# Lectures on Heavy-Flavor Probes of Quark-Gluon Plasma

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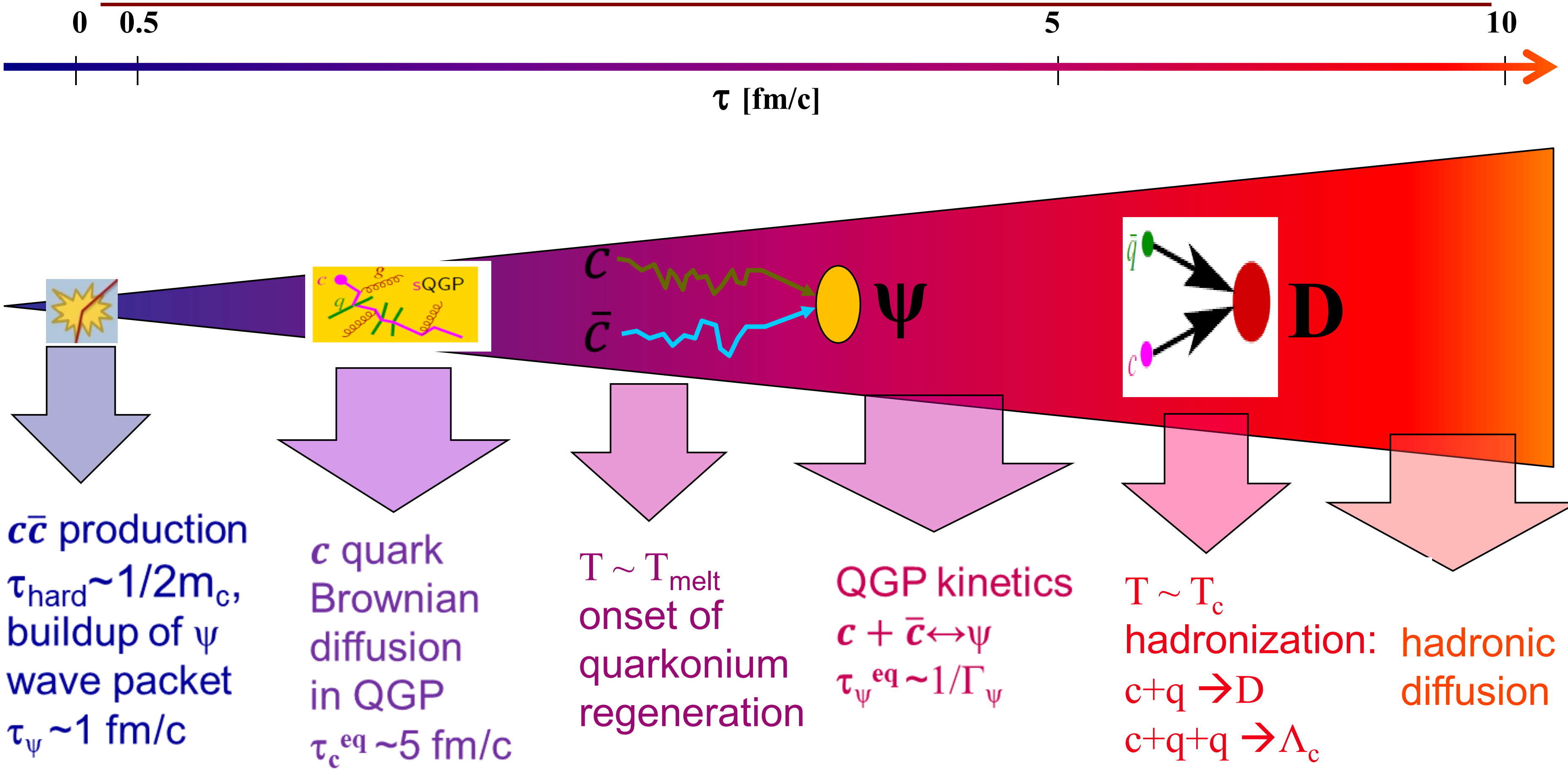
# Lecture II

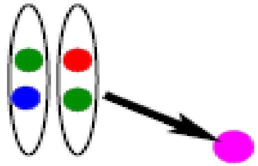
# Open heavy flavor production in AA collisions

- Simulation of HQ diffusion in QGP: Langevin vs Boltzmann
- Microscopic interactions of HQs in QGP
- HQ Hadronization
- Heavy hadron interaction in hot hadronic medium
- Phenomenology

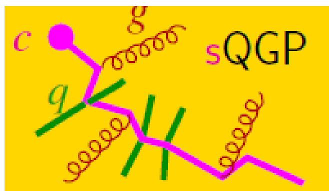
# Heavy flavor transport as probes of QGP



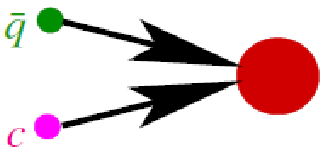
# HQ evolution in hot QCD medium



- c-cbar (b-bbar) produced in pairs in early hard processes t~1/2$m_Q$, calculable via pQCD

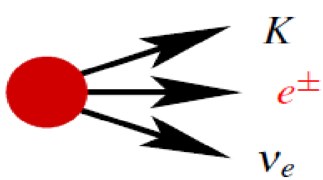



- single HQs diffusion and rescattering in sQGP via elastic or inelastic/radiative interactions, simulated by Boltzmann/Langevin equations




- HQ hadronization via coalescence c+qbar→D, c+q+q→$\Lambda_c$, or independent fragmentation


- D, $\Lambda_c$ further diffusion in HRG via hadronic interactions with bulk hadrons




- D, $\Lambda_c$ decay long after freezeout of the system via weak interactions (semileptonic or hadronic e.g. B→J/ψ+K (nonprompt J/ψ))

# HQ transport in QGP: Boltzmann eq.

- describe heavy-quark scattering in the QGP by (semi-)classical transport equation
- $f_Q(t,\vec{x},\vec{p})$: phase-space distribution of heavy quarks
- equation of motion for HQ-fluid cell at time $t$ at $(\vec{p},\vec{x})$:

$$\mathrm{d}f_Q = \mathrm{d}t\left(\frac{\partial}{\partial t} + \vec{v}\frac{\partial}{\partial \vec{x}} + \vec{F}\cdot\frac{\partial}{\partial \vec{p}}\right) f_Q$$

  - change of phase-space distribution with time (non-equilibrium)
  - drift of HQ-fluid cell with velocity $\vec{v} = \vec{p}/E_{\vec{p}}$, $E_{\vec{p}} = \sqrt{m_Q^2 + \vec{p}^2}$
  - change of momentum with mean-field force, $\vec{F}$
- change must be due to collisions with surrounding medium

$$\mathrm{d}f_Q = C[f_Q] = \int \mathrm{d}^3\vec{k}[\underbrace{w(\vec{p}+\vec{k},\vec{k})f_Q(t,\vec{x},\vec{p}+\vec{k})}_{\text{gain}} - \underbrace{w(\vec{p},\vec{k})f_Q(t,\vec{x},\vec{p})}_{\text{loss}}]$$

  - $w(\vec{p},\vec{k})$: transition rate for collision of a heavy quark with momentum, $\vec{p}$ with a heat-bath particle with momentum transfer, $\vec{k}$

# Boltzmann equation (2→2)

- relation to cross sections of microscopic scattering processes
- e.g., elastic scattering of heavy quark with light quarks

$$w(\vec{p},\vec{k}) = \gamma_q \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3} f_q(\vec{q}) v_{\mathrm{rel}}(\vec{p},\vec{q} \rightarrow \vec{p}-\vec{k},\vec{q}+\vec{k}) \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$$

- $\gamma_q = 2 \times 3 = 6$: spin-color-degeneracy factor
- $v_{\mathrm{rel}} := \sqrt{(p \cdot q)^2 - (m_Q m_q)^2}/(E_Q E_q)$; covariant relative velocity
- in terms of invariant matrix element

$$C[f_Q] = \frac{1}{2E_Q} \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3 2E_q} \int \frac{\mathrm{d}^3\vec{p}\,'}{(2\pi)^3 2E_p'} \int \frac{\mathrm{d}^3\vec{q}\,'}{(2\pi)^3 2E_q'}$$
$$\times \frac{1}{\gamma_Q} \sum_{c,s} \left|\mathcal{M}_{(\vec{p}\,',\vec{q}\,')\leftarrow(\vec{p},\vec{q})}\right|^2$$
$$\times (2\pi)^4 \delta^{(4)}(p+q-p'-q')[f_Q(\vec{p}\,')f_q(\vec{q}\,') - f_Q(\vec{p})f_q(\vec{q})]$$

  - $\vec{p}$, $\vec{q}$ ($\vec{p}\,'$, $\vec{q}\,'$) initial (final) momenta of heavy and light quark
  - momentum transfer: $\vec{k} = \vec{q}\,' - \vec{q} = \vec{p} - \vec{p}\,'$

# Reduction to Fokker-Planck equation

- heavy quarks $\leftrightarrow$ light quarks/gluons: momentum transfers small
- $w(\vec{p}+\vec{k},\vec{k})$: peaked around $\vec{k}=0$
- expansion of collision term around $\vec{k}=0$

$$w(\vec{p}+\vec{k},\vec{k})f_Q(\vec{p}+\vec{k},\vec{k}) \simeq w(\vec{p},\vec{k})f_Q(\vec{p}) + \vec{k}\cdot\frac{\partial}{\partial\vec{p}}[w(\vec{p},\vec{k})f_Q(\vec{p})] + \frac{1}{2}k_i k_j \frac{\partial^2}{\partial\vec{p}_i\vec{p}_k}[w(\vec{p},\vec{k})f_Q(\vec{p})]$$

- collision term

$$C[f_Q] = \int \mathrm{d}^3\vec{k}\left[k_i\frac{\partial}{\partial p_i} + \frac{1}{2}\frac{\partial^2}{\partial p_i \partial p_j}\right][w(\vec{p},\vec{k})f_Q(\vec{p})].$$

# Reduction to Fokker-Planck equation

- Boltzmann equation $\Rightarrow$ simplifies to Fokker-Planck equation

$$\partial_t f_Q(t,\vec{x},\vec{p}) + \frac{\vec{p}}{E_{\vec{p}}} \cdot \frac{\partial}{\partial \vec{x}} f_Q(t,\vec{x},\vec{p}) = \frac{\partial}{\partial p_i}\left\{ A_i(\vec{p}) f_Q(t,\vec{x},\vec{p}) + \frac{\partial}{\partial p_j}\left[B_{ij}(\vec{p}) f_Q(t,\vec{p})\right]\right\}$$

- with drag and diffusion coefficients

$$A_i(\vec{p}) = \int \mathrm{d}^3\vec{k}\, k_i w(\vec{p},\vec{k}), \quad B_{ij}(\vec{p}) = \frac{1}{2}\int \mathrm{d}^3\vec{k}\, k_i k_j w(\vec{p},\vec{k})$$

- equilibrated light quarks and gluons: coefficients in heat-bath frame
- matter homogeneous and isotropic

$$A_i(\vec{p}) = A(p) p_i, \quad B_{ij}(\vec{p}) = B_0(p) P_{ij}^{\perp} + B_1(p) P_{ij}^{\|}$$

$$\text{with} \quad P_{ij}^{\|}(\vec{p}) = \frac{p_i p_j}{\vec{p}^2}, \quad P_{ij}^{\perp}(\vec{p}) = \delta_{ij} - \frac{p_i p_j}{\vec{p}^2}$$

$$A_i(\boldsymbol{p}) = \frac{1}{2E(\boldsymbol{p})}\int \frac{d^3\boldsymbol{q}}{(2\pi)^3 2E(\boldsymbol{q})}\int \frac{d^3\boldsymbol{q}'}{(2\pi)^3 2E(\boldsymbol{q}')} \times \int \frac{d^3\boldsymbol{p}'}{(2\pi)^3 2E(\boldsymbol{p}')}\frac{1}{\gamma}\sum |\mathcal{M}|^2 \hat{f}(\boldsymbol{q}) (2\pi)^4\delta^4(p+q-p'-q')\,[(\boldsymbol{p}'-\boldsymbol{p})_i] \equiv \langle\langle(\boldsymbol{p}'-\boldsymbol{p})_i\rangle\rangle;$$

$$B_{ij}(\boldsymbol{p}) = \frac{1}{2}\langle\langle(\boldsymbol{p}'-\boldsymbol{p})_i(\boldsymbol{p}'-\boldsymbol{p})_j\rangle\rangle.$$

A,B can be calculated from scattering amplitudes, but are not independent of each other!

# Einstein relation & equilibrium limit

- FP equation cast into a continuity equation in momentum space

$$\frac{\partial f_Q(t,\boldsymbol{p})}{\partial t} + \frac{\partial}{\partial p_i} S_i(t,\boldsymbol{p}) = 0. \quad S_i(t,\boldsymbol{p}) = -\left\{ A_i(\boldsymbol{p}) f(t,\boldsymbol{p}) + \frac{\partial}{\partial p_j}[B_{ij}(\boldsymbol{p}) f(t,\boldsymbol{p})] \right\}$$

- In the equilibrium limit ➔ Boltzmann-Juttner distribution

$$f_{\mathrm{eq}}(p,T) = N \exp[-E(p)/T] \qquad E(p) = \sqrt{p^2 + m^2}$$

particle current/flux vanishes $S_i$=0 ➔ fluctuation-dissipation theorem between drag and diffusion coefficient

$$A_i(\boldsymbol{p},T) = B_{ij}(\boldsymbol{p},T)\frac{1}{T}\frac{\partial E(p)}{\partial p_j} - \frac{\partial B_{ij}(\boldsymbol{p},T)}{\partial p_j}$$

- Non-relativistic limit with diagonal diffusion

$$A(p) = \frac{1}{E(p)}\left(\frac{D[E(p)]}{T} - \frac{\partial D[E(p)]}{\partial E}\right) \qquad B_0(p) = B_1(p) = D(p)$$

in the non-relativistic limit, D(p)=D, Γ(p)=γ ➔ Einstein relation: $D = m\gamma T$

- Analytical solution

$$f(p,t) = \left[\frac{\gamma}{2\pi D}(1-e^{-2\gamma t})\right]^{-1/2} \times \exp\left[-\frac{\gamma}{2D}\frac{(p-p_0 e^{-\gamma t})^2}{1-e^{-2\gamma t}}\right]$$

- drag on mean momentum: $\langle p \rangle = p_0 e^{-\gamma t}$
- diffusion in momentum & coordinate space:

$$\langle p^2\rangle - \langle p\rangle^2 = \frac{D}{\gamma}(1-e^{-2\gamma t}) \qquad \langle x^2\rangle - \langle x\rangle^2 \sim \frac{2D}{m^2\gamma^2}t = 2D_s t$$

$$D_s = \frac{T}{m_Q\gamma} = \frac{T^2}{D}$$

# Langevin equation

- FP equation stochastically realized by Langevin equation

$$dx_j = \frac{p_j}{E}dt,$$

$$dp_j = -\Gamma(p,T)p_j dt + \sqrt{dt}C_{jk}(\boldsymbol{p}+\xi d\boldsymbol{p},T)\rho_k$$

$\longrightarrow$

$$\frac{\partial f(t,\boldsymbol{p})}{\partial t} = \frac{\partial}{\partial p_j}\left[\left(\Gamma(p)p_j - \xi C_{lk}(\boldsymbol{p})\frac{\partial C_{jk}(\boldsymbol{p})}{\partial p_l}\right)f(t,\boldsymbol{p})\right] + \frac{1}{2}\frac{\partial^2}{\partial p_j \partial p_k}[C_{jl}(\boldsymbol{p})C_{kl}(\boldsymbol{p})f(t,\boldsymbol{p})]. \quad (10)$$

Γ(p,T) --- deterministic drag/friction force;
$C_{jk}$--- stochastically fluctuating force with independent Gaussian noise $P(\boldsymbol{\rho}) = (2\pi)^{-3/2}e^{-\boldsymbol{\rho}^2/2}$
no correlation at different times $\langle F_j(t)F_k(t')\rangle = C_{jl}C_{kl}\delta(t-t')$

- Pre-point scheme: $\xi = 0$

$$dx_j = \frac{p_j}{E}dt,$$

$$dp_j = -\Gamma(p)p_j dt + \sqrt{2dt\,D(p)}\rho_j$$

with equilibrium condition: $\Gamma(p) = \frac{1}{E(p)}\left(\frac{D[E(p)]}{T} - \frac{\partial D[E(p)]}{\partial E}\right) = A(p)$

➔ F-P equation: $\frac{\partial}{\partial t}f(t,\boldsymbol{p}) = \frac{\partial}{\partial p_i}\left\{\Gamma(p)p_i f(t,\boldsymbol{p}) + \frac{\partial}{\partial p_i}[D(p)f(t,\boldsymbol{p})]\right\}$

- Post-point scheme: $\xi = 1$

$$dx_j = \frac{p_j}{E}dt,$$

$$dp_j = -\Gamma(p)p_j dt + \sqrt{2dt\,D(|\boldsymbol{p}+d\boldsymbol{p}|)}\rho_j$$

with equilibrium condition: $D[E(p)] = \Gamma(p)E(p)T,$ $\quad \Gamma(p) = A(p) + \frac{1}{E(p)}\frac{\partial D[E(p)]}{\partial E}$

➔ F-P equation: $\frac{\partial}{\partial t}f(t,\boldsymbol{p}) = \frac{\partial}{\partial p_i}\left\{\Gamma(p)p_i f(t,\boldsymbol{p}) + D(p)\frac{\partial}{\partial p_i}f(t,\boldsymbol{p})\right\}$

In terms of A and D, the F-P equations in pre- and post-points are the same!

**MH et al., PRE88,032138(2013)**

Langevin eqs. are not covariant! So updates of x, p need to be done in fluid rest frame when there's collective flow!

# Equilibrium limit check

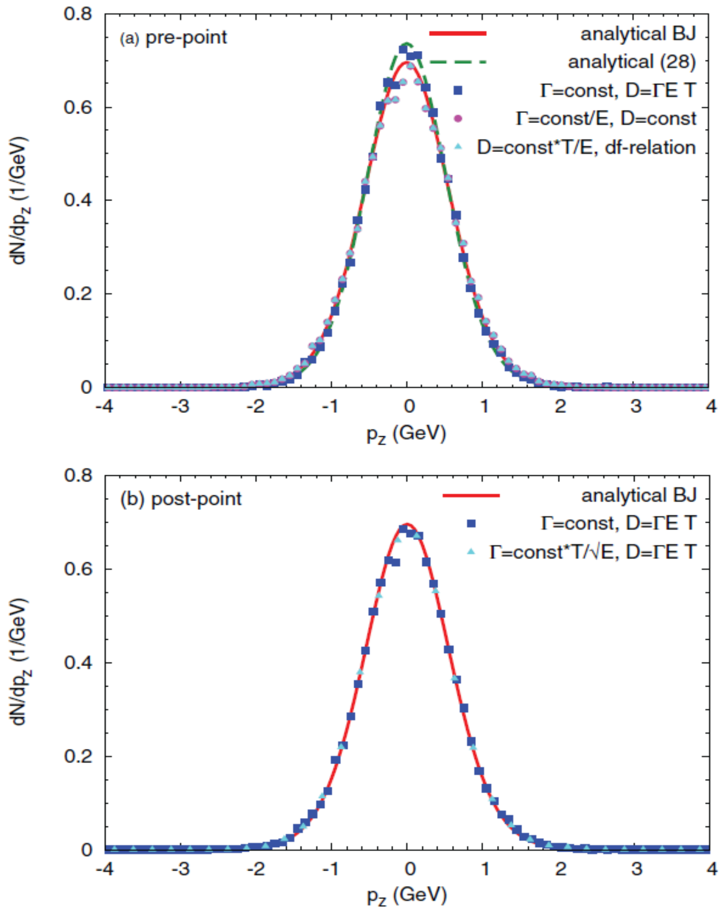



FIG. 1. (Color online) Distribution $dN/dp_z$ from Langevin simulations for heavy quarks with mass $m = 1.5$ GeV, diffusing in a static medium at temperature $T = 0.18$ GeV, compared to calculations with the corresponding analytical phase-space distributions: (a) prepoint Langevin scheme; (b) postpoint Langevin scheme. See text for more details.

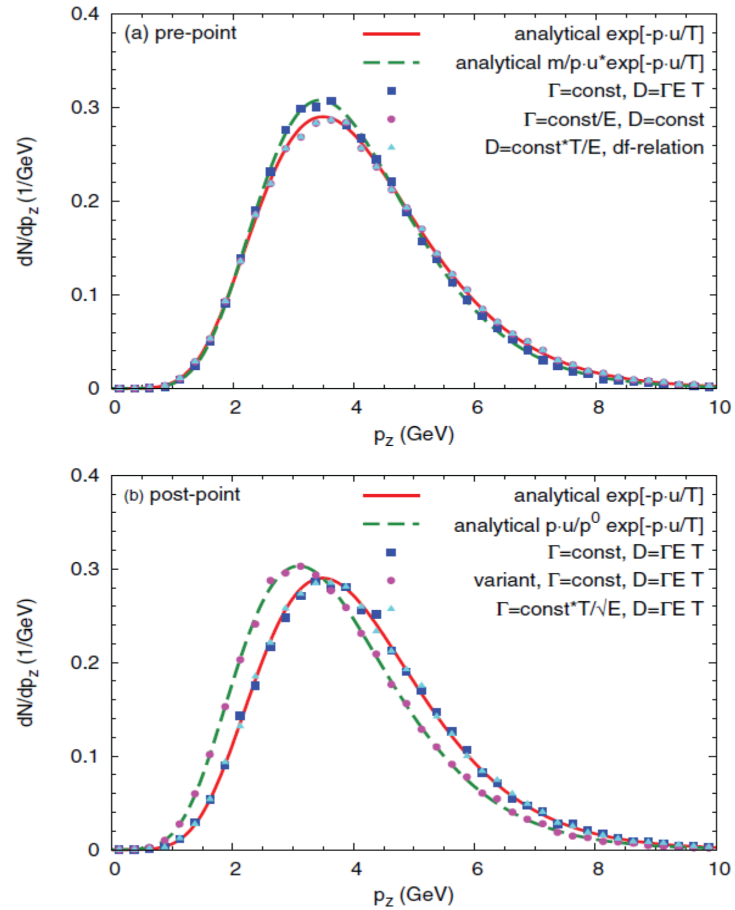



FIG. 2. (Color online) Langevin simulation results for heavy quarks ($m = 1.5$ GeV) diffusing in a flowing medium ($T = 0.18$ GeV, $v_z = 0.9$) compared to calculations with analytical phase-space distributions: (a) prepoint Langevin scheme; (b) postpoint Langevin scheme. The distribution obtained with a variant of the postpoint scheme and the corresponding blast-wave distribution are also shown. See text for more details.

# Equilibrium limit check (elliptic fireball)

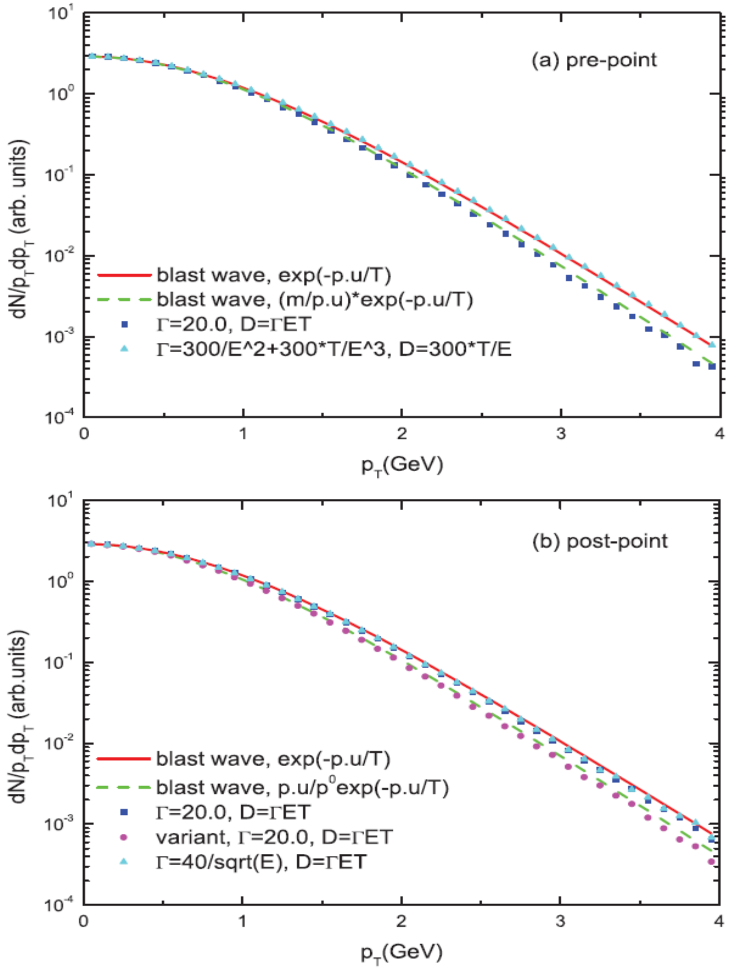



FIG. 3. (Color online) Langevin simulation of the $p_T$ spectrum of a heavy quark ($m = 1.5$ GeV) diffusing in a fireball ($T_i = 0.33$ GeV, $T_f = 0.18$ GeV), compared to the direct blast-wave calculations. (a) Prepoint Langevin scheme; (b) postpoint Langevin scheme. See text for more details.

