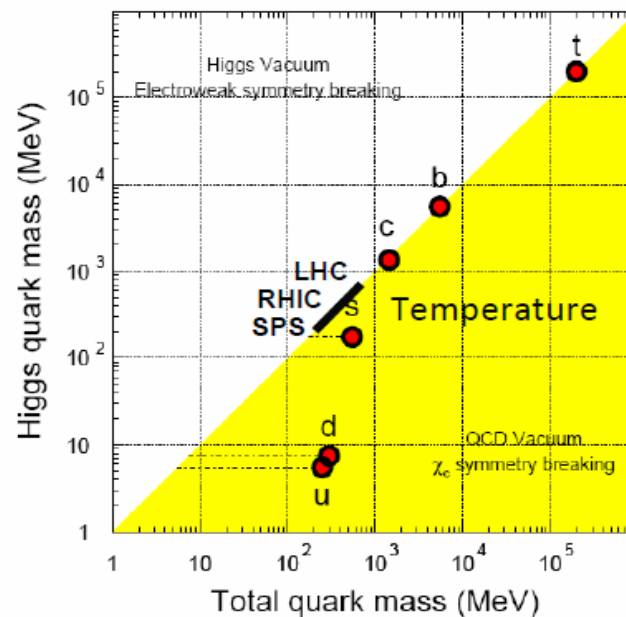
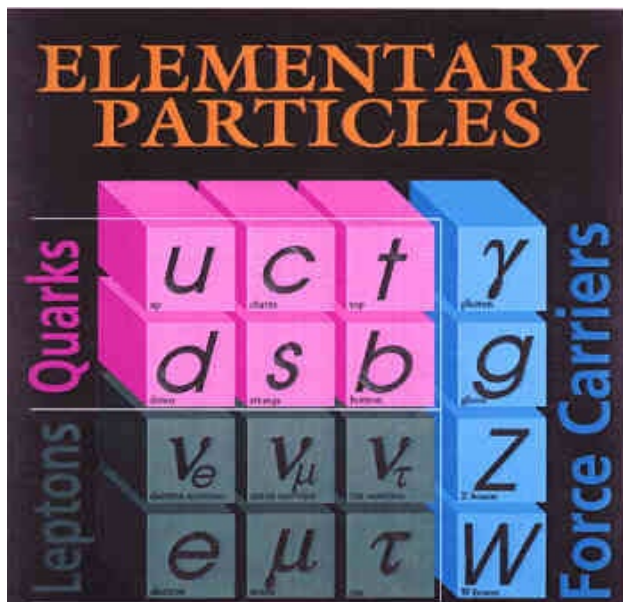



Lectures on Heavy-Flavor Probes of Quark-Gluon Plasma

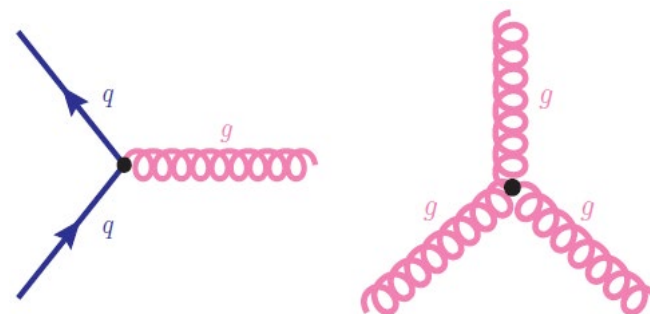
Min He

Nanjing Univ. of Sci. & Tech., Nanjing, China

Quarks & Forces

Weak	Electromagnetic	Strong
W^+, W^-, Z^0	Photon	Gluon
Quarks and Leptons	Quarks and Charged Leptons and W^+, W^-	Quarks and Gluons



QCD & Symmetries

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} \mathcal{F}_{\mu\nu}^a \mathcal{F}_a^{\mu\nu} + \bar{\psi}(i\not{D} - M)\psi = \mathcal{L}_{\text{chiral}} - (m_u \bar{u}u + m_d \bar{d}d) + \mathcal{L}_{\text{scbt}}$$

❖ 2-flavor light sector: chiral symmetry

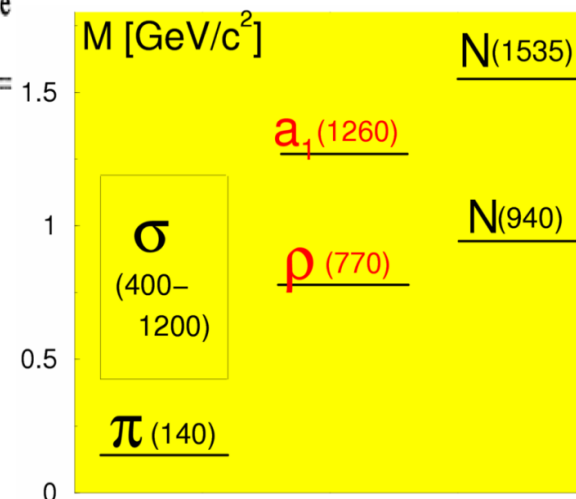
$$\mathcal{L}_{\text{QCD}} = (\bar{u}_L, \bar{d}_L) i\not{D} \begin{pmatrix} u_L \\ d_L \end{pmatrix} + (\bar{u}_R, \bar{d}_R) i\not{D} \begin{pmatrix} u_R \\ d_R \end{pmatrix} + \mathcal{O}(m_q) - \frac{1}{4} G_{a\mu\nu}^2$$

Symmetry	Transformation	Current	Name	Manifestation
$SU_V(2)$	$\psi \rightarrow e^{-i\tau \cdot \omega/2} \psi$	$J_\mu^k = \bar{\psi} \gamma_\mu \tau^k \psi$	isospin	approx. conserved
$U_V(1)$	$\psi \rightarrow e^{-i\alpha} \psi$	$j_\mu = \bar{\psi} \gamma_\mu \psi$	baryonic	always conserved
$SU_A(2)$	$\psi \rightarrow e^{-i\tau \cdot \theta \gamma_5/2} \psi$	$J_{5\mu}^k = \bar{\psi} \gamma_\mu \gamma_5 \tau^k \psi$	chiral	CSB; Goldstone mode
$U_A(1)$	$\psi \rightarrow e^{-i\beta \gamma_5} \psi$	$j_{5\mu} = \bar{\psi} \gamma_\mu \gamma_5 \psi$	axial	$U_A(1)$ “puzzle”

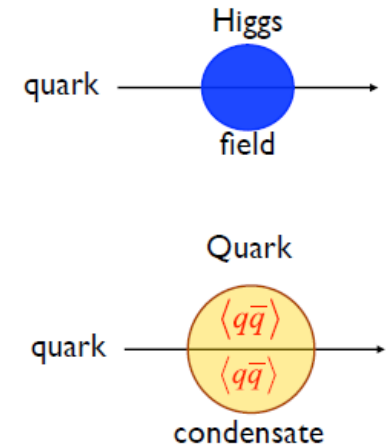
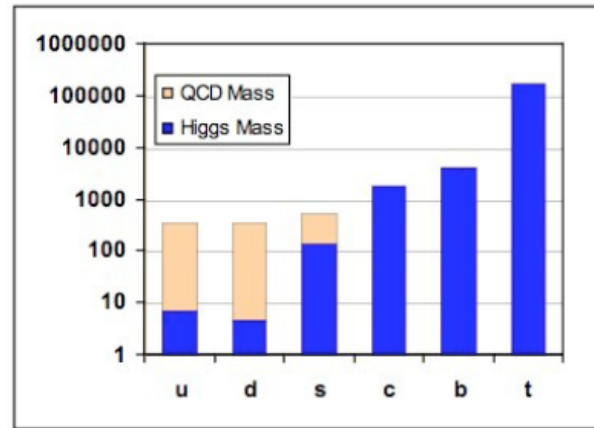
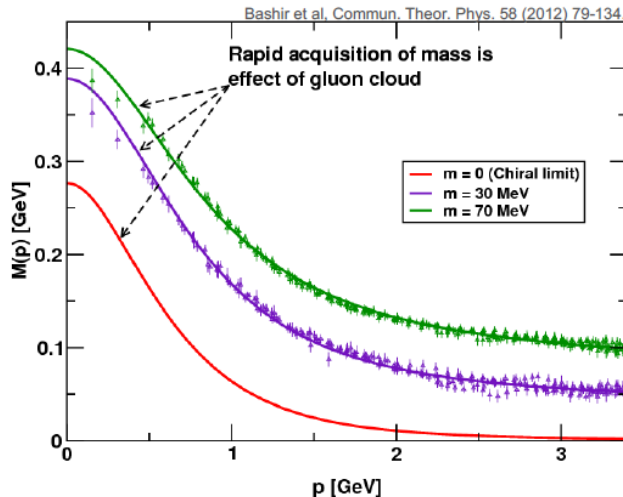
❖ DCSB: condensate vs dynamical breaking

$$\langle 0 | \bar{q} q | 0 \rangle = \langle 0 | \bar{q}_L q_R + \bar{q}_R q_L | 0 \rangle \approx 5 \text{ fm}^{-3}$$

$$SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V$$



Higgs vs QCD masses



❖ Nucleon mass decomposition $M = \frac{1}{2M} \langle N(p) | T^\mu_\mu | N(p) \rangle$

$$M = \frac{1}{2M} \langle N(p) | \frac{\beta(\alpha_s)}{2\alpha_s} G^{a\mu\eta} G^a_{\mu\eta} | N(p) \rangle + \sum_{q=u,d,s} \gamma_{m_q} m_q \bar{q}q | N(p) \rangle + \frac{1}{2M} \langle N(p) | \sum_q m_q \bar{q}q | N(p) \rangle$$

- dynamically generated via mainly gluon self-interaction
- Higgs (current quark) mass

❖ Heavy flavor hadrons: heavy quark symmetry \rightarrow HQEF

Experimental mass splittings between 1^- and 0^- mesons

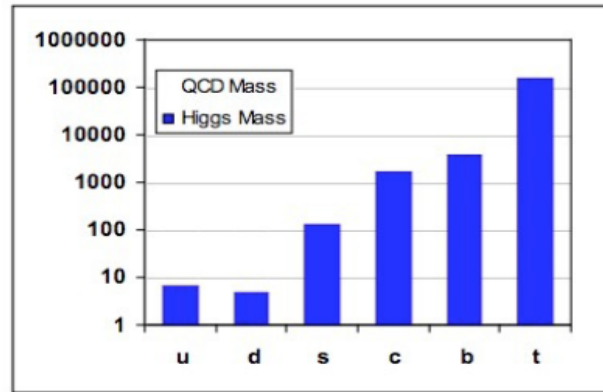
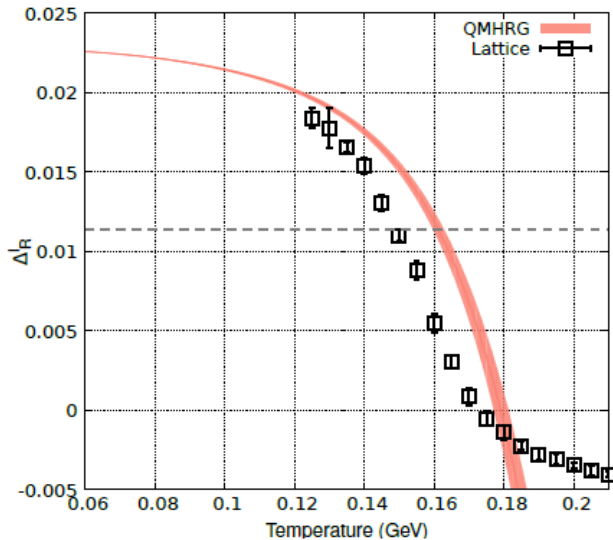
$$M_H = m_Q + \Lambda_H + O(1/m_Q) + \dots$$

$$\Lambda_H = \frac{1}{2M_H} \left\langle H \left| \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \right| H \right\rangle$$

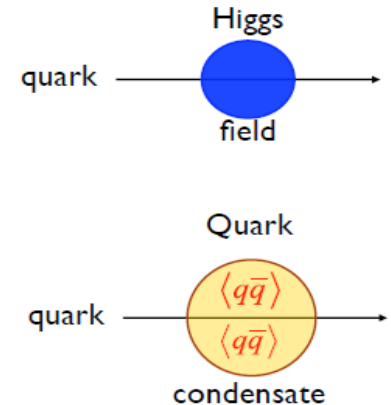
- Hyperfine/spin $\propto 1/m_Q$ splitting small

		Δ (MeV)
$M_{D^{*+}} - M_{D^+}$		140.64 ± 0.09
$M_{D^{*0}} - M_{D^0}$		142.12 ± 0.07
$M_{D_s^{*+}} - M_{D_s^+}$	$[Q \uparrow \downarrow \bar{q}]_{1S_0}$	141.6 ± 1.8
$M_{B^{*+}} - M_{B^+}$	$[Q \uparrow \uparrow \bar{q}]_{3S_1}$	46.0 ± 0.6
$M_{B_s^{*+}} - M_{B_s^+}$		47.0 ± 2.6

At high temperatures ...



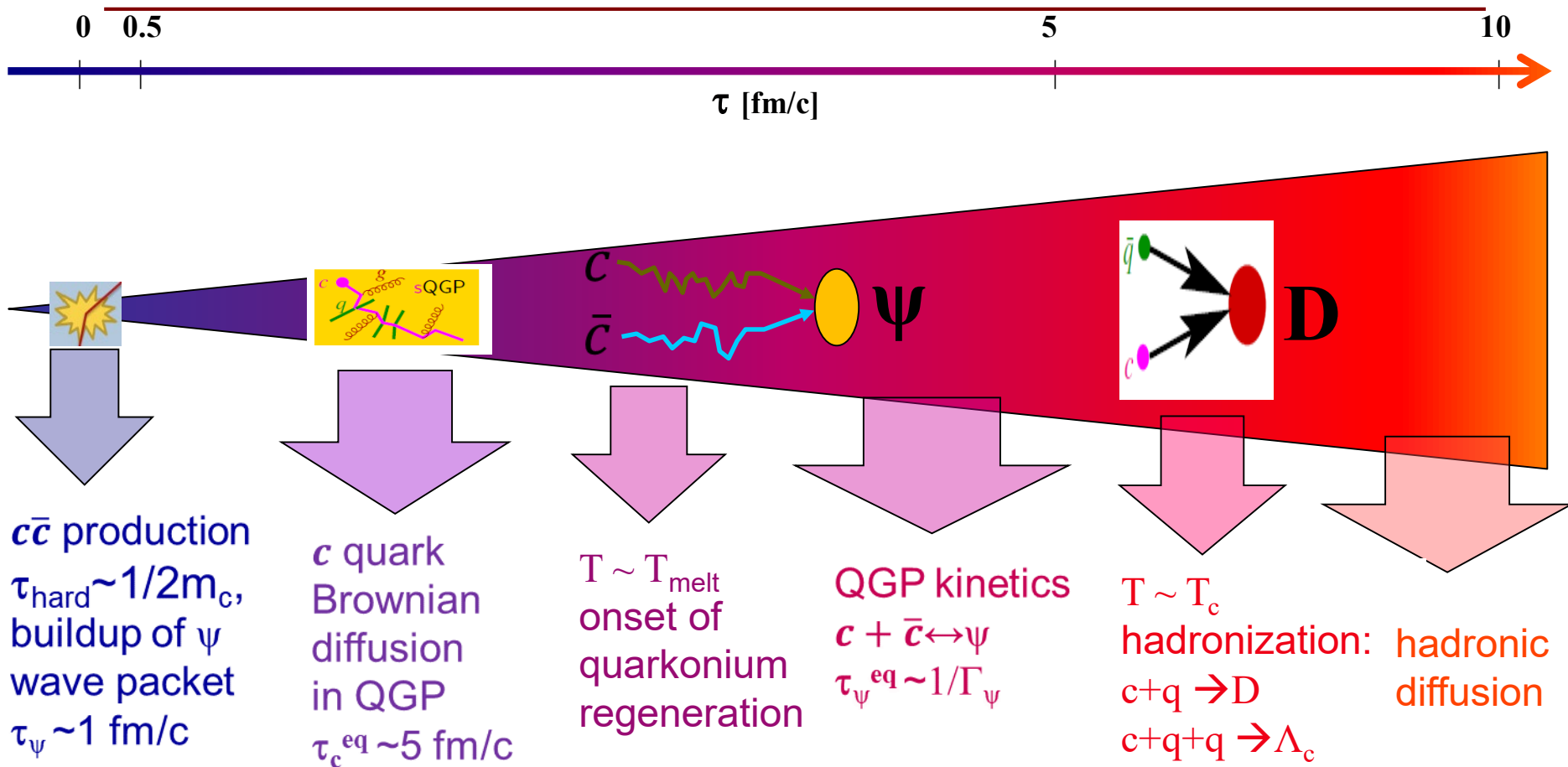
Heat “melts” the quark condensate:
QCD mass disappears above T_c .
(Partial) chiral symmetry restoration



❖ Unique role and advantage of HQs enabled by its large mass

- $m_{c,b} \gg \Lambda_{QCD}$ produced by pQCD processes (out of equil.)
- $\tau_0 \ll \tau_{QGP}$ they go through all the QGP lifetime
- $m_{c,b} \gg T_0$ no thermal production
- $\tau_{eq} > \tau_{QGP} \gg \tau_{q,g}$ carry more information
- $m \gg T \rightarrow q^2 \ll m^2$ transport reduced to Brownian motion
- $q_0 \ll |\vec{q}|$ Concept of potential $V(r) \leftrightarrow IQCD$

Heavy flavor transport as probes of QGP



Outline

Lecture I: Open heavy flavor production in pp

- pQCD production
- hadronization: empirical fragmentation functions
- SHM approach

Lecture II: Open heavy flavor probes of QGP

- HQ interactions in QGP: pQCD vs T-matrix
- diffusion: Langevin vs Boltzmann
- Hadronization: recombination vs fragmentation
- Phenomenology: R_{AA} & v_2 vs hadro-chemistry
- extracting $\mathcal{D}_s(2\pi T)$

Lecture III: Heavy quarkonium probes of QGP

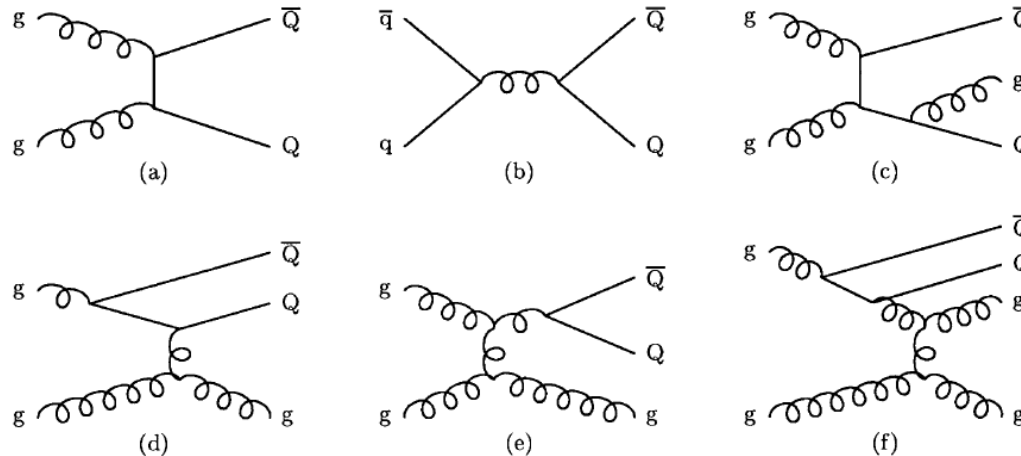
- Vacuum bound states, potential models
- HQ potential: remnants of confining force
- Reaction rates, transport approaches
- open quantum approach to Y states

Lecture I

Open heavy flavor production in pp collisions

- pQCD production
- hadronization: empirical fragmentation functions
- SHM approach

