MeV)	$\beta_3$	$E_{3_1^-}$ (MeV)
$^{96}\text{Ru}$	0.154	0.83	-	3.08
$^{96}\text{Zr}$	0.062	1.75	0.202,0.235,0.27	1.90



ADNDT107,1 (2016) ADNDT80,35(2002)

# Hydrodynamic calculation with initially deformed nuclei



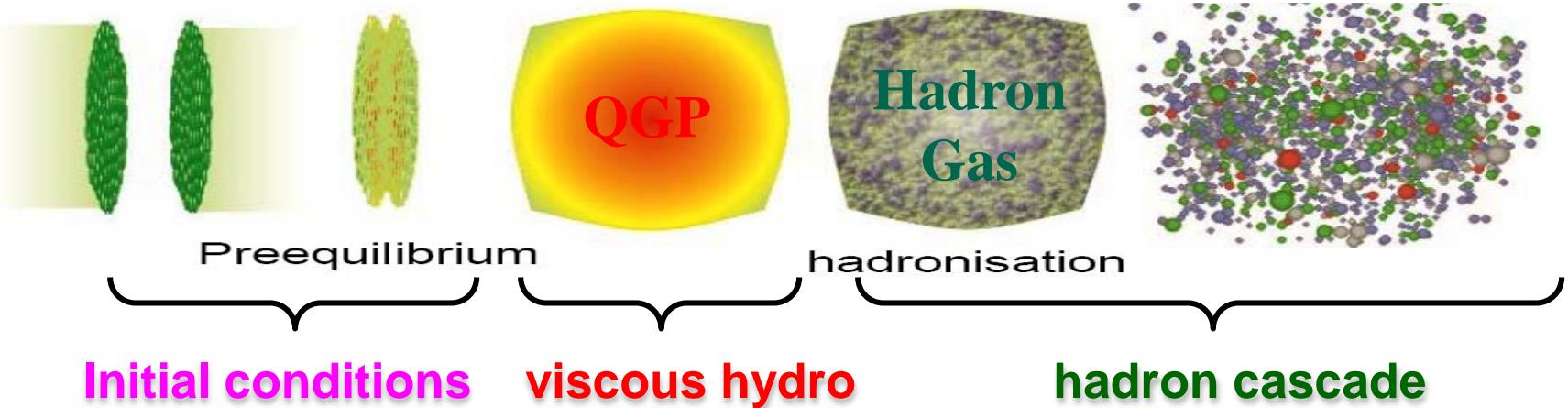
## Initial conditions (TRENTO)

-Sample nucleon position in deformed nuclei with:

$$\rho(r, \theta, \phi) = \frac{\rho_0}{1 + e^{(r - R(\theta, \phi))/a_0}}$$

$$R(\theta, \phi) = R_0 \left( 1 + \beta_2 [\cos \gamma Y_{2,0} + \sin \gamma Y_{2,2}] + \beta_3 \sum_{m=-3}^3 \alpha_{3,m} Y_{3,m} + \beta_4 \sum_{m=-4}^4 \alpha_{4,m} Y_{4,m} \right)$$

# Hydrodynamic calculation with initially deformed nuclei



## Initial conditions (TRENTO)

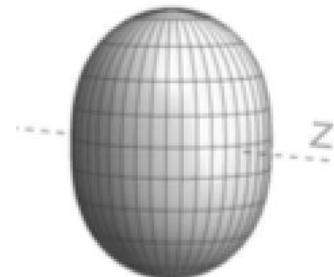
-Sample nucleon position in deformed nuclei with:

$$\rho(r, \theta, \phi) = \frac{\rho_0}{1 + e^{(r - R(\theta, \phi))/a_0}}$$

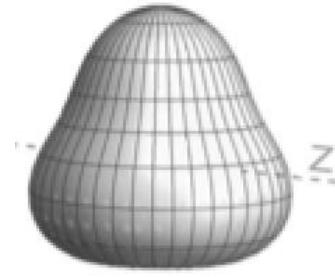
$$R(\theta, \phi) = R_0(1 + \beta_2 Y_{2,0} + \beta_3 Y_{3,0})$$

$$1 + \beta_2 Y_{2,0}(\theta, \phi)$$

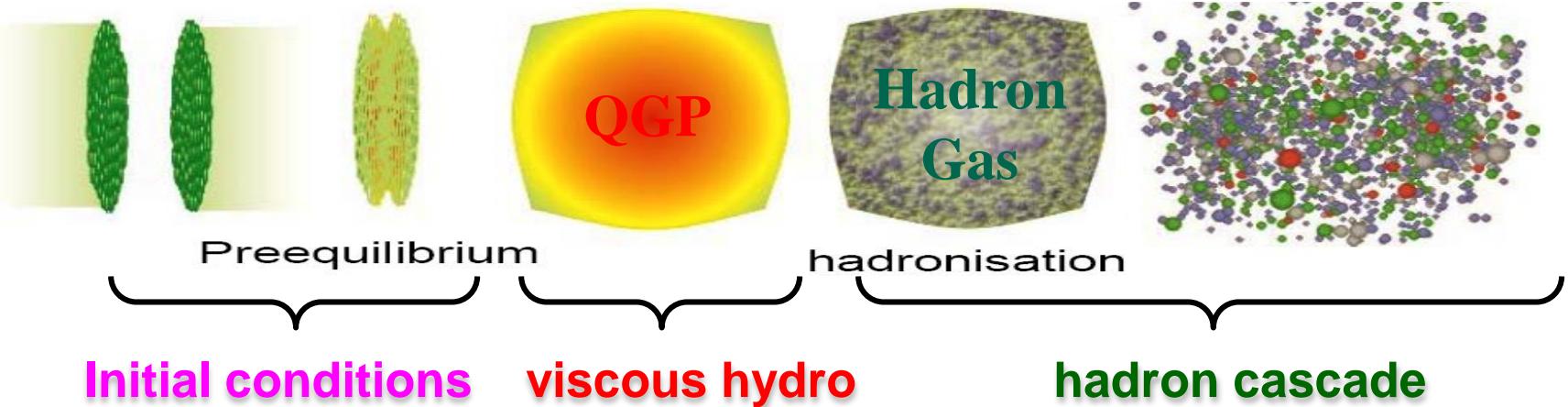
Quadrupole:



Octupole:



# Hydrodynamic calculation with initially deformed nuclei



## Initial conditions (TRENTO)

- Sample nucleon position in deformed nuclei with:

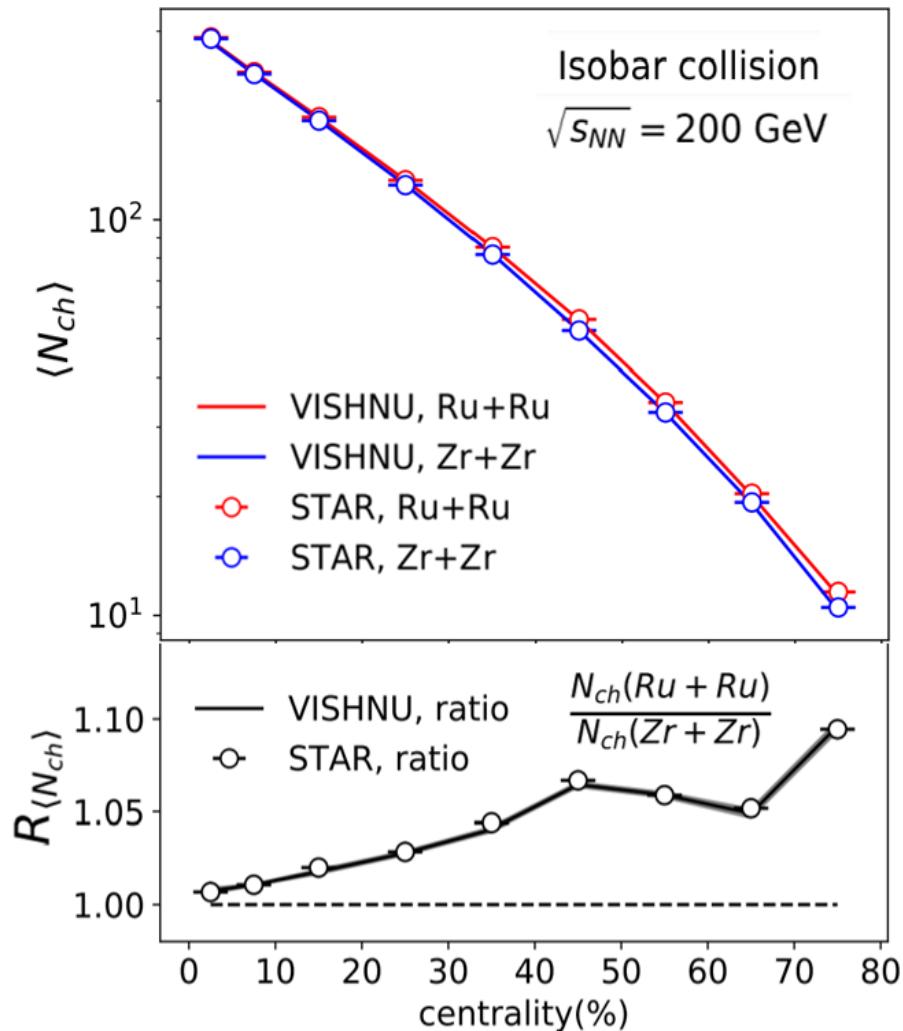
$$\rho(r, \theta, \phi) = \frac{\rho_0}{1 + e^{(r - R_0(1 + \beta_2 Y_{2,0} + \beta_3 Y_{3,0})) / a}}$$

	$\beta_2$	$\beta_3$	$R_0$	$a$
Ru-para-I	0.12	0.00	5.093	0.487
Ru-para-II	0.16	0.00	5.093	0.471
Zr-para-I	0.00	0.16	5.021	0.524
Zr-para-II	0.00	0.20	5.021	0.517

Parameters are refer to:

G. Fricke, et al. Atom. Data Nucl. Data Tabl. 60, 177 (1995).B. Pritychenko, et al. Atom. Data Nucl. Data Tabl. 107, 1 (2016).T Kib'edi and R. H Spear, Atom. Data Nucl. Data Tabl. 80, 35 (2002).  
(H. Xu, et al., Phys. Lett. B 819, 136453 (2021)J. Jia, et al. arXiv: 2111.15559 [nucl-th])

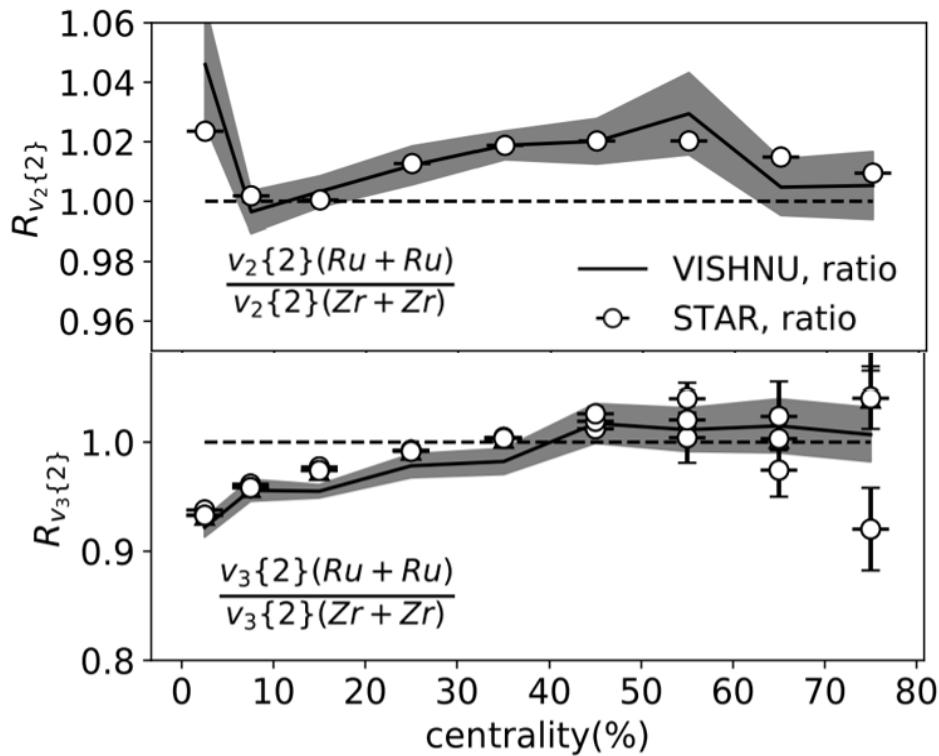
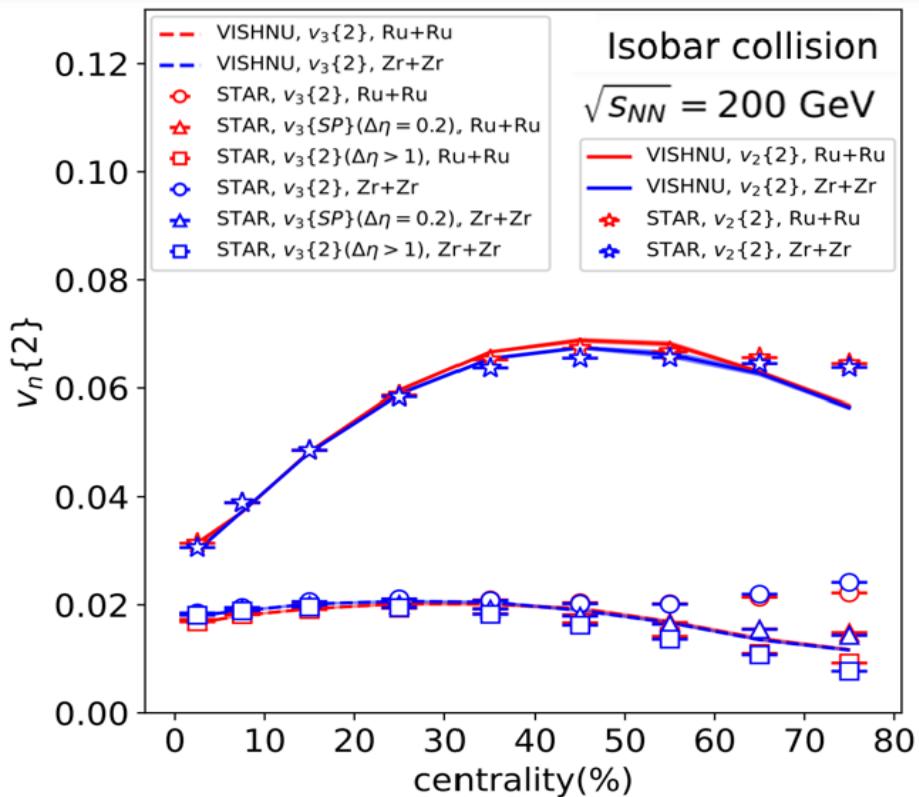
# Nch for Ru+Ru and Zr+Zr collisions