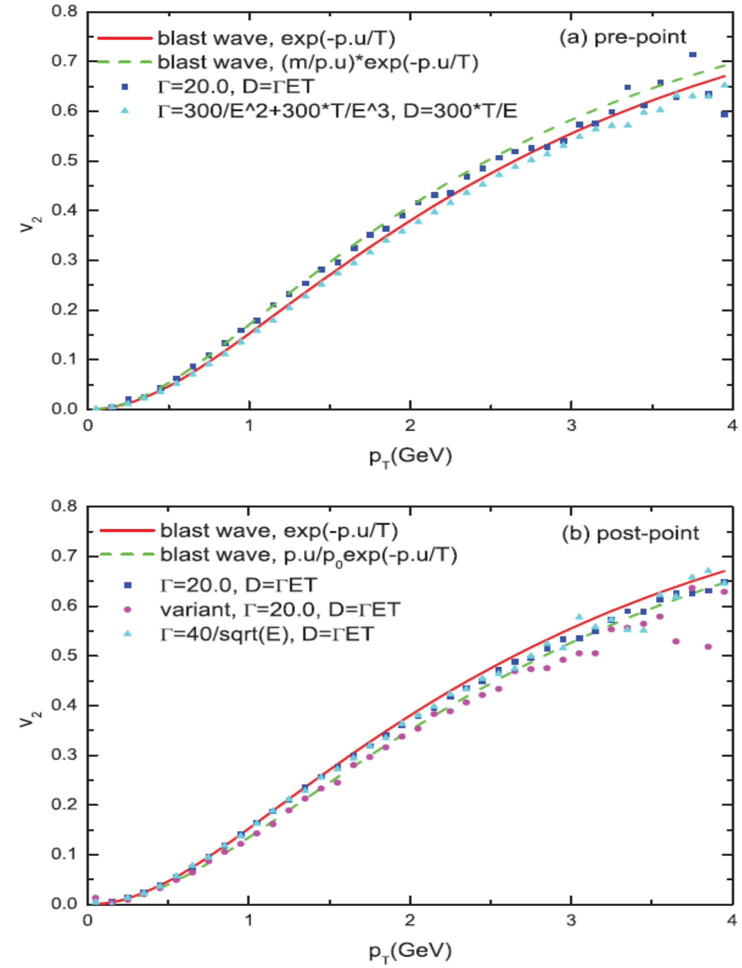
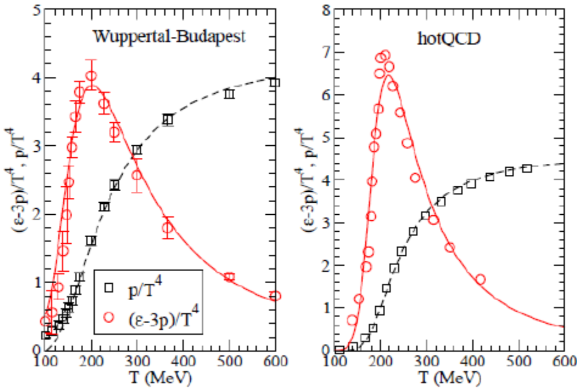
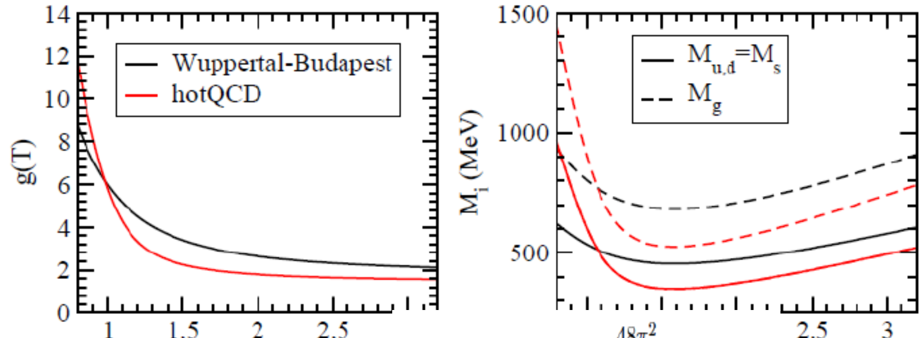



FIG. 4. (Color online) Langevin simulation results for $v_2(p_T)$ for a heavy quark ($m = 1.5$ GeV) diffusing in a QGP ($T_f = 0.18$ GeV) compared to direct blast-wave calculations with the flow field and temperature of the background medium (fireball). (a) Prepoint Langevin scheme; (b) postpoint Langevin scheme.

# Full Boltzmann implementation

- Catania group: quasi-particles for bulk fitted to lattice-EoS + HQ elastic scattering




$$g^2_{QP}(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left[\lambda\left(\frac{T}{T_c} - \frac{T_s}{T_c}\right)\right]^2}$$

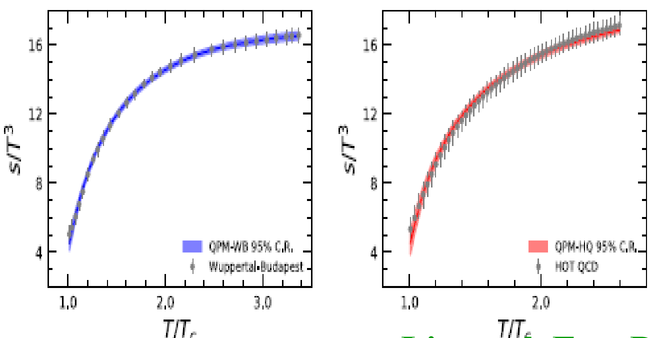
**Plumari et al., PRD, 84, 094004 (2011)**

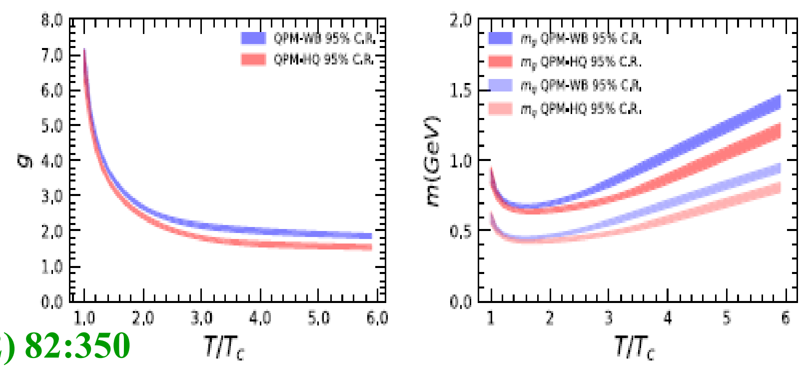
$$p^\mu \partial_\mu f(x,p) = C_{22}$$

**Das et al., PRC90, 044901(2014)**

$$C_{22} = \frac{1}{2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int\frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 |\mathcal{M}_{1'2'\to 12}|^2 (2\pi)^4\delta^{(4)}(p'_1+p'_2-p_1-p_2)$$
$$-\frac{1}{2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int\frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f_1 f_2 |\mathcal{M}_{12\to 1'2'}|^2 (2\pi)^4\delta^{(4)}(p_1+p_2-p'_1-p'_2)$$

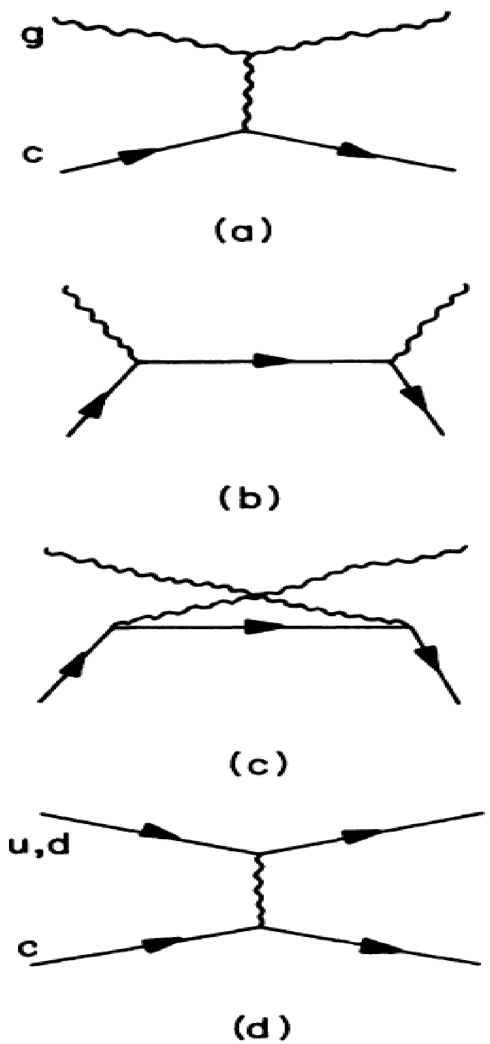
- Similarly by (Q)LBT with both elastic+radiative collisions: $p_1 \cdot \partial f_1(x_1, p_1) = E_1(C_{el} + C_{inel})$,




**Liu et al, Eur. Phys. J. C (2022) 82:350**

# HQ scattering in QGP: pQCD




Svetitski, PRD37, 2484(1988)

$$\sum |\mathcal{M}_a|^2 = 3072\pi^2\alpha_s^2 \frac{(m^2-s)(m^2-u)}{(t-\mu^2)^2} ,$$

$$\sum |\mathcal{M}_b|^2 = \frac{2048}{3}\pi^2\alpha_s^2 \frac{(m^2-s)(m^2-u)-2m^2(m^2+s)}{(m^2-s)^2}$$

$$\sum |\mathcal{M}_c|^2 = \frac{2048}{3}\pi^2\alpha_s^2 \frac{(m^2-u)(m^2-s)-2m^2(m^2+u)}{(m^2-u)^2}$$

$$\sum \mathcal{M}_a\mathcal{M}_b^* = \sum \mathcal{M}_b\mathcal{M}_a^* = 768\pi^2\alpha_s^2 \frac{(m^2-s)(m^2-u)+m^2(u-s)}{(t-\mu^2)(m^2-s)} ,$$

$$\sum \mathcal{M}_a\mathcal{M}_c^* = \sum \mathcal{M}_c\mathcal{M}_a^* = 768\pi^2\alpha_s^2 \frac{(m^2-u)(m^2-s)+m^2(s-u)}{(t-\mu^2)(m^2-u)} ,$$

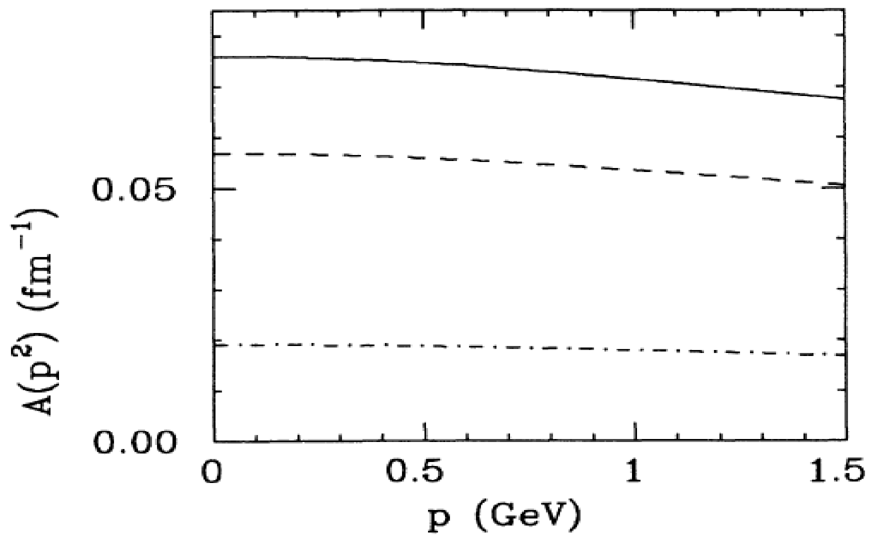
$$\sum \mathcal{M}_b\mathcal{M}_c^* = \sum \mathcal{M}_c\mathcal{M}_b^* = \frac{256}{3}\pi^2\alpha_s^2 \frac{m^2(t-4m^2)}{(m^2-u)(m^2-s)}$$

for gluon scattering, and

$$\sum |\mathcal{M}_d|^2 = 256N_f\pi^2\alpha_s^2 \frac{(m^2-s)^2+(m^2-u)^2+2m^2t}{(t-\mu^2)^2}$$

- t-channel scattering dominates, regularized by gluon screening mass $\mu_D$~gT

# LO-pQCD is far from enough



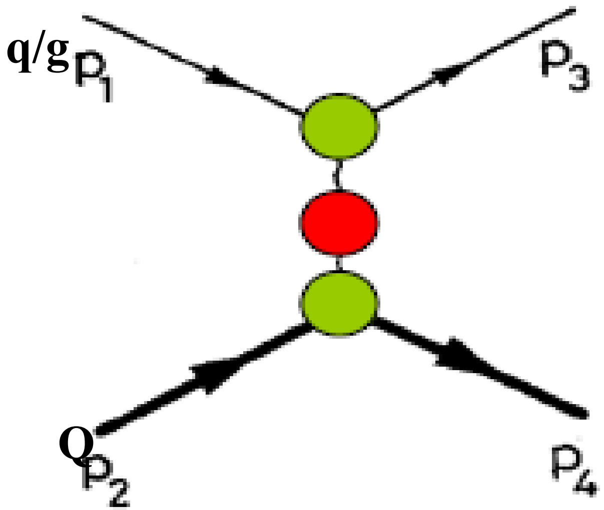

Svetitski, PRD37, 2484(1988)

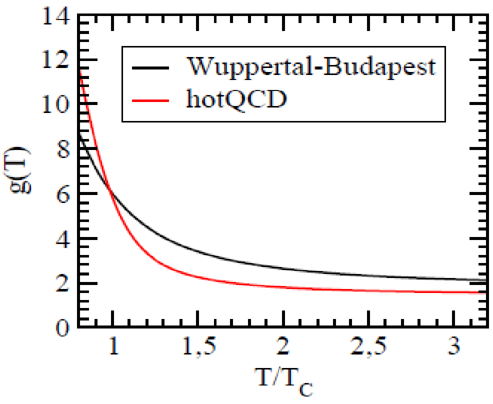
FIG. 2. Drag coefficient $A(p^2)$ at temperature $T=200$ MeV, assuming QCD coupling $\alpha_s=0.6$ and Debye screening mass $\mu=200$ MeV. The dashed-dotted curve is the contribution of quark and antiquark scattering, the dashed curve that of gluon scattering, and the solid line the sum of the two.

- Here, $\alpha_s$=0.6, $\mu_D$=200 MeV, $A \propto \alpha_s^2$. If using more realistic $\alpha_s$~0.3, A(p=0,T=200 MeV) ~0.02/fm ➔ need $\triangle t=1/A=50$ fm/c for c-quark thermalization >> QGP lifetime

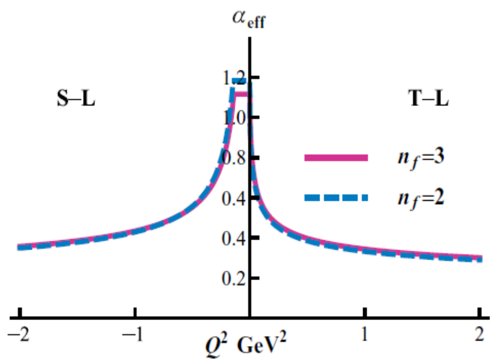
- LO-pQCD t-channel dominated by forward scattering, $d\sigma/d\Omega \sim 1/\theta^4$, not efficient for momentum isotropilization/thermalization ➔ resonant scattering more efficient

# Variants of LO-pQCD/Born diagram



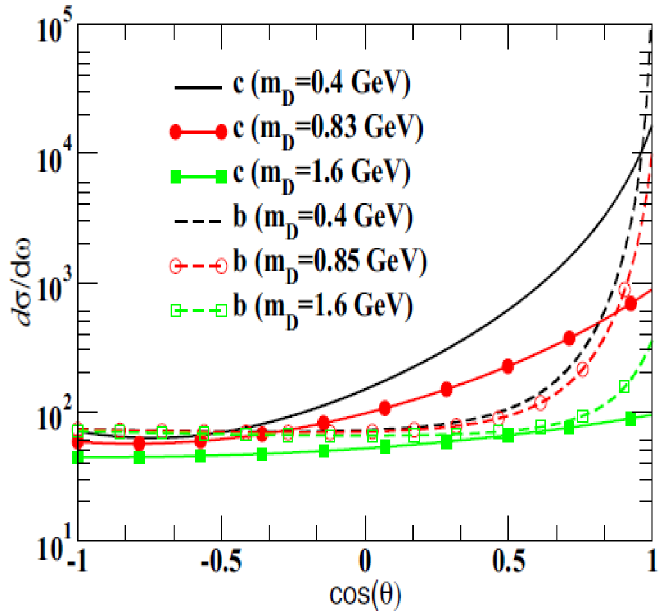

$$\frac{\alpha}{t} \rightarrow \frac{\alpha}{t-\mu^2} \rightarrow \frac{\alpha eff}{t-\kappa m_D^2(T)}$$

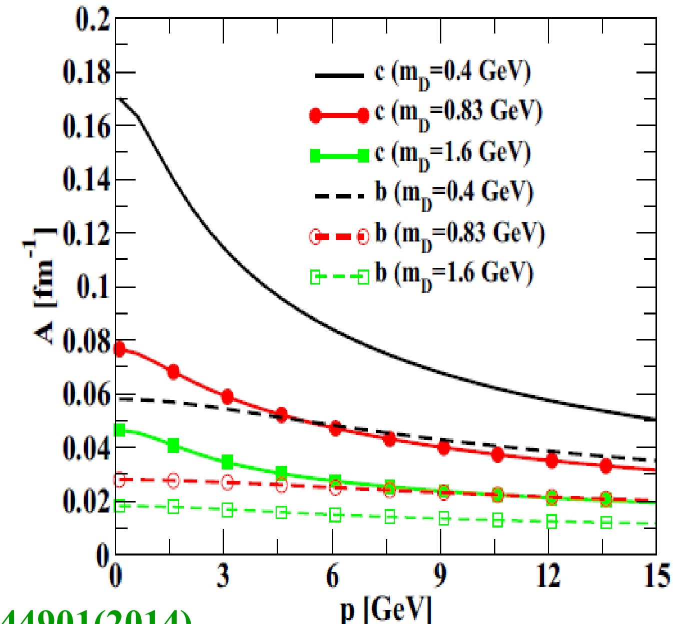



- Nantes group: reduced screening κ~ 0.11 to fit e-loss ➔ matrix element squared much enhanced (t<0) **Gossiaux et al., PRC 79, 044906 (2009)**

- Nantes/Catania group: quasi-particles with running/effective coupling const. $\alpha_{eff}$ ~1 at low T and/or low $Q^2$ ➔ inconsistent with LO approximation
**Das et al., PRC90, 044901(2014)**

# Dependence of charm drag on screening mass



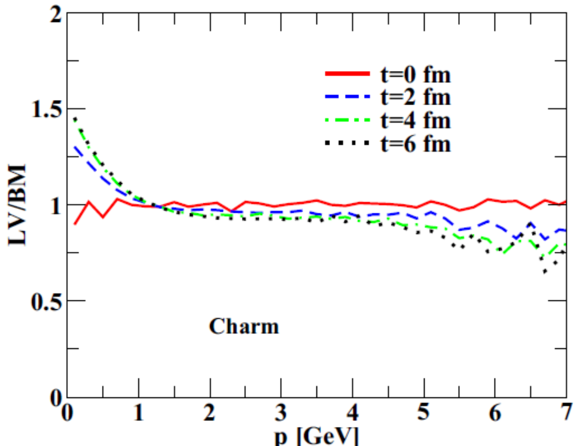

**Das et al., PRC90, 044901(2014)**

FIG. 1. (Color online) Angular dependence of the cross section for different values of $m_D$ for charm quarks (solid lines) and for bottom quarks (dashed lines).

FIG. 3. (Color online) Variation of drag coefficients with $p$ at $T=$ 400 MeV for different values of $m_D$.

- smaller $m_D$ ➔ more forward LO-pQCD
- Matrix element squared increases so does thermal relaxation rate

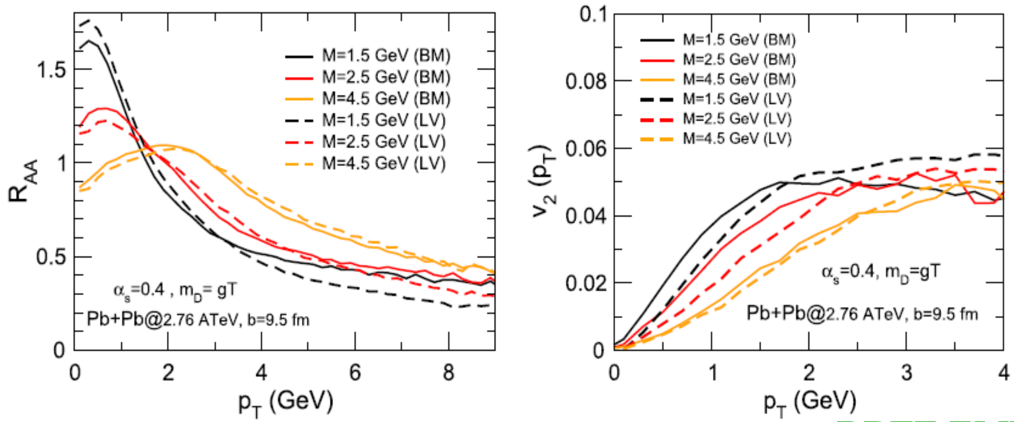
# Boltzmann vs Langevin using LO-pQCD




Das et al., PRC90, 044901(2014)

FIG. 6. (Color online) Ratio between the Langevin (LV) and Boltzmann (BM) spectra for charm quark as a function of momentum for $m_D = 0.4$ GeV

- At high $p_T$, Gauss distribution of e-loss underlying Langevin becomes less accurate ➔ deviation from full Boltzmann
- Charm quark $R_{AA}$ is ~25% smaller from Langevin than Boltzmann at high $p_T$ while v2 from Langevin a bit larger
- Discrepancies between Lagevin vs Boltzmann vanishes for bottom




RRTF-EMMI, NPA979(2018)21-86

Fig. 24. Nuclear modification factor (left panel) and elliptic flow (right panel) for heavy quarks in semi-central Pb+Pb($\sqrt{s_{NN}} = 2.76$ TeV) collisions (at $b = 9.5$ fm) for different values of the HQ mass, $M_Q$ (indicated by the different line colors), in a Boltzmann (solid lines) and in a Langevin approach (dashed lines).