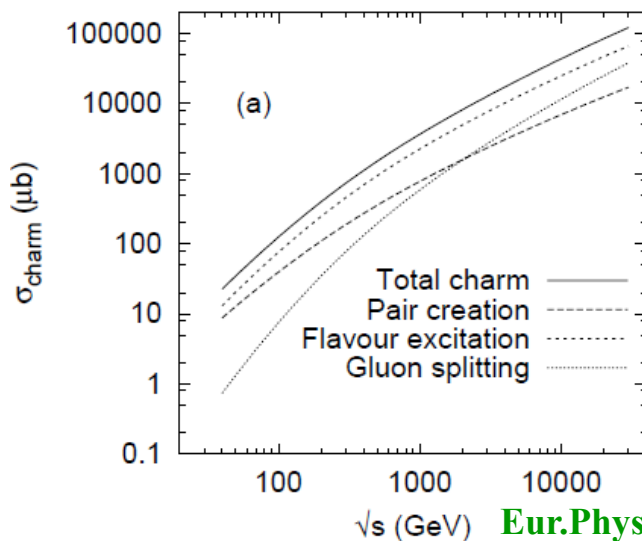
Heavy quark production: pQCD



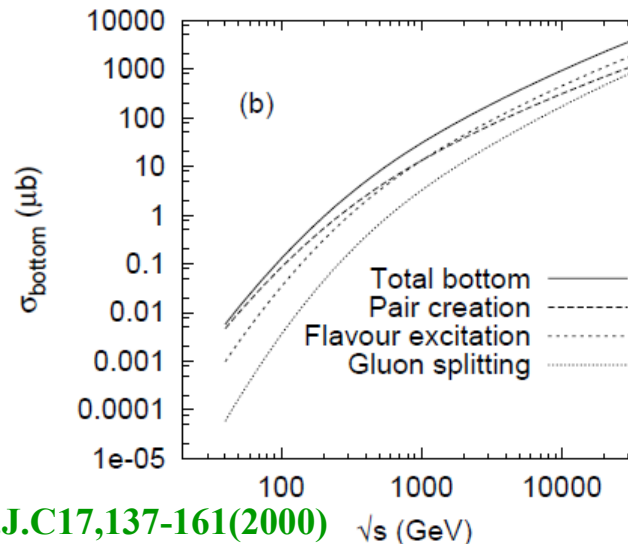
Norrbin and Sjostrand,
Eur.Phys.J.C17,137-161
(2000)

Combridge, NPB151
(1979) 429-456

Fig. 1a–f. Examples of heavy-flavor production diagrams. a,b Leading order. c Pair creation (with gluon emission). d Flavor excitation. e Gluon splitting. f Events classified as gluon splitting but of flavor-excitation character



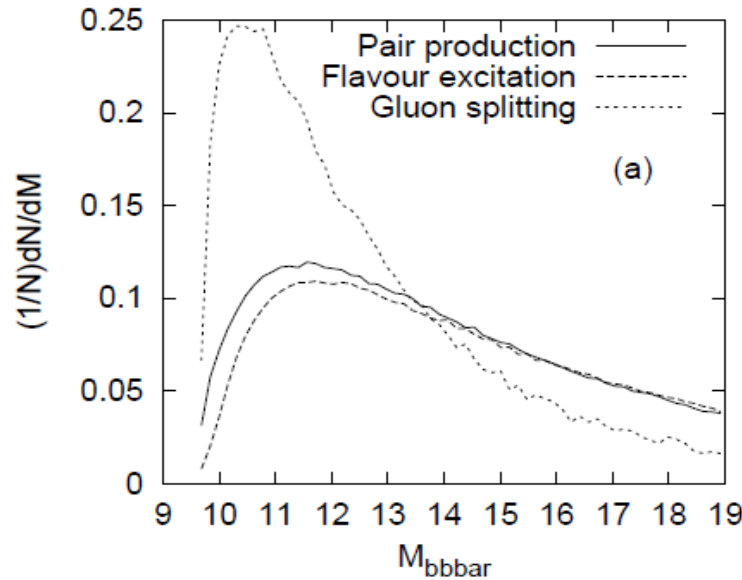
Eur.Phys.J.C17,137-161(2000)



- total cross sections in pp collisions: pair creation dominant at low root(s), flavor excitation important, gluon splitting comparable at high root(s)

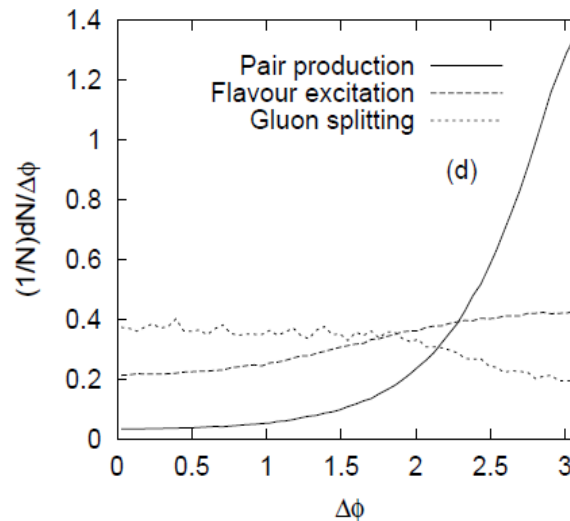
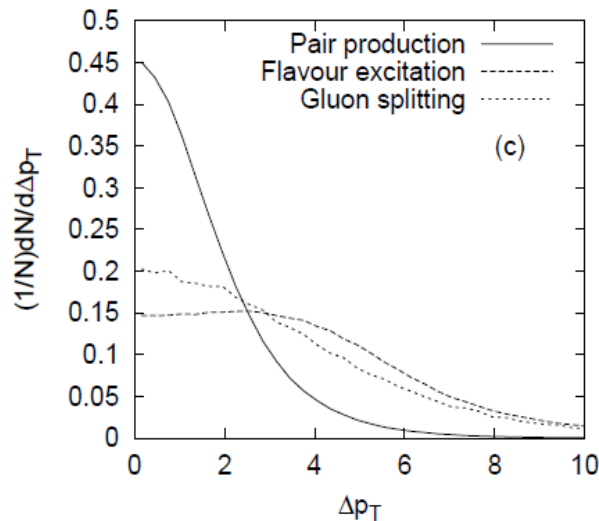
- no non-perturbative effects contributing to the total cross section

b-bbar correlations @ Tevatron



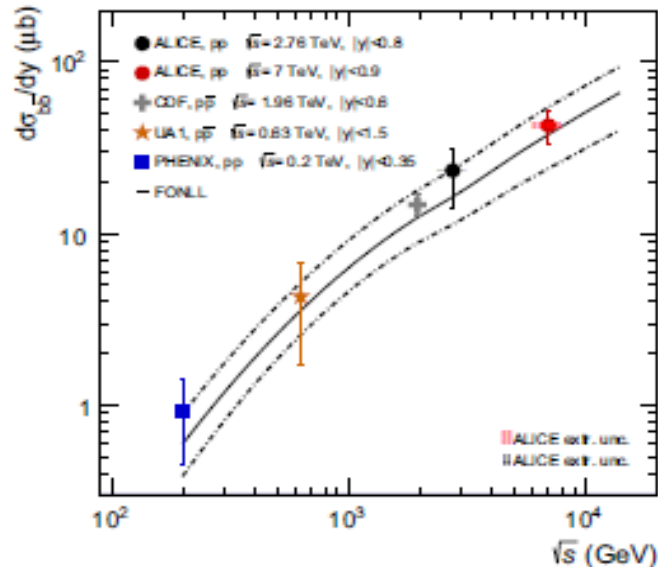
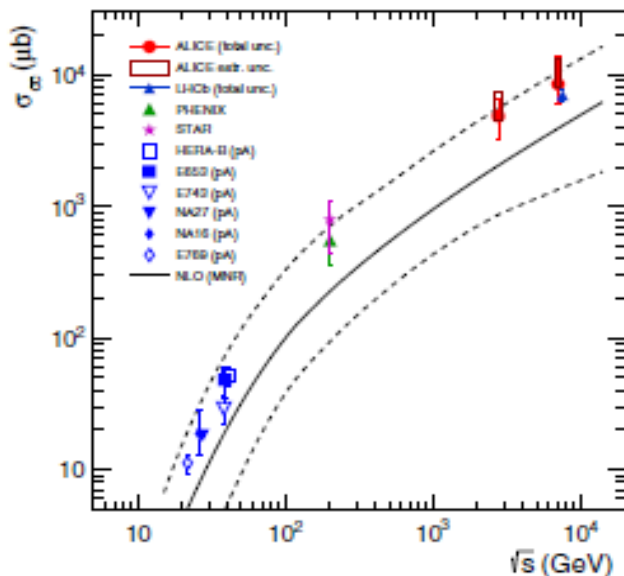
- pair invariant mass distribution:
gluon splitting \sim s-channel
gluon exchange $\rightarrow \sigma \sim 1/s$
suppressed at large $s \sim M_{bbbar}$
- pair creation & flavor excitation
 \sim t-channel contributions

Norrbin and Sjostrand,
Eur.Phys.J.C17,137-161 (2000)

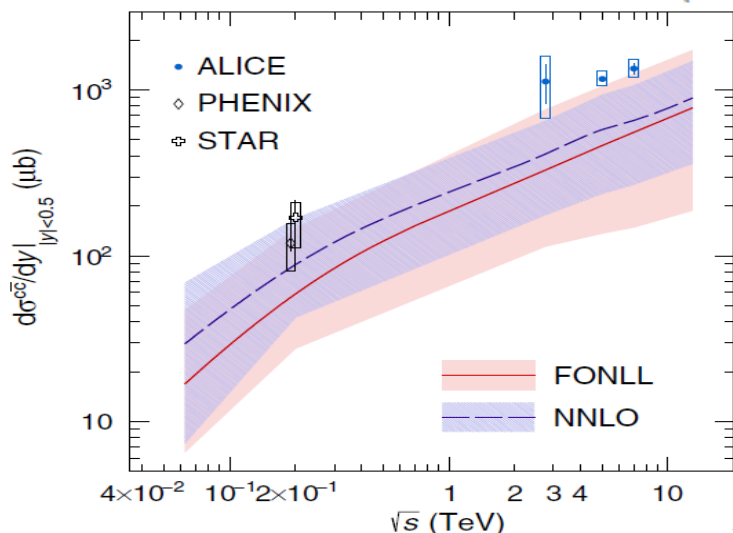


- LO pair creation
 \sim opposite & compensating
 $p_T \rightarrow$ peaked at $\Delta p_T = 0$
- gluon-splitting b-bbar
 \sim collinear \rightarrow peaked
at $\Delta\phi = 0$

Measurements of Q-Qbar cross sections



Andronic et al.,
Eur.Phys.J.C
(2016) 76:107



- ALICE, 5.02 TeV pp, $d\sigma^{ccbar}/dy \sim 1\text{mb}$ at $y=0$, significantly higher than before \rightarrow impact on modelling of charmonium transport in QGP [direct measurement of all charm hadrons]
- ALICE, 7 TeV pp, $d\sigma^{bbbar}/dy \sim 57\text{ mub}$ at $y=0$ [indirect measurements of HF muons]
- LHCb, 7 TeV pp, $d\sigma^{bbbar}/dy \sim 27\text{ mub}$ at $y=2-2.5$ [indirect measurements of HF muons]

ALICE, PRD105, L011103 (2022)

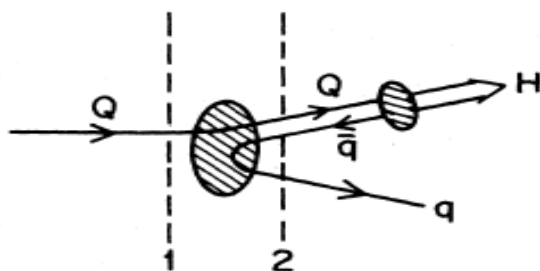
Heavy quark fragmentation

❖ From HQ to heavy hadron (pp): non-perturbative fragmentation

$$\frac{d\sigma^{H_c}}{dp_T^{H_c}}(p_T; \mu_F, \mu_R) = \underbrace{\text{PDF}(x_1, \mu_F) \cdot \text{PDF}(x_2, \mu_F)}_{\text{Parton distribution functions (PDFs)}} \otimes \underbrace{\frac{d\sigma^c}{dp_T^c}(x_1, x_2, \mu_R, \mu_F)}_{\text{Hard scattering cross section (pQCD)}} \otimes \underbrace{D_{c \rightarrow H_c}(z = p_{H_c}/p_c, \mu_F)}_{\text{Fragmentation function (hadronization)}}$$

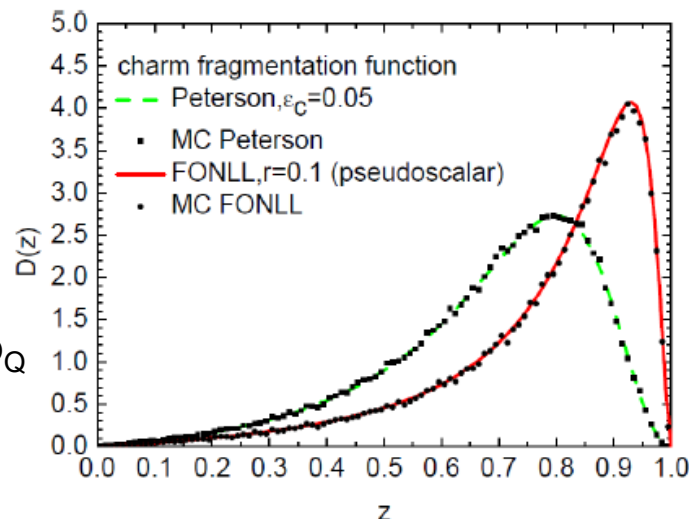
- HQ large mass $\gg \Lambda_{\text{QCD}} \rightarrow$ hadronization & production well separated \rightarrow factorization

❖ Fragmentation functions, e.g. Peterson FF [Peterson et al., PRD27,105 \(1983\)](#)



$$D_Q^H(z) = \frac{N}{z[1 - (1/z) - \epsilon_Q/(1-z)]^2} \quad \text{with } z = p_H/p_Q$$

$$\sum \int dz D_Q^H(z) = 1$$



Heavy quark FF fitted to e^+e^-

❖ In e^+e^- collisions, there's no initial pdf. E.g. for $e^+e^- \rightarrow (\gamma, Z) \rightarrow D^{*\pm} + X$

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma(e^+e^- \rightarrow D^{*\pm} + X)}{dx} = \frac{1}{\sigma_{\text{tot}}} \sum_a \int_x^1 \frac{dz}{z} D_a\left(\frac{x}{z}, M_f^2\right) \frac{d\sigma_a}{dz}(z, \mu^2, M_f^2)$$

- $x=E(D^*)/(\sqrt{s}/2)=2E(D^*)/\sqrt{s}$ **Kniesl et al., Z.Phys.C677(1997); PRD71, 094013 (2005)**
- Parameterizing the FF at starting scale $M_f=\mu$, then evolving the FF with DGLAP eq. to $M_f=M_Z$, such that it fits the e^+e^- data when convoluted with hard-scattering $d\sigma_a/dz$

X_c	Order	Q	N	α	β	ϵ
D^0	LO	c	0.998	0.163
		b	71.8	1.65	5.19	...
	NLO	c	1.16	0.203
		b	97.5	1.71	5.88	...
D^+	LO	c	0.340	0.148
		b	48.5	2.16	5.38	...
	NLO	c	0.398	0.187
		b	64.9	2.20	6.04	...
D_s^+	LO	c	0.0704	0.0578
		b	40.0	2.05	4.93	...
	NLO	c	0.0888	0.0854
		b	21.8	1.64	4.71	...
Λ_c^+	LO	c	0.0118	0.0115
		b	44.1	1.97	6.33	...
	NLO	c	0.0175	0.0218
		b	27.3	1.66	6.24	...

Heavy quark FF fitted to e^+e^- (continued)

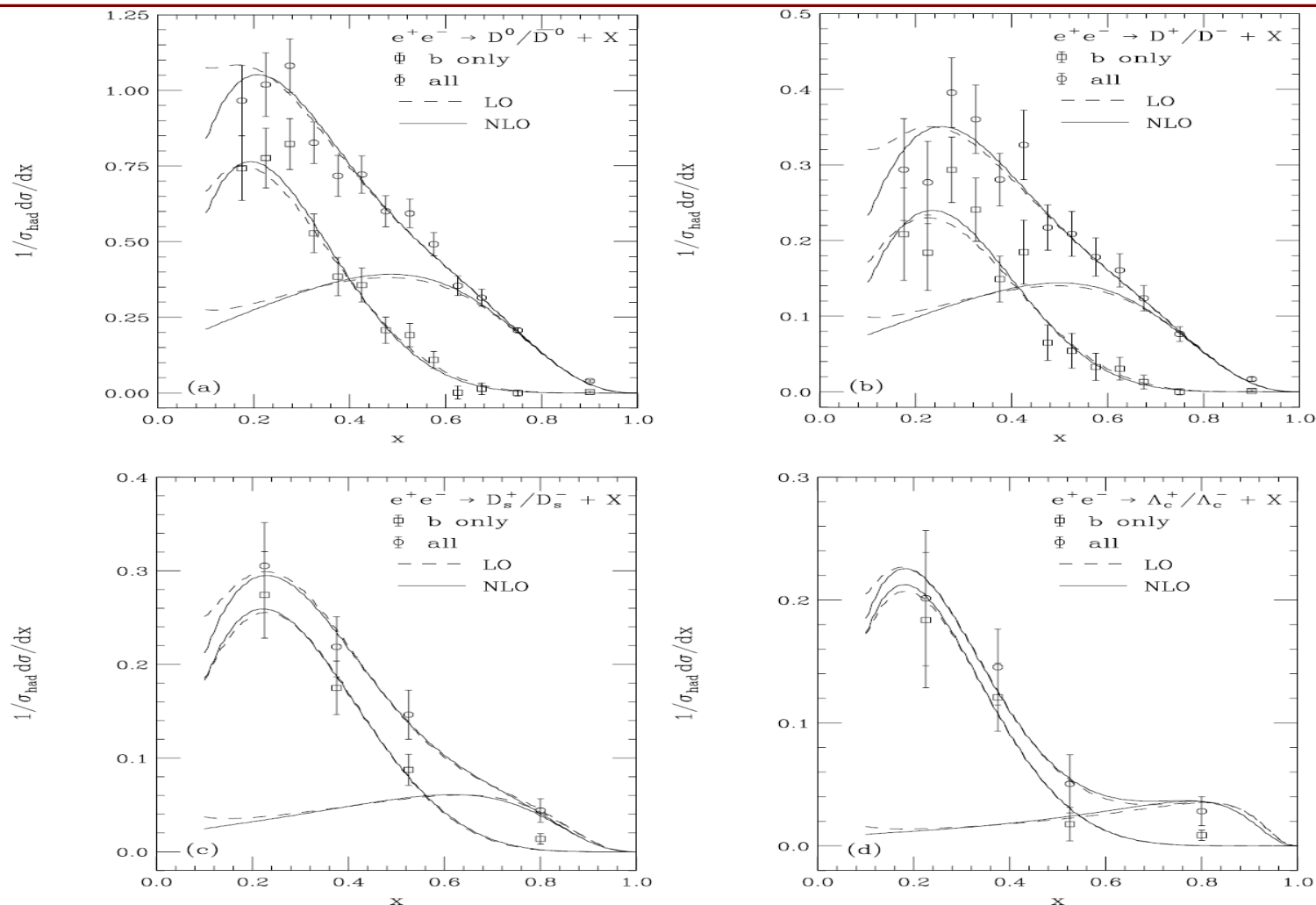
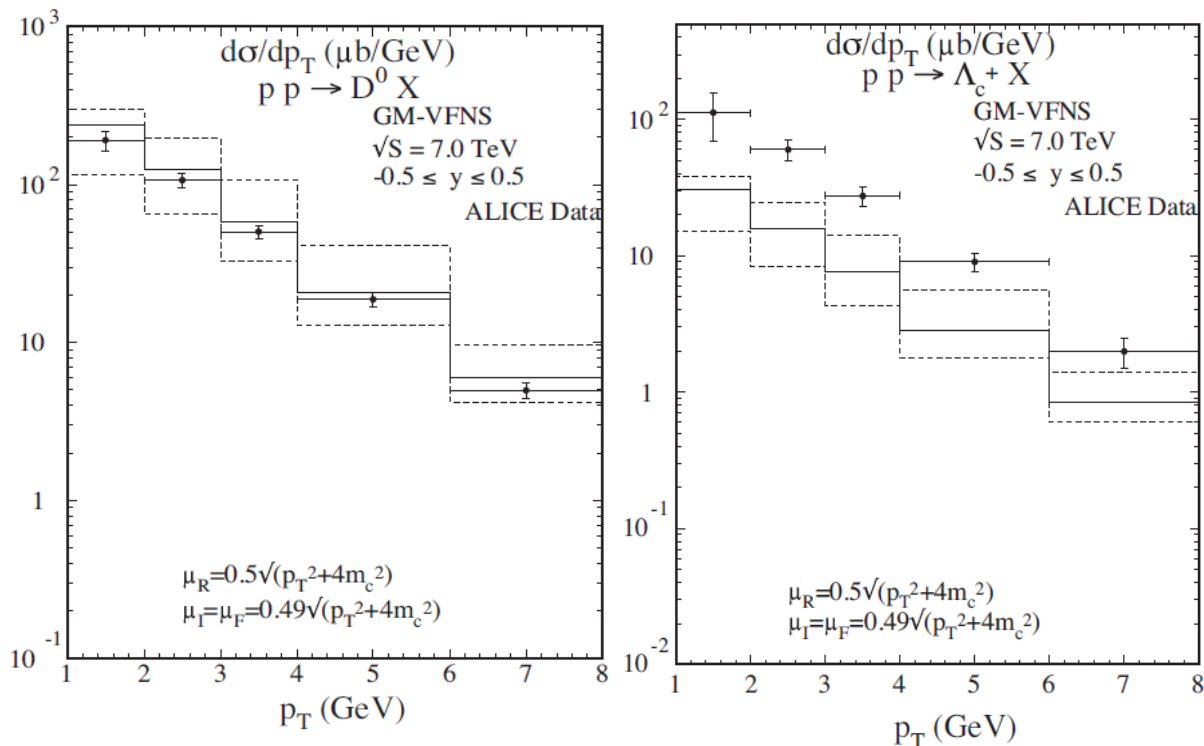


FIG. 1. The normalized differential cross sections ($1/\sigma_{\text{tot}} d\sigma/dx$) of inclusive (a) D^0/\bar{D}^0 , (b) D^\pm , (c) D_s^\pm , and (d) $\Lambda_c^\pm/\bar{\Lambda}_c^\pm$ production in e^+e^- annihilation on the Z-boson resonance evaluated at LO (dashed lines) and NLO (solid lines) with our respective FF sets are compared with the OPAL data [8] renormalized as explained in the text (circles). The same is also done for the $Z \rightarrow b\bar{b}$ subsamples (squares). In addition, our LO and NLO fit results for the $Z \rightarrow c\bar{c}$ contributions are shown. In each case, the X_c hadron and its charge-conjugate counterpart are summed over.