"standard"	Ru	Zr
$a_0$	0.46	0.52
$\beta_2$	0.162	0.060
$\beta_3$	0.00	0.200

- With fine tuning parameters, iEBE-VISHNU fits N<sub>ch</sub> for Ru+Ru collisions
- Using  $\beta_2\beta_3$  in table1, it “predicts” N<sub>ch</sub> for Zr+Zr collisions & the N<sub>ch</sub> ratio between the two systems (the data are nicely described).

# $V_2$ and $V_3$ for Ru+Ru and Zr+Zr collisions

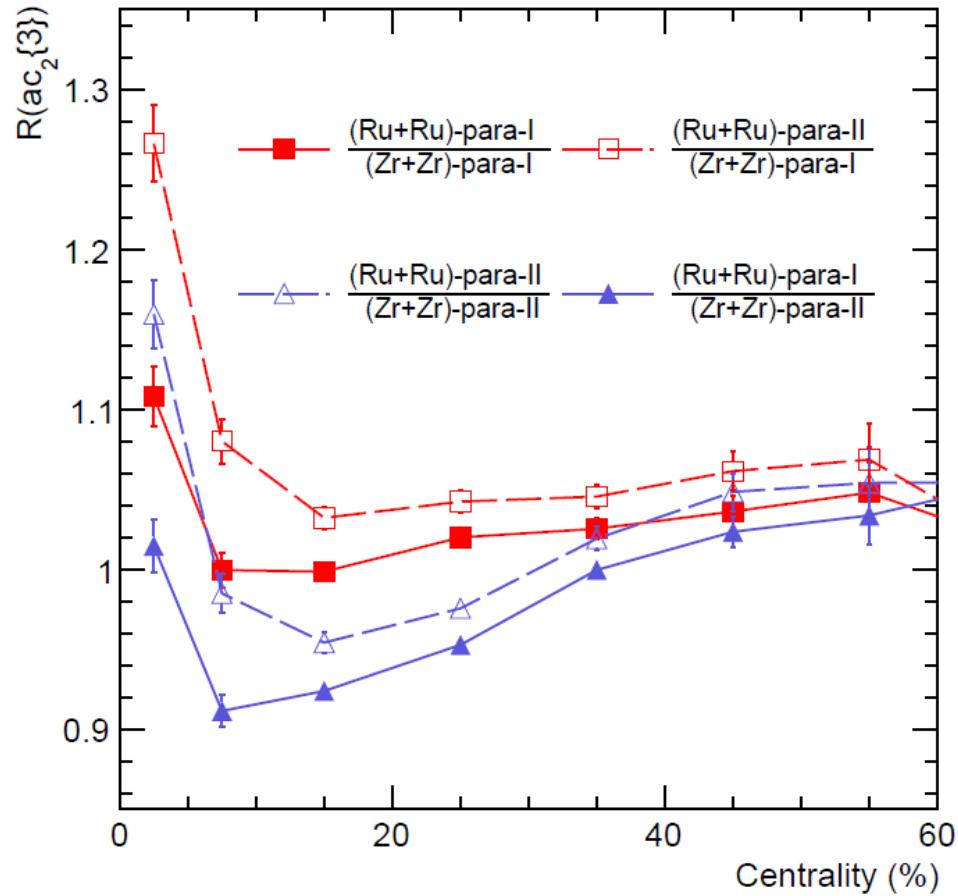


-With fine tuning parameters, iEBE-VISHNU fits  $V_2$  &  $V_3$  for Ru+Ru collisions

-Using  $\beta_2$   $\beta_3$  in table 1, it “predicts”  $V_2$  &  $V_3$  for Zr+Zr collisions & the related ratio -- (the data are roughly described).

“standard”	Ru	Zr
$a_0$	0.46	0.52
$\beta_2$	0.162	0.060
$\beta_3$	0.00	0.200

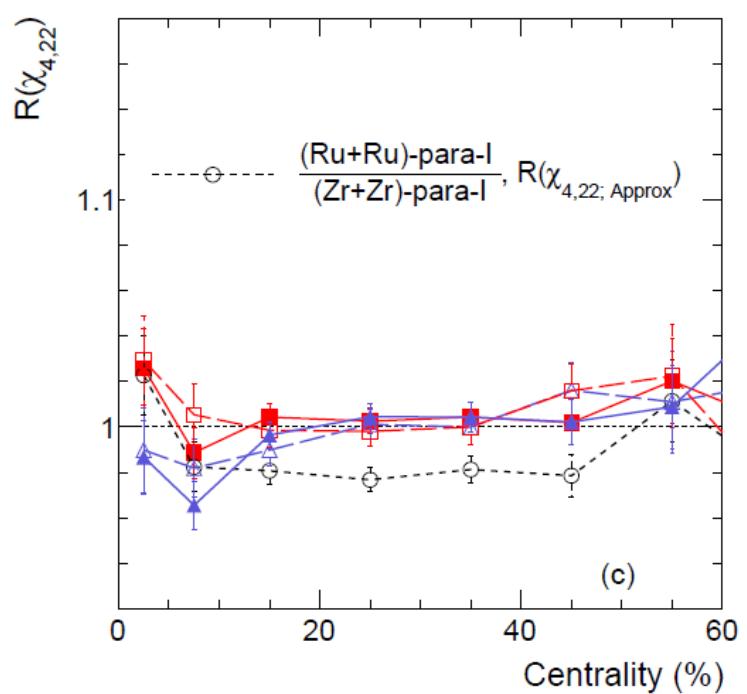
# ac{3} for Ru+Ru and Zr+Zr collisions



ac{3} is sensitive to quadrupole and octupole deformations

$$ac_2\{3\} = \langle v_2^2 v_4 \cos 4(\Phi_2 - \Phi_4) \rangle,$$

$$\chi_{4,22} \equiv \frac{ac_2\{3\}}{\langle v_2^4 \rangle} = nac_2\{3\} \sqrt{\frac{v_4\{2\}^2}{2v_2\{2\}^4 - v_2\{4\}^4}}.$$