# pQCD: NLO

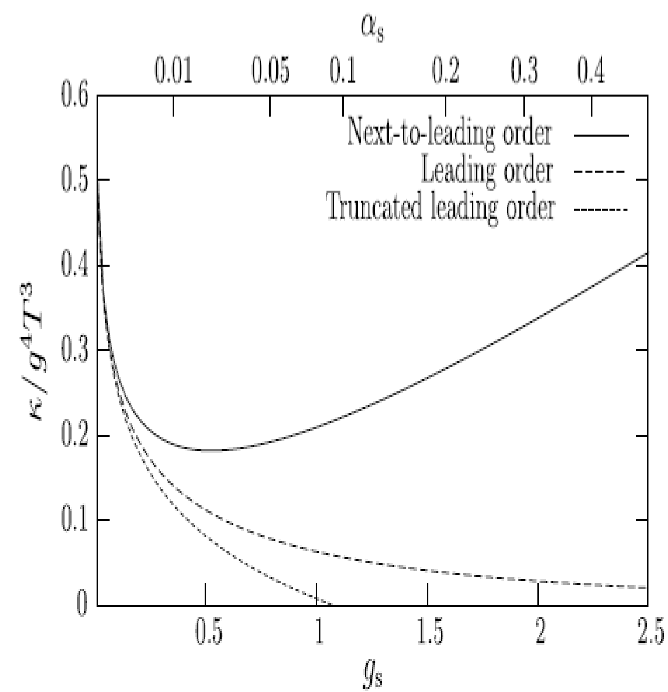


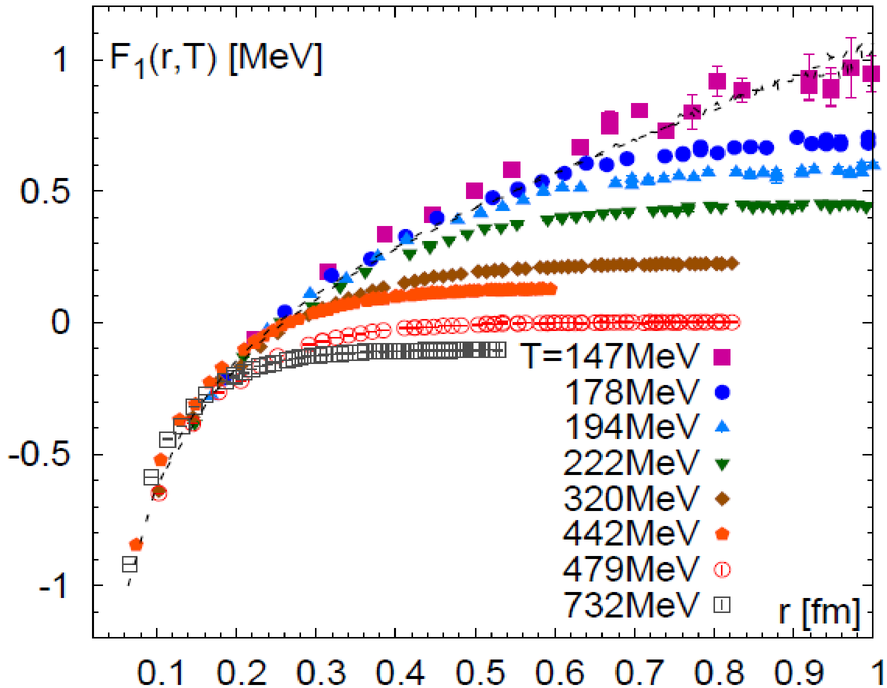

Fig. 20. Comparison of leading and next-to-leading order inverse heavy-quark diffusion coefficient, $\kappa/T^3 = 2/(\mathcal{D}_s T)$, scaled by the leading-order coupling constant dependence. The subleading corrections are large even at coupling values usually considered to be very small.

**Caron-Huot & Moore, JHEP02(2008)081**

- NLO >> LO ➔ poor convergence!

# Non-perturbative effects

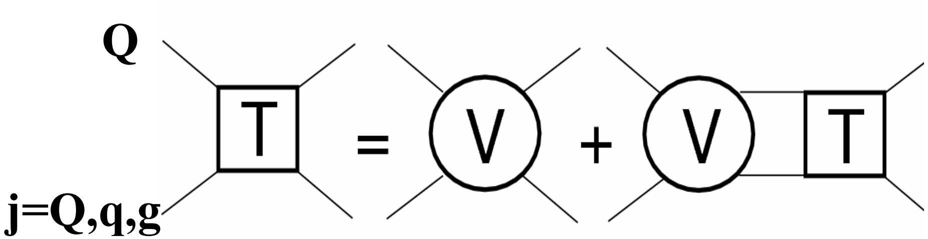
- Large running coupling at small $Q^2$ in LO-pQCD/Born diagram with one-gluon exchange ➔ strong Coulomb potential at large distance
- But at large distance, Q-Qbar potential is linear-confining term (in vacuum: $V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r$ )
- At finite T (<2*Tc) significant remnants linear part in Q-Qbar free energy [yet not potential]
  ➔ residual confining force may be important for HQ interactions in QGP
  ➔ non-perturbative potential model needed!

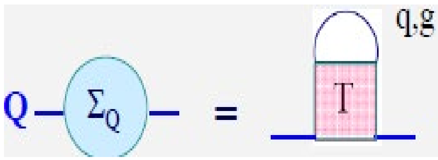



$$\exp(-F_1(r,T)/T) = \frac{1}{N}\langle \mathrm{Tr}[L^\dagger(x)L(y)]\rangle$$

- If there's only Coulomb➔ F<0
- F>0 for T<2*$T_c$, remnant of linear confining term

# T-matrix resuming HQ potential




- 4D Bethe-Salpeter ➔ 3D Lippmann-Schwinger with relativistic corrections

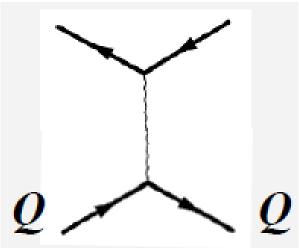
$$
\begin{aligned}
T_{l,a}(E;q',q) \\
&= \mathcal{V}_{l,a}(q',q) + \frac{2}{\pi}\int_0^\infty dk\,k^2\,\mathcal{V}_{l,a}(q',k) \\
&\times G_{12}(E;k)\,T_{l,a}(E;k,q)\{1-n_F[\omega_1(k)]-n_F[\omega_2(k)]\}
\end{aligned}
$$

$$G_{12}^{\mathrm{Th}}(E;q) = \frac{m(q)}{E-\omega^q(q)-\omega^Q(q)-\Sigma_q-\Sigma_Q},$$

$$G_{12}^{\mathrm{BbS}}(E;q) = \frac{2\,m(q)\,(\omega^q(q)+\omega^Q(q))}{E^2-[\omega^q(q)+\omega^Q(q)+\Sigma_q+\Sigma_Q]^2}$$

$$m(q) = \frac{m_q\,m_Q}{\omega^q(q)\,\omega^Q(q)}.$$

Riek et al., 2010





- **momentum transfer**
  $q^2 = q_0^2 - \vec{q}^{\,2} \approx -\vec{q}^{\,2}$
  $q_0 \sim \vec{q}^{\,2}/2m_Q << |\vec{q}\,|$

➔ static/space-like potential interactions, with relativistic corrections

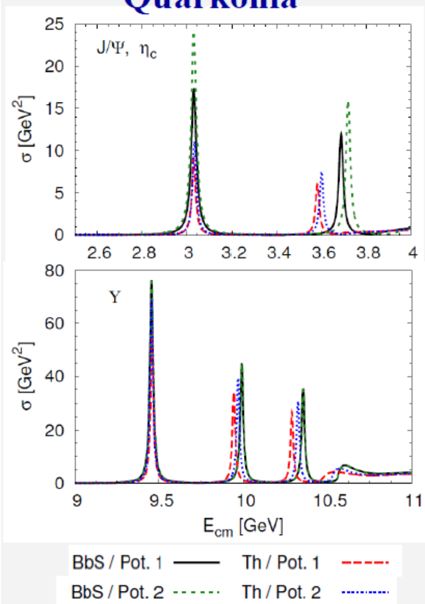
- **single heavy quark in QGP**
  $p_{\mathrm{th}}^2 \sim 2m_Q T >> T^2$

➔ soft interactions q~T << $p_Q$

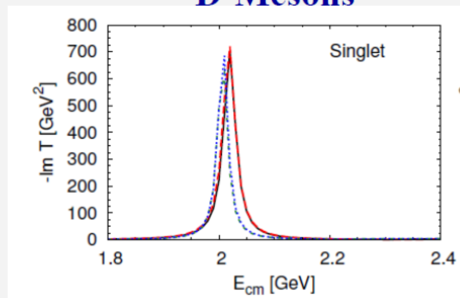
- Soft & (approximately) static Q-Qbar and Q-q/g interactions in QGP
- Common description of heavy quarkonia and single Q transport
- Thermodynamic T-matrix (bound states, scattering states, and resummation)

# T-matrix: vacuum constraints

## Quarkonia




## D-Mesons




- no hyperfine splitting
- (bare) masses adjusted to ground state
- ~ **±50 MeV** accuracy

| | | BbS-scheme | Th-scheme |
|---|---|---|---|
| Potential 1 | $m_c^0$ | 1.355 GeV | 1.264 GeV |
| | $m_b^0$ | 4.712 GeV | 4.662 GeV |
| Potential 2 | $m_c^0$ | 1.402 GeV | 1.293 GeV |
| | $m_b^0$ | 4.718 GeV | 4.668 GeV |

$m_q = 0.4$ GeV, $m_s = 0.55$ GeV

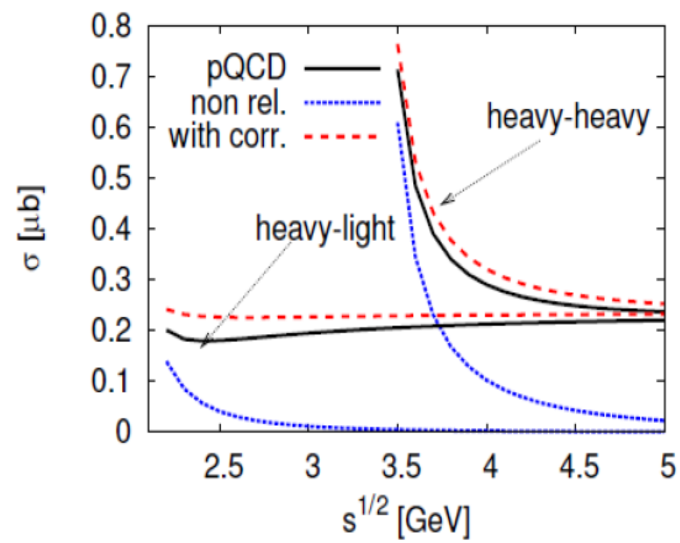
**Riek et al., 2010**

➔ vacuum spectroscopy well reproduced

➔ Born approximation compared to pQCD

➔ Breit correction essential

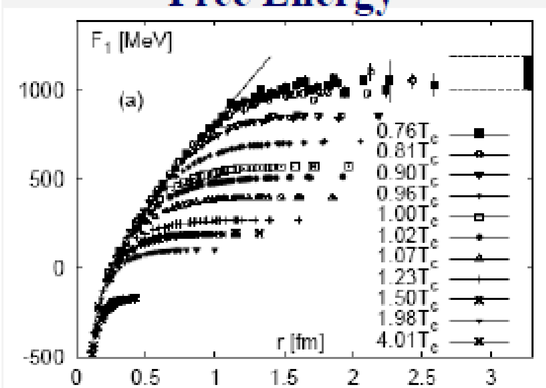
$$V_{Q1Q2}(r) \rightarrow V_{Q1Q2}(r) \ (1 - v_1 \cdot v_2)$$



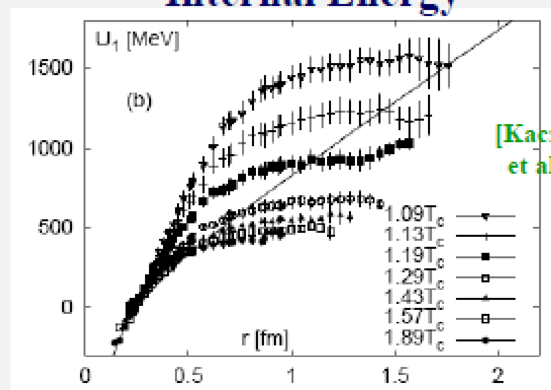
# In-medium HQ potential: F or U

$$F_1(r,T) = U_1(r,T) - T\,S_1(r,T)$$

**Free Energy**




**Internal Energy**




[Kaczmarek et al '05]

- "**weak**" $Q\bar{Q}$ potential
- "**strong**" $Q\bar{Q}$ potential, $U = \langle H_{int} \rangle$

- **F, U, S thermodynamic quantities**
- **Entropy**: many-body effects

(a) Free energy $\mathbf{F_1}$
=> weak $\bar{\mathbf{Q}}\mathbf{Q}$ potential,
small **Q** "selfenergy" $\mathbf{F_1(r{=}\infty,T)/2}$

(b) Internal Energy $\mathbf{U_1}$ ($\mathbf{U = \langle H_{int} \rangle}$)
=> strong $\bar{\mathbf{Q}}\mathbf{Q}$ potential,
large **Q** "selfenergy" $\mathbf{U_1(r{=}\infty,T)/2}$

- U as proxy for HQ potential is favored by (a) Υ(1S) suppression & (b) charm quark $v_2$
  ➔ this will be discussed later

# Self-consistent HQ potential: beyond F or U

- For static Q-Qbar in medium: F(Q-Qbar)= F(Q-Qbar + medium) – F(medium)

$$F_{Q\bar{Q}}(T,r) = -T\ln(\tilde{Z}_{Q\bar{Q}}) = (-T\ln(Z_{Q\bar{Q}})) - (-T\ln(Z))$$

$$\tilde{Z}_{Q\bar{Q}} = \frac{Z_{Q\bar{Q}}}{Z} = \frac{{}_n\langle n|\chi(r_2)\psi(r_1)e^{-\beta H}\psi^+(r_1)\chi^+(r_2)|n\rangle}{Z}$$

$$= G^>(-i\tau, r_1, r_2|r_1, r_2)|_{\tau=\beta} = \tilde{G}^>(-i\tau, r\,)|_{\tau=\beta}$$

$$r = r_1 - r_2, \qquad \beta = 1/\mathrm{T}$$

$$\boldsymbol{F_{Q\bar{Q}}(T,r) = -T\,ln\left(\tilde{G}^>(-i\tau,r)\right)|_{\tau=\beta}}$$

**McLerran and Svetitsky, PRD 24 (1981) 450**

**Blazoit et al., NPA806 (2008) 312–338**

- Two-particle Green's function: full vs free, z=-iτ: time → energy representation

$$G(z,r) = \frac{1}{[\hat{G}_0(z)]^{-1} - V(z,r)} \qquad \left[G_0^{(2)}\right]^{-1} = z - \Sigma(z)$$

$$\tilde{G}^>(-i\tau,r) = \int_{-\infty}^{\infty} dE\, \frac{-1}{\pi}\mathrm{Im}\left[\frac{1}{\left[\hat{G}_0(E+i\epsilon)\right]^{-1} - V(E+i\epsilon,r)}\right] e^{-\tau E}$$

- Q-Qbar free energy F related to Q-Qbar potential and self-energies

**Liu&Rapp, NPA 941 (2015) 179**

$$F_{Q\bar{Q}}(r;T) = -T\ln\left[\int_{-\infty}^{\infty}\frac{dE}{\pi}e^{-\beta E}\mathrm{Im}\left(\frac{-1}{E - \tilde{V}(r;T) - 2\Delta m_Q - \Sigma_{Q\bar{Q}}(E,r;T)}\right)\right]$$

➔ weakly coupled limit, self-energy Σ→iε ➔δ(E-V-2$m_Q$) ➔ V=F
strongly coupled limit, large ImΣ ➔ need a stronger V to match F

# Self-consistent HQ potential

- Now heavy-light T-matrix with full spectral functions (off-shell Q, q, g)

$$T^a_{Qi}(E,\mathbf{p},\mathbf{p}') = V^a_{Qi} + \int \frac{d^3k}{(2\pi)^3} V_{Qi}(\mathbf{p},\mathbf{k}) G^0_{Qi}(E,\mathbf{k}) T^a_{Qi}(E,\mathbf{k},\mathbf{p}')$$

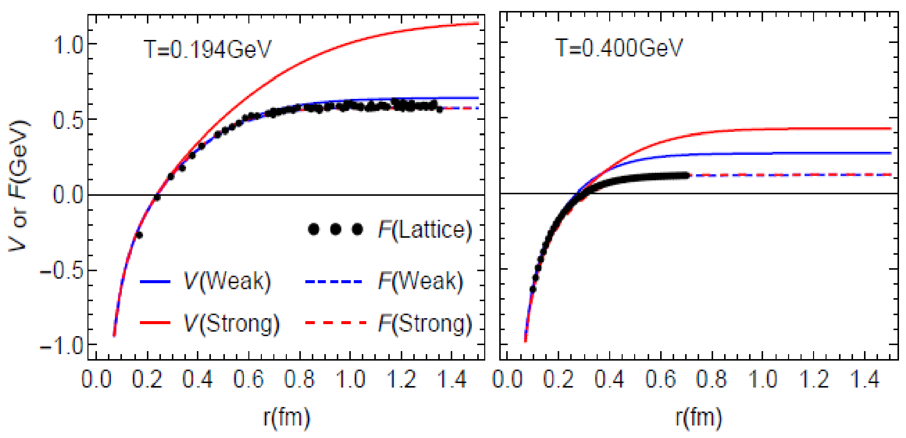
$$G^0_{iQ}(E,\mathbf{k}) = \int d\omega_1 d\omega_2 \frac{\rho_i(\omega_1,\mathbf{k})\rho_Q(\omega_2,\mathbf{k})}{E-\omega_1-\omega_2+i\epsilon}[1 \pm f_i(\omega_1) - f_Q(\omega_2)]$$

- Single Q spectral function related with T-matrix again

$$G_Q = 1/[\omega - \omega_Q(k) - \Sigma_Q(\omega,k)] \ , \quad \rho_Q = -\frac{1}{\pi}\mathrm{Im}\ G_Q$$

$$\Sigma_Q(\omega,k) = \int \frac{d^3p_2}{(2\pi)^3} \int d\omega_2 \sum_i \rho_i(\omega_2,p_2) \int \frac{dE}{\pi} \frac{\mathrm{Im}T^{al}_{Qi}(E,\mathbf{k},\mathbf{p_2})}{E-\omega-\omega_2+i\epsilon}[f_i(\omega_2) \mp f(E)]$$

➔ Finally aided with a relation between 2-particle(Q-Qbar) self-energy and 1-particle(Q) self-energy, self-consistent solution of HQ potential and T-matrix with spectral functions /off-shell effects can be performed by fitting the lattice-QCD free energy

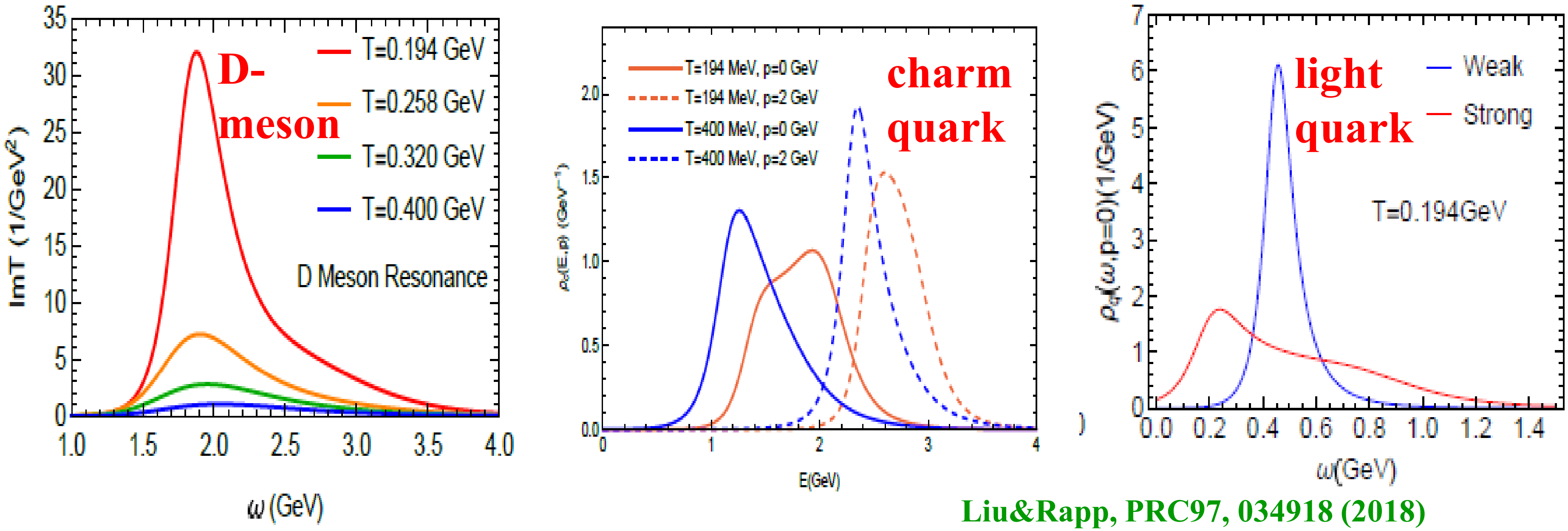



$$V_a(r) = C_a \frac{\alpha_s}{r} e^{-\mu_D T} + \frac{\sigma}{\mu_s}(1 - e^{-\mu_s T})$$

Solutions not unique:
weakly coupled V~ F
VS
Strongly coupled V➔ long range remnant confining Force, not much screening

**Liu&Rapp, NPA 941 (2015) 179**

# Charm-light T-matrix

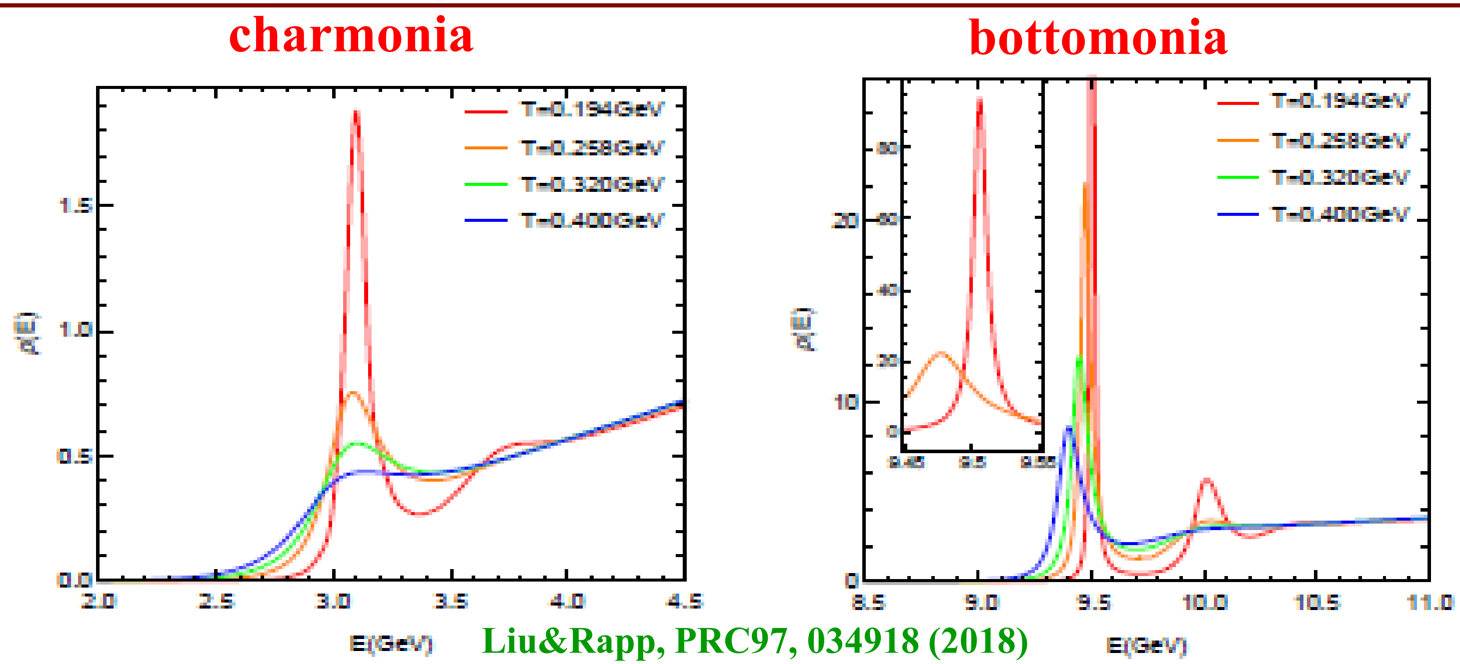



Liu&Rapp, PRC97, 034918 (2018)

- In strongly coupled solution: at low T~194 MeV,
  light q width~600 MeV > nominal thermal mass ~ 500 MeV ➔ melted, no quasiparticles
  charm c collisional width similar 500-700 MeV< mass ➔ still good Brownian markers
  [shoulder in c-spectral due to off-shell scattering with thermal light quarks]

- Charm-light ImT: as T is reduced ➔ V becomes stronger (less screeded) ➔ broad D-meson linear near-threshold resonances ➔ enhancing c-quark thermal relaxation rate [resonances also form in color-antitriplet diquark cq/qq channels]