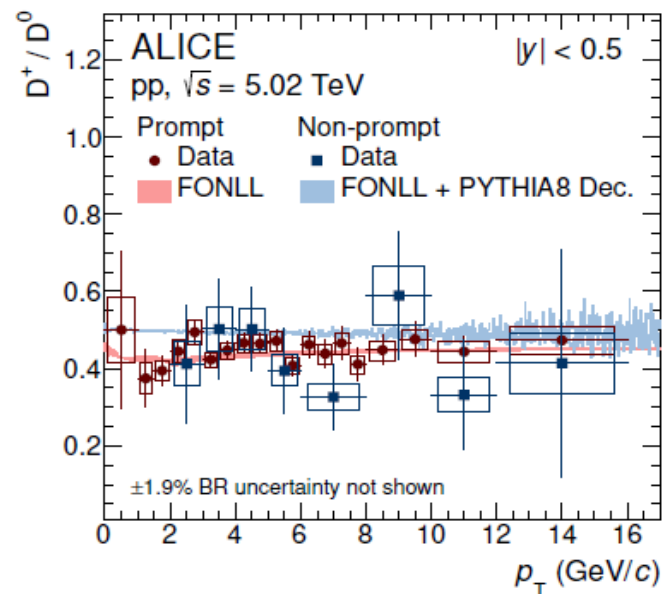
FFs universal from e^+e^- to pp?

- ❖ phenomenological FF: constrained by e^+e^- & usually assumed universal



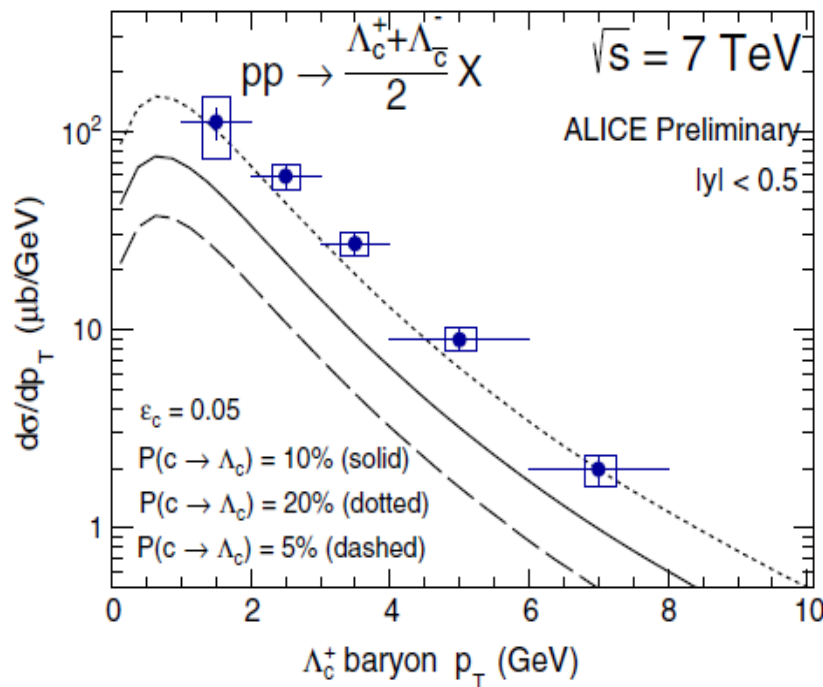
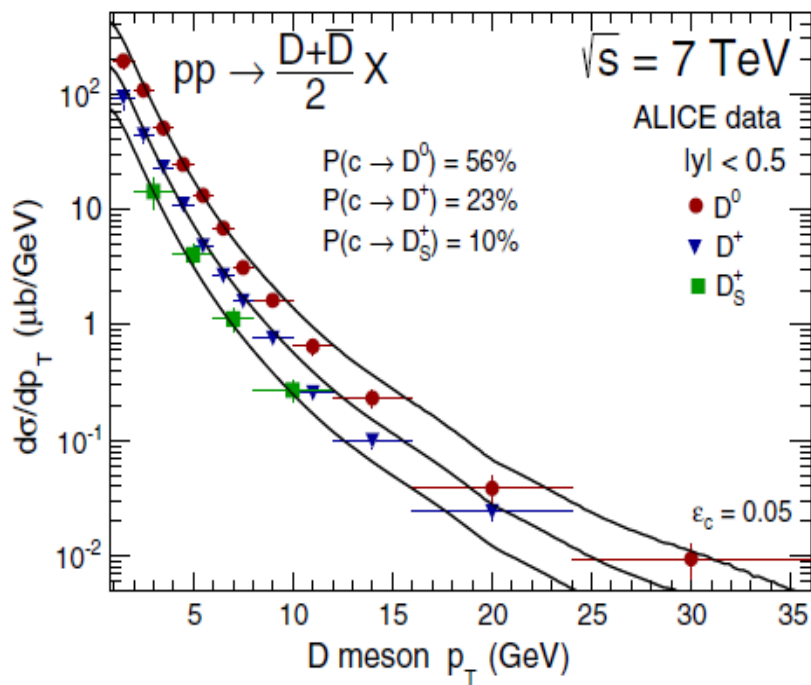
- FF extracted from e^+e^- applied to pp: GM-VFNS scheme
 D^0 -meson well reproduced but Λ_c much underestimated
Kniesl et al., PRD101, 114021 (2020)

- FF extracted from e^+e^- applied to pp: FONLL scheme, D^+/D^0 reproduced



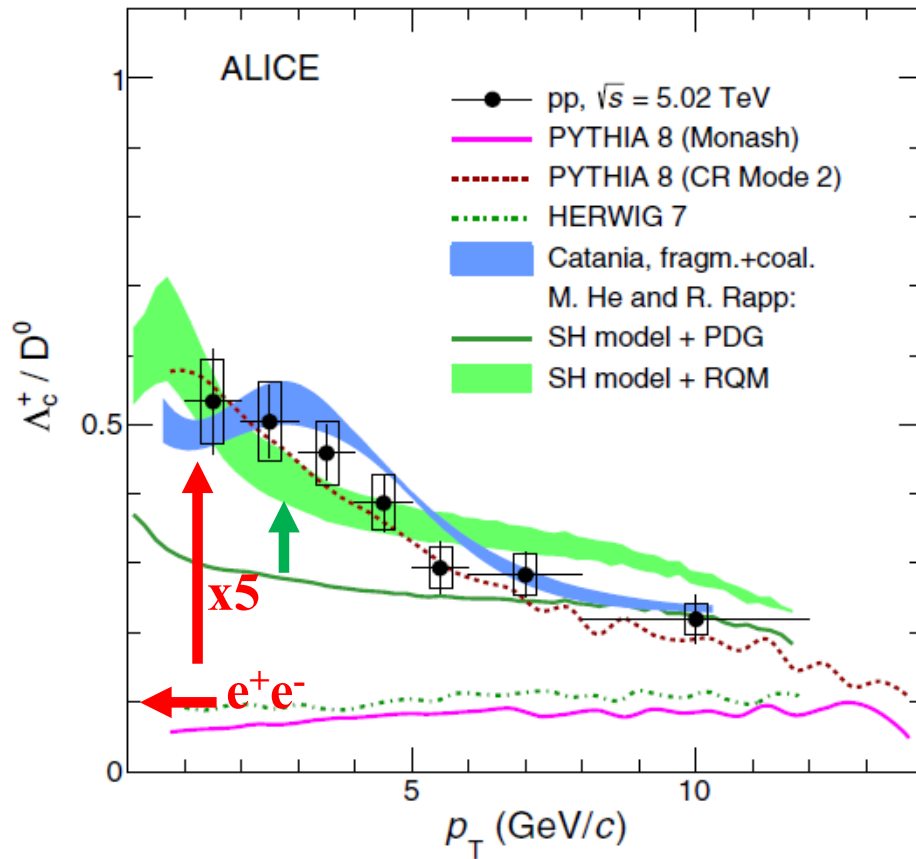
FFs universal from e^+e^- to pp?

- ❖ phenomenological FF: constrained by e^+e^- & usually assumed universal



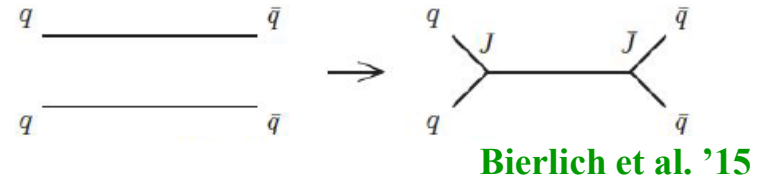
- FF extracted from e^+e^- applied to pp: k_T -factorization scheme
D-mesons well reproduced but Λ_c much underestimated
Maciula and Szczurek., PRD98, 014016 (2018)
- new production channel for Λ_c baryons in high-energy pp collisions ?

Λ_c^+/D^0 @ 5 TeV pp collisions



ALICE, PRL 127, 202301(2021)
ALICE, PRC 104, 054905(2021)

- ❖ PYTHIA8: Color-reconnection with junctions frag. into baryons

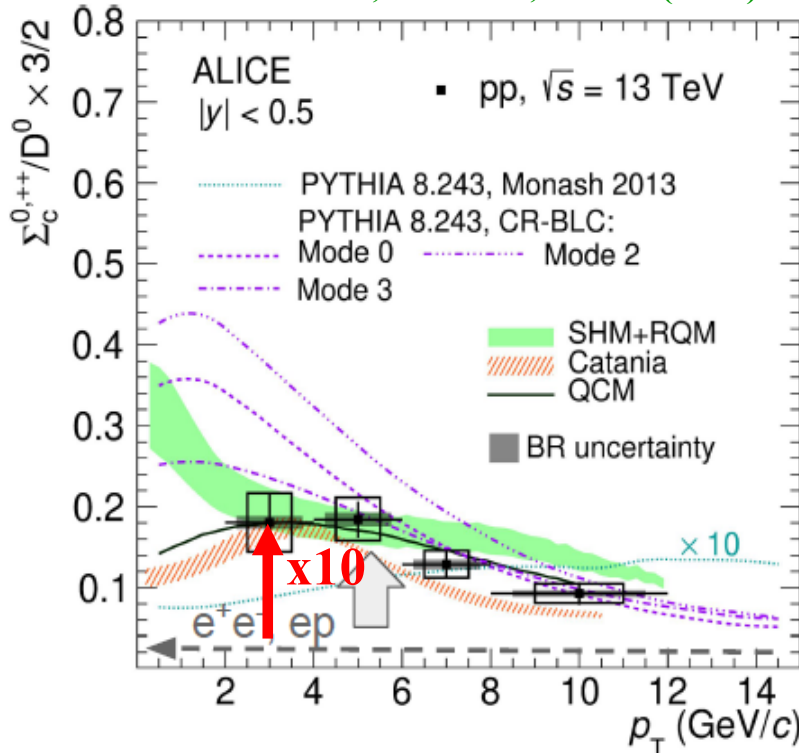


- ❖ Catania: c-q(-q) coalescence in a small QGP fireball Minissale et al. '21
- ❖ Statistical hadronization in q-rich environment (unlike e⁺e⁻)
 - augmented by “missing” charm-baryons assuming *relative* chemical equilibrium

MH & Rapp '19

Σ_c/D^0 & Ξ_c/D^0

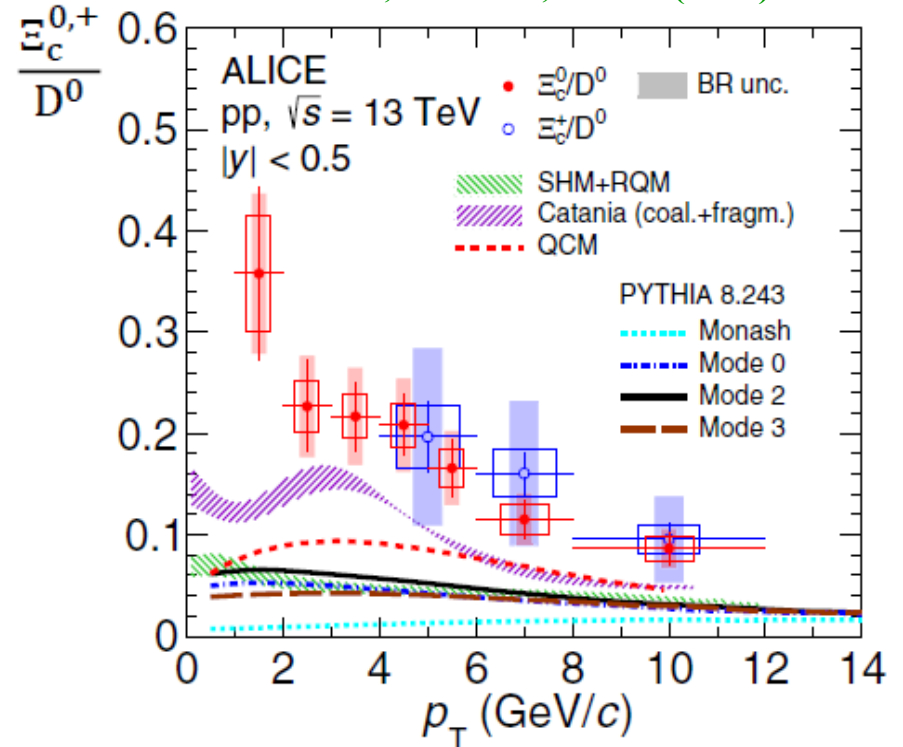
ALICE, PRL 128, 012001(2022)



- ❖ $\Sigma_c/D^0 \times 10$ enhanced despite more massive spin-1 diquark



ALICE, PRL 127, 272001(2021)

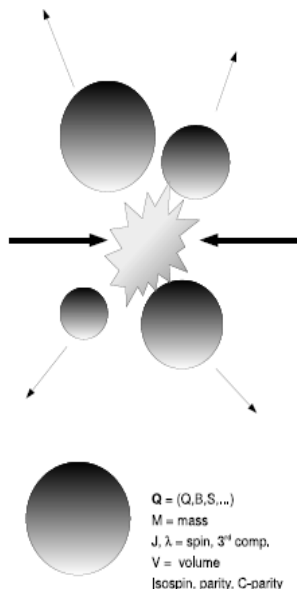


- ❖ Ξ_c/D^0 ratio underestimated by all models



Statistical Hadronization of HQs

❖ Statistical hadronization model



- Thermodynamic equilibrium = filling accessible phase space uniformly
P. B.-Munzinger et al., nucl-th/0304013
F. Becattini, 0901.3643
- High-energy collisions → clusters/fireballs at hadronization → collectively equivalent global cluster (EGC): colorless & endowed with conserved Abelian charges
- Occurring at a critical energy density [microscopic canonical ensemble] → replaced by a hadronization temperature $T_H \sim 160-170$ MeV

Figure 1: High energy collisions are assumed to give rise to multiple clusters at the hadronization stage [top]. Each cluster [bottom] is a colorless extended massive object endowed with abelian charges (electric, strange, baryonic etc.), intrinsic angular momentum and other quantum numbers such as parity, C -parity and isospin.

- SHM applies also to elementary collisions vs heavy-ion collisions (where multiparton scatterings leading to kinetic thermalization)
F. Becattini, Eur. Phys. J. C (2008) 56: 493–510

Hadrons born into equilibrium?!

Eur. Phys. J. C 52, 187–201 (2007)
DOI 10.1140/epjc/s10052-007-0368-6

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

Thermal hadronization and Hawking–Unruh radiation in QCD

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Abstract. We conjecture that, because of color confinement, the physical vacuum forms an event horizon for quarks and gluons, which can be crossed only by quantum tunneling, i.e., through the QCD counterpart of Hawking radiation at black holes. Since such radiation cannot transmit information to the outside, it must be thermal, of a temperature determined by the chromodynamic force at the confinement surface, and it must maintain color neutrality. We explore the possibility that the resulting process provides a common mechanism for thermal hadron production in high energy interactions, from e^+e^- annihilation to heavy ion collisions.

PACS. 04.70.Dy; 12.38.Aw; 12.38.Mh; 12.40.Ee; 25.75.Nq; 97.60.Lf

- Hadron thermodynamic state is not reached by dynamical equilibration among constituents (partons or hadrons), ➔
but rather a generic fingerprint of hadronization or a feature of QCD vacuum

Grand-canonical SHM in HIC

- ❖ CG-SHM: charges are not exactly conserved but only on average by fugacities/chemical potentials

$$Z^{GC}(T, V, \mu_Q) = \text{Tr}[e^{-\beta(H - \sum_i \mu_{Q_i} Q_i)}], \quad \ln Z(T, V, \vec{\mu}) = \sum_i \ln Z_i(T, V, \vec{\mu})$$

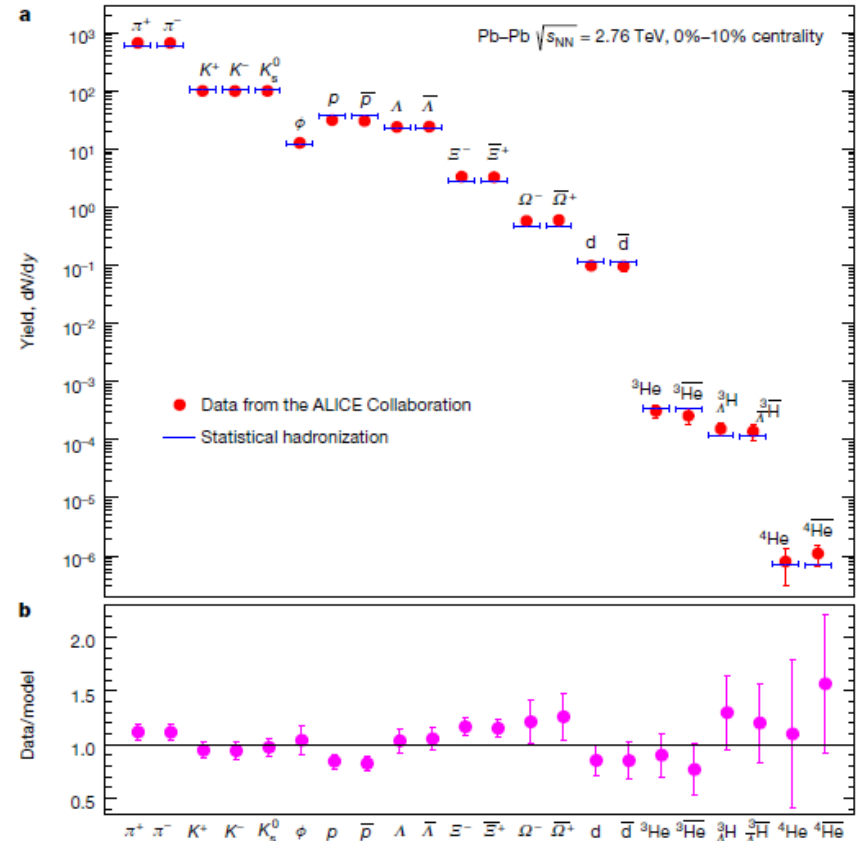
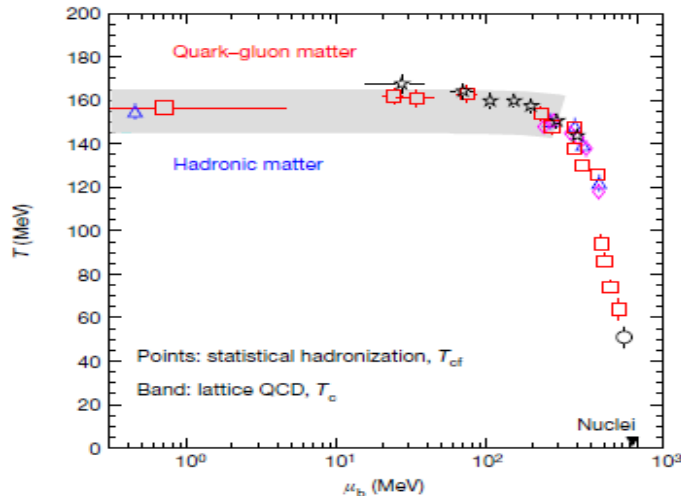
A. Andronic et al., Nature, 561, 321 (2018)

$$\ln Z_i(T, V, \vec{\mu}) = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln[1 \pm \lambda_i \exp(-\beta \epsilon_i)],$$

$$= \frac{VT g_i}{2\pi^2} \sum_{k=1}^\infty \frac{(\pm 1)^{k+1}}{k^2} \lambda_i^k m_i^2 K_2\left(\frac{km_i}{T}\right)$$

$$\lambda_i(T, \vec{\mu}) = \exp\left(\frac{B_i \mu_B + S_i \mu_S + Q_i \mu_Q}{T}\right)$$

$$n_i(T, \vec{\mu}) = \frac{\langle N_i \rangle}{V} = \frac{T g_i}{2\pi^2} \sum_{k=1}^\infty \frac{(\pm 1)^{k+1}}{k} \lambda_i^k m_i^2 K_2\left(\frac{km_i}{T}\right).$$



