Quantum kinetic theory

and its applications to chiral transports and spin polarizations

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Recent review:

- Y. Hidaka, SP, Q. Wang, D.L. Yang, *Foundations and Applications of Quantum Kinetic Theory,* Progress in Particle and Nuclear Physics, 127, 103989 (2022).

Outline

- **Part 1:**
  Chiral magnetic effect, Berry phase and kinetic theory
- **Part 2:**
  Wigner functions and the master equations
- **Part 3:**
  Quantum kinetic theory in massless limit and collisions
- **Part 4:**
  Applications to heavy ion physics

Part 1

Chiral magnetic effect, Berry phase and kinetic theory

1. Chiral magnetic effect and chiral separation effect
   (1a) Strong magnetic fields in HIC and CME
   (1b) Other topics related to the CME
2. Kinetic theory and chiral kinetic theory
   (2a) Standard kinetic theory
   (2b) Chiral kinetic theory: a quick look
3. Berry phase, Berry monopole and chiral anomaly
4. Non-trivial Lorentz symmetry for chiral system

Part 2

Wigner functions and the master equations

1. Definition of gauge invariant covariant Wigner function
   (1a) Gauge invariant covariant Wigner function
   (1b) Vector, chiral currents
   (1c) The choice of gauge link
2. Master equation for covariant Wigner function
   (2a) Closed-Time-Path formalism
   (2b) From Dirac equation to master equations for chiral fermions
   (2c) Master equations in general case (massive case)
3. Equal-time formulism for Wigner function

Part 3

Quantum kinetic theory in massless limit and collisions

1. Solve quantum kinetic theory in gradient expansion
   (1a) Gradient expansion
   (1b) Leading order results and constrains from QKT
   (1c) $\hbar$ order results
   (1d) $\hbar^2$ order results
2. Discussions on the solution of Wigner function
   (2a) CME, CVE, energy-momentum tensor and chiral anomaly
   (2b) Chiral kinetic theory
   (2c) Lorentz transformation and side jump
3. Collision effects
   (3a) Kadanoff-Baym equation
   (3b) General solution of Wigner function with collisions
   (3c) Collision term for QED in HTL approximation

Part 4

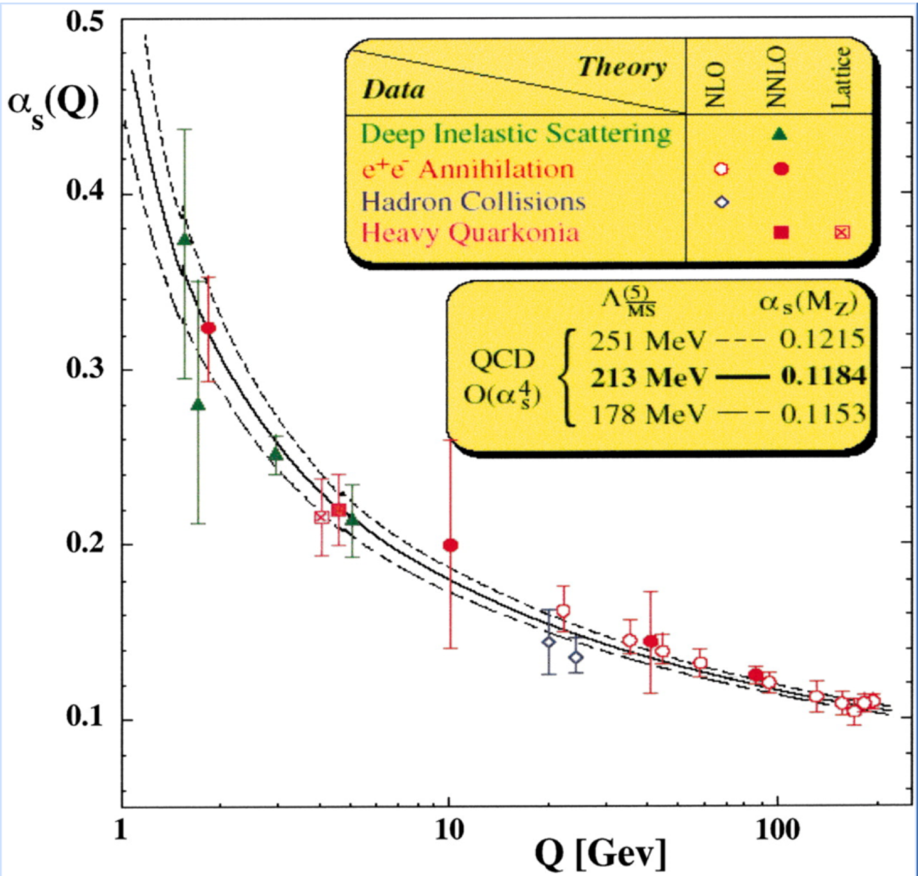
Applications to heavy ion physics

1. Spin polarization in relativistic heavy ion collisions
2. Recent development on QKT
3. Applications to spin polarization

1. Chiral magnetic effect and chiral separation effect

(1a) Strong magnetic fields in HIC and CME

Asymptotic freedom of QCD



The Nobel Prize in Physics 2004

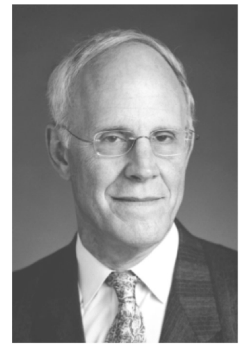


Photo from the Nobel Foundation archive.

David J. Gross

Prize share: 1/3

Photo from the Nobel Foundation archive.

H. David Politzer

Prize share: 1/3

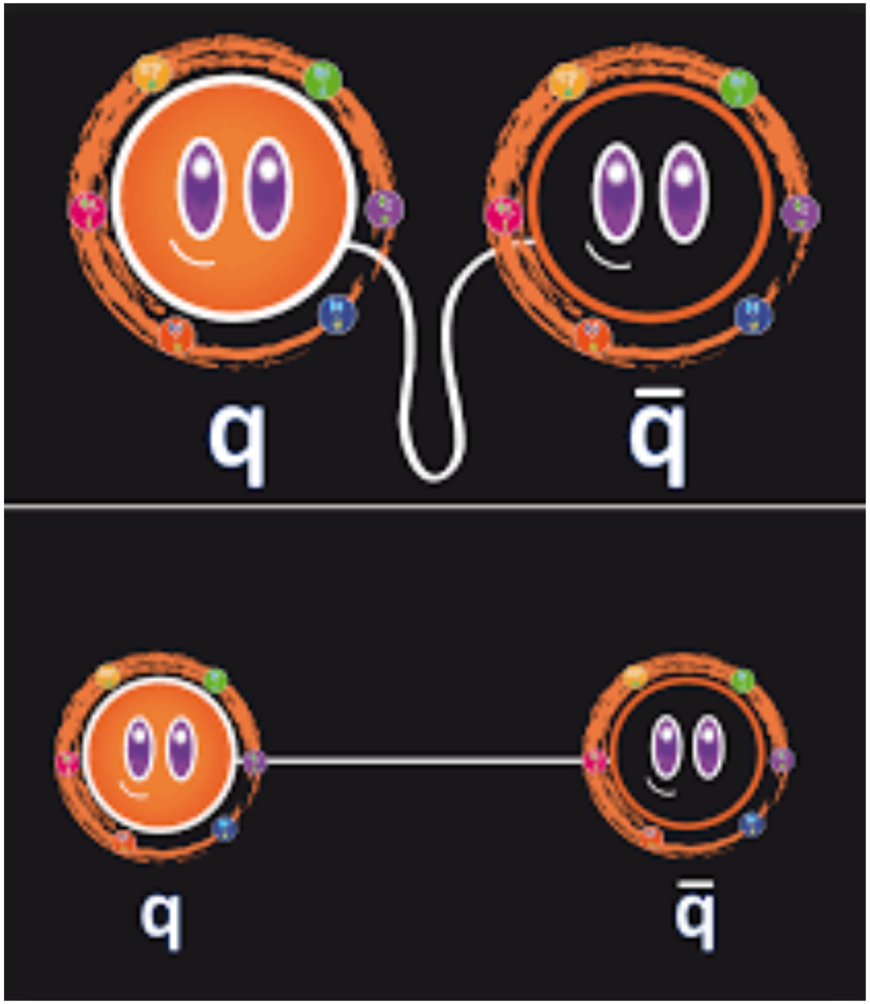
Photo from the Nobel Foundation archive.

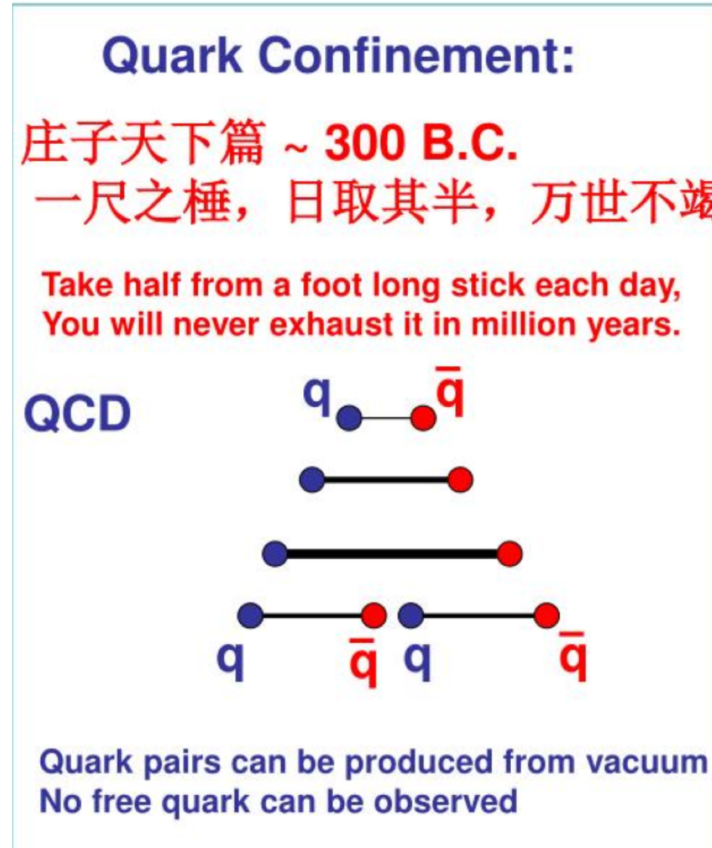
Frank Wilczek

Prize share: 1/3

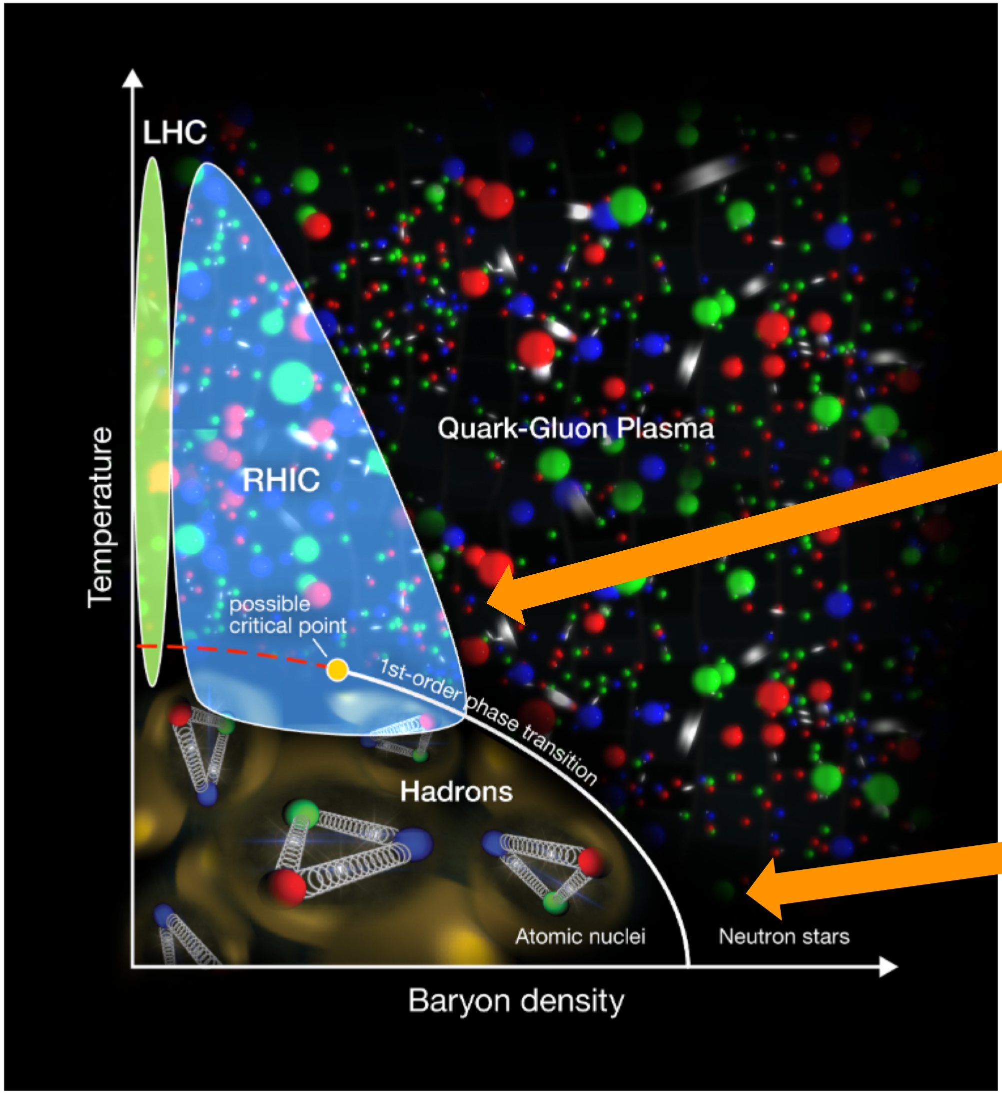
The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek "for the discovery of asymptotic freedom in the theory of the strong interaction."

Quark Confinement





Deconfinement phase transition



High temperature

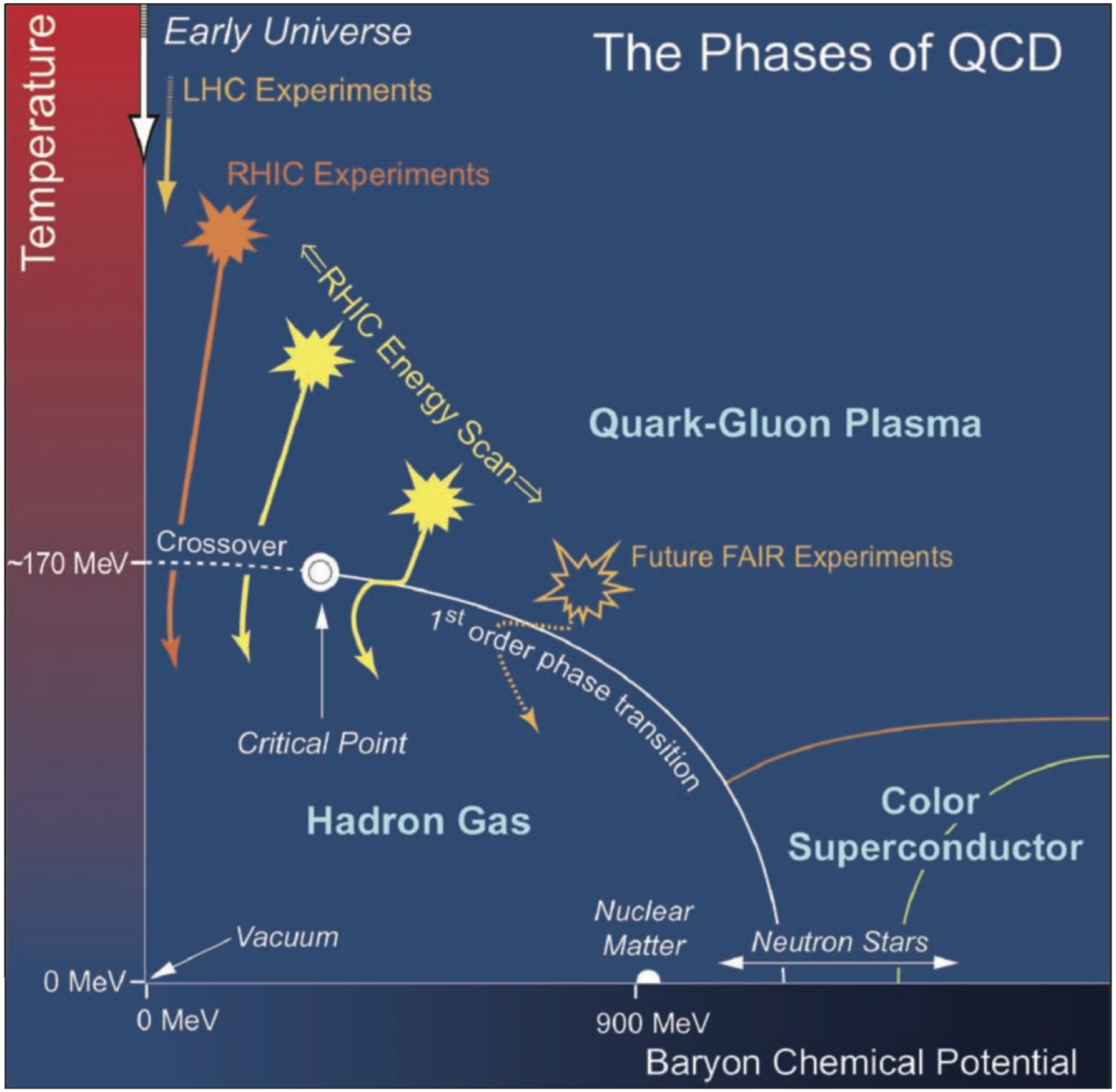
High pressure

核子重如牛 对撞生新态

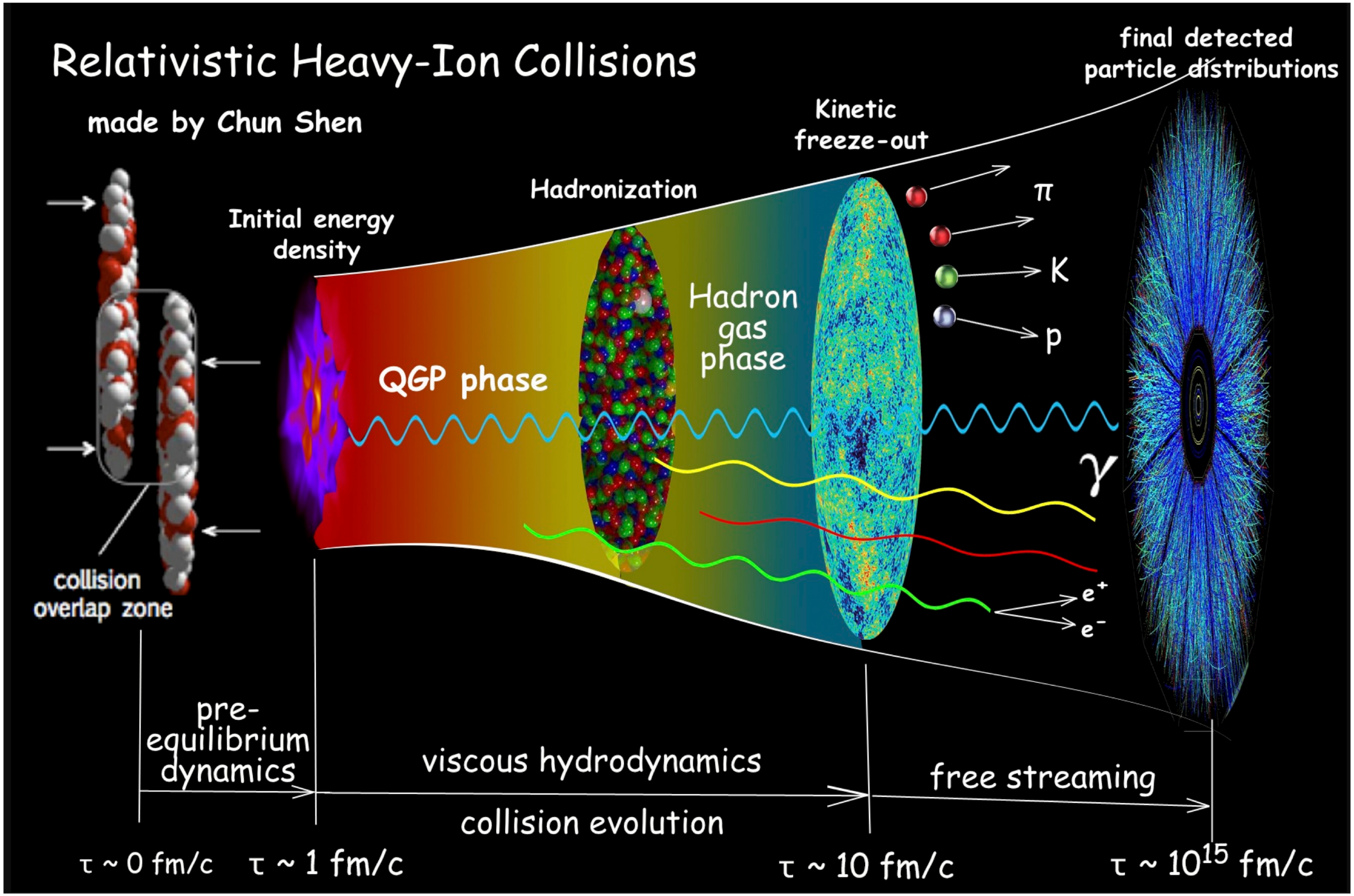
李可染大师 1986年，
为 李政道先生作。

T.D. Lee (1974) and Collins (1975): Heavy ion collision to create a new form of matter!