# Quark Gluon Plasma

Hottest Matter on Earth

Most Perfect Liquid

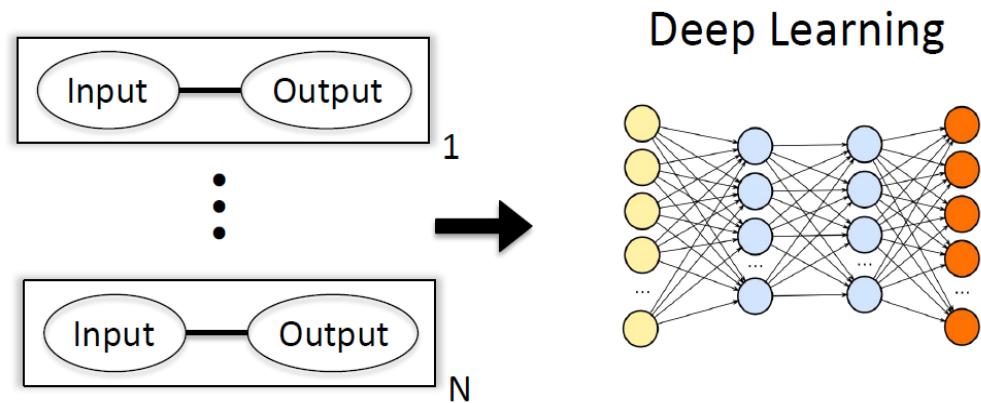
# **Applications of Deep Learning in Relativistic Hydrodynamics**



# Why Deep Learning in Physics?



*“Unlike earlier attempts ... Deep Learning systems can see patterns and spot anomalies in data sets far larger and messier than human beings can cope with.”*

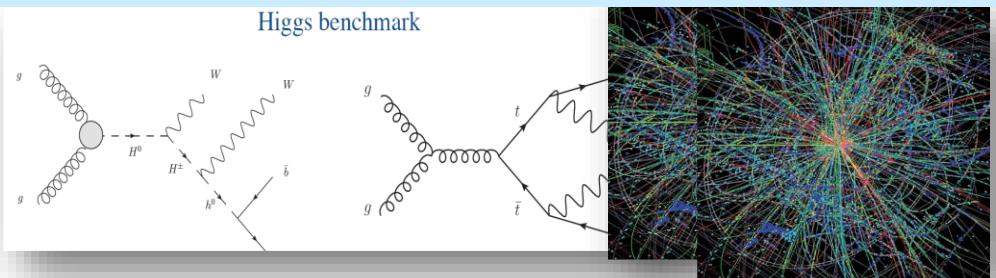


Can “Black–box” models learn patterns and models solely from data without relying on scientific knowledge?

# Applications of Deep Learning in Physics

- Y. D. Hezaveh, L. Perreault Levasseur and P. J. Marshall, Nature 548, 555 (2017)
- J. Carrasquilla and G. R. Melko, Nature Phys. 13, 431 (2017)
- Carleo et al., Science 355, 602-606 (2017)
- E. P. L. van Nieuwenburg, Y. H. Liu, S. Huber, Nature Phys. 13, 435 (2017)
- Pierre Baldi, Peter Sadowski, and Daniel Whiteson, Nature Commun. 5 (2014) 4308
- Luke de Oliveira, Michela Paganini, and Benjamin Nachman, Comput Softw Big Sci (2017) 1: 4
- Long-Gang Pang et al., Nature Commun. 9 (2018) no.1, 210
- ..., ...,
- ...

# Searching for Exotic Particles in High-Energy Physics



Deep learning can improve the power for the collider search of exotic particles

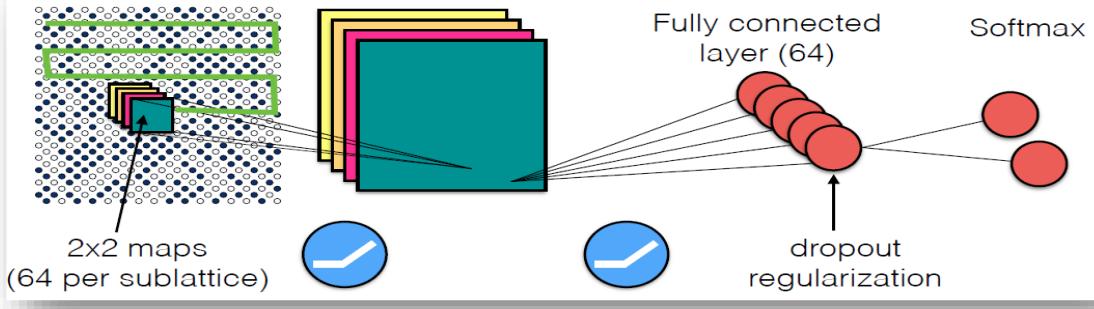
P. Baldi, P. Sadowski, & D. Whiteson  
Nature Commun. 5, 4308 (2014)

## Classifying the Phase of Ising Model

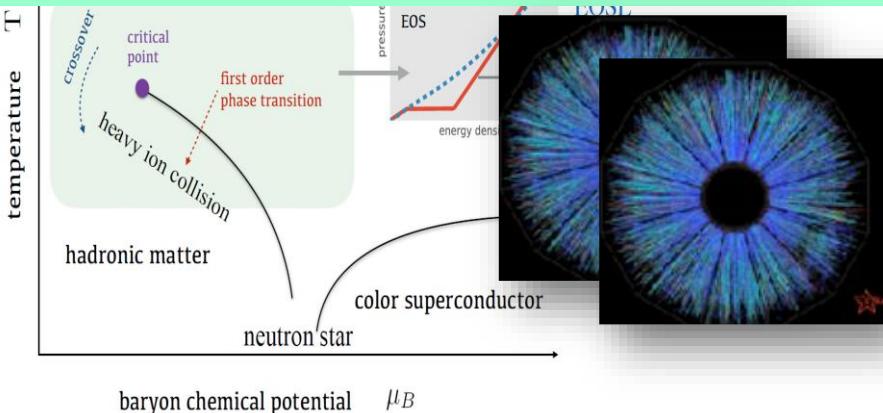
For the case of Ising gauge theory

$$H = -J \sum_p \prod_{i \in p} \sigma_i^z$$

J. Carrasquilla and R. G. Melko.  
Nature Physics 13, 431–434 (2017)



## Identify QCD Phase Transition with Deep Learning



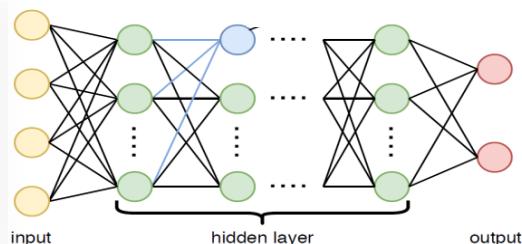
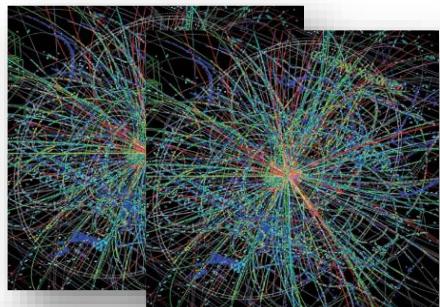
DNN efficiently decode the EOS information from the complex final particle info event by event