Augmented SHM for charm-hadrons in pp

- PDG: 5 Λ_c ($I=0$), 3 Σ_c ($I=1$), 8 Ξ_c ($I=1/2$), 2 Ω_c ($I=0$) **missing baryons?!**
 RQM: 18 extra Λ_c , 42 extra Σ_c , 62 extra Ξ_c , 34 extra Ω_c up to 3.5 GeV → supported by lattice [PRD 84 \(2011\) 014025](#); [PoS LAT. 2014 \(2015\) 084](#); [PLB 737 \(2014\) 210](#)

- Grand-canonical SHM density $T_H=170$ MeV
$$n_i = \frac{d_i}{2\pi^2} m_i^2 T_H K_2\left(\frac{m_i}{T_H}\right)$$

$n_i (\cdot 10^{-4} \text{ fm}^{-3})$	D^0	D^+	D^{*+}	D_s^+	Λ_c^+	$\Xi_c^{+,0}$	Ω_c^0
<u>PDG(170)</u>	1.161	0.5098	0.5010	0.3165	<u>0.3310</u>	0.0874	0.0064
<u>RQM(170)</u>	1.161	0.5098	0.5010	0.3165	<u>0.6613</u>	0.1173	0.0144

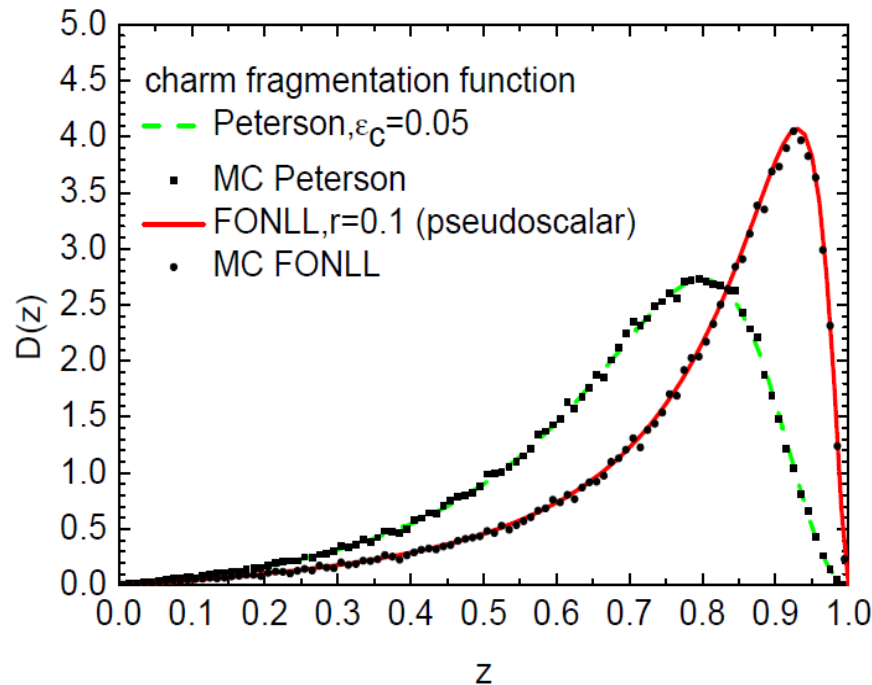
r_i	D^+/D^0	D^{*+}/D^0	D_s^+/D^0	Λ_c^+/D^0
<u>PDG(170)</u>	0.4391	0.4315	<u>0.2736</u>	<u>0.2851</u>
<u>RQM(170)</u>	0.4391	0.4315	<u>0.2726</u>	<u>0.5696</u>

MH, Rapp, PLB795 (2019) 117–121

- Here, strong feeddowns of excited states all included: BR=100% to Λ_c^+ for all Λ_c & Σ_c even above DN (2805 MeV) threshold
- Strangeness supp. $\gamma_s=0.6$ & charm fugacity $\gamma_c=1$

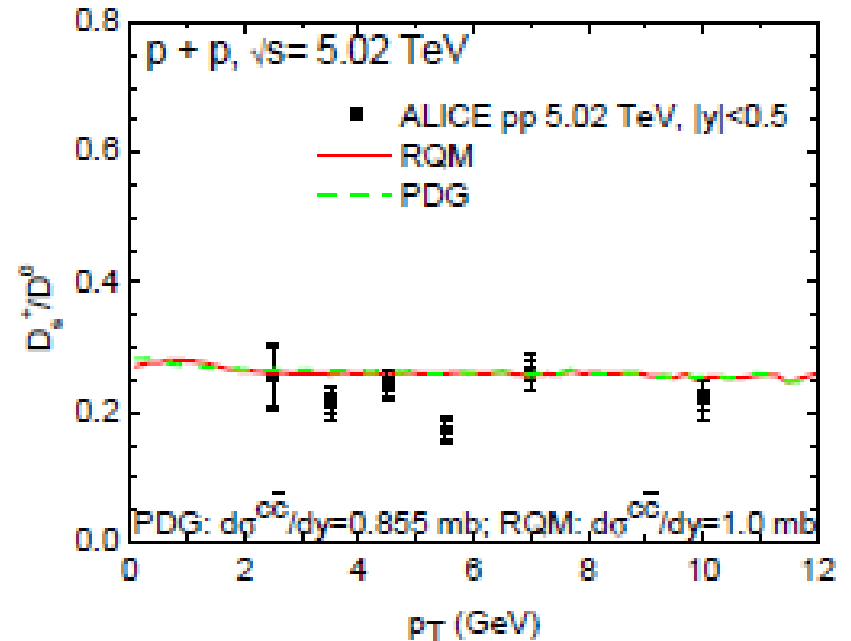
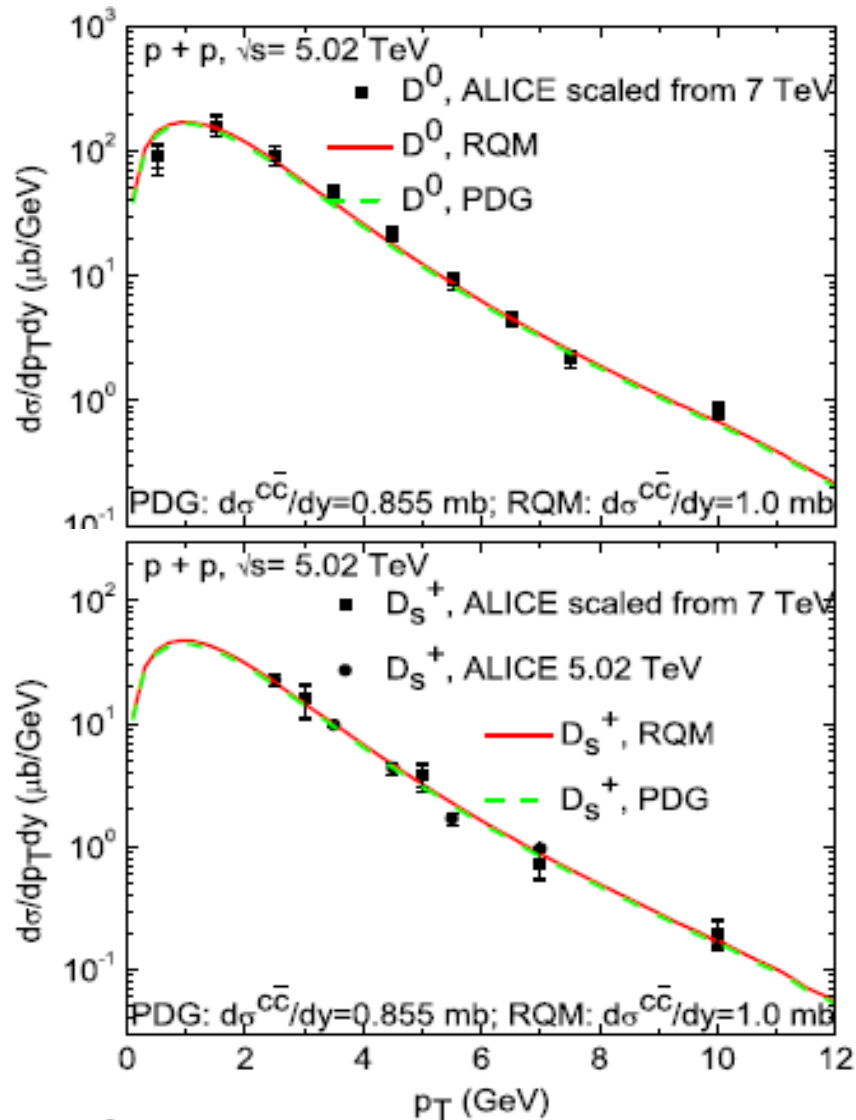
Fragmentation & decay simulations

- FONLL fragmentation of charm quarks into **all** kinds of charm-hadrons: **relative weight** \leftrightarrow SHM thermal densities



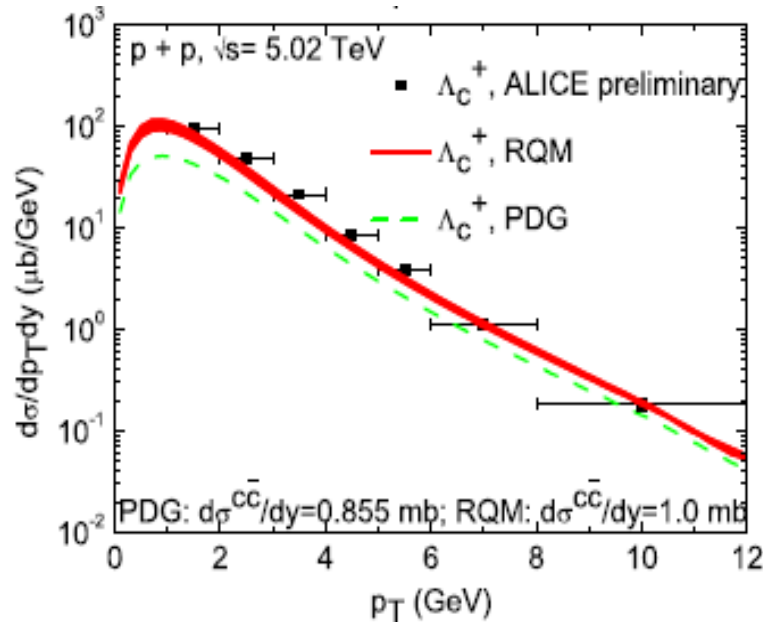
- Decay simulations of **all** excited states to ground states D^0 , D^+ , D_s^+ , Λ_C^+ , Ξ_C & Ω_C

Results: charm-mesons

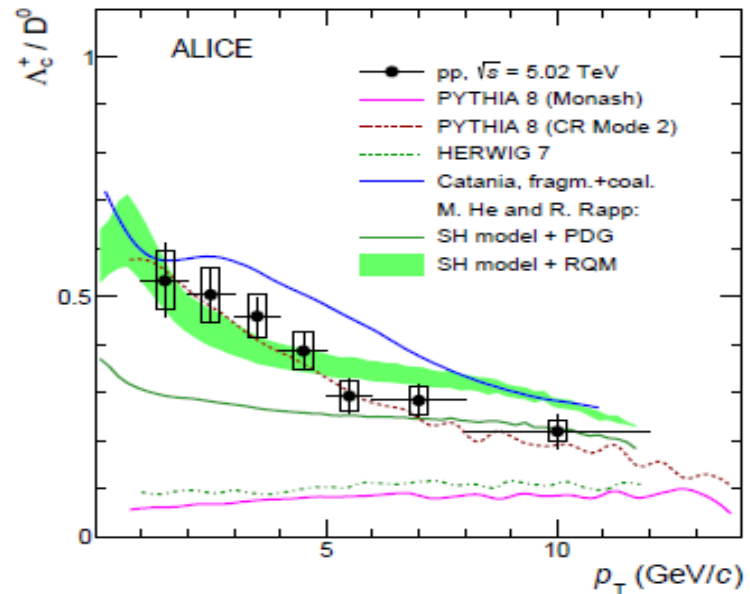


- both PDG & RQM work for charm-mesons
- but with different fitted charm $d\sigma/dy = 0.855$ vs 1.0 mb
 MH, Rapp, PLB795 (2019) 117–121

Results: charm-baryons

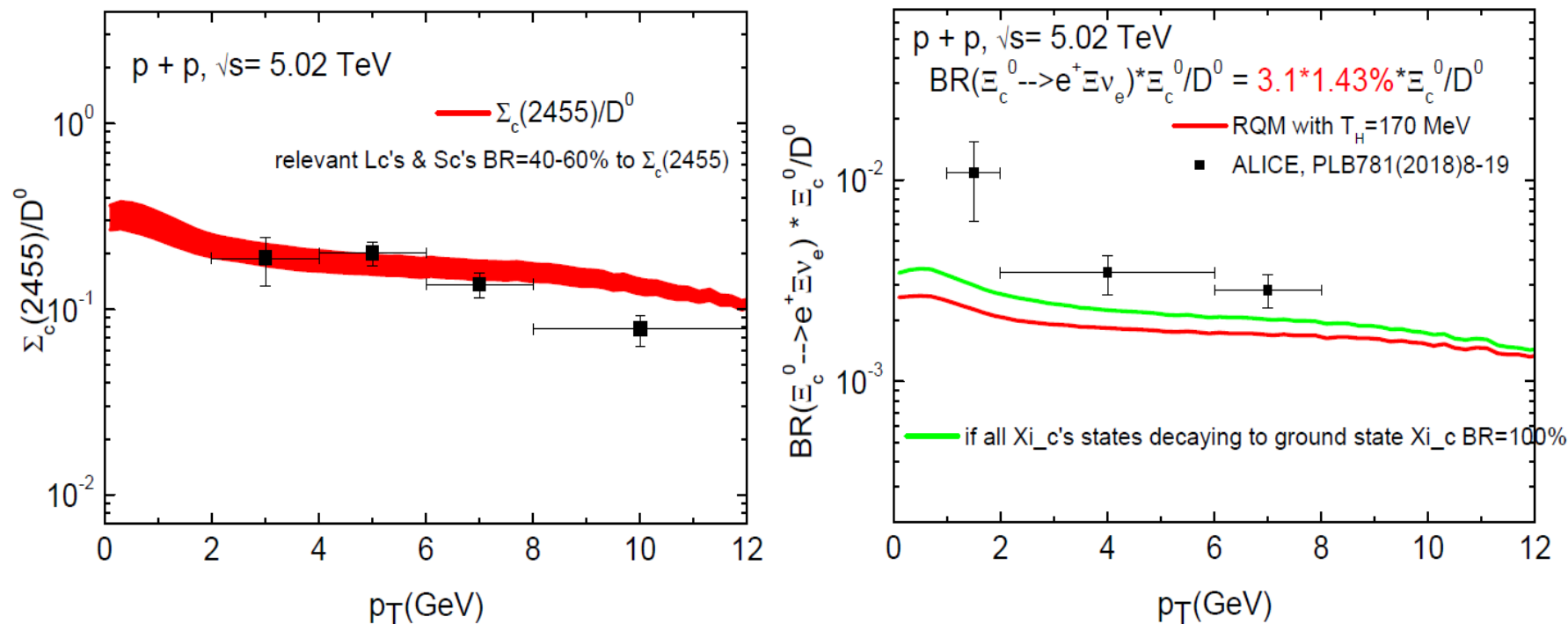


ALICE: PRL127 (2021) 20, 202301



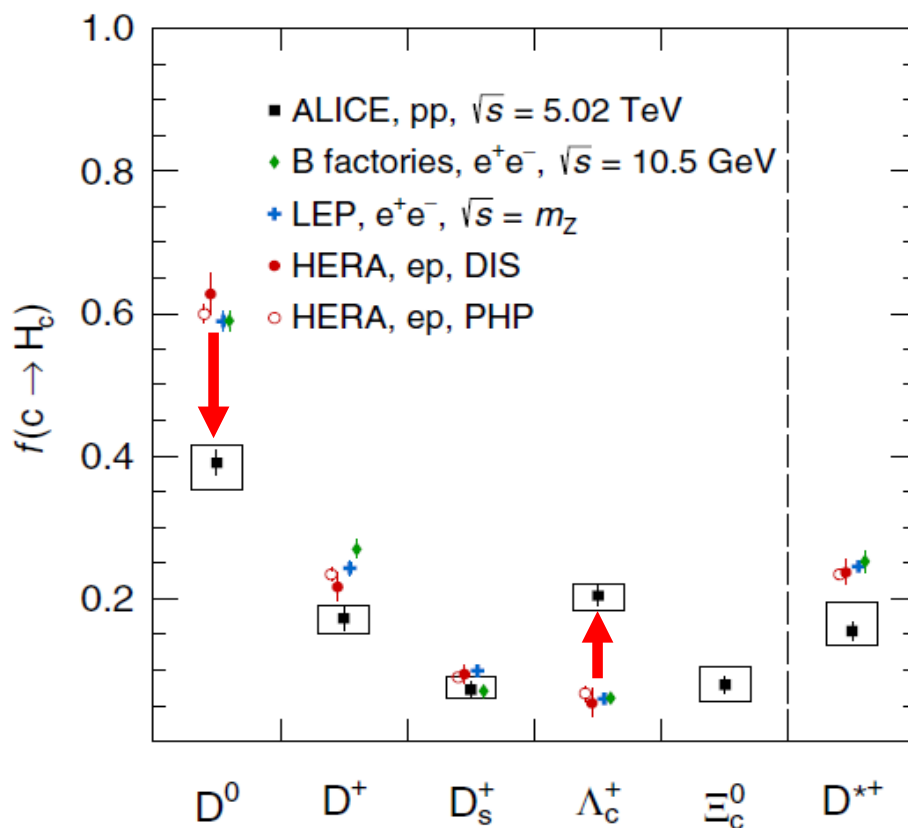
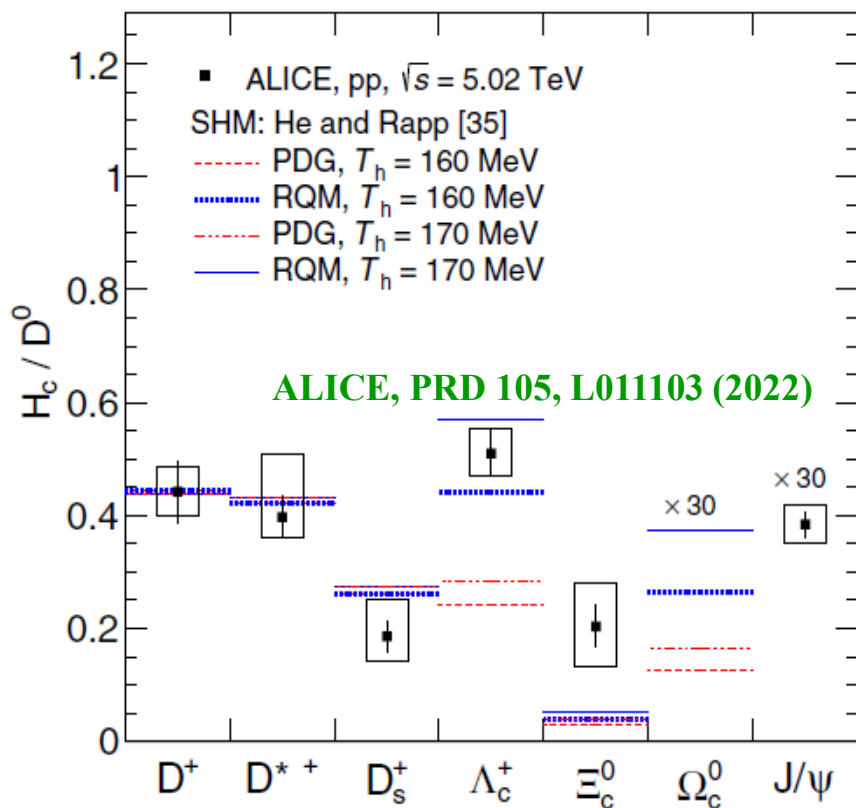
- Λ_C^+ favors RQM with $d\sigma/dy=1.0 \text{ mb}$: **low p_T enhancement** from feeddowns of RQM augmented baryons
- uncertainty band: BR=50%-100% to ground state Λ_C^+ for Λ_C 's & Σ_C 's above DN (2805 MeV) threshold

Results: charm-baryons (cont.)