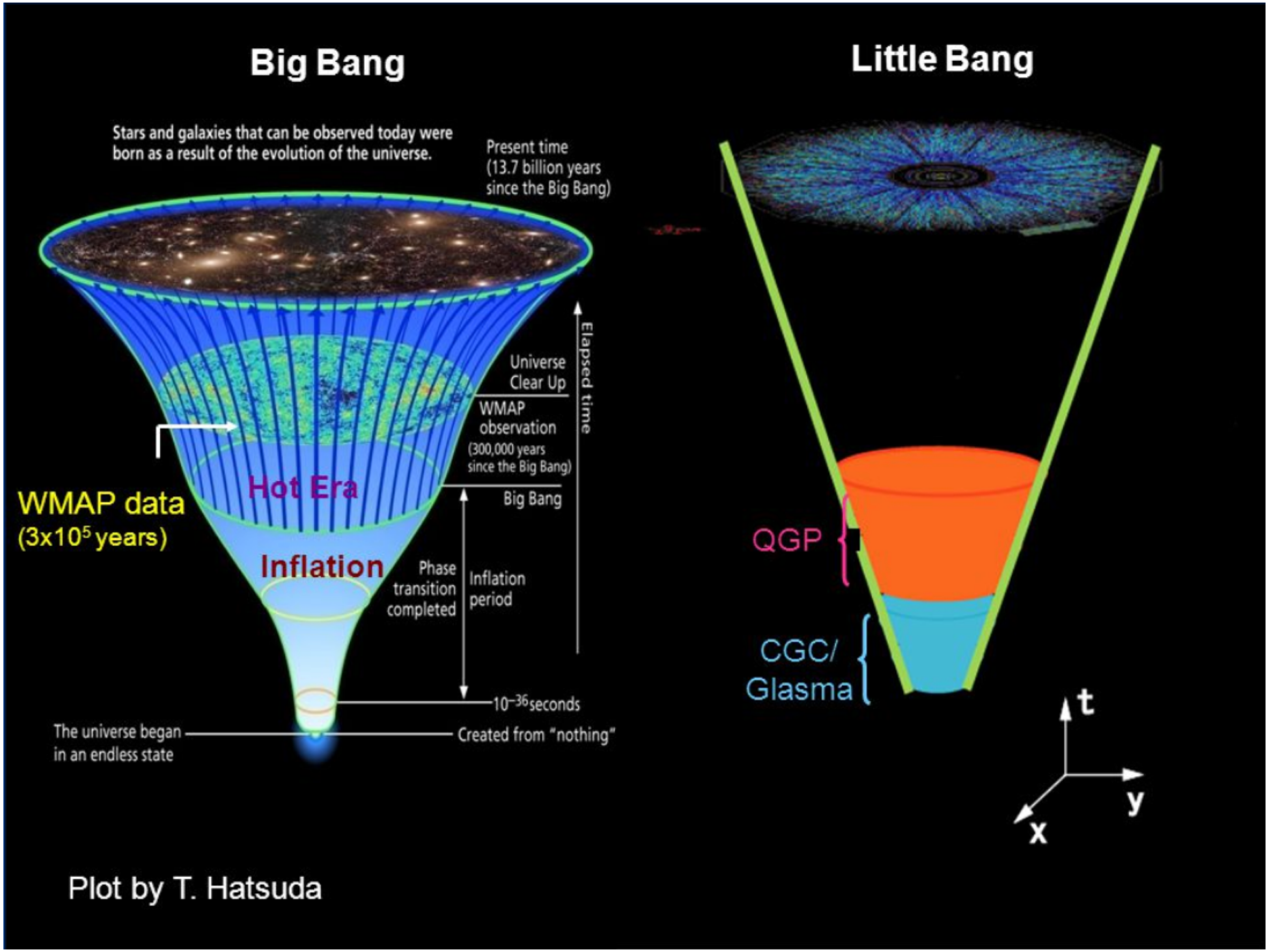
Phases of QCD



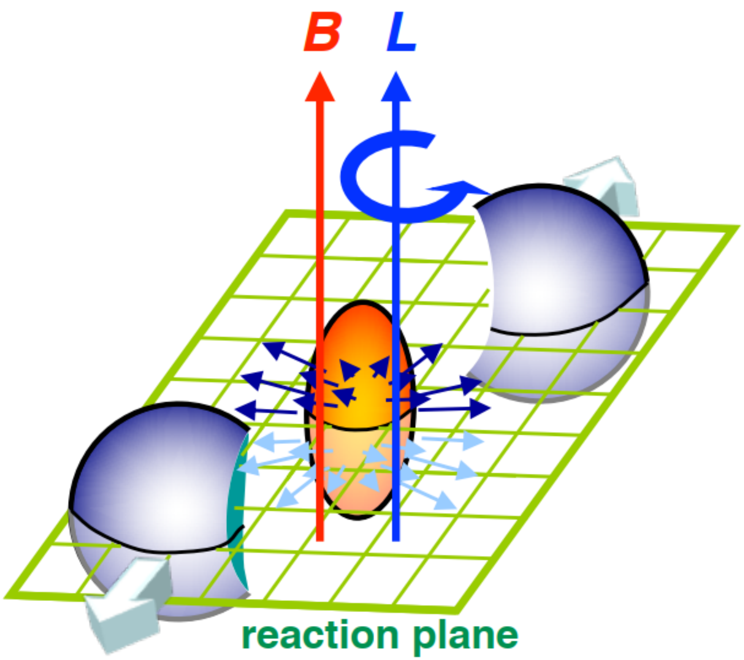
Relativistic heavy ion collisions



Little Bang VS Big Bang



Strong EB fields in HIC (I)



- Two charged nuclei moving alone z direction generate the EB fields.

- EB fields can be computed by Lienard-Wiechert potential.

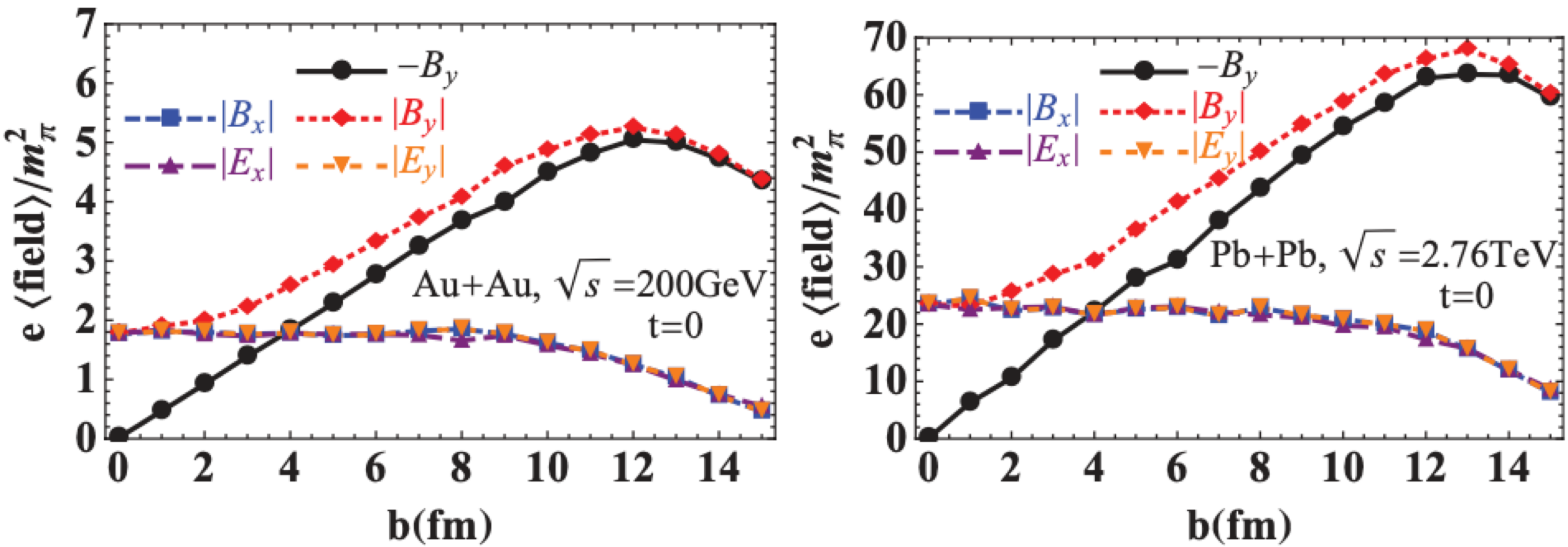
$$\vec{E}(\vec{r},t) = \frac{e}{4\pi}\sum_{i=1}^{N_{\text{proton}}} Z_i \frac{\vec{R}_i - R_i\vec{v}_i}{(R_i - \vec{R}_i\cdot\vec{v}_i)^3}(1-v_i^2),$$

$$\vec{B}(\vec{r},t) = \frac{e}{4\pi}\sum_{i=1}^{N_{\text{proton}}} Z_i \frac{\vec{v}_i \times \vec{R}_i}{(R_i - \vec{R}_i\cdot\vec{v}_i)^3}(1-v_i^2),$$

Strong EB fields in HIC (II)

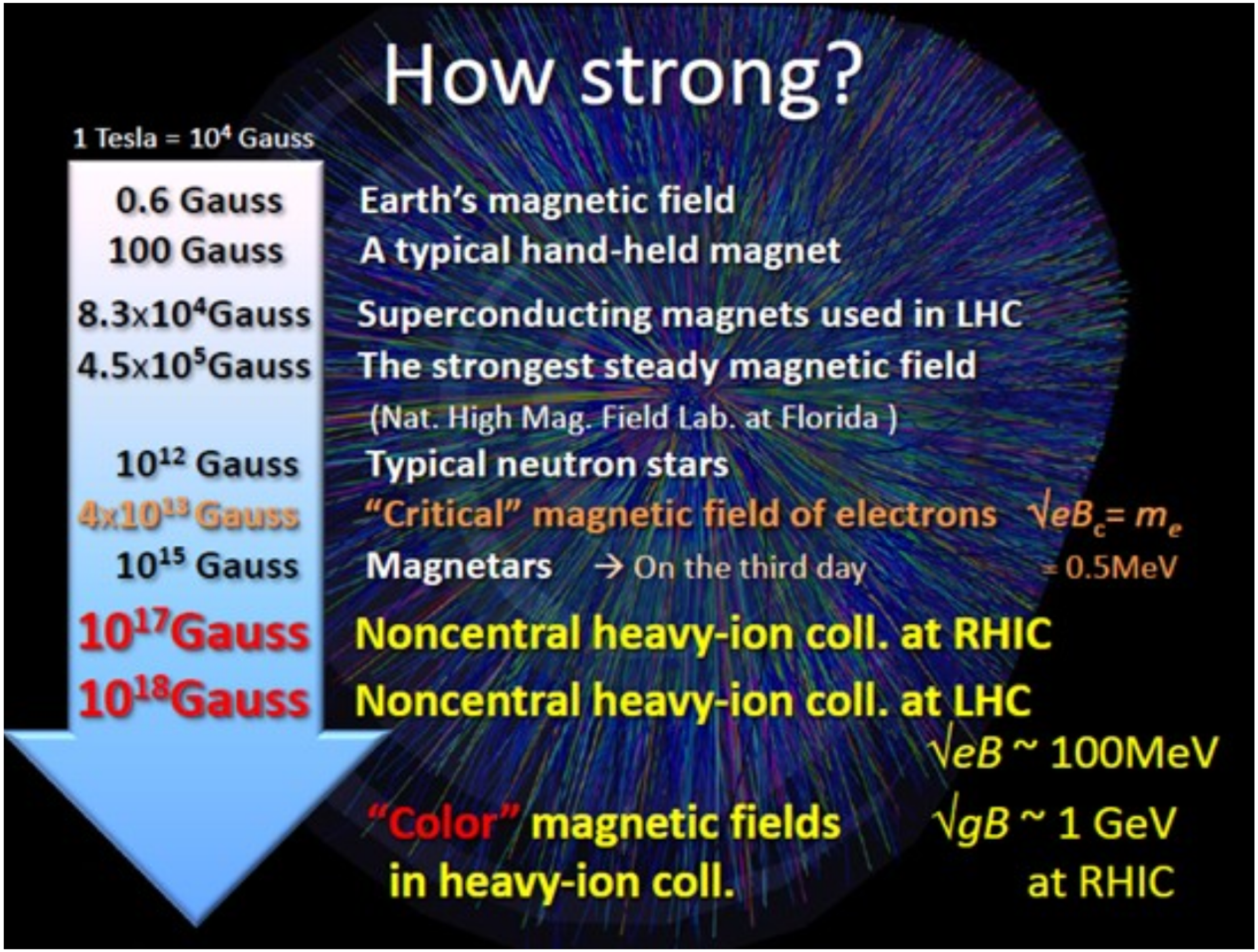
- **Theoretical estimation:**

Lienard-Wiechert potential + Event-by-event

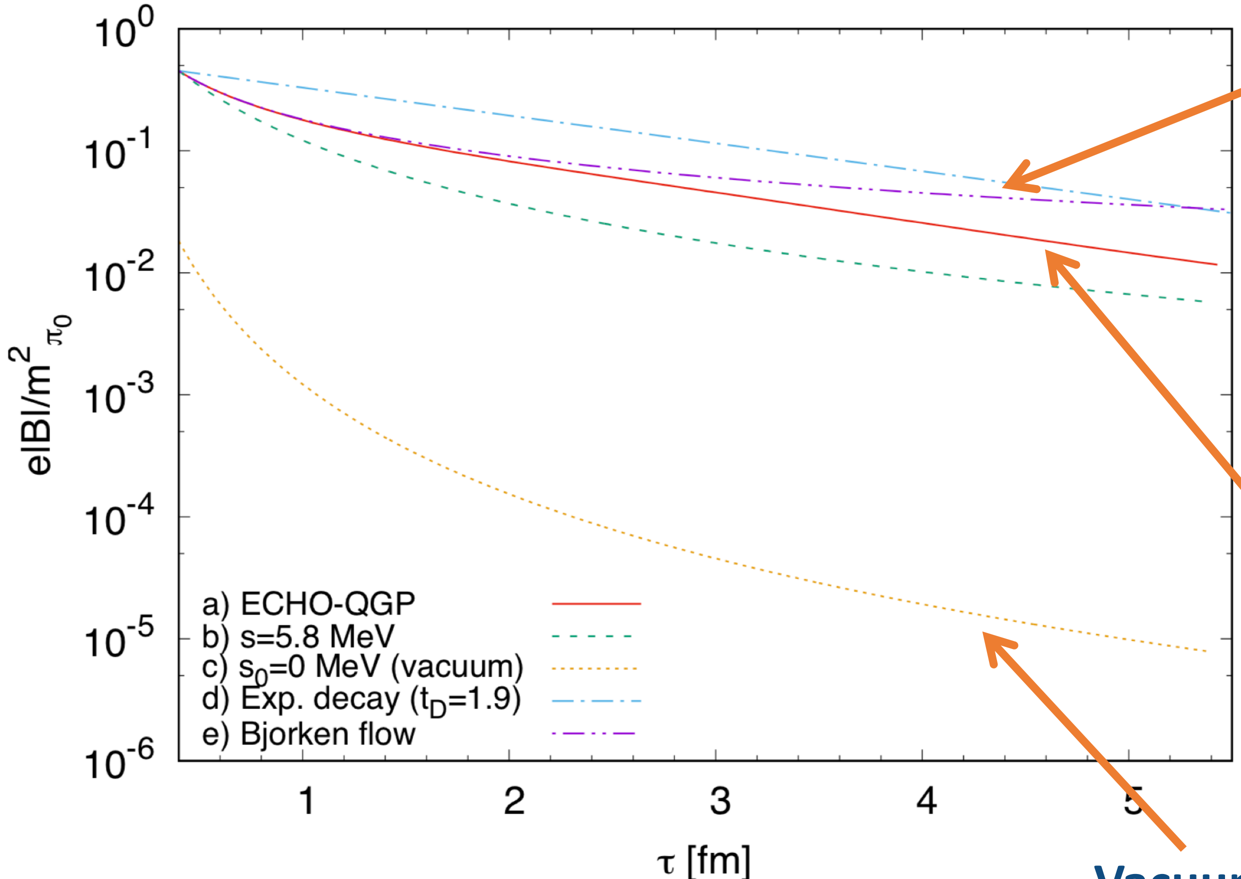


A. Bzdak, V. Skokov PRC 2012 ;W.T. Deng, X.G. Huang PRC 2012;V.Roy, SP, PRC 2015; H. Li, X.l. Sheng, Q.Wang, 2016; etc. / review: K. Tuchin 2013