# Charmonia/Bottomonia spectral functions



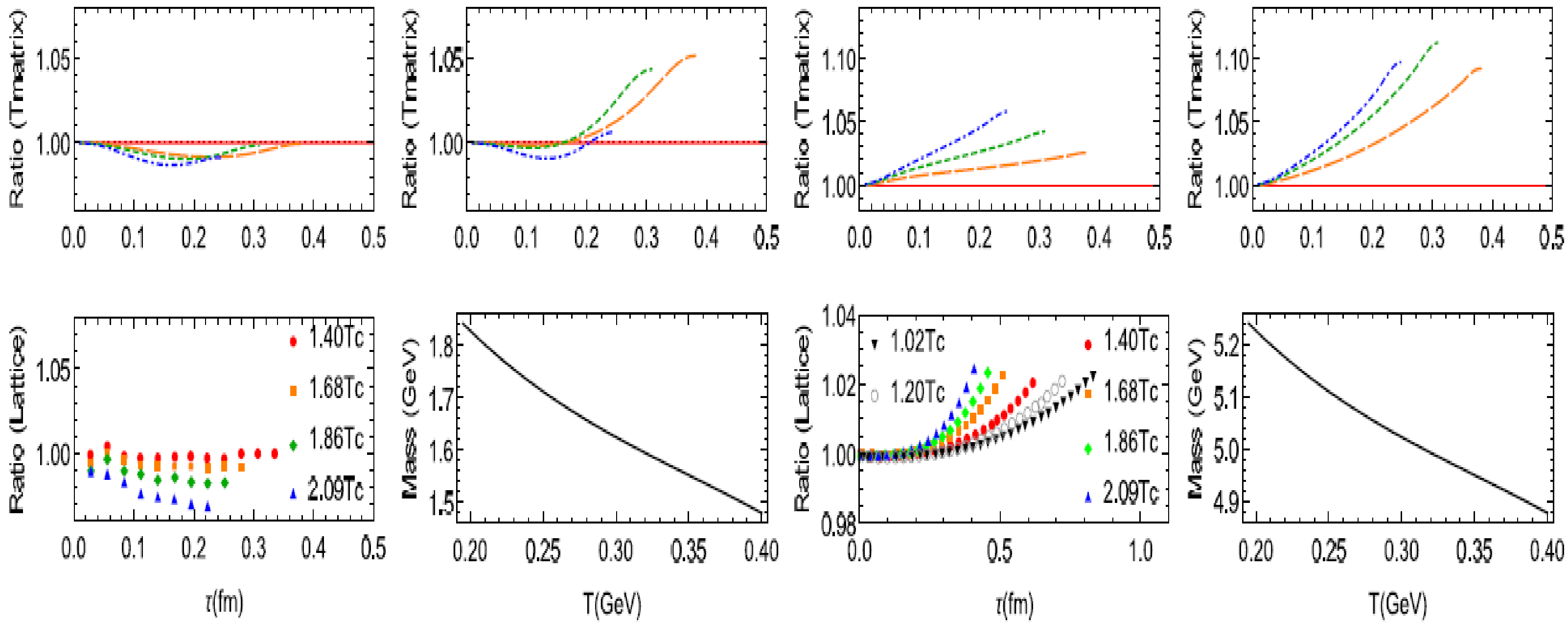

Liu&Rapp, PRC97, 034918 (2018)

Strongly coupled solution with interference effect

- A strong potential V(r,T) ➔ deep bound states for ground states J/ψ, $\eta_c$ [dissolve at T~300 MeV] and especially for Y(1S) and $\eta_b$ [persist for T>400 MeV]
  ➔ their spectral width significantly < $2^*\Gamma_c$ or $2^*\Gamma_b$ (strong interference effect)
- For excited states ψ(2S), Y(3S) [dissolves at T~194 MeV] and Y(2S) [dissolves at T>200 MeV] ➔ their $2^*\Gamma_c$ or $2^*\Gamma_b$ > $E_{binding}$ thus cannot survive
  spectral width ~ $2^*\Gamma_c$ or $2^*\Gamma_b$ (interference effect small) MH et al, 2204.09299

Heavy quarkonia dissociated not by pure static screening (which is rather weak at T not too far from Tc), but continuously by collisional widths due to single Q width

# Constraint from lattice correlator




- Euclidean point-to-point correlator accessible to lattice

**Liu&Rapp, PRC97, 034918 (2018)**

$$G_\alpha(\tau, P; T) = \int d^3 r e^{i\mathbf{P}\cdot\mathbf{r}} \langle j_\alpha(\tau, \mathbf{r}, j_\alpha(0,0)\rangle$$

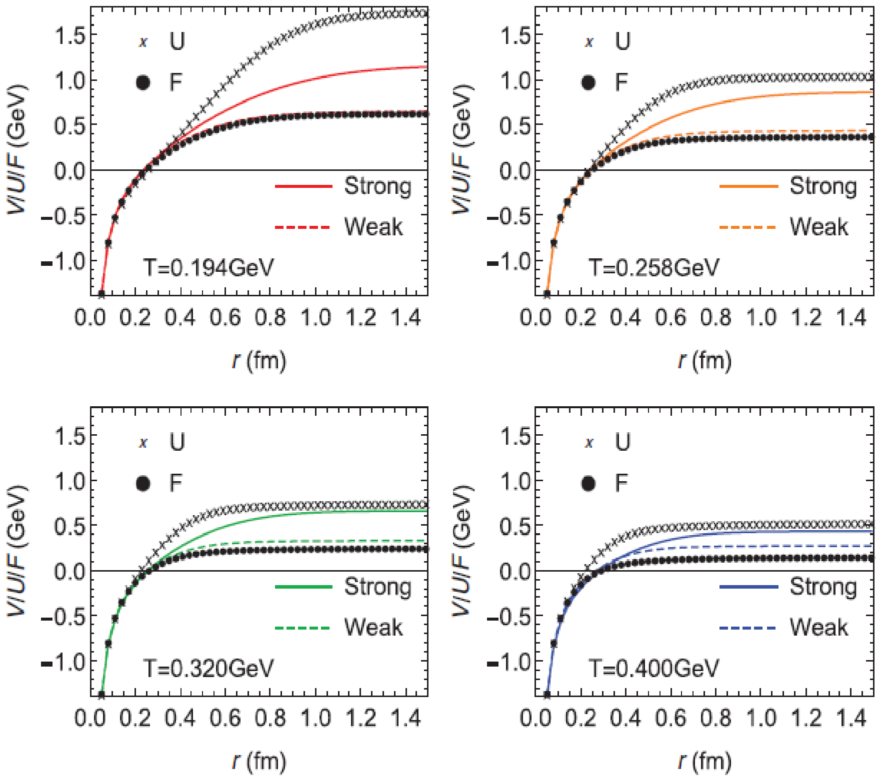
$$\mathcal{K}(\tau, E; T) = \cosh[E(\tau - 1/2T)] / \sinh[E/2T].$$

$$G_\alpha(\tau; T) = \int_0^\infty dE \rho_\alpha(E, T) \mathcal{K}(\tau, E; T) \qquad R_G^\alpha = \frac{G_\alpha(\tau; T)}{G_\alpha^{\rm rec}(\tau; T)}$$

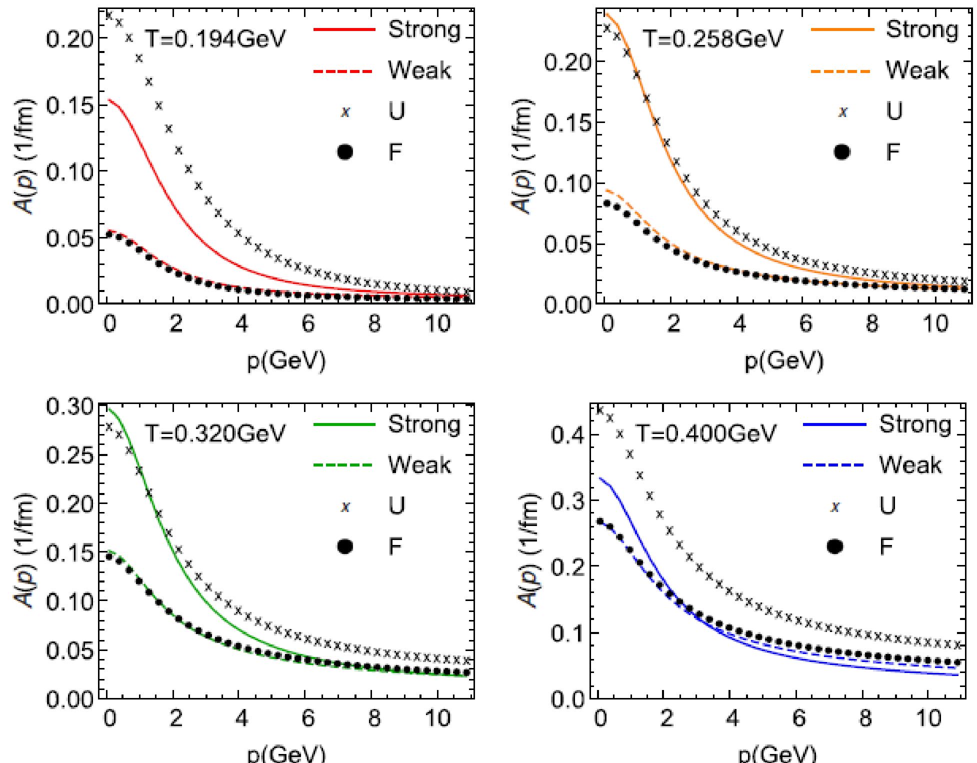
- Spectral functions with interference effects (1st, 3rd columns) a bit better than without(2nd, 4th), but not decisive ➔ point-split/extended correlators needed to better exhibit spectral modifications **Petreczky et al., PRD100(2019)074506**
- T-dependence of correlator mainly from zero-mode (transport peak), not melting of bound states **Petreczky Eur. Phys. J. C (2009) 62: 85–93**

# Charm quark relaxation rate/drag

U vs F vs V(strong vs weak solution)



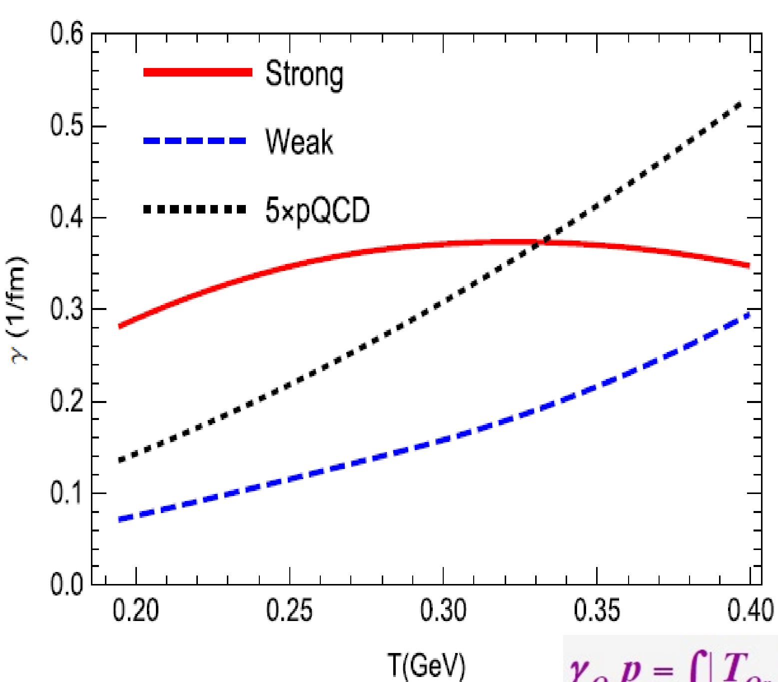

corresponding c-quark thermal relaxation rate

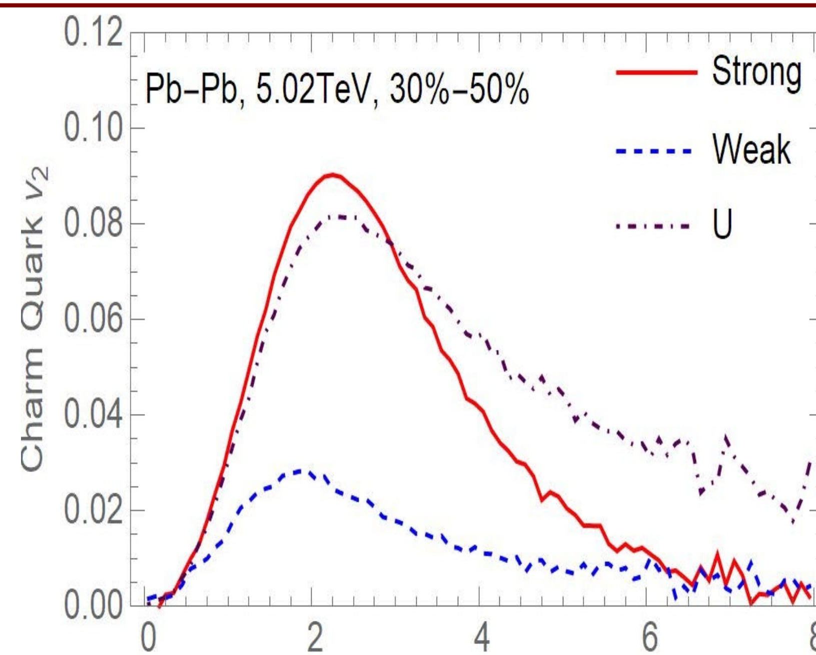



Liu,He&Rapp, PRC99, 055201 (2019)

- F ~ V(weak) ➔ similar A(p,T)
- U & V(strong) ➔ stronger A(p,T) especially at low T and low p [resonant interaction]
- V(strong) features long-range remnant confining force (-dV/dr) to encompass more near-by thermal partons ➔ instrumental for large A(p=0)!

# Charm quark A(p=0,T)



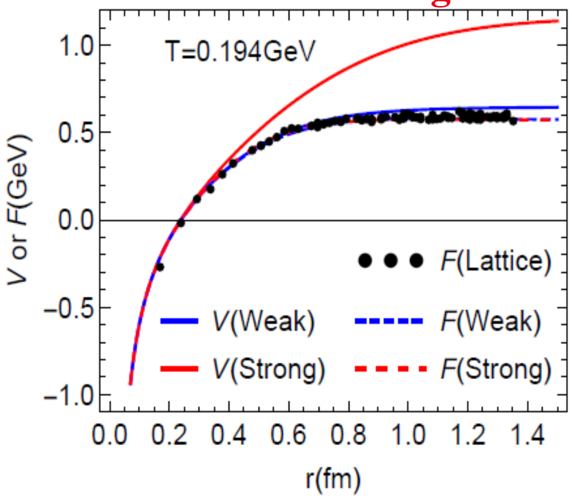

$$\gamma_Q\, p = \int |T_{Qq}|^2\,(1-\cos\theta)\; f^{q,g}$$

- γ=A(p=0,T) ~ 0.3/fm from strongly coupled solution ➔ thermal relaxation time τ ~ 3 fm/c for very low p charm quarks (p-dependence converging to pQCD at high p)
- 5*LO-pQCD is still not enough; weakly coupled interaction also far off
- Strongly coupled solution γ rather stable vs T: resonant enhancement at low T
- Charm quark $v_2$ at $p_T$~ 2 GeV: sensitive measure of charm coupling strength

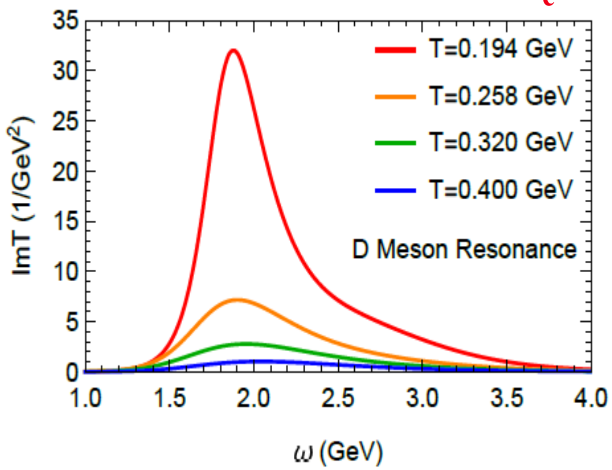
Liu,He&Rapp, PRC99, 055201 (2019)

# Landscape of T-matrix HQ & quarkonium interactions

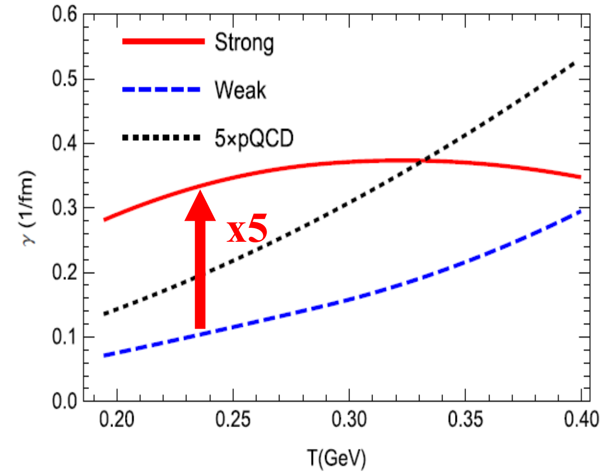
**Remnant confining force**



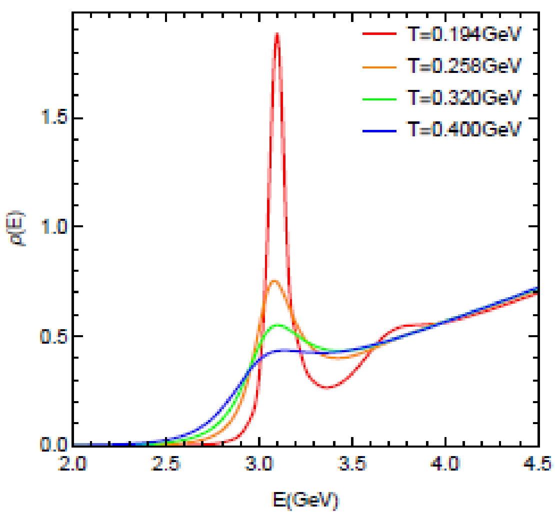

**Resonances form near $T_c$**

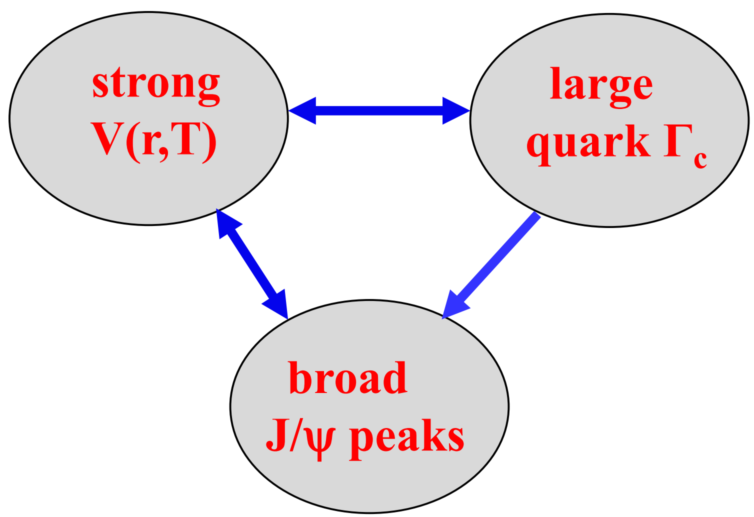



**Accelerates thermalization**

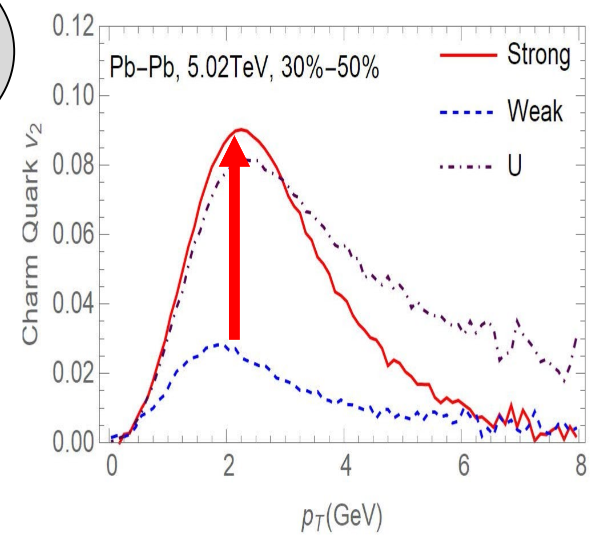



**J/ψ survives in QGP**




**Coupling strength ↔ HQ $v_2$**



# Comparison of A(p,T) in different models

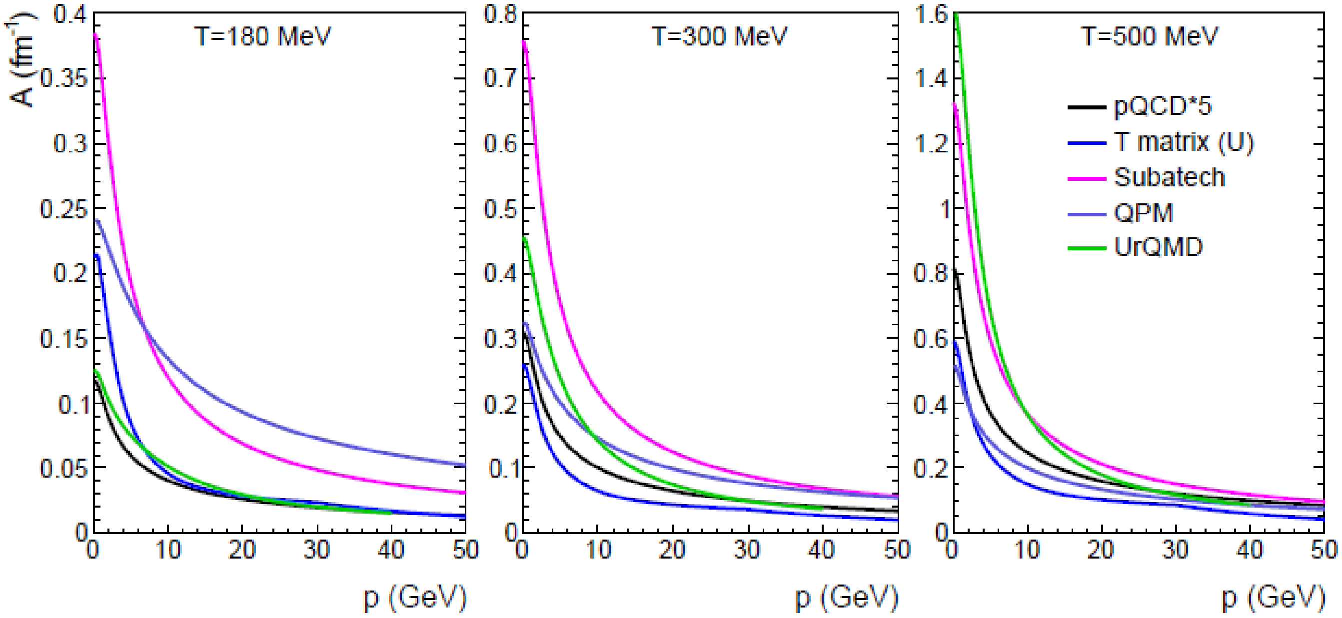



Figure 3.1: Friction coefficients for charm-quark diffusion in the QGP as a function of three-momentum for three different temperatures from various model calculations: black lines: pQCD Born diagrams with $\alpha_s$=0.4, multiplied with and overall $K$-factor of 5, blue lines: $T$-matrix results using the internal-energy ($U$) as potential proxy [20, 89], pink lines: pQCD with running coupling constant and reduced Debye mass [24, 25], purple lines: quasiparticle model with coupling constant fitted to the lQCD EOS [63], and green lines: $D$-meson resonance model [26]; figure taken from Ref. [61].