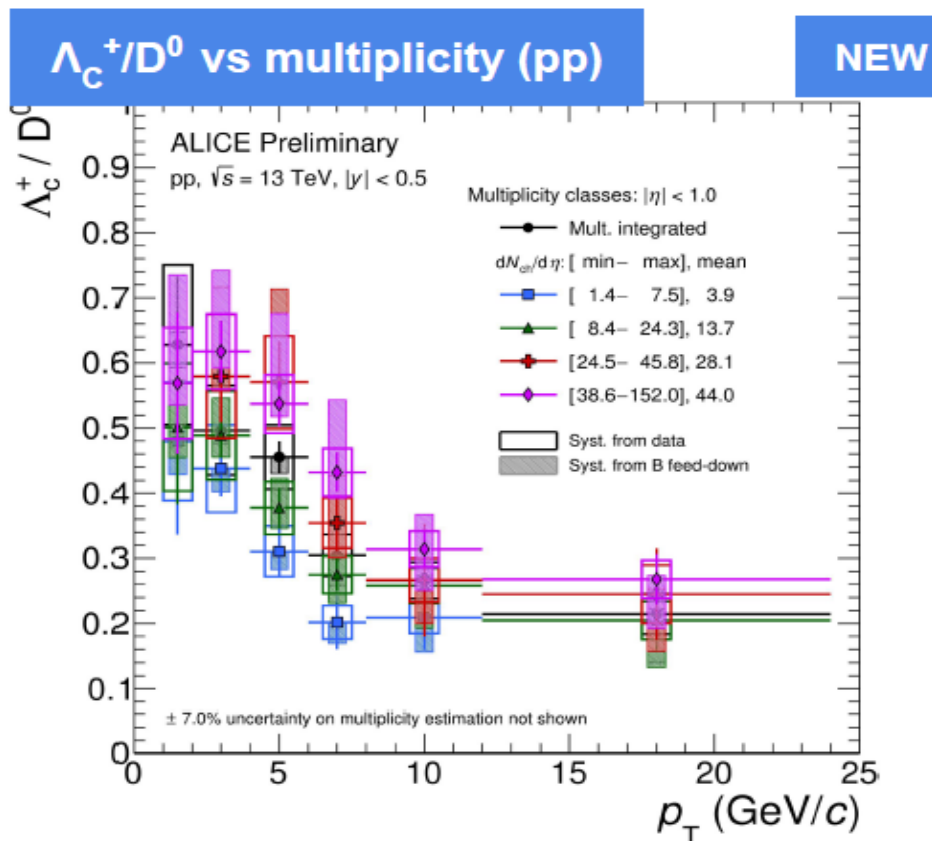
- $\Sigma_c(2455)/D^0$ can also be accounted for within uncertainties
- But Ξ_c^0/D^0 much underestimated, although already twice as large as PYTHIA8(CR) ~ 0.001

Charm-hadron fractions in 5 TeV pp



- ❖ charm quark fragmentation is **non-universal** from e^+e^- to pp
- ❖ charm content shuffled from meson (D^0) to baryon (Λ_c) sector
- ❖ full charm-hadrons measured, $d\sigma^{cc}/dy \sim 1.16$ mb at mid-y
 ➔ significant impact on charmonia production in HIC

$\Lambda_c^+/\text{D}^0 : \text{d}N_{\text{ch}}/\text{d}\eta$ dependence



- Significant enhancement with increasing charged-particle multiplicity
- Might be straightforwardly consistent with statistical coalescence:
 $\Lambda_c^+ \sim cqq$, $\text{D}^0 \sim cq \Rightarrow \Lambda_c^+/\text{D}^0 \sim q \sim \text{d}n_{\text{ch}}/\text{d}\eta$

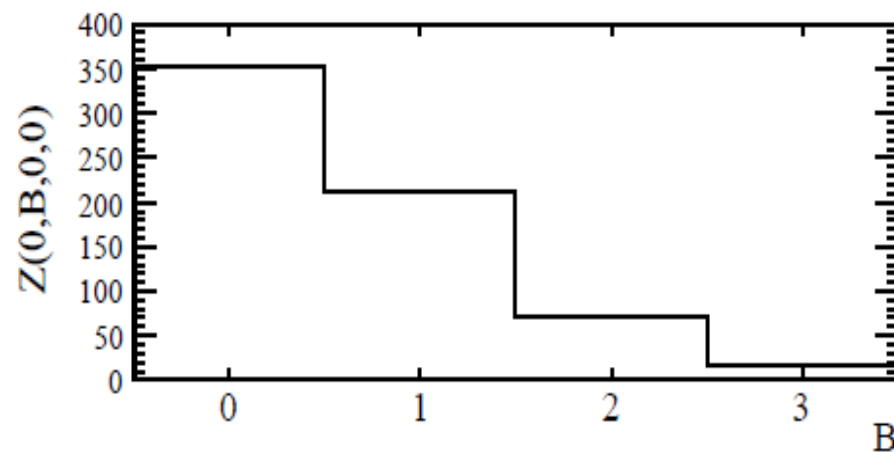
Canonical-ensemble SHM

- Canonical partition function conserving charge $\vec{Q} = (Q, B, S, C)$

$$Z(\vec{Q}) = \frac{1}{(2\pi)^4} \int_0^{2\pi} d^4\phi e^{i\vec{Q} \cdot \vec{\phi}} \exp\left[\sum_i \gamma_s^{N_{sj}} \gamma_c^{N_{cj}} e^{-i\vec{q}_j \cdot \vec{\phi}} z_j\right].$$

where $z_j = (2J_j + 1) \frac{VT_H}{2\pi^2} m_j^2 K_2\left(\frac{m_j}{T_H}\right)$

- Pair creation → energy-expensive:
once a baryon created, simultaneous
creation of an antibaryon is required
to fulfill baryon-number conservation



- Hadron mean number

$$\langle N_j \rangle^{CE} = \langle N_j \rangle^{GCE} \frac{Z(\vec{Q} - \vec{q}_j)}{Z(\vec{Q})}$$

Y. Chen and MH, PLB 815 (2021) 136144

chemical factor
characterizing canonical
suppression for charged
hadron with $\vec{q}_j \neq 0$

Chemical factors: canonical suppression

CF	$V=10 \text{ fm}^3$	20	50	100	200
D^0	0.025877	0.066239	0.190294	0.373107	0.627886
D^+	0.025439	0.065891	0.190002	0.372841	0.627669
D_s^+	0.015805	0.053178	0.178586	0.362376	0.619125
Λ_c^+	0.016956	0.055485	0.182039	0.365923	0.622147
Ξ_c^{+0}	0.009884	0.042956	0.167943	0.352535	0.611073
Ω_c	0.003495	0.022604	0.130312	0.312514	0.576383
Λ_c^+ / D^0	0.655254	0.837649	0.956620	0.980745	0.990860
D_s^+ / D^0	0.610774	0.802820	0.938474	0.971239	0.986047

pp mid-rapidity neutral (Q,B,S,C)=(0,0,0,0) $T_H = 170 \text{ MeV}$, $\gamma_s = 0.6$, $\gamma_c = 15$

- Column** at given V : CF progressively smaller for hadrons containing more charges (S, B, BS, BSS)
- Row**: CF increases with volume, tending to the **common** residual canonical charm supp. (canonical B/S supp. diminishing)
- Relative CF**: Λ_c^+ / D^0 & D_s^+ / D^0 increases with volume toward unity

CE-SHM densities with feeddowns

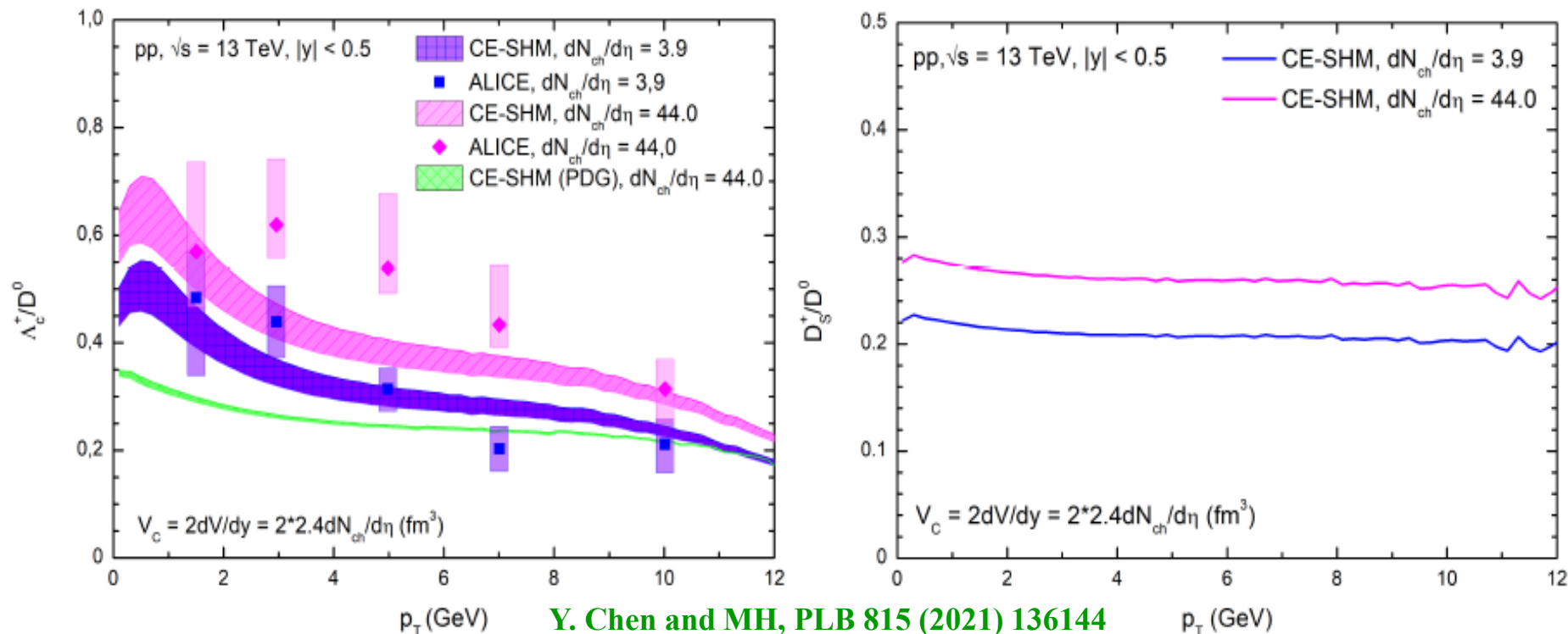
n_j ($\cdot 10^{-4} \text{fm}^{-3}$)	$V=10 \text{ fm}^3$	20	50	100	200	GCE
D^0	0.445553	1.148287	3.310131	6.495330	10.934662	17.420949
D^+	0.194705	0.503847	1.453016	2.851351	4.800262	7.647869
D_s^+	0.075040	0.252484	0.847910	1.720531	2.939551	4.747914
Λ_c^+ (BR50%)	0.126963	0.439135	1.497132	3.045487	5.207572	8.415360
Λ_c^+ (BR100%)	0.149573	0.519555	1.776775	3.617118	6.187127	10.001702
Ξ_c^{+0}	0.016539	0.071955	0.281624	0.591389	1.025276	1.678110
Ω_c	0.000756	0.004889	0.028184	0.067592	0.124662	0.216283
Λ_c^+ / D^0 (BR50%)	0.284956	0.382426	0.452288	0.468873	0.476244	0.483060
Λ_c^+ / D^0 (BR100%)	0.335702	0.452461	0.536769	0.556880	0.565827	0.574119
D_s^+ / D^0	0.168420	0.219879	0.256156	0.264887	0.268829	0.272540

pp mid-rapidity neutral (Q,B,S,C)=(0,0,0,0)

$T_H = 170 \text{ MeV}$, $\gamma_s = 0.6$, $\gamma_c = 15$

- Row: density of each charm-hadron increases with volume
- Λ_c^+ / D^0 & D_s / D^0 : marked system-size dependence: a $\sim 40\%$ reduction from $V=200$ (\sim GCE-SHM) to $V=10 \text{ fm}^3$

Fragmentation & decay: p_T -dependent ratios



- Splitting of Λ_c^+/D^0 between $dN_{ch}/d\eta=3.9$ vs 44.0 roughly reproduced, due to **additional canonical baryon supp. on charm-baryons**, which becomes stronger toward smaller system-size
- Similar splitting of D_s^+/D^0 by additional canonical strangeness supp.

Augmented SHM for bottom-hadrons in pp

- “Missing” bottom baryons: Ebert et al., PRD 84 (2011) 014025





PDG: 5 B, 4 B_s, 3 Λ_b , 2 Σ_b , 4 Ξ_b , 1 Ω_b

RQM: 25 B, 20 B_s, 30 Λ_b , 46 Σ_b , 75 Ξ_b , 42 Ω_b

- Relativistic quark model (RQM): Q-light diquark bound states

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_{d,B}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M) \Psi_{d,B}(\mathbf{q}) \quad \text{Ebert et al., PRD 84 (2011) 014025}$$

TABLE I. Masses M and form factor parameters of diquarks. S and A denote scalar and axial-vector diquarks which are anti-symmetric $[\cdots]$ and symmetric $\{\cdots\}$ in flavor, respectively [4].

	Quark content	Diquark type	M (MeV)	ξ (GeV)	ζ (GeV ²)
Λ_b 	$[u, d]$	S	710	1.09	0.185
Σ_b 	$\{u, d\}$	A	909	1.185	0.365
Ξ_b 	$[u, s]$	S	948	1.23	0.225
	$\{u, s\}$	A	1069	1.15	0.325
Ω_b 	$\{s, s\}$	A	1203	1.13	0.280

PDG vs RQM heavy baryons (Λ_Q & Σ_Q)

TABLE II. Masses of the Λ_Q ($Q = c, b$) heavy baryons (in MeV).

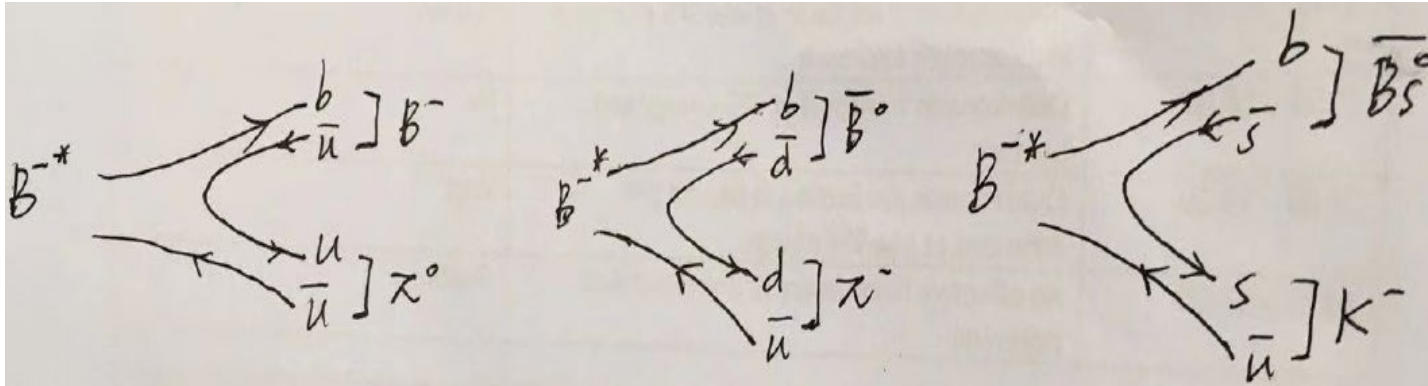
$I(J^P)$	Qd state	$Q = c$		$Q = b$	
		M	$M^{\text{exp}} [1]$	M	$M^{\text{exp}} [1]$
$0(\frac{1}{2}^+)$	$1S$	2286	2286.46(14)	5620	5620.2(1.6)
$0(\frac{1}{2}^+)$	$2S$	2769	2766.6(2.4)?	6089	
$0(\frac{1}{2}^+)$	$3S$	3130		6455	
$0(\frac{1}{2}^+)$	$4S$	3437		6756	
$0(\frac{1}{2}^+)$	$5S$	3715		7015	
$0(\frac{1}{2}^+)$	$6S$	3973		7256	
$0(\frac{1}{2}^-)$	$1P$	2598	2595.4(6)	5930	
$0(\frac{1}{2}^-)$	$2P$	2983	2939.3($\frac{1}{1,5}$)?	6326	
$0(\frac{1}{2}^-)$	$3P$	3303		6645	
$0(\frac{1}{2}^-)$	$4P$	3588		6917	
$0(\frac{1}{2}^-)$	$5P$	3852		7157	
$0(\frac{3}{2}^-)$	$1P$	2627	2628.1(6)	5942	
$0(\frac{3}{2}^-)$	$2P$	3005		6333	
$0(\frac{3}{2}^-)$	$3P$	3322		6651	
$0(\frac{3}{2}^-)$	$4P$	3606		6922	
$0(\frac{3}{2}^-)$	$5P$	3869		7171	
$0(\frac{3}{2}^+)$	$1D$	2874		6190	
$0(\frac{3}{2}^+)$	$2D$	3189		6526	
$0(\frac{3}{2}^+)$	$3D$	3480		6811	
$0(\frac{3}{2}^+)$	$4D$	3747		7060	
$0(\frac{5}{2}^+)$	$1D$	2880	2881.53(35)	6196	
$0(\frac{5}{2}^+)$	$2D$	3209		6531	
$0(\frac{5}{2}^+)$	$3D$	3500		6814	

TABLE III. Masses of the Σ_Q ($Q = c, b$) heavy baryons (in MeV).