Strong magnetic fields



Evolution of EB fields



Ideal Bjorken MHD

SP, V. Roy, L. Rezzolla, D.H. Rischke, PRD(2015)

MHD + CME + Chiral anomaly

Siddique, Wang, SP, Wang, PRD (2019)

ECHO-QGP

Inghirami, Zanna, Moghaddam, Becattini, Bleicher, EPJC(2016)

Vacuum

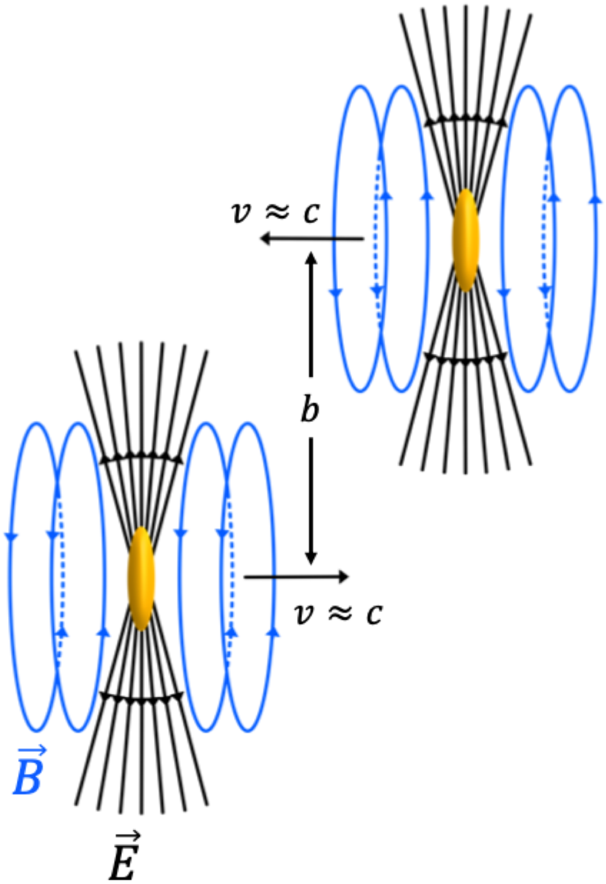
Kharzeev, McLerran, Warringa, NPA(2008)

Two ways to study the effects to EM fields

- Consider EM fields as the (real) photon fields
  - Photo-photon, photon-nuclear interaction

- Consider EM fields as the background fields
  - Quantum transport phenomena
  - …

Ultra-Peripheral Collisions



- Ultra-Peripheral Collisions (UPC): the impact parameter is larger than 2 times the radius of a nucleus
- Since the QCD effects are higher orders and QED effects are enhanced by the Ze, UPC provides a nice platform to study the strong EB effects.

  $Z\alpha \approx 1$ → High photon density

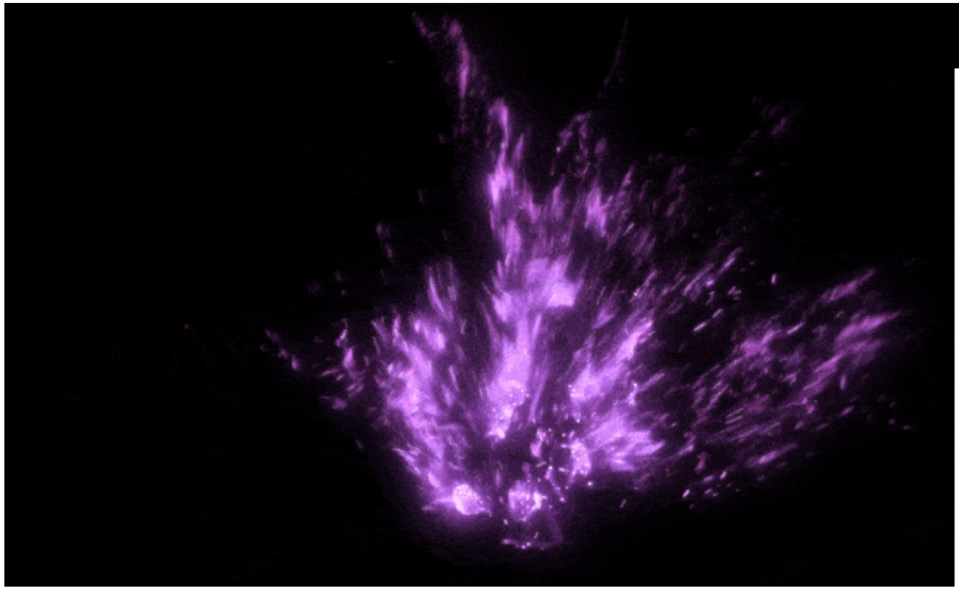
  Magnetic field strength B ≈ $10^{12} - 10^{14}$ T
- Because the relativity, the photon (EB fields) are almost real.
- Photon-photon, photon-nuclear interactions

Generation matter directly from lights

Scientists Generate Matter Directly From Light – Physics Phenomena Predicted More Than 80 Years Ago

TOPICS: Antimatter Atomic Physics Brookhaven National Laboratory DOE Popular

By BROOKHAVEN NATIONAL LABORATORY JULY 30, 2021

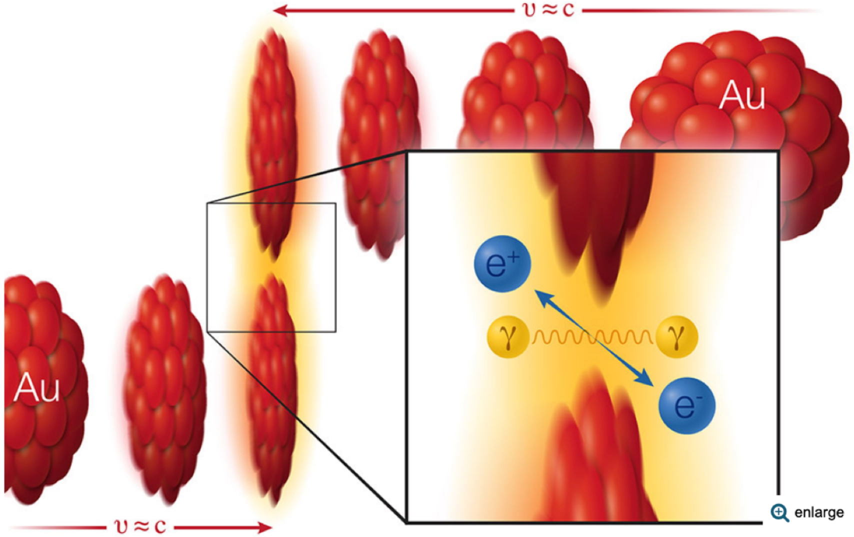


Abstract energy concept illustration.

Collisions of Light Produce Matter/Antimatter from Pure Energy

Study demonstrates a long-predicted process for generating matter directly from light — plus evidence that magnetism can bend polarized photons along different paths in a vacuum

July 28, 2021

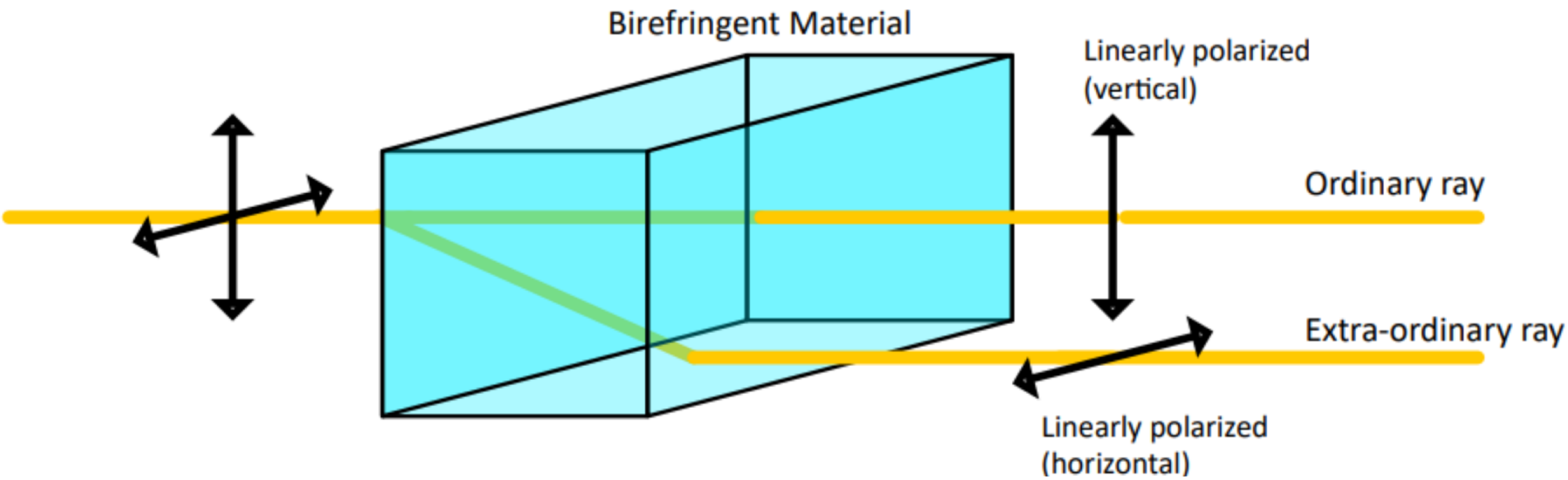


Making matter from light: Two gold (Au) ions (red) move in opposite direction at 99.995% of the speed of light (v, for velocity, = approximately c, the speed of light). As the ions pass one another without colliding, two photons (γ) from the electromagnetic cloud surrounding the ions can interact with each other to create a matter-antimatter pair: an electron (e^-) and positron (e^+).

J. Adam *et al*. (STAR Collaboration), Measurement of e+e− Momentum and Angular Distributions from Linearly Polarized Photon Collisions, *Phys. Rev. Lett* 127, 052302

Vacuum birefringence (I)

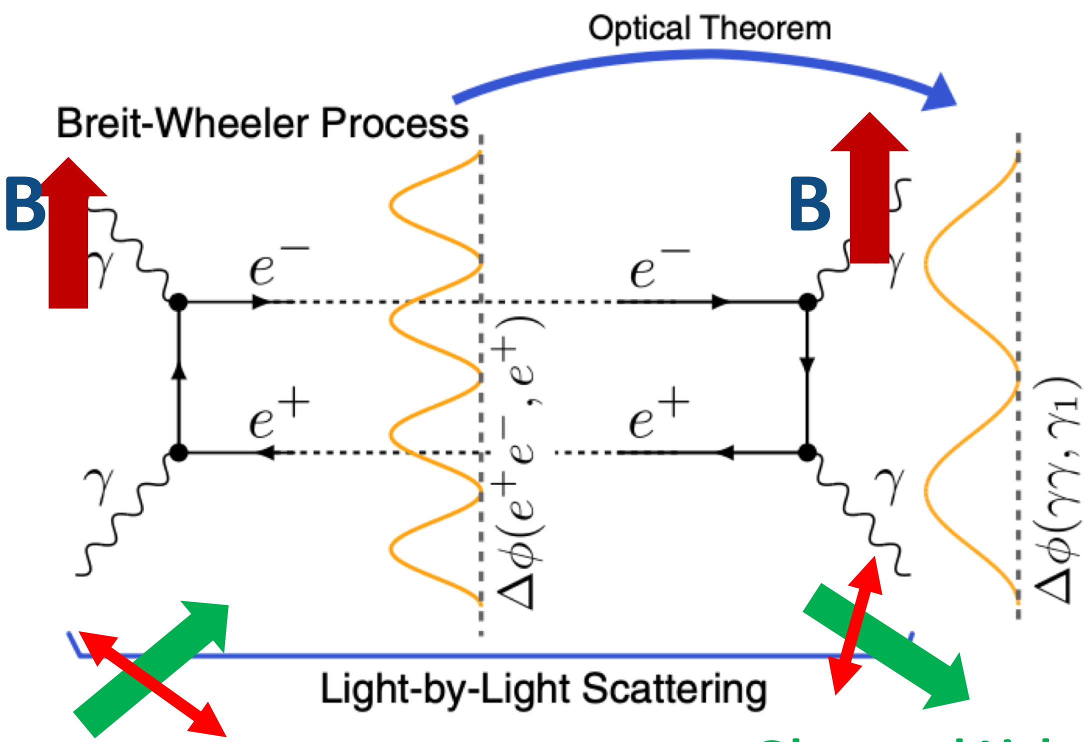
- **Optical birefringence:**
  Different index of refraction for light polarized parallel vs. perpendicular to material's ordinary axis



Figures from the talks given by Daniel Brandenburg and Zhangbu Xu

Vacuum birefringence (II)

- **Vacuum birefringence: Index of refraction for photon interaction with B field depends on relative polarization angle**



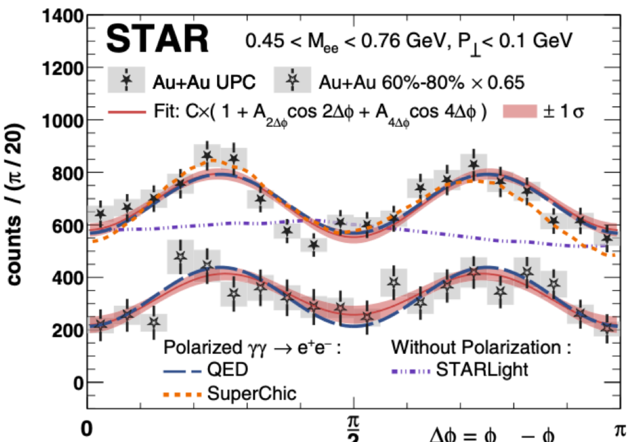
Linear Polarized Probe Light

Observed Light polarization direction is modified

- The difference of linear polarization between probe and observed lights leads to $\sim \cos(n\phi)$ type correction to differential cross section.

$$\Delta\phi = \Delta\phi[(e^+ + e^-), (e^+ - e^-)]$$
$$\approx \Delta\phi[(e^+ + e^-), e^+]$$

Li, Zhou, Zhou, PLB 795, 576 (2019)

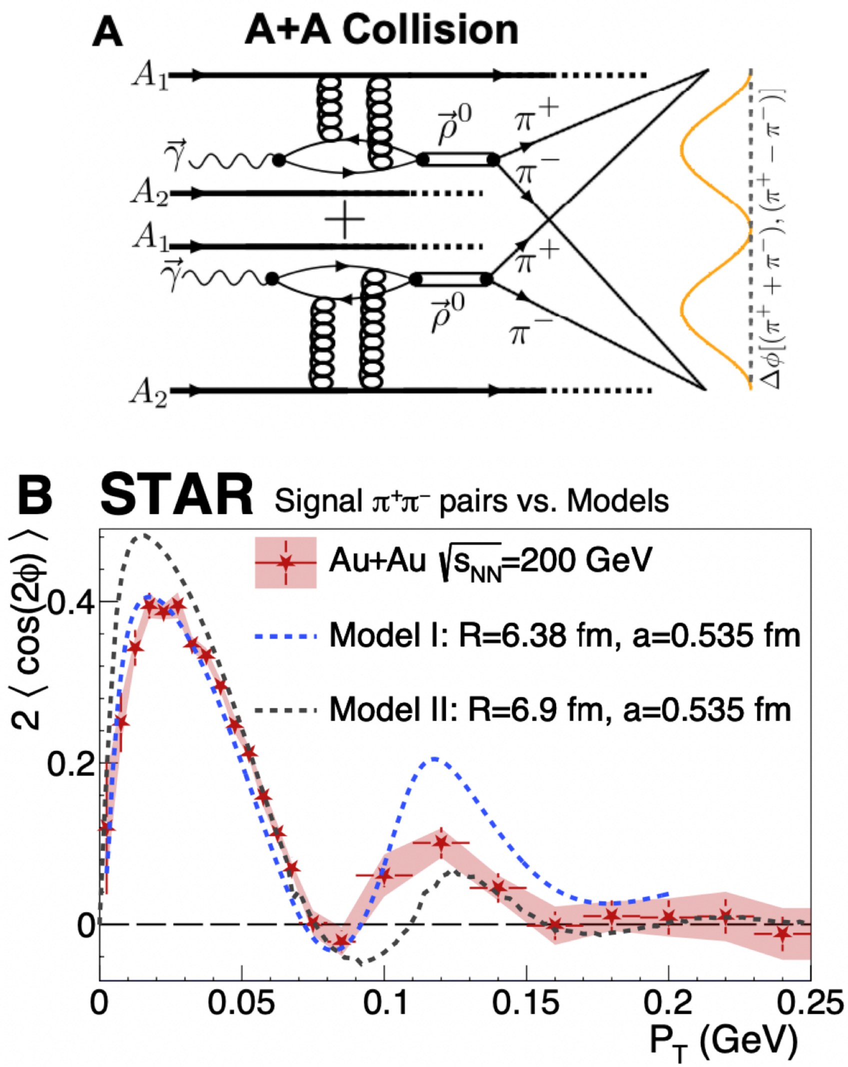


STAR, PRL 127, 052302

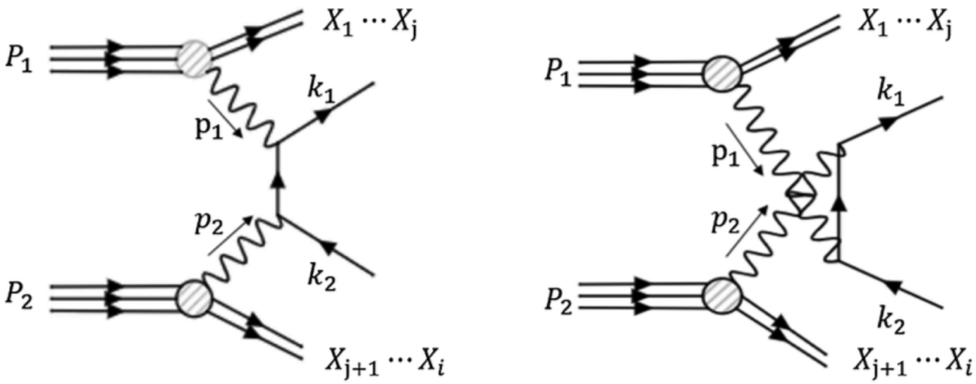
Strong QED fields studies: Hattori, Itakura, Annals.Phys. (2013)

Polarization dependent vector meson production

- **Azimuthal asymmetries $\cos(2\phi)$ in diffractive vector meson production in UPC**
- $\rho^0 \to \pi^+\pi^-$
  - **STAR: arXiv:2204.01625.**
  - **Theory:**
    - **Model I: Zha, Brandenburg, Ruan, Tang, Xu, PRD 2021**
    - **Model II: Xing, Zhang, Zhou, Zhou, JHEP 2020**
  - **For $\cos(\phi)$ and $\cos(3\phi)$ related to ρ^0, see Hagiwara, Zhang, Zhou, Zhou, PRD 2021**
- **Also see studies for J/ψ :**
Brandenburg, Xu, Zha, Zhang, Zhou, Zhou, PRD 2022