- Subatech/Nantes (Born diagram with running coupling and reduced $m_D$) largest at low p – driven by large coupling strength for soft momentum transfers
- T-matrix with U-potential smaller at low p, and drop-off vs p due to transition from a resumed string force at large distance to color-Coulomb force at small distances
- QPM(Catania) comparable to T-matrix at low p, but much flatter toward high p > Nates at T~180 MeV

# Lattice-QCD HQ diffusion

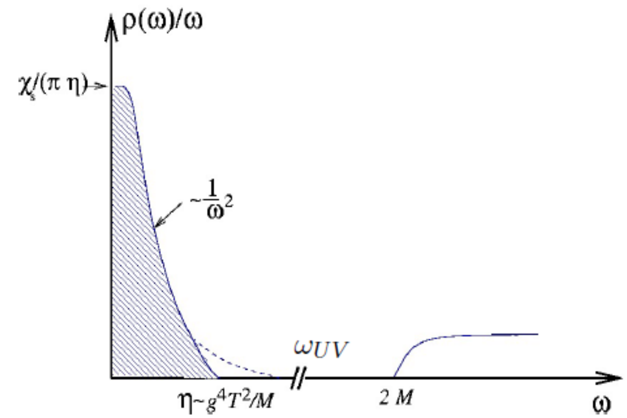
- Vector meson spectral function encoding a transport peak

$$\rho_V^{\mu\nu}(\omega) \equiv \int_{-\infty}^{\infty} dt e^{i\omega t} \int d^3x \left\langle [\hat{J}^\mu(t,\vec{x}), \hat{J}^\nu(0,\vec{0})] \right\rangle$$

$$\sum_i \frac{\rho_V^{ii}(\omega)}{\omega} \simeq 3\chi_2^q \frac{T}{M} \frac{\eta}{\eta^2+\omega^2}, \quad \omega < \omega_{UV}, \quad \eta = \frac{T}{M}\frac{1}{D}$$

Petreczky, Teaney, PRD 72 ('06) 014508

↑ drag constant




- HQ drag coefficient ~ width of the peak in the limit ω→0

$$\kappa = 2\, MT\eta = 2\, T^2/Ds$$

For large quark mass the transport peak is very narrow even for strong coupling and its difficult to reconstruct it accurately from Euclidean correlator calculated on the lattice

TABLE V. Estimated ranges for $\eta/T$ using the thermal ratio $R^{2,0}$ and resultant $2\pi TD$ with a mass of $M_c = 1.28$ GeV and $M_b = 4.18$ GeV. For some temperatures, the method did not work out to yield a result.

| | Charm | | Bottom | |
|---|---|---|---|---|
| $T/T_c$ | $\eta/T$ | $2\pi TD$ | $\eta/T$ | $2\pi TD$ |
| 1.1 | – | – | – | – |
| 1.3 | <0.27 | >7.48 | – | – |
| 1.5 | 0.85–2.78 | 0.84–2.73 | – | – |
| 2.25 | 3.32–5.28 | 0.66–1.05 | 0.29–1.10 | 0.97–3.66 |

**Ding et al., PRD104, 114508 (2022)**

- Using mid-point correlator (τT=0.5) and a model spectral function, Ding et al. extracted a larger drag for charm than bottom at T=2.25Tc $\rho_{ii}^{\text{mod}}(\omega) = A\rho_V^{\text{pert}}(\omega - B)$
- In the heavy quark limit, Ds=T/mγ independent of heavy quark mass

# Lattice-QCD HQ diffusion

- In the heavy quark limit (quenched approximation) ➔ chromo-electric field correlators

$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{ReTr}\left[U(\beta,\tau)\, gE_i(\tau,\vec{0})\, U(\tau,0)\, gE_i(0,\vec{0})\right]\right\rangle}{\left\langle \mathrm{ReTr}[U(\beta,0)]\right\rangle}$$

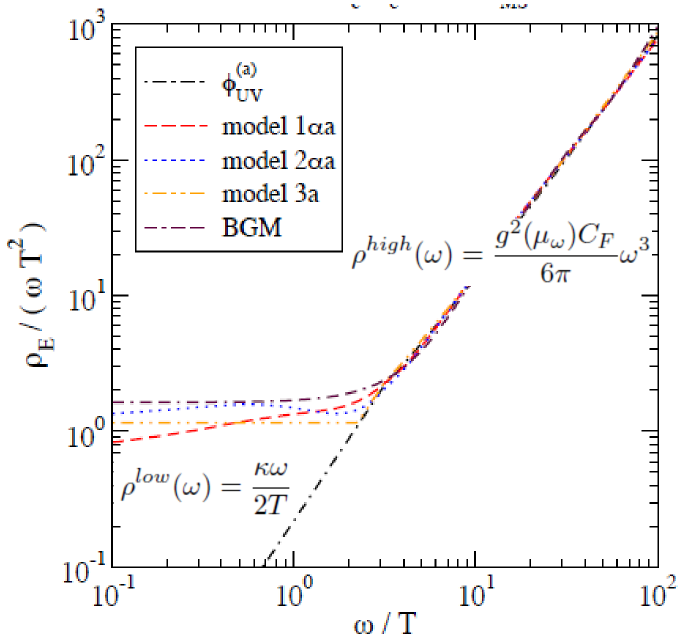
Transport coefficient ~ intercept of the spectral function not its width

$$\kappa = \lim_{\omega\to 0} \frac{2T}{\omega}\rho_E(\omega)$$

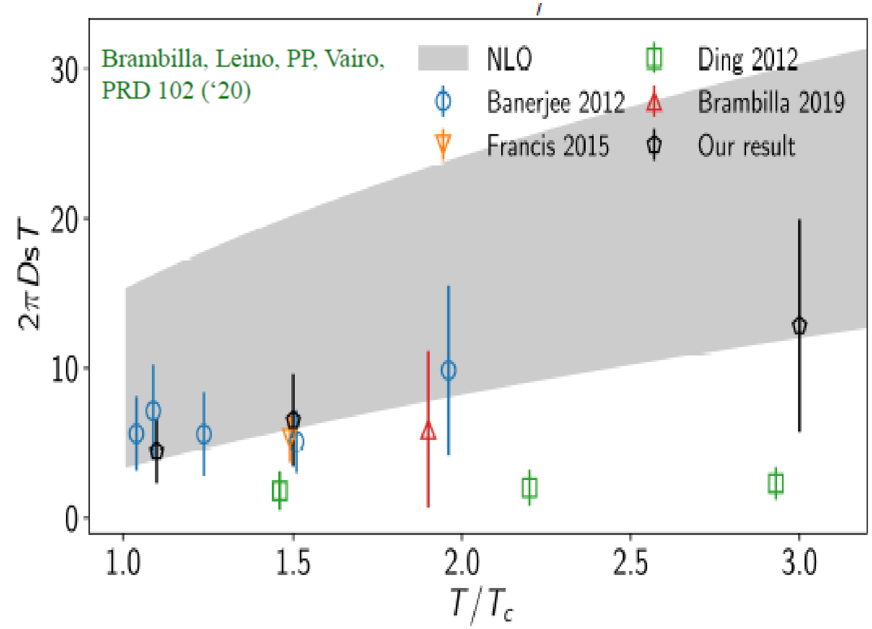
$$G_E(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho_E(\omega)\frac{\cosh\left(\tau - \frac{1}{2T}\right)\omega}{\sinh\frac{\omega}{2T}}$$

Caron-Huot, Laine, Moore, JHEP 0904 ('09) 053

Francis, Kaczmarek, Laine, et al, PRD 92 ('15) 116003



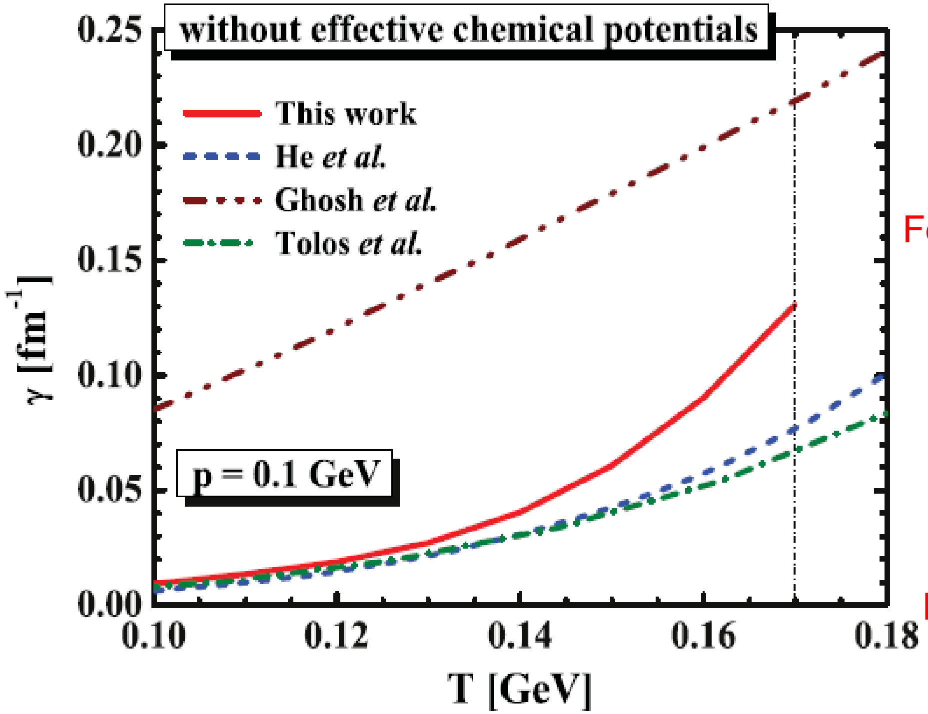

- Can lattice go to finite momentum ?



# D-meson diffusion in HRG

- D mesons diffusing in HRG, via interactions with light hadrons

Tolos et al., PRC90, 054909(2014)




$$A(p,T) = \frac{\gamma_h}{2E_D}\int \frac{1}{(2\pi)^9}\frac{d^3q}{2E_h}\frac{d^3p'}{2E'_D}\frac{d^3q'}{2E'_h} f^h(E_h;T) \times \overline{|\mathcal{M}_{Dh}|^2}(2\pi)^4\delta^{(4)}(p+q-p'-q')\left(1-\frac{\vec{p}\cdot\vec{p}\,'}{\vec{p}^2}\right) \equiv \left\langle\!\left\langle 1-\frac{\vec{p}\cdot\vec{p}\,'}{\vec{p}^2}\right\rangle\!\right\rangle$$

For a pion gas (dominates at low T~100 MeV):

- Laine (2011) heavy-meson ChPT➔ $\gamma_D$~0.05/fm
  Ghosh(2011) Born diagrams ➔ $\gamma_D$~0.08/fm
- He,Fries,Rapp(2011) empirical scattering amplitudes (resonance) ➔ $\gamma_D$~0.005/fm
- Tolos et al.(2011) Unitarized ChPT ➔resonance ➔ $\gamma_D$~0.08/fm

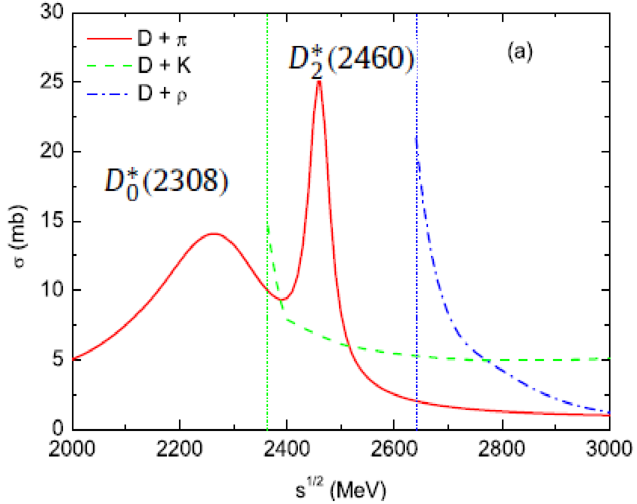
For full HRG:

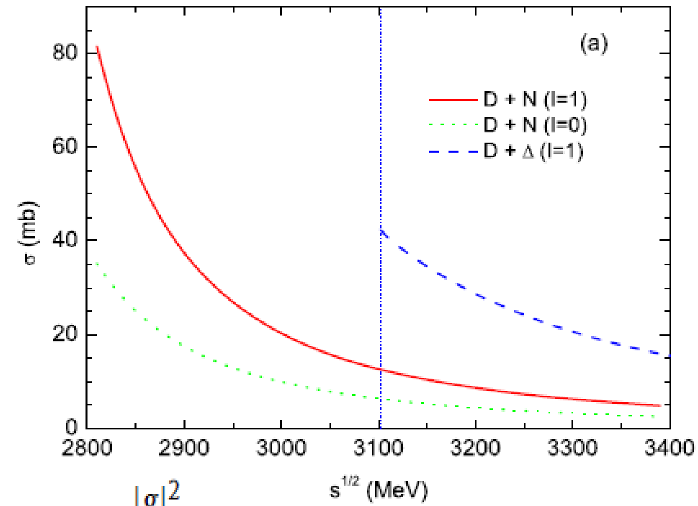
- Substantial contributions from higher states as temperature tends to Tc from below

**Figure 3.** Thermal relaxation rate of low-momentum *D* mesons in hadronic matter in chemical equilibrium, computed in the approaches of [98] (dashed line), [99] (dash-double-dotted line), [101] (dash-dotted line) and [102] (solid line). Figure taken from [102].

# D-meson diffusion in HRG

- D + light hadron empirical scattering amplitudes/cross sections



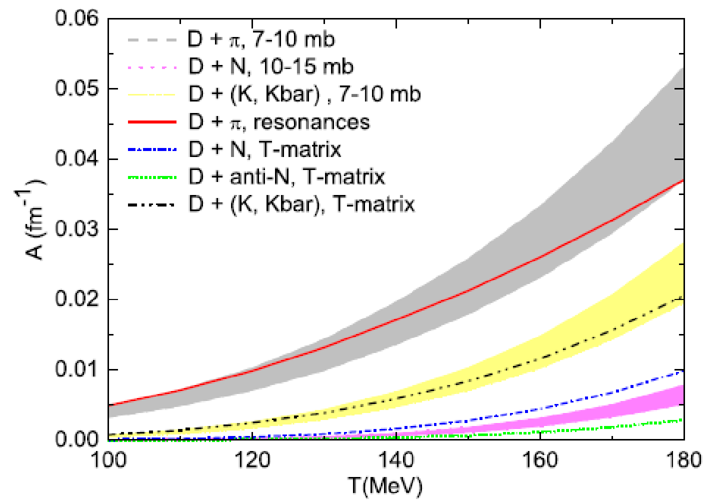




$$\mathcal{M}(s,\Theta=0)=\sum_{j=0}^{2}\frac{8\pi\sqrt{s}}{k}\frac{(2j+1)\sqrt{s}\Gamma_j^{D\pi}}{s-M_j^2+i\sqrt{s}\Gamma_j^{tot}}$$

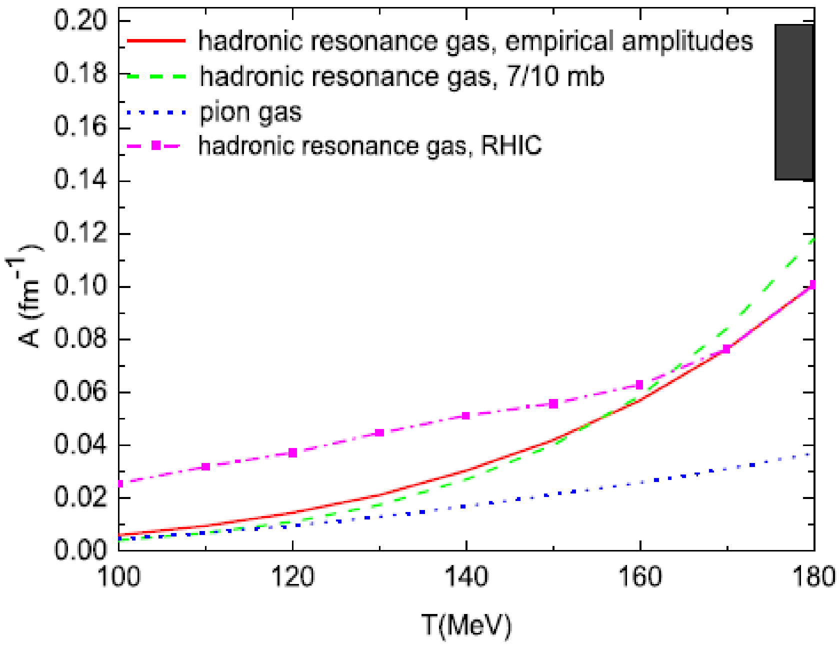
$$T(\sqrt{s})=\frac{|g|^2}{\sqrt{s}-m_R+i\Gamma_R/2}$$

- Contributions to $\gamma_D$ from different light mesons/baryons; well approximated by light constituent q-q scattering with σ=3-4 mb
- At high T~Tc, baryon excitations important

**He,Fries,Rapp, PLB701(2011)445**



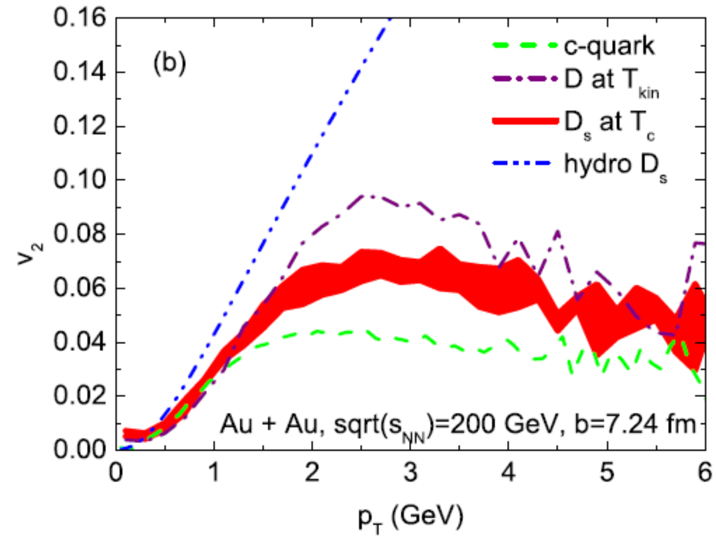
# D-meson diffusion in HRG




Contributions to the thermal $D$-meson thermal relaxation rate at $T = 180$ MeV indicating the quantum numbers of the included scattering channels with $L$: partial wave, $I$: isospin and $J$: total angular momentum.

| Hadrons | $L_{I,2J}$ | $A$ [fm$^{-1}$] |
|---|---|---|
| $\pi$ | $S_{1/2,0}$, $P_{1/2,2}$, $D_{1/2,4}$, $S_{3/2,0}$ | 0.0371 |
| $K+\eta$ | $S_{0,0}$, $S_{1,0}$ | 0.0236 |
| $\rho+\omega+K^*$ | $S_{1/2,2}$, $S_{0,2}$, $S_{1,2}$ | 0.0129 |
| $N+\bar{N}$ | $S_{0,1}$, $S_{1,1}$ | 0.0128 |
| $\Delta+\bar{\Delta}$ | $S_{1,3}$ | 0.0144 |

- D-meson thermal relaxation rate at low momentum near Tc ~0.1 /fm ➔ relaxation time ~ 10 fm/c, already comparable to the fireball lifetime

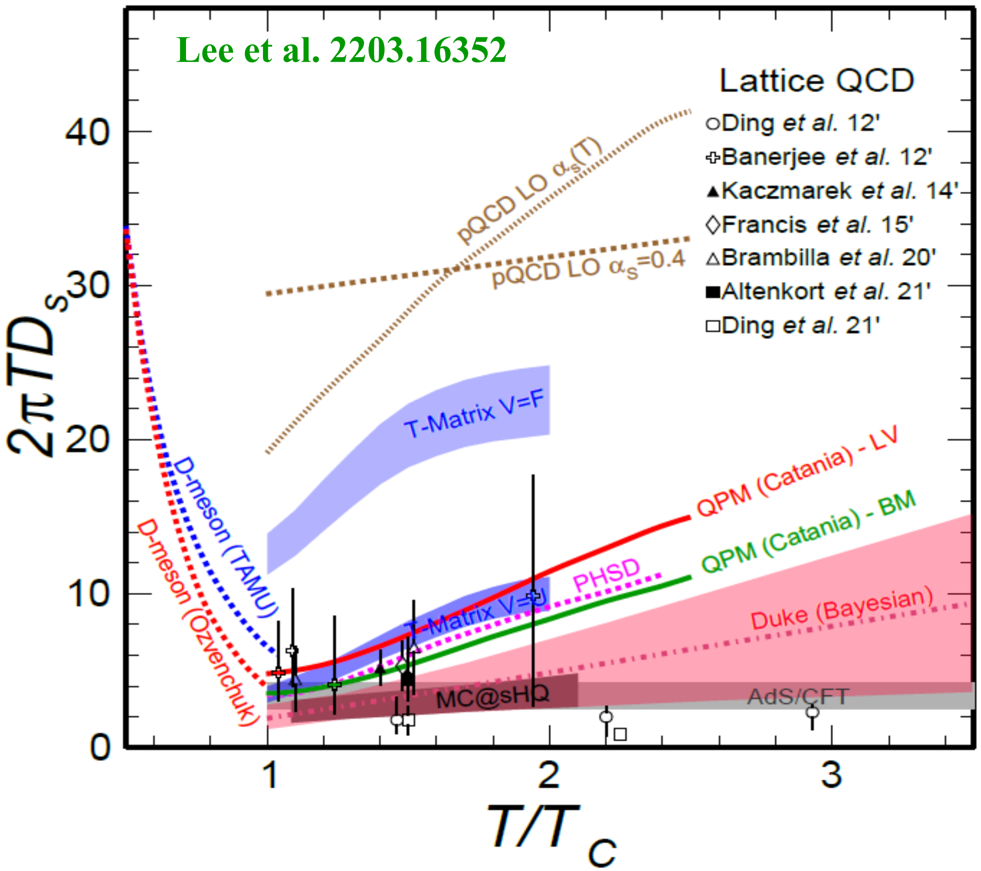



- Enhancing the D-meson $v_2$ by 20-30% at intermediate $p_T$, because the background bulk $v_2$ already almost fully built up by Tc

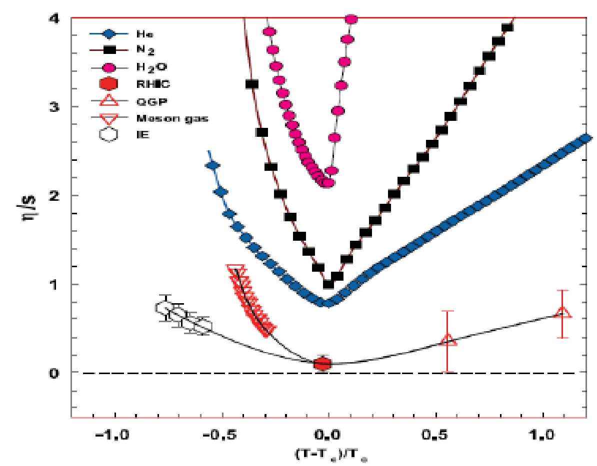
**He,Fries,Rapp, PRL110, 112301 (2013)**

# Summary: transport coefficient: $\mathcal{D}_s(2\pi T)$

❖ HQ spatial diffusion coefficient: $\mathcal{D}_s = T/m_Q A(p=0) = T/m_Q \gamma$

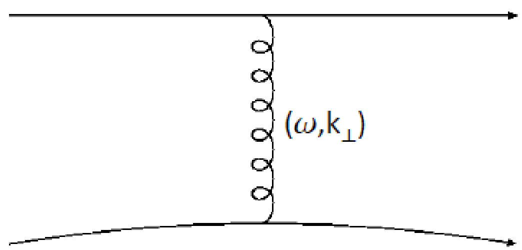


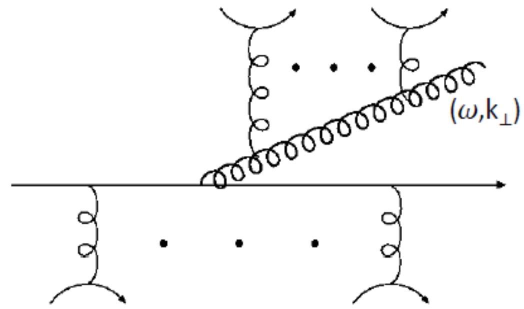

- models & lattice $\mathcal{D}_s(2\pi T)\sim$2-4 near $T_c$, x10 smaller than pQCD, scattering rate $\Gamma_{coll}$ ~3/$\mathcal{D}_s$ ~1 GeV > $M_{q,g}$ ➔ thermal partons melt, Brownian markers survive
- maximum coupling strength near $T_c$, remnant of confining force?!
- minimum structure across $T_c$, analogous to η/s



# HQ radiative e-loss

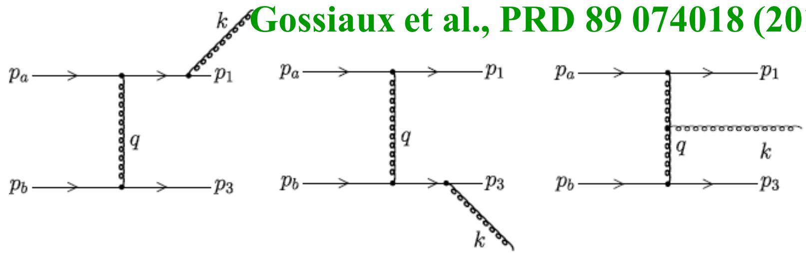
- elastic collisions VS medium-induced gluon radiation




- Factorized elastic * radiation probability

Gossiaux et al., PRD 89 074018 (2014 )




$$\frac{\omega d^4\sigma^{rad}}{dx d^2k_t dq_t^2} = \frac{N_c\alpha_s}{\pi^2}(1-x)\cdot\frac{d\sigma^{el}}{dq_t^2}\cdot P_{rad}$$

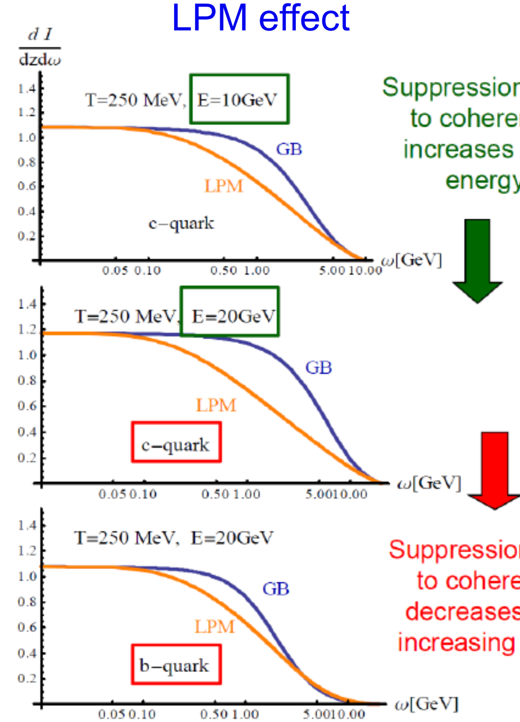
$k_t$ , ω = transv mom/ energy of gluon  E = energy of the heavy quark

$$P_{rad} = C_A\left(\frac{\vec{k}_t}{k_t^2+(\omega/E)^2m^2} - \frac{\vec{k}_t-\vec{q}_t}{(\vec{q}_t-\vec{k}_t)^2+(\omega/E)^2m^2}\right)^2$$