$I(J^P)$	Qd state	$Q = c$		$Q = b$	
		M	$M^{\text{exp}} [1]$	M	$M^{\text{exp}} [1]$
$1(\frac{1}{2}^+)$	$1S$	2443	2453.76(18)	5808	5807.8(2.7)
$1(\frac{1}{2}^+)$	$2S$	2901		6213	
$1(\frac{1}{2}^+)$	$3S$	3271		6575	
$1(\frac{1}{2}^+)$	$4S$	3581		6869	
$1(\frac{1}{2}^+)$	$5S$	3861		7124	
$1(\frac{3}{2}^+)$	$1S$	2519	2518.0(5)	5834	5829.0(3.4)
$1(\frac{3}{2}^+)$	$2S$	2936	2939.3($\frac{1}{1,5}$)?	6226	
$1(\frac{3}{2}^+)$	$3S$	3293		6583	
$1(\frac{3}{2}^+)$	$4S$	3598		6876	
$1(\frac{3}{2}^+)$	$5S$	3873		7129	
$1(\frac{1}{2}^-)$	$1P$	2799	2802($\frac{4}{7}$)	6101	
$1(\frac{1}{2}^-)$	$2P$	3172		6440	
$1(\frac{1}{2}^-)$	$3P$	3488		6756	
$1(\frac{1}{2}^-)$	$4P$	3770		7024	
$1(\frac{1}{2}^-)$	$1P$	2713		6095	
$1(\frac{1}{2}^-)$	$2P$	3125		6430	
$1(\frac{1}{2}^-)$	$3P$	3455		6742	
$1(\frac{1}{2}^-)$	$4P$	3743		7008	
$1(\frac{3}{2}^-)$	$1P$	2798	2802($\frac{4}{7}$)	6096	
$1(\frac{3}{2}^-)$	$2P$	3172		6430	
$1(\frac{3}{2}^-)$	$3P$	3486		6742	
$1(\frac{3}{2}^-)$	$4P$	3768		7009	
$1(\frac{5}{2}^-)$	$1P$	2773	2766.6(2.4)?	6087	
$1(\frac{5}{2}^-)$	$2P$	3151		6423	
$1(\frac{5}{2}^-)$	$3P$	3469		6736	
$1(\frac{5}{2}^-)$	$4P$	3753		7003	
$1(\frac{5}{2}^-)$	$1P$	2789		6084	

Strong decay systematics: BR's estimation

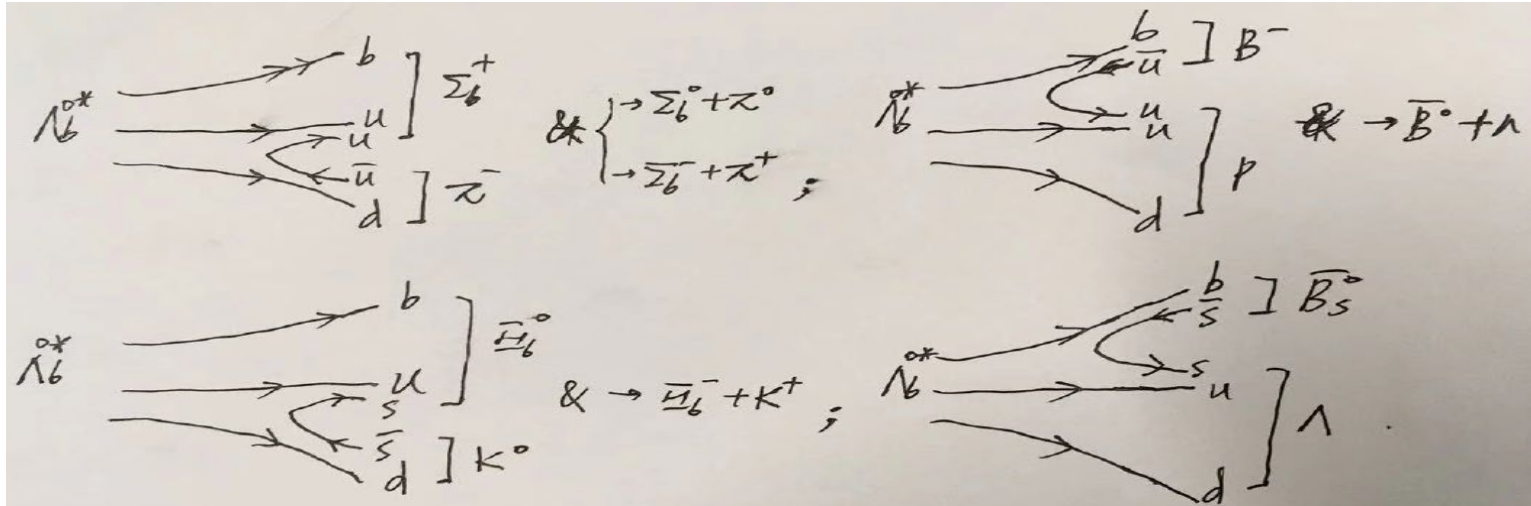
- Counting all possible diagrams once above the threshold



- Probability of producing a q - q bar pair $\propto \exp(-2m/T_H)$
 - ➔ $\exp(-2m_q/T_H) : \exp(-2m_s/T_H) = 1 : 1/3$ [$m_q \sim 8$, $m_s \sim 100$ MeV]
 - ➔ diagrams involving s - s bar counted as $1/3$
- E.g. $\text{BR}(B^{*-} \rightarrow B^- + \pi^0) = 1/(1+1+1/3) = 43\%$
 $\text{BR}(B^{*-} \rightarrow \bar{B}^0 + \pi^-) = 1/(1+1+1/3) = 43\%$
 $\text{BR}(B^{*-} \rightarrow \bar{B}_s^0 + \pi^0) = 1/3/(1+1+1/3) = 14\%$

Strong decay systematics: BR's estimation

- Counting all possible diagrams once above the mass threshold



- E.g. $\text{BR}(\Lambda_b^{0*} \rightarrow \Sigma_b + \pi^- \rightarrow \Lambda_b^0 + 2\pi) = 3/(3+2+2*1/3+1/3) = 54\%$
 $\text{BR}(\Lambda_b^{0*} \rightarrow B^- + p) = 1/(3+2+1/3+2*1/3) = 16\%$
 $\text{BR}(\Lambda_b^{0*} \rightarrow \Xi_b + K) = 2/3/(3+2+1/3+2*1/3) = 11\%$
 $\text{BR}(\Lambda_b^{0*} \rightarrow B_s^0 \text{bar} + \Lambda) = 1/3/(3+2+1/3+2*1/3) = 6\%$
- Results comparable to (limited) results computed in 3P_0 model
Ferretti et al., PRD 97114020 (2018); Yu et al., 2206.08128

Augmented SHM for bottom hadrons in pp

➤ Grand-canonical SHM densities $n_i^{\text{primary}} = \frac{d_i}{2\pi^2} \gamma_s^{N_s^i} m_i^2 T_H K_2\left(\frac{m_i}{T_H}\right)$

➔ Weakly decaying b-hadrons
(ground states)

- strangeness supp. $\gamma_s=0.6$
bottom fugacity $\gamma_b=1$

$$n_\alpha = n_\alpha^{\text{primary}} + \sum n_i^{\text{primary}} \cdot BR(i \rightarrow \alpha)$$

n_α ($\cdot 10^{-12} \text{ fm}^{-3}$)	B^-	\bar{B}^0	\bar{B}_s^0	Λ_b^0	$\Xi_b^{0,-}$	Ω_b^-
PDG(170)	1.0094	1.0089	0.29308	0.31591	0.10097	0.002341
PDG(160)	0.12655	0.12649	0.036622	0.034241	0.010520	0.00023076
RQM(170)	1.2045	1.2041	0.32513	0.61702	0.19548	0.0063204
RQM(160)	0.14567	0.14561	0.039664	0.061914	0.018819	0.00061087

MH & Rapp,
arXiv:2209.13419

f_α	B^-	\bar{B}^0	\bar{B}_s^0	Λ_b^0	$\Xi_b^{0,-}$
PDG(170)	0.3697	0.3695	0.1073	0.1157	0.03698
PDG(160)	0.3782	0.3780	0.1094	0.1023	0.03144
<u>RQM(170)</u>	<u>0.3391</u>	<u>0.3389</u>	<u>0.09152</u>	<u>0.1737</u>	<u>0.05503</u>
RQM(160)	0.3533	0.3532	0.09620	0.1502	0.04565

r_α	\bar{B}^0/B^-	\bar{B}_s^0/B^-	Λ_b^0/B^-	$\Xi_b^{0,-}/B^-$
PDG(170)	0.9995	0.2904	0.3129	0.1000
PDG(160)	0.9995	0.2894	0.2706	0.08313
<u>RQM(170)</u>	<u>0.9994</u>	<u>0.2699</u>	<u>0.5122</u>	<u>0.1623</u>
RQM(160)	0.9996	0.2723	0.4250	0.1292

- RQM vs PDG: Λ_b & Ξ_b fraction both enhanced by $\sim 50\%$

- RQM f_α very comparable to p-pbar data by Tevatron

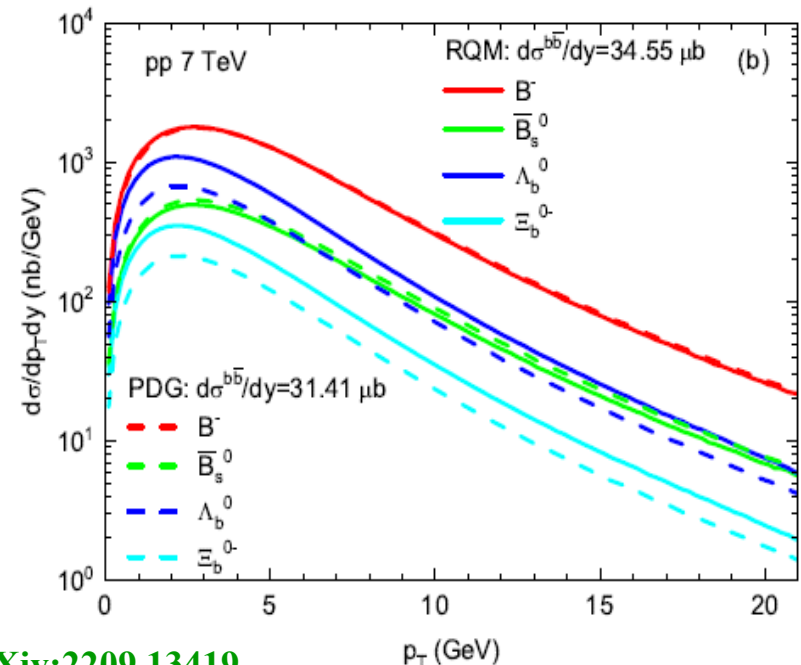
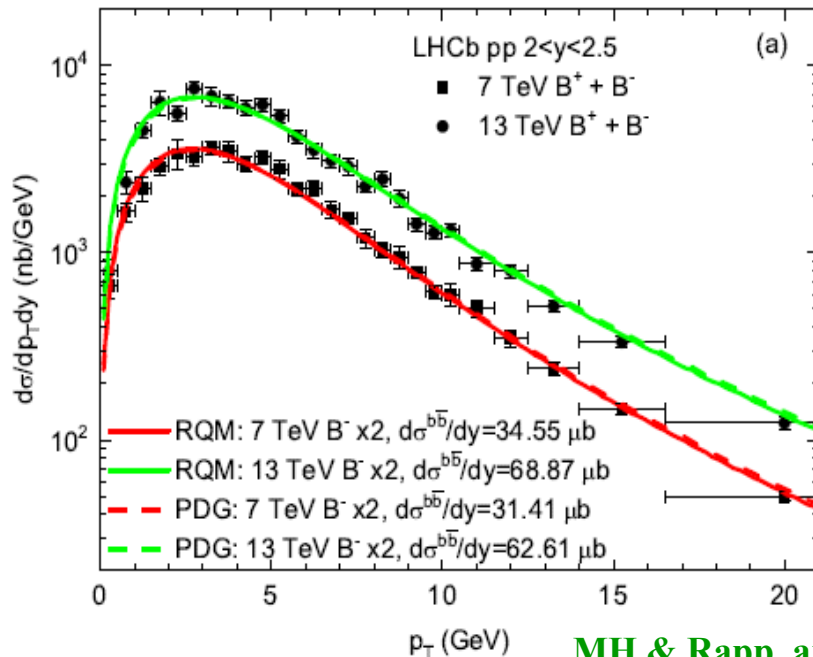
$$f_u = f_d = 0.340 \pm 0.021, f_s = 0.101 \pm 0.015$$

$$f_{\text{baryon}} = 0.220 \pm 0.048$$

HFLAG, Eur. Phys. J. C (2021) 81:226

Fit of p_T -spectra & cross sections

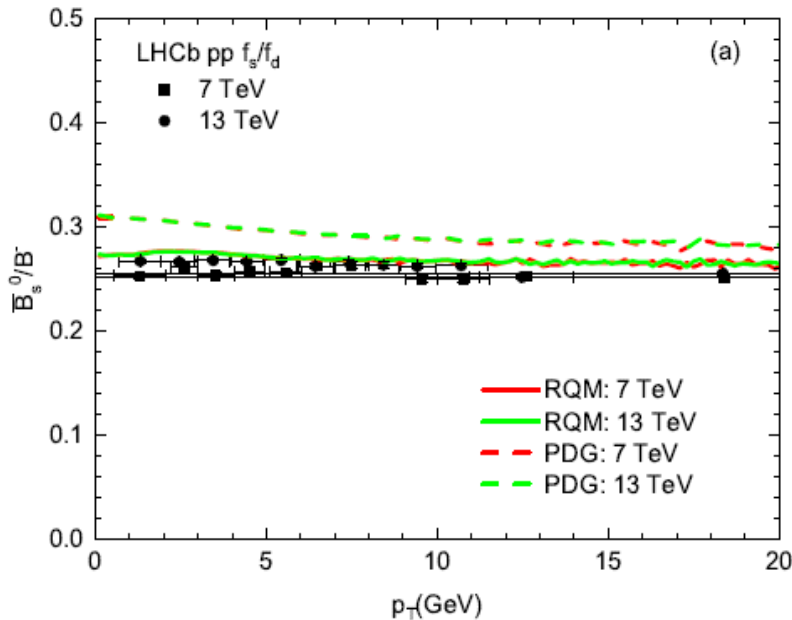
- FONLL b-quark spectrum + FF $D_{b \rightarrow H_b}(z) \propto z^\alpha(1-z)$ to **all** states (weight \propto density) + decay simulations \rightarrow ground states p_T -spectra



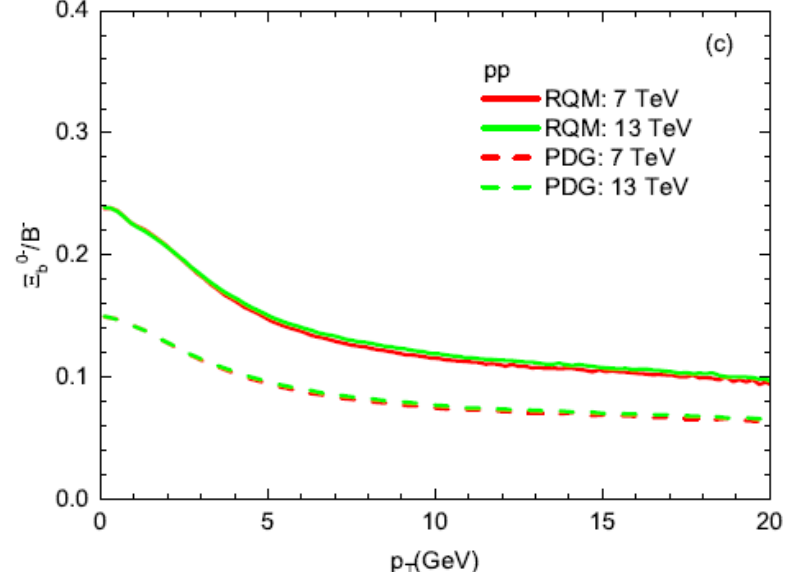
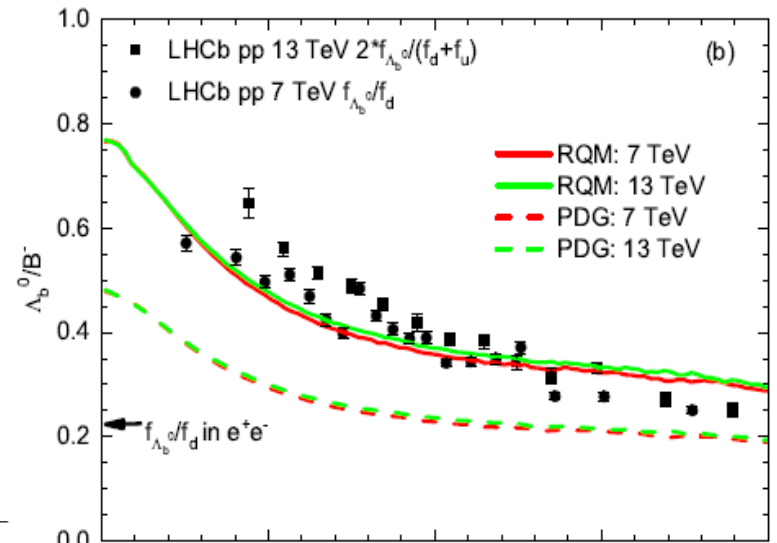
MH & Rapp, arXiv:2209.13419

- $B^+ + B^-$ data equally well described by RQM & PDG, but $\sim 10\%$ smaller $d\sigma^{b\bar{b}}/dy$ for PDG
- The decrease is due to reduction of bottom content in the baryon sector

Bottom hadro-chemistry: ratios



- \bar{B}_s^0/B^- & Λ_b^0/B^- almost unchanged from 7 to 13 TeV
- PDG-only curves far off; RQM states needed
- f_{Λ_b}/f_d in pp significantly larger than e^+e^- MH & Rapp, arXiv:2209.13419



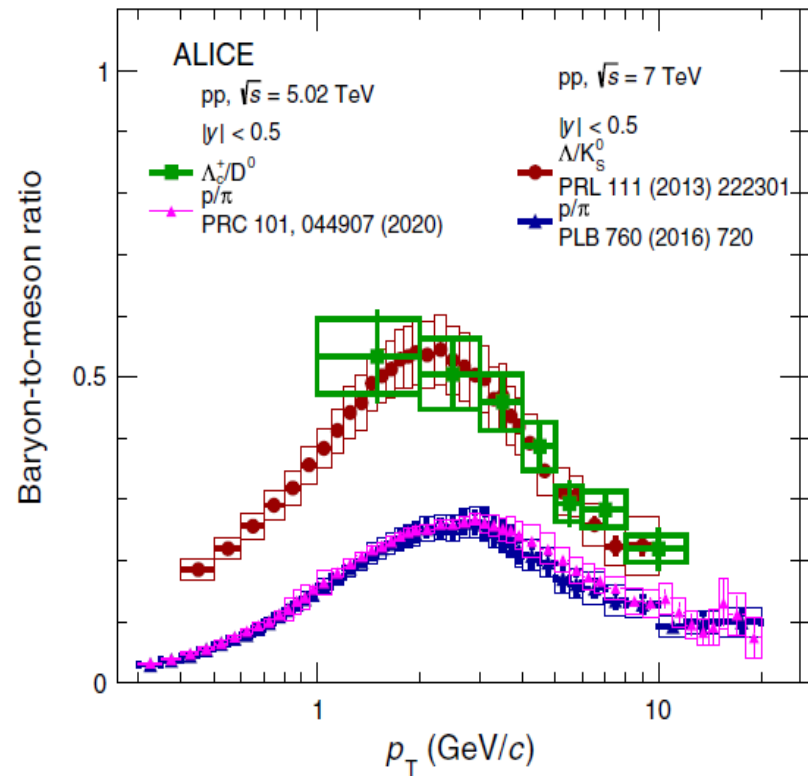
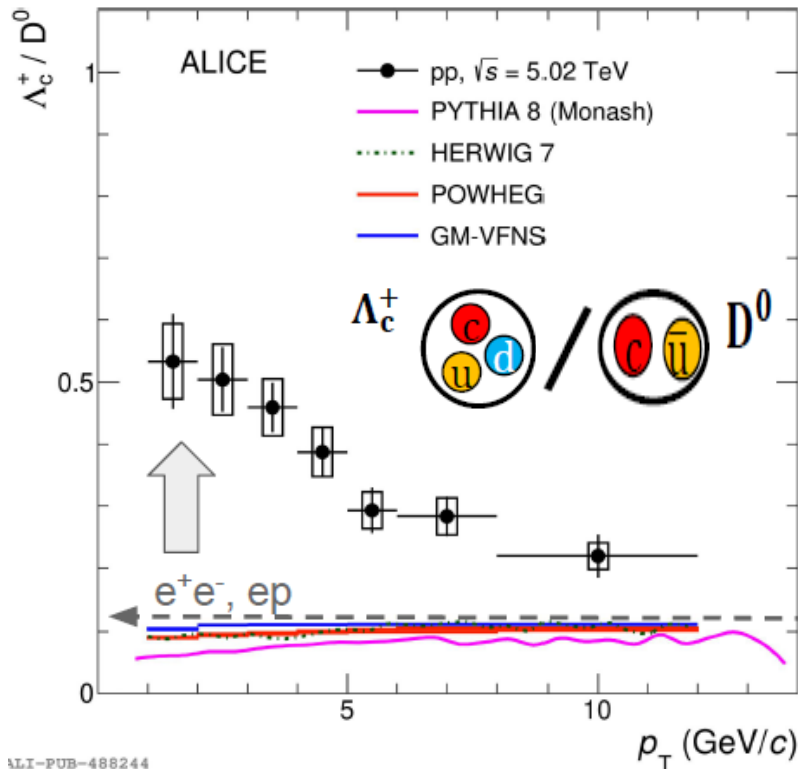
Summary: HQ hadronization in pp

- Statistical hadronization of HQs in hadronic collisions works well
 - ➔ heavy hadron fragmentation fractions & ratios well explained, especially for bottom for the first time, by augmenting PDG states with many more “missing” states/baryons predicted by RQM/lattice QCD
- Baryon/meson ratio significantly enhanced in hadronic (pp, pp-bar) than e^+e^-
 - ➔ shuffling of charm/bottom content from meson to baryon sector
 - ➔ universality assumption of HQ fragmentation is not true
- More intuitively, high-energy pp, pp-bar provide a quark-rich environment conducive to (statistical) recombination of HQ with light quarks, especially to the formation of heavy baryons, VS e^+e^- with less phase space