Dilepton photoproduction



Equivalent photon approximation (EPA)

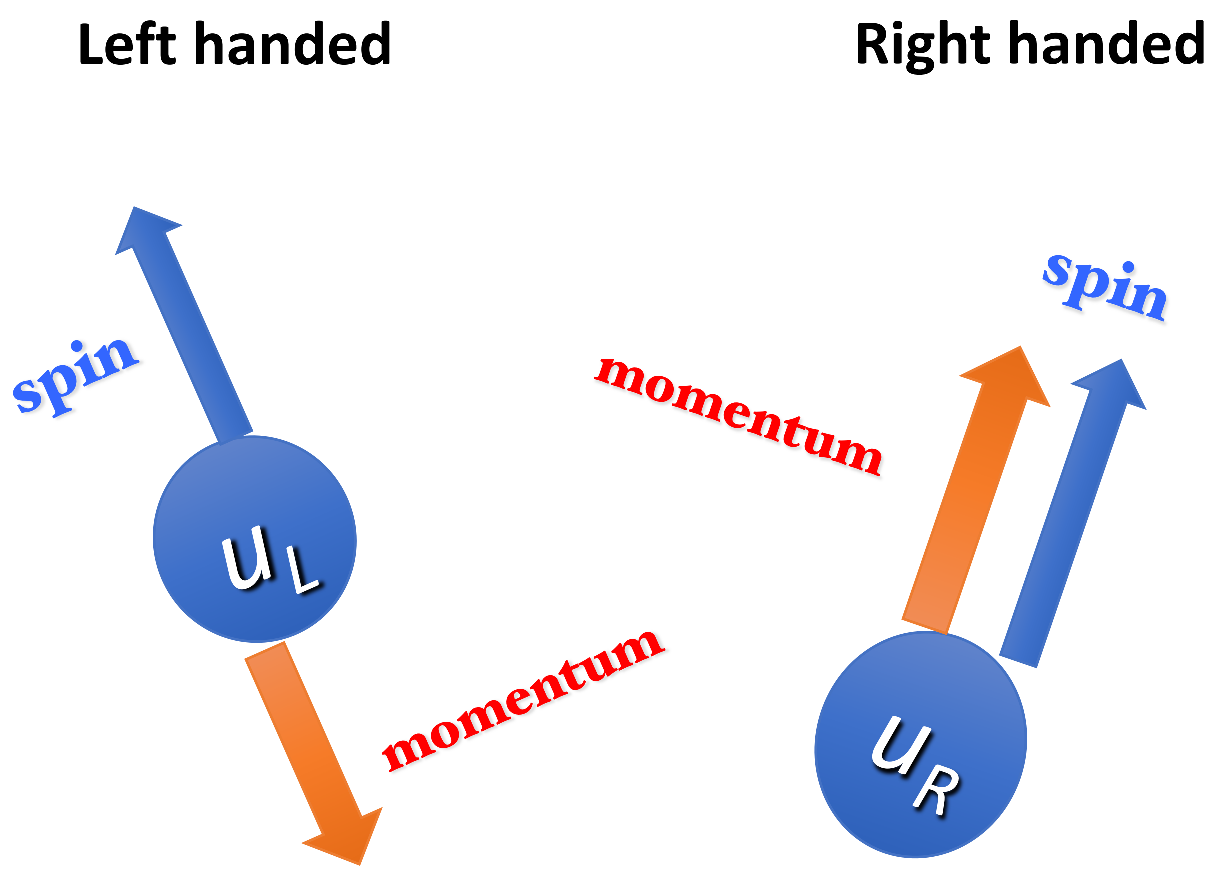
- A. J. Baltz, Y. Gorbunov, S. R. Klein and J. Nystrand, PRC 80, 044902 (2009)
- W. Zha, L. Ruan, Z. Tang, Z. Xu and S. Yang, PLB 781, 182 (2018)
- W. Zha, J. D. Brandenburg, Z. Tang and Z. Xu, PLB 800 (2020) 135089

Based on QED calculations

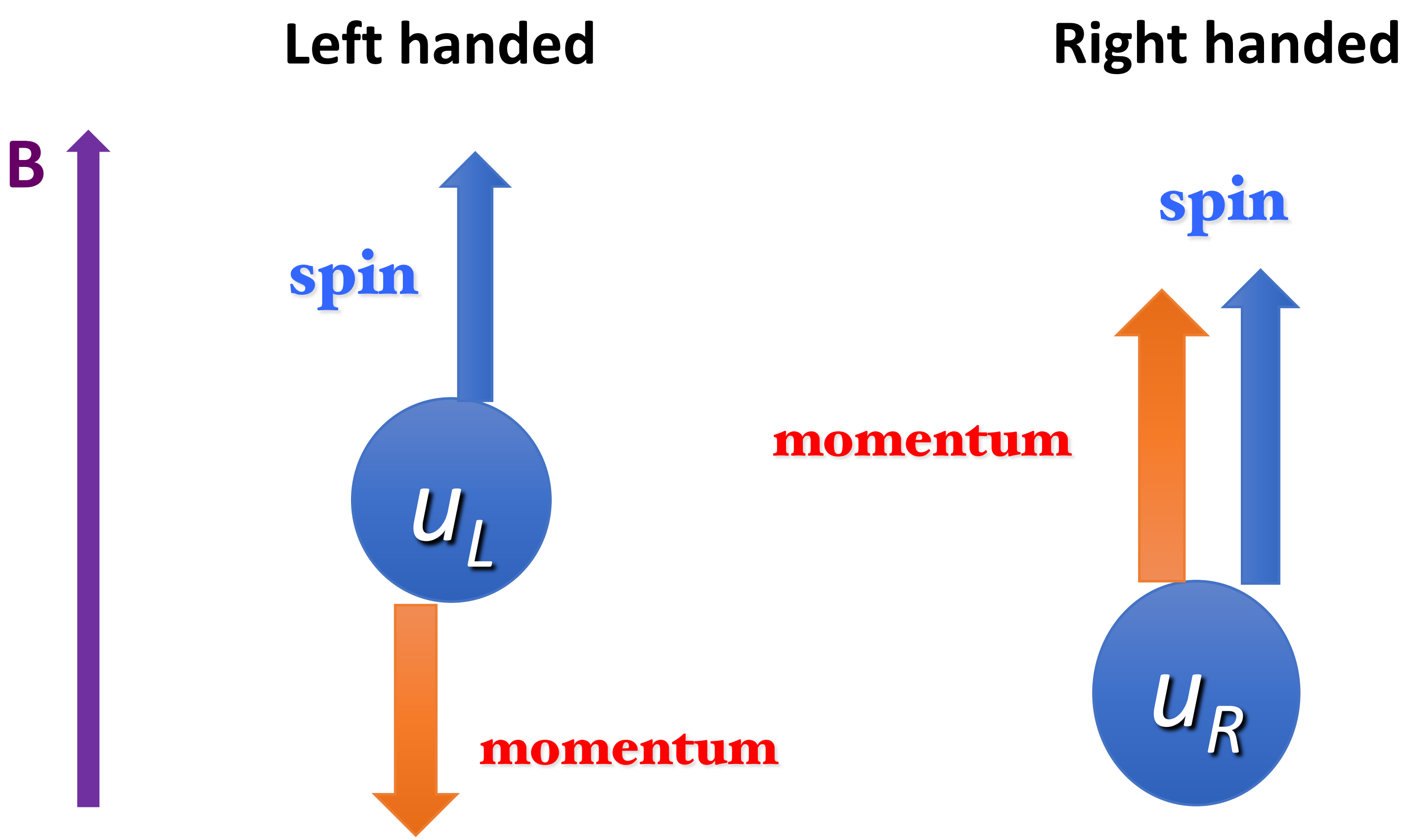
- Transverse momentum dependent (TMD) formulism
  - C. Li, J. Zhou and Y. J. Zhou, Phys. Lett. B 795, 576 (2019) ; arXiv:1911.00237 [hep-ph]].
  - Klein, Muller, Xiao, Yuan, PRL 122 (2019) 13, 132301; PRD 102 (2020) 9, 094013
  - Xiao, Yuan, Zhou, PRL 125 (2020) 23, 232301
- QED in classical field approximation
  - Vidovic, Greiner, Best, Soff, PRC (1993)
  - W. Zha, J. D. Brandenburg, Z. Tang and Z. Xu, PLB 800 (2020) 135089
- R.J. Wang, SP, Q. Wang, PRD (2021)

....

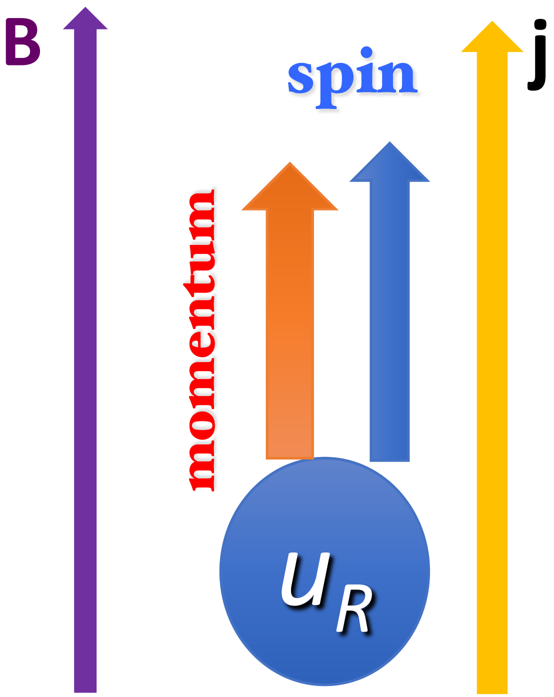
Chirality and massless fermions



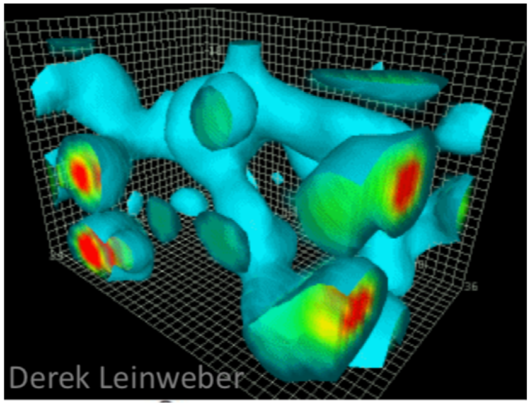
Polarization by magnetic fields

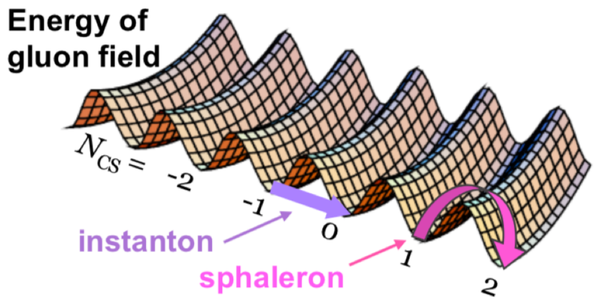


Chiral Magnetic Effect



- **Magnetic fields**
- **Nonzero axial chemical potential**
- **Number of Left handed fermions ≠ Number of Right handed fermions**



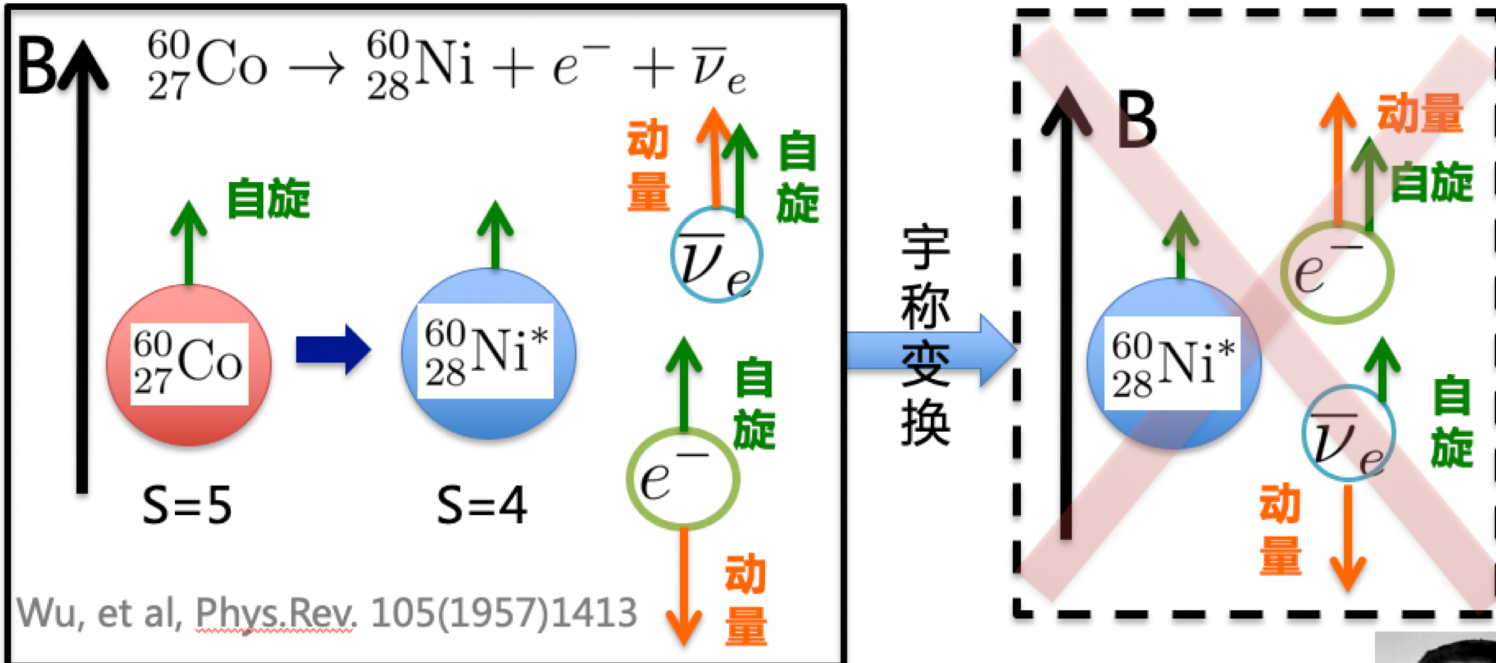


- **Charge current: charge separation**

$$\mathbf{j} = \frac{e^2}{2\pi^2}\mu_5 \mathbf{B},$$

Kharzeev, Fukushima, Warrigna, (08,09), etc. ...

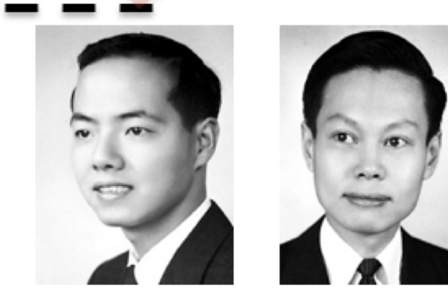
Lee-Yang-Wu Parity violation in weak decay



吴建雄, 1957

实验结果：电子出射与核极化反向

(1)反中微子是右旋的;　　(2)中微子是左旋的



开启人类认识弱作用的历史并最终导致弱电作用的统一!

(1957年以来有5次有关对称性诺贝尔奖颁发)

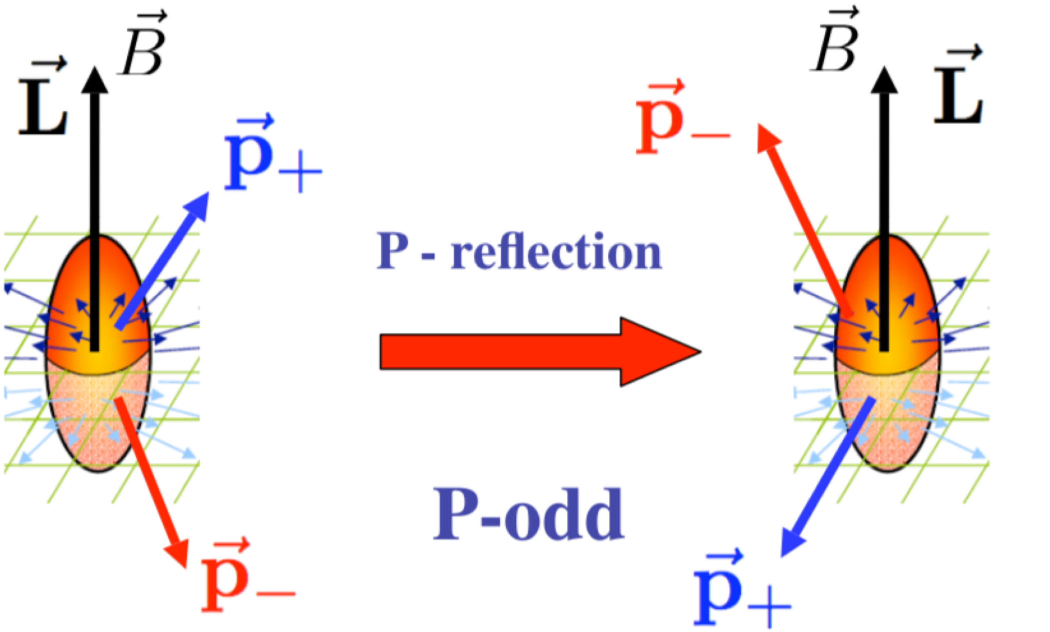
李政道, 杨振宁

诺贝尔奖1957

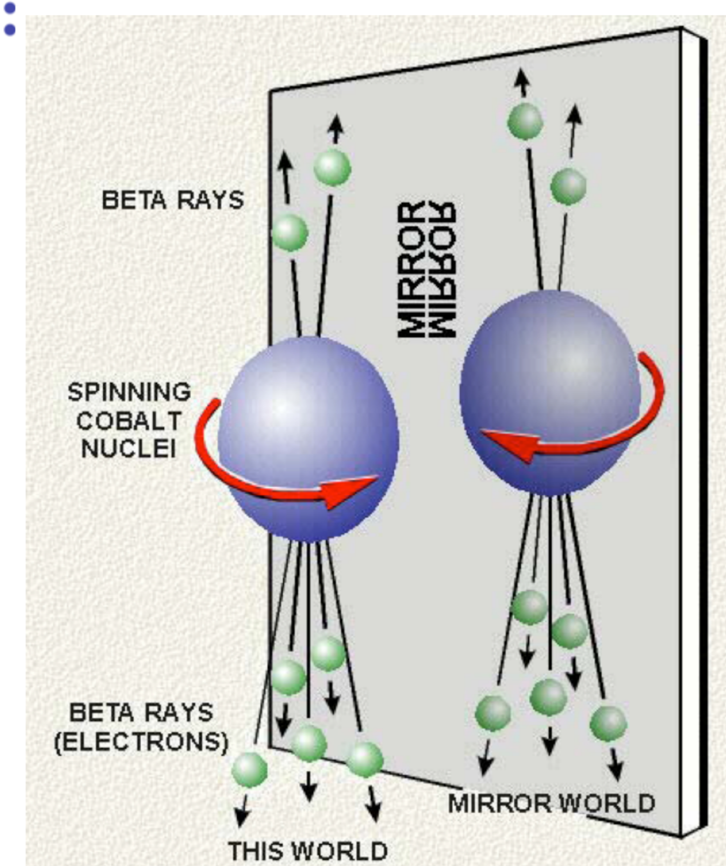
Charge separation ?= Parity Violation

Slides from Kharzeev's talk at 26th Winter Workshop on Nuclear Dynamics (2010)