L.G. Pang, K. Zhou, N. Su,  
H. Petersen, H. Stoecker, XN. Wang.  
Nature Commun. 9 (2018) no. 1, 210

# More Comments

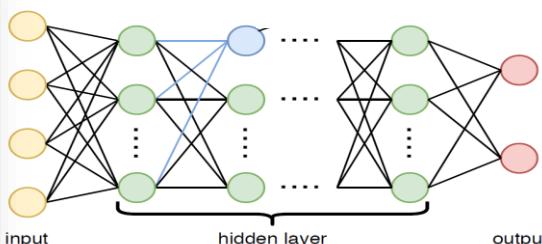
on several examples of supervised learning

## Image identification



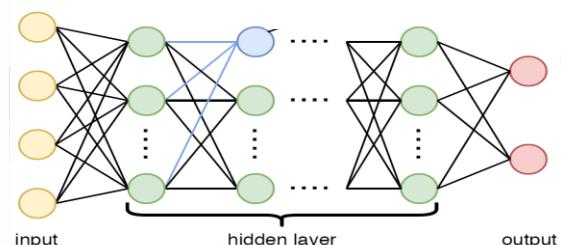
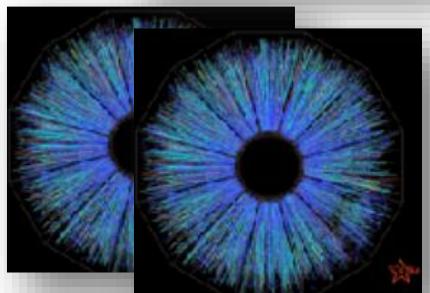
Higgs signal or background?

P.Baldi,et al,Nature Commun.(2014)



High temperature or low temperature phase?

Carrasquilla & Melko. Nature Physics (2017)



EoS L or EOSQ ?

Pang,et al Nature Commun.(2018)

*“Unlike earlier attempts ... Deep Learning systems can see patterns and spot anomalies in data sets far larger and messier than human beings can cope with.”*

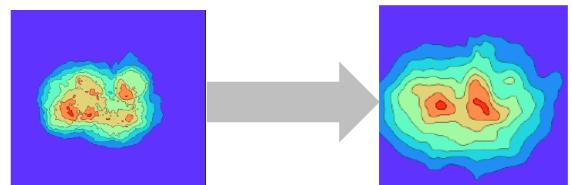
## Image generation



For hydrodynamics can we use deep learning to learn/predict the pattern transformation between initial and final profiles?

**Initial energy density profiles**

----- > **final energy density velocity profiles**



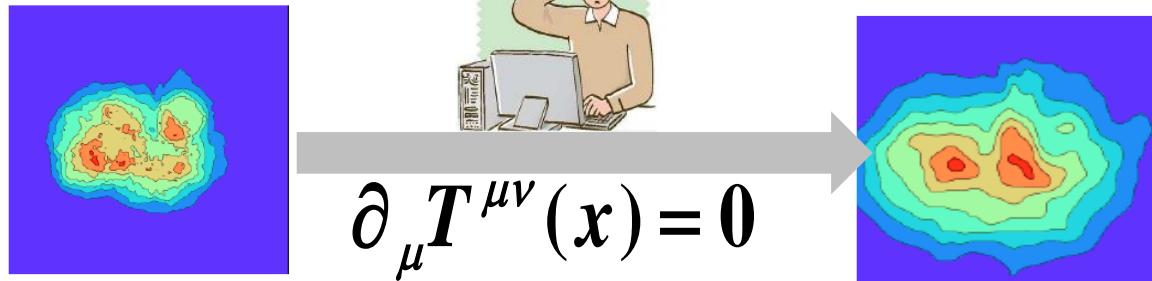
For the non-linear hydro system, can the black-box network could learn pattern transformations solely from data without relying on scientific knowledge?

( conservation laws)

# Applications of deep learning to relativistic hydrodynamics

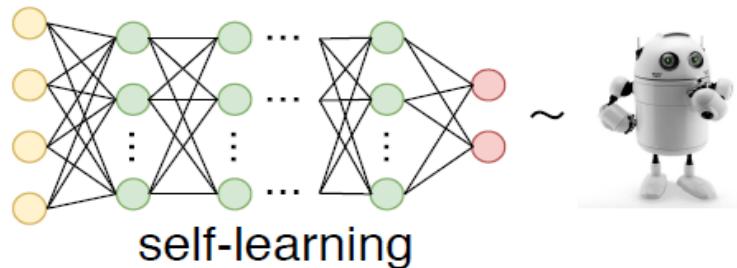
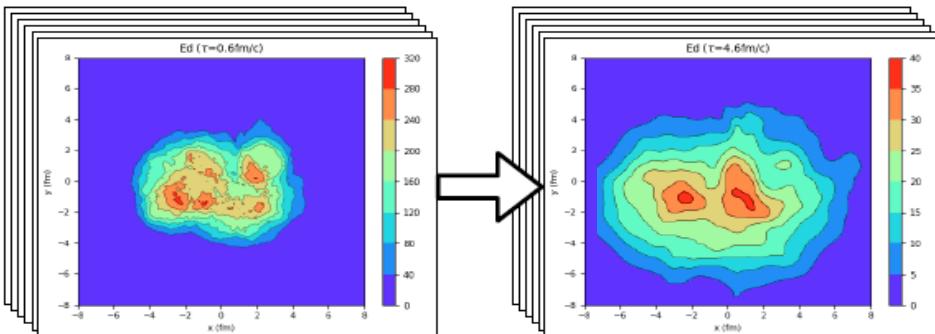
H. Huang, B. Xiao, H. Xiong, Z. Wu, Y. Mu and H. Song; NPA 2019  
Phys. Rev. Res. 3 2 023256(2021)

# Traditional hydrodynamics

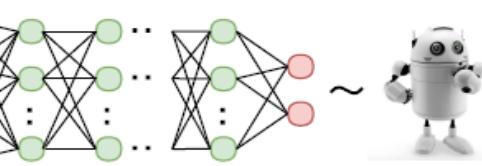
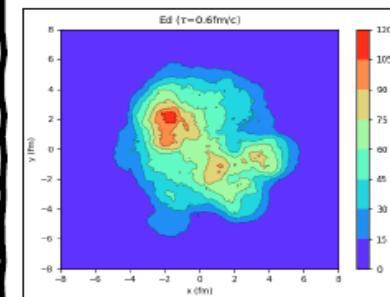