Back-up

The following are back-up pages

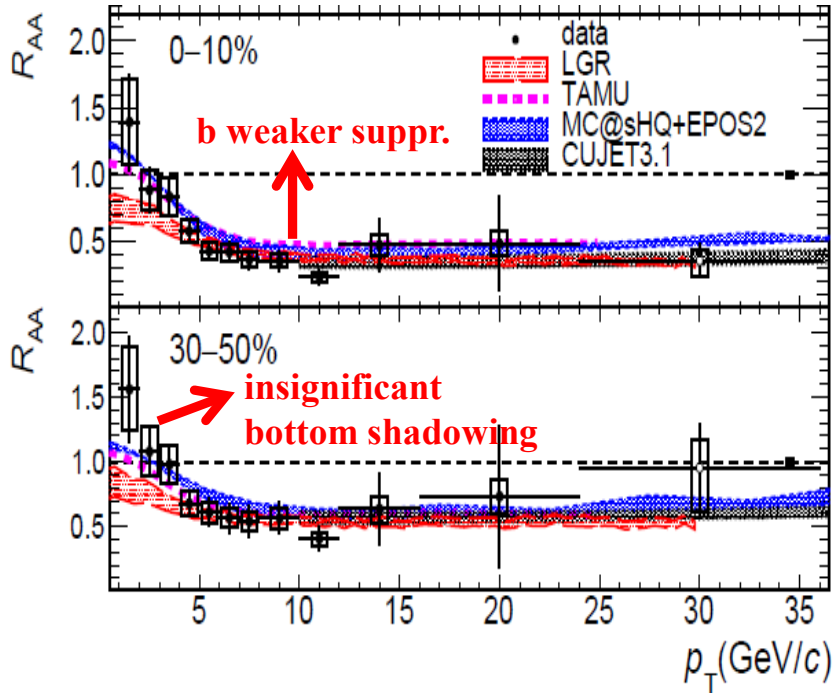
Λ_c^+/D^0 enhancement surprise



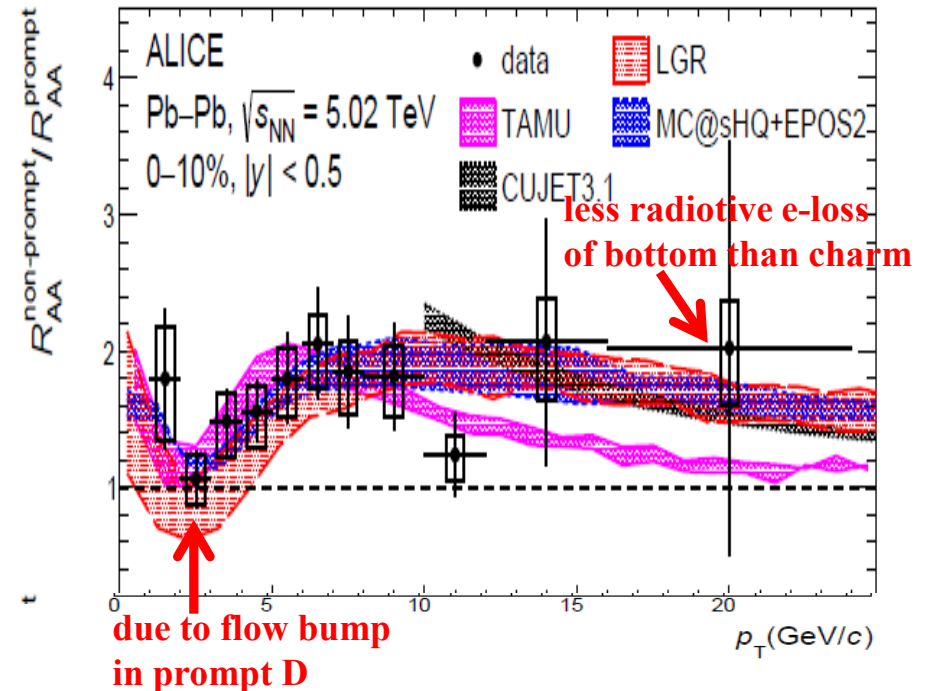
- ❖ a factor ~ 5 enhancement w.r.t. e^+e^- at low p_T , much underestimated by FFs tuned to e^+e^-
- ❖ decreasing toward high p_T , trend similar to Λ/K and p/π

Flavor dependence: charm vs bottom

nonprompt D (\leftarrow B) R_{AA} , ALICE:2202.00815

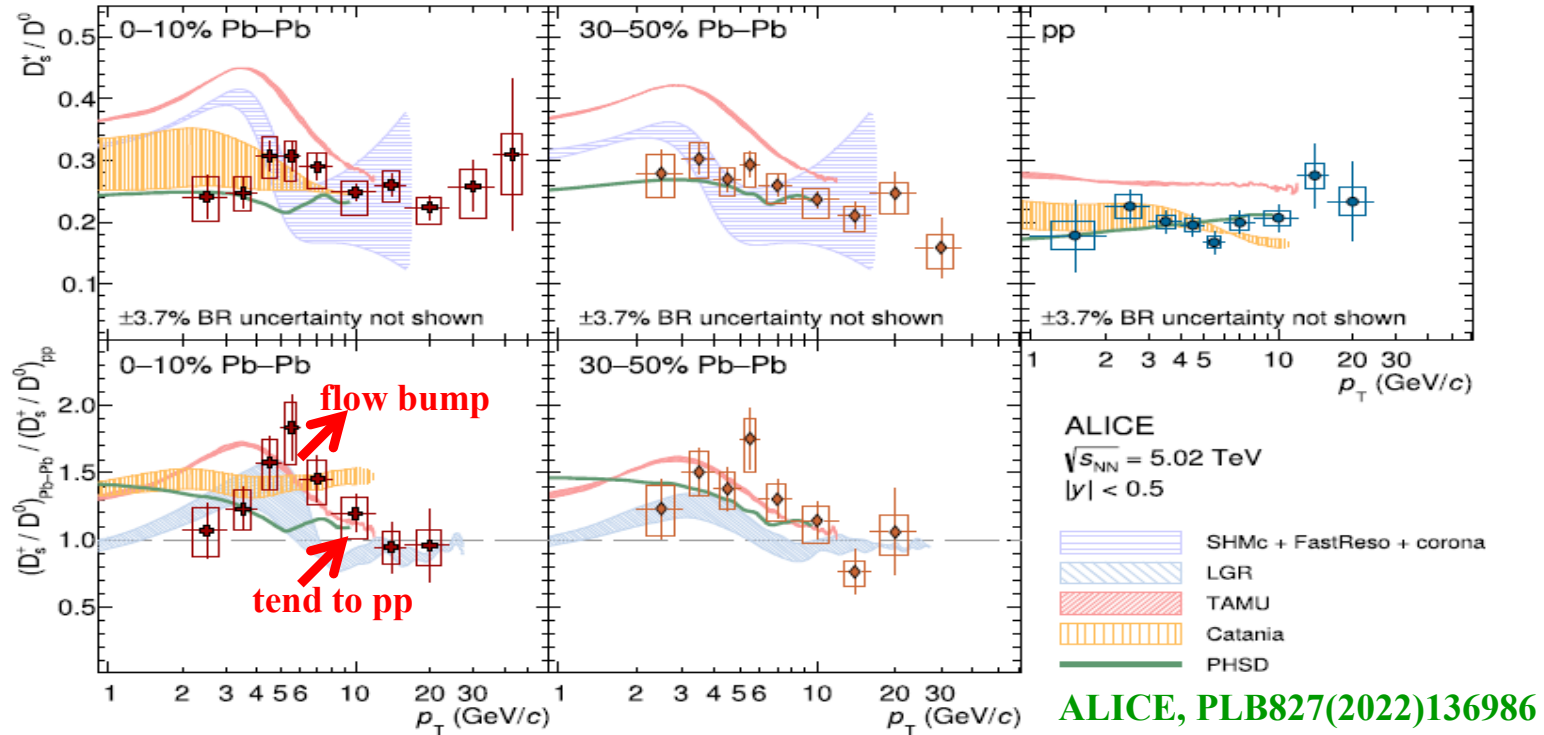


nonprompt-to-prompt D-meson R_{AA} ratio



- ❖ x3 mass: b-quark longer thermalization time at low p_T than charm
less flow added to b from recombination with u/d/s
- ❖ high $p_T > 15$ GeV: b-quark less radiative e-loss \leftarrow stronger “dead cone”

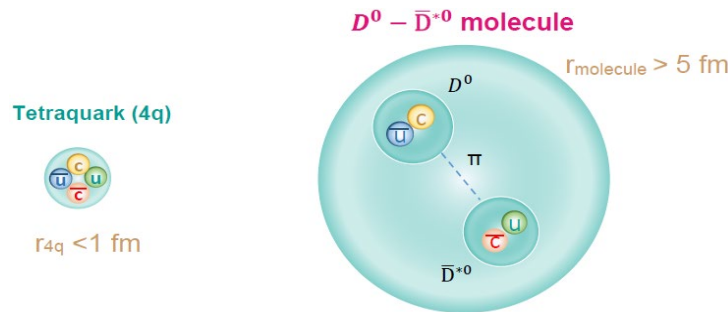
Charm hadro-chemistry: D_s/D^0



- low p_T : enhancement due to charm recombination in a strangeness-equilibrated QGP reproduced by Catania & PHSD; overestimated by TAMU in both pp and PbPb
- high p_T : tending to pp value as fragmentation takes over
- flow bump due to recombination with flowing s-quark heavier than u/d, predicted by TAMU (RRM w/ SMCs) & SHMc (hydro blastwave spectrum)

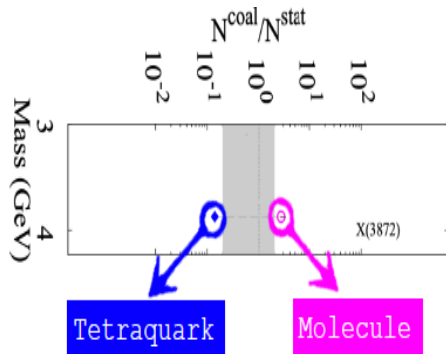
X(3872) production in HIC

❖ inner structure: compact tetraquark vs loosely bound molecule

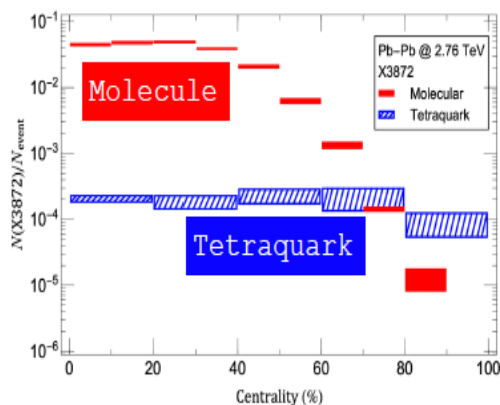


coalescence model

Cho et al. '11

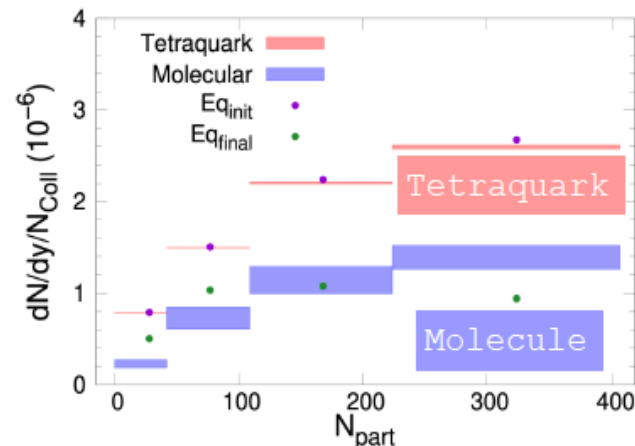


coalescence within AMPT zhang et al. '21



transport model

Wu et al. '21



- $N_{\text{molecule}} > N_{\text{tetraquark}}$ by x10 or 100, yet no account of hadron phase reactions $\pi X \leftrightarrow DD^*$
→ to be better constrained

- $N_{\text{tetraquark}} > N_{\text{molecule}}$ by x2, molecule regenerated in late hadronic phase, tetraquark chem. freezeout at T_c

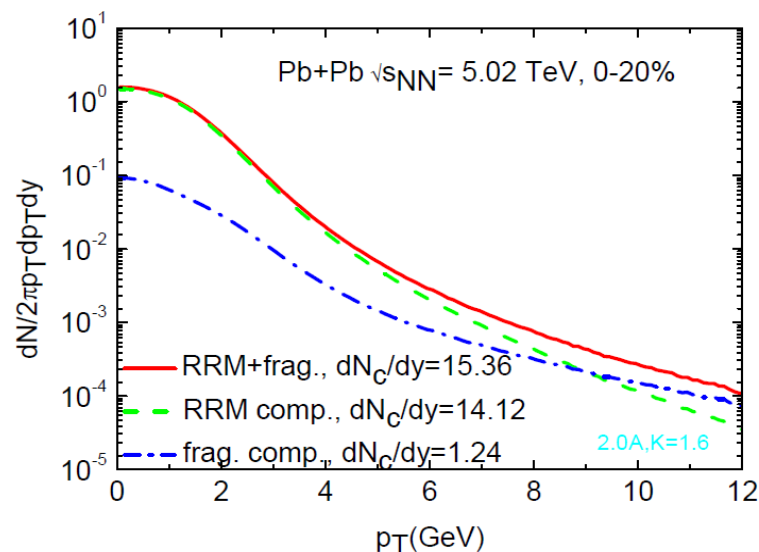
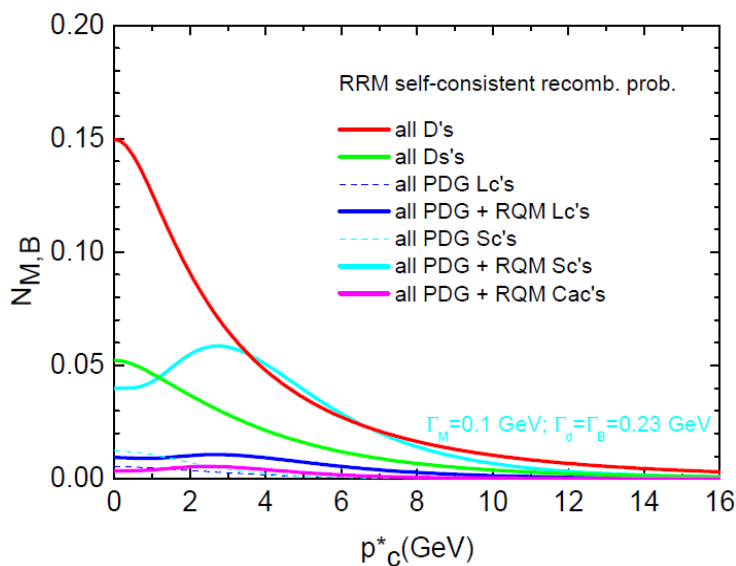
Charm quark recombination probability

- No. of mesons/baryons formed from a single c-quark of rest frame p_c^*

$$N_M(p_c^*) = \int \frac{d^3 \vec{p}_1^*}{(2\pi)^3} g_q e^{-E(\vec{p}_1^*)/T_{pc}} \frac{E_M(\vec{p}^*)}{m_M \Gamma_M} \sigma(s) v_{\text{rel}},$$

$$N_B(p_c^*) = \int \frac{d^3 p_1 d^3 p_2}{(2\pi)^6} g_1 e^{-E(\vec{p}_1)/T_c} g_2 e^{-E(\vec{p}_2)/T_c} \frac{E_d(\vec{p}_{12})}{m_d \Gamma_d} \sigma(s_{12}) v_{\text{rel}}^{12}(\vec{p}_1, \vec{p}_2) \frac{E_B(\vec{p})}{m_B \Gamma_B} \sigma(s_{d3}) v_{\text{rel}}^{d3}(\vec{p}_{12}, \vec{p}_{30}),$$

- Renormalizing $N_M(p_c^*)$ and $N_B(p_c^*)$ by a **common** factor ~ 4 for all charmed mesons/baryons such that $\sum_M P_{\text{coal},M}(p_c^* = 0) + \sum_B P_{\text{coal},B}(p_c^* = 0) = 1$



➔ **charm conservation** consistently built in, in an (e-by-e) way without spoiling the relative chemical equilibrium realized by RRM

Space-momentum correlations: light-q

- hydro: a manifestation of SMCs

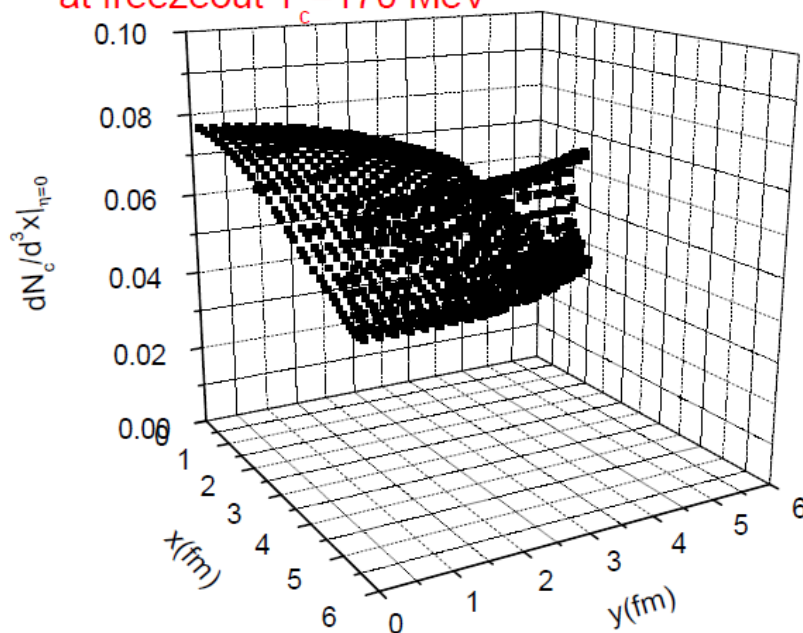
$$f_q^{eq}(\vec{x}, \vec{p}) = g_q e^{-p \cdot u(x)/T(x)} = g_q e^{-\gamma_T(x)[m_T \cosh(y-\eta) - \vec{p}_T \cdot \vec{v}_T(x)]/T(x)}$$

longitudinal boost invariance: $y - \eta$

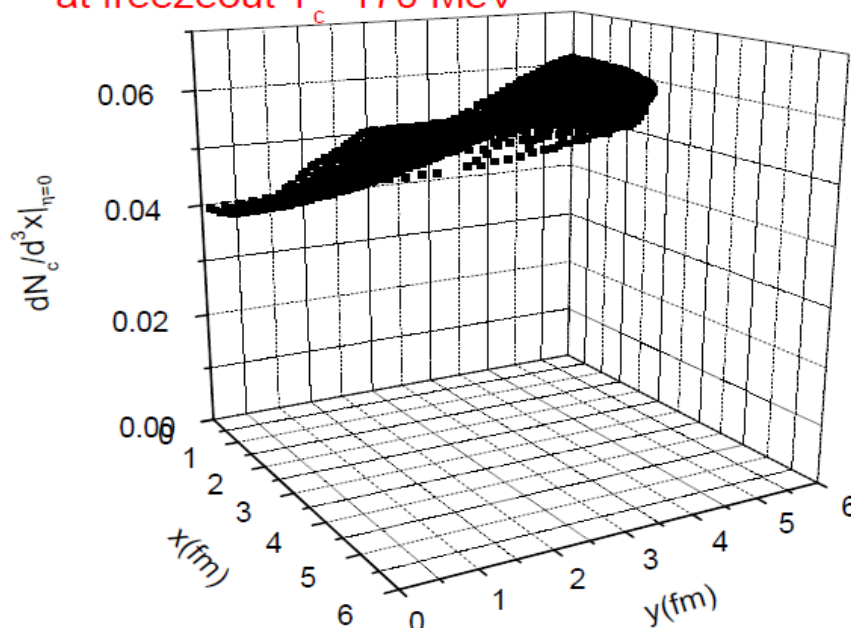
transverse SMCs $p_T \cdot v_T$

- hydro-q: low (high) p_T more concentrated in center (boundary)

hydro light quarks $p_T = 0.0 - 0.3$ GeV,
at freezeout $T_c = 170$ MeV



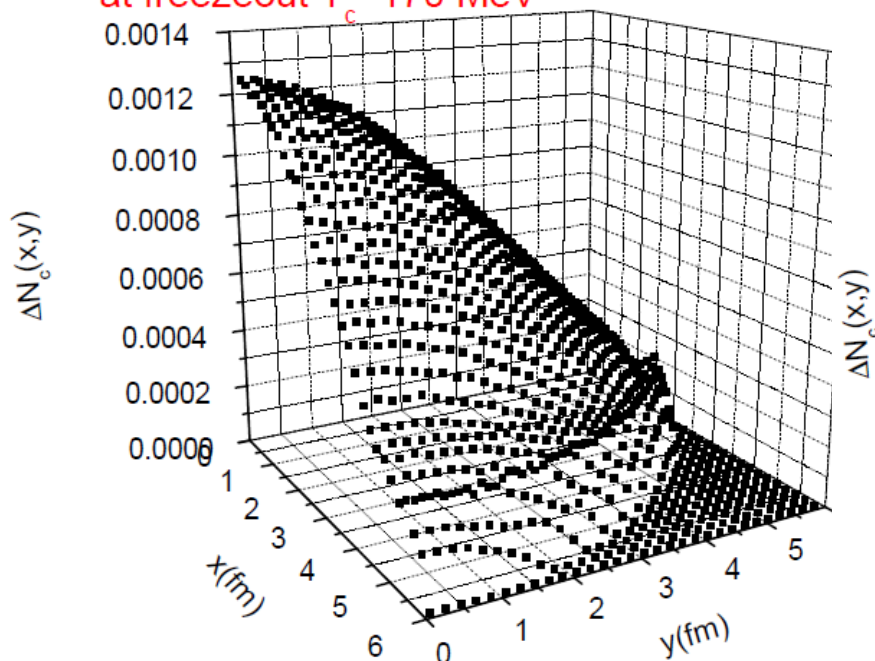
hydro light quarks $p_T = 0.6 - 0.9$ GeV,
at freezeout $T_c = 170$ MeV