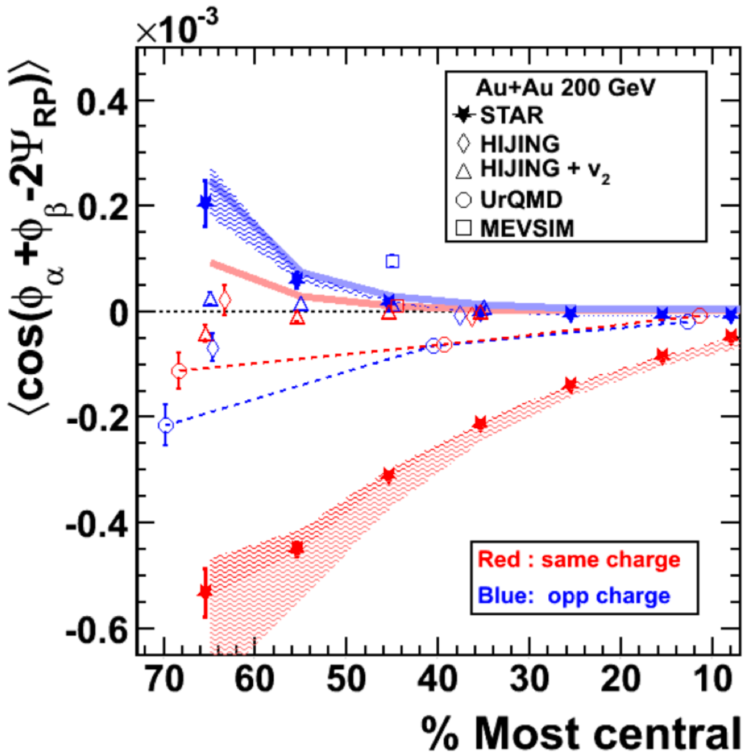
Charge separation = parity violation:

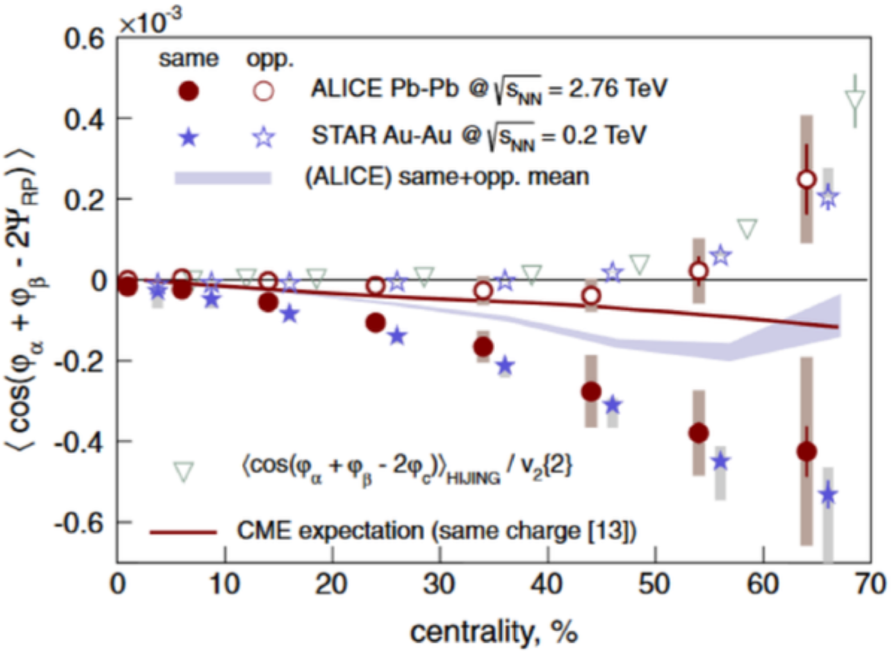


$$\mathcal{P}: \quad \vec{p} \to -\vec{p}; \quad \vec{B} \to \vec{B}; \quad \vec{L} \to \vec{L}$$



Experiments: signal VS background

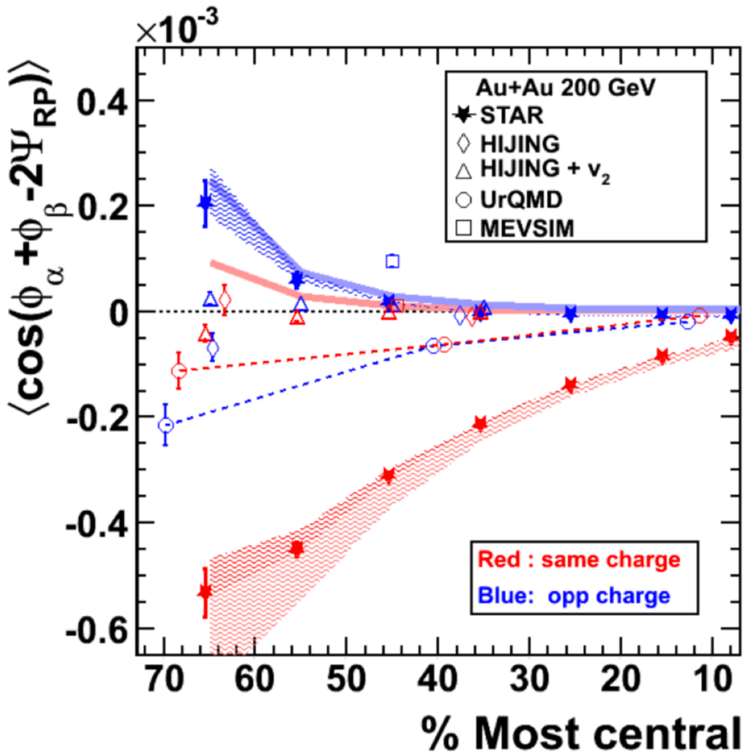


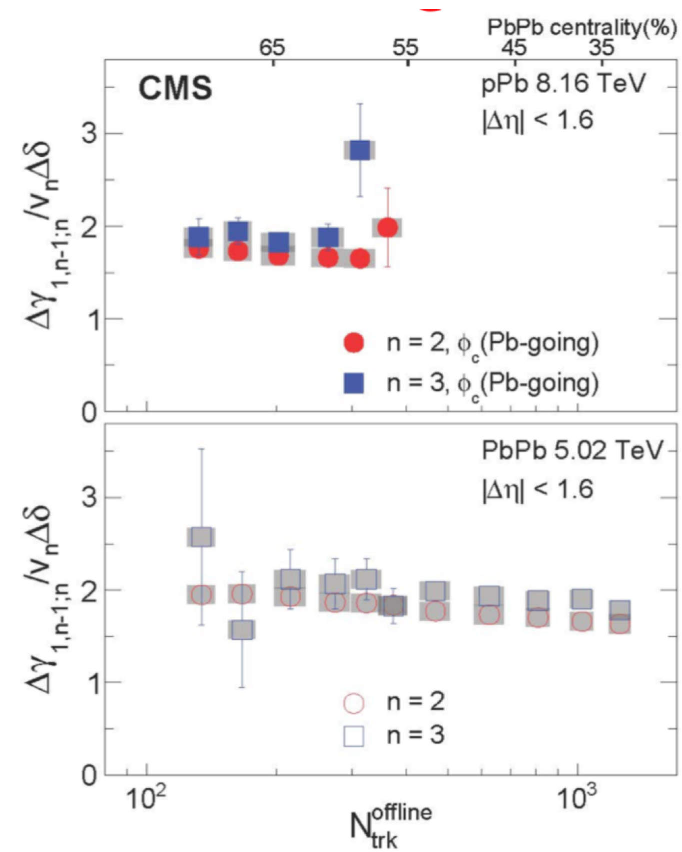


STAR PRL 103, 251601(2009);
PRC 81, 054908

ALICE PRL 110, 012301(2013)

Experiments: signal VS background

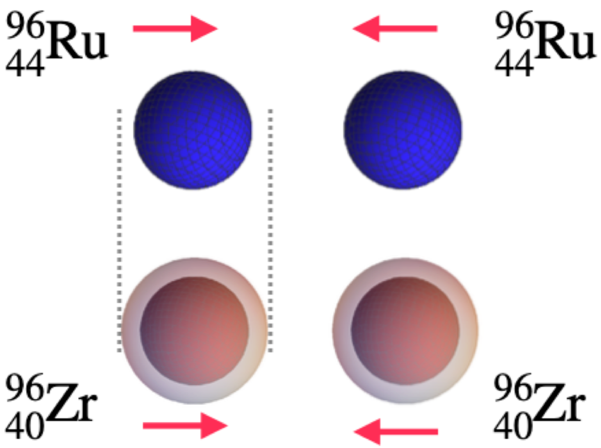




STAR PRL 103, 251601(2009); PRC 81, 054908

CMS PRL 118, 122301 (2016); PRC 97, 044912

Isobaric collisions



- Same multiplicity distributions, eccentricities => same background
- Different magnetic field => different CME signals

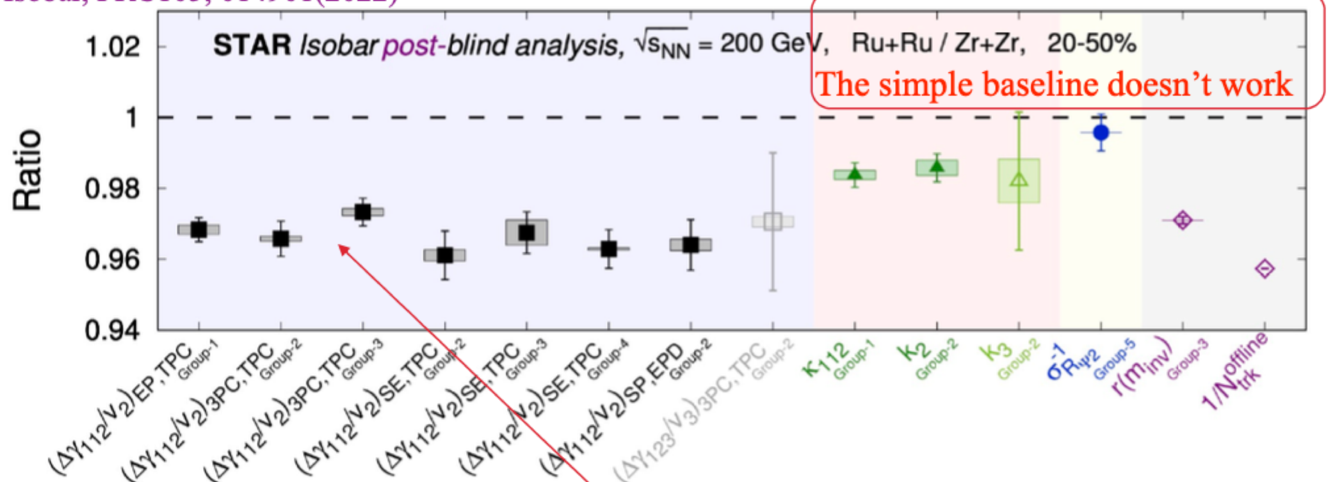
Isobar structure difference

Copy from Hao-jie Xu's slides

e.g. see Xu, et al., PRL 2018; Li, et al, PRC 2018; Zhang, Jia, PRL 2022; Deng, Huang, Ma, Wang PRC 2016

Results from Isobaric collisions

STAR, Isobar, PRC105, 014901(2022)



$$\Delta\gamma_{\rm bkg} = \langle\cos(\varphi_\alpha+\varphi_\beta-2\Psi_{RP})\rangle = \frac{N_{\rm cluster}}{N_\alpha N_\beta}\times\langle\cos(\varphi_\alpha+\varphi_\beta-2\Psi_{\rm cluster})\rangle\times v_{2,\rm cluster}$$

Multiplicity differences

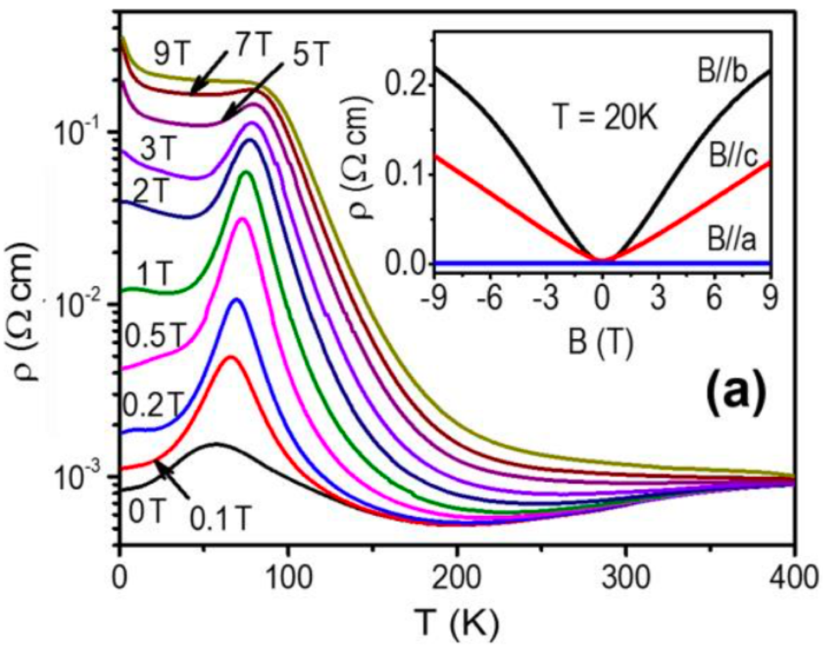
Flow differences

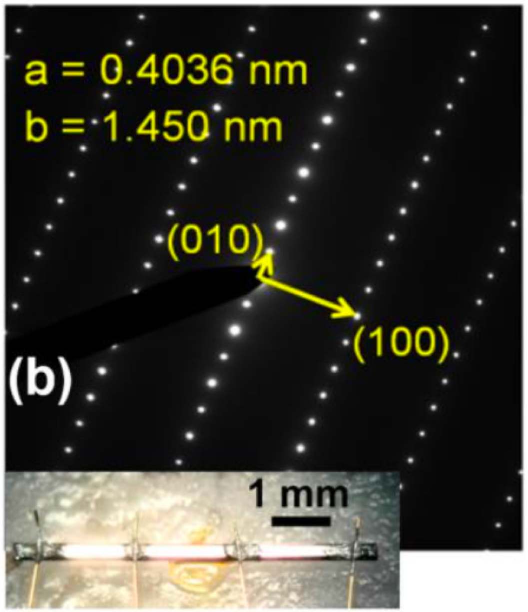
The multiplicity and v2 differences from isobar structure are crucial for the CME search in the isobar collisions at RHIC

Copy from Hao-jie Xu's slides

CME in condense matter

- **Weyl Semi-metal: new transport effects**





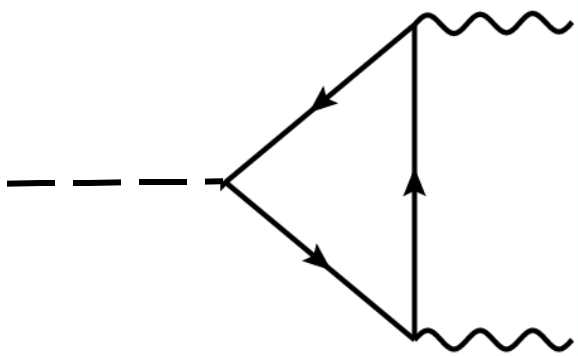
$ZrTe_5$: Nature Physics, 12 , 550–554, (2016)

1. Chiral magnetic effect and chiral separation effect

(1b) Other important topics related to the CME

Chiral separation effect

$$\partial_\mu j_5^\mu$$



Standard Chiral anomaly

$$\partial_\mu j_5^\mu = C F_{\mu\nu} \tilde{F}_{\alpha\beta}$$
$$= C E \cdot B$$

Adler-Bell-Jackiw anomaly, Phys. Rev. 1969

$$\partial_\mu j_5^\mu = CE \cdot B = C\partial_\mu(\epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta)$$
$$0 = \partial_\mu(j_5^\mu - C\epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta)$$
$$\mathbf{j}_5 = CA^0\mathbf{B} \rightarrow C\mu\mathbf{B}$$

Chiral separation effect

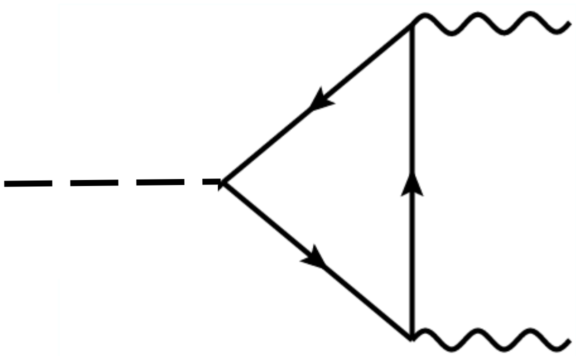
$$\mathbf{j}_5 = C\mu\mathbf{B}$$

In the Dirac equations or Lagrangian, A^0 and chemical potential always appear at the same place.