Emission from heavy q

Emission from g

LPM effect




Suppression due to coherence increases with energy

Suppression due to coherence decreases with increasing mass

# HQ radiative e-loss

- Higher Twist formalism

$$\frac{dN_g}{dx dk_\perp^2 dt} = \frac{2\alpha_s C_A P(x)}{\pi k_\perp^4}\hat{q}\left(\frac{k_\perp^2}{k_\perp^2 + x^2 M^2}\right)^4 \sin^2\left(\frac{t-t_i}{2\tau_f}\right)$$

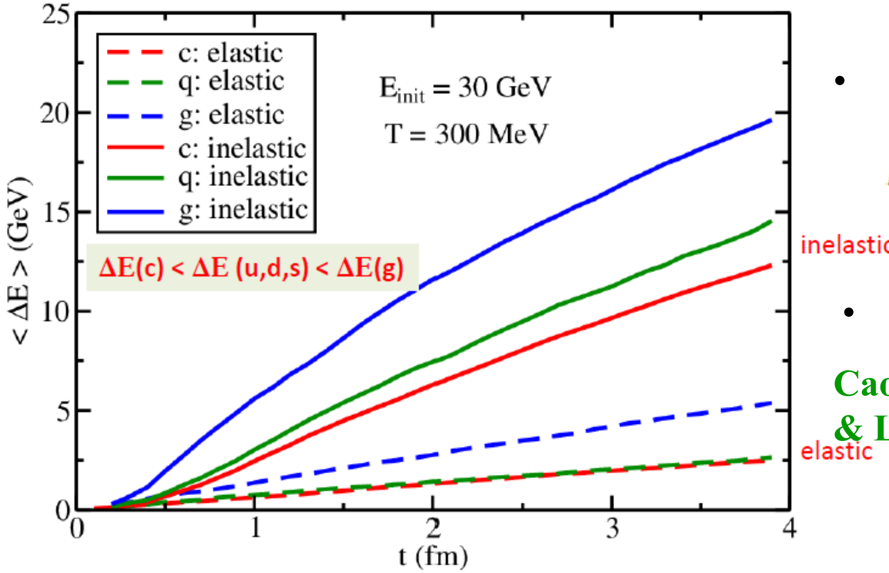
$$\tau_f = 2Ex(1-x)/(k_\perp^2 + x^2 M^2)$$

**Zhang, Wang, and Wang, PRL93, 072301 (2004)**

- Average number of radiated gluons & probability of radiation within △t

$$\langle N_g \rangle (t, \Delta t) = \Gamma_{\rm inel} \Delta t = \Delta t \int dx dk_\perp^2 \frac{dN_g}{dx dk_\perp^2 dt}$$

$$P_{inel} = 1 - e^{-\langle N_g \rangle}$$




- Implemented in LBT

$$p_1 \cdot \partial f_1(x_1, p_1) = E_1(C_{\rm el} + C_{\rm inel})$$

- Flavor hierarchy of e-loss quantified

**Cao, Qin and Bass, PLB777(2018) 255**
**& Liu et al, Eur. Phys. J. C (2022) 82:350**

# HQ hadronization (I): SHMc

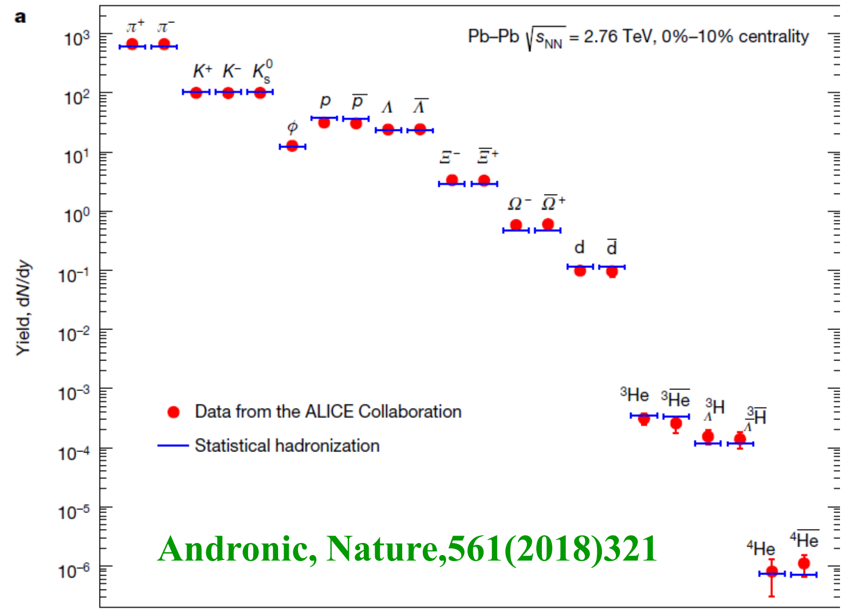
- Statistical hadronization model (SHM) for light quarks:

grand-canonical partition function for HRG

$$\ln Z_i = \frac{V g_i}{2\pi^2}\int_0^\infty \pm p^2 \mathrm{d}p \ln[1 \pm \exp(-(E_i-\mu_i)/T)]$$

$$T_{CF} = 156.6 \pm 1.7 \text{ MeV}$$

$$\mu_B = 0.7 \pm 3.8 \text{ MeV} \quad V = 4175 \pm 380 \text{ fm}^3$$




Andronic, Nature,561(2018)321

- Statistical hadronization model for charm quarks (SHMc) :
  - charm balance equation $N_{c\bar{c}} = \frac{1}{2} g_c V \sum_{h^i_{oc,1}} n_i^{\mathrm{th}} + g_c^2 V \sum_{h^j_{hc}} n_j^{\mathrm{th}} + \frac{1}{2} g_c^2 V \sum_{h^k_{oc,2}} n_k^{\mathrm{th}}$

where $N_{c\bar{c}} \equiv \mathrm{d}N_{c\bar{c}}/\mathrm{d}y$ = $T_{AA}$*$d\sigma^{ccbar}$/dy is the charm quark number per unit rapidity,

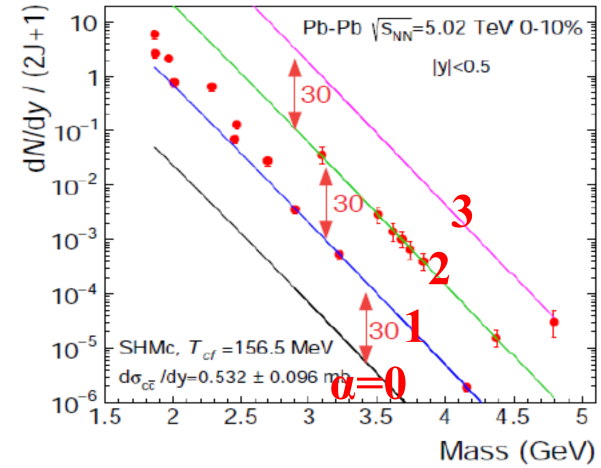
- thermalized c-quarks hadronized at $T_c$ (strict charm conservation➔ canonical correction)

$$\frac{\mathrm{d}N(h^i_{oc,\alpha})}{\mathrm{d}y} = g_c^\alpha V n_i^{\mathrm{th}} \frac{I_\alpha(N_c^{\mathrm{tot}})}{I_0(N_c^{\mathrm{tot}})}$$ $\propto g_c^\alpha \leftarrow d\sigma^{cc}/dy$

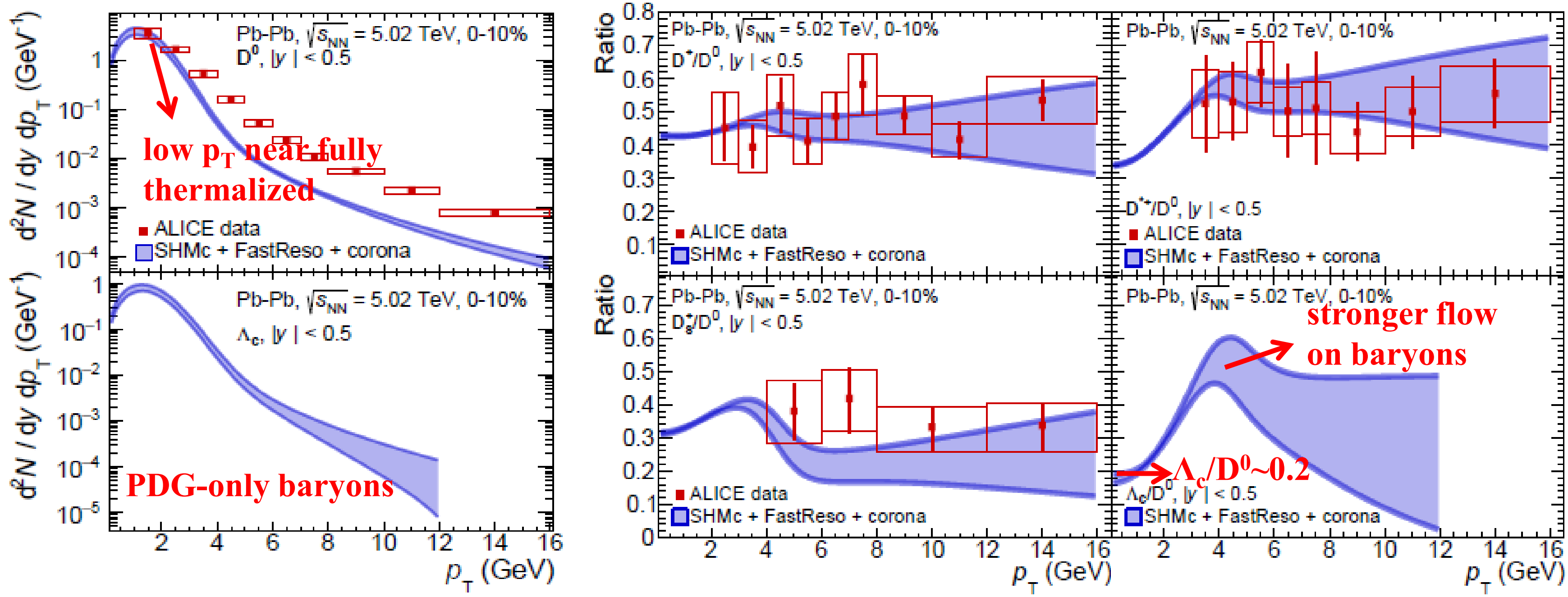
- $p_T$-spectrum modelled by hydrodynamic blast wave at $T_c$

# Hadronization: SHMc Andronic, et al., JHEP07(2021)035

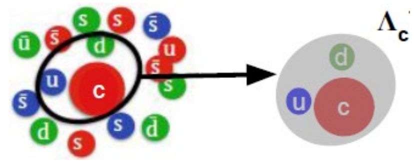
- SHMc: open-charm statistical hadronization at $T_c$ $\frac{dN(h^i_{oc,\alpha})}{dy} = g_c^\alpha\, V\, n_i^{\rm th} \frac{I_\alpha(N_c^{\rm tot})}{I_0(N_c^{\rm tot})}$
  - multicharm baryons $\alpha$=1,2,3 emerging pattern
  - yields enhanced by $g_c^\alpha \sim 30^\alpha$ than pure thermal →strong signal of deconfinement

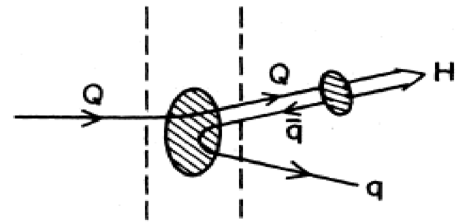



- SHMc yields + blast wave→$p_T$ spectra



# HQ hadronization (II): coalescence

Coalescence: 

vs. Fragmentation 


- recombination of low $p_T$ HQs with thermal light quarks nearby in (x&p) phase space ➔ add momentum & flow to the parent HQ
- HQ independent fragmentation into HF hadrons ➔ dominant at high $p_T$

❖ Instantaneous coalescence models (ICM)

**Fries et al., Greco et al., Voloshin '03**

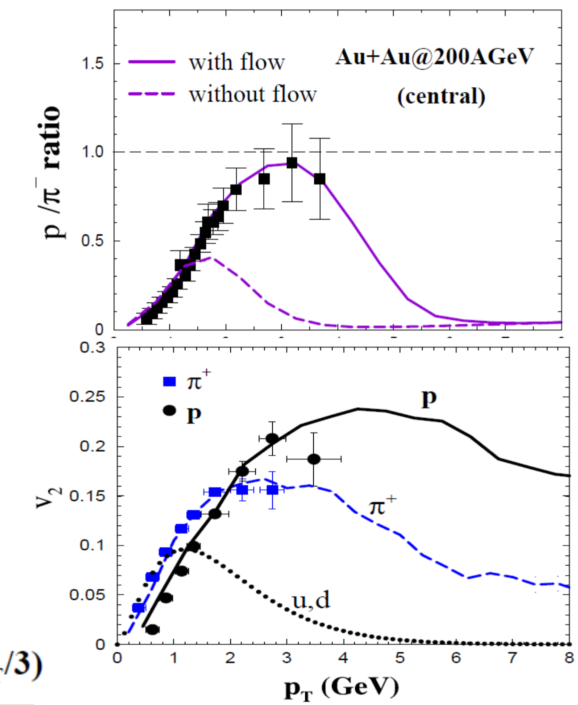
$$\frac{d^2N_H}{dP_T^2} = g_H \int \prod_{i=1}^{n} \frac{d^3p_i}{(2\pi)^3 E_i} p_i \cdot d\sigma_i \, f_{q_i}(x_i, p_i)$$
$$\times \; f_H(x_1..x_n, p_1..p_n)\, \delta^{(2)}\left(P_T - \sum_{i=1}^{n} p_{T,i}\right)$$

- Instantaneous projection via Wigner function

$$f_M(x_1, x_2; p_1, p_2) = \frac{9\pi}{2} \Theta\left(\Delta_x^2 - x_r^2\right) \times \Theta\left(\Delta_p^2 - p_r^2 + \Delta m_{12}^2\right)$$

$$f_B = \frac{81\pi^2}{4} \Theta\left(\Delta_x^2 - \frac{1}{2}x_{r1}^2\right) \times \Theta\left(\Delta_p^2 - \frac{1}{2}p_{r1}^2\right) \Theta\left(\Delta_x^2 - x_{r2}^2\right) \times \Theta\left(\Delta_p^2 - p_{r2}^2 - \Delta m_{123}^2\right),$$

- Heinsenberg's uncertaity principle: $\Delta_x \cdot \Delta_p \approx 1$
- successful in explaining B/M, $v_2$ scaling




**$f_M(p_T) \sim f_q(p_T/2) * f_{qbar}(p_T/2)$ VS $f_B(p_T) \sim f_q(p_T/3) * f_q(p_T/3) * f_q(p_T/3)$**

# Instantaneous coalescence model

❖ ICM: usually implemented only in momentum space

Oh et al., PRC79, 044905 (2009)

- with a Gaussian Wigner function, $\triangle_x=\sigma\sim$hadron radius, $\triangle_p=1/\sigma$

$$\frac{dN_M}{d\boldsymbol{p}_M} = g_M \frac{(2\sqrt{\pi}\sigma)^3}{V} \int d\boldsymbol{p}_1 d\boldsymbol{p}_2 \frac{dN_1}{d\boldsymbol{p}_1}\frac{dN_2}{d\boldsymbol{p}_2} \times \exp(-\boldsymbol{k}^2\sigma^2)\delta(\boldsymbol{p}_M - \boldsymbol{p}_1 - \boldsymbol{p}_2),$$

relative momentum **k** measured in meson rest frame
degeneracy $g_M=(2s_M+1)/[(2s_1+1)*3_c*(2s_2+1)*3_c]$,
e.g. $g_{D0}=1/36$, $g_{D*}=3/36=1/12$

$$\boldsymbol{k} = \frac{1}{m_1+m_2}(m_2\boldsymbol{p}_1' - m_1\boldsymbol{p}_2')$$

- for charm baryons

$$\frac{dN_B}{d\boldsymbol{p}_B} = g_B \frac{(2\sqrt{\pi})^6(\sigma_1\sigma_2)^3}{V^2} \int d\boldsymbol{p}_1 d\boldsymbol{p}_2 d\boldsymbol{p}_3 \frac{dN_1}{d\boldsymbol{p}_1}\frac{dN_2}{d\boldsymbol{p}_2}\frac{dN_3}{d\boldsymbol{p}_3} \times \exp\left(-\boldsymbol{k}_1^2\sigma_1^2 - \boldsymbol{k}_2^2\sigma_2^2\right)\delta(\boldsymbol{p}_B - \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3),$$

$$\boldsymbol{k}_1 = \frac{1}{m_1+m_2}(m_2\boldsymbol{p}_1' - m_1\boldsymbol{p}_2'),$$

$$\boldsymbol{k}_2 = \frac{1}{m_1+m_2+m_3}[m_3(\boldsymbol{p}_1' + \boldsymbol{p}_2') - (m_1+m_2)\boldsymbol{p}_3']$$

$g_{\Lambda c}=2/216=1/108$

- By construction, only 3-mom. conserved, energy not conserved ➔ equilibrium limit challenging to reach at low $p_T$

# HQ hadronization (III): RRM

- **Hadronization = Resonance formation** $c\bar{q} \rightarrow D$

→ ***consistent with T-matrix findings of resonance correlations towards $T_c$***

- **Realized by Boltzmann equation** Ravagli & Rapp,2007

$$p^{\mu}\partial_{\mu} f_M(t,\vec{x},\vec{p}) = -m\Gamma f_M(t,\vec{x},\vec{p}) + p^0\beta(\vec{x},\vec{p})$$

$$\beta(\vec{x},\vec{p}) = \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} f_q(\vec{x},\vec{p}_1) f_{\bar{q}}(\vec{x},\vec{p}_2) \times \sigma(s) v_{\rm rel}(\vec{p}_1,\vec{p}_2)\delta^3(\vec{p}-\vec{p}_1-\vec{p}_2)$$

gain term

Breit-Wigner

$$\sigma(s) = g_{\sigma}\frac{4\pi}{k^2}\frac{(\Gamma m)^2}{(s-m^2)+(\Gamma m)^2}$$

- **Equilibrium limit** $$f_M(\vec{x},\vec{p}) = \frac{\gamma_{p,M}}{\Gamma_M}\int \frac{d^3\vec{p}_1 d^3\vec{p}_2}{(2\pi)^3} f_q(\vec{x},\vec{p}_1) f_{\bar{q}}(\vec{x},\vec{p}_2) \times \sigma_M(s) v_{\rm rel}(\vec{p}_1,\vec{p}_2)\delta^3(\vec{p}-\vec{p}_1-\vec{p}_2),$$
- **Energy conservation (finite width, not on-shell) + detailed balance**

→ **equilibrium mapping between quark & meson distributions**

# Generalization to 3-body RRM

- **The 1st step: $q_1(p_1) + q_2(p_2) \rightarrow$ diquark ($p_{12}$)**

$$f_d(\vec{x},\vec{p}_{12}) = \frac{E_d(\vec{p}_{12})}{\Gamma_d m_d}\int \frac{d^3p_1 d^3p_2}{(2\pi)^3} f_1(\vec{x},\vec{p}_1) f_2(\vec{x},\vec{p}_2)\sigma_{12}(s_{12}) v_{\rm rel}^{12}(\vec{p}_1,\vec{p}_2)\delta^3(\vec{p}_{12}-\vec{p}_1-\vec{p}_2)$$

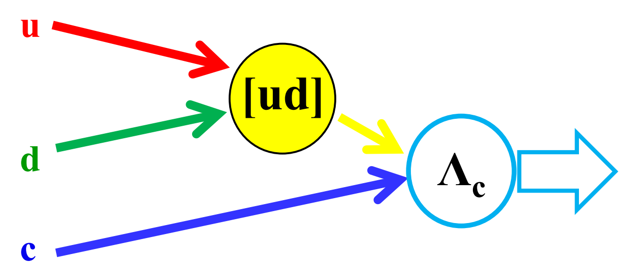
- **The 2nd step: diquark ($p_{12}$) $+ q_3(p_3) \rightarrow$ baryon (p)**

$$f_B(\vec{x},\vec{p}) = \frac{E_B(\vec{p})}{\Gamma_B m_B}\int \frac{d^3p_{12} d^3p_3}{(2\pi)^3} f_d(\vec{x},\vec{p}_{12}) f_3(\vec{x},\vec{p}_3)\sigma_B(s_{d3}) v_{\rm rel}^{d3}(\vec{p}_{12},\vec{p}_3)\delta^3(\vec{p}-\vec{p}_{12}-\vec{p}_3)$$

$$= \frac{E_B(\vec{p})}{\Gamma_B m_B}\int \frac{d^3p_1 d^3p_2 d^3p_3}{(2\pi)^6}\frac{E_d(\vec{p}_{12})}{\Gamma_d m_d} f_1(\vec{x},\vec{p}_1) f_2(\vec{x},\vec{p}_2) f_3(\vec{x},\vec{p}_3)$$

$$\times\, \sigma_{12}(s_{12}) v_{\rm rel}^{12}(\vec{p}_1,\vec{p}_2)\sigma_B(s_{d3}) v_{\rm rel}^{d3}(\vec{p}_{12},\vec{p}_3)|_{\vec{p}_{12}=\vec{p}_1+\vec{p}_2}\delta^3(\vec{p}-\vec{p}_1-\vec{p}_2-\vec{p}_3)$$

| diquark type | mass (MeV) | wave func. | charm-baryon |
|---|---|---|---|
| Scalar [u,d] | 710 | $\bar{3}_{\rm color}\bar{3}_{\rm flavor}0^+_{\rm spin}$ | $\Lambda_c$: c[ud] |
| Axialvector {u,d} | 909 | $\bar{3}_{\rm color}6_{\rm flavor}1^+_{\rm spin}$ | $\Sigma_c$: c{ud} |



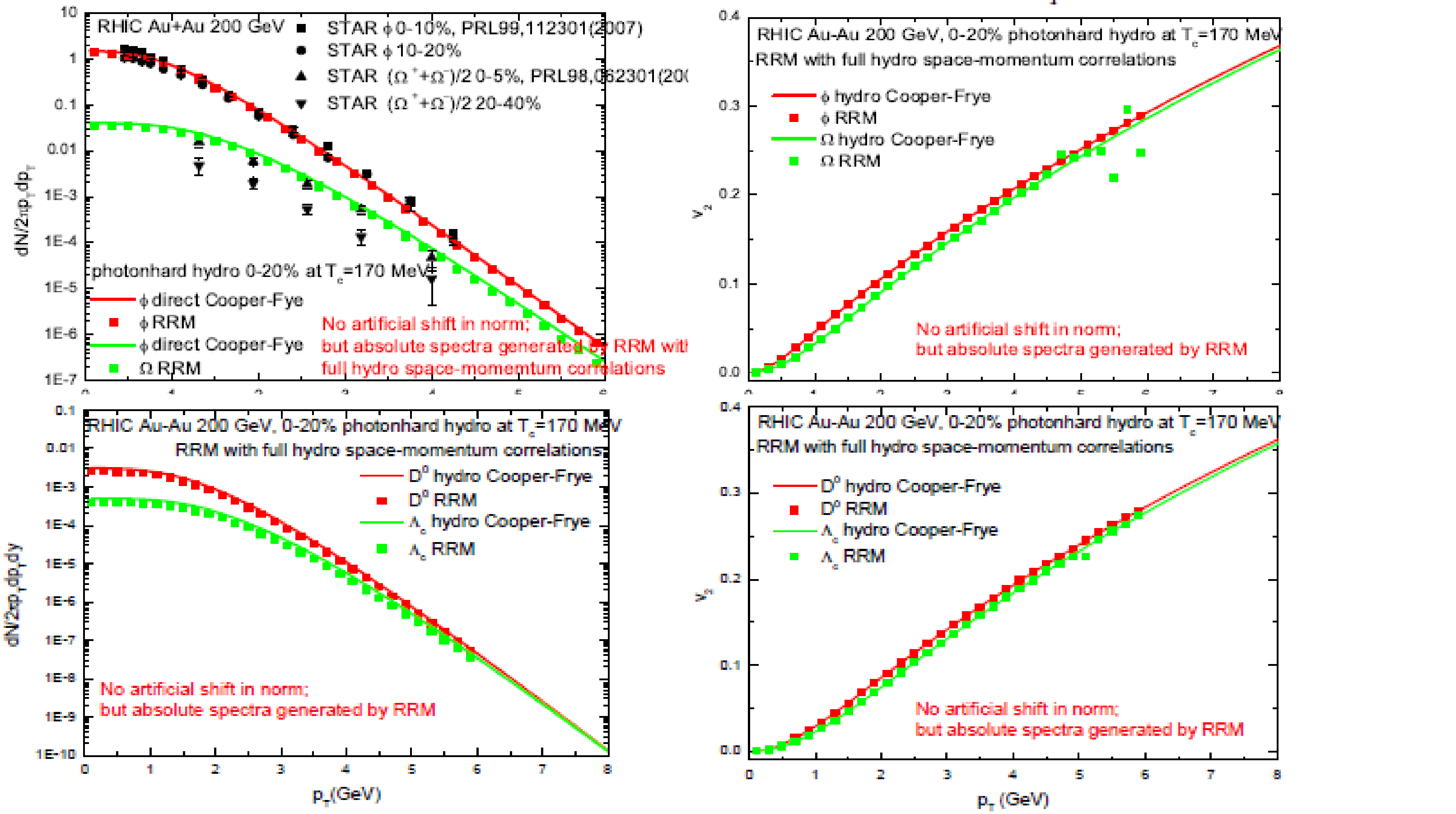

- Meson/baryon invariant spectra on hydrodynamic Cooper-Frye hypersurface at $T_H$=170 MeV

$$\frac{dN_{M,B}}{p_T dp_T d\phi_p dy} = \int \frac{p\cdot d\sigma}{(2\pi)^3} f_{M,B}(\vec{x},\vec{p})$$