## Deep Learning

training



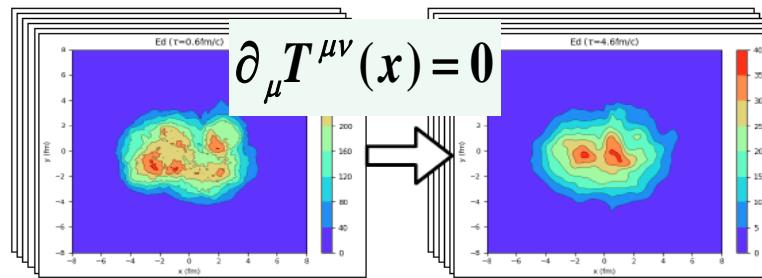
testing



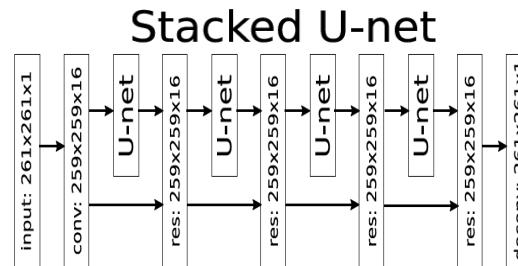
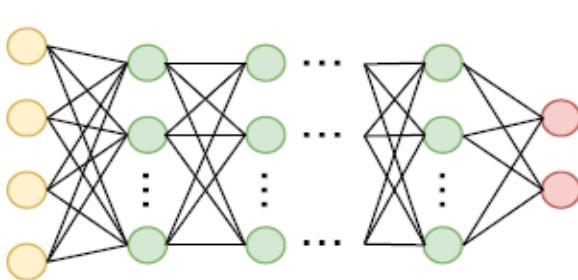
-Such deep learning systems do not need to be programmed with the hydro equation  $\partial_\mu T^{\mu\nu}(x) = 0$  Instead, they learn on their own