Connection to the chiral anomaly

$\partial_\mu j_5^\mu$

Chiral current



E field

B field

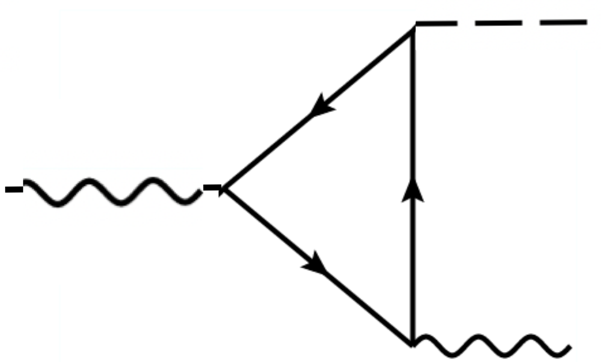
Standard Chiral anomaly

$$\partial_\mu j_5^\mu = C F_{\mu\nu} \tilde{F}_{\alpha\beta}$$

$$= C E \cdot B$$

$\partial_\mu j^\mu$

Vector current



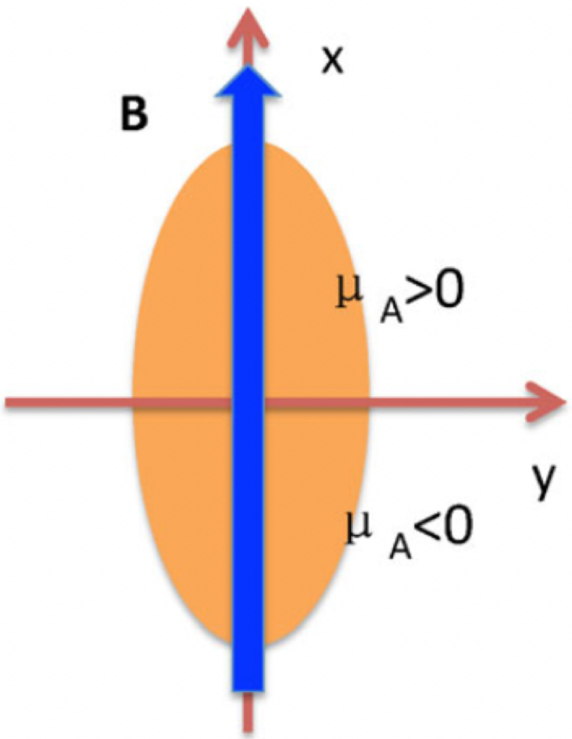
"fake" $E_5 \sim \nabla \mu_5$ field

B field

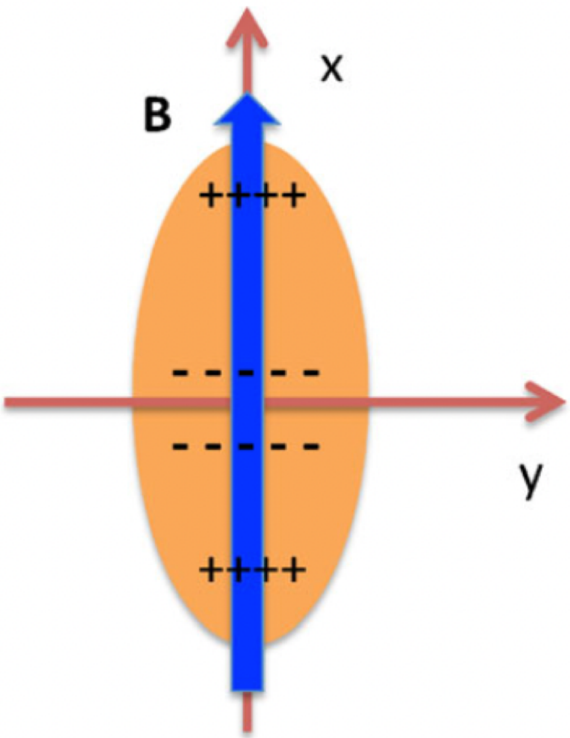
Chiral magnetic effect

$$\mathbf{j} = C \mu_5 \mathbf{B}$$

Chiral magnetic wave: Electric Quadrupole Moment



$$\mathbf{j}_5 = C\mu\mathbf{B}$$



$$\mathbf{j} = C\mu_5\mathbf{B}$$

Theory:

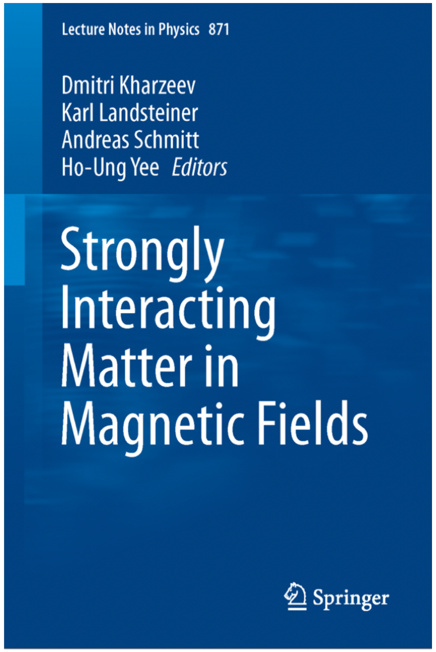
Kharzeev, Yee, PRD 83, 085007 (2011).

Burnier, Kharzeev, Liao, Yee, PRL 107, 052303 (2011).

A brief history for CME

Views of the Chiral Magnetic Effect; K. Fukushima ;
Lect. Notes Phys. 871 (2013) 241-259

Equation (2) is as a meaningful formula as Eq. (1), but the determination of $\gamma(x)$ requires some assumptions. Besides, since the formula involves Q_w, it is unavoidable to think of topologically non-trivial gauge configurations. As a matter of fact, Harmen and I once tried to compute Q concretely on top of the real-time topological configuration, namely, the Lüscher-Schechter classical solution, which turned out to be too complicated to be of any practical use. Then, Harmen hit on a brilliant idea to deal with Q_w, or strictly speaking, an idea to skirt around Q_w. [He invented another nice trick later to treat Q_w more directly. I will come to this point later.] The crucial point is the following; it is not the topological charge Q_w but the chirality N_5 that causes the charge separation. It is tough to think of Q_w, then what about starting with N_5 not caring too much about its microscopic origin? If one wants to fix a value of some number, one should introduce a chemical potential conjugate to the number. In this case of N_5, the necessary ingredient is the chiral chemical potential μ_5 that couples the chiral-charge operator $\bar{\psi}\gamma^0\gamma^5\psi$. In my opinion the introduction of μ_5 was a simple and great step to make the CME transparent to everybody. In this way the CME has eventually gotten equipped with enough simplicity and clarity.



Other methods to derive the CME:
Fukushima, Kharzeev, Warringa, The Chiral magnetic effect. PRD78, 074,033 (2008).
Also see more discussions on the dynamical mu5 and other corrections:
Bo Feng, D.F. Hou, H.C. Ren, PRD99 (2019) 3, 036010; M. Horvath, D.F. Hou, H.C. Ren, PRD 101 (2020) 7, 076026; …

Chirality Production

- **We obtained the chirality production rate:**

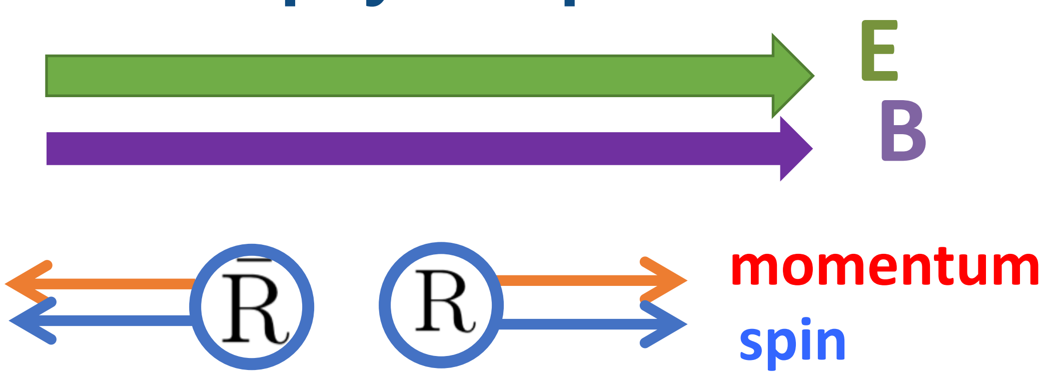
$$\partial_\mu j_5^\mu = \frac{e^2 EB}{2\pi^2} \exp\left(-\frac{\pi m^2}{eE}\right)$$

smooth massless limit:

$m \to 0$,Chiral anomaly

Copinger, Fukushima, SP, PRL(2018)

- **Consistent with physical picture**



$$\frac{1}{2}\partial_t n_5 = \text{Schwinger Pair Production rate}$$

K. Fukushima, D.Kharzeev, H. Warringa PRL 2010

Mass correction to CME

- Assuming E,B at z direction, we obtain the current

$$j^3 = \frac{e^2 EB}{2\pi^2} \coth\left(\frac{B}{E}\pi\right) \exp\left(-\frac{\pi m^2}{eE}\right) t$$

- Non-perturbative: $\sim \frac{1}{eE}$
- Sum over all Landau levels: $Coth(\frac{B}{E}\pi)$

Copinger, Fukushima, SP, PRL(2018)

Also see recent review:
Copinger, SP, IJMPA (2020)

Chiral vorticial effect Vs CME

$$\begin{aligned} j^\mu &= \xi_B B^\mu + \xi \omega^\mu, \\ j_5^\mu &= \xi_{5B} B^\mu + \xi_5 \omega^\mu, \end{aligned}$$

$$\xi = \frac{1}{\pi^2}\mu\mu_5,\ \xi_B = \frac{e}{2\pi^2}\mu_5,$$

$$\xi_5 = \boxed{\frac{T^2}{6}} + \frac{1}{2\pi^2}(\mu^2+\mu_5^2),\ \xi_{B5} = \frac{e}{2\pi^2}\mu.$$

Son, Surowka, PRL 2009;…

QFT: Vilenkin, PRD 22 (1980) 3080

- **If we replace the magnetic field by the Coriolis force,**

  Stephanov, Yin, PRL 2012

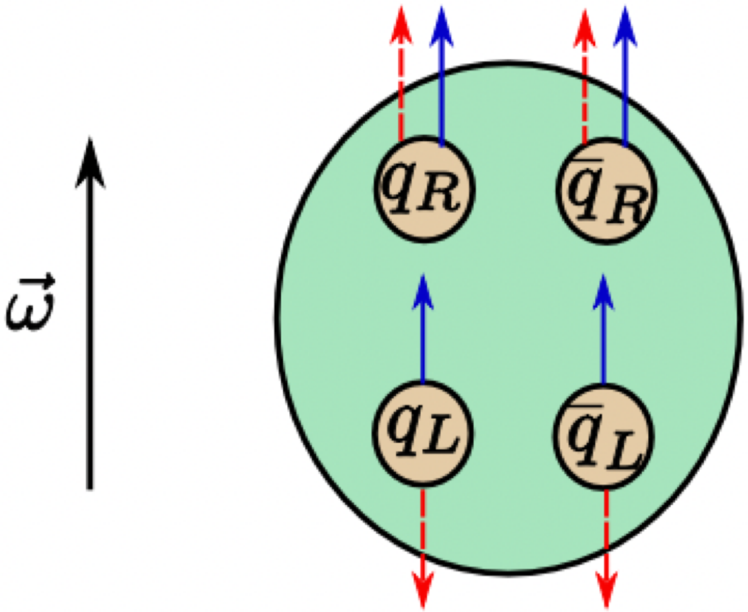
$$\mathbf{B} \to 2m\omega \sim 2|\mathbf{p}|\omega \to 2\mu\omega$$

CVE can be " derived" from CME expect T^2/6.

- **Why have we gotten T^2/6?**
  - Holographic models: it is the gravitational anomaly
  - Curved space QED: No…
- **Will the T^2/6 Non-renormalizable?**

  No. D.F. Hou, H. Liu, H.C. Ren, PRD 2012; S. Golkar, D.T. Son, JHEP 2015

CVE and local polarization



$$j^\mu = \xi_B B^\mu + \xi \omega^\mu,$$
$$j_5^\mu = \xi_{5B} B^\mu + \xi_5 \omega^\mu,$$

$$\xi = \frac{1}{\pi^2}\mu\mu_5,\ \xi_B = \frac{e}{2\pi^2}\mu_5,$$
$$\xi_5 = \frac{T^2}{6} + \frac{1}{2\pi^2}(\mu^2+\mu_5^2),\ \xi_{B5} = \frac{e}{2\pi^2}\mu.$$

- **In the HIC (high temperature limit), $\xi_5 \sim T^2$. The quarks will be polarized by the vorticity. Local spin polarization.**

Gao, Liang, SP, Wang, Wang, PRL 2012

2. kinetic theory and chiral kinetic theory

(2a) Standard kinetic theory

Kinetic theory

- **Assumptions:**

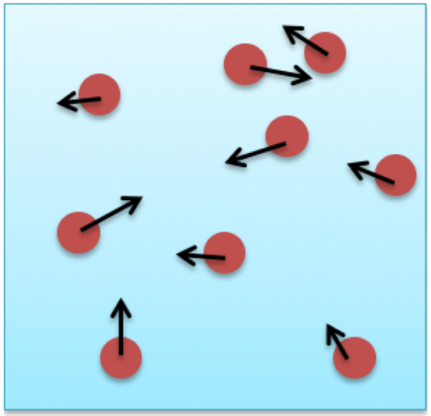
Mean free path >> collision length scaling

- **“distribution function” f(x,p,t)**

how many particles in a small

volume of phase space (x+dx, p+dp)

e.g. Fermi-Dirac distribution function



- **Ordinary kinetic theory: Boltzmann equation**

Dynamical evolution equation for f(x,p,t)

Ordinary Boltzmann equation

Particle's velocity:

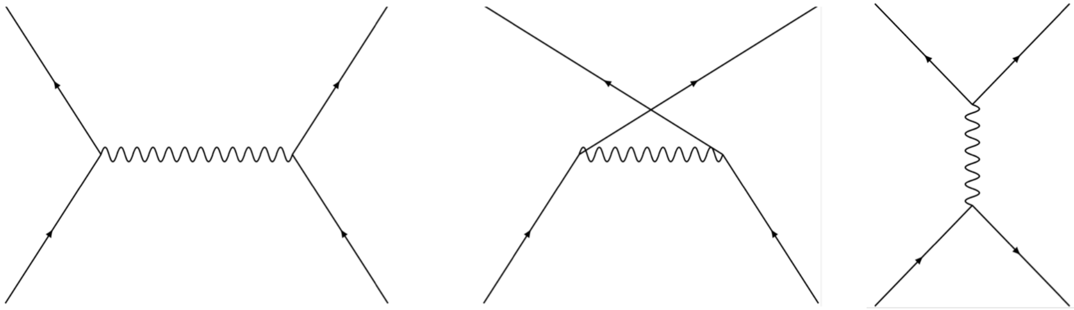
$$\dot{\mathbf{x}} = \frac{\partial \varepsilon}{\partial \mathbf{p}},$$

ε : Particle's energy

Lorentz force:

$$\dot{\mathbf{p}} = \mathbf{E} + \frac{\partial \varepsilon}{\partial \mathbf{p}} \times \mathbf{B},$$

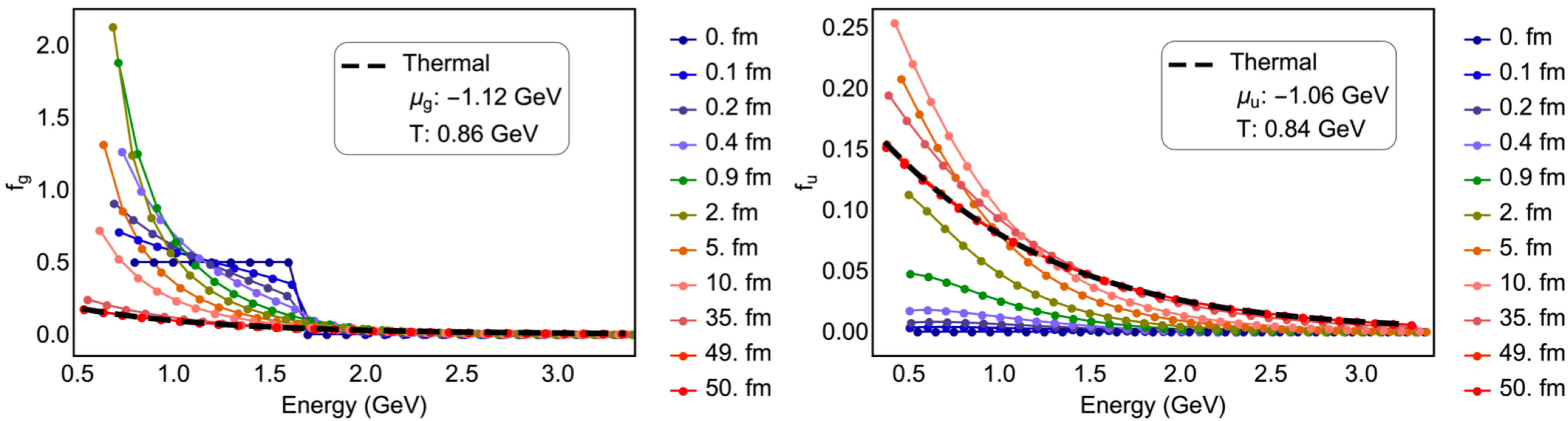
$$\partial_t f + \dot{\mathbf{x}} \cdot \nabla_x f + \dot{\mathbf{p}} \cdot \nabla_p f = C[f],$$



Collison term:

An example: evolution of a quark-gluon system

Gluons + quarks with Leading-Log order QCD scatterings



Phase space box is of size $[-3\text{fm}, 3\text{fm}]^3 \times [-2\text{GeV}, 2\text{GeV}]^3$.

- **Grids:** space: 1 grid; momentum: 30x30x30=27,000
- **Phase space size:** $[-3\text{fm},3\text{fm}]^3$ x $[-2\text{GeV},2\text{GeV}]^3$
- **Time step:** dt=0.0005fm； 100,000 steps
- **Time cost:** around 50 hours on a Nvidia Tesla V100 card

Jun-jie Zhang, Hong-zhong Wu, SP, Guang-you Qin, Qun Wang, PRD 2020

2. kinetic theory and chiral kinetic theory

(2b) chiral kinetic theory: a quick look

What is Chiral kinetic theory?