**MH & Rapp, PRL124, 042301 (2020)**

# RRM: equilibrium mapping

- RRM on hydrofreezeout hypersurface at $T_c$ with $f_q^{eq}(\vec{x},\vec{p}) = g_q e^{-p\cdot u(x)/T(x)}$




- Equilibrium mapping: ensured by 4-momentum conservation in RRM

  $m_q$=0.3, $m_s$=0.4, $m_c$=1.5, $\Gamma_M$~0.1 GeV, $\Gamma_d$~0.2 GeV, $\Gamma_B$~0.3 GeV

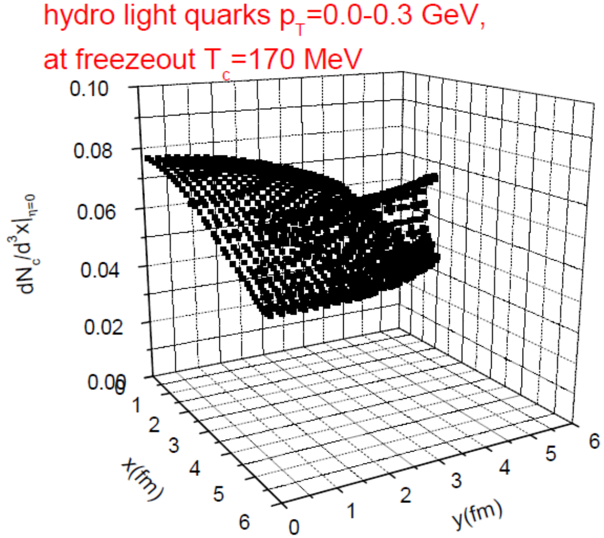
# Space-momentum correlations (SMCs)

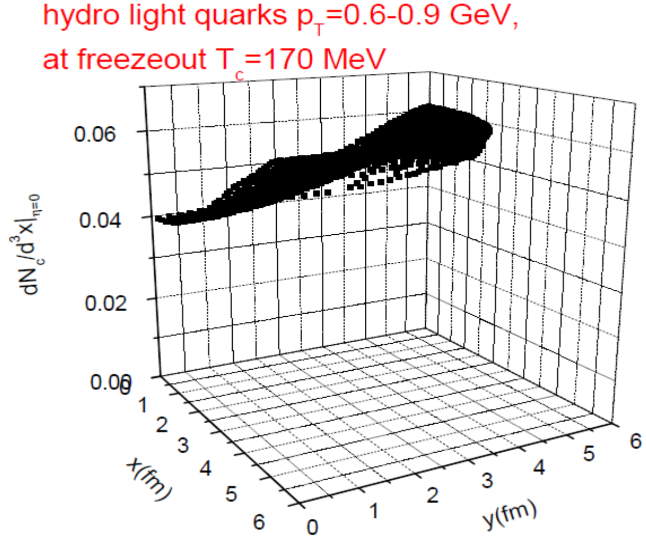
- **hydro: a manifestation of SMCs**

$$f_q^{eq}(\vec{x},\vec{p}) = g_q e^{-p\cdot u(x)/T(x)} = g_q e^{-\gamma_T(x)[m_T\cosh(y-\eta)-\vec{p}_T\cdot\vec{v}_T(x)]/T(x)}$$

**longitudinal boost invariance: y- η** **transverse SMCs $p_T \cdot v_T$**

- **hydro-q density: low (high) $p_T$-q more concentrated in center (boundary)**



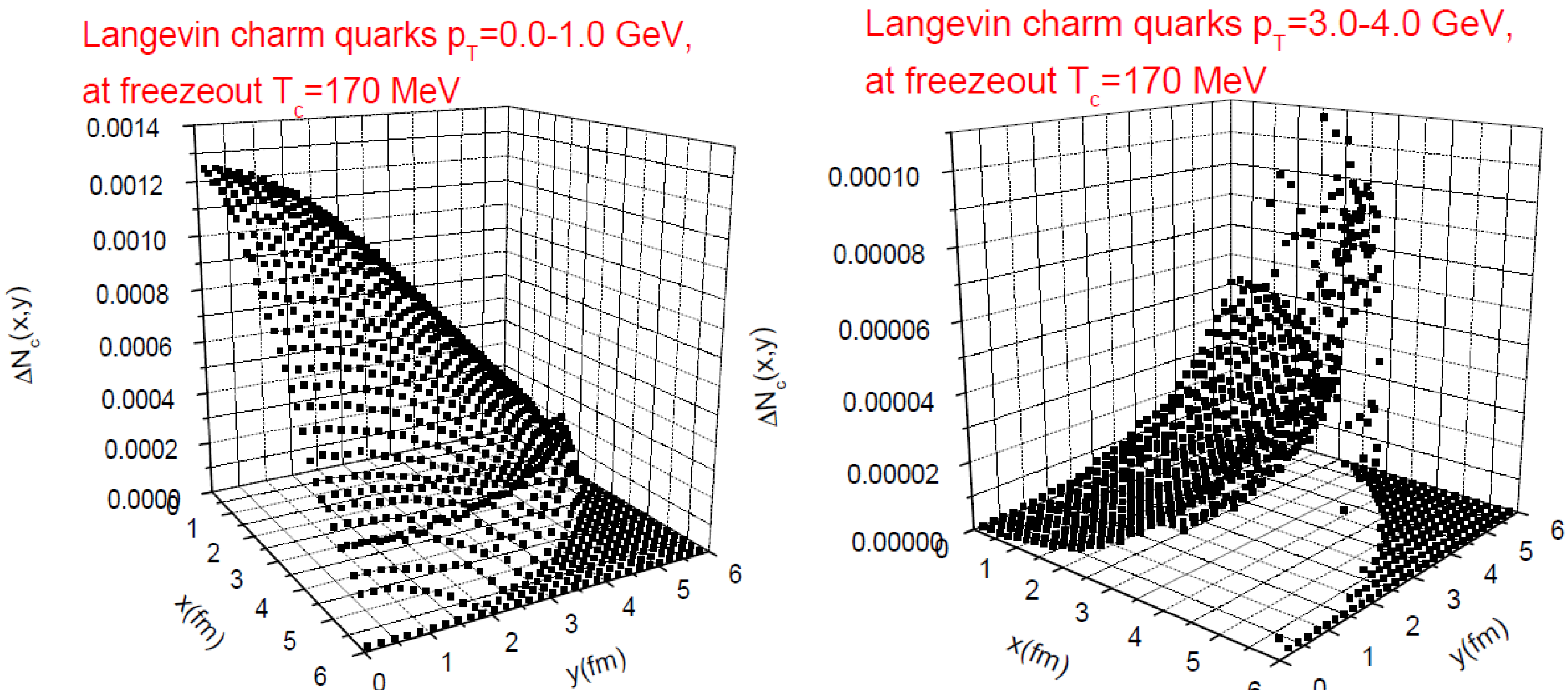




- **what if neglecting SMCs: uniformly distributed independent of $p_T$ as usually done in conventional instantaneous coalescence models**

$$f_{c,q}(\vec{x},\vec{p}) = (2\pi)^3\frac{dN_{c,q}}{d^3\vec{x}d^3\vec{p}} = \frac{(2\pi)^3}{VE_{(\vec{p})}}\frac{dN_{c,q}}{p_T dp_T d\phi_q dy} \quad \& \quad \int p\cdot d\sigma = E(\vec{p})V$$

# SMCs: Langevin charm quarks

- Langevin simulation of charm quark diffusion in a hydrodynamically expanding QGP with T-matrix charm thermalization rate
- c-quarks: low (high) $p_T$-c more populated in central (outer) region




- SMCs usually neglected in ICMs: uniformly distributed independent of $p_T$
- what will be the role of SMCs in recombination/RRM?

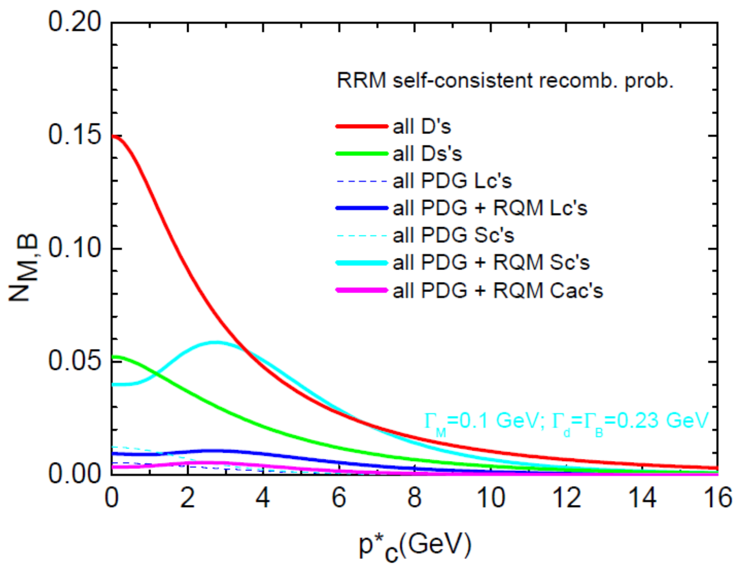
# Charm quark recombination probability

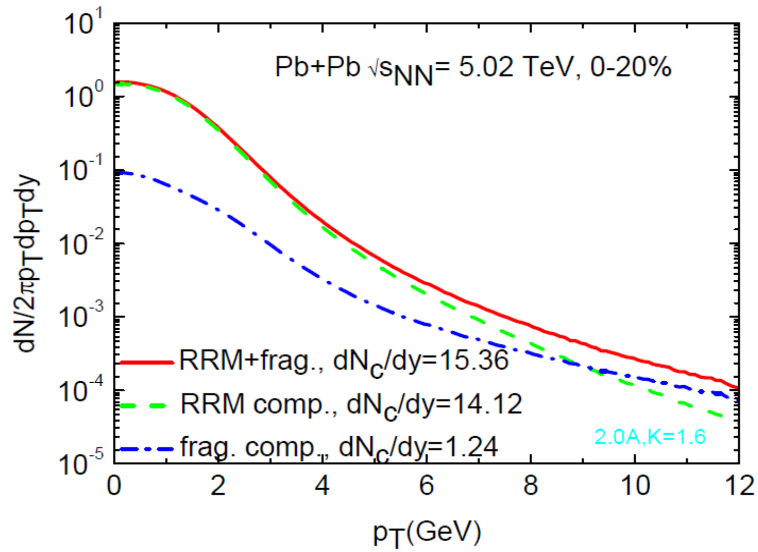
- No. of mesons/baryons formed from a single c-quark of rest frame $p_c$*

$$N_M(p_c^*) = \int \frac{d^3\vec{p}_1^*}{(2\pi)^3} g_q e^{-E(\vec{p}_1^*)/T_{pc}} \frac{E_M(\vec{p}^*)}{m_M \Gamma_M} \sigma(s) v_{\rm rel},$$

$$N_B(p_c^*) = \int \frac{d^3 p_1 d^3 p_2}{(2\pi)^6} g_1 e^{-E(\vec{p}_1)/T_c} g_2 e^{-E(\vec{p}_2)/T_c} \frac{E_d(\vec{p}_{12})}{m_d \Gamma_d} \sigma(s_{12}) v_{\rm rel}^{12}(\vec{p}_1, \vec{p}_2) \frac{E_B(\vec{p})}{m_B \Gamma_B} \sigma(s_{d3}) v_{rel}^{d3}(\vec{p}_{12}, \vec{p}_{30}),$$

- Renormalizing $N_M(p_c^*)$ and $N_B(p_c^*)$ by a common factor ~4 for all charmed mesons/baryons such that $\sum_M P_{{\rm coal},M}(p_c^* = 0) + \sum_B P_{{\rm coal},B}(p_c^* = 0) = 1$




--- charm conservation separately built in, in a (e-by-e) way without spoiling the relative chemical equilibrium realized by RRM

# Event-by-event Langevin-RRM simulation

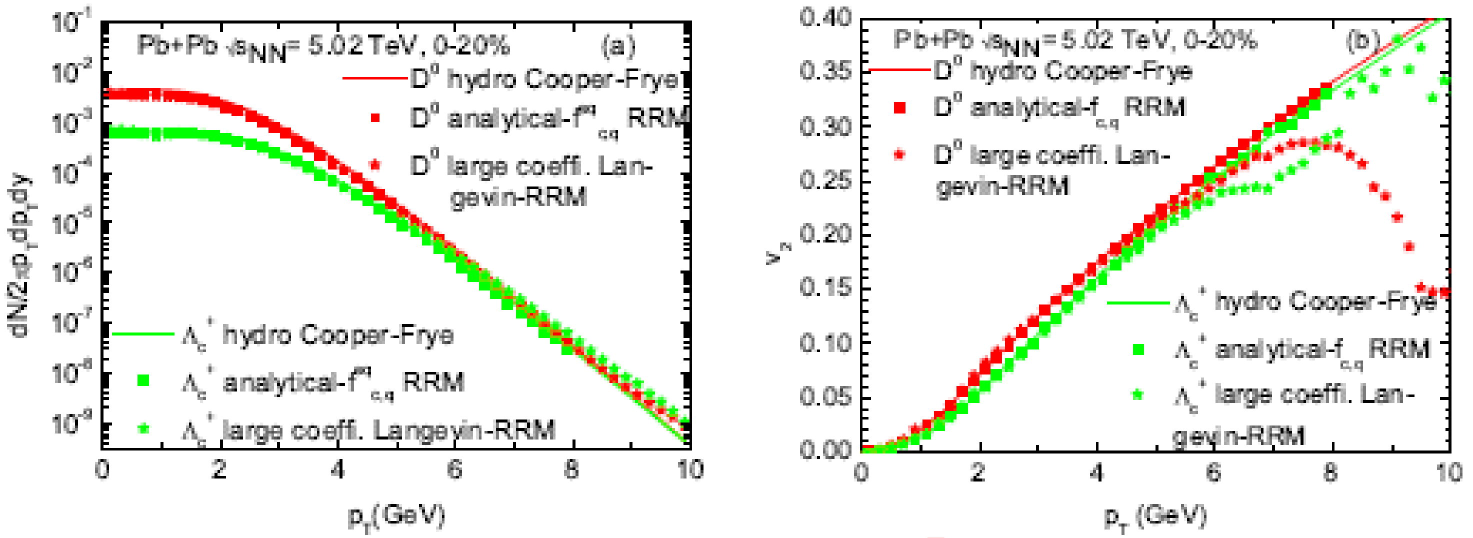
- for a single Langevin c-quark, sample a/two thermal light-q distribution(s)

$$\frac{dN_M}{d\eta}|_{\eta=0} \equiv \sum_n \Delta N_M[n] = \sum_n \frac{p\cdot d\sigma(j_0)}{m_M\Gamma_M}\sigma(s)v_{\rm rel} \qquad \vec{p} = \vec{p}_{1n} + \vec{p}_c$$

$$\frac{dN_B}{d\eta}|_{\eta=0} \equiv \sum_{n_1}\sum_{n_2} \Delta N_B[n_1, n_2]$$

$$= \sum_{n_1}\sum_{n_2} \frac{p\cdot d\sigma(j_0)}{m_B\Gamma_B}\frac{E_d(\vec{p}^*_{12})}{m_d\Gamma_d}\sigma(s_{12})v^{12}_{\rm rel}(\vec{p}^*_{1n_1}, \vec{p}^*_{2n_2})\sigma(s_{d3})v^{d3}_{rel}. \qquad \vec{p} = \vec{p}_{1n_1} + \vec{p}_{2n_2} + \vec{p}_c$$

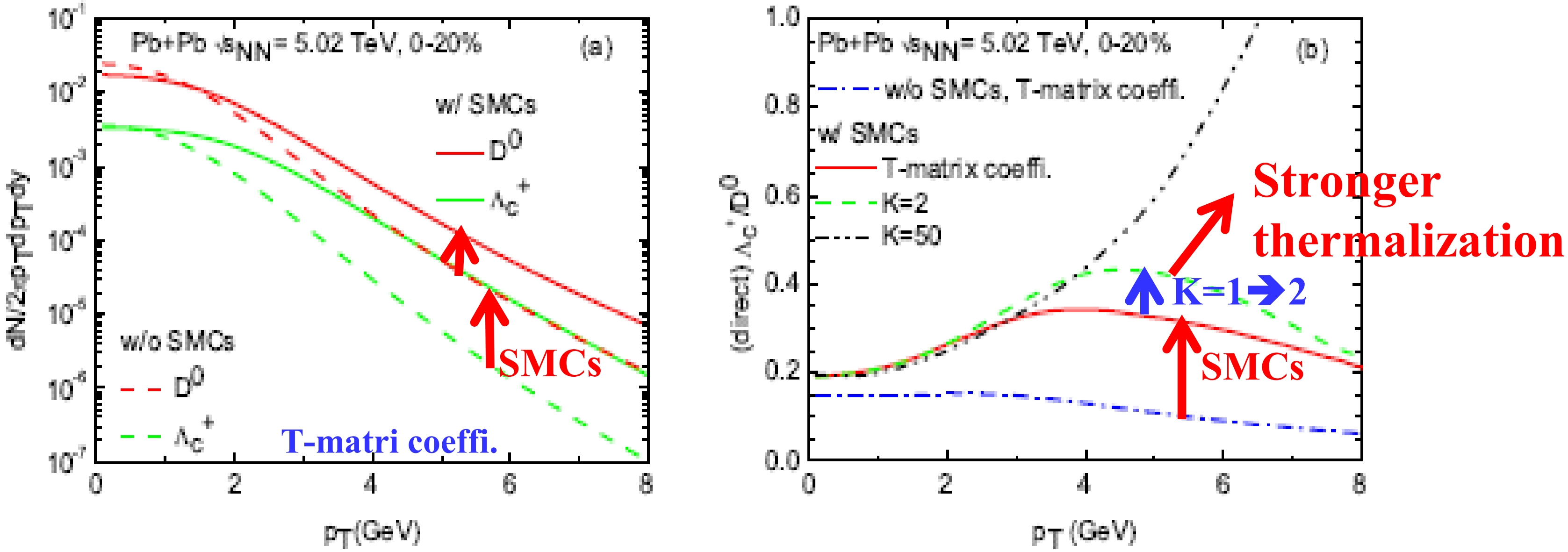
- equil. mapping with large transport coeffi. checks out: SMCs incorporated




- equil. mapping : both kinetic & chemical ➔ observables come out as RRM predictions with realistic T-matrix transport coefficient

# Direct $D^0$ & $\Lambda_c^+$ production via RRM

- **Including SMCs makes the spectra harder & enhances the ratio $\Lambda_c^+/D_0$**




- **Consider RRM formation of $D^0$ (3.5+0.7) & $\Lambda_c^+$ (3.0+0.6+0.6) of $p_T$~4.2 GeV:**

  **enhancement of density of light-q of $p_T$~ 0.6-0.7 GeV & c of $p_T$~3.0-3.5 GeV**

$$\Delta N_{D^0}(4.2) \sim \frac{\Delta N_c(3.0-3.5)}{V_{c,\mathrm{eff}}} \cdot \frac{\Delta N_q(0.6-0.7)}{V_{q,\mathrm{eff}}} \qquad (15)$$

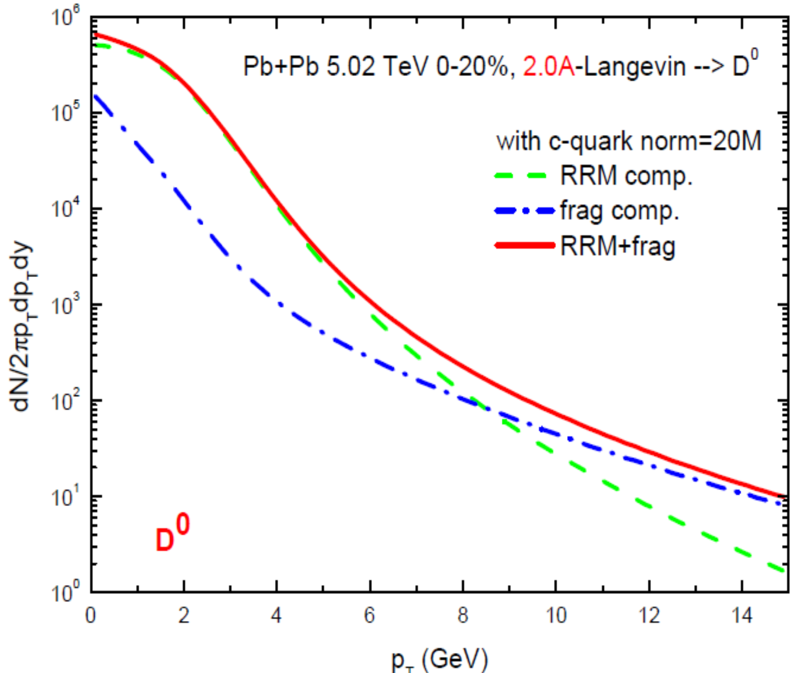
--- **Rencombinant quark density enhanced vs w/o SMCs: $V_{eff}<V_{fb}$**

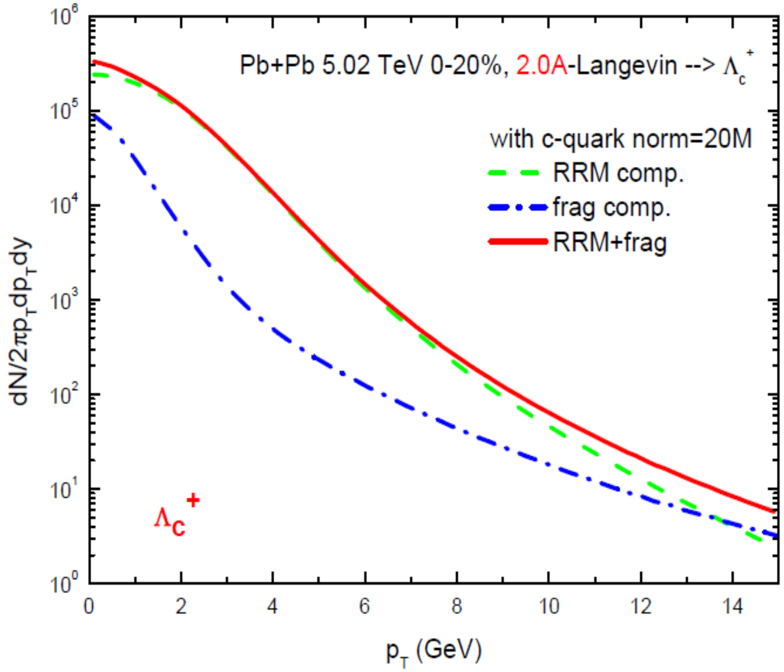
$$\Delta N_{\Lambda_c^+}(4.2) \sim \frac{\Delta N_c(3.0-3.5)}{V_{c,\mathrm{eff}}} \cdot \frac{\Delta N_q(0.6-0.7)}{V_{q,\mathrm{eff}}} \cdot \frac{\Delta N_q(0.6-0.7)}{V_{q,\mathrm{eff}}}$$

--- **Enhanced light-q density entering $D^0$ RRM only once vs twice (squared) for $\Lambda_c^+$ RRM ➔ the ratio $\Lambda_c^+/D^0$ enhanced!**

# Recombinant vs fragmenting spectra

- Hydro-Langevin-RRM(+fragmentation): for all charm-mesons/baryons
  ➔ higher states decay into ground state $D^0$ , $D^+$ , $D_s^+$ , $\Lambda_C^+$