# Deep Learning

Step1 ) Generate the training/testing data sets from hydro



Step2 ) Design & train the deep neural network

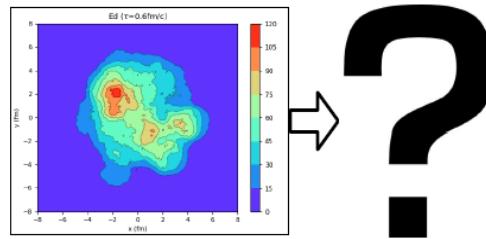


**The Training Data Sets**

**hydro  
VISH2+1**

**MC-GI  
10000**

Step3 ) Test the deep neural network



**The Testing Data Sets**

**hydro  
VISH 2+1**

**MC-GI**

**MC-KLN**

**AMPT**

**Trento**

**10000**

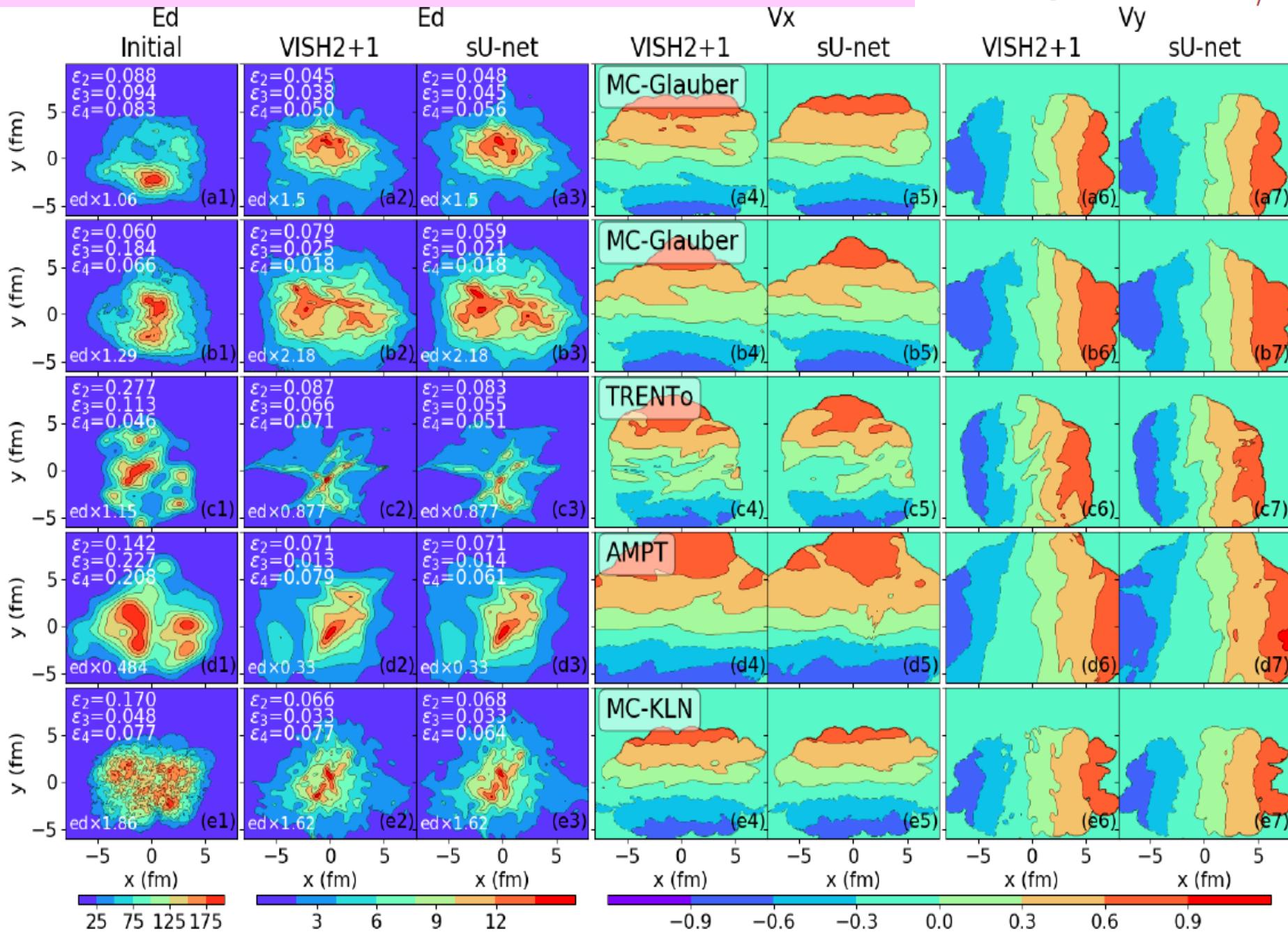
**10000**

**10000**

**10000**

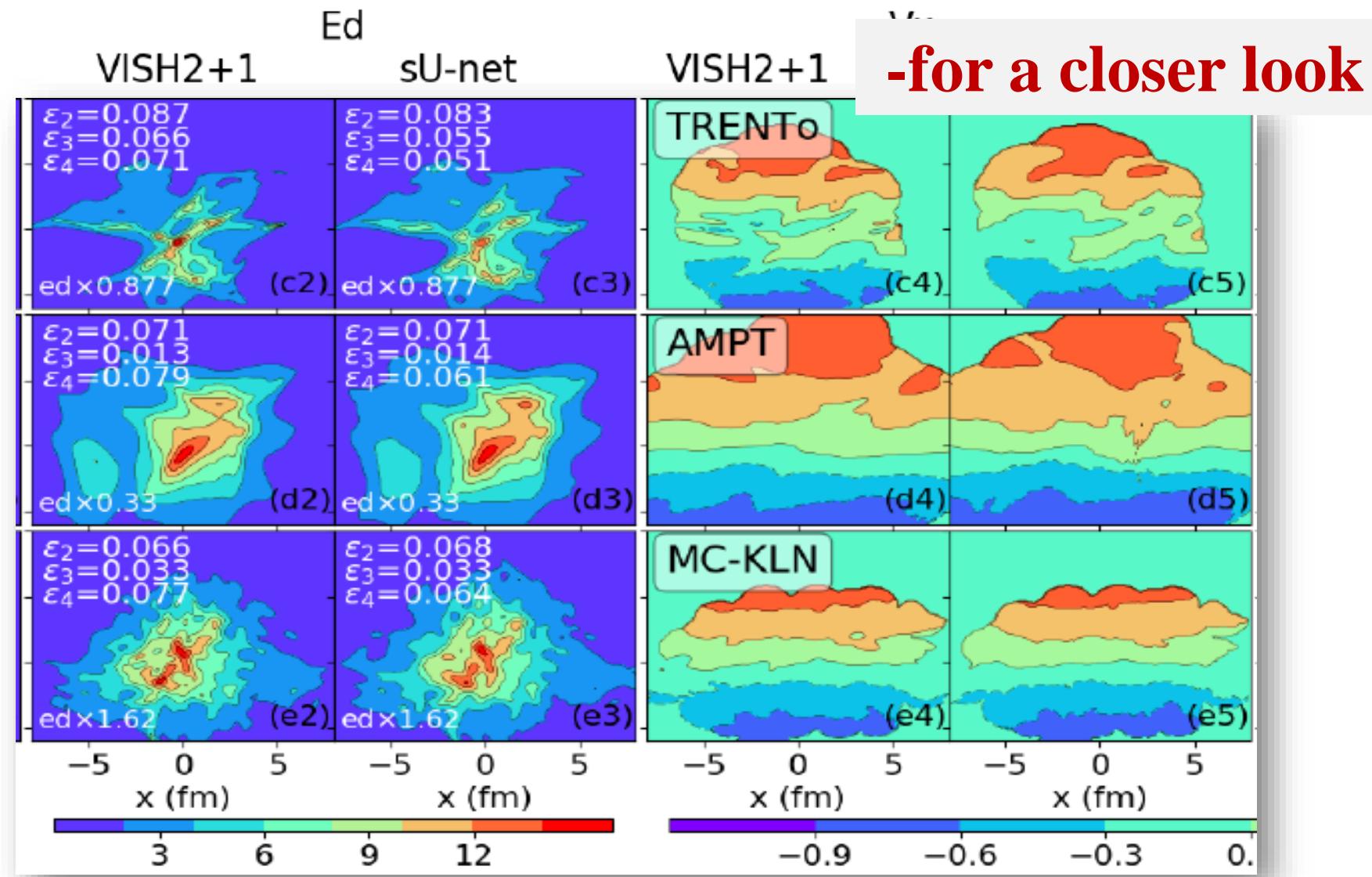
# sUnet prediction vs. hydro simulations

$$\tau - \tau_0 = 6.0 \text{fm}/c$$



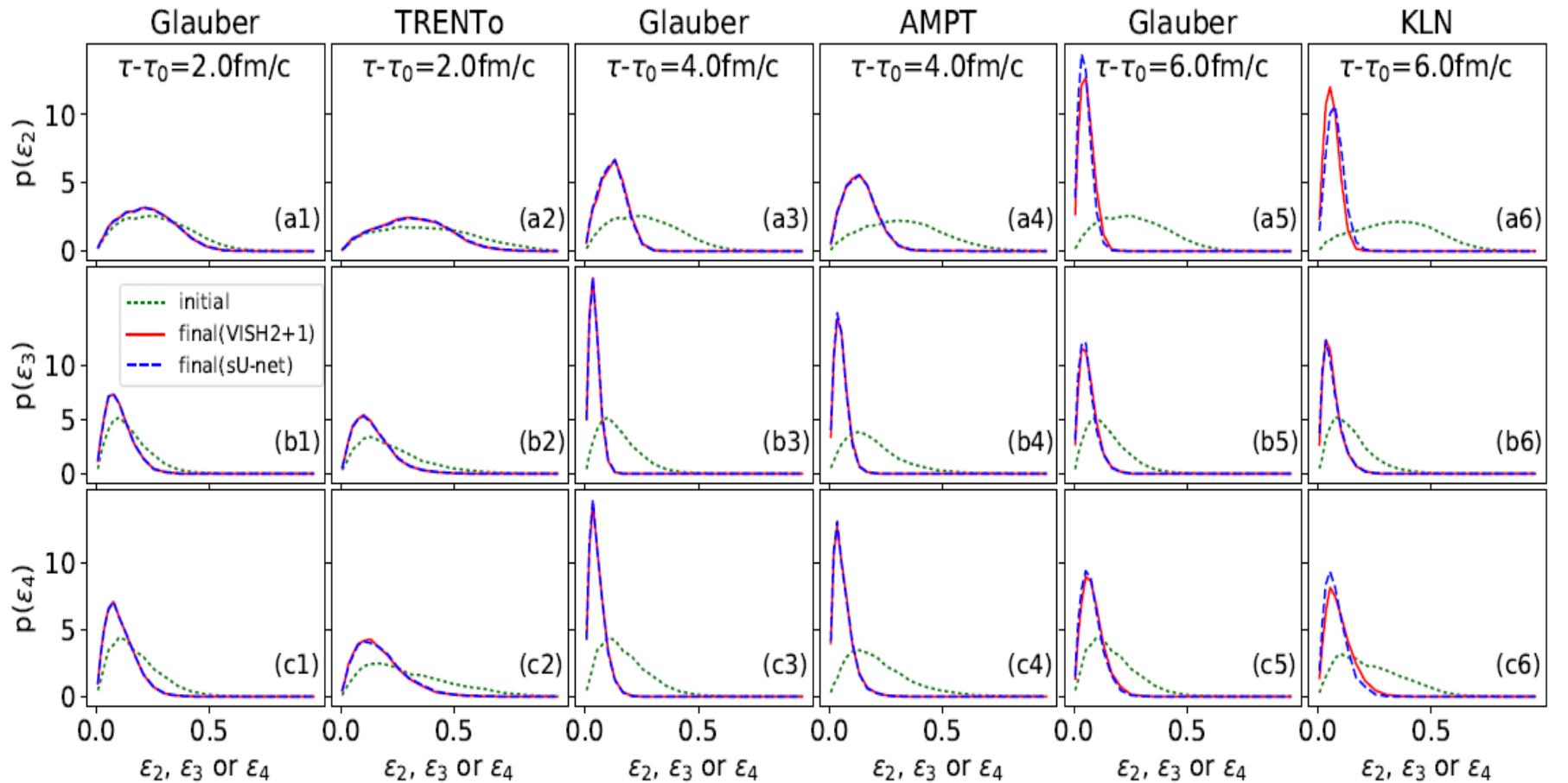
# sUnet prediction vs. hydro simulations