Boltzmann equation ⟷

$$\partial_t f + \dot{\mathbf{x}} \cdot \nabla_x f + \dot{\mathbf{p}} \cdot \nabla_p f = C[f]$$

\+

Chiral Kinetic Theory = Massless fermions

\+

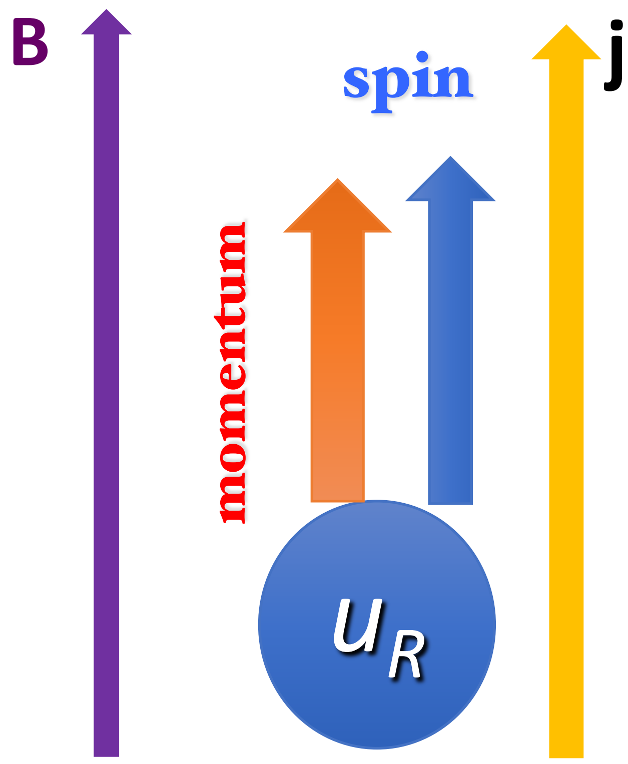
Quantum Corrections ⟷

$$\varepsilon = |\mathbf{p}| + \hbar \ldots$$

$$\dot{\mathbf{x}} = \frac{\partial \varepsilon}{\partial \mathbf{p}} + \hbar \ldots$$

$$\dot{\mathbf{p}} = \mathbf{E} + \frac{\partial \varepsilon}{\partial \mathbf{p}} \times \mathbf{B} + \hbar \ldots$$

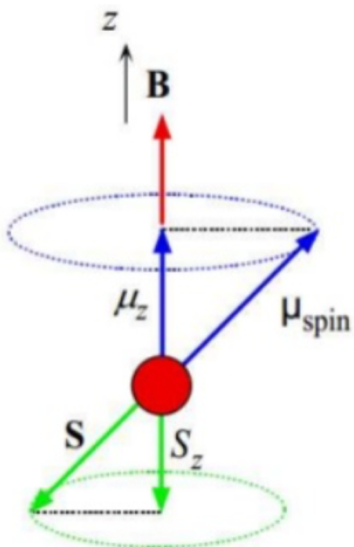
Let us “Guess” what the corrections are



Chiral Magnetic Effect

Spin coupled with magnetic fields

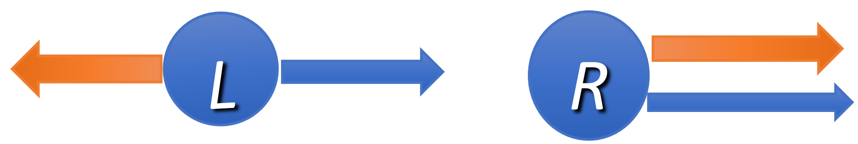
Quantum correction (I)



Spin magnetic moment:

$$\mu_S = -g_S \frac{e}{2m_e}\mathbf{S} \rightarrow -\frac{e}{|\mathbf{p}|}\mathbf{S} \rightarrow \mp \frac{e}{|\mathbf{p}|}\frac{\mathbf{p}}{2|\mathbf{p}|}$$

Massless Chirality

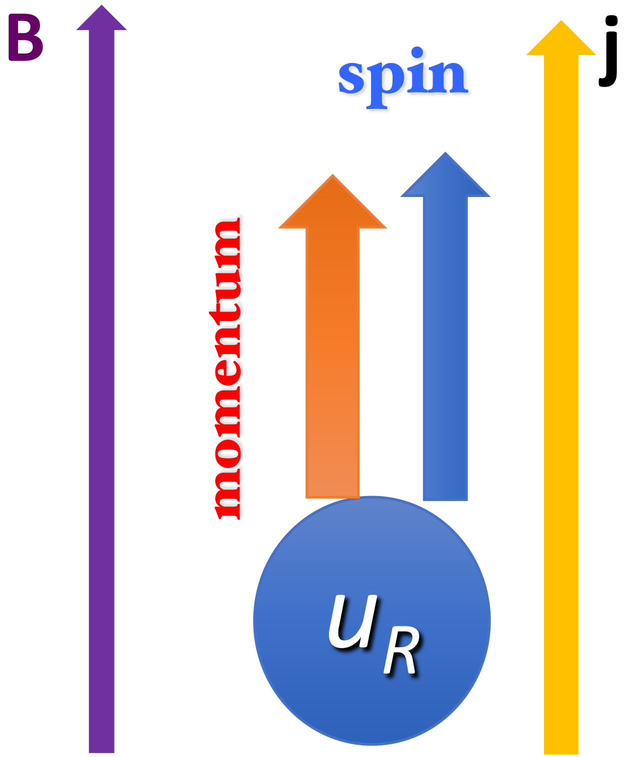


Zeeman effect:

$$\Delta\varepsilon = -\hbar\mu_S \cdot \mathbf{B} = \mp\hbar\frac{|e|}{|\mathbf{p}|}\frac{\mathbf{p}\cdot\mathbf{B}}{2|\mathbf{p}|}$$

Quantum correction (II)

- Correction to effective velocity/w.o. E fields



Particles move parallel or anti-parallel to B

$$\Delta\dot{\mathbf{x}} \propto \mathbf{B}$$

Dimension analysis

$$\Delta\dot{\mathbf{x}} \propto \frac{\mathbf{B}}{|\mathbf{p}|^2}$$

Final results:

$$\Delta\dot{\mathbf{x}} = \hbar\frac{\mathbf{B}}{2|\mathbf{p}|^2}$$

Quantum correction (II)

- **Correction to effective velocity with E fields**

$$\Delta \dot{\mathbf{x}} = \hbar \frac{\mathbf{B}}{2|\mathbf{p}|^2}$$

For moving particles, they feel like:

$$\mathbf{B} \to \mathbf{B} + \mathbf{E} \times \mathbf{v}$$

$$\Rightarrow \quad \Delta \dot{\mathbf{x}} = \hbar \frac{\mathbf{B}}{2|\mathbf{p}|^2} + \hbar \frac{1}{2|\mathbf{p}|^2} \mathbf{E} \times \mathbf{v}$$

Quantum correction (III)

- Are there corrections to effective force?

$$\dot{\mathbf{p}} = \mathbf{E} + \frac{\partial \varepsilon}{\partial \mathbf{p}} \times \mathbf{B} + \hbar ...$$

- History: in condensate matter physics:

Could be neglected!

D. Xiao, M.C. Chang, Q. Niu, Rev. Mod. Phys. 82, 1959 (2010)

- QFT: Chiral anomaly!

Son, Yamamoto, PRL, (2012); PRD (2013)

Stephanov, Yin, PRL (2012);

J.W. Chen, SP, Q. Wang, X.N. Wang, PRL (2013);

Chiral kinetic equation

$$\sqrt{G}\partial_t f + \sqrt{G}\dot{\mathbf{x}} \cdot \nabla_x f + \sqrt{G}\dot{\mathbf{p}} \cdot \nabla_p f = C[f].$$

- **Particle's effective velocity:**

$$\sqrt{G}\dot{\mathbf{x}} = \frac{\partial \varepsilon}{\partial \mathbf{p}} + \hbar \left(\frac{\partial \varepsilon}{\partial \mathbf{p}} \cdot \boldsymbol{\Omega} \right) \mathbf{B} + \hbar \mathbf{E} \times \boldsymbol{\Omega},$$

- **Effective force:**

$$\sqrt{G}\dot{\mathbf{p}} = \mathbf{E} + \frac{\partial \varepsilon}{\partial \mathbf{p}} \times \mathbf{B} + \hbar (\mathbf{E} \cdot \mathbf{B}) \boldsymbol{\Omega},$$

- **Berry curvature**

$$\sqrt{G} = 1 + \hbar \mathbf{B} \cdot \boldsymbol{\Omega}, \qquad \boldsymbol{\Omega} = \frac{\mathbf{p}}{2|\mathbf{p}|^3},$$

Chiral kinetic theory (massless fermions)

- Hamiltonian formulism, effective theory

  Son, Yamamoto, PRL, (2012); PRD (2013)

- Path integration

  Stephanov, Yin, PRL (2012);

  Chen, Son, Stephanov, Yee, Yin, PRL, (2014);

  J.W. Chen, J.Y. Pang, SP, Q. Wang, PRD (2014)

- Wigner function (Quantum field theory)
  - hydrodynamics, equilibrium

    J.W. Chen, SP, Q. Wang, X.N. Wang, PRL (2013);
  - out-of-equilibrium, quantum field theory

    Y. Hidaka, SP, D.L. Yang, PRD(RC) (2017)
  - Other studies

    A.P. Huang, S.Z. Su, Y. Jiang, J.F. Liao, P.F. Zhuang, PRD (2018)

- World-line formulism

  N. Muller, R, Venugopalan PRD 2017

  Also see recent review:

  Gao, Liang, Wang, Int.J.Mod.Phys A 36 (2021), 2130001

  Hidaka, SP, D.L. Yang, Q. Wang, arXiv:2201.07644

3. Berry phase, Berry monopole and chiral anomaly

Definition of Berry phase (I)

- We consider a Hamiltonian H(t) whose time dependence is through a set of parameters R(t),

$$H = H(\mathbf{R}(t)).$$

- We consider an adiabatic evolution of the system: R(t) moves so slowly along a path C that the instantaneous orthonormal basis can be defined at any time t,

$$H(\mathbf{R}(t))\,|n(\mathbf{R}(t))\rangle = \varepsilon_n(\mathbf{R}(t))\,|n(\mathbf{R}(t))\rangle\,,$$

- The wave function has the form,

$$|\psi_n(t)\rangle = e^{i\gamma_n(t)} e^{-ih(t)}\,|n(\mathbf{R}(t))\rangle\,,$$

$$e^{-ih(t)} = \exp\left[-i\int_0^t dt'\varepsilon_n(\mathbf{R}(t'))\right]$$

Dynamical phase factor

$$\gamma_n = \int_0^t dt' \frac{d\mathbf{R}(t')}{dt'}\cdot\langle n(\mathbf{R}(t'))|\,\frac{\partial}{\partial\mathbf{R}}\,|n(\mathbf{R}(t'))\rangle \equiv \int_c d\mathbf{R}\cdot\mathbf{a}_n(\mathbf{R}),$$

New phase ?

Definition of Berry phase (II)

- For a normal path (with time from 0 to t),

$$\gamma_n = \int_0^t dt' \frac{d\mathbf{R}(t')}{dt'} \cdot \langle n(\mathbf{R}(t'))| \frac{\partial}{\partial \mathbf{R}} |n(\mathbf{R}(t'))\rangle \equiv \int_c d\mathbf{R} \cdot \mathbf{a}_n(\mathbf{R}),$$

$$\mathbf{a}_n(\mathbf{R}) = i \langle n(\mathbf{R}(t'))| \frac{\partial}{\partial \mathbf{R}} |n(\mathbf{R}(t'))\rangle .$$

Berry connection

such a phase can be gauged away (i.e. Not physical)

$$|n(\mathbf{R})\rangle \to e^{i\xi(\mathbf{R})} |n(\mathbf{R})\rangle \qquad \gamma_n \to \gamma_n + \xi(\mathbf{R}(0)) - \xi(\mathbf{R}(t))$$

- However, if we choose a closed path, the phase cannot be removed.

$$\gamma_c = \oint_C d\mathbf{R} \cdot \mathbf{a}_n(\mathbf{R}),$$

Berry phase

$$\gamma_c = \int_S d\mathbf{S} \cdot \mathbf{\Omega}_n(\mathbf{R}), \quad \mathbf{\Omega}_n(\mathbf{R}) = \nabla_{\mathbf{R}} \times \mathbf{a}_n(\mathbf{R}),$$

Berry curvature

Berry, Proc. Roy. Soc. Lond. A 392 (1984) 45–57
D. Xiao, M.-C. Chang, Q. Niu, Rev. Mod. Phys. 82 (2010) 1959–2007
Y. Hidaka, SP, Q.Wang, D.L. Yang, arXiv:2201.07644, Chapter 6

Gauge field VS. Berry phase

Gauge theory	Berry "things"
local at x space	at p space
gauge field $\vec{A}$	Berry connection $\vec{a_p}$
magnetic field $\vec{B} = \nabla \times \vec{A}$	Berry curvature $\vec{\Omega_p} = \nabla_p \times \vec{a_p}$
Aharonov–Bohm phase $\int_V d\vec{x} \cdot \vec{A} = \iint_S d\vec{S} \cdot \vec{B}$	Berry phase $\int_V d\vec{p} \cdot \vec{a_p} = \iint_S d\vec{S_p} \cdot \vec{\Omega_p}$
Dirac monopole (magnetic charge) $\int d^3x \nabla \cdot \vec{B} = const.$	Berry monople $\int d^3p \nabla_p \cdot \Omega_p = const.$

Path integrals for a "classical" Weyl fermion (I)

- The Hamiltonian for the right-handed fermions reads

$$H \quad = \quad \boldsymbol{\sigma}\cdot[\mathbf{p}_c - e\mathbf{A}(\mathbf{x})] + e\phi(\mathbf{x}),$$

pc = p + eA(x):canonical momentum, p: the mechanical one.

- The transition matrix element in the path integral is

$$K_{\mathrm{fi}} \quad = \quad \langle \mathbf{x}_{\mathrm{f}} | e^{-iH(t_{\mathrm{f}}-t_{\mathrm{i}})} | \mathbf{x}_{\mathrm{i}} \rangle \;=\; \int [D\mathbf{x}][D\mathbf{p}_c] P \exp\left[i \int_{t_{\mathrm{i}}}^{t_{\mathrm{f}}} dt (\mathbf{p}_c \cdot \dot{\mathbf{x}} - H) \right],$$

- We can to diagonalize H

$$\begin{aligned} U_{\mathbf{p}}^{\dagger} H U_{\mathbf{p}} &= \begin{bmatrix} |\mathbf{p}| + e\phi(\mathbf{x}) & 0 \\ 0 & -|\mathbf{p}| + e\phi(\mathbf{x}) \end{bmatrix} \\ &= \sigma_3 \epsilon(\mathbf{p}_c - e\mathbf{A}) + e\phi(\mathbf{x}), \end{aligned} \qquad U_{\mathbf{p}} = (\chi_+, \chi_-) = \begin{pmatrix} e^{-i\phi}\cos\frac{\theta}{2} & -e^{-i\phi}\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

$$\sigma\cdot\mathbf{p}\chi_{\pm}(\mathbf{p}) = \pm|\mathbf{p}|\chi_{\pm}(\mathbf{p}), \qquad \chi_{\pm}^{\dagger}\chi_{\pm} = 1,$$

Here, +, - denotes the particle and anti-particle, respectively.

Path integrals for a "classical" Weyl fermion (II)

- The amplitude becomes

$$K_{\rm fi} = \int [D\mathbf{x}][D\mathbf{p}_c]U(\mathbf{x}_{\rm f},\mathbf{p}_{\rm f}^c)P\exp\left\{i\int_{t_{\rm i}}^{t_{\rm f}} dt\,[\mathbf{p}\cdot\dot{\mathbf{x}}+e\mathbf{A}(\mathbf{x})\cdot\mathbf{x} -\sigma_3\epsilon(\mathbf{p})-e\phi(\mathbf{x})-\boldsymbol{\mathcal{A}}(\mathbf{p})\cdot\dot{\mathbf{p}}]\right\}U^\dagger(\mathbf{x}_{\rm i},\mathbf{p}_{\rm i}^c),$$

$$\boldsymbol{\mathcal{A}}(\mathbf{p}) \equiv -iU_{\mathbf{p}}^\dagger\nabla_{\mathbf{p}}U_{\mathbf{p}} = -\frac{1}{2|\mathbf{p}|}\begin{pmatrix} \mathbf{e}_\phi\cot\frac{\theta}{2} & \mathbf{e}_\phi-i\mathbf{e}_\theta \\ \mathbf{e}_\phi+i\mathbf{e}_\theta & \mathbf{e}_\phi\tan\frac{\theta}{2}\end{pmatrix}$$

- We can read out the effective action for the right-handed particle (adiabatic approximation),

$$S_\pm = \int dt\,[\mathbf{p}\cdot\dot{\mathbf{x}}+e\mathbf{A}(\mathbf{x})\cdot\mathbf{x}\mp\epsilon(\mathbf{p})-e\phi(\mathbf{x})-\mathbf{a}_\pm(\mathbf{p})\cdot\dot{\mathbf{p}}]\,,$$

$$\mathbf{a}_\pm(\mathbf{p}) = \boldsymbol{\mathcal{A}}_{11/22}(\mathbf{p})$$

Berry connection

Here, +, - denotes the particle and anti-particle, respectively.

Stephanov, Yin, PRL 2012; Chen, Son, Stephanov Yee, Yin, PRL 2014
Chen, Pang, SP, Wang, PRD 2014;

EOM for classical Weyl fermion

- Using the Euler-Lagrangian equations, we get

$$\sqrt{G}\dot{\mathbf{x}} = \hat{\mathbf{p}} + \mathbf{E} \times \boldsymbol{\Omega}_{\mathbf{p}} + \mathbf{B}(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega}_{\mathbf{p}}),$$

$$\sqrt{G}\dot{\mathbf{p}} = \mathbf{E} + \widehat{\mathbf{p}} \times \mathbf{B} + \boldsymbol{\Omega}_{\mathbf{p}}(\mathbf{E} \cdot \mathbf{B}),$$

$$\widehat{\mathbf{p}} \equiv \mathbf{p}/|\mathbf{p}|, \sqrt{G} = 1 + \boldsymbol{\Omega}_{\mathbf{p}} \cdot \mathbf{B}$$

$$\boldsymbol{\Omega}_{\mathbf{p}} = \pm \frac{\mathbf{p}}{2|\mathbf{p}|^3},$$

Berry curvature

$$\boldsymbol{\nabla}_{\mathbf{p}} \cdot \boldsymbol{\Omega}_{\mathbf{p}} = 2\pi\delta^3(\mathbf{p}),$$

It is a monopole!

Classical kinetic theory for Weyl fermions

- In a non-interacting system of fermions without collisions

$$\frac{df(t,x,p)}{dt}=0$$

$$\left[\sqrt{G}\partial_t+\sqrt{G}\dot{\mathbf{x}}\cdot\nabla_{\mathbf{x}}+\sqrt{G}\dot{\mathbf{p}}\cdot\nabla_{\mathbf{p}}\right]f(t,\mathbf{x},\mathbf{p})=0.$$

- Integral over momentum, we get

Chiral anomaly

$$\partial_t\rho+\boldsymbol{\nabla}\cdot\mathbf{J}=\frac{1}{2\pi}(\mathbf{E}\cdot\mathbf{B})f(\mathbf{p}=0)=\frac{1}{4\pi^2}(\mathbf{E}\cdot\mathbf{B}),$$

$$\rho = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\sqrt{G}f(t,\mathbf{x},\mathbf{p}),$$

Number density for right handed fermions

$$\mathbf{J} = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\sqrt{G}\dot{\mathbf{x}}f(t,\mathbf{x},\mathbf{p}),$$

Right handed fermions current

Symplectic form

- If we rewrite the effective action in a compact form,

$$S = \int dt \left[\mathbf{p} \cdot \dot{\mathbf{x}} + e\mathbf{A}(\mathbf{x}) \cdot \mathbf{x} - \mathbf{a_p} \cdot \dot{\mathbf{p}} - H(\mathbf{x}, \mathbf{p}) \right] = \int dt \left[-w_a \dot{\xi}^a - H(\xi) \right]$$

$$\xi^a = (\mathbf{x}, \mathbf{p}) \qquad w_a = (-\dot{\mathbf{p}} - \dot{\mathbf{A}}, \mathbf{a_p})$$

- We can "define" the metric in the phase space

$$\omega_{ab} \equiv \partial_a w_b - \partial_b w_a = \begin{pmatrix} -\epsilon_{ijk}\Omega_k & -\delta_{jk} \\ \delta_{jk} & \epsilon_{ijk}B_k \end{pmatrix}$$

Anti-symmetric!

which is anti-symmetric and is very different with the normal symmetric metric. It is called symplectic form.