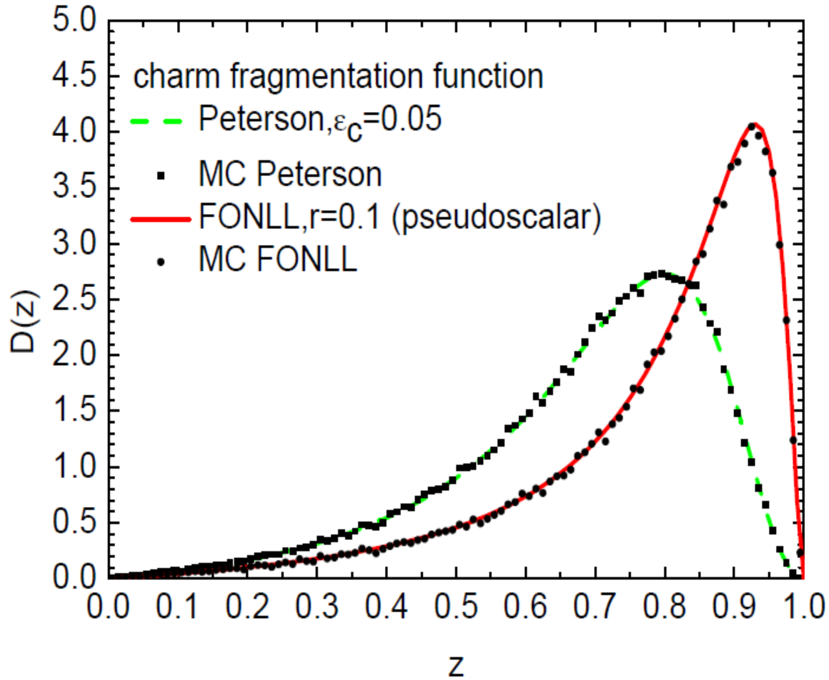



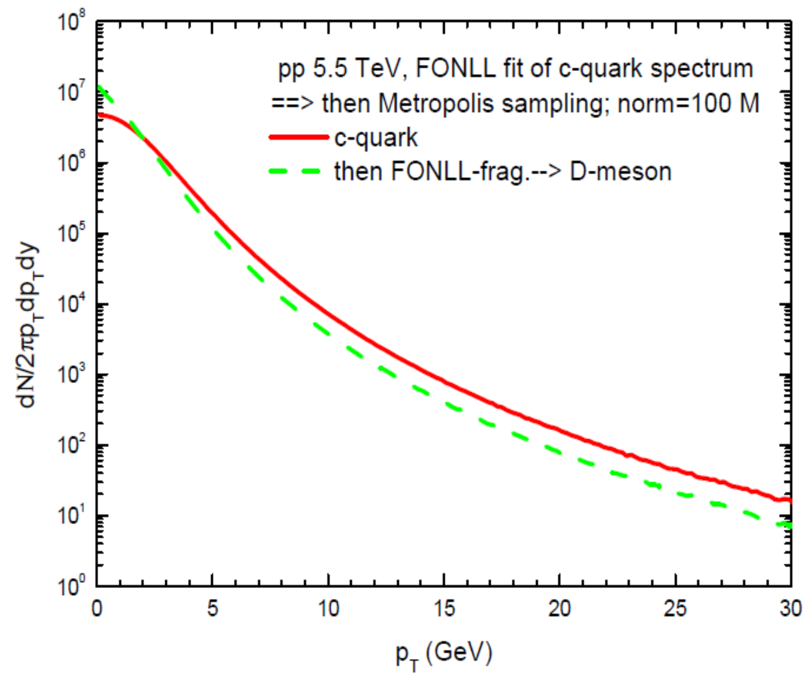

- SMCs extend the recombinant component toward (quite) high $p_T$;
  RQM augmented higher states' RRM spectra even harder (also thanks to SMCs) ➔ RRM & frag. cross at $p_T$ ~8.5 (13) GeV for $D^0$ ($\Lambda_C^+$)

- Helpful for large total $v_2$ (weighted between RRM vs frag. components)

# HQ hadronization (IV): fragmentation

- charm quarks that are not consumed via recombination ($P_{coal}(p_{c^*})<1$ is a decreasing function toward high $p_{c^*}$ ) go to independent fragmentation



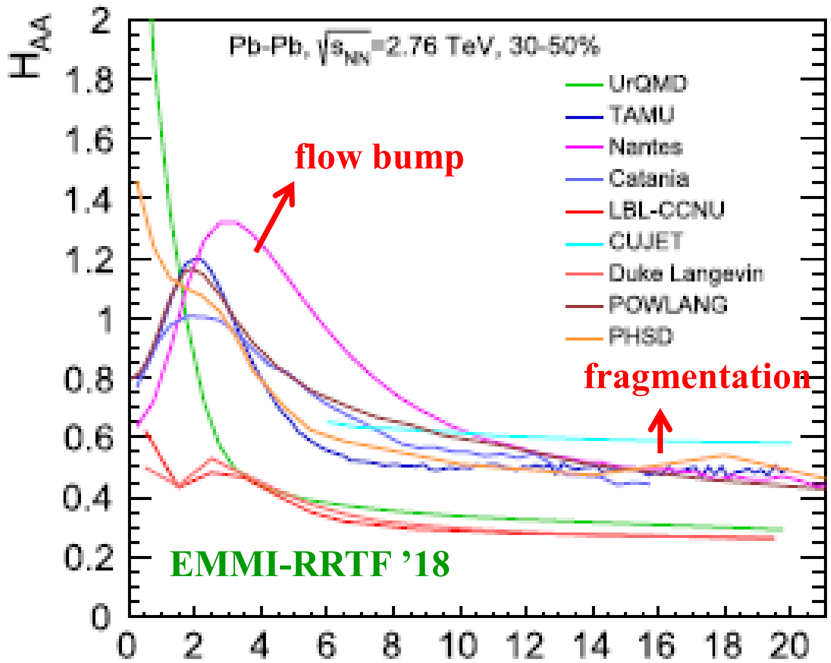

- z= $p_{TH}/p_{Tc}$

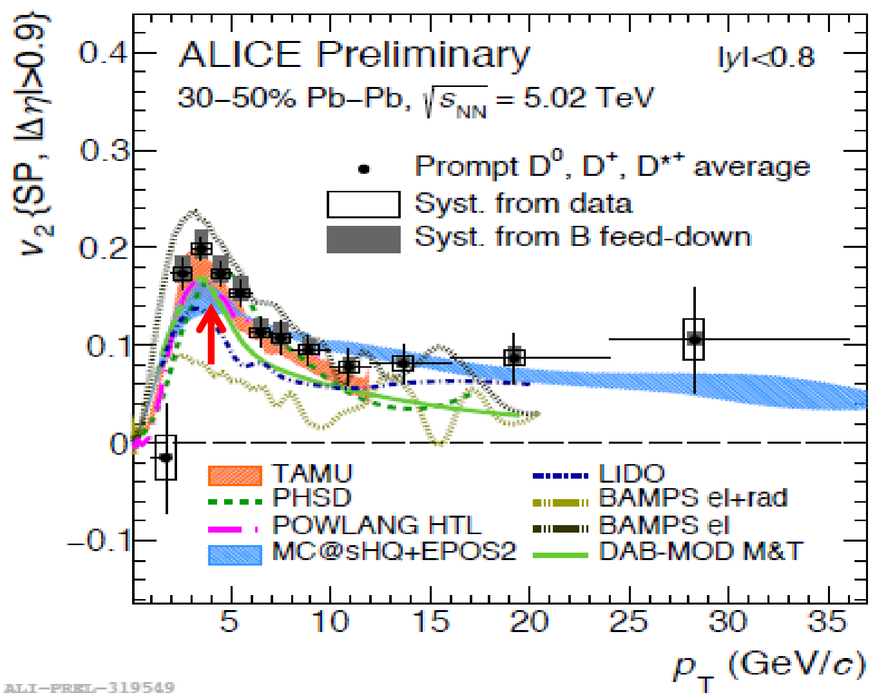
$$r_M/r_{D^0} = ((m_M - m_c)/m_M)/((m_{D^0} - m_c)/m_{D^0})$$

$$r_B/r_{\Lambda_c^+} = ((m_B - m_c)/m_B)/((m_{\Lambda_c^+} - m_c)/m_{\Lambda_c^+})$$

$$D_{c\to H}(z) = N_H \frac{rz(1-z)^2}{[1-(1-r)z]^6}[(6-18(1-2r)z + (21-74r+68r^2)z^2 - 2(1-r)(6-19r+18r^2)z^3 + 3(1-r)^2(1-2r+2r^2)z^4],$$

# Recombination vs fragmentation

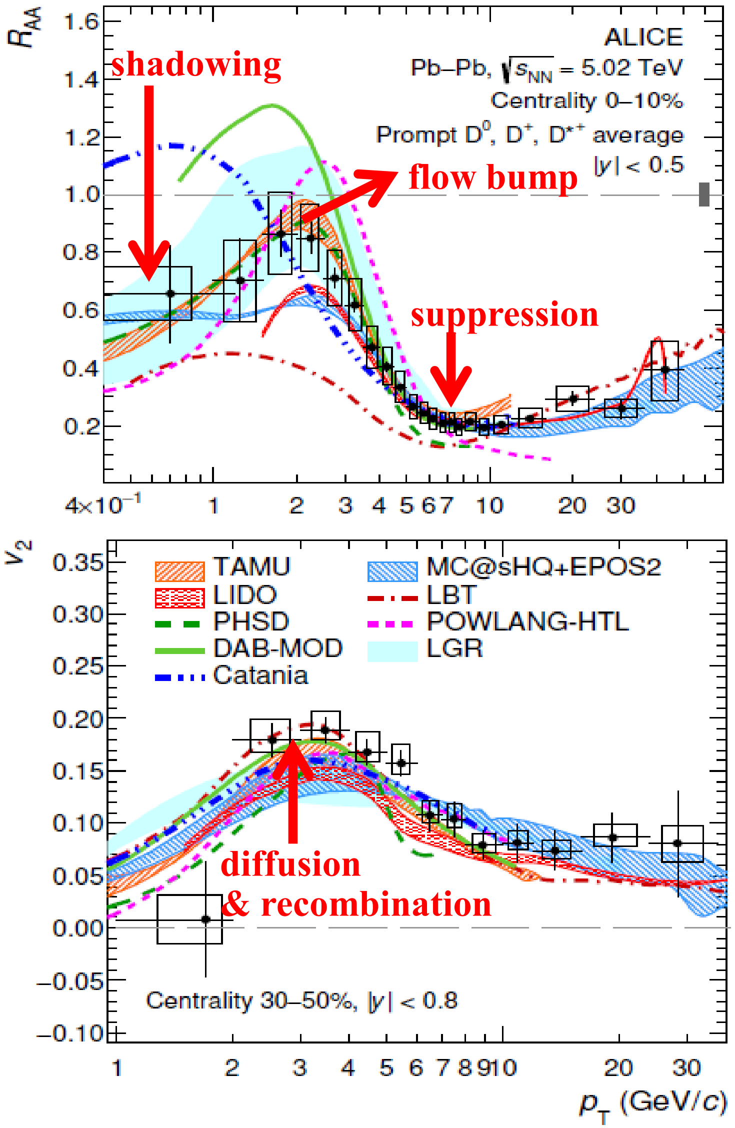



$$H_{AA}(p_T, p_t = p_T) = \frac{dN_D/dp_T}{dN_c/dp_t} = \frac{dN_D^{coal}/dp_T + dN_D^{frag}/dp_T}{dN_c/dp_t}$$

- Recombination dominant at low $p_T$: adding flow from light quark to charm quark ➔ flow bump in the $H_{AA}$ and also in $R_{AA}$ & $v_2$ increased from c to D
- Fragmentation dominant at high $p_T$: $p_T$ shift from c to D

# D-meson $R_{AA}$ & $v_2$: extracting $\mathcal{D}_s(2\pi T)$

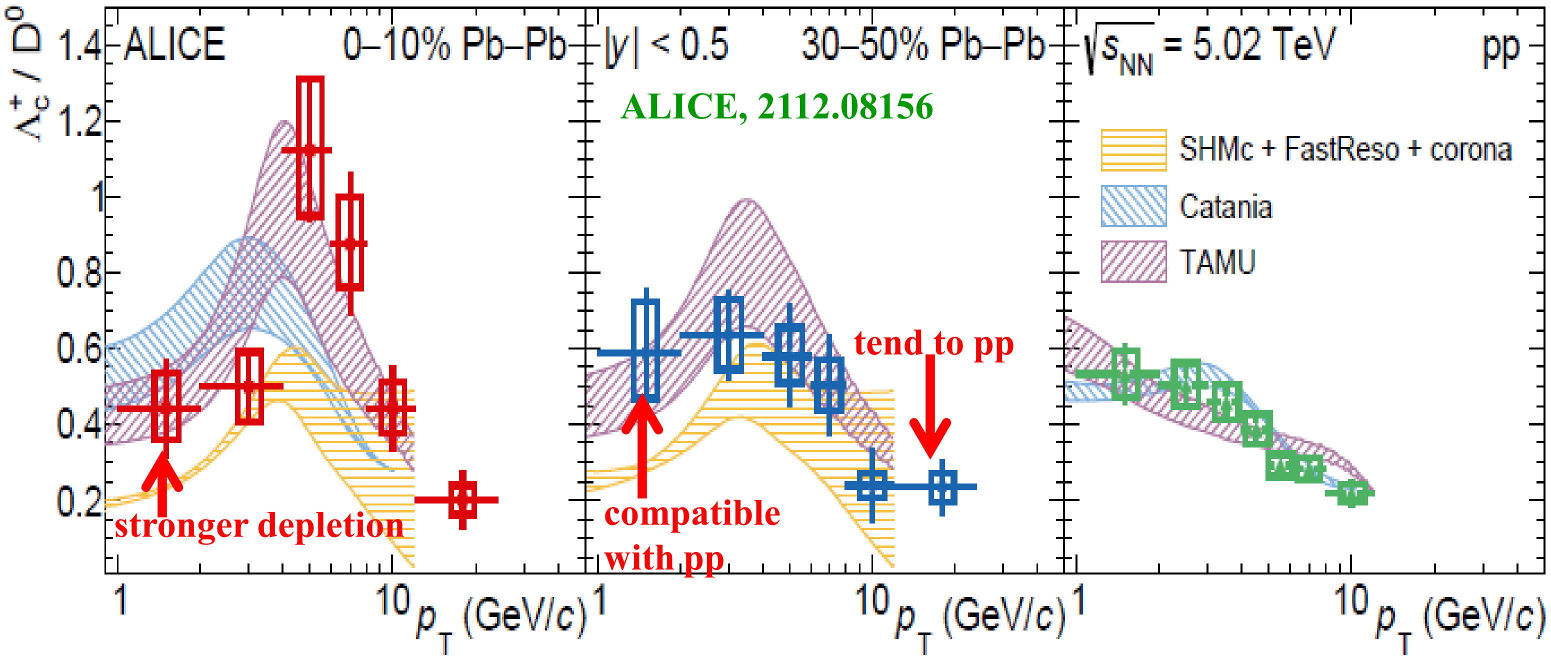



ALICE, JHEP01(2022)174; PLB 813(2021)136054

| *Model* | $\chi^2$/ndf | |
|---|---|---|
| | $R_{AA}$ | $v_2$ |
| Catania [6, 7] | 143.8/30 | 14.0/8 |
| DAB-MOD [9] | 234.1/30 | 9.8/6 |
| LBT [10, 11] | 411.8/30 | 15.8/12 |
| LIDO [13] | 46.4/26 | 62.0/11 |
| LGR [12] | 9.2/30 | 15.5/11 |
| MC@sHQ+EPOS2 [8] | 56.6/30 | 5.7/12 |
| PHSD [5] | 294.7/30 | 19.6/11 |
| POWLANG-HTL [3, 4] | 468.6/30 | 13.5/8 |
| TAMU [2] | 30.2/30 | 8.15/9 |

- models with $\chi^2$/ndf < 5 (2) for $R_{AA}$ ($v_2$)
  ➔ $\mathcal{D}_s(2\pi T)$=1.5-4.5 near $T_c$
- caveat: also affected by hadronization, hydro, hadronic phase

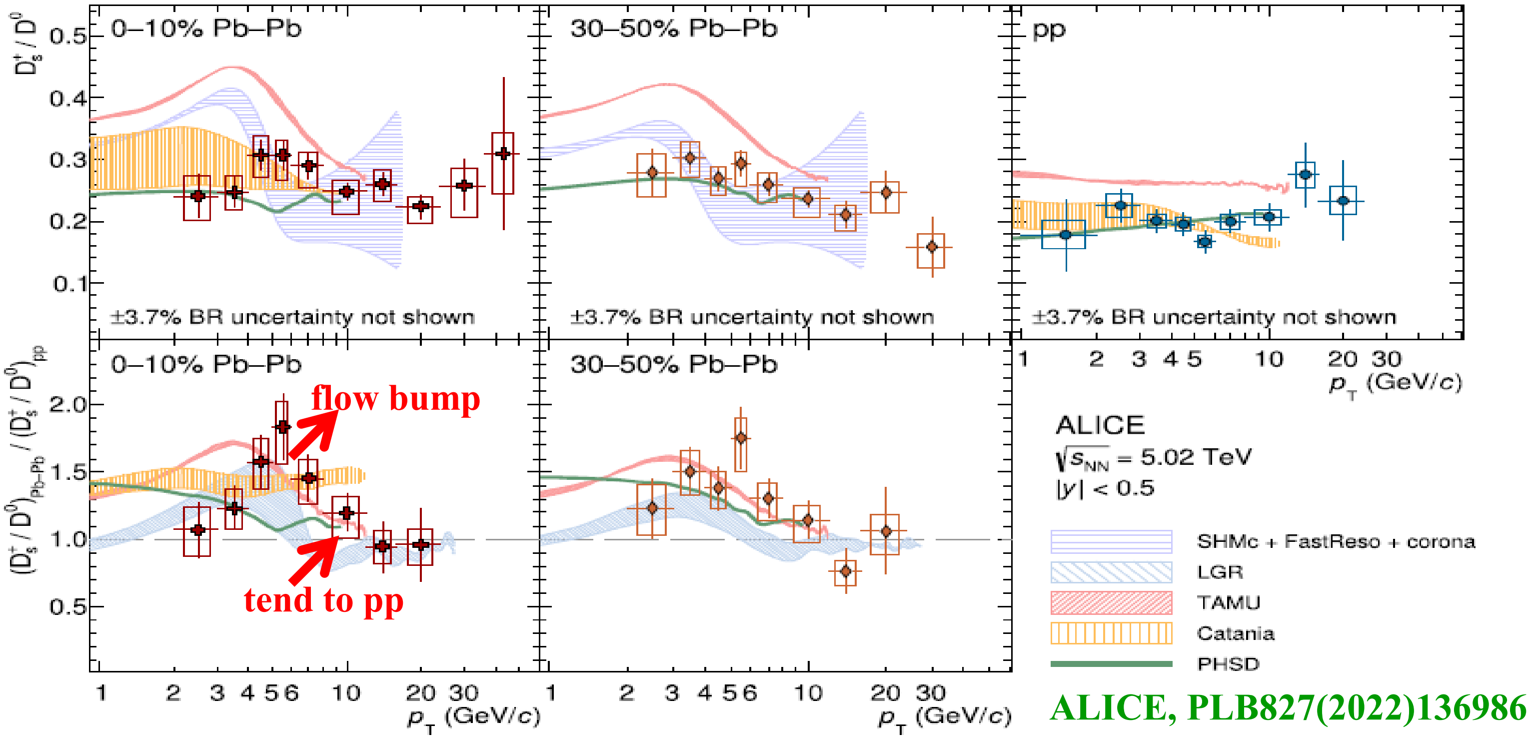
# Charm hadro-chemistry: $\Lambda_c/D^0$




- Catania: instantaneous coalescence + fragmentation **Plumari et al. '18**
- SHMc: hydrodynamic blast wave spectrum on PDG-only baryons + corona pp **Andronic et al. '21**
- TAMU: RQM charm-baryons + RRM w/ SMCs
  integrated ratio compatible with pp **MH & Rapp '20**

$p_T$-integrated $\Lambda_c/D^0$ compatible with pp → kinematic redistribution in $p_T$ in AA

# Charm hadro-chemistry: $D_s/D^0$

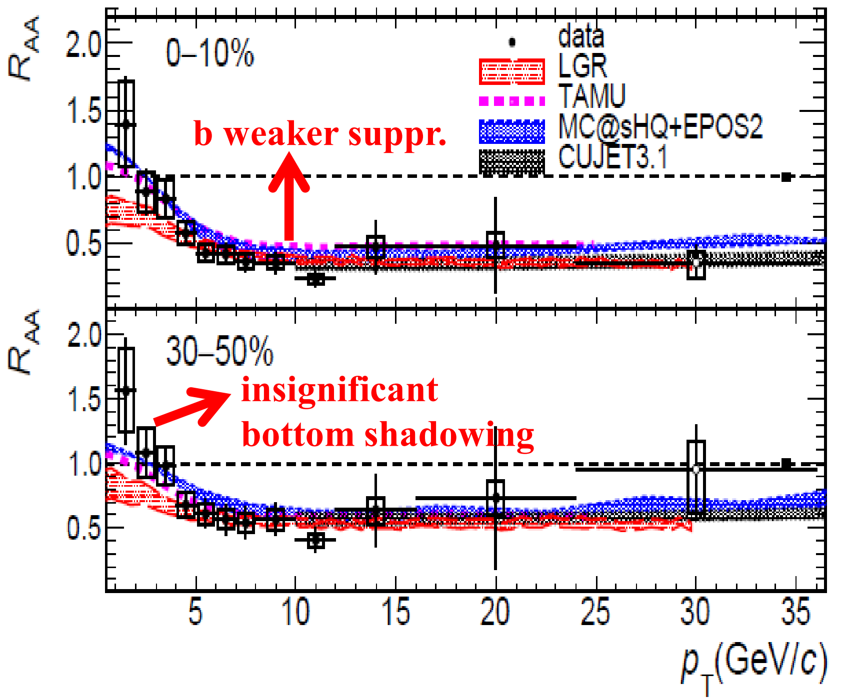



ALICE, PLB827(2022)136986

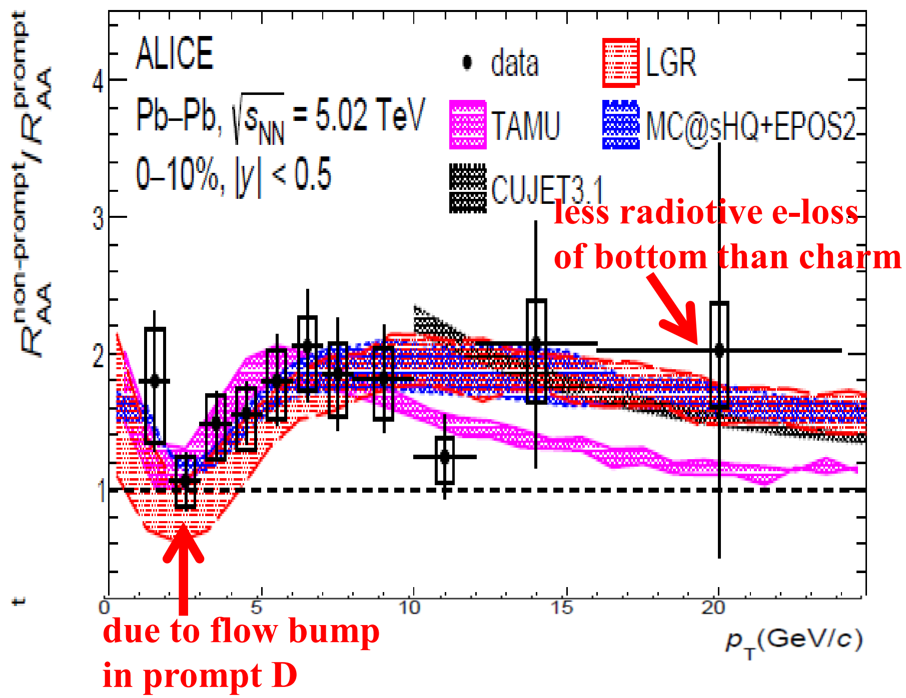
- low $p_T$: enhancement due to charm recombination in a strangeness-equilibrated QGP reproduced by Cantania & PHSD; overestimated by TAMU in both pp and PbPb
- high $p_T$: tending to pp value as fragmentation takes over
- flow bump due to recombination with flowing s-quark heavier than u/d, predicted by TAMU (RRM w/ SMCs) & SHMc (hydro blastwave spectrum)

# Flavor dependence: charm vs bottom

nonprompt D (← B) $R_{AA}$ , ALICE:2202.00815




nonprompt-to-prompt D-meson $R_{AA}$ ratio




❖ x3 mass: b-quark longer thermalization time at low $p_T$ than charm
less flow added to b from recombination with u/d/s

❖ high $p_T$>15 GeV: b-quark less radiative e-loss← stronger “dead cone”