$$\tau - \tau_0 = 6.0 \text{fm}/c$$



# sUnet prediction vs. hydro simulations

## Eccentricity distributions:



$$\varepsilon_n e^{in\Phi_n} = - \frac{\int dx dy r^2 e^{in\phi} e(x,y)}{\int dx dy r^2 e(x,y)}$$

--- initial  
— final(VISH2+1)  
--- final(sU-net)

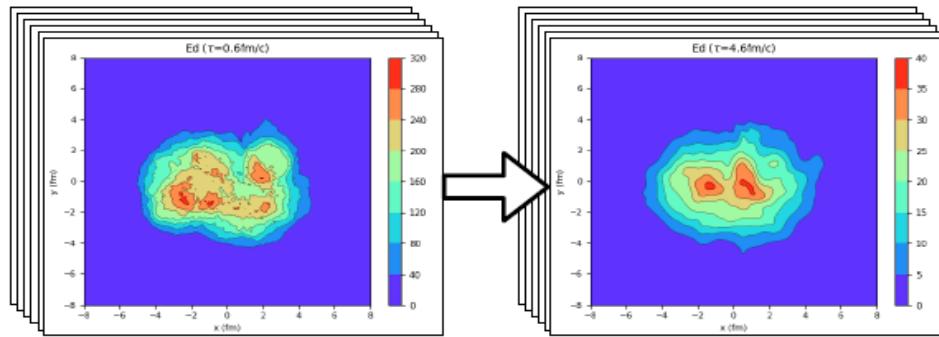
# Simulation time: sUnet vs. hydro

VISH2+1

network

10~20 minute  
with one CPU

1~2 second  
with P40 GPU



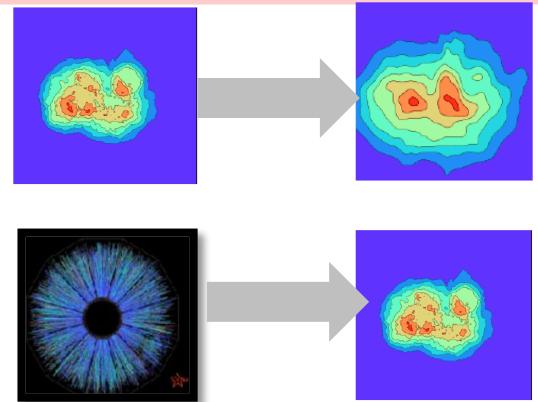
With the well trained  
network, the final state  
profiles can be quickly  
generated from the initial  
profiles.

# Outlook

## For hydrodynamics

Initial energy density profiles

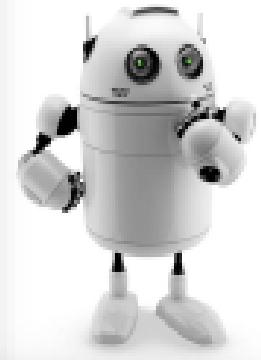
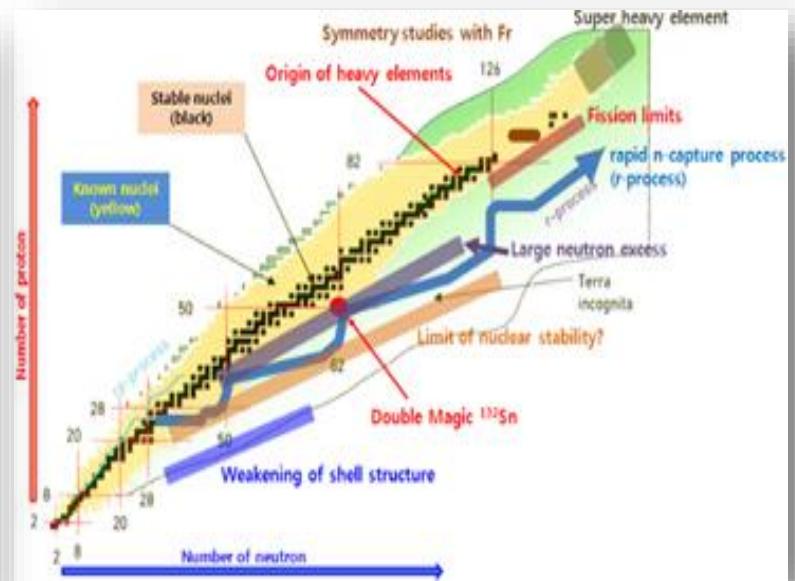
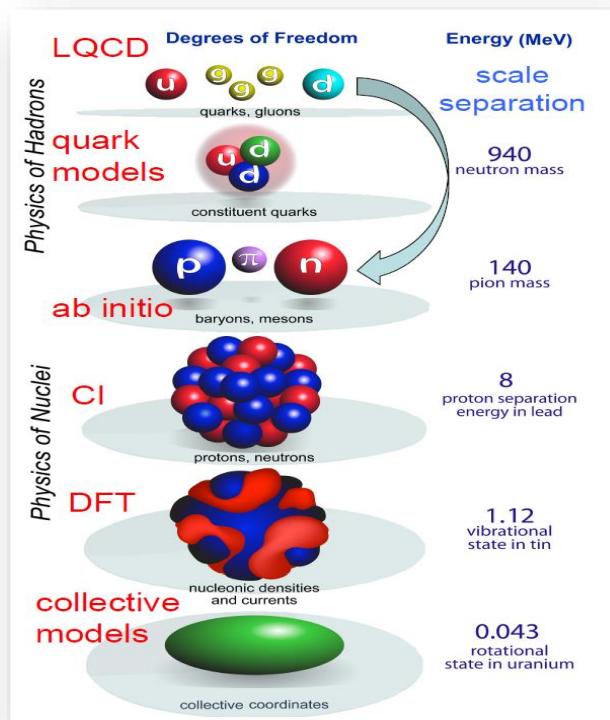
----- > final energy density velocity profiles



Final particle profiles

----- > Initial energy density profiles

## For Nuclear Physics



**Many many more to explore . . . .  
Enjoy it! have fun!**