Duval, Horvath, Horvathy, Martina, Stichel, Phys. Lett. B 742 (2015) 322–326

Poisson brackets in symplectic form

- **Hamilton equation is then written by modified Poisson brackets**

$$\dot{\xi}^a = \{H, \xi^a\}_\omega = \{\xi^b, \xi^a\}_\omega \frac{\partial H}{\partial \xi^b}.$$

$$\begin{aligned}
\{\mathbf{p}_i, \mathbf{p}_j\}_\omega &= -\frac{\epsilon_{ijk} B_k}{1 + \mathbf{\Omega_p} \cdot \mathbf{B}}, \\
\{\mathbf{x}_i, \mathbf{x}_j\}_\omega &= \frac{\epsilon_{ijk} \Omega_k}{1 + \mathbf{\Omega_p} \cdot \mathbf{B}}, \\
\{\mathbf{p}_i, \mathbf{x}_j\}_\omega &= \frac{\delta_{ij} + \Omega_i B_j}{1 + \mathbf{\Omega_p} \cdot \mathbf{B}},
\end{aligned}$$

x_i, x_j and p_i, p_j do not commute with each other!

CKT from Poisson bracket

- Next, let us consider the Heisenberg equation for density operator,

$$\partial_t n_{\mathbf{p}}(\mathbf{x}) = i[H, n_{\mathbf{p}}(\mathbf{x})].$$

in the symplectic form (modified Poisson brackets)

$$[\widehat{A}_1, \widehat{A}_2] = -\frac{i}{2}\int d\xi \sqrt{\det \omega_{cd}}\, \omega^{ba} \left(\frac{\delta A_1}{\delta n(\xi)} \partial_b \frac{\delta A_2}{\delta n(\xi)} - \frac{\delta A_2}{\delta n(\xi)} \partial_b \frac{\delta A_1}{\delta n(\xi)} \right) \partial_a n(\xi).$$

$$\partial_t \rho(\mathbf{x}) \quad = \quad i[H, \rho(\mathbf{x})] = -\nabla \cdot \mathbf{j} + \boxed{\frac{\mathcal{K}}{4\pi^2} \mathbf{B} \cdot \mathbf{E},}$$

Chiral anomaly

$$\mathbf{J}(\mathbf{x}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[\frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} n_{\mathbf{p}} - \left(\mathbf{\Omega}_{\mathbf{p}} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} \right) \epsilon_{\mathbf{p}} \mathbf{B} - \epsilon_{\mathbf{p}} \mathbf{\Omega}_{\mathbf{p}} \times \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{x}} \right] + \mathbf{E} \times \boldsymbol{\sigma}(\mathbf{x}),$$

Son, Yamamoto, PRL 2012; PRD 2013

Other approaches: effective theories

- **High-density effective theory**

$$p^\mu = \mu v^\mu + l^\mu,$$

$$\psi(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\psi(p) = \sum_v e^{i\mu\mathbf{v}\cdot\mathbf{x}}\psi_v(x) = \sum_v e^{i\mu\mathbf{v}\cdot\mathbf{x}}[\psi_{+v}(x) + \psi_{-v}(x)],$$

$$\psi_{\pm v}(x) \equiv P_\pm(\mathbf{v})\psi_v(x) = \int_{|l|<\mu} \frac{d^4l}{(2\pi)^4} e^{-il\cdot x}\psi_{\pm v}(l),$$

+: slow mode

-: fast mode

- **By using EoM, one can express the fast modes in terms of slow modes, and then derive the effective theory in large (right-handed) chemical potential limit**

$$\mathcal{D}^{(0)} = iv\cdot D,$$

$$\mathcal{D}^{(1)} = \frac{(\sigma\cdot D_\perp)^2}{2\mu},$$

$$D^{(2)} = -\frac{i}{4\mu^2}(\sigma\cdot D^\perp)(\overline{v}\cdot D)(\sigma\cdot D^\perp).$$

$$\mathcal{L}_{\text{EFT}} = \sum_n \mathcal{L}^{(n)},\ \mathcal{L}^{(n)} = \sum_v \psi^\dagger_{+v}\mathcal{D}^{(n)}\psi_{+v},$$

Son, Yamamoto, PRD 2013

Revisited by Lin, Shukla, JHEP 2019

Also see the on-shell effective field theory

$$p^\mu = p^0 v^\mu + l^\mu,$$

Manuel, Torres-Rincon, PRD 2014; PRD 2018, PRD 2020

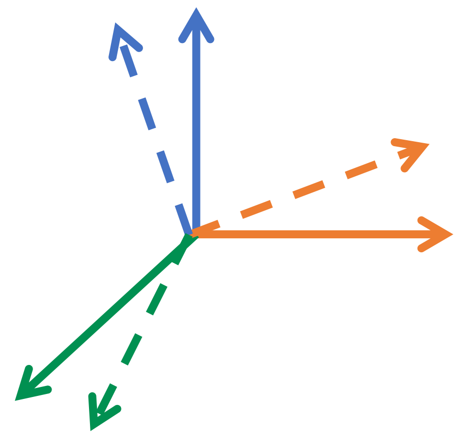
4. Non-trivial Lorentz symmetry

Subgroup of Lorentz symmetry

- Massive particles: Rest frame

$$p^\mu = (m, 0, 0, 0)$$

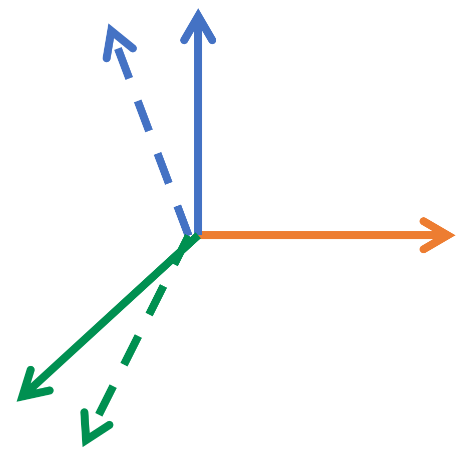
Subgroup: SO(3)



- Massless particles: No rest frame

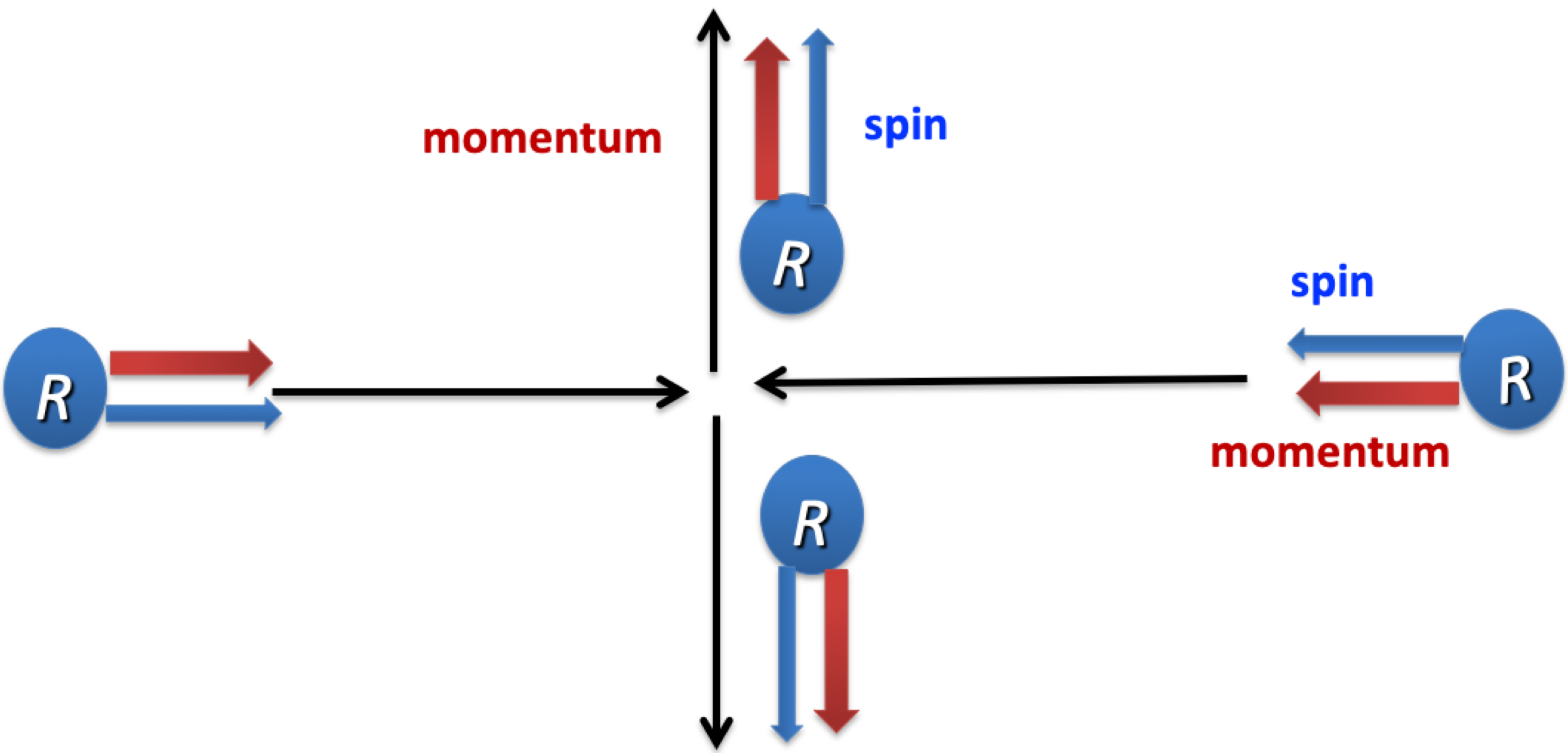
$$p^\mu = (|p_z|, 0, 0, p_z)$$

Subgroup: ISO(2)



Side-jump (I)

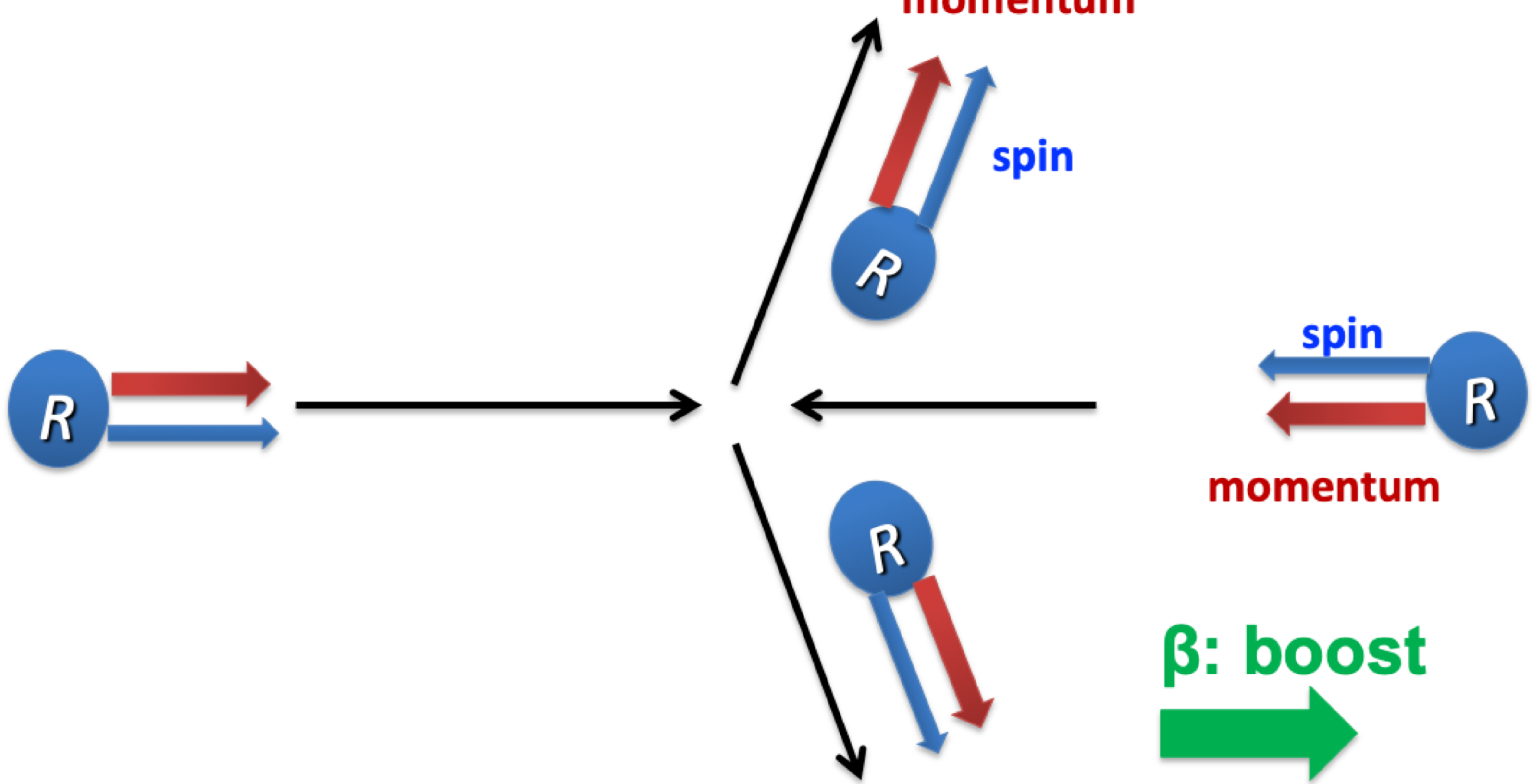
Orbital angular momentum and spin are conserved separately



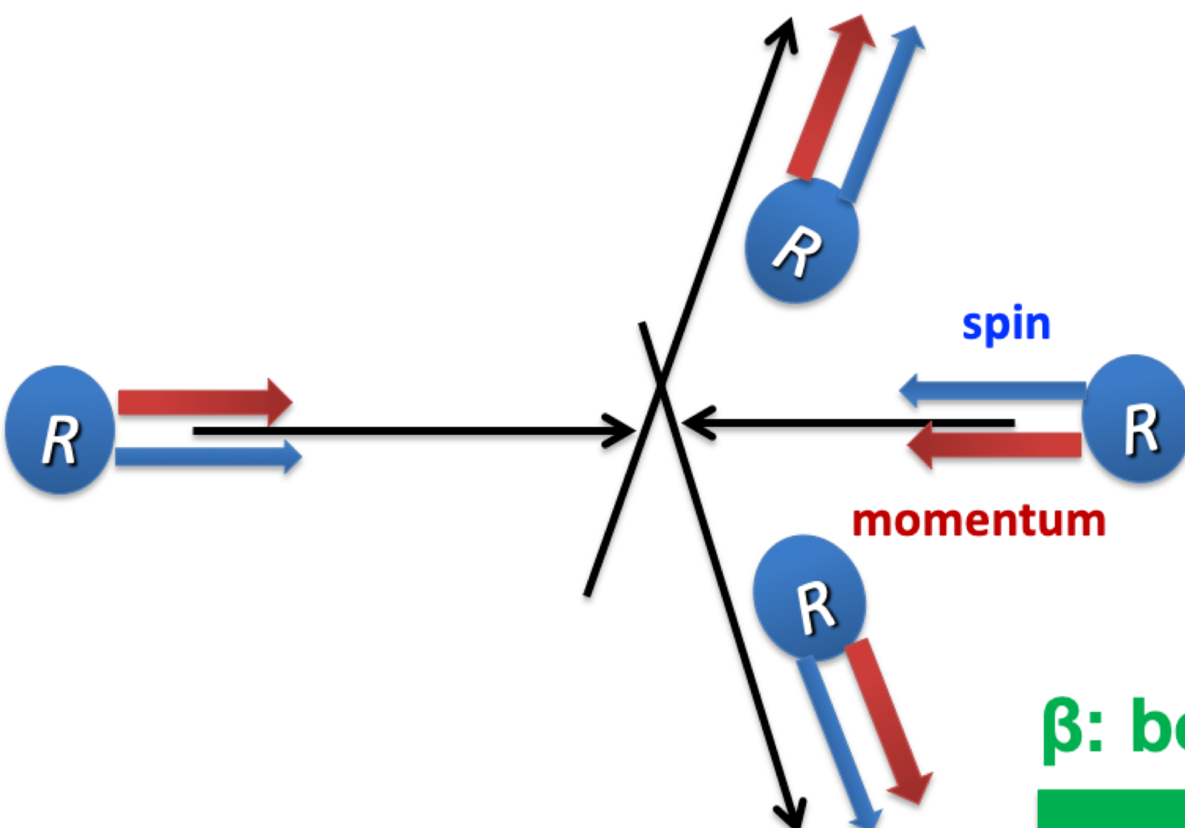
Chen, Son, Stephanov, Yee, Yin, PRL, (2014)

Side-jump (II)

Orbital angular momentum seems to be conserved separately.
Spin is NOT conserved!



Side-jump (III)



x has a shift!!!

"Side-jump" :

$$\mathbf{x}' = \mathbf{x} + \boldsymbol{\beta} t + \delta\mathbf{x},$$
$$\mathbf{p}' = \mathbf{p} + \boldsymbol{\beta}\varepsilon + \delta\mathbf{p},$$

$$\delta\mathbf{x} = \hbar\frac{\boldsymbol{\beta}\times\hat{\mathbf{p}}}{2|\mathbf{p}|},$$
$$\delta\mathbf{p} = \hbar\frac{\boldsymbol{\beta}\times\hat{\mathbf{p}}}{2|\mathbf{p}|}\times\mathbf{B}$$

β: boost

Chen, Son, Stephanov, PRL, (2015);
Y. Hidaka, SP, D.L. Yang, PRD (2016)

Non-trivial Lorentz symmetry

- **Quantum field theory**

$$j^\mu = \overline{\psi}\sigma^\mu\psi \to \Lambda^\mu_\nu j^\nu$$

- **Lorentz transformation**

$$x^{\mu\prime} = \Lambda^\mu_\nu x^\nu,\ p^{\mu\prime} = \Lambda^\mu_\nu p^\nu,$$

$$\boxed{f'(x',p',t') = f(x',p',t') + \hbar N^\mu(\partial^x_\mu + F_{\nu\mu}\partial^\nu_p)f,}$$

Infinitesimal Lorentz Transform

$$\begin{aligned}\delta\mathbf{x} &= \hbar\frac{\boldsymbol{\beta}\times\hat{\mathbf{p}}}{2|\mathbf{p}|},\\ \delta\mathbf{p} &= \hbar\frac{\boldsymbol{\beta}\times\hat{\mathbf{p}}}{2|\mathbf{p}|}\times\mathrm{B}\end{aligned}$$

Chen, Son, Stephanov, PRL, (2015);
Y. Hidaka, SP, D.L. Yang, PRD (2016)

Part 1

Chiral magnetic effect, Berry phase and kinetic theory

1. Chiral magnetic effect and chiral separation effect

   (1a) Strong magnetic fields in HIC and CME

   (1b) Other topics related to the CME

2. Kinetic theory and chiral kinetic theory

   (2a) Standard kinetic theory

   (2b) Chiral kinetic theory: a quick look

3. Berry phase, Berry monopole and chiral anomaly

4. Non-trivial Lorentz symmetry for chiral system

Thank you for your time!

欢迎批评指正！