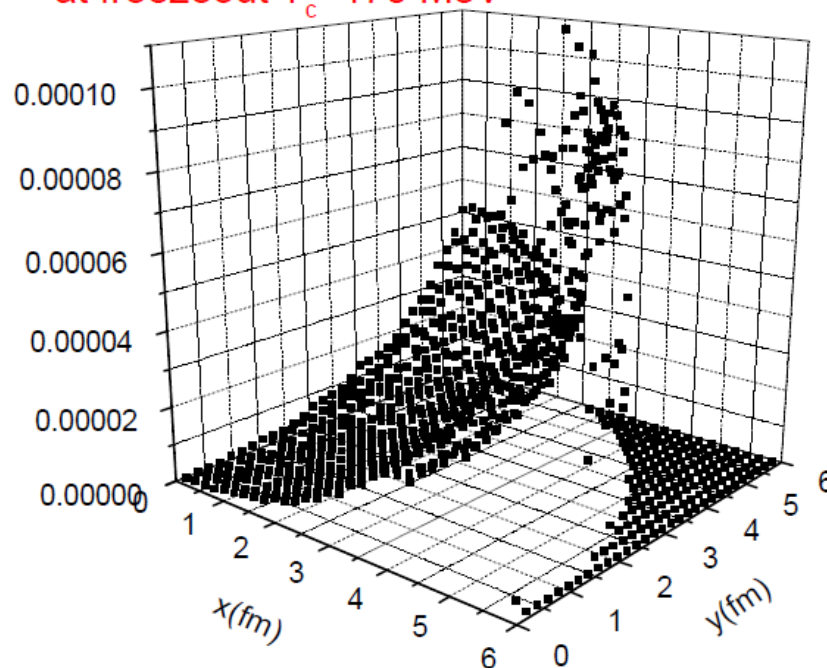
SMCs: Langevin charm quarks

- Langevin-c: low (high) p_T more populated in central (outer)

Langevin charm quarks $p_T=0.0-1.0$ GeV,
at freezeout $T_c=170$ MeV



Langevin charm quarks $p_T=3.0-4.0$ GeV,
at freezeout $T_c=170$ MeV



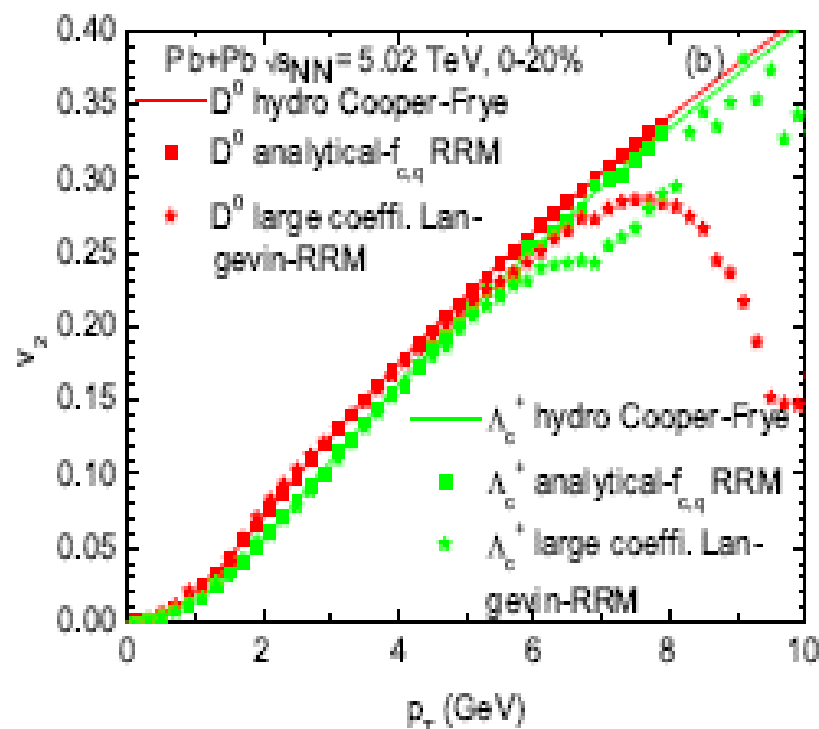
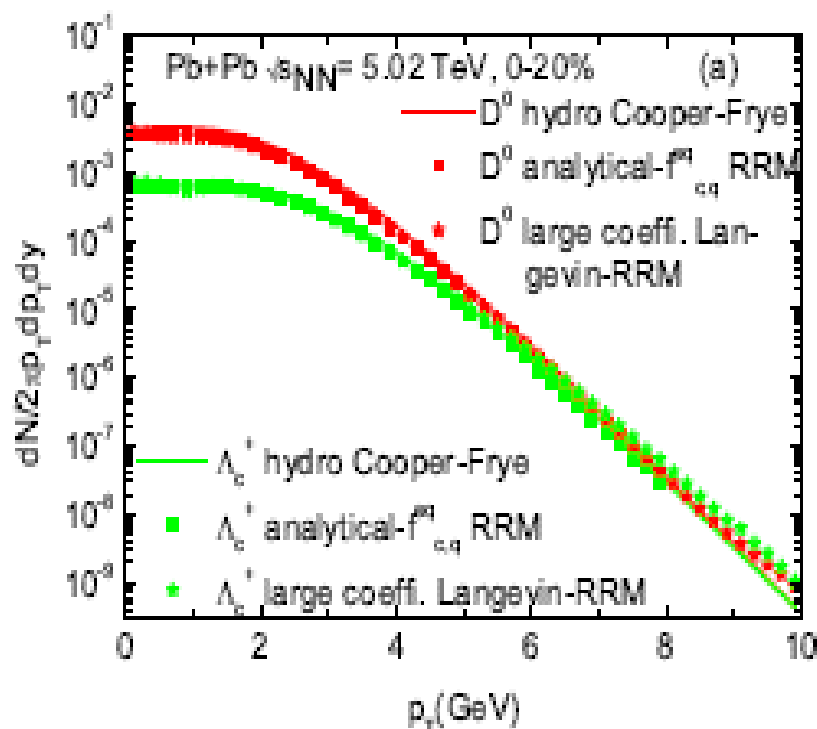
- SMCs usually neglected in ICMs: uniformly distributed independent of p_T

$$f_{c,q}(\vec{x}, \vec{p}) = (2\pi)^3 \frac{dN_{c,q}}{d^3\vec{x}d^3\vec{p}} = \frac{(2\pi)^3}{V E(\vec{p})} \frac{dN_{c,q}}{p_T dp_T d\phi_q dy}$$

RRM equilibrium mapping

- Event-by-event Langevin-RRM simulation with **very large trans. coeffi.** & with **SMCs** properly incorporated

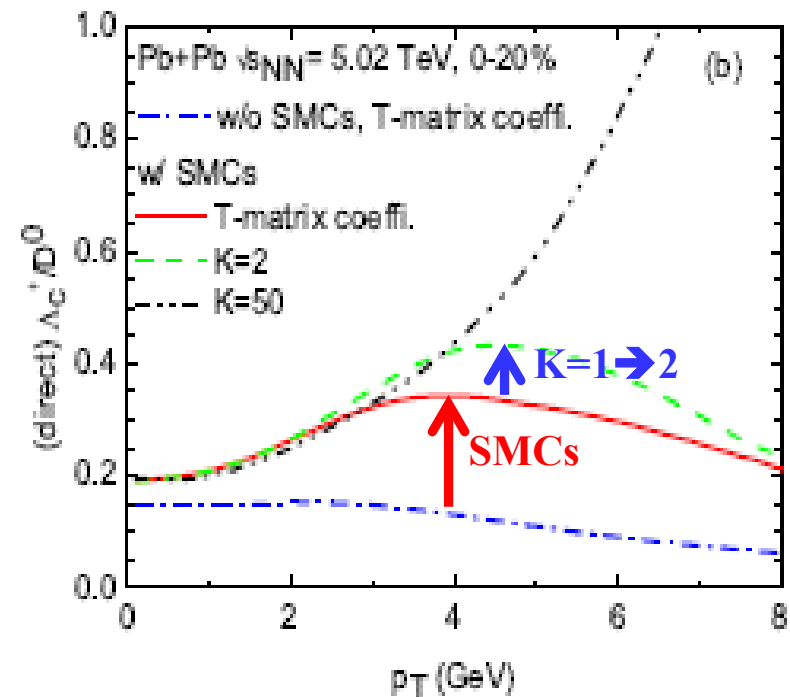
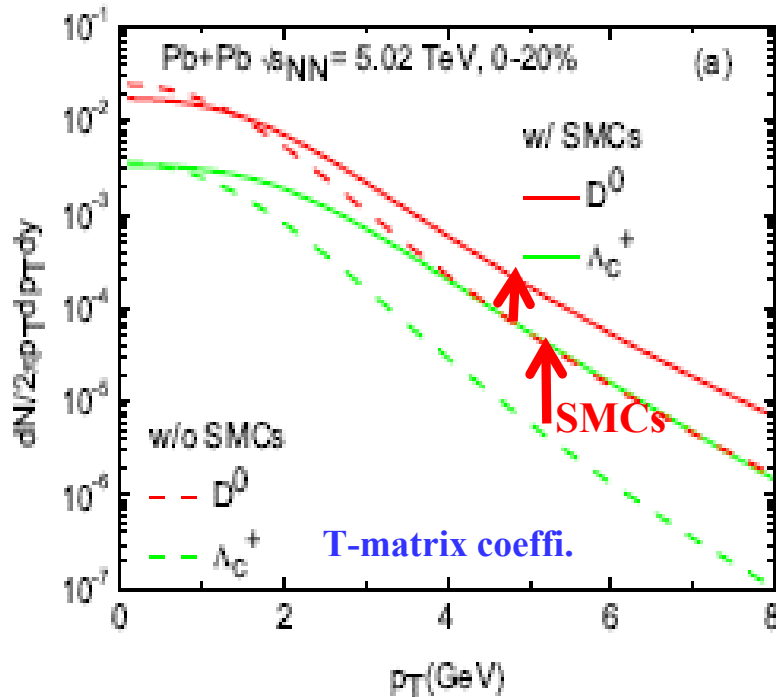
➔ kinetic & chemical equil. mapping



➔ Observables come out as RRM predictions with realistic T-matrix coeffi.

Direct D^0 & Λ_c^+ production via RRM

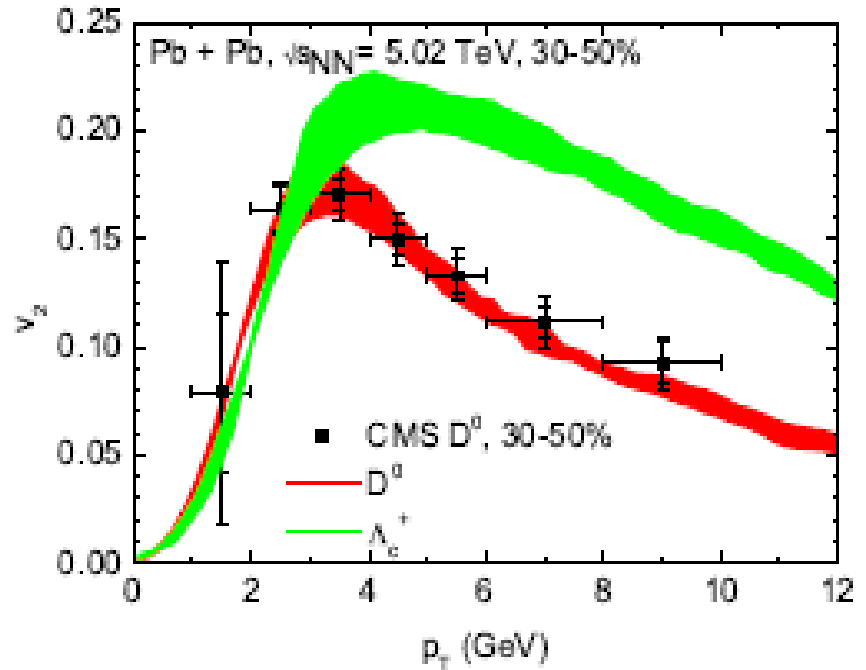
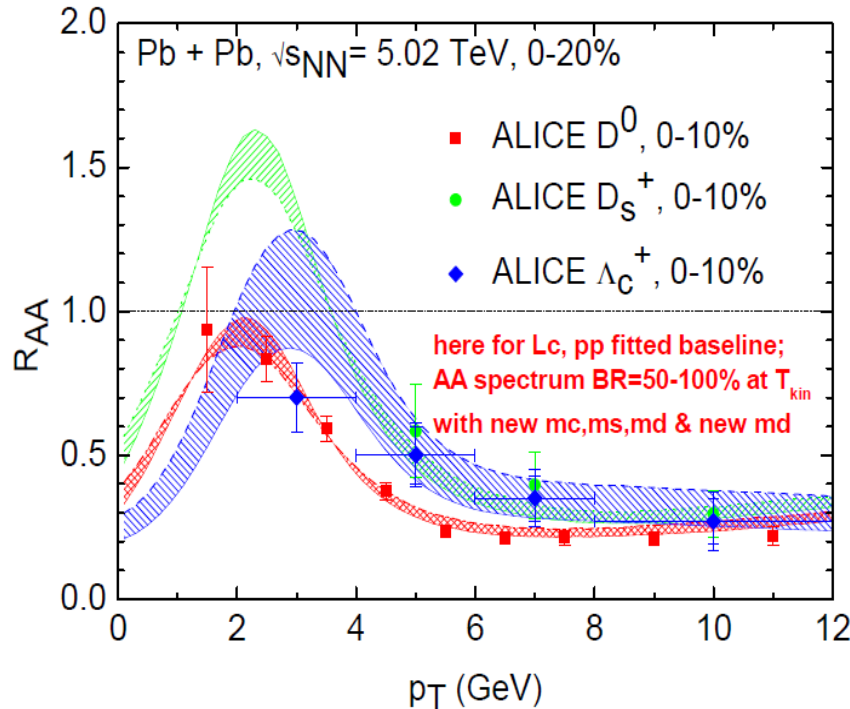
- Including SMCs makes spectra **harder** & **enhances** the Λ_c^+/D^0



- Fast-moving c-quarks [$p_T \sim 3-4$ GeV] moving to outer part of fireball find higher-density of harder [$p_T \sim 0.6-0.9$ GeV] light quarks for recombination
- An effect entering **squared** for the recombination production of Λ_c^+
→ larger enhancement for $\Lambda_c^+ \rightarrow \Lambda_c^+/D^0$ ratio enhanced!

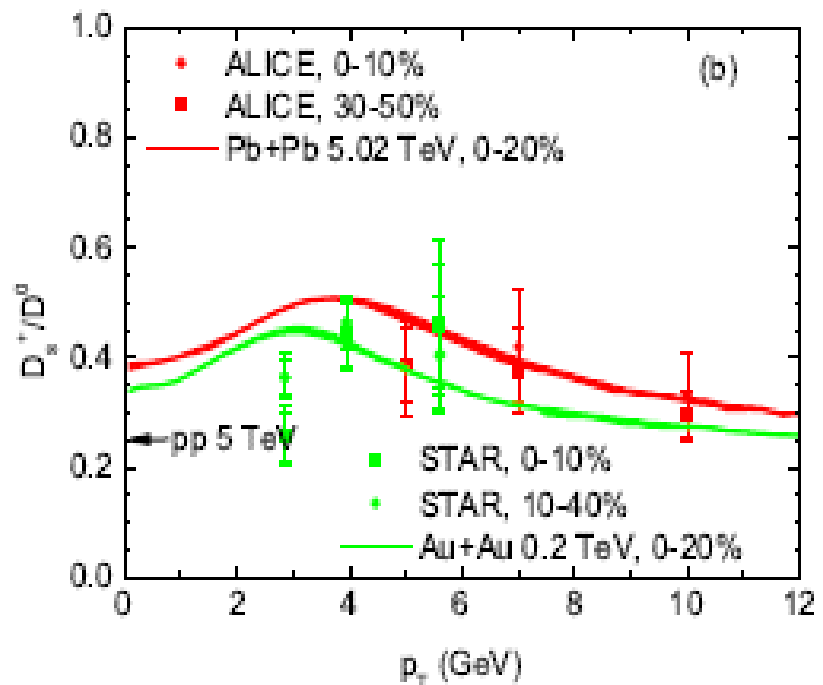
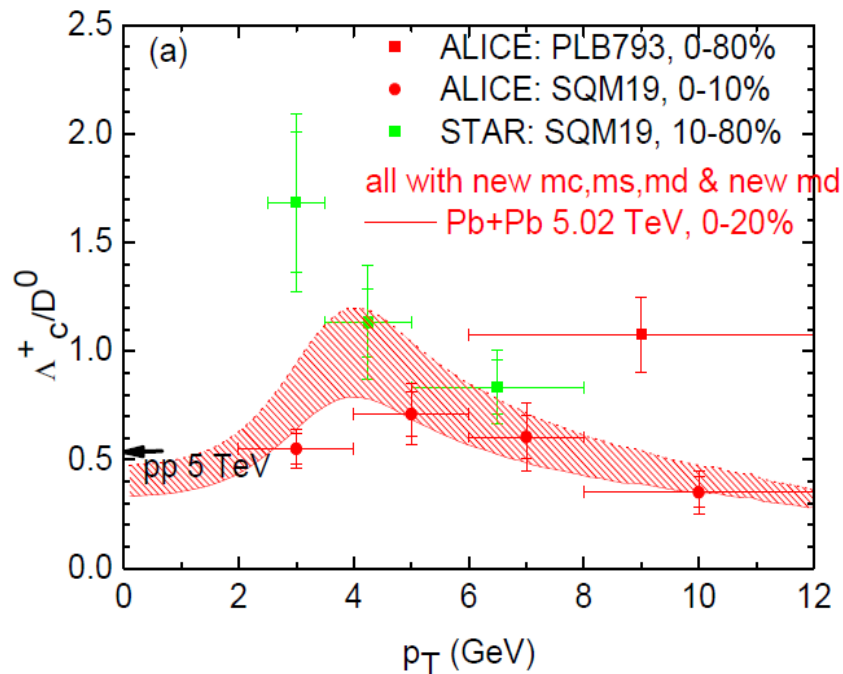
D^0 , D_s^+ & Λ_c^+ suppression & elliptic flow

➤ Final D^0 , D_s^+ & Λ_c^+ , including feeddowns from all RQM baryons



- T-matrix coefficient*K-factor(=1.6), to compensate for radiative e-loss; uncertainty: BR=50-100% to Λ_c^+ for Λ_c 's & Σ_c 's above DN (2805 MeV)
- Hadronic phase diffusion also included: seamlessly connected to hadronization (RRM+frag), increasing D-meson v_2 by $\sim 15\%$

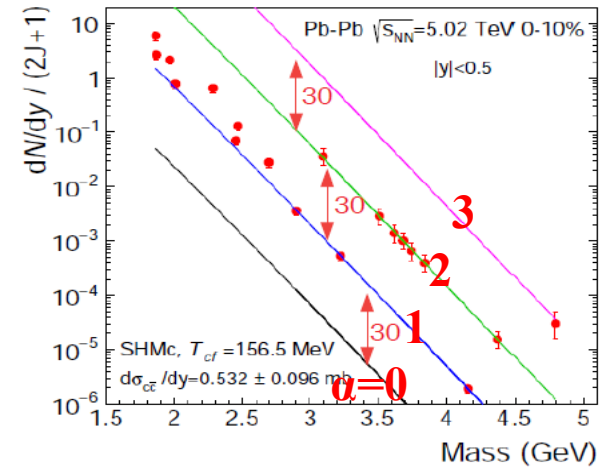
Charm-hadron ratios: Λ_c^+/D^0 & D_s^+/D^0



- Λ_c^+/D^0 : low p_T approaching RRM equil. limit = SHM pp; intermediate p_T enhancement from RRM with SMCs; high p_T fragmentation tending to pp value
- D_s^+/D^0 enhancement: recombination of charm in a strangeness-equilibrated QGP

Hadronization: SHMc Andronic, PBM et al. 2104.12754

- SHMc: open-charm statistical hadronization at T_c $\frac{dN(h_{oc,\alpha}^i)}{dy} = g_c^\alpha V n_i^{th} \frac{I_\alpha(N_c^{tot})}{I_0(N_c^{tot})}$
 - ❑ multicharm baryons $\alpha=1,2,3$ emerging pattern
 - ❑ yields enhanced by $g_c^\alpha \sim 30^\alpha$ than pure thermal \rightarrow strong signal of deconfinement



- SHMc yields + blast wave $\rightarrow p_T$ spectra

